

## DOPED PARTON DISTRIBUTIONS

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Calculations of high-energy processes involving the production of  $b$  quarks are typically performed in two different ways, the massive four-flavour scheme (4FS) and the massless five-flavour scheme (5FS). For processes where the combination of the 4FS and 5FS results into a matched calculation is technically difficult, it is possible to define a hybrid scheme known as the *doped* scheme, where above the  $b$ -quark threshold the strong coupling runs with  $n_f = 5$ , as in the massless calculation, while the DGLAP splitting functions are those of the  $n_f = 4$  scheme. In this contribution we present NNPDF3.0 PDF sets in this *doped* scheme, compare them with the corresponding 4FS and 5FS sets, and discuss their relevance for LHC phenomenology.

**Motivation** The accurate modeling of heavy quark mass effects is a crucial ingredient of LHC phenomenology. In processes involving  $b$ -quarks, two calculational schemes are usually employed: the massless  $n_f = 5$  scheme (5FS), where the  $b$ -quark mass is neglected and potentially large logarithms of the type  $\ln Q^2/m_b^2$  are resummed into a  $b$ -quark PDF  $b(x, Q^2)$ , and the massive  $n_f = 4$  scheme (4FS), where the non-zero value of  $m_b$  is kept in the calculation. Each scheme has its own advantages and weakness.

A general framework for the combination of 4FS and 5FS results into a single matched calculation, FONLL, is available, and has been applied to several processes from heavy quark hadroproduction<sup>1</sup> to DIS structure functions<sup>2</sup> and Higgs production via bottom quark fusion<sup>3</sup>. The implementation of FONLL is however technically challenging for processes for which analytic expressions are not available (such as in semi-automated NLO codes) and therefore a hybrid scheme that captures the essence of this matching may be useful, even though it will be necessarily less accurate than the full matched calculation.

As discussed in Ref.<sup>4</sup>, for many LHC processes the 4FS is perfectly adequate, since potentially large collinear logarithms in  $m_b$  are suppressed both by the smallness of the  $b$ -PDF at large- $x$  and by universal phase-space factors. However, a known drawback of this scheme is that with the 4FS evolution of the  $\beta$  function for the running of the strong coupling, above the  $b$ -quark threshold  $Q = m_b$  the value of  $\alpha_s(Q)$  is not accurately reproduced, and therefore the strong coupling running cannot be simultaneously correct at relatively low scales, relevant for many of the datasets that enter global PDF fits, and at the higher scales relevant for the majority of LHC processes. This difference induces a suppression in the predicted cross-sections, which can be especially important for multi-leg processes at the Born level where the calculation

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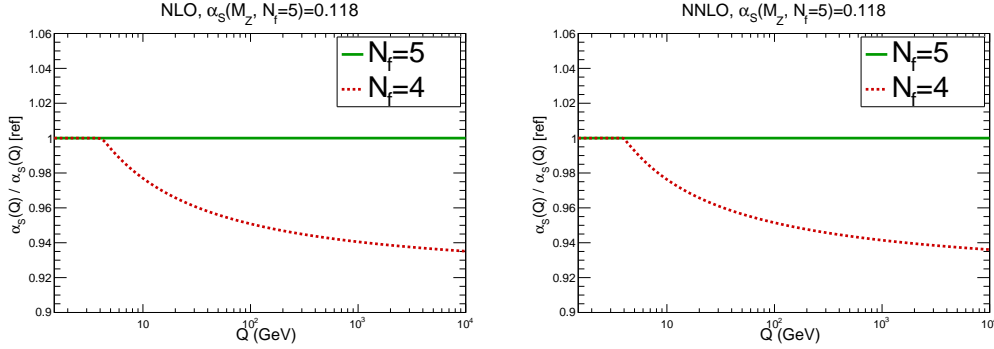


Figure 1 – Running of the strong coupling  $\alpha_s(Q)$  in the 5FS and 4FS, normalized to the 5FS result.

starts with a high power of  $\alpha_s$ .

To overcome these difficulties, a hybrid scheme for the treatment of heavy quark mass effects has been proposed<sup>5</sup>, named the *doped* scheme, where  $\alpha_s(Q)$  runs with  $n_f = 5$  flavors above  $m_b$ , thus having the standard value at  $M_Z$  (consistent with the global PDG average), while DGLAP evolution is still performed in the  $n_f = 4$  scheme, and in particular no  $b$ -quark PDF is introduced, as in the massive scheme. This scheme allows the combination of the useful properties of the 4FS, in particular accounting for bottom quark mass effects, while also using a value of  $\alpha_s(Q)$  consistent with the global average for all values of  $Q$ .

