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Using pixel intensity as a self-regulating threshold for deterministic image sampling in Milano Retinex: the T-Rex algorithm

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Abstract. Milano Retinexes are spatial color algorithms, part of the Retinex family, usually employed for image enhancement. They modify the color of each pixel taking into account the surrounding colors and their position, catching in this way the local spatial color distribution relevant to image enhancement. We present T-Rex (from the words threshold and Retinex), an implementation of Milano Retinex, whose main novelty is the use of the pixel intensity as a self-regulating threshold to deterministically sample local color information. The experiments, carried out on real-world pictures, show that T-Rex image enhancement performance are in line with those of the Milano Retinex family: T-Rex increases the brightness, the contrast, and the flatness of the channel distributions of the input image, making more intelligible the content of pictures acquired under difficult light conditions. [DOI: 10.1117/1.JEI.27.1.011005]

Keywords: Milano Retinex; spatial color sampling; image enhancement.

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1 Introduction

Milano Retinex is a family of spatial color algorithms called in this way since they implement the original Retinex computational model[1] in a slightly different way.[2]

Retinex principles derive from a series of experiments demonstrating that human color vision is a local spatial process.[3,4] In fact, the color sensation, as reported by humans, does not depend exclusively on the spectral properties of the observed point but also on those of the surrounding regions. Due to this spatial dependency of the color sensation, the human vision system is able to lower gradients and enhance edges while partially removing possible color dominants due to the ambient light.

The Retinex model estimates the color sensation from an RGB image by three steps: precalibration, computation of the channel color lightness, and postcalibration. The first and third steps are very important to model color sensation, because they map the calibrated RGB digital values to the human color appearance. The precalibration step transforms the digital values of the input image into the reflectance values effectively detected by the vision system. The second step independently processes the color channels of the precalibrated image and outputs an RGB image, termed color lightness. For each channel, the information about local spatial color distribution, relevant to form the color sensation, are fetched, exploring the image support, usually by random paths. The locality of the algorithm, i.e., the range of mutual interaction among the image colors, is determined by number, geometry, and length of the paths. For each path, the Retinex algorithm computes the ratios between the intensities of adjacent pixels. Then, the channel lightness is obtained as a mobile average over each path of the products of the path ratios, preventing of course division by zero. Ratios close to 1.0 and cumulative products exceeding 1.0 are cast to 1.0. Finally, the postcalibration step maps the RGB values of the color lightness into a scale of color appearance.

When pre- and postcalibrations are skipped, the Retinex algorithm works as image enhancer: the filtered image is an enhanced version of the input one, with adjusted brightness, contrast and channel distributions, and possible chromatic dominants attenuated. The algorithm locality strongly influences these enhancement effects.

The algorithms of the Milano Retinex family propose alternative computational models for the lightness. Milano Retinexes can, as well, implement pre- and postcalibration, but since here we want to focus on the image enhancement characteristics of Milano Retinex the output image will be termed color lightness and will refer to the application of Milano Retinex, skipping pre- and postcalibration steps.

Like all Retinex algorithms, Milano Retinexes process the color channels independently. For each channel, Milano Retinexes rescale the intensity of each pixel (treated as a target) by an intensity level, termed local reference white, and computed by reworking the intensities of a set of pixels close to the target. These neighboring pixels form the so-called figure sampling, and they define the locality of Milano Retinexes. Several sampling figures are proposed: for instance, random paths,[5,6] constrained random paths,[7–9] random sprays,[10–12] and edges picked up from random squares around the target.[13] In these cases, the pixelwise random sampling is repeated many times, and the lightness is computed by averaging the results obtained over each sampled set. These operations reduce the chromatic noise possibly affecting the output image due to the random

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sampling. This undesired noise is completely avoided by the Milano Retinex approach GREAT,[14] which computes the spatial and color features relevant to the lightness from a set of strong, deterministically defined edges. With the exception of the methods based on random paths[5,6] that use a computational scheme similar to that of the original Retinex algorithm, the other methods mentioned above compute the local reference white by averaging the maxima of the intensity[7–12] or the maximals of the target intensity[13,14] over the sampled sets in this framework, the maximals of a real-value v over a set $S \subseteq \mathbf{R}$ are the elements of S strictly greater than v. If no maxima or maximals exist, the local reference white equals the target intensity. In some implementations, the contributions of the maxima or of the maximals are weighted by function of the spatial distance between the target and the pixels realizing the maxima or the maximals. Probablistic formulations[15–17] inspired by previous Milano Retinexes[10,18] have been recently proposed. As GREAT,[14] these methods avoid the random sampling while they compute the local reference white at each pixel based on statistical features extracted from the intensity probability density functions weighted by spatial information.

This paper contributes to the state-of-the-art on image enhancement by proposing an innovative sampling-based Milano Retinex algorithm, named T-Rex. A preliminary version of T-Rex has been recently presented at a conference.[19] This work provides an extended description of the algorithm with additional experiments.

T-Rex scans the image region around the target in search of pixels whose intensity value, weighted by a Gaussian function of the distance from the target, exceeds the target intensity. In this framework, the target intensity acts as a self-regulating threshold for the selection of the spatial and color features relevant to image enhancement. The name T-Rex comes from the words "threshold and Retinex," which just refer to the intensity thresholding strategy for image sampling. The locality of T-Rex is determined by the Gaussian functions, whose width is an input parameter of T-Rex: this means that the algorithm locality can be tuned by the user. The local reference white is computed as the average of the sampled intensities, weighted by the Gaussian functions.

The main novelty of T-Rex is the pixelwise mechanism of self-regulating intensity thresholding. This mechanism makes T-Rex very different from Milano Retinexes based on random image exploration:[5–13] in fact, T-Rex implements a determistic image sampling, thus it avoids the generation of chromatic noise and allows the exact reproducibility of the results. T-Rex shares this characteristic with GREAT.[14] In addition, differently from the random paths, whose pixels are constrained to be connected, and from the random sprays, whose elements are radially distributed around the target, the sampling figures of T-Rex and GREAT do not exhibit any specific geometric structure. Nevertheless, T-Rex and GREAT strongly differ to each other. In fact, the sampling figure of GREAT is defined by image gradient, and it is independent of the target, whereas that of T-Rex is defined by spatial and intensity features and it is specific for each pixel. Moreover, T-Rex and GREAT implement different equations of the local reference white.

The image enhancement performance of T-Rex has been numerically measured and compared with other deterministic Milano Retinex algorithm on real-world images, captured by commercial cameras under different light conditions. This evaluation is based on a set of features (i.e., image brightness, contrast, and histogram flatness) related to the image content understanding. The experiments show that T-Rex actually works as an image enhancer and its performance is in line with those of other Milano Retinex approaches.

