

UNIVERSITY OF MILAN  
LOMBARDY ADVANCED SCHOOL OF  
ECONOMIC RESEARCH

PhD in Economics  
XXVIII Cycle

**Three Essays on  
Borrowing-Constrained Agents  
in a DSGE model**

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Academic Year 2014-2015

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## Introduction

This thesis is an attempt to contribute to the literature on New-Keynesian Dynamic Stochastic General Equilibrium models - henceforth, NK-DSGE models - with heterogeneous households. In this respect, the economies are characterized by the presence of savers and borrowers that interact in the credit market. Borrowers and savers are modeled using a modified version of the mechanism proposed by Bilbiie, Monacelli and Perotti (2012 - BMP henceforth). They indeed differ in their degree of impatience: both agents are intertemporal maximizers - since borrowing and lending take place in equilibrium; and, financial markets are imperfect. Particularly, we focus on the interaction between fiscal and monetary policy and their redistributive effects. More in the detail, the thesis is composed of three papers. The second one profits from the contribution of my supervisor Prof.ssa Lorenza Rossi, while the first and third papers are not co-authored. The first and the second paper can be read as separated works, with their rationale and motivations. However, they are part of a research program and therefore they share a common ground, based on borrowing constraints and redistributive issues. While the third paper consists of a Bayesian estimation of the theoretical model presented in the second paper.

The three papers are structured as follows. In the first paper, we consider a NK-DSGE model with heterogeneous agents and distortive taxation. Heterogeneous agents are modeled using a modified version of the mechanism proposed by BMP. In particular, we introduce government expenditure in the utility function of both agents, and the budget constraints are also characterized by distortive taxes. We study the dynamics of the model in response to: (i) a positive productivity shock; (ii) a positive public expenditure shock; (iii)

a negative interest rate shock; (iv) a negative saver tax shock and (v) a negative borrower tax shock (redistributive shocks). We consider these shocks in presence of three different assumptions on the labor income tax rates of the two agents: a) *equal taxes*, both agents face the same labor income tax rate; b) *partial redistribution*, both agents pay a tax but the tax rate on borrower labor income is lower than the tax rate on saver labor income; c) *full redistribution*, saver labor income is taxed while borrower labor income is subsidized at the same rate. In the analysis of expansionary fiscal policy, public debt increases more in a context of partial redistribution than in a context of full redistribution, due to the internalization of government budget constraint by savers. In addition, a negative saver tax shock has a negative impact on redistribution, which is exacerbated under partial redistribution. Finally, a negative borrower tax shock has a negative impact on redistribution when borrowers receive subsidies, because savers are completely discouraged to save.

In the second paper we consider a NK-DSGE model with distortive taxation and heterogeneous agents, modeled using a modified version of the mechanism proposed by BMP. Following Galì (2014), we study the effects of a shock to government purchases under two alternative financing regime: (i) monetary financing; (ii) debt financing. Particularly, we focus on the redistributive effects of the two regimes and we find the following. Both regimes imply a redistributive effect from savers to borrowers, measured in terms of the ratio between the consumption of borrower and that of saver. The redistribution is much greater in the money-financed fiscal stimulus, where the consumption ratio is more than three times higher than the implied one in the debt-financed fiscal stimulus. Borrowers are better off also in terms of their relative labor supply. Finally, with respect to the representative agent

model, the presence of borrowers enhances the impact of the fiscal intervention on aggregate output, when spending is debt financed. Remarkably, with respect to Galí (2014), the same regime implies a reduction of the debt burden instead of an increase.

The third paper is a Bayesian estimation of the model presented in the second paper with heterogeneous agents and a debt-financed fiscal stimulus. We estimate the model using US data, we choose five series (output, consumption, real wage, hours and policy rate) for the period 1966Q1-2004Q4 and five shocks. We consider the economy to be affected by a government spending shock, together with four other shocks: a TFP shock, a labor supply shock, a price markup shock and a preference shock, which are quite standard. Our results convey a key message: a debt-financed fiscal stimulus played no role in determining the US area business cycle (see also, Albonico, Paccagnini and Tirelli (2016)). Another important result is the main role played by productivity and mark-up shocks, as the most important drivers of borrower consumption and output. The preference shock adds to these two ones in explaining the volatility of savers' consumption.

Redistribution has been largely overlooked in the literature on DSGE models, which has been mostly based on the paradigm of a representative-agent economy with perfect financial market. At the heart of macroeconomic literature dealing with monetary policy issues lies some form of “aggregate Euler equation” or “IS” curve: an inverse relationship between aggregate consumption today and the expected real interest rate. This relationship is derived from the households' individual Euler equation assuming that all households substitute consumption intertemporally – for example using assets. Normative prescriptions are then derived by using this equation as a building block. But direct data on asset

holdings show that a low fraction of US population holds assets in various form. Hence, models incorporating this insight have been recently used in the macroeconomic literature. They show that the presence of liquidity constrained consumers alters the standard results on the dynamics of the NK-DSGE models. Mankiw (2000) has introduced a fraction of agents which does not hold physical capital. Galì et al. (2007) have extended this insight for fiscal policy issues. Rule of thumb behavior results from consumers who face binding borrowing constraints. They demonstrate that the presence of liquidity constrained consumers can explain consumption crowding in, which follows an increase in government spending. Bilbiie (2008) shows that limited asset market participation can lead to an inverted aggregate demand logic (the IS curve has a positive slope). Di Bartolomeo and Rossi (2007) show that the effectiveness of monetary policy increases as limited asset market participation becomes more important. Galì et al. (2004) study the determinacy properties in a model with limited asset market participation and capital accumulation under different Taylor rules, showing that the presence of liquidity constrained consumers may alter the determinacy properties of a standard NK model. However, none of these papers compares a money-financed fiscal stimulus with a debt-financed fiscal stimulus. Also they do not consider the dynamic effects of fiscal rules in the presence of public debt and borrowing-constrained agents. Households heterogeneity explains also the rapid increase in gross household debt in a number of countries in the years leading up to the 2008 crisis. This debt set the stage for the crisis, and the overhang of debt continues to act as a drag on recovery. Debt is also invoked as a reason to dismiss calls for expansionary fiscal policy as a response to unemployment: you cannot solve a problem created by debt running up even

more debt. Given the prominence of debt in popular discussion of our current economic difficulties and the long tradition of invoking debt as a key factor in major economic contractions, one might have expected debt at the heart of most mainstream macroeconomic models - especially the analysis of monetary and fiscal policy. Perhaps somewhat surprisingly, however, it is quite common to abstract altogether from debt issue. One exception is represented by BMP which analyze the effects of two types of fiscal policy rules in a model where a fraction of households are borrowing-constrained. However, this paper does not investigate the role played by distortive labor income tax rules, and does not analyze the effects of a government spending financed through seigniorage. In the Eurozone, avoiding monetary finance of public debt was the absolute core of inherited Bundesbank philosophy. This is the reason why the pre-crisis dominant tool has always been represented by the policy rate. It could influence the price of credit, or in other words the price of money. The channel to influence the price of credit was generally considered indirect by passing through the movements of the policy rate. And, no role was expected by fiscal policies. One reason for the lack of interest was the general belief that the lags in implementing fiscal policies were typically too long to be useful for combating recessions. However, this long period of crisis has opened a wide spectrum of policy tools because interest rates have already been reduced close to zero bounds. Nowadays, many money creation policies are considered. The extreme end of this spectrum of possible tools is represented by the overt money finance of fiscal deficit - "helicopter money", permanent monetization of government debt. But a more moderate example of money creation has already been implemented by central banks: quantitative easing operation. However, Giavazzi and Tabellini (2014)

argue that measures as quantitative easing should take place, but together with fiscal easing. Combining a monetary and fiscal expansion is the key for the success of aggregate demand management, as shown in the recent experience of other advanced countries. Fiscal policy can be an effective tool of demand management in circumstances when interest rates are at the zero bound. But, fiscal expansion without monetary easing would be almost impossible, because of public debt in circulation is already too high in many countries. The main objection to the combined monetary and fiscal stimulus is not economic, but political. It would be opposed by Germany, and perhaps a few other member states, because it runs counter to the principle of monetary and fiscal separation of the Treaty. The academic literature has reacted with a renewed interest in monetary and fiscal policy interactions (See Woodford (2011) and Kirsanova et al. (2009)). Ascari and Rankin (2013) have analyzed the potentially drastic effect of a Taylor Rule on the effectiveness of fiscal policy in a non-Ricardian model with overlapping generations. However, they do not analyze the effects of distortive labor income taxes and they do not consider monetary policies alternative to the standard Taylor-type rules. In this scenario, Galì (2014) has analyzed the effects of an alternative and not conventional *monetary* policy to recover the economy: a fiscal stimulus, in the form of temporary increase in government purchases, financed entirely through money creation. However, Galì (2014) does not consider the redistributive effects of this policy which is instead the main objective of our thesis. Finally, this literature considers lump-sum taxes and the distortive effect of the labor income tax rules is not taken into account.



# Heterogeneous Households and Debt Dynamics\*

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## Abstract

We consider a NK-DSGE model with distortive taxation and heterogeneous agents, modeled using a modified version of the mechanism proposed by Bilbiie, Monacelli and Perotti (2012). We study the dynamics of the model in response to five shocks, under three different assumptions on the labor income tax rates: a) *equal taxes*, both agents face the same labor income tax rate; b) *partial redistribution*, both agents pay a tax but the tax rate on borrower labor income is lower than the tax rate on saver labor income; c) *full redistribution*, saver labor income is taxed while borrower labor income is subsidized at the same rate. In the analysis of expansionary fiscal policy, public debt increases more in a context of partial redistribution than in a context of full redistribution, due to the internalization of government budget constraint by savers. In addition, a negative saver tax shock has a negative impact on redistribution, which is exacerbated under partial redistribution. Finally, a negative borrower tax shock has a negative impact on redistribution when borrowers receive subsidies, because savers are completely discouraged to save.

## 1 Introduction

This paper contributes to the literature on heterogeneous agents. While this literature has concentrated on the preferences of agents for consumption and labor, it abstracts from the role played by the presence of public expenditure in utility function. Furthermore, most of the papers

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\*Preliminary Draft (Not Be Quoted)

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with fiscal stimula consider lump-sum taxes and do not analyze the interaction between distortive taxes and fiscal stimula. This paper tries to fill this gap, by considering a New-Keynesian Dynamic Stochastic General Equilibrium model - henceforth, NK-DSGE model - characterized by heterogeneous agents and distortive taxation, together with imperfect competitive firms and sticky prices. In this respect, the economies are characterized by the presence of savers and borrowers that interact in the credit market and given intrinsic characteristics of the model economy this gives rise to dynamics which are influenced by the balance sheet of agents and feed back to the rest of the economy. Heterogeneous agents differ in their degree of impatience: both agents are intertemporal maximizers - s.t. borrowing and lending take place in equilibrium; and, financial markets are imperfect. Heterogeneous agents are modeled using a modified version of the mechanism proposed by Bilbiie, Monacelli and Perotti (2013). In particular, we introduce government expenditure in the utility function of both agents, and the budget constraints are also characterized by distortive taxes.

In this context, we study the dynamics of the model in response to: (i) a positive productivity shock; (ii) a positive public expenditure shock; (iii) a negative interest rate shock; (iv) a negative saver tax shock and (v) a negative borrower tax shock. We consider these shocks in presence of three different steady-state levels of exogenous taxes: a) *equal taxes*, tax rates on both labor incomes are equal; b) *partial redistribution*, both agents pay a tax but the tax rate on borrower labor income is lower than the tax rate on saver labor income; c) *full redistribution*, borrowers receive a subsidy proportional to their labor incomes and savers pay a tax. The main results of the paper can be summarized as follows.

In this model, although aggregate wealth is mainly in the hands of the rich, poor agents have a large influence on aggregate consumption. Thus, in the aggregate, we observe a significant departure from permanent income behavior, in contrast to standard representative-agent models. In this paper we show how the presence of impatient consumers may alter dramatically the consequences of stimulus programmes, particularly in light of the upward trajectory of public debt and distortive taxes, and overturn some of the conventional results found in the literature. The assumption that government bonds are perceived as net wealth by the private sector plays an important role in theoretical analyses of monetary and fiscal effects. For example, in response to a negative saver tax shock, savers choose to work less when borrowers pay taxes, while they choose to work more when borrowers receive a subsidy, in steady state. Why? Because there is a strong incentive for savers to internalize the government budget constraint through their public debt holdings and so

recognize that a lower saver tax today implies an higher tax on themselves, today or in the future. On the other side, borrowers in any case decide to work more, because they internalize the government budget constraint due to distortive taxation on labor income. This is the reason why they also decrease their consumption. But borrowers reaction depends also on saver impulse responses. When savers work less, borrowers decide to work more and consume less. When savers work more, viceversa. Saver consumption, instead, always increases but proportionally to saver labor supply. Hence, there is an expansion when borrowers receive subsidies, otherwise economy is stabilized. The sticky price environment explains why labor demand increases when borrowers receive subsidies in steady state, while it decreases when borrowers pay taxes. Remember that savers are the owners of firms. And, it explains also the corresponding effects on inflation. Nominal interest rate fluctuations are explained by a standard Taylor rule. And, finally, public debt depends on distortive taxation on labor incomes. When savers work less, public debt increases more.

We wanted also to compare the equilibrium dynamics concerning the ratio between borrower consumption and saver consumption (and their relative wealth) under the different fiscal and monetary policies. A positive public expenditure shock and a negative interest rate shock increase borrowers' consumption more than savers' one. On the other side, a negative saver tax shock increases saver consumption more than borrower one but the effect in the event of partial redistribution is enormously greater than other cases. Finally, the IRFs in the event of a negative borrower tax shock are very different. In the case borrowers pay taxes, a borrower tax shock increases borrower consumption more than savers one. When borrowers receive a subsidy, a borrower tax shock increases saver consumption more than borrowers one.

New Keynesian (NK) models of last generation, featuring imperfect competition, and price stickiness as central building blocks, have recently become a workhorse reference for the analysis of monetary policy (Clarida, Gali and Gertler, 1999). Surprisingly, most of these models have largely ignored the interaction between heterogeneous agents and the role of government and Central Bank. Redistribution has been largely overlooked in the recent literature, which has been mostly based on the paradigm of a representative-agent economy with perfect financial market. At the heart of modern macroeconomic literature dealing with monetary policy issues lies some form of "aggregate Euler equation" or "IS" curve: an inverse relationship between aggregate consumption today and the expected real interest rate. This relationship is derived from the households' individual Euler equation assuming that all households

substitute consumption intertemporally - for example using assets. Normative prescriptions are then derived by using this equation as a building block. Direct data on asset holdings show that a low fraction of US population holds assets in various forms. Models incorporating this insight have been recently used in the macroeconomic literature. Some version of this assumption - whereby a fraction of agents does not hold physical capital - has been proposed by Mankiw (2000) and extended by Gali et al. (2007) for fiscal policy issues. Both the Barro-Ramsey model (Barro, 1974) and the Diamond-Samuelson model (Diamond, 1965) assume that all households use financial markets to smooth consumption over time. But neither the Barro-Ramsey nor the Diamond-Samuelson model is adequate for analyzing fiscal policy. Consumption smoothing is far from perfect. One can view the rule-of-thumb behavior as resulting from consumers who face binding borrowing constraints. Of the consumers who participated in Federal Reserve Board's 1983 *Survey of Consumers Finances*, 43 said that being prepared for emergencies was the most important reason for saving. Only 15 percent said that preparing for retirement was the most important saving motive. These are not the answers that standard interpretations of the Life Cycle/Permanent Income Hypothesis (LC/PIH) model of saving would lead one to expect. Carroll (1997), however, argues that such responses, and a wide range of other evidence, are consistent with a version of the LC/PIH model in which consumers face important income uncertainty, but are also "prudent" in sense that have a precautionary saving motive, and "impatient" in the sense that if future income were known with certainty they would choose to consume more than their current income. Under these conditions, consumers may engage in what Carroll calls "buffer-stock" saving behavior. In addition, Mankiw argues that many people have net worth near zero and bequests are an important factor in wealth accumulation.

During this period of Great Recession, one of the most important political debates concerns with the effects of so-called fiscal and monetary stimulus programmes. And, if there is a single word that appears most frequently in discussion of the economic problems now afflicting United States and Europe, that word is surely *debt*. It is important to understand the distributional consequences of stimulus programmes, particularly in light of the upward trajectory of public debt. There was a rapid increase in gross household debt in a number of countries in the years leading up to the 2008 crisis. This debt, it is widely argued, set the stage for the crisis, and the overhang of debt continues to act as a drag on recovery. Debt is also invoked - wrongly, Krugman and Eggertsson (2012) argue - as a reason to dismiss calls for expansionary fiscal policy as a response to unemployment: you cannot solve a problem created by

debt running up even more debt, say the critics. The current preoccupation with debt harks back to a long tradition in economic analysis. And, given the prominence of debt in popular discussion of our current economic difficulties and the long tradition of invoking debt as a key factor in major economic contractions, one might have expected debt at the hearth of most mainstream macroeconomic models - especially the analysis of monetary and fiscal policy. Perhaps somewhat surprisingly, however, it is quite common to abstract altogether from this feature of the economy.

The remainder of this paper is organized as follows. Next section introduces the model, while Section 3 presents the different policy regimes and analyzes the steady states and model dynamics. Section 4 concludes.

## 2 The Model

We consider a New-Keynesian model with imperfectly competitive goods markets and sticky prices. A closed production economy is populated by a continuum of monopolistically competitive producers and a continuum of households  $[0,1]$  all having the same utility function deriving from consumption goods, government expenditure and leisure. Each firm produces a differentiated good by using as input the labor services supplied by the household in a perfectly competitive labor market. Prices of consumption goods are assumed to be sticky à la Rotemberg (1982).

There are two Policymakers. We assume that the monetary authority decides on the nominal interest rate as in the cashless limit economy. The fiscal authority is responsible for choosing the level of government expenditure, levying distortive taxes on labor income and issuing one-period nominal non state-contingent government debt. By no arbitrage, the interest rate on bonds has to equalize the monetary policy rate in equilibrium. Finally, we assume that the central bank and the fiscal authority are fully independent, i.e. they do not act cooperatively and they do not share a budget constraint.

This section briefly describes our economy and defines competitive equilibria.

### 2.1 Households

All households have preferences defined over private consumption,  $C_{i,t}$ , public expenditure,  $G_t$ , and labor services,  $N_{i,t}$ , according to the following utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta_t^t \left[ (1 - \chi) \ln C_{i,t} + \chi \ln G_t - \psi_i \frac{N_{i,t}^{1+\varphi}}{1 + \varphi} \right], \quad (1)$$

where  $\varphi$  is the inverse of the labour supply elasticity. The agents differ in their discount factors  $\beta_\iota \in (0, 1)$  and possibly in their preference for leisure  $\psi_\iota$ . Specifically, we assume that there are two types of agents  $\iota = s, b$ , and  $\beta_s > \beta_b$ .  $E_0$  denotes expectations conditional on the information available at time 0 and  $\chi$  measures the weight of public spending relatively to private consumption. Also, as we show below,  $\chi$  determines the share of government expenditure over GDP, computed at the non-stochastic steady state of the Pareto efficient equilibrium.  $C_{\iota,t}$  is a CES aggregator of the quantity consumed  $C_t(z)$  of any of the infinitely many varieties  $z \in [0, 1]$  and it is defined as

$$C_{\iota,t} = \left[ \int_0^1 C_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (2)$$

$\epsilon > 1$  is the elasticity of substitution between varieties.

A  $1 - \lambda$  share is represented by households who are patient: we label them savers, discounting the future at  $\beta_s$ . Consistent with the equilibrium outcome the patient agents are savers (and hence will hold the bonds issued by impatient agents), we impose that patient agents also hold all the shares in firms.

In each period  $t \geq 0$  and under all contingencies each saver chooses consumption, hours worked and asset holdings (bonds and shares), subject to:

$$C_{s,t} + B_{s,t+1} + A_{s,t+1} + \Omega_{s,t+1} V_t \leq \frac{1 + i_{t-1}}{\pi_t} B_{s,t} + \frac{1 + i_{t-1}}{\pi_t} A_{s,t} + \Omega_{s,t} (V_t + \Gamma_t) + w_t N_{s,t} (1 - \tau_{s,t}), \quad (3)$$

where  $w_t$  is the real wage,  $A_{s,t}$  is the real value at beginning of period  $t$  of total private assets held in period  $t$  ( $\pi_t = P_t/P_{t-1}$  is the net inflation rate), a portfolio of one-period bonds issued in  $t-1$  on which the household receives the nominal interest  $i_{t-1}$ .  $V_t$  is the real market value at time  $t$  of shares in intermediate good firms,  $\Gamma_t$  are real dividend payoffs of these shares,  $\Omega_{s,t}$  are share holdings,  $w_t N_{s,t} (1 - \tau_{s,t})$  is the after-tax real saver labor income, and  $B_{s,t}$  are the savers' holdings of nominal public bonds which deliver the same nominal interest as private bonds. The nominal debt  $B_t$  pays one unit in nominal terms in period  $t+1$ . To prevent Ponzi games, the following condition is assumed to hold at all dates and under all contingencies

$$\lim_{T \rightarrow \infty} E_t \left\{ \prod_{k=0}^T (1 + i_{t+k})^{-1} B_{s,t+T} \right\} \geq 0. \quad (4)$$

Given prices, policies and transfers  $\{P_t(z), w_t, i_t, G_t, \tau_{s,t}, V_t, \Gamma_t, T_t\}_{t \geq 0}$  and the initial condition  $B_{-1}$ , the saver chooses the set of processes  $\{C_{s,t}(z), C_{s,t}, N_{s,t}, B_{s,t}, \Omega_{s,t}, A_{s,t}\}_{t \geq 0}$ , so as to maximize (1) subject to (2) - (4). After defining the aggregate price level as:

$$P_t = \left[ \int_0^1 P_t(z)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}, \quad (5)$$

as well as real debt as,  $b_t \equiv B_t/P_t$ , optimality is characterized by the standard first-order conditions:

$$C_{s,t}(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} C_s, \quad (6)$$

$$\beta_s E_t \left\{ \frac{C_{s,t}(1+i_t)}{C_{s,t+1}\pi_{t+1}} \right\} = 1, \quad (7)$$

$$\beta_s E_t \left\{ \frac{C_{s,t}}{C_{s,t+1}} \frac{V_{t+1} + \Gamma_{t+1}}{V_t} \right\} = 1, \quad (8)$$

$$\frac{\psi_s N_{s,t}^\varphi C_{s,t}}{1-\chi} = w_t(1-\tau_{s,t}), \quad (9)$$

together with transversality:

$$\lim_{T \rightarrow \infty} E_t \left\{ \beta^T \frac{b_{s,t+T}}{C_{s,t+T}} \right\} = 0. \quad (10)$$

Equation 9 shows that the labor income tax drives a wedge between the marginal rate of substitution between leisure and consumption and the real wage.