Therefore, the basic idea of the doped scheme is to use, above the  $b$ -quark threshold, a four-flavor factorization scheme (therefore a 4FS for the DGLAP evolution) with a five-flavor renormalization scheme (thus, a 5FS for the running of  $\alpha_s(Q)$ ). However, as explained in Ref.<sup>6</sup>, this choice is inconsistent, because then there would be a mismatch between terms in the splitting functions (specifically  $P_{gg}$ ) which depend on the beta function, and those governing the evolution of the strong coupling. Recently, it was suggested in Refs.<sup>5,7</sup> that this inconsistency can be cured by subtracting order by order from the partonic matrix elements the difference between the 4FS and 5FS running of  $\alpha_s(Q)$ , up to the order at which the hard cross-section is computed. Once this subtraction is implemented, one consistently restores the 4FS up to the finite order at which the matrix element has been computed.

**NNPDF3.0 sets in the doped scheme** We now present results for a version of the NNPDF3.0 NLO and NNLO global PDF sets<sup>8</sup> that are suitable to be used the doped scheme calculations, and compare them with their 4FS and 5FS counterparts. Following the standard method adopted in the NNPDF framework<sup>9</sup>, the doped sets are constructed starting from the NNPDF3.0  $n_f = 5$  global fit at a scale equal to the  $b$ -quark mass,  $Q^2 = m_b^2$ , and then evolving upwards in  $Q^2$  using the DGLAP equations but now with  $n_f = 5$  in the  $\beta$ -function for the running of  $\alpha_s(Q^2)$  and with  $n_f = 4$  in the splitting functions. The DGLAP upwards evolution is performed with the APFEL program<sup>10</sup>, using the same truncated solution of the evolution equations as that used in the NNPDF3.0 fits.

First of all, we show in Fig. 1 a comparison of the running of the strong coupling  $\alpha_s(Q)$  in the 5F and 4F schemes. The different running in the 4F as compared to the 5F scheme induces variations on the value of  $\alpha_s(Q)$  between 3% at  $Q \sim 10$  GeV, and 6% at  $Q \sim 10$  TeV, which in turns would lead to a shift  $\delta\sigma/\sigma \sim n\delta\alpha_s/\alpha_s$  for processes whose 4FS calculation starts as  $\sigma \sim \alpha_s^n$  at the Born level.

In Fig. 2 we compare the gluon PDF, as function of  $Q$ , for two values of  $x$ ,  $x = 10^{-4}$  and  $x = 0.1$ , at NLO and at NNLO, between the 5F (reference), 4F and doped schemes. As can be seen from the comparison, at large- $x$  the doped scheme leads to intermediate results between the 4F and 5F schemes. On the other hand, at small- $x$ , the doped scheme leads to a stronger rise of the gluon at small- $x$  as compared to the 4F calculation, due to the larger value of  $\alpha_s(Q)$