The paper is organized as follows: Sec. 2 reports on related works; Sec. 3 describes T-Rex; Sec. 4 presents the experiments measuring the image enhancement performance of T-Rex; finally, Sec. 5 outlines our conclusions and future work.

2 Related Work

This section reviews three Milano Retinex algorithms that share with T-Rex the implementation of a deterministic approach for the computation of the color lightness. These algorithms are GREAT[14] (Sec. 2.1), RSR-P[16] (Sec. 2.2), and QBRIX[17] (Sec. 2.3). As already pointed out in Sec. 1, GREAT implements a point-based image sampling approach exploiting image edges, whereas RSR-P and QBRIX are probabilistic Milano Retinexes, both derived from the point-based sampling algorithm random spray Retinex.[10]

The following notation will be used throughout this paper. Any RGB image will be denoted by $\overline{I}$ and any chromatic channel of $\overline{I}$ by I. The channel I is represented as a function from S to (0,1], where S indicates the image support, i.e., the set of the spatial coordinates of the pixels composing the image. This representation is obtained by rescaling the intensity values of I, usually ranging over $\{0, \dots, 255\}$, on the unit interval (0, 1], where zero has been excluded to prevent division by zero in the subsequent processing. In particular, null intensity values are mapped on to 10^{-6}. The cardinality of the image support will be indicated by $|S|$. The color lightness of $\overline{I}$ will be denoted by $\overline{L}$ and any chromatic channel of $\overline{L}$ by L. The Milano Retinex channel lightness L at x is given by

$$L(x) = \frac{I(x)}{w(x)}, \tag{1}$$

where $w(x)$ is the local reference white at x.

2.1 *GREAT*

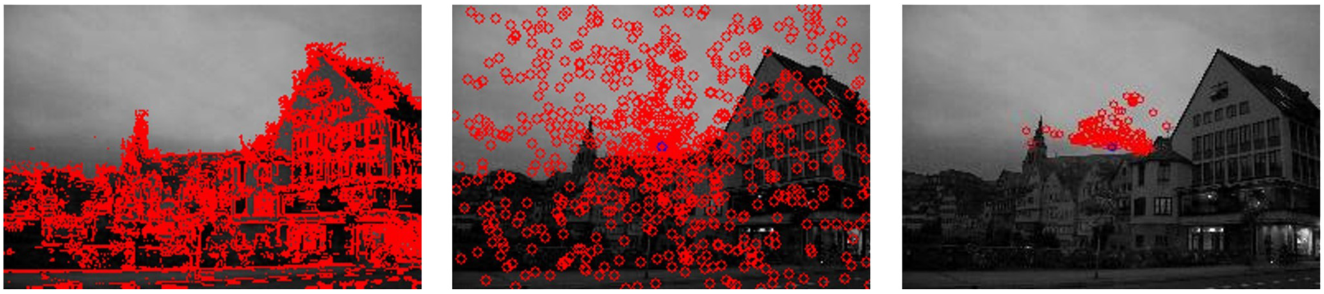
The Milano Retinex algorithm GREAT implements a deterministic spatial color sampling based on gradient information. The name GREAT just derives from the words "gradient relevance in Retinex," that characterizes the method.

For each channel, GREAT computes a sampling figure Σ independent of the target. Σ is composed by image edges whose high gradient magnitude exceeds a predefined threshold $\tau \leq 1.0$, named gradient relevance parameters. Precisely

$$\Sigma = \{y \in S : \|\nabla I(y)\| \geq \tau\},$$

where ∇I indicates the image gradient.

The use of edges as key-points for lightness computation is justified by two main reasons. First, edges play a very important role in the human color sensation.[3] Second, the structural properties of a scene depicted in a digital image



(a) (b) (c)

Fig. 1 Figure sampling (red points on the first picture and red circles on the other pictures) obtained by (a) GREAT,[14] (b) RSR,[10] and (c) T-Rex ($\lambda = 1.0$) on the red channel of an image from TEST35COLOR. While the figure sampling of GREAT is independent of the target, i.e., it is the same for each pixel, those of RSR and T-Rex vary from pixel to pixel: in this example, the figure samplings of RSR and T-Rex are computed with respect to the barycenter of the image support, highlighted by a blue circle.

are typically encoded in the pixels around the edges, whereas the information in the rest of the picture is less relevant. Therefore, focusing the attention on the edge zones allows collection of the most relevant information.

Based on these observations, GREAT computes the maximum intensity over the dilation of Σ by one pixel and stores this information as an image $M: S \to (0,1]$ defined as follows:

$$M(y) = \begin{cases} \max_{\{u \in \mathcal{N}(y)\}} I(u) & \text{if } y \in \Sigma \\ 0 & \text{otherwise} \end{cases},$$

where $\mathcal{N}(y)$ is the 3×3 window centered at y and acting as structuring element.

The color lightness is computed by processing Σ and M as follows. For each $x \in S$, GREAT computes the set $P(x) = \{u \in S: M(u) > I(x)\}$. The intensity values of the pixels in $P(x)$ are maximals of the target intensity over the dilation of Σ. These maximals are used to compute the local reference white at x

$$w(x) = \begin{cases} \frac{\sum_{u \in P(x)} [1 - d(u,x)] \|\nabla I(u)\| I(u)}{\sum_{u \in P(x)} [1 - d(u,x)] \|\nabla I(u)\|} & \text{if } y \in \Sigma \\ I(x) & \text{otherwise} \end{cases}, \tag{2}$$

where $d(u, x)$ indicates the Euclidean distance between u and x, normalized by the length D of the diagonal of S, in order to range over [0, 1], i.e.,

$$d(x, y) = \frac{\|x - y\|}{D}.$$

The value of τ determines the size of Σ. When $\tau = 0$, the set Σ is empty, and in this case $w(x)$ is set up to $I(x)$ for any $x \in S$, whereas when $\tau = 1.0$, Σ is composed by the pixels with the maximum gradient magnitude. When $\tau \in (0,1]$ and the input image is nonuniform, Σ is a proper subset of S. The value of τ recommended by the authors of GREAT[14] is image-dependent and it is given as

$$\tau(I) = \frac{1}{|I|} \sum_{u \in S} \|\nabla I(u)\|.$$

An example of GREAT figure sampling is provided in Fig. 1.

2.2 *RSR-P*

The Milano Retinex approach RSR-P[16] is a probabilistic formalization of the point-based sampling algorithm random spray Retinex[10] (RSR for short).

For each channel I of any color image, RSR samples the neighborhood of each target x by N random sprays $S_1, \ldots, S_N$. A random spray is a set of m pixels randomly sampled with radial density from a circular region centered at x and having radius $R > 0$. In the most applications, R is set up as the length of the diagonal of the image support.[10] Many random sprays are generated in order to reduce as much as possible the chromatic noise due to the random sampling. The constraint about the radial distribution of the pixels around x is introduced to model the empirical evidence that the colors of pixels close to the target influence are more relevant to those of pixels located further.[3,20] An example of figure sampling provided by RSR is shown in Fig. 1.