In each period  $t \geq 0$  and under all contingencies the rest of the households on the  $[0, \lambda]$  interval are impatient (and will borrow in equilibrium, hence we index them by  $b$  for borrowers) faces the following budget constraint:

$$C_{b,t} + A_{b,t+1} \leq \frac{1+i_{t-1}}{\pi_t} A_{b,t} + w_t N_{b,t}(1-\tau_{b,t}), \quad (11)$$

as well as the additional borrowing constraint (on borrowing in real terms) at all times  $t$ :

$$-A_{b,t+1} \leq \bar{D}. \quad (12)$$

Given prices, policies and transfers  $\{P_t(z), w_t, \phi_t, i_t, G_t, \tau_{b,t}, T_t\}_{t \geq 0}$ , the borrower chooses the set of processes  $\{C_{b,t}(z), C_{b,t}, N_{b,t}, A_{b,t}\}_{t \geq 0}$ , so

as to maximize (1) subject to (2) and (11). Optimality is characterized by the standard first-order conditions:

$$C_{b,t}(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} C_b, \quad (13)$$

$$C_{b,t}^{-1} = \beta_b E_t \left( \frac{(1+i_t)}{\pi_{t+1}} C_{b,t+1}^{-1} \right) + \phi_t, \quad (14)$$

$$\frac{\psi_b N_{b,t}^\varphi C_{b,t}}{1-\chi} = w_t(1-\tau_{b,t}). \quad (15)$$

Equation (15) shows that the labor income tax drives a wedge between the marginal rate of substitution between leisure and consumption and the real wage. While, in (14),  $\phi_t$  takes a positive value whenever the constraint is binding. Indeed, because of our assumption on the relative size of the discount factors, the borrowing constraint will bind in steady state.

## 2.2 Firms

There are infinitely many firms indexed by  $z$  on the unit interval  $[0,1]$  and each of them produces a differentiated variety with a constant return to scale technology

$$Y_t(z) = x_t N_t(z), \quad (16)$$

where productivity  $x_t$  is identical across firms and  $N_t(z)$  denotes the quantity of labor hired by firm  $z$  in period  $t$ . Following Rotemberg(1982), we assume that firms face quadratic price adjustment costs:

$$\frac{\gamma}{2} \left( \frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 \quad (17)$$

expressed in the units of the consumption good defined in (2) and  $\gamma \geq 0$ . Nominal profits read as:

$$E_t \left\{ \sum_{i=0}^{\infty} Q_{t,t+i} \left[ P_{t+i}(z) Y_{t+i}(z) - W_{t+i} N_{t+i}(z) - P_{t+i} \frac{\gamma}{2} \left( \frac{P_{t+i}(z)}{P_{t+i-1}(z)} - 1 \right)^2 \right] \right\}, \quad (18)$$

where  $Q_{t,t+i}$  is the discount factor in period  $t$  for nominal profits  $i$  periods ahead.

Assuming that firms discount at the same rate as savers implies

$$Q_{t,t+i} = \beta_s^i \frac{C_{s,t}}{C_{s,t+i} \pi_{t+i}}. \quad (19)$$



Each firm faces the following demand function:

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t^d, \quad (20)$$

where  $Y_t^d$  is aggregate demand and it is taken as given by any firm  $z$ .

Cost minimization taking the wage as given implies that real marginal cost is  $w_t$ . Firms choose processes  $\{P_t(z), N_{b,t}(z), N_{s,t}(z), Y_t(z)\}_{t \geq 0}$  so as to maximize (18) subject to (16) and (20), taking as aggregate prices and quantities  $\{P_t, W_t, Y_t^d\}_{t \geq 0}$ . Let the real marginal cost be denoted by

$$mc_t = w_t/x_t \quad (21)$$

Then, at a symmetric equilibrium where  $P_t(z) = P_t$  for all  $z \in [0, 1]$ , profit maximization and the definition of the discount factor imply:

$$\pi_t(\pi_t - 1) = \beta_s E_t \left[ \frac{C_{s,t}}{C_{s,t+1}} \pi_{t+1} (\pi_{t+1} - 1) \right] + \frac{\epsilon x_t N_t}{\gamma} \left( mc_t - \frac{\epsilon - 1}{\epsilon} \right) \quad (22)$$

(22) is the standard Phillips curve according to which current inflation depends positively on future inflation and current marginal cost.

The profit function in real terms is given by

$$\Gamma_t(z) = \left[ \frac{P_t(z)}{P_t} \right] Y_t(z) - w_t N_t(z) - \frac{\gamma}{2} \left( \frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 \quad (23)$$

which aggregated over firms gives total profits

$$\Gamma_t(z) = (1 - mc_t) Y_t - \frac{\gamma}{2} (\pi_t - 1)^2. \quad (24)$$

### 2.3 Policymakers

In the economy there are two benevolent policy makers. A monetary authority is responsible for setting the nominal interest rate  $i_t$  in response to fluctuations in interest rate, output and inflation (we assume for simplicity that target inflation is one).

$$\ln \hat{i}_t = \phi_r \ln \hat{i}_{t-1} + (1 - \phi_r) \left( \phi_y \ln \hat{Y}_t + \phi_\pi \ln \hat{\pi}_t \right) \quad (25)$$

The fiscal authority provides the public good  $G_t(z)$  for any  $z \in [0, 1]$  and aggregating them according to:

$$G_t = \left[ \int_0^1 G_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}} \quad (26)$$

so that total government expenditure in nominal terms is  $P_t G_t$  and the public demand of any variety is:

$$G_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} G_t$$

Expenditures are financed by levying a distortive labor income tax on savers  $\tau_{s,t}$  and on borrowers  $\tau_{b,t}$  or by issuing one period, risk-free, non state contingent nominal bonds  $B_{s,t}$ , which are held only by the savers. Hence, the budget constraint of the government is:

$$B_{s,t+1} + (1 - \lambda)\tau_{s,t}W_tN_{s,t} + \lambda\tau_{b,t}W_tN_{b,t} = \frac{1 + i_{t-1}}{\pi_t}B_{s,t} + G_tP_t \quad (27)$$

where  $\tau_t$  are total tax revenues, that is,  $\tau_t = \lambda\tau_{b,t} + (1 - \lambda)\tau_{s,t}$ .

Therefore, the central bank and the fiscal authority determine the sequence  $\{i_t, G_t, \tau_{b,t}, \tau_{s,t}\}_{t \geq 0}$  that, at the equilibrium prices, uniquely determines the sequence  $\{B_{s,t}\}_{t \geq 0}$  via (27). For what follows, the government budget constraint can be written in real terms

$$b_{s,t+1} + (1 - \lambda)\tau_{s,t}mc_t x_t N_{s,t} + \lambda\tau_{b,t}mc_t x_t N_{b,t} = \frac{1 + i_{t-1}}{\pi_t}b_{s,t} + G_t \quad (28)$$

after substituting for  $w_t$  from the expression for the real marginal cost.

## 2.4 Competitive Equilibrium with constant taxes

We take as benchmark the case of competitive equilibrium with constant taxes. Our benchmark economy features three distortions: a) imperfect competition in the goods market; b) price-adjustment costs; c) distortive taxation. In an equilibrium of this economy, all agents take as given prices, as well as the evolution of exogenous processes. Specifically, labour market clearing requires that labour demand equal total labor supply,

$$N_t = \lambda N_{b,t} + (1 - \lambda)N_{s,t}. \quad (29)$$

Private debt is in zero net supply  $\int_0^1 A_{l,t+1} = 0$ , and hence, since agents of a certain type make symmetric decisions:

$$\lambda A_{b,t+1} + (1 - \lambda)A_{s,t+1} = 0 \quad (30)$$

Equity market clearing implies that share holdings of each saver are

$$\Omega_{s,t+1} = \Omega_{s,t} = \Omega = \frac{1}{1-\lambda} \quad (31)$$

Finally, by Walras' Law the goods market also clears. At a symmetric equilibrium where  $P_t(z) = P_t$  for all  $z \in [0, 1]$ ,  $Y_t(z) = Y_t^d$  and the feasibility constraint is:

$$x_t N_t = C_t + G_t + \frac{\gamma}{2}(\pi_t - 1)^2 \quad (32)$$

where

$$C_t = \lambda C_{b,t} + (1-\lambda)C_{s,t} \quad (33)$$

All bonds issued by the government will be held by savers. Market clearing for public debt implies:

$$(1-\lambda)B_{s,t+1} = B_{t+1} \quad (34)$$

In our equilibrium, productivity, public expenditure, interest rate, borrower tax and saver tax are stochastic and evolve according to the following process

$$\ln \hat{x}_t = \rho_x \ln \hat{x}_{t-1} + \epsilon_t^x \quad (35)$$

$$\ln \hat{G}_t = \rho_g \ln \hat{G}_{t-1} + \epsilon_t^g \quad (36)$$

$$\ln \hat{i}_t = \phi_r \ln \hat{i}_{t-1} + (1-\phi_r) \left( \phi_y \ln \hat{Y}_t + \phi_\pi \ln \hat{\pi}_t \right) + \ln \hat{\epsilon}_t^m \quad (37)$$

where

$$\ln \hat{\epsilon}_t^m = \rho_r \ln \hat{\epsilon}_{t-1}^m - u_t^m \quad (38)$$

$$\ln \hat{\tau}_{b,t} = \rho_b \ln \hat{\tau}_{b,t-1} - \epsilon_t^b \quad (39)$$

$$\ln \hat{\tau}_{s,t} = \rho_s \ln \hat{\tau}_{s,t-1} - \epsilon_t^s \quad (40)$$

where  $\epsilon^x$ ,  $\epsilon^g$ ,  $\epsilon^m$ ,  $\epsilon^b$  and  $\epsilon^s$  are i.i.d. shocks and  $\rho_x$ ,  $\rho_g$ ,  $\rho_r$ ,  $\rho_b$  and  $\rho_s$  are autoregressive coefficients.

We define the notion of competitive equilibrium as in Barro (1979) and Lucas and Stokey (1983), where decisions of the private sector and policies are described by collections of rules mapping the history of exogenous events into outcomes, given the initial state. Let  $s^t =$

## Summary of the model

|                                      |  |
|--------------------------------------|--|
| Budget constraint, S                 | $C_{s,t} + b_{s,t+1} + A_{s,t+1} + \Omega_{s,t+1}V_t = \frac{1+i_{t-1}}{\pi_t}b_{s,t} + \frac{1+i_{t-1}}{\pi_t}A_{s,t} + \Omega_{s,t}(V_t + \Gamma_t) + w_tN_{s,t}(1 - \tau_{s,t})$    |
| Euler equation for bond, S           | $C_{s,t}^{-1} = \beta_s \left[ C_{s,t+1}^{-1} \frac{1+i_t}{\pi_{t+1}} \right]$   |
| Euler equation for share holdings, S | $C_{s,t}^{-1}V_t = \beta_s \left[ C_{s,t+1}^{-1}(V_{t+1} + \Gamma_{t+1}) \right]$  |
| Labour supply, S                     | $\frac{\psi_s N_{s,t}^\varphi C_{s,t}}{1-\chi} = w_t(1 - \tau_{s,t})$  |
| Borrowing constraint, B              | $-A_{b,t+1} = \bar{D}$   |
| Euler equation for bond, B           | $C_{b,t}^{-1} = \beta_b E_t \left[ \frac{1+i_t}{\pi_{t+1}} C_{b,t+1}^{-1} \right] + \phi_t$  |
| Labour supply, B                     | $\frac{\psi_b N_{b,t}^\varphi C_{b,t}}{1-\chi} = w_t(1 - \tau_{b,t})$  |
| Production function                  | $Y_t = x_t N_t$  |
| Phillips curve                       | $\pi_t(\pi_t - 1) = \beta_s E_t \left[ \frac{C_{s,t}}{C_{s,t+1}} \pi_{t+1}(\pi_{t+1} - 1) \right] + \frac{\epsilon x_t N_t}{\gamma} \left( mc_t - \frac{\epsilon-1}{\epsilon} \right)$ |
| Aggregated real profits              | $\Gamma_t = Y_t - w_t N_t - \frac{\gamma}{2} (\pi_t - 1)^2$  |
| Government budget constraint         | $b_{s,t+1} + (1 - \lambda)\tau_{s,t}mc_t x_t N_{s,t} + \lambda\tau_{b,t}mc_t x_t N_{b,t} = \frac{1+i_{t-1}}{\pi_t}b_{s,t} + G_t$   |
| Resource constraint                  | $Y_t = C_t + G_t + \frac{\gamma}{2}(\pi_t - 1)^2$  |
| Productivity process                 | $\ln \hat{x}_t = \rho_x \ln \hat{x}_{t-1} + \epsilon_t^x$  |
| Public expenditure process           | $\ln \hat{G}_t = \rho_g \ln \hat{G}_{t-1} + \epsilon_t^g$  |
| Interest rate process                | $\ln \hat{\epsilon}_t^m = \rho_r \ln \hat{\epsilon}_{t-1}^m - u_t^m$   |
| Private Debt Market Clearing         | $\lambda A_{b,t+1} + (1 - \lambda)A_{s,t+1} = 0$   |
| Labour Demand                        | $mc_t = w_t/x_t$   |
| Aggregate labor input                | $N_t = \lambda N_{b,t} + (1 - \lambda)N_{s,t}$   |
| Aggregate consumption                | $C_t = \lambda C_{b,t} + (1 - \lambda)C_{s,t}$   |
| Taylor rule                          | $\ln \hat{i}_t = \phi_r \ln \hat{i}_{t-1} + (1 - \phi_r) \left( \phi_y \ln \hat{Y}_t + \phi_\pi \ln \hat{\pi}_t \right)$   |
| Borrower tax process                 | $\ln \hat{\tau}_{b,t} = \rho_b \ln \hat{\tau}_{b,t-1} - \epsilon_t^b$  |
| Saver tax process                    | $\ln \hat{\tau}_{s,t} = \rho_s \ln \hat{\tau}_{s,t-1} - \epsilon_t^s$  |

Table 1: Summary of the Model

$(x_0, \dots, x_t; G_0 \dots G_t, i_0 \dots i_t; \tau_{b,0} \dots \tau_{b,t}; \tau_{s,0} \dots \tau_{s,t})$  be the history of the exogenous events.

Given a particular history  $s^t$ , the endogenous state  $b_{s,t-1}$ , and  $j_t = (C_{b,t}, C_{s,t}, C_t, A_{b,t}, A_{s,t}, \phi_t, N_{b,t}, N_{s,t}, N_t, Y_t, b_t, w_t, mc_t, V_t, \Gamma_t, \pi_t)$ ,  $j_r(s^r | s^t, b_{s,t-1})$  denotes the rules describing current and future decisions for any possible history  $s^r, r \geq t, t \geq 0$ . Finally, we can define a continuation competitive equilibrium as a set of sequences  $\Upsilon_t = \{j_r\}_{r \geq t}$  satisfying equations (3), (7)-(10), (12), (14)-(16), (21)-(22), (24)-(25), (28)-(30) and (32)-(33) for any  $s^r$ . Obviously, a competitive equilibrium  $\Upsilon_0$  is simply a continuation competitive equilibrium starting at  $s^0$ , given  $b_{s,-1}$ .

### 3 Calibration

In this section we analyze different policy regimes. First, we will analyze as benchmark the Competitive problem with constant taxes presented before. We consider three steady-state levels of taxes. In the first steady-state, taxes on both agents are equal. The second steady state features a redistribution where both agents pay a tax but borrower tax is lower than saver tax. The third steady state features a full redistribution where borrowers receive a subsidy while savers pay a tax, both proportional to worked hours and wages.

Then, a different policy regime will be presented: the Competitive equilibrium with fiscal rule. The plan prescribes a tax path that depends on public debt. We think that this is an interesting case, even though it is a simple one, because an endogenous tax response to public debt roughly agrees with the intentions declared by most of public debt-targeting governments. We keep all our previous assumptions.

The deep parameters of the model are set according to Table (2). The weight  $\chi$  in the utility function has been chosen to roughly match U.S.

post-war government spending-to-GDP ratio. We set the serial correlation of the shocks equal to 0.9, except for the interest rate shock where we set it equal to 0.5. After substituting the aggregate production function  $Y_t = x_t N_t$ , the log-linearized Phillips curve (22) reads as follow:

$$\hat{\pi}_t = \frac{\pi - 1}{2\pi - 1} \beta_s (\hat{C}_t - \hat{C}_{t-1}) + \beta_s E_t \hat{\pi}_{t+1} + \frac{\varepsilon Y mc}{\gamma \pi (2\pi - 1)} \hat{m}c_t + \frac{\varepsilon Y}{\gamma \pi (2\pi - 1)} \left[ mc - \frac{\varepsilon - 1}{\varepsilon} \right] \hat{Y}_t \quad (41)$$

where a circumflex denotes log-deviations from steady state, variables without a time subscript denote steady-state values. The effect of

| <b>Description</b>                | <b>Parameter</b> | <b>Value</b> | <b>Source</b>   |
|-----------------------------------|------------------|--------------|---|
| Saver's discount factor           | $\beta_s$        | 0.99         | Bilbiie, Monacelli and Perotti (2013) and Gnocchi and Lambertini (2014) |
| Borrower's discount factor        | $\beta_b$        | 0.95         | Bilbiie, Monacelli and Perotti (2013)                                   |
| Weight of G in utility            | $\chi$           | 0.15         | Gnocchi and Lambertini (2014)   |
| Weight of C in utility            | $1 - \chi$       | 0.85         | Gnocchi and Lambertini (2014)   |
| Saver's preference for leisure    | $\psi_s$         | 0.3          |   |
| Borrower's preference for leisure | $\psi_b$         | 0.6          |   |
| Elast. subst. goods               | $\epsilon$       | 6            | Gali et al. (2004)  |
| Calvo Parameter                   | $\alpha$         | 0.75         |   |
| Frisch elasticity                 | $\varphi^{-1}$   | 1            | Gnocchi and Lambertini (2014)   |
| Borrowing constraint              | $\bar{D}$        | 0.5          | Bilbiie, Monacelli and Perotti (2013)                                   |
| Share of impatient household      | $\lambda$        | 0.35         | Bilbiie, Monacelli and Perotti (2013)                                   |
| Serial corr. tech.                | $\rho_x$         | 0.9          |   |
| Serial corr. public expenditure   | $\rho_G$         | 0.9          |   |
| Serial corr. interest rate        | $\rho_r$         | 0.5          |   |
| Output coefficient                | $\phi_y$         | 0.125        |   |
| Inflation coefficient             | $\phi_\pi$       | 1.5          |   |
| Interest rate coefficient         | $\phi_r$         | 0.8          |   |
| Serial corr. borrower tax         | $\rho_b$         | 0.09         |   |
| Serial corr. saver tax            | $\rho_s$         | 0.09         |   |

Table 2: Benchmark Calibration

| Variable                        | Symbol   | No Red. | Red.    | Full Red. |
|---------------------------------|----------|---------|---------|-----------|
| Borrower's consumption          | $C_b$    | 0.7751  | 0.8281  | 1.5013    |
| Saver's consumption             | $C_s$    | 1.2491  | 1.0191  | 0.9031    |
| Aggregate consumption           | $C$      | 1.0832  | 0.9523  | 1.1125    |
| Borrower's hours worked         | $N_b$    | 1.0661  | 0.9979  | 1.1795    |
| Saver's hours worked            | $N_s$    | 1.3232  | 1.1584  | 1.3072    |
| Aggregate worked hours          | $N$      | 1.2332  | 1.1023  | 1.2625    |
| Real debt                       | $b$      | 15.6719 | 24.855  | 3.1701    |
| Real market value of shares     | $V$      | 20.3479 | 18.1872 | 20.8313   |
| Real dividend payoffs of shares | $\Gamma$ | 0.2055  | 0.1837  | 0.2104    |

Table 3: Steady State

variations in the marginal cost on current inflation depends on the parameters  $\gamma$  and  $\varepsilon$  but also on steady-state output and inflation. Around a zero net inflation steady state, equation (41) boils down to:

$$\hat{\pi}_t = \beta_s E_t \hat{\pi}_{t+1} + \frac{(\varepsilon - 1) Y}{\gamma} \hat{m}c_t \quad (42)$$

taking the same form as in the Calvo model. Hence, we can establish a mapping between our parametrization and average price duration. We set parameter

$$\gamma = \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)Y}$$

where  $\alpha = 0.75$ , for our benchmark calibration, which implies a price duration of roughly two quarters.