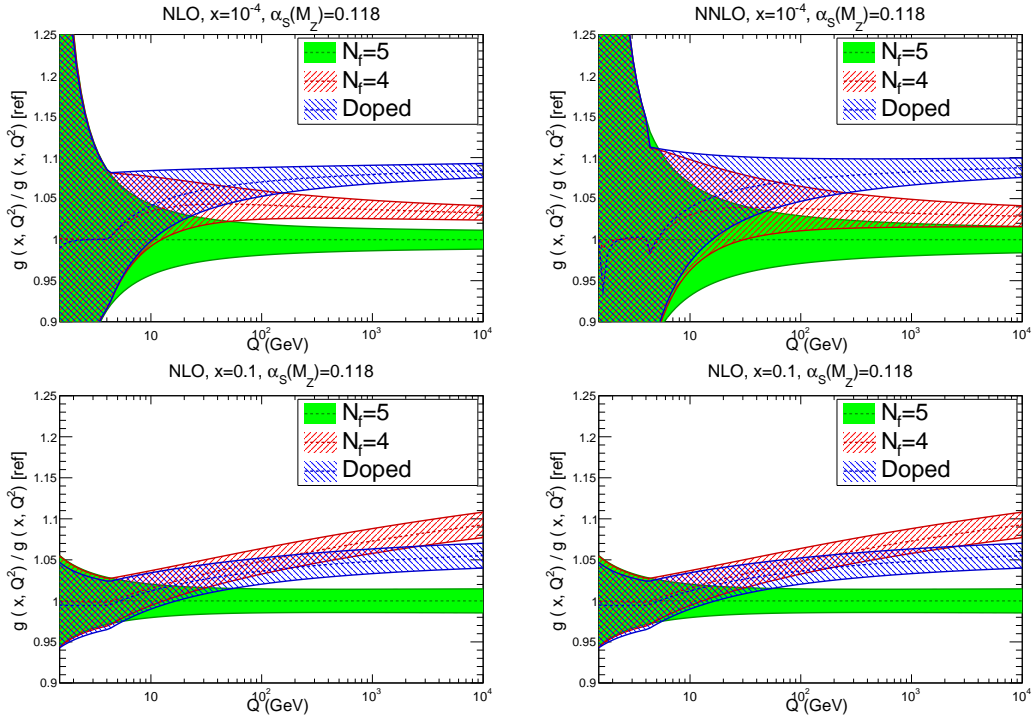


Figure 2 – Comparison of the NNPDF3.0 global fits in the 5F, 4F and doped heavy quark schemes, as a function of the factorization scale  $Q$  for two different values of  $x$ ,  $x = 10^{-4}$  (upper plots) and  $x = 0.1$  (lower plots). We show results normalized to the central value of the 5FS PDF set, for both NLO (left plots) and NNLO (right plots).

at high scales, which drives the small- $x$  DGLAP evolution. The results are qualitatively the same at both perturbative orders, NLO and NNLO. The same comparison, now for the down quark PDF  $d(x, Q^2)$  is shown in Fig. 3. At large values of  $x$ , where effects of DGLAP evolution are moderate, the doped scheme coincides essentially with the 5F scheme, and the differences with the 4FS are a few percent at most. At small- $x$ , the doped scheme also leads to a steeper rise of the light quark sea.

**Implications for LHC phenomenology** A first study of the use of the doped scheme at the LHC for the production of vector bosons associated with a  $b\bar{b}$  pair was presented in Ref. <sup>5</sup>. As an illustration of these results, in Fig. 4 we show differential distributions for  $Vb\bar{b}$  production at the LHC for  $\sqrt{s} = 7$  TeV, in particular the  $p_T^W$  of the  $W$  boson in  $Wb\bar{b}$  production and the invariant mass of the  $b\bar{b}$  pair,  $m_{b\bar{b}}$  in  $Zb\bar{b}$  production.

These results were obtained in the framework of the **Sherpa+OpenLoops** <sup>11,12</sup> Monte Carlo generator, using the 5FS, 4FS and doped versions of NNPDF2.3 NLO <sup>13</sup>. In each case, a consistent treatment of the partonic matrix elements was adopted. From Fig. 4 we see that the differential distributions in the doped scheme are typically intermediate between the results from the 4F and 5F schemes, as one could expect from the comparison at the level of PDFs shown in Figs. 2 and 3. These comparisons imply that the (resummed) change in  $\alpha_s(Q)$  and in the PDFs is more significant than the finite-order correction in the matrix element implemented in the doped scheme. The results of the calculations in the three schemes are consistent with the scale uncertainties of the 5FS calculation.

Doped PDFs were also used in the  $t\bar{t}b\bar{b}$  calculation of Ref. <sup>7</sup>, where it was found that a resummation of the logarithms of  $\mu_R/m_b$  arising from the running of  $\alpha_s$  in the doped scheme increased to total NLO cross-section by almost 10% as compared to the 4FS calculation.

The NNPDF3.0 doped PDFs presented in this contribution have been recently used <sup>14</sup> in the **Sherpa** framework to compare the predictions of the 4F, 5F and doped schemes with recent