The lightness $L(x)$ is computed as the average of the ratios between $I(x)$ and the maximum intensity over each spray, extended so that to include x

$$L(x) = \frac{1}{N} \sum_{i=1}^{N} \frac{I(x)}{\max\{I(y): y \in S_i \cup \{x\}\}}. \tag{3}$$

RSR-P is the exact mapping of the RSR approach in to a population-based model. The letter "P" in the algorithm name just stands for "population." RSR-P completely avoids the random sampling, thus it provides a noise-free color lightness.

RSR-P observes that the factor $\frac{1}{w(x)} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\max\{I(y): y \in S_i \cup \{x\}\}}$ rescaling $I(x)$ in Eq. (3) can be estimated deterministically from the probability to sample m intensity values $I_1, \ldots, I_m$ such that, for any $j = 1, \ldots, m$, (i) $I_j \geq I(x)$, and (ii) I_j is the maximum intensity over a spray, i.e., over a set of pixels with radial density around x.

This probability is computed as follows. For each channel, and for each x, RSR-P models locality by weighting the intensity $I(y)$ of each $y \in S$ by a function inversely proportional to the squared Euclidean distance of the target from y. This information is compactly encoded by the target-dependent histogram f_x given as

$$f_x(i) = \frac{1}{W} \sum_{y \in \{S \setminus \{x\}, I(y)=i\}} d(x,y)^{-2}, \tag{4}$$

where i is an intensity level, and W is a normalization factor so that f_x ranges over [0,1].

Let F_x be the probability distribution function of f_x. Therefore

$$\frac{1}{w(x)} = \frac{\{F_x[I(x)]\}^m}{I(x)} + \sum_{i>I(x)} \frac{1}{i} b_x(i), \tag{5}$$

where

$$b_x(i_j) = [F_x(i_j)]^m - [F_x(i_{j-1})]^m.$$

Here, i_{j-1}, i_j indicate two subsequent, discrete values of the 256 levels of the channel intensity discretized over (0, 1], as explained at the beginning of Sec. 2. The first and the second terms in Eq. (5) measure the probability to sample from f_x pixels with intensity values equal to $I(x)$ and pixels with intensity values strictly greater than $I(x)$, respectively.

RSR-P requires to set up the parameter m, corresponding to the number of pixels per spray in RSR, while, ideally, the value of $L(x)$ output by RSR-P corresponds to the output by RSR when the number of sprays N tends to infinity.

2.3 *QBRIX*

As RSR-P, the quantile-based approach to Retinex (QBRIX for short)[17] is a deterministic Milano Retinex based on statistical features and born out of RSR.

QBRIX observes that to achieve an accurate computation of the color lightness, the following requirements have to be implemented: (a) an accurate image exploration takes into account a high number of image colors; (b) the colors of pixels close to the target influence are more relevant to those of pixels located further.[3,20] In RSR, the first issue is accomplished by considering many pixels per spray, whereas the second one is achieved by constraining the sampled pixels to be radially distributed around the target.

QBRIX proposes two different implementations based respectively on (a) and on (a) plus (b).

The first implementation does not consider any spatial information, thus it does not model locality. For this reason, this implementation is termed global-QBRIX.

Global-QBRIX computes the pdf f of the intensity of each channel I and estimates the local reference white at any target x as the maximum value between $I(x)$ and the intensity value q corresponding to a high quantile Q of the intensity pdf, i.e.,

$$\int_{i=0}^{q} f(i) = Q$$

and

$$w(x) = \max[I(x), q]. \tag{6}$$

The max operation is needed to set the value of $w(x)$ when q is smaller than $I(x)$.

The second implementation of QBRIX considers both the issues (a) and (b). Since it models spatial information, it is called local-QBRIX. For any channel I, global-QBRIX computes the spatially weighted pdf f_x defined in Eq. (4) and estimates the local reference white at any target x by Eq. (6), where q is computed from f_x instead of from f.

In both global- and local-QBRIX, the value of Q is a user input, but it is usually set up as a number greater than 0.90. The higher Q, the higher the number of the pixels considered in the lightness computation is.

3 T-Rex

T-Rex (from the words "threshold" and "Retinex") proposes an innovative, deterministic, point-based sampling scheme that selects the spatial and color information relevant to image enhancement by means of an intensity threshold strategy.

The color lightness is computed by three steps, performed for each channel I:

Modeling intensity and spatial information: T-Rex defines a target-dependent function that weights the intensity of each pixel by a function inversely proportional to the distance of the pixel from the target. This function takes into account both the intensity and spatial features around the target. Precisely, for every $x \in S$, T-Rex computes the target-dependent function $v_x : S \to \mathbf{R}$ such that

$$v_x(y) = I(y) \exp[-\lambda d(x,y)^2], \tag{7}$$

where, as in GREAT, $d(x,y)$ is the Euclidean distance between x and y, normalized by the length of the diagonal of S, while $\lambda \in [0, +\infty)$ is an input user parameter weighting the importance of the intensity versus the spatial information. When $\lambda = 0$, v_x coincides with the channel intensity. When λ tends to $+\infty$, v_x tends to the null function. When $\lambda \in (0, +\infty)$, v_x acts as a penalty term: the intensity of the pixels far from x are weighted less than that of the pixels close to x, according to the higher relevance that colors close to x have with respect to colors located further.[3,20]

Computing the sampling figure: The function v_x determines the T-Rex sampling figure $\Omega(x)$ at x. Precisely, $\Omega(x)$ is composed by the elements $y \in S$ satisfying the following conditions:

$$v_x(y) > I(x) \tag{8}$$

and

$$d(x,y) = \min\{d(u,x) : u \in S \text{ and } v_x(u) = v_x(y)\}. \tag{9}$$

The first condition implements the T-Rex thresholding strategy: the target intensity is used as threshold to detect intensity values relevant to the lightness. The pixels, selected by the inequality in Eq. (8), have the following characteristics: their intensity values, even penalized by the Gaussian exponential function in v_x [Eq. (7)], are maximals of the target intensity over S.

Equation (9) specifies that the pixels of $\Omega(x)$ are the closest to x among the pixels satisfying Eq. (8). Again, this choice is justified by the experiments on the relationship between colors and distance[3,20] mentioned above.