### 3.1 Steady States

We now set  $x_t = 1$ ,  $G_t = 0.15$  for all  $t$  and analyze the non-stochastic steady state of the competitive equilibrium. We focus on a deterministic steady state where inflation is one. As the constraint binds in steady state ( $\phi = C_b^{-1} [1 - (\beta_b/\beta_s)] > 0$  whenever  $\beta_s > \beta_b$ ), patient agents are net borrowers and steady-state private debt is  $A_b = -\bar{D}$ ; by debt market clearing, then the patient agents are net lenders and their private bond holdings are  $A_s = \lambda\bar{D}/(1 - \lambda)$ . We consider three steady-state levels of taxes. In the first steady-state:  $\tau_b = \tau_s = 0.3$ . The values of the macroeconomic variables of interest are reported in the third column of Table(3). In words, public liabilities must be 15 times GDP to be sufficiently high to finance government spending. The assumption of commitment to repay on the side of public agents may appear unrealistic with such high level of indebtedness. But we consider the

competitive steady state with no redistribution as a theoretical benchmark and maintain the assumption that all debts are repaid - private or public.

The second steady state features a redistribution where  $\tau_s > \tau_b > 0$ . In the third steady-state borrowers receive a subsidy while savers pay a tax, both proportional to worked hours and wages. These two steady state are summarized in the fourth and fifth column of Table (3). In the economy with  $\tau_s > \tau_b > 0$  the public debt is 23 times GDP at the steady state and hours worked, profits, and consequently the value of shares, are well below the first case. Aggregate consumption, and in particular savers' consumption, decreases. Only borrowers' consumption increases. The economy with  $\tau_s > 0$  and  $\tau_b < 0$  has a steady state public debt of 2.5%, which implies higher hours, output, profits, value of share as well as aggregate consumption, in particular borrowers' consumption, relative to the economy with  $\tau_s > \tau_b > 0$ . The cost of this economic expansion is paid by savers' consumption.

### 3.2 Analysis of equilibrium dynamics

To illustrate how taxes affect stimulus programmes, we analyze the dynamic responses of the economy starting at the three steady states specified in Table (3). Consider first the case of a technological shock. Figure (1) presents the impulse responses of our key variables to a **positive technological shock**. We fix the size of the shocks to 0.01. In response to a positive technological shock, the impulse responses of the economies starting at three different initial conditions are similar but, in some cases, also proportional to the size of the gap between the levels of taxes. The reduction on labor supply by the savers more than compensates the increase in labor supply by the borrowers, leading to an overall contraction in spending and output. But an interest rate rule that satisfies the Taylor principle will generate lower nominal interest rates along the adjustment path, and hence, will call for a higher level of consumption for savers. Two elements are typical of the sticky-price environment. First, as firms cannot optimally adjust prices, the reduction in consumption ensuing from the productivity shock generates a decrease in labor demand. Second, the decrease in the real wage that results from the reduction in labor demand generates, for one, a further income effect on borrowers and hence a further reduction in their consumption; it also results in a rise in profits, with an additional negative income effect on the saver's labor supply. Public debt increase results from lower public revenues. Hence, a positive technological shock generates paradoxical results: an increase in productivity is contractional.



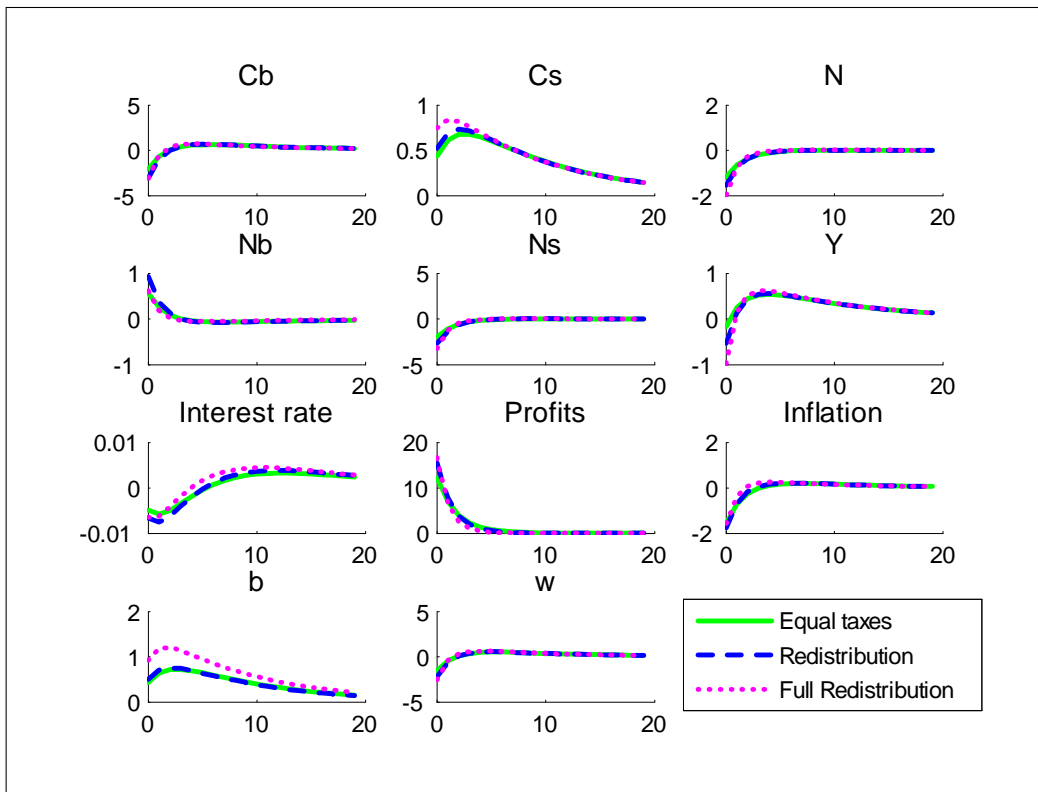


Figure 1: IRFs Productivity shock - Constant taxes

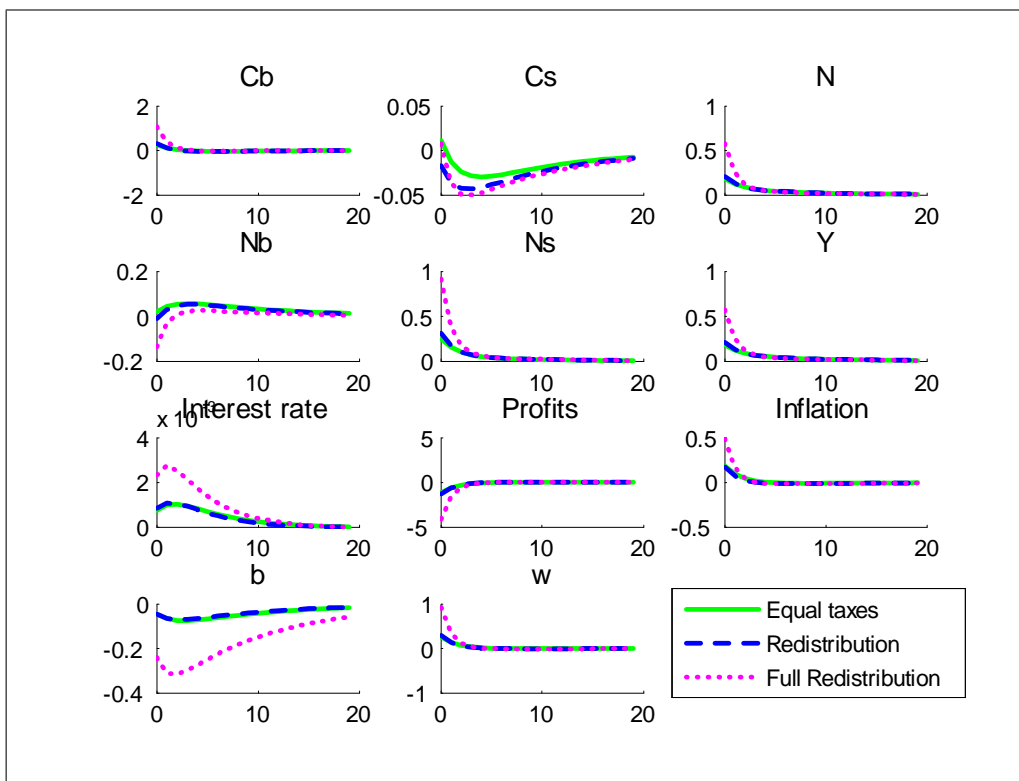


Figure 2: IRFs Public expenditure shock - Constant taxes

The key extra element is that these contradicting forces depend on the share of borrowers. The results for an alternative share associated to the technological shock in the Competitive problem with constant taxes are reported in Appendix to demonstrate the importance of borrowers share on economic activity.

Public expenditure shock alters these findings. The impulse responses to a **positive public expenditure shock** are reported in Figure(2); the magnitude of the shock is the same as in the productivity shock. In response to a positive public expenditure shock, savers choose to work more but the intensity of the income effect on labor supply varies with the size of the gap between the levels of taxes. On the other side, borrowers choose to work more when taxes are uniform but they choose to work less when they receive a subsidy. In any case, the income effect on savers' labor supply prevails over the income effect on borrowers' labor supply, leading to an overall expansion in spending and output proportional to the size of the gap between the levels of taxes. But an interest rate rule that satisfies the Taylor principle will generate higher nominal interest rates along the adjustment path, and hence, will call for a lower level of consumption for savers. As firms cannot

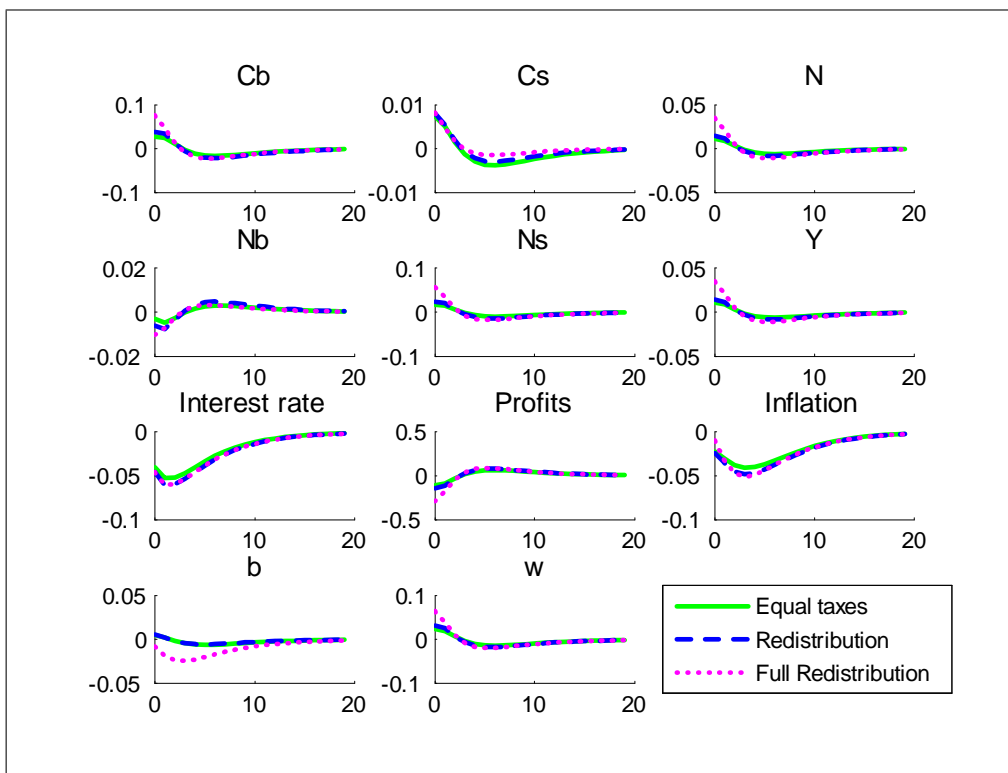


Figure 3: IRFs Interest rate shock - Constant taxes

optimally adjust prices, the improvement in borrowers' consumption ensuing from the public expenditure shock generates an increase in labor demand. The rise in real wage that results from the expansion in labor demand generates, for one, a further income effect on borrowers and further expansion in their consumption; it also results in a fall in profits, and in a fall of public debt due to higher public revenues.

Consider now the case of a interest rate shock. Figure (3) presents the impulse responses of our key variables to a negative interest rate shock. We fix the size of the shocks to 0.01. In response to a **negative interest rate shock**, the impulse responses of the economies starting at three different initial conditions are similar but vary with the size of the gap between the levels of taxes. Savers choose to work more because of the substitution effect between labor and financial activities incomes, leading to an overall expansion in spending and output. On the other side the negative interest rate shock will call for an higher level of consumption for both, savers and borrowers. And, because of an income effect, borrowers' labor supply will decrease. As firms cannot optimally adjust prices, the expansion in consumption ensuing from the interest rate shock generates an increase in labor demand. The increase in the

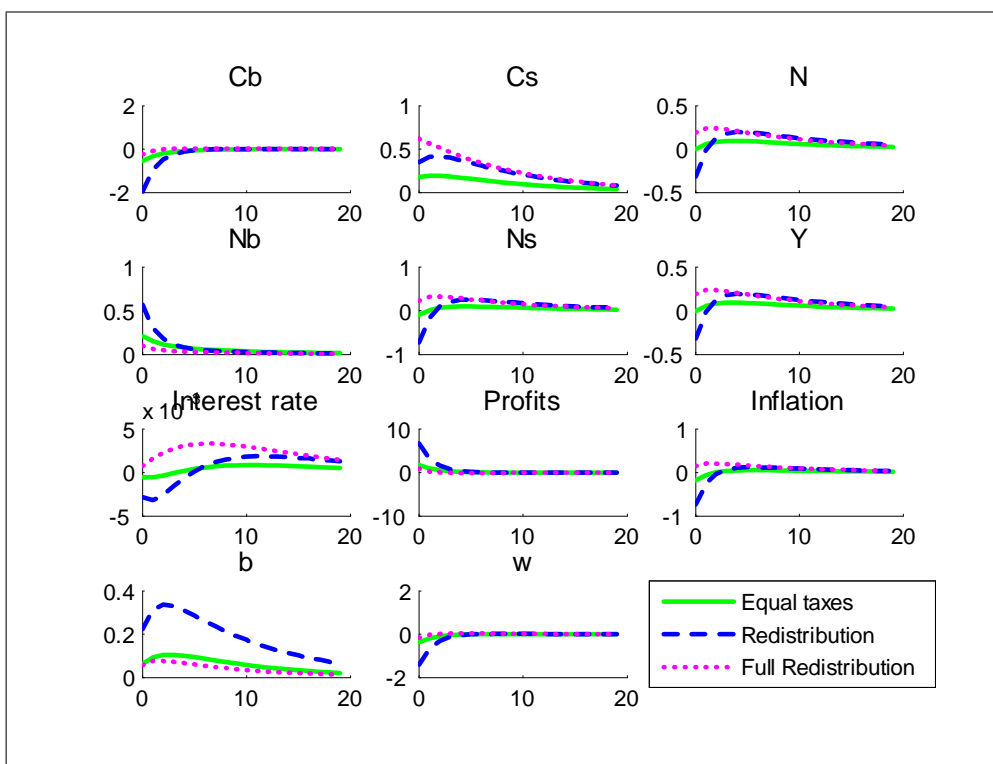


Figure 4: **IRFs Saver tax shock - Constant taxes**

real wage that results from the expansion in labor demand generates, for one, a further income effect and hence a further increase in consumption; it also results in a decrease in profits, with an additional positive income effect on the saver's labor supply. Public debt reduction results from higher public revenues.

Consider now the case of a saver tax shock. Figure (4) presents the impulse responses of our key variables to a **negative saver tax shock**; the magnitude of the shock is the same as previous shocks. In response to a negative saver tax shock, savers choose to work less when borrowers pay taxes, while they choose to work more when borrowers receive a subsidy. Savers internalize the government budget constraint through their public debt holdings and so recognize that a lower saver tax today implies a tax on themselves, today or in the future. On the other side, borrowers choose to work more but the intensity of the income effect on labor supply varies with the size of the gap between the levels of taxes. Hence, when taxes are uniform, the income effect on borrowers' labor supply prevails over the income effect on savers' labor supply, leading to an overall expansion in spending, output, and consequently in labor demand. But, in this particular case, the expansion is not big

enough to generate a rise in real wage and higher nominal interest rates along the adjustment path. Lower real wages generate a further income effect on borrowers and further reduction in their consumption; it also results in an increase of profits, and in an increase of public debt due to lower public revenues. When there is a redistribution but borrowers continue to pay a tax, the income effect on savers' labor supply prevails over the income effect on borrowers' labor supply, leading to an overall contraction in spending, output, and consequently in labor demand. An interest rate rule that satisfies the Taylor principle will generate lower nominal interest rates along the adjustment path. The reduction in real wage that results from the reduction in labor demand generates a further income effect on borrowers and further reduction in their consumption; it also results in an increase of profits, and in an increase of public debt due to lower public revenues. Finally, when there is a positive income effect on both labor supplies - the case in which savers internalize the budget constraint - a negative saver tax shock leads to the bigger expansion in spending, output, and consequently in labor demand. But also in this case, as in the case of uniform taxes, the expansion is not big enough to generate a rise in real wage. Lower real wages generate a further income effect on borrowers and further reduction in their consumption; it also results in an increase of profits, and in an increase of public debt due to lower public revenues. The Taylor principle will generate higher interest rate; however, the increase in interest rate is not so high to call a lower level of consumption for savers, an income effect prevails over the common substitution effect in this particular case. While, in the other ones, lower nominal interest rates will call higher level of consumption for savers.

Finally, consider the case of a borrower tax shock. Figure (5) presents the impulse responses of our key variables to a **negative borrower tax shock**; the magnitude of the shock is the same as previous shocks. In response to a negative borrower tax shock, savers labor choose to work more when borrowers pay a tax, while they choose to work less when borrowers receive a subsidy. On the other hand, borrowers choose to work more but the intensity of the income effect on labor supply varies with the size of the gap between the levels of taxes. Hence, when borrowers pay the labor tax, a negative borrower tax shock leads to the expansion in spending and output. But an interest rate rule that satisfies the Taylor principle will generate higher nominal interest rates along the adjustment path, and hence, will call for a lower level of consumption for savers. As firms cannot optimally adjust prices, the improvement in borrowers' consumption ensuing from the borrower tax shock generates an increase in labor demand. The rise in real wage that results from the

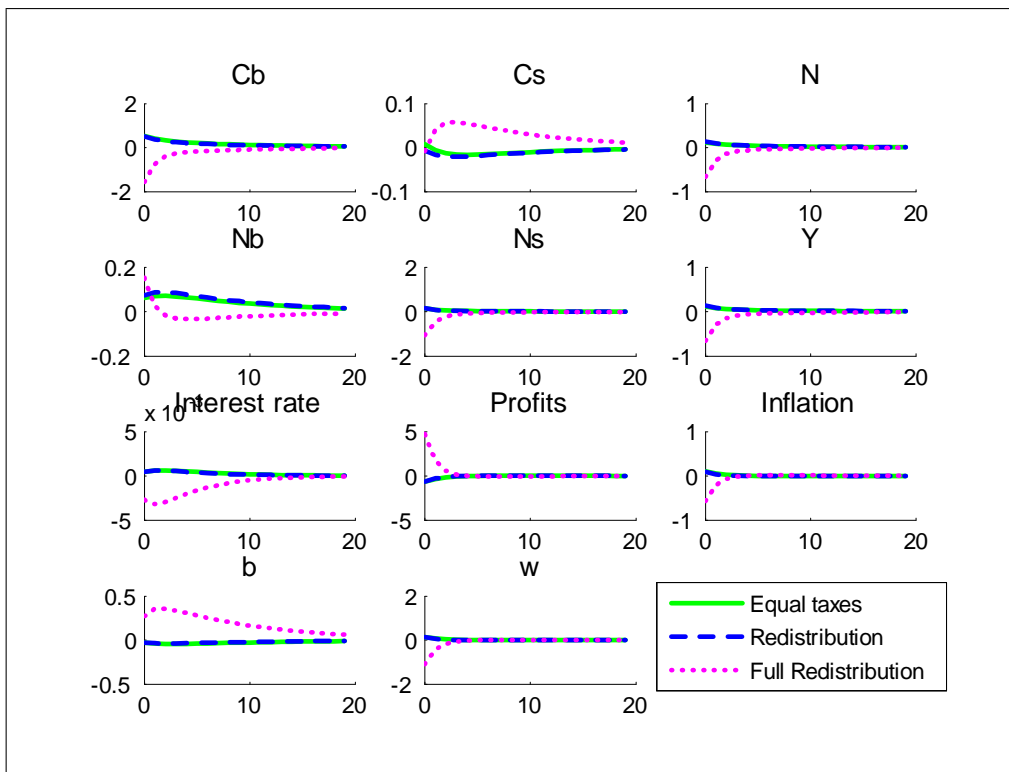


Figure 5: IRFs Borrower tax shock - Constant taxes

expansion in labor demand generates, for one, a further income effect on borrowers and further expansion in their consumption; it also results in a fall in profits, and in a fall of public debt due to higher public revenues. On the other hand, when borrowers receive a subsidy, a negative borrower tax shock leads to the contraction in spending and output because the income effect on savers' labor supply prevails over the income effect on borrowers' labor supply. But an interest rate rule that satisfies the Taylor principle will generate lower nominal interest rates along the adjustment path, and hence, will call for a higher level of consumption for savers. As firms cannot optimally adjust prices, the reduction in consumption ensuing from the borrower tax shock generates a decrease in labor demand. The reduction in the real wage that results from the decrease in labor demand generates, for one, a further income effect on borrowers and hence a further reduction in their consumption; it also results in a rise in profits, with an additional negative income effect on the saver's labor supply. Public debt increase results from lower public revenues.