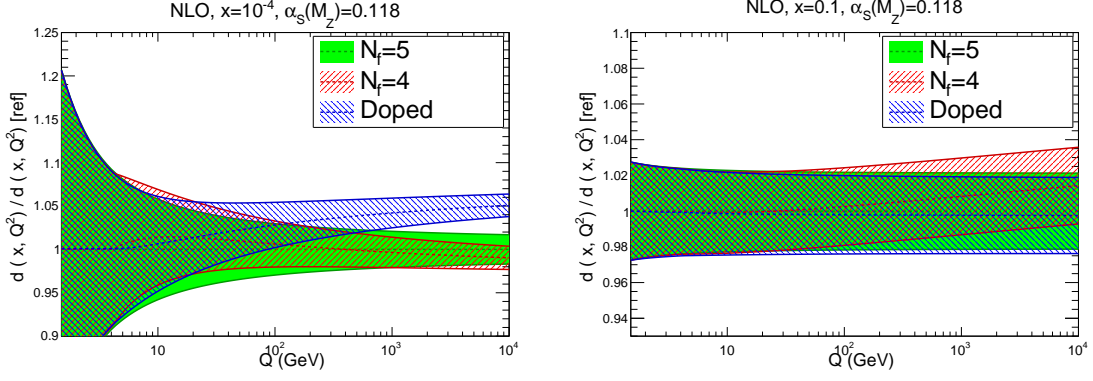


Figure 3 – Same as Fig. 2 for the NLO down quark PDF.

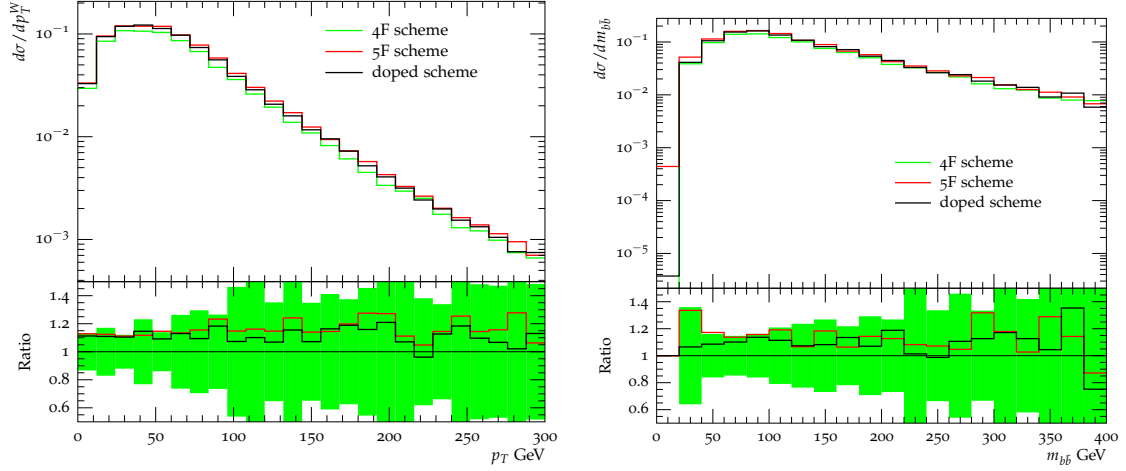


Figure 4 – Differential distributions for vector boson production with a pair of  $b$ -quarks at the LHC 7 TeV: the  $p_T^W$  of the  $W$  boson in  $Wb\bar{b}$  production (left) and the invariant mass of the  $b\bar{b}$  pair in  $Zb\bar{b}$  production (right). The results of the 4F and 5F schemes are compared with the doped scheme.

ATLAS<sup>15</sup> and CMS<sup>16</sup> measurements at 7 TeV for the differential distributions of  $Z$  bosons produced in association with  $b$ -jets.<sup>b</sup> These comparisons are illustrated in Fig. 5, showing that in general the doped scheme leads to a better agreement with the LHC data as compared to the 4F and 5F schemes; for instance for the ATLAS  $y_{\text{boost}}(Z, b)$  distribution, the doped scheme leads to the correct normalization and shape while the 4FS reproduces the shape but undershoots the data.

Summarizing, the doped scheme is a hybrid scheme for the treatment of heavy quark masses in perturbative QCD calculations, which has the potential to improve some of the drawbacks of 4FS calculations, in particular for processes with many colored partons in the final state. The NLO and NNLO doped versions of NNPDF3.0 will be made available in LHAPDF6.

## References

1. M. Cacciari, M. Greco and P. Nason, JHEP **9805**, 007 (1998) [hep-ph/9803400].
2. S. Forte, E. Laenen, P. Nason and J. Rojo, Nucl. Phys. B **834**, 116 (2010) [arXiv:1001.2312 [hep-ph]].
3. S. Forte, D. Napoletano and M. Ubiali, arXiv:1508.01529 [hep-ph].
4. F. Maltoni, G. Ridolfi and M. Ubiali, JHEP **1207**, 022 (2012) [JHEP **1304**, 095 (2013)] [arXiv:1203.6393 [hep-ph]].