Computing the lightness: The local reference white $L(x)$ is computed as follows:

$$w(x) = \begin{cases} \frac{1}{\sum_{y\in\Omega(x)} \exp[-\lambda d(x,y)^2]} \sum_{y\in N(x)} v_x(y) & \text{if } \Omega(x) \neq \emptyset \\ I(x) & \text{otherwise} \end{cases}, \tag{10}$$

When $\lambda = 0$ and $\lambda \to +\infty$, T-Rex does not model locality, i.e., it has a global behavior: the computed lightness does not depend on the spatial color distribution around the target. In particular, when $\lambda = 0$, the lightness at x is the average of the maximals of the target intensity, regardless of their position, whereas for $\lambda \to +\infty$, $L(x)$ tends to one for every $x \in S$. In the first case, the image output by T-Rex is again an enhanced version of the input one, but the spatial constraints of the Retinex theory are not considered. In the second case, no enhancement is achieved, because the image details are completely removed. In order to perform a local spatial color processing, the parameter λ must be strictly greater than zero and $\lambda \ll +\infty$. Some examples of sampling figures obtained for different values of λ are shown in Fig. 2.

The computation of the T-Rex sampling figure requires scanning the image to search for the pixels satisfying the conditions Eqs. (8) and (9). In the worst case, this search may require consideration of all the image pixels, thus the computational complexity of T-Rex is $\mathcal{O}(|I|^2)$.

Table 1 summarizes the main characteristics of the deterministic Milano Retinex algorithms considered in this work, allowing a theoretical comparison among them. In particular,

- Approach: both T-Rex and GREAT implement an image sampling scheme to extract features relevant to lightness, whereas RSR-P and QBRIX replace the feature sampling process with a statistical analysis of the image visual cues;
- Relevant features: T-Rex, GREAT, RSR-P, and local-QBRIX compute the lightness based on intensity and spatial information. GREAT also uses edges, whereas global-QBRIX uses intensity only.
- Sampling figure: apart from some limit cases (e.g., in T-Rex, when $\lambda = 0$ or when $\Omega(x) = \emptyset$ for any x in S) the sampling figure of T-Rex depends on the target and it is characterized by the geometric constraint expressed by the Gaussian weights in Eq. (7). On

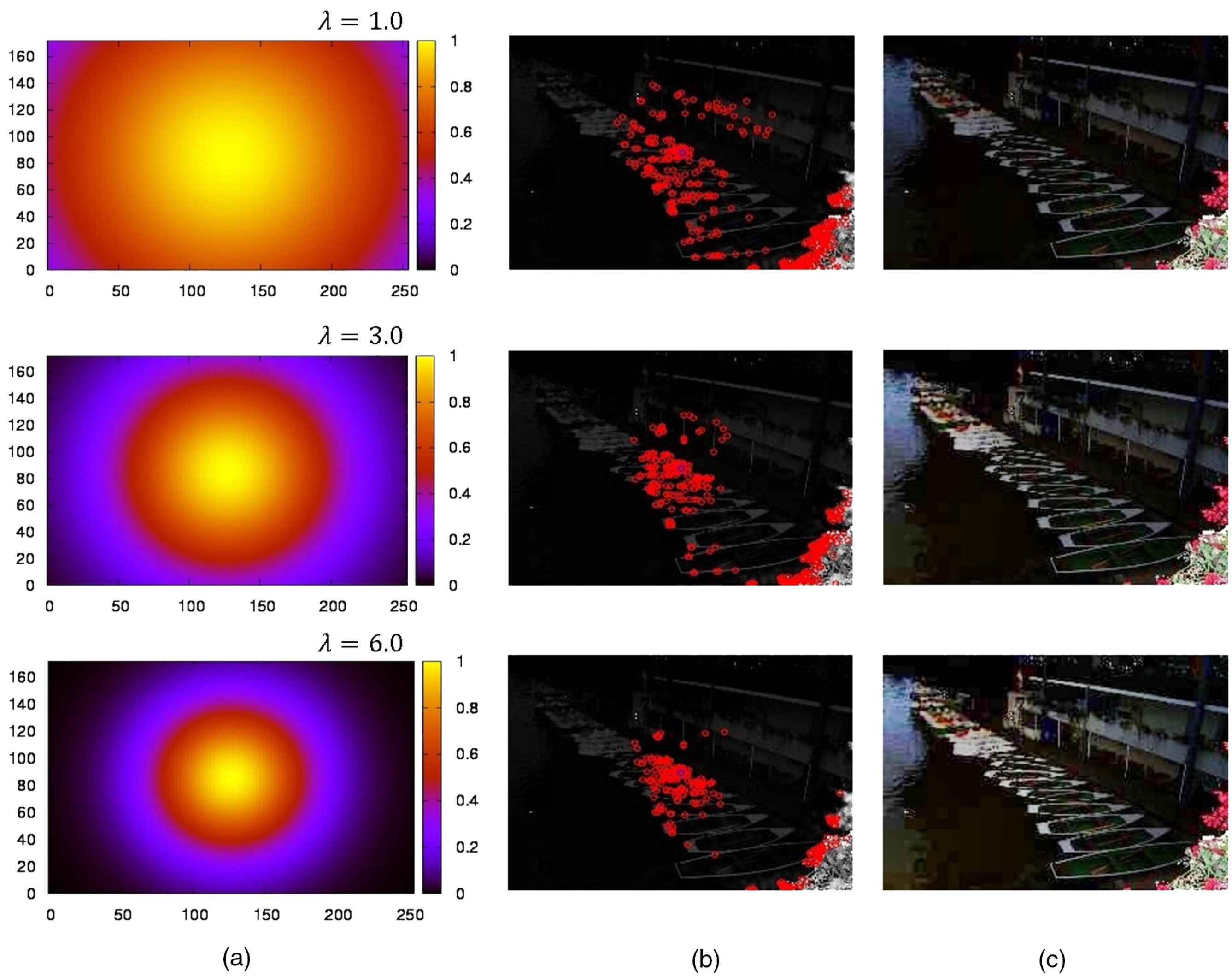


Fig. 2 (a) The exponential Gaussian weight $g(y) = \exp(-\lambda \frac{\|x-y\|^2}{D^2})$ evaluated at the barycenter x of the support of an image from TEST35COLOR for $\lambda = 1.0, 3.0, 6.0$; (b) the corresponding sampling figure of T-Rex at x (from top to bottom); and (c) the color lightnesses by T-Rex.

Table 1 Summary of the theoretical comparison between T-Rex and the deterministic Milano Retinexes GREAT, RSR-P, global-QBRIX, and local-QBRIX.