### 3.3 Competitive Equilibrium with Fiscal Rule

Assume a general financing scheme whereby taxes on each agent increase to repay the outstanding debt but only gradually so:

$$\tau_{\iota,t} = \Phi_B^\iota b_t - \epsilon_{\iota,t}, \quad (43)$$

where  $\iota = b, s$ , and  $\epsilon_t$  is an i.i.d. shock.

This tax rule is general enough to allow taxes on each agent to react to stabilize government debt ( $\Phi_B^\iota = 0.09$  is the debt feedback coefficient), and asymmetric changes in taxation for the two agents ( $\epsilon_{\iota,t}$  is a random innovation). The plan prescribes a tax path that depends on public debt. We think that this is an interesting case, even though it is a simple one, because an endogenous tax response to public debt roughly agrees with the intentions declared by most of public debt-targeting governments. We keep all our previous assumptions.

#### 3.3.1 Steady State

As in the previous section, we first set  $x_t = 1$  and  $G_t = 0.15$  for all  $t$  and analyze the non-stochastic steady state of the competitive equilibrium with fiscal rule as a function of the steady-state inflation rate  $\pi_t = 1$ . The values of the macroeconomic variables of interest are reported in the third column of Table(4). In words, public liabilities must be 20% of GDP, which implies higher hours, output, profits, value of share as well as aggregate consumption, in particular savers' consumption, relative to the economy where taxes are constant. Only borrower' consumption is

| Variable                        | Symbol           | Fiscal rule |
|---------------------------------|------------------|-------------|
| Borrower's consumption          | $C_b$            | 0.8573      |
| Saver's consumption             | $C_s$            | 1.4045      |
| Aggregate consumption           | $C$              | 1.2130      |
| Borrower's hours worked         | $N_b$            | 1.1919      |
| Saver's hours worked            | $N_s$            | 1.4551      |
| Aggregate worked hours          | $N$              | 1.3630      |
| Taxes                           | $\tau_b, \tau_s$ | 0.1345      |
| Real debt                       | $b$              | 0.2689      |
| Real market value of shares     | $V$              | 22.4890     |
| Real dividend payoffs of shares | $\Gamma$         | 0.2272      |

Table 4: Steady State

lower in this economy than in the case in which he receive a subsidy.

### 3.3.2 Analysis of equilibrium dynamics

Figure (6) presents the impulse responses of our key variables to a **positive technological shock**. We fix the size of the shock to 0.01. In a competitive economy with fiscal rule, both labor supplies decrease, while in a competitive economy with constant taxes borrower labor supply increases. But in this event the reduction is so small that the shock leads, in any case, to an overall expansion in spending and output, while in the previous section we have observed an overall contraction. Hence, in this particular case, nominal interest rate increases along the adjustment path, according to the Taylor principle. The productivity shock generates a decrease in labor demand and an increase in real wage. And, according to the element typical of a sticky-price environment, the increase in the real wage generates, for one, a further income effect on borrowers and a further improvement in their consumption. However, in this particular case, the positive technological shock generates not only a rise in wages, but also in profits, with an additional negative effect on the saver's labor supply. Finally, the increase in wages more than compensates the reduction on labor supply, leading to an overall expansion in public revenues, and consequently a reduction in public debt.

The impulse response to a **positive public expenditure shock** are reported in Figure (7); the magnitude of the shock is the same as in productivity shock. In this scenario, we can observe two important differences relative to the context with constant taxes:

- both agents choose to work more;



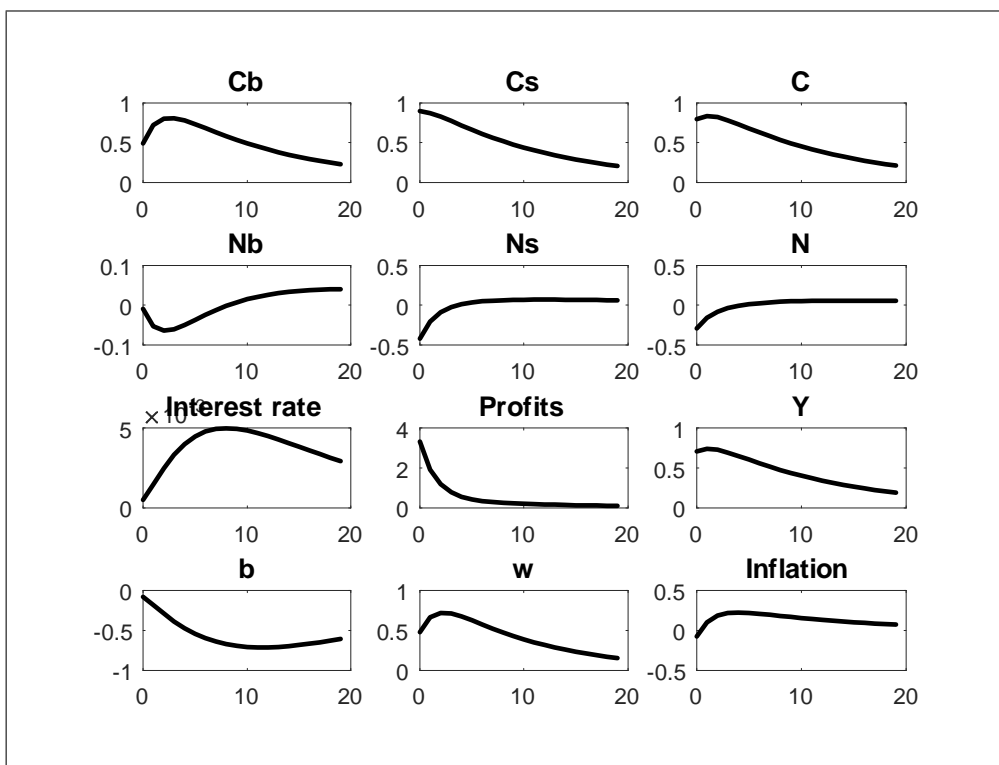


Figure 6: Fiscal rule with productivity shock

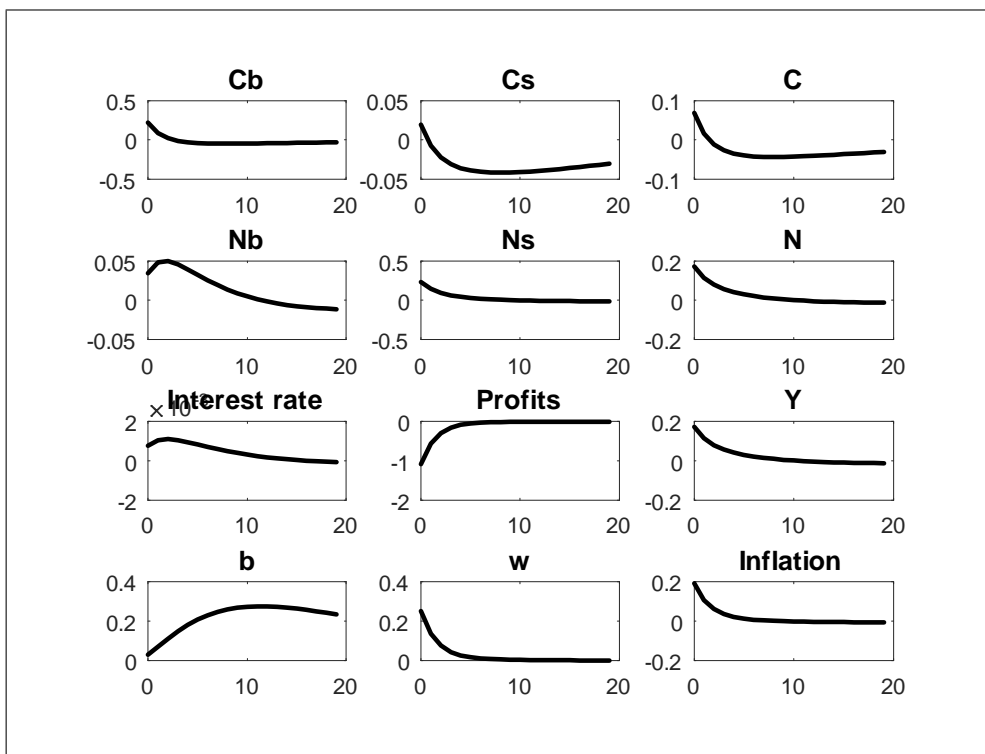


Figure 7: **Fiscal rule with government expenditure shock**

- public debt increases.

While it is pretty obvious that, in absence of subsidies, an increase in government expenditure support labor supplies by both agents, as we have already seen in the previous section, the second element can be object of discussion. Public debt increases because, in this particular case in which taxes depend on public debt, firms internalize the government budget constraint limiting the rise in labor demand and wages, and consequently generating an increase in public debt.

Consider now the case of a interest rate shock. Figure (8) presents the impulse responses of our key variables to a negative interest rate shock. We fix the size of the shocks to 0.01. Dynamics of an interest rate shock in a competitive scenario with fiscal rule are equal to dynamics observed for the same shock in a context with constant taxes, as seen in the previous Section.

Consider now the event of a saver tax shock. Figure (9) presents the impulse responses of our key variables to a **negative saver tax shock**; the magnitude of the shock is the same as previous shocks. In response to a negative tax shock, savers choose to increase their labor supply because they internalize the government budget constraint more than

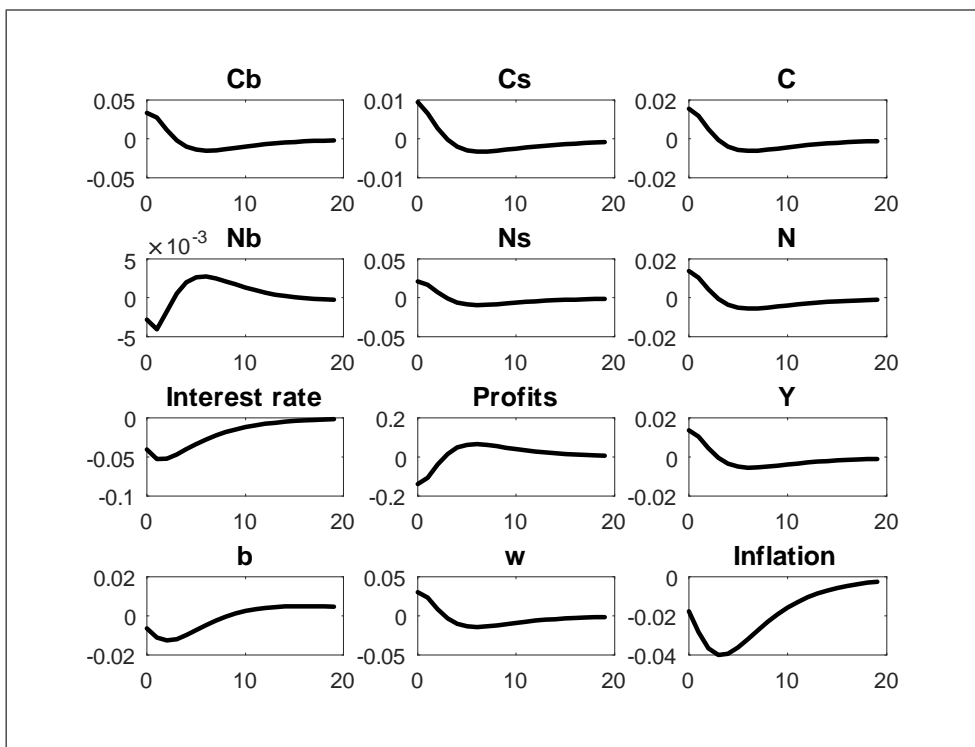


Figure 8: **Fiscal rule with interest rate shock**

in the event of constant taxes, and so recognize that a lower saver tax today implies a tax on themselves in the future. On the other side, as in the event of constant taxes, borrowers choose to work more, leading to an overall expansion in spending, output, and consequently in labor demand. An interest rate rule that satisfies the Taylor principle will generate higher nominal interest rates along the adjustment path. The increase in real wage that results from the expansion in labor demand generates a further income effect on borrowers and further expansion in their consumption; it also results in a decrease of profits, and in a decrease of public debt due to higher public revenues.

Finally, consider the case of a borrower tax shock. Figure (10) presents the impulse responses of our key variables to a **negative borrower tax shock**; the magnitude of the shock is the same as previous shocks. In response to a negative borrower tax shock, savers and borrowers choose to work more, leading to an overall expansion in spending, output and labor demand. But an interest rate rule that satisfies the Taylor principle will generate higher nominal interest rates along the adjustment path. The rise in real wage that results from the expansion in labor demand generates, for one, a further income effect on borrowers and further expansion in their consumption; it also results in a fall in

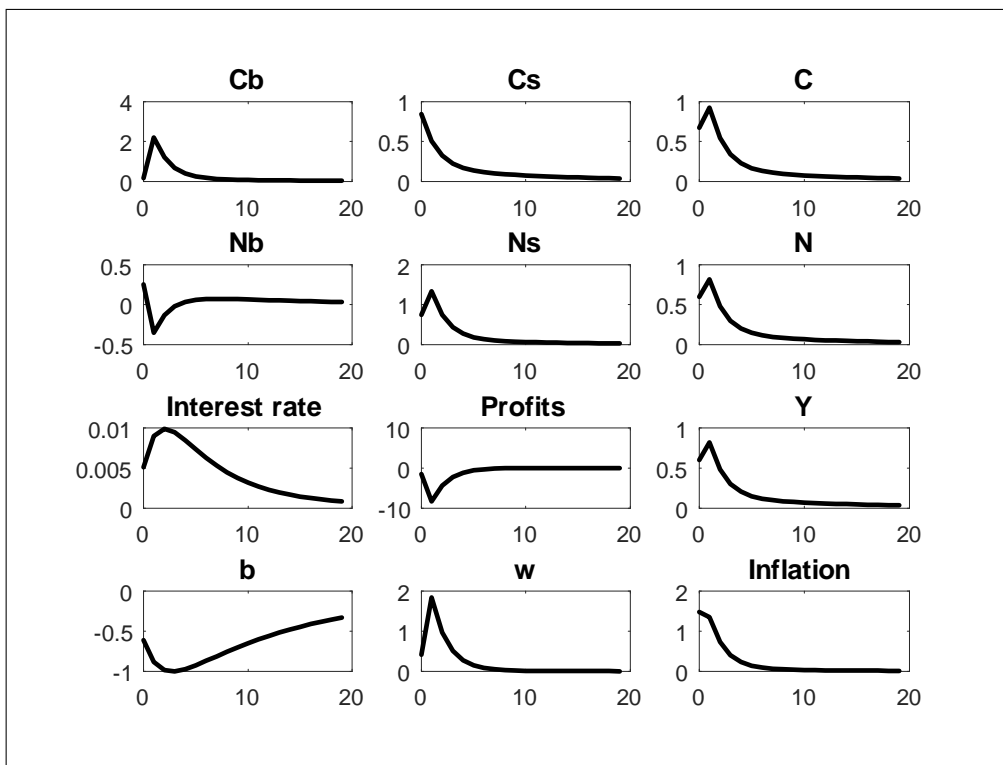


Figure 9: Fiscal rule with saver tax shock

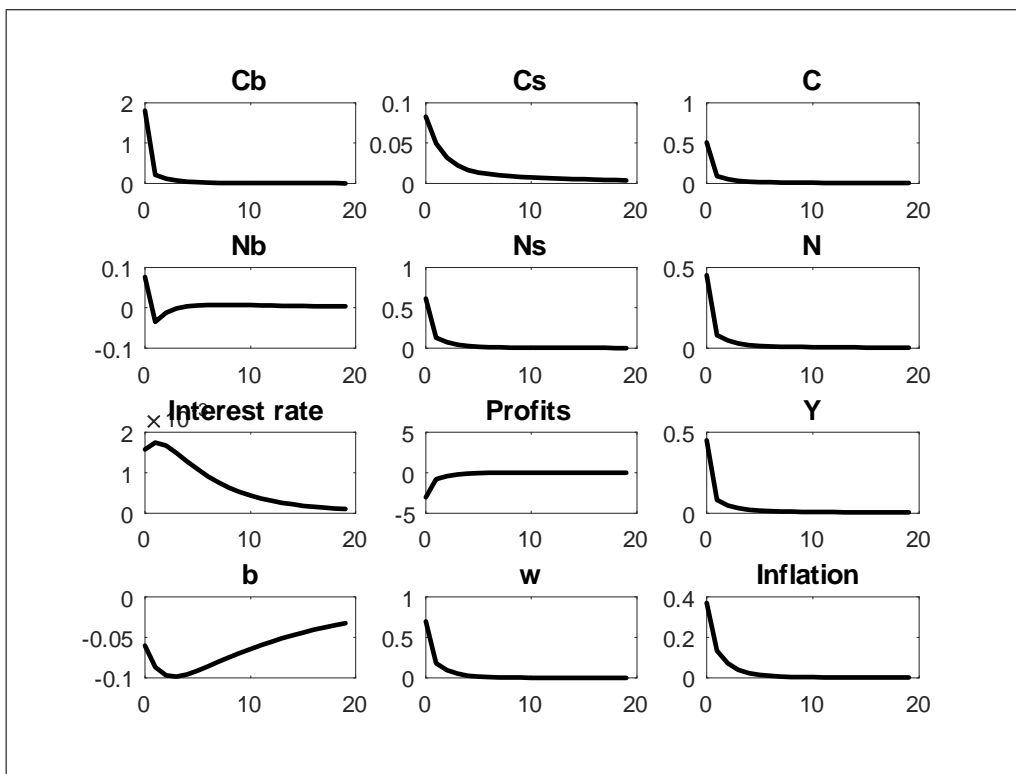


Figure 10: Fiscal rule with borrower tax shock

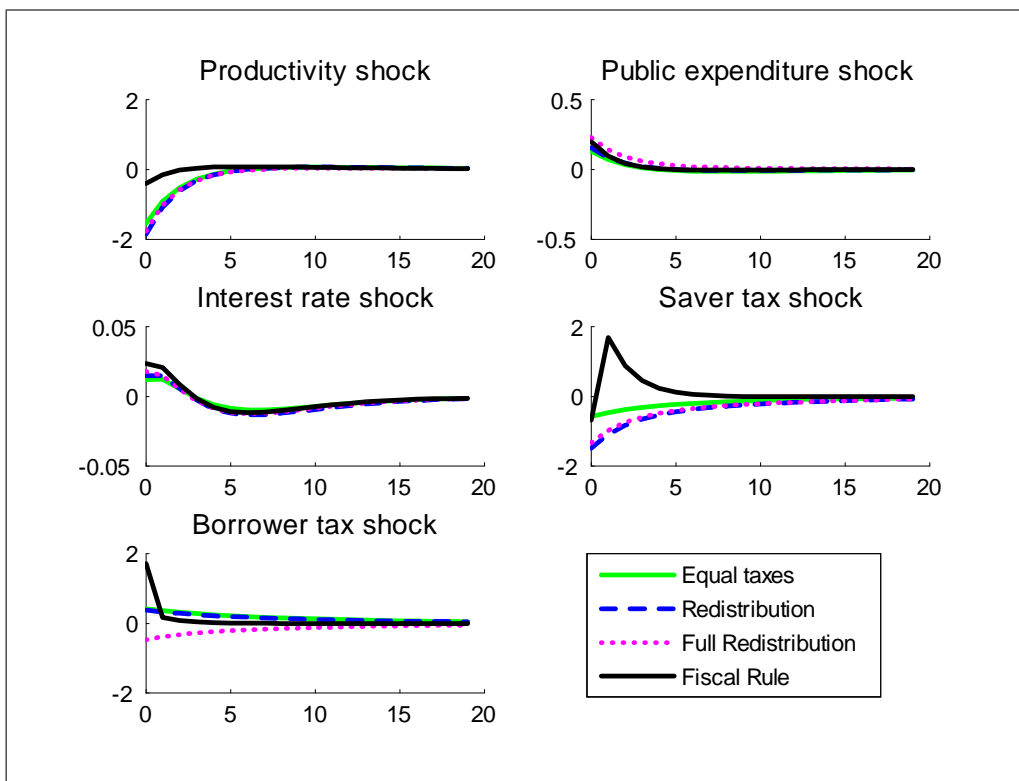


Figure 11: IRFs  $c\_ratio$

profits, and in a fall of public debt due to higher public revenues.

In the end, we want to compare the equilibrium dynamics concerning the ratio between borrower consumption and saver consumption (and their relative wealth) in the four cases shown in detail before:

- no redistribution;
- redistribution;
- full redistribution;
- fiscal rule.

A positive productivity shock obviously increases savers consumption more than borrowers one. Figure (11) shows that the effect, when taxes are constant, is enormously greater than the effect in the presence of the fiscal rule. It is not the same in the event of a public expenditure shock. In any case, a positive public expenditure shock increases borrowers consumption more than savers consumption in the same amount. In the event of a negative interest rate shock the IRFs are slightly different

but in any case the shock increases borrowers consumption more than savers one. On the other side, a negative saver tax shock increases savers consumption more than borrowers one but the effect in the event savers receive a subsidy is enormously greater than all other cases.

Finally, the IRFs in the event of a negative borrower tax shock are very different. In the case borrowers pay taxes and taxes are constant, and in the presence of a fiscal rule, a borrower tax shock increases borrowers consumption more than savers one, but the effect in the presence of the fiscal rule is enormously greater than the effect when taxes are constant. On the other side, when borrowers receive a subsidy a borrower tax shock increases savers consumption more than borrowers one.