<sup>b</sup>We thank Davide Napoletano for providing these comparison plots.

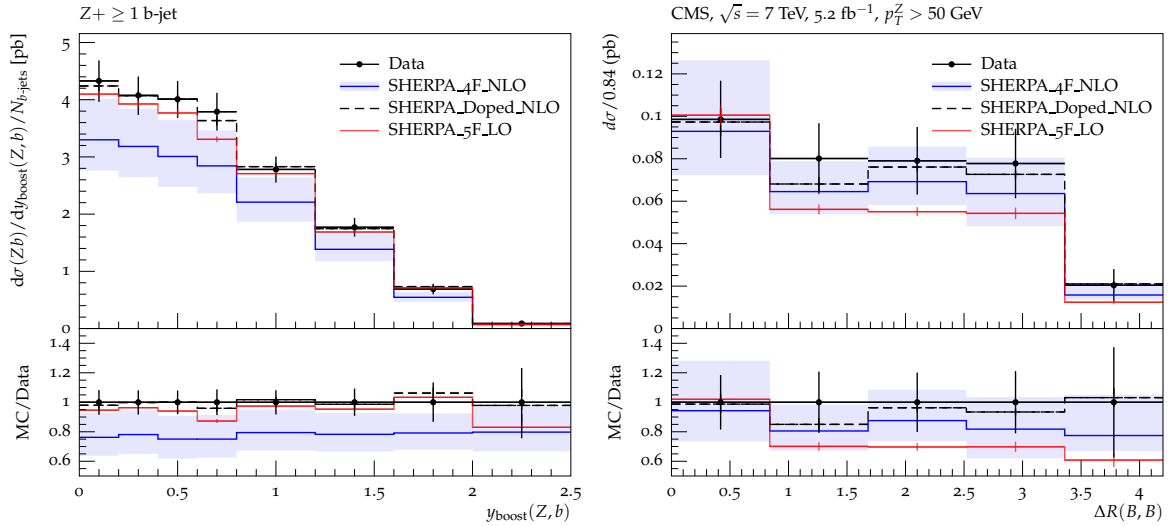


Figure 5 – Preliminary results for the comparison between the 4FS NLO, doped NLO and 5FS LO calculations with *Sherpa* using the corresponding NNPDF3.0 PDFs, and differential distributions from the recent ATLAS and CMS 7 TeV measurements of  $Z$  production in association with  $b$ -jets.

5. D. Napoletano, *A new hybrid scheme for the treatment of heavy quarks in perturbative QCD*, Master Thesis, Milano University, 2014, available online at <http://nnpdf.hepforge.org/docs/theses/NapoletanoTesi.pdf>.
6. A. D. Martin, W. J. Stirling and R. S. Thorne, *Phys. Lett. B* **636**, 259 (2006) [hep-ph/0603143].
7. F. Cascioli, P. Maierhöfer, N. Moretti, S. Pozzorini and F. Siegert, *Phys. Lett. B* **734**, 210 (2014) [arXiv:1309.5912 [hep-ph]].
8. R. D. Ball *et al.*, *JHEP* **1504**, 040 (2015) [arXiv:1410.8849 [hep-ph]].
9. R. D. Ball *et al.*, *Nucl. Phys. B* **849**, 296 (2011) [arXiv:1101.1300 [hep-ph]].
10. V. Bertone, S. Carrazza and J. Rojo, *Comput. Phys. Commun.* **185**, 1647 (2014) [arXiv:1310.1394 [hep-ph]].
11. T. Gleisberg, S. Hoeche, F. Krauss, M. Schonherr, S. Schumann, F. Siegert and J. Winter, *JHEP* **0902**, 007 (2009) [arXiv:0811.4622 [hep-ph]].
12. F. Cascioli, P. Maierhöfer and S. Pozzorini, *Phys. Rev. Lett.* **108**, 111601 (2012) [arXiv:1111.5206 [hep-ph]].
13. R. D. Ball *et al.*, *Nucl. Phys. B* **867**, 244 (2013) [arXiv:1207.1303 [hep-ph]].
14. F. Krauss, D. Napoletano and S. Schumann, in preparation.
15. G. Aad *et al.* [ATLAS Collaboration], *JHEP* **1410**, 141 (2014) [arXiv:1407.3643 [hep-ex]].
16. S. Chatrchyan *et al.* [CMS Collaboration], *JHEP* **1312**, 039 (2013) [arXiv:1310.1349 [hep-ex]].