Algorithms characteristics	T-Rex	GREAT	RSR-P	Global-QBRIX	Local-QBRIX
Approach	Sampling-based	Sampling-based	Statistical	Statistical	Statistical
Relevant features	– Intensity	– Intensity	– Intensity	– Intensity	– Intensity
	– Distance	– Distance	– Distance		– Distance
		– Gradient			
Sampling figure	Target-dependent	Target-independent	Target-dependent	Target-independent	Target-dependent
Local reference white	Target-dependent	Target-dependent	Target-dependent	Target-independent	Target-dependent
Lightness equation	Maximals over sampling figure	Maximals over sampling figure	Maximum over f_x	Quantile over f	Quantile over f_x
Explicit model of locality in lightness equation	Yes	Yes	No	No	No

the contrary, the sampling figure of GREAT is independent of the target, and it is extracted from the image gradient without considering spatial information. The statistical approaches do not compute sampling figures.

- Local reference white: The local reference whites of T-Rex, GREAT, RSR-P, and local-QBRIX vary from pixel to pixel, whereas that of global-QBRIX is the same for each image pixel, i.e., it is independent of the target.
- Lightness equation: the lightness equations of T-Rex and GREAT present a similar mathematical form: both T-Rex and GREAT compute the lightness as a weighted average of maximals of the target intensity. Nevertheless, the maximals and the weights used in T-Rex and GREAT are different: they are computed over different sampling figures, and weights have different mathematical expressions. RSR-P computes the lightness by estimating the probability to sample m maximum intensities with radial density around the target. Both the QBRIX versions define lightness in terms of quantile of a given pdf (f or f_x).
- Explicit model of locality in lightness equation: No locality information is modeled in global-QBRIX. In RSR-P and local-QBRIX, the lightness equation does not include spatial terms: locality is modeled during the spatial sampling. In T-Rex, locality is modeled both in the image sampling step through the pixel selection based on v_x and in the lightness equation by the Gaussian weights. In GREAT, locality is modeled only by the spatial weights in the lightness equation.

4 Performance Evaluation

Evaluating the performance of an image enhancer is a hard task due to the lack of agreed measures for assessing the quality and/or the accuracy of image enhancement algorithms.[3,21] In this work, the enhancement performance of T-Rex is numerically quantified by comparing three image features that are related to the image content understanding (e.g., visibility of the details) before and after applying T-Rex: brightness, contrast, and intensity distribution.

The features measuring brightness, contrast, and characterizing the intensity probability density function of a channel I are computed as follows:

1. Image brightness (f_0^I): it is the mean value of the image intensities, i.e.,

$$f_0^I = \frac{1}{|S|}\sum_{x\in S} I(x); \tag{11}$$

2. Multiresolution contrast (f_1^I): it is the average of the local contrast computed over a set of rescaled versions of the input image.[22] Precisely, the image I is rescaled by $\frac{1}{2}, \ldots, \frac{1}{2^K}$. Let $I_0, I_1, \ldots, I_K$ be the rescaled versions of I, where $I_0 = I$, and I_k is the image I_{k-1} rescaled by $\frac{1}{2}$ for any $k = 1, \ldots, K$. The value of f_1^I is obtained as follows:

$$f_1^I = \frac{1}{K+1}\sum_{k=0}^{K} \overline{c}_k, \tag{12}$$

where $\overline{c}_k$ is the mean value of the local contrast over I_k

$$\overline{c}_k = \frac{1}{|S_k|}\sum_{x\in S_k} c_k(x). \tag{13}$$

Here, S_k is the image support of I_k, whereas $c_k(x)$ is the contrast at $x \in S_k$ over the 3×3 window $\mathcal{N}(x)$ centered at x, i.e.,

$$c_k(x) = \frac{1}{8} \sum_{u \in \mathcal{N}(x)} |I_k(u) - I_k(x)|. \tag{14}$$

3. Histogram flatness (f_2^I): it is the L^1 distance between the histogram h representing the pdf of I and the histogram U of an uniform pdf defined over the discrete set of the image intensity values, that in the following equation range over $\{0, \dots, 255\}$:

$$f_2^I = \frac{1}{255} \sum_{b=0}^{255} |h(b) - U(b)|. \tag{15}$$

This value measures how much the histogram of I is flat. The channel histogram returned by Milano–Retinexes has usually smoothed peaks and a higher number of adjacent bins with nonnull value.

The features f_0^I and f_1^I (f_2^I, resp.) are usually increased (decreased, resp.) by image enhancers. The word *usually* is reported here in italic, because the amount of the increment introduced by image enhancers such as Milano Retinexes depends on the input image, and specifically, it is proportional to the image content readability: the enhancement effects on an image with an already clearly understandable content are less evident or even null than those on an image with poorly visible details. The evaluation of the image enhancement performance must be carried on by considering simultaneously all the measures f_0^I, f_1^I, and f_2^I: in fact, the single increment of the image brightness is not enough to state that an algorithm improves the readability of the image content, e.g., whitening an image may also cancel important details, thus it worse the image quality.

In addition to measures f_i^I's, the performance of T-Rex is here evaluated also by the CIELAB distance ΔE between the input image $\overline{I}$ and its color lightness $\overline{L}$. Precisely, the value of ΔE is the mean value of the pixelwise L^1 distances between the colors of $\overline{I}$ and those of $\overline{L}$ expressed in the CIELAB color coordinates

$$\Delta E = \frac{1}{|S|} \sum_{x \in S} |L^* a^* b^*_{\overline{I}}(x) - L^* a^* b^*_{\overline{L}}(x)|, \tag{16}$$

where $L^* a^* b^*_{\overline{I}}(x)$ and $L^* a^* b^*_{\overline{L}}(x)$ indicate the Lab coordinates of the color at position x in $\overline{I}$ and $\overline{L}$, respectively.

ΔE is a measure of the differences between colors in a perceptual color space. Although devised for isolated colors, it is widely used in the literature to quantify perceptual color changes in images. Besides the possible masking effects, the value of E is in many cases proportional to the perceptual evidence of the color changes. Here, we present the results and as a rule of thumb for the reader, if $\Delta E < 2$, the images could be considered close to be indistinguishable. This is not a precise threshold, rather a kind of reference value to scale the presented results.

Many other metrics for the numerical assessment of image enhancement algorithms are available in the literature (e.g., the work of Le Moan and Urban[23]). Here we have chosen the measures f_i's, because they have been employed to evaluate Milano-Retinex algorithms previously published.[8,9,13,14]

The image enhancement performance of T-Rex has been evaluated and compared with that of GREAT, RSR-P, and QBRIX on the dataset TEST35COLOR that has been already used to test previous Milano Retinexes.[9,14] This dataset consists of 35 real-world color pictures, taken under different light conditions, in indoor and outdoor environments. Some examples of such pictures are displayed in Fig. 3.