## 4 Concluding Remarks

The analysis herein has shown how the interaction between borrowing-constrained behavior by some households and sticky prices make it possible generate an increase in consumption in response to fiscal stimulus programs, in a way consistent with much of the recent evidence. Borrowing-constrained consumers partly insulate aggregate demand from the negative wealth effects generated by the higher levels of (current and future) taxes needed to finance the fiscal expansion, while making it more sensitive to current disposable income. Sticky prices make it possible for real wages to increase. The combined effect of a higher real wage and higher employment raises current labor income and hence stimulates the consumption of borrowing-constrained households.

Our theoretical analysis assumes that government spending is financed by means of distortive labor income tax on both agents or by issuing one-period, risk free, non state contingent nominal bonds, which are held only by the savers. Allowing for staggered nominal wage setting or some form of real wage rigidity constitutes another potentially useful extension of our framework, one that is likely to have a significant effect on the response of real wages and, hence, of labor income and consumption to any fiscal shock (Gali et al., 2007).

This article contributes to a vast literature on the relation between public debt and redistribution through fiscal policy, in a model with heterogeneous agents; see Bilbiie, Monacelli and Perotti (2013) for a survey. The novel element is that our analysis introduces public expenditure variable in the utility function of both agents analyzing a fiscal regime with distortive taxes on labor income. In this economy, a positive technological shock leads to an overall contraction in the event of constant taxes, while it leads to an overall expansion in the presence of the fiscal rule. Hence, a positive technological shock generates paradoxical results: an increase in productivity is contractional. The key extra element is

that these contradicting forces depend on the share of borrowers. A positive public expenditure shock and a negative interest rate shock lead to an overall expansion, in any case. The model suggests not only that a temporary rise in government spending will not crowd out private spending, it will lead to increased spending on the part of liquidity-constrained debtors (Krugman and Eggertsson, 2012). In the analysis of expansionary fiscal policy, public debt increases more in a context of partial redistribution in steady state than in a context of full redistribution, due to the internalization of government budget constraint by the fraction of households which holds public bonds. In addition, a negative saver tax shock has a negative impact on redistribution, but this effect is exacerbated by the presence of partial redistribution in steady state. Finally, a negative borrower tax shock has a negative impact on redistribution when borrowers receive subsidies in steady state, because savers are completely discouraged to save.

In the paper we constructed a model of dynamic economy that, at the aggregate level, is deterministic; and we then hit the economy with an unexpected temporary shock. Although this approach succeeds in keeping the analysis tractable, it skirts around some central issues. The key question is, To what extent can *contingent* debt contracts be written? There are a number of explanations for why it may be impossible to condition debt repayments on idiosyncratic shock. However, it is less clear why the terms of a contract cannot be made sensitive to aggregate events. This is a difficult matter to resolve.

Let us turn to less thorny issues. A weakness of our model is that it provides no analysis of who becomes credit constrained, and when. we merely rely on the assumption that different agents have different discount factors and preferences for leisure (Kiyotaki and Moore, 1997).

We conclude by describing several areas where future research would quite useful. Firstly, it is always the case that more knowledge of the way the macroeconomic works can improve the performance of coordination between fiscal and monetary policy. Particularly critical, however, is a better understanding of the determinants of inequality. Secondly, our analysis of coordination, as in much of the literature, was restricted to closed economy frameworks. Extensions to open economy frameworks are likely to provide new insights on the desirability of alternative kinds of coordination between fiscal and monetary rules, and raise a number of issues of great interest, including: the choice of exchange rate regime, the potential benefits from monetary and fiscal coordination among different countries, the optimal response to shocks originating abroad. Finally, one would want to consider some of the normative implications of our framework. In a model with two types of consumers considered herein,



the monetary and fiscal policy responses to shocks of different nature can be expected to have distributional effects, which should be taken into account in the design of those policies. Exploring the implications of the present model for optimal monetary and fiscal policy design constitutes an additional interesting avenue for future research (Clarida, Galì and Gertler, 1999).

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## A Appendix

We have seen that a positive technological shock generates paradoxical results: an increase in productivity is contractional. The key extra element is that these contradicting forces depend on the share of borrowers.

| Variable                        | Symbol   | No Red. | Red.    | Full Red. |
|---------------------------------|----------|---------|---------|-----------|
| Borrower's consumption          | $C_b$    | 0.7828  | 0.85    | 1.5735    |
| Saver's consumption             | $C_s$    | 1.2273  | 1.0109  | 0.9471    |
| Aggregate consumption           | $C$      | 1.1384  | 0.9787  | 1.0723    |
| Borrower's hours worked         | $N_b$    | 1.0556  | 0.9722  | 1.1254    |
| Saver's hours worked            | $N_s$    | 1.3466  | 1.1678  | 1.2466    |
| Aggregate worked hours          | $N$      | 1.2884  | 1.1287  | 1.2223    |
| Real debt                       | $b$      | 17.0388 | 28.501  | 17.0017   |
| Real market value of shares     | $V$      | 21.2592 | 18.6238 | 20.1685   |
| Real dividend payoffs of shares | $\Gamma$ | 0.2147  | 0.1881  | 0.2037    |

Table 5: Steady State - Lower Borrower share

Assume a lower share of impatient households  $\lambda$  than we have assumed before. We set the parameter equal to 0.2. We keep all our previous assumptions. In Table (5), we can observe that public liabilities must be higher than in Table(3). And, Figure (12) shows that the increase on consumption by the savers more than compensates the reduction on consumption by the borrowers, leading to an overall expansion in output, and confirming standard results.

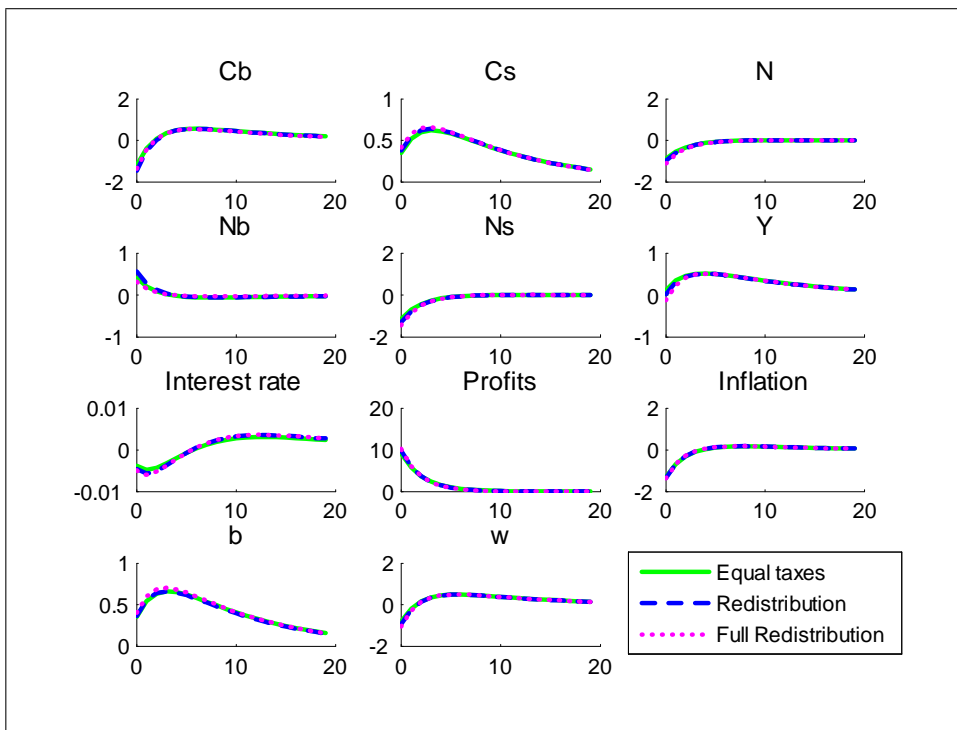


Figure 12: IRFs Productivity shock - Lower Borrower Share

# Money-Financed versus Debt Financed Fiscal Stimulus with Borrowing Constraints

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November 2016

## Abstract

We consider a NK-DSGE model with distortive taxation and heterogeneous agents, modeled using a modified version of the mechanism proposed by Bilbiie, Monacelli and Perotti (2012). Following Galì (2014), we study the effects of a shock to government purchases under two alternative financing regimes: (i) monetary financing; (ii) debt financing. Particularly, we focus on the redistributive effects of the two regimes and we find the following. Both regimes imply a redistributive effect from savers to borrowers, measured in terms of the ratio between the consumption of borrowers and that of savers. The redistribution is much greater in the money-financed fiscal stimulus, where the consumption ratio is more than three times higher than the implied one in the debt-financed fiscal stimulus. Borrowers are better off also in terms of their relative labor supply. Finally, with respect to the representative agent model, the presence of borrowers enhances the impact of the fiscal intervention on aggregate output, when spending is debt financed. Remarkably, with respect to Galì (2014) the same regime implies a reduction of the debt burden instead of an increase.

## 1 Introduction

This paper contributes to the literature of money-financed fiscal stimulus. This literature has concentrated on the comparison between the Classical and the New Keynesian framework (Galì (2014)), particularly focusing on the effects of a shock to government purchases financed entirely through seigniorage or more conventionally through public debt. However, it abstracts from the role played by borrowing constraints. Furthermore, as for most of the papers with fiscal stimulus, it considers lump-sum taxes and do not analyze the interaction between distortive taxes and fiscal stimulus. This paper tries to fill this gap, by considering a New-Keynesian Dynamic Stochastic General Equilibrium model -

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henceforth, NK-DSGE model - characterized by distortive taxation and heterogeneous agents. In this respect, the economies are characterized by the presence of savers and borrowers that interact in the credit market. Borrowers and savers are modeled using a modified version of the mechanism proposed by Bilbiie, Monacelli and Perotti (2013 - BMP henceforth). They indeed differ in their degree of impatience: both agents are intertemporal maximizers - since borrowing and lending take place in equilibrium; and, financial markets are imperfect. Differently from them, we introduce real balances in the utility function of both agents, and the budget constraints are also characterized by real money holdings, together with distortive taxes on labor income of both agents. In particular, as in BMP, we assume that the labor income tax follows a simple feedback rule that reacts to stabilize government debt. In this context, we study the dynamics of the model in response to an exogenous increase in government purchases under two alternative financing regimes: (i) monetary financing and (ii) debt financing, with the central bank's decision bound by an interest rate rule in the latter case. The main results of the paper can be summarized as follows.

A key finding from our analysis lies on the redistributive effect of a money-financed fiscal stimulus. The redistributive effect is measured in terms of the ratio between borrower and saver consumption. We show that, in the money-financed fiscal stimulus the consumption ratio is more than three times higher than that implied by the debt-financing regime. Borrowers are also better off in terms of their relative labor supply. Indeed, the money-financed fiscal stimulus implies that borrowers supply less labor than savers.

The intuition for these findings is the following. As in Galì (2014), a government spending shock, financed through money, implies a consumption crowding in, followed by an high and persistent increase in inflation, which is responsible for a long lasting decline in the real debt ratio owned by savers. The increase in consumption and that of inflation is then followed by an increase in the money demand, which leads to an increase in the nominal interest rate. Furthermore, differently from Galì (2014), the reduction in the real public debt implies lower income tax rates for both types of households, due to the fiscal rule considered. This reduces government revenues and partially counteracts the initial reduction of the debt. Both lower revenues and higher interest rates imply an higher debt burden, which is immediately internalized by savers. They indeed fear that an increase in the debt would be followed by an increase in their labor income tax and thus, *ceteris paribus*, they decide to consume less than borrowers, which do not own any debt and, by construction, cannot internalize the negative effect of the debt.

Another key finding turns out when we compare our model characterized by distortive labor income tax rules with the same model with lump-sum taxes. Remarkably, in the distortive labor income tax model, we find that the debt financed regime implies a reduction in the public debt owned by savers and thus, a reduction in the government debt burden instead of the increase found in Galì (2014). As will be clear in the paper, this counterintuitive result is due to the presence of a distortive labor income tax rule, while households

heterogeneity plays no role in determining the debt reduction. Indeed, with a distortive labor income tax rule, savers internalize the government budget constraint, and recognize that an increase in public expenditure today effectively implies an increase in the debt and thus in their labor income tax, today and in the future. To avoid this, they sell their holdings of government debt to the central bank. In doing this, they reduce their debt beyond the newly issued debt required to finance the fiscal stimulus.

Finally, we find that when the government spending shock is debt-financed and taxes are distortive, the behavior of aggregate variables presents an important departure from that of the representative agent model. In particular, we show that the presence of borrowers enhances the impact of the fiscal intervention on aggregate output, when spending is financed through debt. Furthermore, we find that the reduction in the debt burden is stronger in the two agents model with distortive taxes than in the same model with a representative agent. The extra-reduction in the debt burden, characterizing the model with borrowing constraints, can be explained as follows. Since in our model savers are the single private owners of public debt, *ceteris paribus*, their percapita internalization of the debt, and consequently the amount of debt sold to the central bank, is higher than in a representative agent economy. This implies a lower debt burden, bringing about lower nominal and real interest rates, which leads to an increase in savers consumption higher than in the single agent economy. Because of the higher increase in the aggregate demand, output increases more than in a representative agent model.

In the recent years, many authors concentrated on the issue of consumers heterogeneity due to the limited asset market participation. They show that the presence of liquidity constrained consumers alters the standard results on the dynamics of the NK-DSGE model. For example, Galì et al. (2007) demonstrate that the presence of liquidity constrained consumers can explain consumption crowding in, which follows an increase in government spending. Bilbiie (2008) shows that limited asset market participation can lead to an inverted aggregate demand logic (the IS curve has a positive slope). Di Bartolomeo and Rossi (2007) show that the effectiveness of monetary policy increases as limited asset market participation becomes more important. Galì et al. (2004) study the determinacy properties in a model with limited asset market participation and capital accumulation under different Taylor rules, showing that the presence of liquidity constrained consumers may alter the determinacy properties of a standard NK model. However, none of these papers compare a money-financed fiscal stimulus with a debt-financed fiscal stimulus. Also they do not consider the dynamic effects of fiscal rules in the presence of public debt and borrowing-constrained agents. One exception is represented by BMP which analyze the effects of two types of fiscal policy rules in a model where a fraction of households are borrowing-constrained. However, this paper does not investigate the role played by distortive labor income tax rules, and does not analyze the effects of a government spending financed through seigniorage.

The academic literature has reacted with a renewed interest in monetary and fiscal policy interactions (See Woodford (2011) and Kirsanova et al. (2009)).

Ascari and Rankin (2013) have analyzed the potentially drastic effect of a Taylor Rule on the effectiveness of fiscal policy in a non-Ricardian model with overlapping generations. However, they do not analyze the effects of distortive labor income taxes and they do not consider monetary policies alternative to the standard Taylor-type rules. Galì (2014) has instead analyzed the effects of an alternative and not conventional monetary policy to recover the economy: a fiscal stimulus, in the form of temporary increase in government purchases, financed entirely through money creation. However, Galì (2014) does not consider the redistributive effects of this policy which is instead the main objective of our paper. Finally, this literature considers lump-sum taxes and the distortive effect of the labor income tax rules is not taken into account.

Money creation to finance the government debt has always been associated with the fears of high inflation and no role was expected by fiscal policies. Also because of the beliefs that the lags in implementing fiscal policies were typically too long to be useful to recover from recessions. However, this long period of crisis has opened a wide spectrum of policy tools, above all because interest rates are already close to zero bound. Nowadays, many money creation policies are considered. The extreme end of this spectrum of possible tools is represented by the over money finance of fiscal deficit - "helicopter money", that is a permanent monetization of government debt (Turner 2013). However, a more moderate example of money creation has already been implemented by central banks: as for example quantitative easing operations. The latter has been accompanied by expansive fiscal policies at least in the US. On this last issue, Giavazzi and Tabellini (2014) argue that measures as quantitative easing should always take place together with fiscal easing. Our paper goes in this direction by considering a more structured fiscal policy than the one considered by Galì (2014) and by also investigating the redistributive effects of money financed spending policies. As we show in the paper the redistributive effects of these policies seem to be important and cannot be neglected by policy makers. Also we point out that when labor income taxes follow a feedback rule, as the one considered in our model, a debt financing fiscal rule is not necessarily so detrimental for the fiscal authority since when the central bank is allowed to buy in the secondary market, private households (consumers or even banks in a more sophisticated financial markets) have the incentive to sell their debt to the monetary policy authority.

The remainder of this paper is organized as follows. Section 2 spells out the model economy, while Section 3 analyzes the effect of a money-financed fiscal stimulus in a NK-DSGE model with savers and borrowers. It then compares the results with those obtained under a debt-financing regime, and then it compares our results with the ones implied by a representative agent model. Section 4 summarizes the main findings and concludes.

## 2 The model

The model considered is a closed economy composed by four agents: households, firms, the fiscal authority and the monetary authority.

## 2.1 Households

All households have preferences defined over private consumption,  $C_{\iota,t}$ , real balances,  $M_{\iota,t}/P_t$ , and labor services,  $N_{\iota,t}$ , according to the following utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta_{\iota}^t \left[ (1 - \chi) \ln C_{\iota,t} + \chi \ln \frac{M_{\iota,t}}{P_t} - \psi_{\iota} \frac{N_{\iota,t}^{1+\varphi}}{1+\varphi} \right], \quad (1)$$

where  $\varphi$  is the inverse of the labour supply elasticity. Following BMP, the agents differ in their discount factors  $\beta_{\iota} \in (0, 1)$  and possibly in their preference for leisure  $\psi_{\iota}$ . Specifically, there are two types of agents  $\iota = s, b$ , and  $\beta_s > \beta_b$ .  $E_0$  denotes expectations conditional on the information available at time 0 and  $\chi$  measures the weight of real balances relatively to private consumption. Also,  $\chi$  determines the share of real balances over GDP.

$C_{\iota,t}$  is a CES aggregator of the quantity consumed  $C_t(z)$  of any of the infinitely many varieties  $z \in [0, 1]$  and it is defined as

$$C_{\iota,t} = \left[ \int_0^1 C_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (2)$$

$\epsilon > 1$  is the elasticity of substitution between varieties.

A  $1 - \lambda$  share is represented by households who are patient: we label them savers, discounting the future at  $\beta_s$ . Consistent with the equilibrium outcome the patient agents are savers (and hence will hold the bonds issued by impatient agents), we impose that patient agents also hold all the shares in firms. Hence, they have access to three different assets: money, one-period nominally riskless bonds and shareholdings. In each period  $t \geq 0$  and under all contingencies each saver chooses consumption, hours worked, money demand asset holdings (bonds and shares), subject to:

$$\begin{aligned} P_t C_{s,t} + B_{s,t}^H + P_t A_{s,t} + M_{s,t} + \Omega_{s,t} P_t V_t &\leq (1 + i_{t-1}) B_{s,t-1}^H + (1 + i_{t-1}) P_t A_{s,t} \\ &\quad M_{s,t-1} + \Omega_{s,t-1} P_t (V_{t-1} + \Gamma_t) \\ &\quad + W_t N_{s,t} (1 - \tau_{s,t}), \end{aligned} \quad (3)$$

where  $W_t$  is the nominal wage,  $A_{s,t-1}$  is the real value at beginning of period  $t$  of total private assets held in period  $t$ , a portfolio of one-period bonds issued in  $t-1$  on which the household receives the nominal interest  $i_{t-1}$ .  $V_{t-1}$  is the real market value at time  $t$  of shares in intermediate good firms,  $\Gamma_t$  are real dividend payoffs of these shares,  $\Omega_{s,t}$  are share holdings,  $w_t N_{s,t} (1 - \tau_{s,t})$  is the after-tax real saver labor income,  $B_{s,t}^H$  are the savers' holdings of nominally riskless one-period government bonds (paying an interest  $i_t$ ). The nominal debt  $B_t^H$  pays one unit in nominal terms in period  $t+1$ . To prevent Ponzi games, the following condition is assumed to hold at all dates and under all contingencies



$\lim_{T \rightarrow \infty} E_t \left\{ \prod_{k=0}^T (1 + i_{t+k})^{-1} B_{s,t+T}^H \right\} \geq 0$ . Given prices, policies and transfers  $\{P_t(z), W_t, i_t, G_t, \tau_{s,t}, V_t, \Gamma_t, T_t\}_{t \geq 0}$ , the saver chooses the set of processes  $\{C_{s,t}(z), C_{s,t}, N_{s,t}, M_{s,t}, A_{s,t}, B_{s,t}^H, \Omega_{s,t}\}_{t \geq 0}$ , so as to maximize (1) subject to (2), (3) and no-Ponzi game condition. After defining the aggregate price level as  $P_t = \left[ \int_0^1 P_t(z)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}$ ,

as well as real debt as,  $b_t^H \equiv B_t/P_t$ , optimality is characterized by the first-order conditions:

$$\beta_s E_t \left\{ \frac{C_{s,t}(1+i_t)}{C_{s,t+1}\pi_{t+1}} \right\} = 1, \quad (4)$$

$$\beta_s E_t \left\{ \frac{C_{s,t}}{C_{s,t+1}} \frac{V_{t+1} + \Gamma_{t+1}}{V_t} \right\} = 1, \quad (5)$$

$$\frac{\psi_s N_{s,t} C_{s,t}}{1-\chi} = w_t (1 - \tau_{s,t}), \quad (6)$$

$$\frac{M_{s,t}}{P_t} = \left( \frac{\chi}{1-\chi} \right) C_{s,t} \left( 1 + \frac{1}{i_t} \right). \quad (7)$$

Equation (6) shows that the labor income tax drives a wedge between the marginal rate of substitution between leisure and consumption and the real wage.

In each period  $t \geq 0$  and under all contingencies the rest of households on the  $[0, \lambda]$  interval are impatient (and will borrow in equilibrium, hence we index them by b for borrowers) faces the following budget constraint:

$$P_t C_{b,t} + P_t A_{b,t} + M_{b,t} \leq (1 + i_{t-1}) P_t A_{b,t-1} + M_{b,t-1} + W_t N_t (1 - \tau_{b,t}), \quad (8)$$

as well as the additional borrowing constraint (on borrowing in real terms) at all times  $t$ :

$$-A_{b,t} \leq \bar{D}, \quad (9)$$

Given prices, policies and transfers  $\{P_t(z), W_t, \phi_t, i_t, G_t, \tau_{b,t}, T_t\}_{t \geq 0}$ , the borrower chooses the set of processes  $\{C_{b,t}(z), C_{b,t}, N_{b,t}, M_{b,t}, A_{b,t}\}_{t \geq 0}$ , so as to maximize (1) subject to (2) and (8). Optimality is characterized by the first-order conditions:

$$C_{b,t}^{-1} = \beta_b E_t \left( \frac{1 + i_t}{\pi_{t+1}} C_{b,t+1}^{-1} \right) + \phi_t, \quad (10)$$

$$\frac{\psi_b N_{b,t}^\varphi C_{b,t}}{1-\chi} = w_t (1 - \tau_{b,t}), \quad (11)$$

$$\frac{M_{b,t}}{P_t} = \left( \frac{\chi}{1-\chi} \right) C_{b,t} \left( 1 + \frac{1}{i_t} \right). \quad (12)$$

Equation (11) shows that the labor income tax drives a wedge between the marginal rate of substitution between leisure and consumption and the real wage. While, in (10),  $\phi_t$  takes a positive value whenever the constraint is binding. Indeed, because of BMP assumptions on the relative size of the discount factors, the borrowing constraint will bind in steady state.

## 2.2 Firms

The economy is characterized by an infinite number firms indexed by  $z$  on the unit interval  $[0, 1]$ . Each firm produces a differentiated variety with a constant return to scale technology,

$$Y_t(z) = N_t(z), \quad (13)$$

where  $N_t(z)$  denotes the quantity of labor hired by firm  $z$  in period  $t$ . Following Rotemberg (1982), we assume that firms face quadratic price adjustment costs  $\frac{\theta}{2} \left( \frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2$  expressed in the units of the consumption good defined in (2) and  $\gamma \geq 0$ . Nominal profits read as:

$$E_t \left\{ \sum_{i=0}^{\infty} Q_{t,t+i}(z) \left[ \begin{array}{l} P_{t+i}(z) Y_{t+i}(z) - W_{t+i} N_{t+i}(z) \\ - P_{t+i} \frac{\theta}{2} \left( \frac{P_{t+i}(z)}{P_{t+i-1}(z)} - 1 \right) \end{array} \right] \right\}, \quad (14)$$

where  $Q_{t,t+i}$  is the discount factor in period  $t$  for nominal profits  $i$  periods ahead.

Assuming that firms discount at the same rate as savers implies  $Q_{t,t+i} = \beta_s^i \frac{C_{s,t}}{C_{s,t+i} \pi_{t+i}}$ , each firm faces the following demand function:

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t^d, \quad (15)$$

where  $Y_t^d$  is aggregate demand and it is taken as given by any firm  $z$ . Cost minimization taking the wage as given implied that real marginal cost is  $w_t$ . Firms choose processes  $\{P_t(z), N_t(z), Y_t(z)\}_{t \geq 0}$  so as to maximize (14) subject to (13) and (15), taking as aggregate prices and quantities  $\{P_t, W_t, Y_t^d\}_{t \geq 0}$ . Let the real marginal cost be denoted by

$$mc_t = w_t \quad (16)$$

Then, at a symmetric equilibrium where  $P_t(z) = P_t$  for all  $z \in [0, 1]$ , profit maximization and the definition of the discount factor imply:

$$\pi_t (\pi_t - 1) = \beta E_t \left[ \frac{C_{s,t}}{C_{s,t+1}} \pi_{t+1} (\pi_{t+1} - 1) \right] + \frac{\epsilon N_t}{\gamma} \left( mc_t - \frac{\epsilon - 1}{\epsilon} \right) \quad (17)$$

(17) is the standard Phillips curve according to which current inflation depends positively on future inflation and current marginal cost. The aggregate real profits are:

$$\Gamma_t = (1 - mc_t) Y_t - \frac{\gamma}{2} (\pi_t - 1)^2. \quad (18)$$

### 2.3 Fiscal Authority and Monetary Authority

We start by introducing the budget constraints of the fiscal and the monetary authorities, we then describe formally the fiscal intervention that is the focus of our analysis.

The fiscal authority provides the public good  $G_t(z)$  for any  $z \in [0, 1]$  and aggregating them according to:

$$G_t = \left[ \int_0^1 G_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (19)$$

so that total government expenditures in nominal terms is  $P_t G_t$  and the public demand of any variety is:

$$G_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} G_t. \quad (20)$$

Expenditures are financed by levying a distortive tax or by issuing one period, risk-free, non state contingent nominal bonds. Hence, the fiscal authority's period budget constraint is given by

$$P_t G_t + B_{t-1} (1 + i_{t-1}) = (1 - \lambda) \tau_{s,t} W_t N_{s,t} + \lambda \tau_{b,t} W_t N_{b,t} + P_t S_t^G + B_t, \quad (21)$$

where  $G_t$  and  $\tau_{\iota,t}$  denote government purchases and distortive taxes (in real terms),  $B_t$  is the stock of one-period nominally riskless government debt issued in period  $t$  and yielding a nominal return  $i_t$ , and  $S_t^G$  denotes a real transfer from the central bank to the fiscal authority. Equivalently, and after letting  $b_t = B_t/P_t$  we can write:

$$G_t + b_{t-1} \frac{(1 + i_{t-1})}{\pi_t} = (1 - \lambda) \tau_{s,t} w_t N_{s,t} + \lambda \tau_{b,t} w_t N_{b,t} + S_t^G + b_t. \quad (22)$$

The central bank's budget constraint is given by

$$B_t^M + P_t S_t^G = B_{t-1}^M (1 + i_{t-1}) + \Delta M_t,$$

where  $B_t^M$  denotes the central bank's holdings of government debt at the end of period  $t$ , and  $M_t$  is the quantity of money in circulation<sup>1</sup>. Equivalently, in real terms

<sup>1</sup>The balance sheet of the central bank is given by

$$B_t^M = M_t$$

$$b_t^M + S_t^G = b_{t-1}^M \frac{(1 + i_{t-1})}{\pi_t} + \frac{\Delta M_t}{P_t}, \quad (23)$$

where  $b_t^M \equiv B_t^M/P_t$  and  $\frac{\Delta M_t}{P_t}$  is the amount of seigniorage generated in period  $t$ .

The amount of government debt held by households (expressed in real terms), and denoted by  $b_t^H \equiv B_t^H/P_t$ , is given by

$$b_t^H = b_t - b_t^M \quad (24)$$

In what follows we often refer to  $b_t^H$  as *net* government debt, for short. Combining (22), (23) and (24) one can derive the government's *consolidated* budget constraint

$$G_t + b_{t-1}^H \frac{(1 + i_{t-1})}{\pi_t} = (1 - \lambda) \tau_{s,t} w_t N_{s,t} + \lambda \tau_{b,t} w_t N_{b,t} + b_t^H + \frac{\Delta M_t}{P_t} \quad (25)$$

which may also be interpreted as a difference equation describing the evolution of net government debt over time. Below, following Galì (2014), we consider equilibria near a steady state with zero inflation, no trend growth, and constant government debt  $b^H$ , government purchases  $G$ , and taxes  $\tau^2$ . On the other hand, constancy of real balances requires that  $\Delta M = 0$  in the steady state. It follows from (25) that

$$\tau_b = \frac{G + ib^H - (1 - \lambda) \tau_s w N_s}{\lambda w N_b}, \quad (26)$$

and

$$\tau_s = \frac{G + ib^H - \lambda \tau_b w N_b}{(1 - \lambda) w N_s}. \quad (27)$$

Note that (23) implies

$$S^G = ib^M, \quad (28)$$

i.e. in that steady state the central bank's transfer to the fiscal authority equals the interest revenue generated by its holdings of government debt. Note that in a neighborhood of the zero inflation steady state, the level of seigniorage (expressed as a fraction of steady state output) can be approximated as

$$\begin{aligned} \left( \frac{\Delta M_t}{P_t} \right) \left( \frac{1}{Y} \right) &= \left( \frac{\Delta M_t}{M_{t-1}} \right) \left( \frac{M_{t-1}}{P_{t-1}} \right) \left( \frac{P_{t-1}}{P_t} \right) \left( \frac{1}{Y} \right) \\ &\cong \left( \frac{1}{V} \right) \Delta m_t \end{aligned} \quad (29)$$

---

<sup>2</sup>The constancy of the net government debt in the steady state implicitly assumes a tax rule designed to stabilize that variable about some target  $b^H$

where  $m_t = \log M_t$  and  $V \equiv \frac{PY}{M}$  is the steady state income velocity of money. In words, the level of seigniorage is proportional to money growth.

Let  $\hat{b}_t^H \equiv \frac{(b_t^H - b^H)}{Y}$ ,  $\hat{g}_t = \frac{(G_t - G)}{Y}$  and  $\hat{\tau}_{\iota,t} \equiv \frac{(\tau_{\iota,t} - \tau_{\iota})}{Y}$  denote, respectively, deviations of net government debt, government purchases and taxes from their steady state values, expressed as a fraction of output. We assume that the fiscal authority implements the following feedback rule

$$\hat{\tau}_{\iota,t} = \Phi_B \hat{b}_t^H. \quad (30)$$

This tax rule is general enough to allow taxes on each agent to react to stabilize government debt ( $\Phi_B = 0.09$  is the debt feedback coefficient). The plan prescribes a tax path that depends on public debt. We think that this is an interesting case, even though it is a simple one, because an endogenous tax response to public debt roughly agrees with the intentions declared by most of public debt-targeting governments.

## 2.4 Money-Financed vs. Debt-Financed Fiscal Stimulus

In "normal" times government purchases are assumed to be constant and equal to  $G$ . The objective of the analysis below is to determine the consequences of deviations of government purchases from that "normal" level, i.e.  $\hat{G}_t = G_t - G$ . We refer to those deviations as "fiscal stimulus" (or "fiscal contraction", if negative). Below we assume that such fiscal stimulus, expressed as a fraction of steady state output and denoted by  $\hat{g}_t \equiv \frac{(G_t - G)}{Y}$ , follows the exogenous process

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g, \quad (31)$$

where  $\rho_g \in [0, 1)$  indexes the "persistence" of the fiscal intervention. The baseline policy experiment analyzed below consists of an increase in government purchases financed entirely through seigniorage. Formally,

$$\frac{\Delta M_t}{P_t} = \hat{G}_t, \quad (32)$$

or, equivalently, using (29),

$$\Delta m_t = V \hat{g}_t, \quad (33)$$

i.e., the growth rate of the money supply is proportional to the fiscal stimulus, inheriting the latter's exogeneity. Note that whether the central bank transfer to the fiscal authority takes the form of a direct transfer of seigniorage (with no counterpart) or a permanent increase in the central bank's holdings of government debt has no bearing on the macroeconomic effects of the fiscal stimulus and is only relevant from an accounting viewpoint.

As an alternative to the fiscal-monetary regime described above, and with the purpose of having a comparison benchmark, we also analyze the effects of a debt-financed fiscal stimulus in a (more conventional) environment in which the central bank follows a simple interest rate rule given by

$$\hat{i}_t = \phi_\pi \hat{\pi}_t, \quad (34)$$

where  $\hat{i}_t = \log \frac{i_t}{\bar{i}}$ ,  $\hat{\pi}_t = \log \frac{\pi_t}{\bar{\pi}}$  and  $\phi_\pi > 1$  determines the strength of the central bank's response of inflation deviations from the zero long-term target. Notice that, in contrast with the money-financing regime,  $\Delta m_t$  is no longer determined by  $\hat{g}_t$ . The interest rate rule requires that the central bank injects or withdraws money from circulation by means of open market operations (in exchange for government debt) in order to accommodate whatever money is demanded by households at the targeted interest rate.

As discussed below, an interest rate rule like (34) gives the central bank a tight control over inflation in response to a fiscal stimulus, through its choice of coefficient  $\phi_\pi$ . Yet, that tighter control comes at the price of a smaller impact of the fiscal stimulus on economic activity (i.e. a smaller "fiscal multiplier").

## 2.5 Equilibrium

The equilibrium allocation  $Y_t = C_t + G_t + \frac{\theta}{2}(\pi_t - 1)^2$  is based on additional markets clearing conditions,

$$C_t = \lambda C_{b,t} + (1 - \lambda)C_{s,t}; \quad (35)$$

$$M_t = \lambda M_{b,t} + (1 - \lambda)M_{s,t}; \quad (36)$$

$$N_t = \lambda N_{b,t} + (1 - \lambda)N_{s,t}; \quad (37)$$

respectively, aggregate consumption, money market clearing condition and labor market clearing condition.

## 3 Model Dynamics

### 3.1 Calibration

Before we start showing our results, we briefly describe the baseline calibration of the model's parameters. That calibration is summarized in the top panel of Table (1). We assume the following settings for the household related parameters: discount factors of borrowers and savers are set respectively  $\beta_b = 0.95$  and  $\beta_s = 0.99$ , values which are in line to those of BMP. Analogously, as in BMP, we set the borrowing constraint  $\bar{D} = 0.05$ , borrower and saver preferences for leisure respectively equal to  $\psi_b = 0.6$  and  $\psi_s = 0.3$ . Parameter  $\lambda$ , denoting the share of impatient agents, is set to 0.35, as in BMP, while the debt feedback coefficient in the labor income rule is set to  $\Phi_B = 0.1$ .

The remaining parameters are kept at their baseline values. We assume that government purchases account for a fraction  $\gamma$  of output in the steady state, i.e.  $G/Y = \gamma$ . We calibrate the curvature of labor disutility  $\varphi = 5$  as in Galí (2014) so as we calibrate the government spending share  $\gamma = 0.2$  (steady state

|                 | <i>Description</i>                  | <i>Value</i> |
|-----------------|-------------------------------------|--------------|
| <i>NK Model</i> |                                     |              |
| $\varphi$       | Curvature of labor disutility       | 5            |
| $\beta_b$       | Borrower's discount factor          | 0.95         |
| $\beta_s$       | Saver's discount factor             | 0.99         |
| $\bar{D}$       | Borrowing Constraint                | 0.05         |
| $\psi_b$        | Borrower's preference for leisure   | 0.6          |
| $\psi_s$        | Saver's preference for leisure      | 0.3          |
| $\pi$           | Inflation                           | 1            |
| $\chi$          | Weight of money in utility function | 0.08         |
| $V$             | Velocity (quarterly)                | 4            |
| $\gamma$        | Government spending share           | 1/5          |
| $\rho_g$        | Fiscal stimulus persistence         | 0.9          |
| $\hat{b}^H$     | Steady state debt ratio (quarterly) | 2.4          |
| $\phi_\pi$      | Taylor rule coefficient             | 1.5          |
| $\epsilon$      | Elasticity of substitution (goods)  | 6            |
| $\alpha$        | Index of price rigidities           | 0.65         |
| $\Phi_B$        | Debt feedback coefficient           | 0.1          |

Table 1: Baseline Calibration

share of government purchases in output), the velocity of money  $V = 4$ , the steady state debt ratio  $b^H = 2.4$ , and that of inflation  $\pi = 1$ . The elasticity of substitution among goods is  $\epsilon = 6$  and the Taylor rule coefficient is equal to 1.5. The persistence of the government spending shock is set to  $\rho_g = 0.9$  and the Calvo price stickiness coefficient used to calibrate the Rotemberg adjustment cost is set to  $\alpha = 0.65$ , which is in the range of values used in calibration of aggregate sticky price models with exogenous staggering and also consistent with most of the Bayesian estimation of this parameters.

### 3.2 The Effects of a Money-Financed Fiscal Stimulus

Next we show the impulse response functions (IRFs) of the two-agents model, in face of a government spending shock entirely financed through money creation. Figures 1 and 2 display selected IRFs to a one percent money-financed fiscal stimulus. The dynamics can be described as follows. In particular, Figure 1 shows that the effects of the money-financed fiscal stimulus on inflation are very large (20 percent on impact). They are, however, extremely short-lived, and concentrated in the first quarter. The increase of consumption (almost 20 percent on impact) contrasts with the crowding out of that variable observed in the classical models (Galí, 2014). A large expansion of money demand due to higher prices and consumption brings to an increase in the nominal rate, which explains the increase in the real interest rate. The gap between the two is, of course due to a persistently higher rate of inflation, resulting from the gradual adjustment of prices. Gradualism in the price response, implied by staggered

price setting, thus seems to play a key role in the transmission mechanism of the money-financed fiscal stimulus in the New Keynesian model. Importantly, as in Galì (2014), the upward response of the nominal interest rate suggests that the existence of a zero lower bound on that variable (whether currently binding or not) should not be an impediment to the implementation and success of a fiscal intervention of the kind considered here. Further, an unambiguous "positive" outcome of the intervention considered pertains to the substantial decrease in the debt ratio (more than 50 percentage points), resulting from erosion of the real value of government debt outstanding at the time the stimulus is initiated, due to the high unanticipated inflation.

Figure 2 underlines the redistributive effects of the policy. Notice in particular, that the effects of a money-financed fiscal stimulus on borrower consumption is much greater than the effect on saver consumption. Indeed borrower consumption increases much more than that of savers. Also, borrowers are better off in terms of their relative labor supply, even if the difference is lower with respect to the consumption gap between the two agents. Another important difference concerns the effect of the policy on the agents money demand. Saver money demand declines much more than borrowers money demand. The reason of these responses depends on the fact that, thanks to the tax rules, savers internalize the government budget constraint through their public debt holdings and so recognize that an expansionary monetary and fiscal policy today effectively implies a tax on themselves, today or in the future, also due to the higher interest rates. For this reason savers consume less than borrowers and overall the public debt works as a mechanism to redistribute wealth among agents.

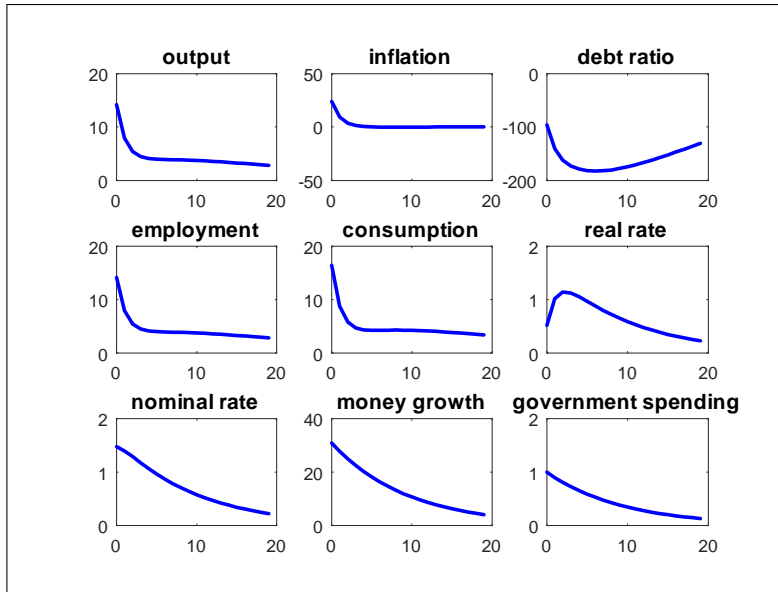


Figure1: Money-Financed Fiscal Stimulus in the baseline model



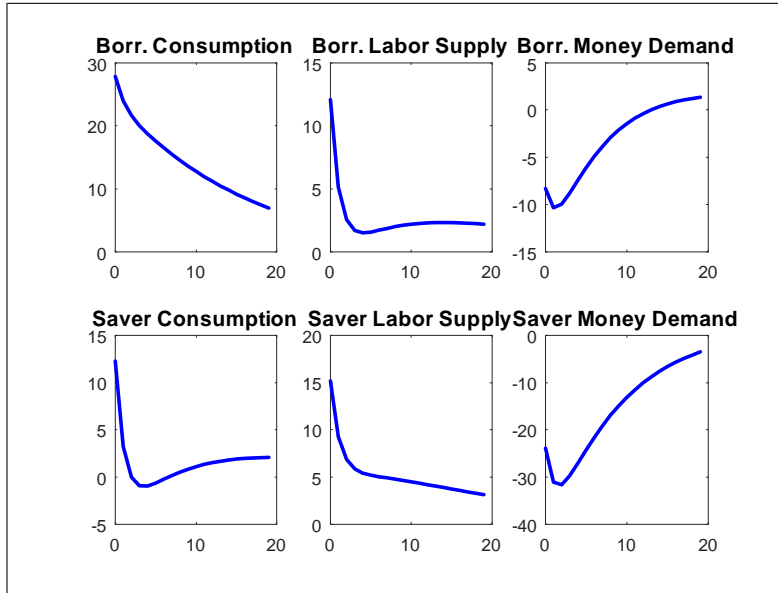


Figure 2: Money-Financed Fiscal Stimulus in the baseline model: disaggregate effects

### 3.3 Money-Financed vs. Debt-Financed Fiscal Stimulus

Consider next the alternative regime of a debt-financed fiscal stimulus, accompanied by a monetary policy described by the simple interest rate Taylor rule (34). A central bank that follows a simple rule like (34) can "control", through an appropriate choice of coefficient  $\phi_\pi$  the extent of the inflationary impact of the fiscal stimulus. In particular, that impact can be made arbitrarily small by having the central bank respond to inflationary pressures sufficiently aggressively, i.e. by choosing a sufficiently large value for  $\phi_\pi$ . Figure 3 displays the dynamic responses of several macro variables to the fiscal stimulus under a debt-financing regime when the inflation coefficient in rule(34) is  $\phi_\pi = 1.5$ . This setting corresponds to the value of the inflation coefficient in Taylor's (1993) celebrated rule, and is meant to capture (in a highly stylized way) an empirically plausible policy response. For the sake of comparability, Figure 3 also displays the dynamic responses obtained under a monetary-financing fiscal stimulus. Because of the monetary policy rule considered, we label the debt-financing regime as *Taylor*.