Table 2 reports the evaluation measures for the algorithms T-Rex, GREAT, RSR-P, global-QBRIX, and local-QBRIX. The parameters of these algorithms have been set up as follows: in T-Rex, many values of λ are considered, i.e., $\lambda \in \{0{,}00, 0{,}25, 0.50, 0.75, 1.00, 2.00, 3.00, 4.00, 5.00, 6.00, 7.00, 8.00, 9.00, 10.00\}$; in GREAT, the value of τ depends on the input image, i.e., $\tau = \tau(I)$ as explained in Sec. 2.1; in RSR-P, $m = 150$; finally, in both the versions of QBRIX, $Q = 0.99$. The values of the evaluation measures in Table 2 are averaged over the number of pictures in TEST35COLOR and broken down by channel, as indicated by the superscript.

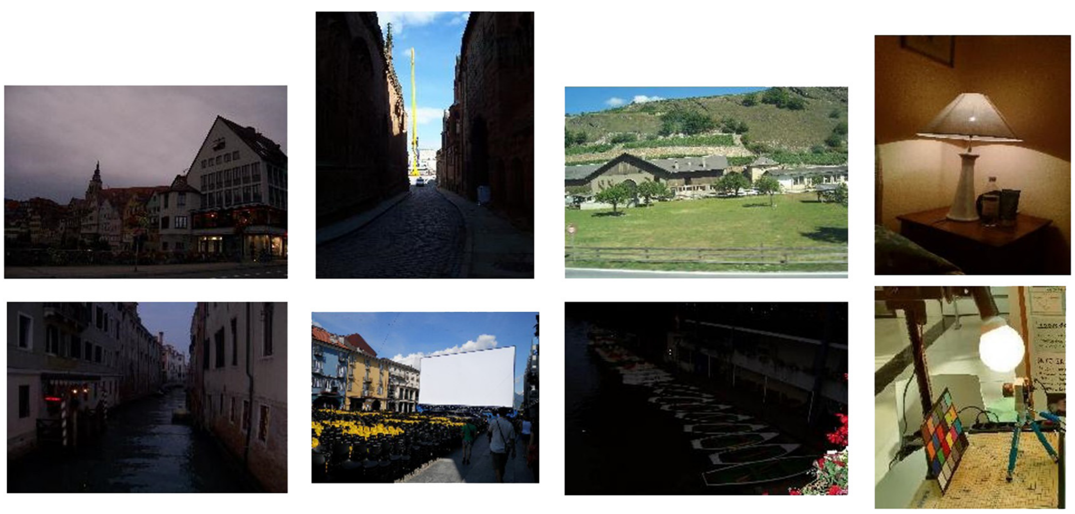


Fig. 3 Some images from the dataset TEST35COLOR used for evaluating the enhancement performance of T-Rex.

Table 2 TEST35COLOR: evaluation measures f_0, f_1, and f_2 over each image channel (see superscript), averaged over the number of images of TEST35COLOR. The algorithm parameters have been set up as follows: in T-Rex, λ varies as indicated in the parentheses, in GREAT, $\tau = \tau(I)$ as indicated in Sec. 2.1, in RSR-P $m = 150$, in global- and local-QBRIX $Q = 0.99$. The highest values of f_0 and f_1 and the lowest values of f_2 for each channel are achieved by T-Rex with $\lambda = 10.00$. RSR-P outputs the smallest values of f_0 and f_1, while global-QBRIX gets the highest values of f_2. These values have been reported in Italic to highlight the variability range of the evaluation measures obtained by the considered algorithms.

Algorithm	f_0^{red}	f_0^{green}	f_0^{blue}	f_1^{red}	f_1^{green}	f_1^{blue}	f_2^{red} ($\times 10^{-3}$)	f_2^{green} ($\times 10^{-3}$)	f_2^{blue} ($\times 10^{-3}$)
None	65.12	65.74	63.00	16.30	15.61	15.38	3.90	4.10	4.38
T-Rex ($\lambda = 0.00$)	94.23	94.67	91.29	19.61	19.05	19.35	4.05	4.26	4.51
T-Rex ($\lambda = 0.25$)	98.82	99.46	96.24	20.31	19.81	20.17	3.18	3.40	3.68
T-Rex ($\lambda = 0.50$)	100.17	100.73	97.45	20.50	20.00	20.37	3.05	3.26	3.53
T-Rex ($\lambda = 0.75$)	101.78	101.97	98.62	20.69	20.21	20.58	2.99	3.17	3.45
T-Rex ($\lambda = 1.00$)	102.72	103.19	99.75	20.89	20.40	20.78	2.93	3.11	3.38
T-Rex ($\lambda = 2.00$)	106.99	107.37	103.56	21.53	21.09	21.50	2.81	2.95	3.22
T-Rex ($\lambda = 3.00$)	110.22	110.57	106.46	22.02	21.62	22.05	2.73	2.86	3.12
T-Rex ($\lambda = 4.00$)	112.84	113.16	108.84	22.40	22.05	22.51	2.68	2.79	3.04
T-Rex ($\lambda = 5.00$)	115.08	115.36	110.89	22.72	22.40	22.91	2.62	2.74	2.98
T-Rex ($\lambda = 6.00$)	117.06	117.31	112.70	23.02	22.71	23.26	2.57	2.68	2.93
T-Rex ($\lambda = 7.00$)	118.83	119.07	114.34	23.29	22.99	23.57	2.52	2.64	2.88
T-Rex ($\lambda = 8.00$)	120.45	120.69	115.84	23.52	23.23	23.85	2.48	2.61	2.84
T-Rex ($\lambda = 9.00$)	121.93	122.15	117.23	23.73	23.46	24.11	2.45	2.57	2.80
T-Rex ($\lambda = 10.00$)	*123.32*	*123.52*	*118.51*	*23.93*	*23.66*	*24.34*	*2.42*	*2.54*	*2.77*
GREAT	97.04	98.12	96.60	20.40	20.10	20.44	3.31	3.49	3.71
RSR-P	*80.30*	*82.45*	*81.52*	*19.74*	*19.45*	*19.53*	3.30	3.44	3.69
Global-QBRIX	81.61	83.77	83.40	20.24	19.94	20.11	*3.95*	*4.14*	*4.42*
Local-QBRIX	94.08	96.54	94.92	23.09	22.94	23.07	2.81	2.91	3.20

All the algorithms reported in Table 2 modify the input image by remarkably increasing its brightness and contrast channel-by-channel. The distance to the flatness of the histograms of the input image is decreased by the algorithms with global behavior, i.e., T-Rex for $\lambda = 0$ and global-QBRIX, whereas it is increased by those exploiting local spatial information. This means that modeling locality is essential to widen the intensity range of the input image. On this dataset, the results obtained by RSR-P and global-QBRIX (GREAT and local-QBRIX, resp.) are close to those obtained by T-Rex for $\lambda = 0$ (for $0.25 < \lambda < 1.00$, resp.). Some examples of image enhancement by T-Rex, GREAT, RSR-P, and the two versions of QBRIX are given in Fig. 4.