As shown in Figure 3, the difference in the responses of inflation is unambiguous: as in Galí (2014), even a moderate inflation coefficient of 1.5 is enough to stabilize inflation on impact. The decrease in the debt ratio under a debt financing regime is a consequence of distortive taxation, as shown in Appendix. Hence, it leads to a sale of household holdings of government debt in the short run and, hence, a temporary increase in the size of the corresponding central bank holdings above and beyond the newly issued debt required to finance the fiscal stimulus. The money supply increases on impact ( more than under

money-financing but less persistently), and it leads to a nominal rate decrease and consequently a real rate decrease. Remarkably, notice that differently from Galì (2014) even though spending is financed through debt, the debt owned by savers reduces on impact as stays below zero for several periods. Even though the debt reduction is lower than that obtained under a money financed fiscal stimulus. As shown in Figure 4, which compares our baseline model with distortive tax rule with the same model characterized by lump-sum taxes, this counterintuitive result is mainly due to the presence of the distortive labor income tax rules.<sup>3</sup> Indeed, savers internalize the government budget constraint through their public debt holdings, and recognize that an increase in public expenditure today effectively implies an increase in the debt and thus of their labor income tax, today and in the future. To avoid this, they sell their holdings of government debt to the central bank, reducing their debt holding beyond the newly issued debt required to finance the fiscal stimulus.<sup>4</sup>

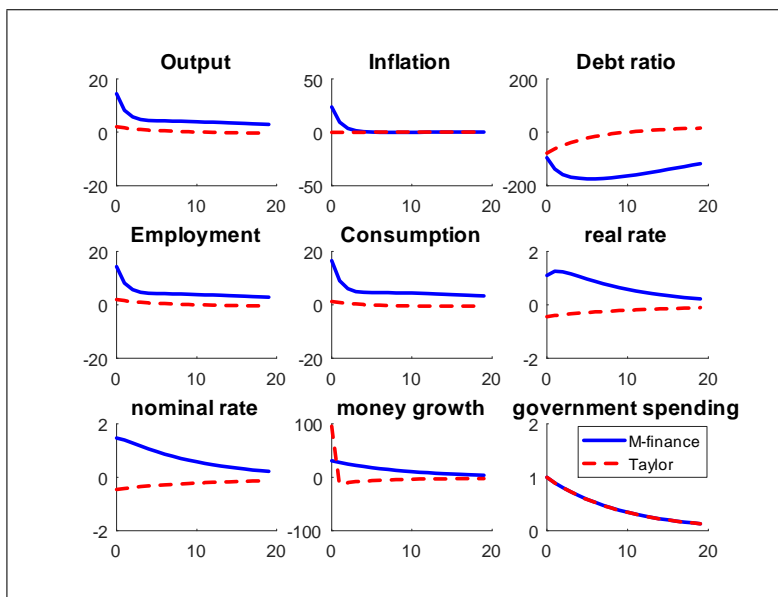
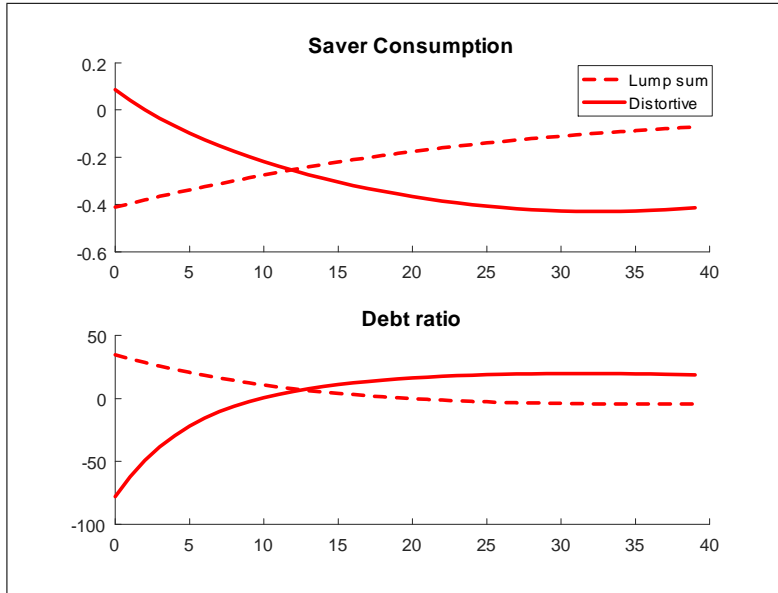


Figure 3: **Fiscal Stimulus in the baseline model**

<sup>3</sup>Figure (B.33) in the appendix shows that the same results hold in the representative agent model when lump-sum taxes are substituted with the labor income tax rule considered in our model, which is the same considered in Galì (2014).

<sup>4</sup>This result does not depend on the persistence of the government spending shock, and indeed it is robust also for  $\rho_g = 0.5$ .



**Figure 4: Debt-financed Fiscal Stimulus in the baseline model: distorsive vs lump-sum taxes**

Finally, notice the impulse responses of the two agents differ more in the case of a money-financed fiscal stimulus than in the case of a debt-financing regime, as Figure 5 makes clear. In particular, notice that both regimes imply a redistributive effect from savers to borrowers, measured in terms of the ratio between the consumption of borrowers and that of savers. The redistribution is much greater in the money-financed fiscal stimulus, where as shown in Figure 6, the consumption ratio is more than three times higher than the implied one in the debt-financed fiscal stimulus. Borrowers are better off also in terms of their relative labor supply and money demand.

The intuition for these findings is the following. As in Galí (2014), a government spending shock, financed through money, implies a consumption crowding in, followed by an high and persistent increase in inflation, which is responsible for a long lasting decline of the debt ratio owned by savers. The increase in consumption and that of inflation is then followed by an increase in the money demand, which leads to an increase in the nominal interest rate. Furthermore, differently from Galí (2014), the reduction in the public debt implies lower income tax rates for both types of households, due to the fiscal rule considered. This reduces Government revenues and partially counteracts the initial reduction of the debt. Finally, *ceteris paribus*, both lower revenues and higher interest rates imply an higher debt burden, which is immediately internalized by savers through their public debt holdings. As a consequence, savers consumption increases much less than that of borrowers, which do not own any debt and, by construction, cannot internalize the negative effect on the government debt burden.

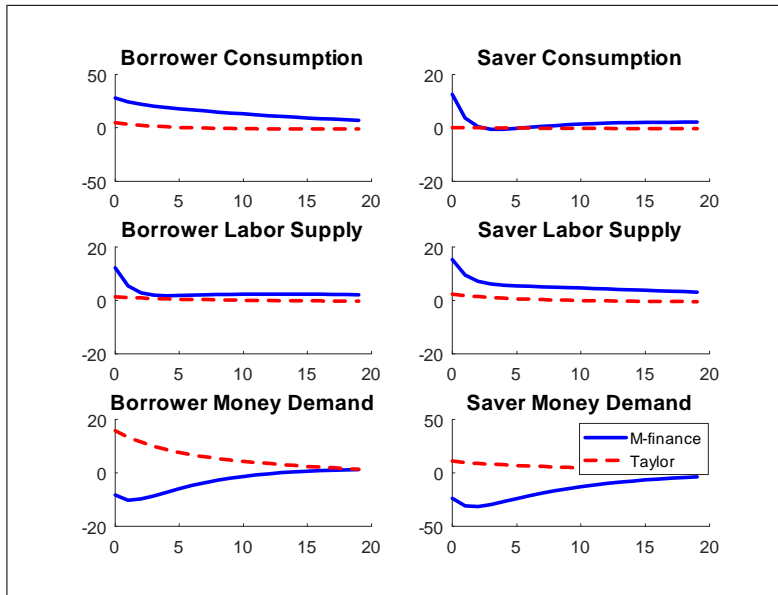


Figure 5: **Fiscal Stimulus in the baseline model: disaggregated effects**

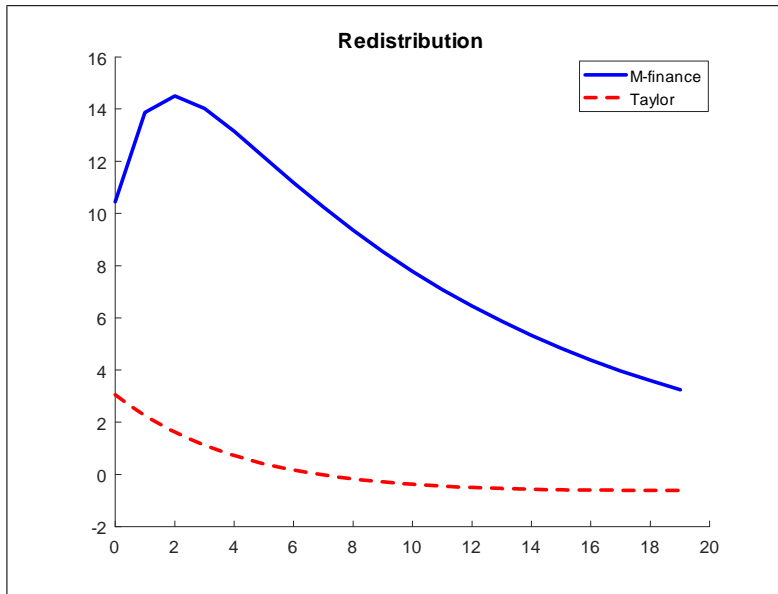


Figure 6: **Consumption Ratio: Money-Financed versus Debt-Financed Stimuls**

Over all, our analysis confirms that the effects of a money-financed fiscal stimulus in a New-Keynesian monetary economy supports a strong case for that intervention, due to its effectiveness in stimulating output and employment, despite its large inflationary consequences. While its impact on inflation is very

limited, a debt-financed fiscal stimulus, accompanied by a simple interest rate rule has the disadvantage of a null impact on activity. Last but not the least, private debt results in a reduction instead of an increase.

### **3.4 Representative *vs.* Heterogeneous Agents**

Some of the key findings of the previous section regarding the effects of a fiscal stimulus under alternative financing schemes depend likely on the assumption of heterogeneous agents. The assumption of heterogeneous agents is likely to be central to the response of real variables to alternative schemes for the financing of the fiscal stimulus (with their implied differences in monetary policy rules). In the present section we compare our baseline model with the same model with a representative agent. Taxes are distortive as in our baseline model. This allows to analyze to what extent our results on the aggregate macro-variables are due to the presence of borrowing constraints.

#### **3.4.1 Money-Financed Fiscal Stimulus**

Next we compare the dynamics of our baseline model with the dynamics of the representative agent model, in response to a spending shock financed through money creation.

Figure 7 displays selected impulse responses to a one percent money-financed fiscal stimulus. As Figure 7 makes clear, the effects of a money-financed fiscal stimulus on output and inflation (as well as on most other macro variables) appear not be affected by the presence of borrowing-constrained households. In contrast with the representative agent economy, in the two agents model the fiscal stimulus requires a slightly higher increase in money growth which explains the corresponding higher increase in the response of nominal interest rate.

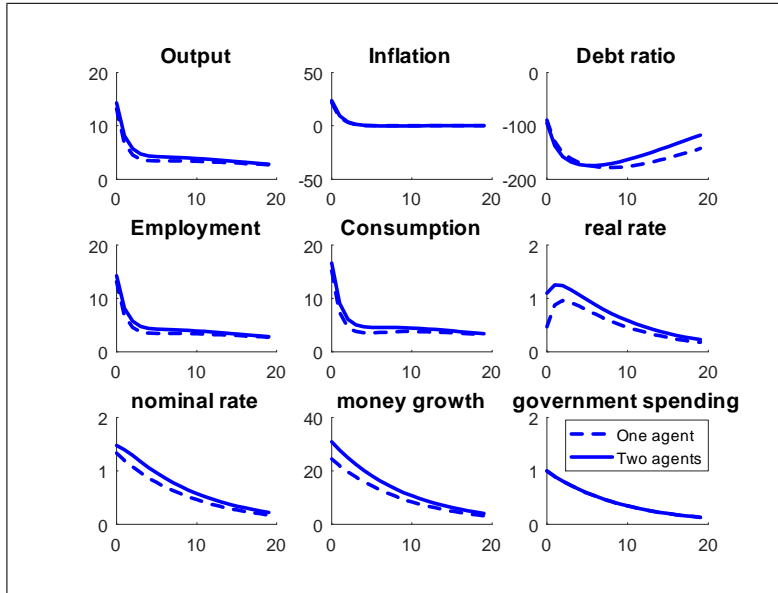


Figure 7: Money-Financed Fiscal Stimulus: two agents vs representative agent model

### 3.4.2 Money-Financed vs. Debt-Financed Fiscal Stimulus

That invariance, resulting in the money-financed fiscal stimulus case, no longer holds in the case of a debt-financed fiscal stimulus, as Figure 8 shows in the resulting IRFs. Notice that, under a Taylor rule, the presence of borrowing-constrained consumers enhances the impact of that fiscal intervention on output due to the increase in overall demand of labor and the increase in overall consumption (due to the crowding in effect of a fiscal stimulus on borrower consumption). Two reasons explain the decrease of public debt held by households: the decrease of overall public debt due to the rise of overall demand of labor and a greater internalization of government budget constraint by the single owners of public debt, the savers. They increase the sale of their public debt holdings with respect to the model with representative agent. The extra-reduction of the debt burden, characterizing the savers-borrowers model, can be explained as follows. When public spending is financed through debt and taxes are distortive, savers internalize the government budget constraint, because of the fear of higher current and future taxes, and immediately sell their own debt to the Central Bank. Since in our model savers are the single private owners of public debt, *ceteris paribus*, their percapita internalization of the debt, and consequently the amount of debt sold to the central bank, is higher than in a representative agent economy. This results in a lower debt burden, accompanied by an increase in savers consumption which is higher than in the single agent economy. Because of the higher increase in the aggregate demand, output increases more than in a representative agent model. This also confirms that our

result on the reduction of the debt ratio does not depend on the introduction of borrowing constraints but simply on the distortive income tax rule. The appendix shows that in a lump-sum tax representative agent economy debt owned by households increases in debt financed fiscal rules, as found in Galì (2014).

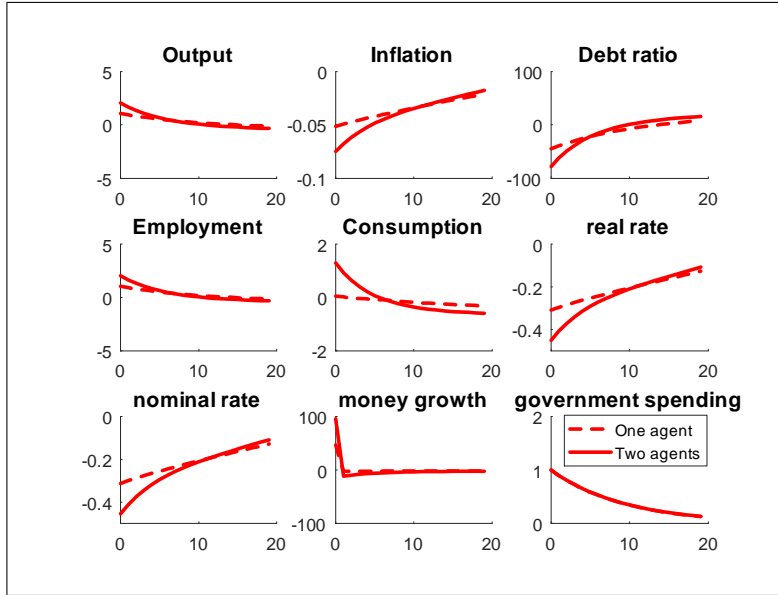


Figure 8: Debt-Financed Fiscal Stimulus: two agents vs. representative agent model

## 4 Conclusions

In the present paper we consider a NK-DSGE model with distortive labor income tax rules and borrowing constrained agents. We analyze the effects of an increase in government purchases financed entirely through seigniorage and compare them with those resulting from a more conventional debt-financed stimulus.

A key finding from our analysis lies on the redistributive effects of a money-financed fiscal stimulus. The redistributive effect of the money-financed fiscal stimulus, measured in terms of the ratio between borrower and saver consumption, is much greater than in the debt-financing regime.

Another key finding turns out when we compare our model with the representative agent model, considered by Galì (2014). While the money-financed fiscal stimulus implies important results in terms of redistribution between the two agents, at aggregate level our model is only slightly different from the same model with representative agent. However, when the government spending shock is debt-financed, the behavior of aggregate variables presents an important departure from the representative agent model. Indeed, in this case, the presence of borrowing-constrained consumers enhances the impact of the fiscal

intervention on aggregate output due to the increase in the aggregate consumption (Galí et al. (2007)). In addition, we observe an higher decrease of the public debt held by households. Furthermore, we show that the reduction in the savers debt, following the debt financed stimulus, is only due to the presence of distortive labor income tax rules.

Future research efforts could embed a deeper welfare analysis, including optimal monetary and fiscal policy, under discretion as well as under commitment.

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## A The Effect of a Money-Financed Fiscal Stimulus: Lump-sum *versus* Distortive Taxes

Some of the key findings of the paper regarding the effects of a fiscal stimulus under alternative financing schemes depend likely on the assumption of distortive taxes. The assumption of distortive taxes is likely to be central to the response of real variables to alternative schemes for the financing of the fiscal stimulus (with their implied differences in monetary policy rules).

In the present section we relax the assumption of distortive taxes on labor income underlying the analysis above. More specifically, we include lump sum taxes into the New Keynesian model with monopolistic competition in goods market and staggered price setting introduced in the paper.

Our objective is to get a sense of the quantitative effects of a money-financed fiscal stimulus on different macro variables, and their differences with those obtained under a more conventional debt-financing scheme, in a model with lump-sum taxes.

Next we describe the key features of the model. We keep all previous assumptions, except for changes we list. Firms sector remains unchanged.

### The Model

#### Households

The household maximizes utility function subject to a sequence of budget constraints

$$P_t C_t + \mathbf{B}_t^H + \mathbf{M}_t = (1 + i_{t-1}) \mathbf{B}_{t-1}^H + M_{t-1} + W_t N_t + P_t \Gamma_t - P_t \tau_t, \quad (38)$$

for  $t = 0, 1, 2, \dots$  where  $\tau_t$  is a lump-sum tax. Hence, the optimality conditions are given by

$$\frac{W_t}{P_t} = C_t N_t^\varphi, \quad (39)$$

$$1 = \beta(1 + i) E_t \left\{ \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\}, \quad (40)$$

$$\frac{M_t}{P_t} = \chi C_t \left( 1 + \frac{1}{i_t} \right). \quad (41)$$

Equation (39) shows that there is no wedge, in the case of lump-sum tax, between the marginal rate of substitution between leisure and consumption and the real wage.

### Fiscal and Monetary Authorities' Budget Constraints

Expenditures are financed by levying lump-sum taxes or by issuing one period, risk-free, non state contingent nominal bonds. Hence, the fiscal authority's period budget constraint is given by

$$P_t G_t + B_{t-1}(1 + i_{t-1}) = P_t(\tau_t + S_t^G) + B_t, \quad (42)$$

where  $\tau_t$  denote lump-sum taxes (in real terms). Equivalently we can write:

$$G_t + b_{t-1} \frac{(1 + i_{t-1})}{\pi_t} = \tau_t + S_t^G + b_t. \quad (43)$$

Combining (43), (23) and (24) one can derive the government's *consolidated* budget constraint with lump-sum taxes

$$G_t + b_{t-1}^H \frac{(1 + i_{t-1})}{\pi_t} = \tau_t + b_t^H + \frac{\Delta M_t}{P_t}. \quad (44)$$

### Money-Financed Fiscal Stimulus

Next we report the predictions of the New Keynesian model with lump-sum taxes regarding the effects of a money-financed fiscal stimulus identical to the one analyzed in the paper in the context of a New Keynesian model with distortive taxes.