In T-Rex, the amount of the increase of brightness and contrast and of the decrease of the histogram flatness is proportional to the value of λ, as visible from the plot in Fig. 5, which shows the triplets $[f_0, f_1, f_2]$ by varying λ, where f_i ($i = 0, 1, 2$) is obtained as the mean value of the f_i^{channel}'s. Figure 6 shows some examples of image enhancement obtained by T-Rex by varying the value of λ, which controls the algorithm locality. In particular, increasing the value of λ means weighting more the intensities of the pixels close to the target than those of the pixels located further. This produces a more remarkable increment of the image brightness in wide region with uniform intensity and of the image contrast along strong image edges. Both these effects are evident by observing the variation of the lightnesses in the pictures of Fig. 6. Increasing locality, i.e., weighting more the spatial features than the intensity, may alter the property of gradient smoothing of the Retinex algorithms, causing halos [see for instance Fig. 6(b)].

In general, the value of λ to be considered optimal varies from application to application. For this reason, we did not

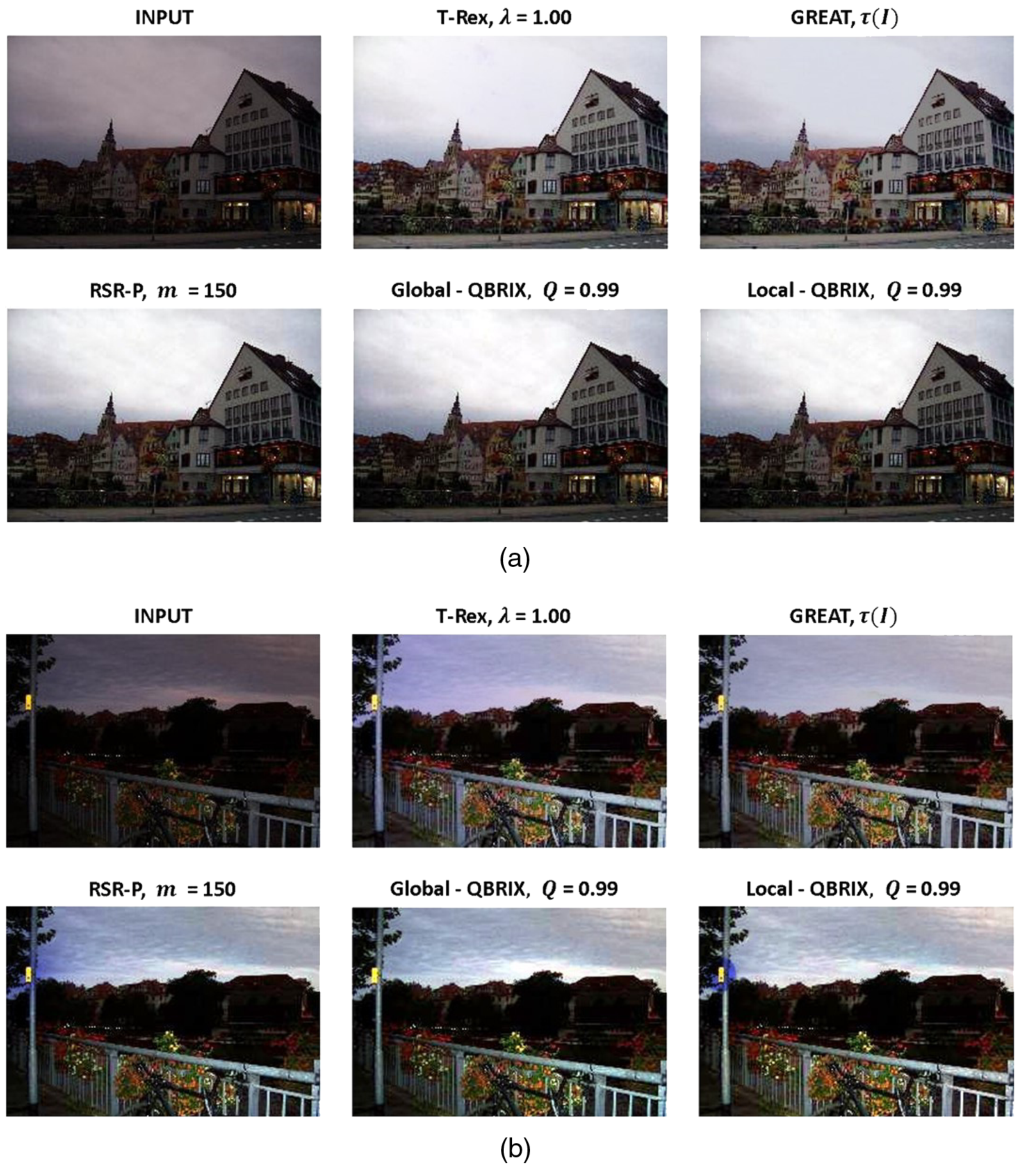


Fig. 4 Image enhancement of two pictures [labeled as "INPUT" in (a) and (b)] from TEST35COLOR obtained by T-Rex, GREAT, RSR-P, and QBRIX. The input picture in (a) is characterized by large, almost uniform bright regions (the sky and the house facades), located on top, and by a central part with many details. The input picture in (b) is darker than that in (a) and it is characterized by a small, very bright region on left (the yellow box). The visibility of the details of (a) and (b) is remarkably improved by all these algorithms.

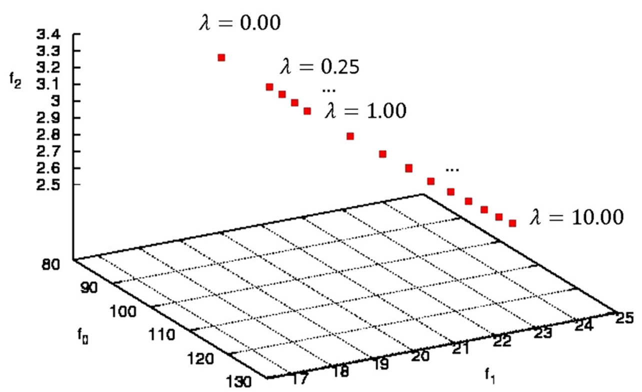


Fig. 5 Scatterplot of the triplets (f_0, f_1, f_2) for different values of λ, where $f_i = \frac{1}{3}\sum_{j=\text{red,green,blue}} f_i^j$, and $i = 0,1,2$.

discuss possibly automatic methods for setting it. This problem could be matter for future research.

Tables 3(a) and 3(b) report respectively the values of ΔE averaged over the number of images of TEST35COLOR for T-Rex and for the Milano Retinexes GREAT, RSR-P, and QBRIX. In these tests, the conversion from the RGB color space to the Lab color space has been implemented by assuming a D65 illuminant. The results reported in Table 3(a) show that the higher the value of λ the higher is the value of ΔE: increasing the contribution of the spatial feature versus the intensity makes the perceptual color changes in Lab space more evident. The comparison between the results reported in Table 3(a) and Table 3(b) yields the following conclusions: the values of ΔE produced by RSR-P and GREAT (global-QBRIX and local-QBRIX,

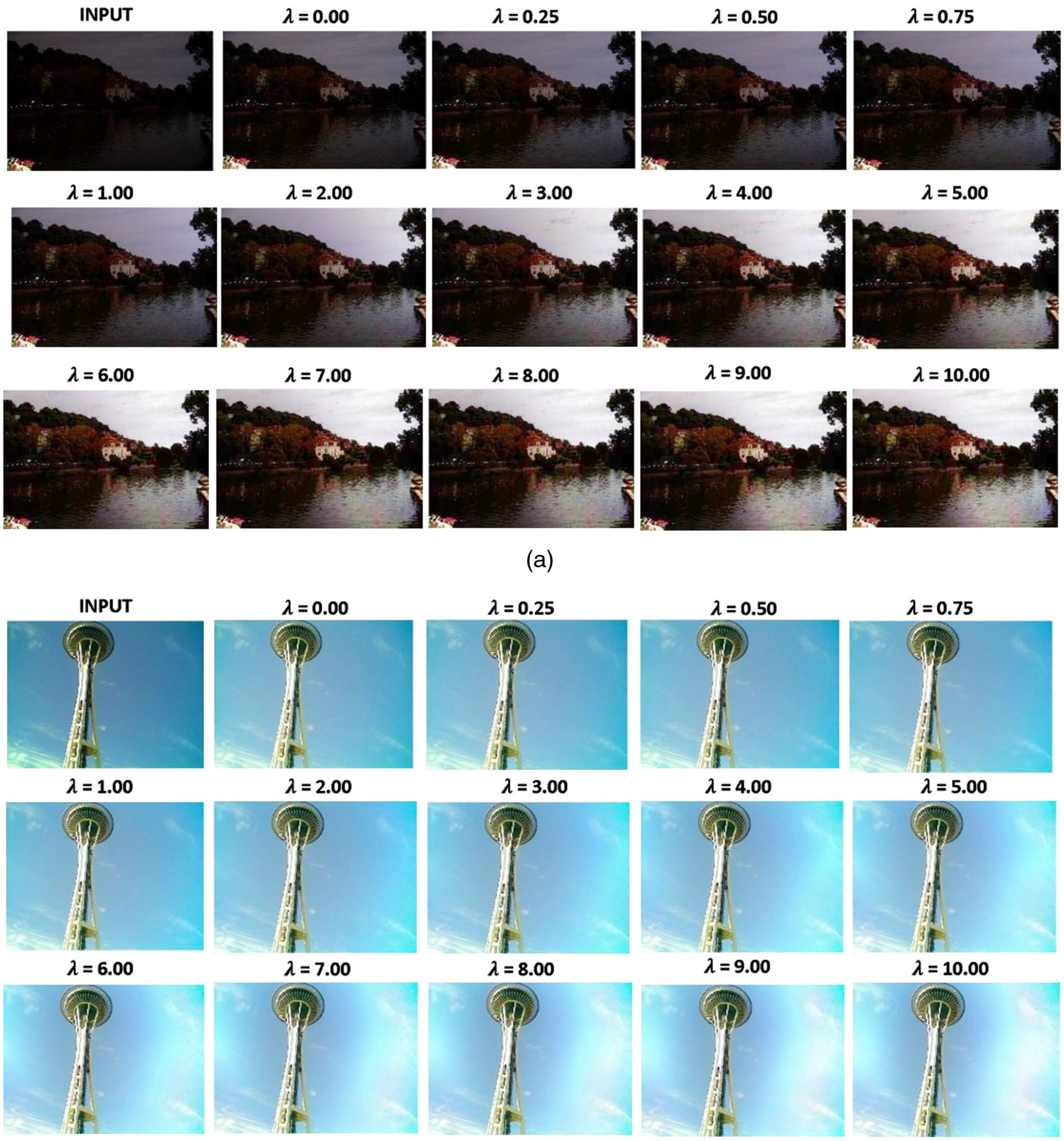


Fig. 6 T-Rex enhancement of two images from TEST35COLOR by varying λ. In both cases, the visual content of the input images in (a) and (b) are remarkably improved by T-Rex, for any value of λ. Nevertheless, we notice that high value of λ may generate halos, as shown in the example in (b).

resp.) are close to (smaller than, resp.) those obtained by T-Rex for $\lambda \leq 0.25$ ($\lambda = 0.0$, resp).

5 Conclusions

T-Rex contributes to the state-of-the-art of Milano Retinexes by introducing a deterministic spatial color sampling scheme, where the intensity values relevant to the lightness computation are extracted by a self-regulating threshold mechanism and locality is modeled by tunable Gaussian functions. The enhancement performance of T-Rex has been numerically measured on a dataset of real-world images by three visual features that are usually modified by Milano Retinexes: brightness, contrast, and histogram flatness. The experiments show that, as a member of Milano Retinex family, T-Rex actually works as an image enhancer. Its performance depends of course on the image at hand (i.e., already clear images remain unchanged), but also on the sampled features and on the algorithm locality. In this framework, the tunable Gaussian weights of T-Rex provide an interesting way to study the effects of locality on image enhancement and to choose the enhancement level according to possible further applications, such as image visual inspection in

Table 3 TEST35COLOR: mean values of ΔE for different values of λ.

(a) T-Rex

Parameter λ	ΔE
$\lambda = 0.00$	16.87
$\lambda = 0.25$	19.33
$\lambda = 0.50$	19.96
$\lambda = 0.75$	20.58
$\lambda = 1.00$	21.17
$\lambda = 2.00$	23.22
$\lambda = 3.00$	24.81
Parameter λ	ΔE
$\lambda = 4.00$	26.10
$\lambda = 5.00$	27.18
$\lambda = 6.00$	28.13
$\lambda = 7.00$	28.97
$\lambda = 8.00$	29.73
$\lambda = 9.00$	30.42
$\lambda = 10.00$	31.05

(b) GREAT, RSR-P, global- and local-QBRIX

Algorithm	ΔE
GREAT	18.92
RSR-P	10.73
Global-QBRIX	11.14
Local-QBRIX	18.15

forensic activities, video restoration, and remote sensing image analysis. Future work will address the problem of estimating automatically the value of λ to drive the image enhancement upon such kind of vision tasks.

Disclosures

The authors declare that they do not have relevant financial interests in the manuscript and no other potential conflicts of interest to disclose.

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