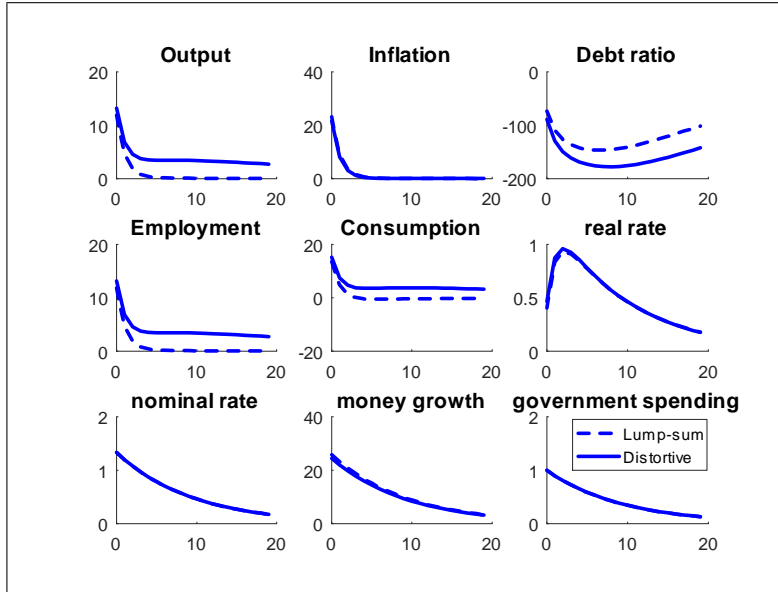


Figure 9: Money Financed Fiscal Stimulus in the representative agent model

Figure 9 displays selected impulse responses to a one percent money-financed fiscal stimulus, under the assumption of  $\rho_g = 0.9$ . The effect on inflation, money growth, nominal and real interest rate is very similar between the lump-sum tax and the distortive tax context. As Figure 9 makes clear, a substantial difference between the New Keynesian model with lump-sum tax and the New Keynesian model with distortive tax lies in the responses of consumption and employment, and consequently in the responses of output, but also in the response of public debt. In contrast with the lump-sum tax economy, in the distortive tax model employment and consumption increase persistently in response to the monetary injection that accompanies the fiscal stimulus. That increase induces a large and persistent expansion of output (almost 15 percent on impact). The debt ratio declines more fast than in the lump-sum economy. This is due to the increase of employment (and hence a larger increase of public revenues).

### Money-Financed vs. Debt-Financed Fiscal Stimulus

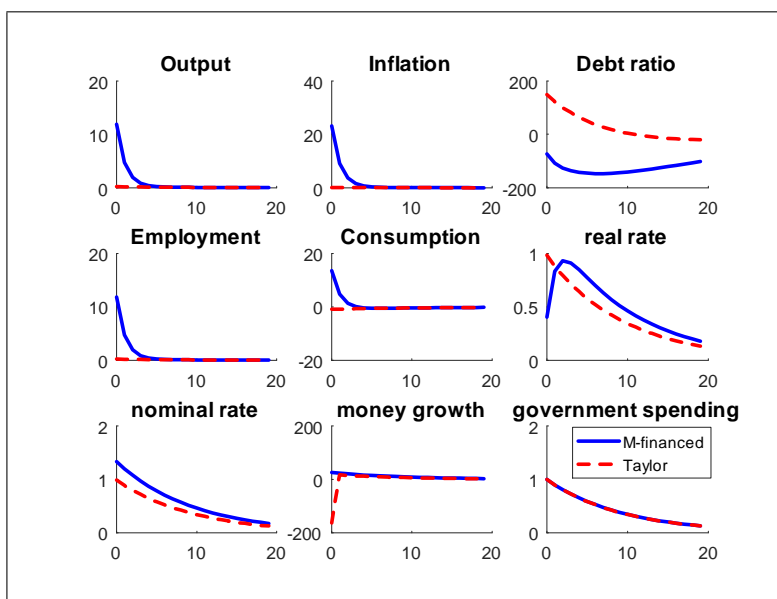


Figure 10: **Fiscal Stimulus in the representative agent with lump-sum taxes**

Figure 10 allows to compare the effects of a money-financed fiscal stimulus to those resulting from a more conventional debt-financed stimulus combined with a monetary policy described by a simple interest rate rule. As in the case of distortive tax, the response of inflation to the fiscal stimulus is much more muted under debt financing, since the central bank has its hands free to counteract the incipient inflation with a more restrictive monetary policy. As shown earlier, a money financed fiscal stimulus is much more effective than a debt-financed one at stimulating economic activity.

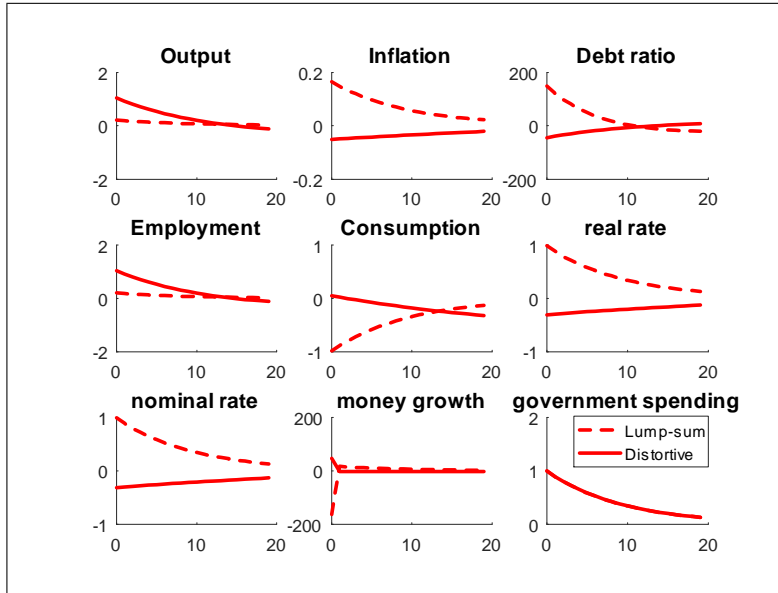


Figure 11: **Debt-financed Fiscal Stimulus in the representative agent model**

However, if we compare the effects of a debt financed fiscal stimulus between the case of lump-sum taxes and the case of distortive taxation, we notice important differences. The decrease of debt ratio, in the case of distortive taxation, is a consequence of the fact that the representative agent internalizes more the government budget constraint through its public debt holdings due to the tax on its labor income, and recognizes that an increase in public expenditure today effectively implies a tax on its labor income, today or in the future. Hence, it leads to a sale of household holdings of government debt in the short run and, hence, a temporary increase in the size of the corresponding central bank holdings above and beyond the newly issued debt required to finance the fiscal stimulus. The money supply increases on impact, and it leads to a nominal rate decrease, and consequently a real rate decrease.

# A Bayesian Estimation of a DSGE Model with Borrowing-Constrained Agents\*

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November 10, 2016

## Abstract

Using a Bayesian likelihood approach, we estimate a dynamic stochastic general equilibrium model for the US economy using five macroeconomic time series. The model incorporates monopolistic competition in goods market, staggered price setting, distortive tax and the presence of borrowing-constrained agent. We use the estimated New-Keynesian model to analyze the main driving forces of output developments in United States. This paper investigates the role of fiscal policy over the aggregate US business cycle. Fiscal policies were substantially muted.

Following Smets and Wouters (2007), this paper estimates an extended version of a New Keynesian model with monopolistic competition in goods market, staggered price setting, distortive tax and heterogeneous agents, largely based on Bilbiie, Monacelli and Perotti (2013) and Galí (2014), on US data covering the period 1966:1 - 2004:4, and using a Bayesian estimation methodology.

To the best of our knowledge, we are the first to estimate a DSGE model with a fraction of Ricardian but borrowing-constrained agents. The stochastic dynamics are driven by five shocks: productivity shock, labor supply shock, mark up shock, preference shock and government expenditure shock. The objective of the paper is to verify whether the model can explain the main features of the US macro data: real GDP, hours worked, consumption, real wages and the short term nominal interest rate. Five shocks allow us to estimate the full model using the five data series mentioned above. Bayesian New-Keynesian models combine a sound, microfounded structure suitable for policy analysis with a good probabilistic description of the observed data and good forecasting performance. We use the estimated New-Keynesian model to analyze the main driving forces of output developments in the United States. We find that the productivity and mark-up shocks explain an important part of the volatility of the main variables, while the most striking result is the irrelevance of fiscal shock.

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\*Preliminary Draft (Not Be Quoted)

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In the next section, we discuss the linearized dynamic, stochastic, general-equilibrium (DSGE) model that is subsequently estimated. Then we present results obtained from the estimation, by showing posterior estimations, posterior distributions and the Bayesian impulse response functions.

## 1 The model

The model allows for a fraction of households to face a suitably defined borrowing limit. We conduct our analysis in a framework featuring heterogeneous agents, who differ in their degree of impatience, and imperfect financial markets. Both agents are intertemporal maximizers - so that borrowing and lending take place in equilibrium (Bilbiie, Monacelli and Perotti (2013)). We label households who are patient savers (and hence will hold the bonds issued by impatient agents); patient agents also hold all the shares in firms. Hence, they have access to three different assets: money, one-period nominally risk less bonds and shareholdings. The rest of households are impatient: they face a borrowing constraint. Households supply labor services to monopolistically competitive producers of goods. At the given wage rate, labor is then supplied on demand to producers of goods. The monetary policy and fiscal policy decisions are respectively allocated to the Central Bank and to the Government. The model features standard nominal frictions, i.e. price stickiness. The technical Appendix provides a full description of the model. In what follows we focus on certain aspects of the model that are crucial to understand our results, i.e. shocks.

All households have preferences defined over private consumption,  $C_{\iota,t}$ , real balances,  $M_t/P_t$ , and labor services,  $N_{\iota,t}$ , according to the following utility function:

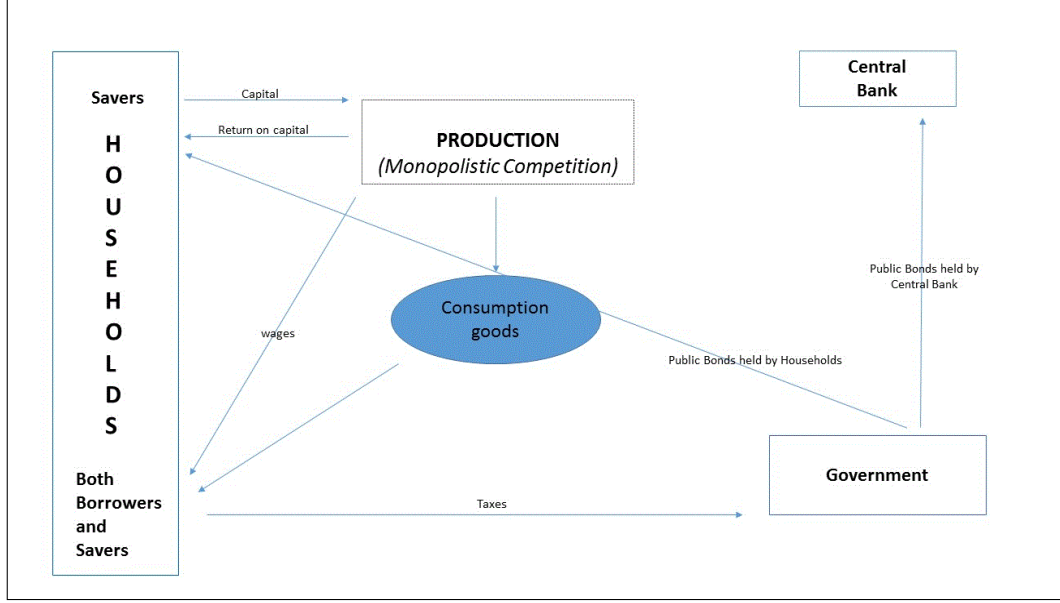
$$U_0 = E_0 \sum_{t=0}^{\infty} \beta_{\iota}^t \left[ (1 - \chi) \ln C_{\iota,t} + \chi \ln \frac{M_{\iota,t}}{P_t} - \psi_{\iota} \frac{N_{\iota,t}^{1+\varphi}}{1+\varphi} \right],$$

where  $\varphi$  is the inverse of labour supply elasticity. We assume that a fraction  $1 - \lambda$  of households (Savers) will hold the bonds issued by impatient agents, own firms and trade government bonds. The remaining  $\lambda$  households (Borrowers) face a borrowing constraint. The agents differ in their discount factors  $\beta_{\iota} \in (0, 1)$  and possibly in their preference for leisure  $\psi_{\iota}$ . Specifically, we assume that there are two types of agents  $\iota = s, b$ , and  $\beta_s > \beta_b$ .  $E_0$  denotes expectations conditional on the information available at time 0 and  $\chi$  measures the weight of real balances relatively to private consumption. Also,  $\chi$  determines the relative weight of real balances in utility.

$C_{\iota,t}$  is a CES aggregator of the quantity consumed  $C_t(z)$  of any of infinitely many varieties  $z \in [0, 1]$  and it is defined as  $C_{\iota,t} = \left[ \int_0^1 C_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}}$ ,  $\epsilon > 1$  is the elasticity of substitution between varieties.

The flow budget constraint of savers is

Figure 1: Model structure



$$P_t C_t + B_{s,t}^H + P_t A_{s,t} + M_{s,t} + \Omega_{s,t} P_t V_t \leq (1 + i_{t-1}) B_{s,t-1}^H + (1 + i_{t-1}) P_t A_{s,t-1} + M_{s,t-1} + \Omega_{s,t-1} P_t (V_t + P_t) + W_t N_{s,t} (1 - \tau_{s,t}),$$

where  $W_t$  is the nominal wage,  $A_{s,t-1}$  is the real value at beginning of period  $t$  of total private assets held in period  $t$ , a portfolio of one-period bonds issued in  $t-1$  on which the household receives the nominal interest rate.  $V_{t-1}$  is the real market value at time  $t$  of shares in intermediate good firms,  $\Gamma_t$  are real dividend payoffs of these shares,  $\Omega_{s,t}$  are share holdings,  $w_t N_{s,t} (1 - \tau_{s,t})$  is the after-tax saver real labor income,  $B_{s,t}^H$  are the savers' holdings of nominally riskless one-period government bonds (paying an interest  $i_t$ ). The nominal debt  $B_t^H$  pays one unit in nominal terms in period  $t+1$ .

Borrowers face the following budget constraint:

$$P_t C_{b,t} + P_t A_{b,t} + M_{b,t} \leq (1 + i_{t-1}) P_t A_{b,t-1} + M_{b,t-1} + W_t N_{b,t} (1 - \tau_{b,t})$$

as well as the additional borrowing constraint (on borrowing in real terms) at all times  $t$ :

$$-A_{b,t} \leq \bar{D}$$

Firms  $z$  are monopolistically competitive and use as input labor services,  $N_t(z)$ . The production technology is:

$$Y_t(z) = x_t N_t(z)$$

where

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \varepsilon_t^x$$

$\hat{x}_t$  evolves as an AR(1) process with an i.i.d Normal innovation term,  $\varepsilon_t^x$  which defines a total factor productivity shock.

## 1.1 Monetary and fiscal policy rules

Let  $\hat{b}_t^H \equiv \frac{(b_t^H - b^H)}{Y}$ ,  $\hat{g}_t \equiv \frac{(G_t - G)}{Y}$  and  $\hat{\tau}_t \equiv \frac{(\tau_t - \tau)}{Y}$  denote, respectively, deviations of net government debt, government purchases and taxes from their steady state values, expressed as a fraction of output. The Central Bank sets the nominal interest according to a log-linear Taylor rule:

$$\frac{i}{1+i} \hat{i}_t = \phi_R \left( \frac{i}{1+i} \right) \hat{i}_{t-1} + (1 - \phi_R) \left[ \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t \right] + \phi_{\Delta y} (\hat{Y}_t - \hat{Y}_{t-1})$$

where the hatted variables define log-deviations from steady state. We assume a set of log-linear fiscal rules such that

$$\hat{\tau}_{b,t} = \Phi_B^b \hat{b}_t^H$$

$$\hat{\tau}_{s,t} = \Phi_B^s \hat{b}_t^H$$

Following Galí (2014), the objective of the analysis below is to determine the consequences of deviations of government purchases from that "normal" level, i.e.  $\hat{G}_t = G_t - G$ . We refer to those deviations as "fiscal stimulus" (or "fiscal contraction", if negative). Below we assume that such fiscal stimulus, expressed as a fraction of steady state output, follows the exogenous process

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g$$

## 1.2 Bayesian estimation

The model presented in the previous section is log-linearized around its steady state and then estimated with Bayesian estimation techniques. According to the Bayesian estimation, we choose prior distributions for the parameters and combine this prior information with the likelihood of the data, following the Bayes rule, to estimate the mode of the posterior distribution. The software used for the estimation is Dynare. The posterior distributions of the parameters are obtained through the Metropolis-Hastings Markov chain Monte Carlo algorithm with a sample of 250,000 draws (dropping the first 20% draws) and 4



parallel chains. According to the literature, we set the scale to be used for the jumping distribution in MH algorithm at 0.2. This option must be tuned obtain, ideally, an acceptance rate of 25%, in order not to reject or accept too often a candidate parameter. Convergence is assessed by means of the convergence statistics proposed by Brooks and Gelman (1998).

We use five key macroeconomic quarterly US time series as observable variables: the log difference of real GDP, real consumption, the real wage, log hours worked and the federal funds rate. We include series from the Smets and Wouters database (2007). The sample period is 1966Q1-2004Q4.

To avoid stochastic singularity, we consider the same number of observables and shocks. Hence, we include five structural shocks: technology shock, labor supply shock, price markup shock, government spending shock and a preferences shock.

### 1.3 Calibration and priors

A subset of parameters is calibrated (Table 1). Borrower and saver discount factors,  $\beta_b$  and  $\beta_s$ , are fixed respectively at 0.95 and 0.99, values which are in line to those of Bilbiie, Monacelli and Perotti (2013). The elasticity of substitution among goods,  $\epsilon$ , is set to 6, in line to that of Galì (2014). The borrowing constraint,  $\bar{D}$ , is fixed at all times  $t$  at 0.05, and Borrower and saver preferences for leisure,  $\psi_b$  and  $\psi_s$ , are set respectively to 0.6 and 0.3, all in line to BMP. The relative weight of real balances in utility,  $\chi$ , is fixed at 0.08. In addition we assume the following fiscal policy settings:  $\gamma = 0.2$  (steady state share of government purchases to output),  $b^H = 2.4$  (corresponding to a 60 percent ratio of debt to *annual* output). The steady state value of productivity is  $x = 1$  and the steady state value of inflation is  $\pi = 1$ .

The remaining parameters are estimated with Bayesian techniques. A description of our prior distribution can be found in Table 2. Overall, our priors are consistent with the literature. When it is not possible to find references in the literature or in the data, our priors are relatively uninformative. In particular, parameters measuring the persistence of the shocks are Beta distributed, with mean 0.5 and standard deviation 0.2 and the standard errors of the innovations are assumed to follow an Inverse-gamma distribution. The parameters governing price setting, interest rate smoothing and the steady state fraction of borrowers are also Beta distributed. The fraction of borrowers  $\lambda$  is assumed to be Beta distributed with mean 0.3 and standard deviation 0.05. The parameters of the Taylor are Normally distributed. Concerning the parameters characterizing the fiscal rules, the prior of feedback parameters is that they are Normally distributed with mean 0.09 and a standard deviation of 0.25<sup>1</sup>. It is important to highlight that we do not impose any restriction on the parameter defining labor utility,  $\varphi$ . We posit that  $\varphi$  is Normally distributed with mean 2 and standard deviation 0.75, thus allowing for the possibility that  $\varphi < 1$ .

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<sup>1</sup>Using a Normal distribution implies that we are not making any assumptions on the signs of these parameters.

| parameter       | Description                             | value |
|-----------------|---|-------|
| $\beta_b$       | Borrower's discount factor              | 0.95  |
| $\beta_s$       | Saver's discount factor                 | 0.99  |
| $\epsilon$      | Elasticity of substitution among goods  | 6     |
| $\bar{D}$       | Borrowing Constraint                    | 0.05  |
| $\psi_b$        | Borrower's preference for leisure       | 0.6   |
| $\psi_s$        | Saver's preference for leisure          | 0.3   |
| $\chi$          | Weight of real balances in utility      | 0.08  |
| $\pi$           | Inflation                               | 1     |
| $\frac{G}{Y}$   | Government purchases-to-output SS ratio | 0.2   |
| $\frac{b^H}{Y}$ | Debt-to-output SS ratio                 | 2.4   |

Table 1: Calibrated parameters

## 2 Results

In this section we show the results of our estimation. We estimate a model with five observable variables and five shocks: a technology shock, a labor supply shock, a mark up shock, a preference shock and a government purchases shock. In Table 2 we present the posterior distributions for the estimation, reporting the posterior mean and Highest Posterior Density (HPD) intervals. Figures 2, 3, 4 show the prior and posterior distributions of the structural parameters and shock processes of the model.

The high persistence of the mark up, technology and preferences processes ( $\rho_\mu = 0.96, \rho_x = 0.88$  and  $\rho_p = 0.92$ , respectively) implies, at long horizons, most of the forecast error variance of the real variables will be explained by those three shocks. In contrast, the persistence of the public expenditure labor supply shock is relatively low ( $\rho_g = 0.08$  and  $\rho_w = 0.21$ , respectively). Standard errors of the shocks show that the technology shock has the highest volatility ( $\varepsilon^x = 1.14$ ).

Turning to the estimates of the main behavioral parameters, it turns out that the degree of price stickiness is estimated to be quite a bit lower than 0.75. The average duration of price contracts is about half year. Turning to the monetary policy reaction function parameters, the mean of the long-run reaction coefficient to inflation is estimated to be relatively high (1.67). There is a considerable degree of interest rate smoothing, as the mean of the coefficient on the lagged interest rate is estimated to be 0.75. Policy does not appear to react very strongly neither to the output gap level nor to changes in the output gap in the short run, but the value for the coefficient on output is  $\phi_y = -0.0625$ , implying a procyclical response by monetary authority to output gap, but not to changes in the output-gap ( $\phi_{\Delta y} = 0.1$ ). The posterior for the fraction of borrowers is about 25% (HDP interval: 23%-26%). For the estimated fiscal policy parameters, we find a value for the debt feedback coefficient related to the fiscal rule on borrower's labor income  $\Phi_B^b = 0.62$ , which is much higher than the prior mean. But, we find that the mean of the debt feedback coefficient related

| parameters        | Prior distribution |       |         | Posterior distribution |                  |         |
|-------------------|--------------------|-------|---------|------------------------|------------------|---------|
|                   | shape              | mean  | std dev | post. mean             | 90% HPD interval |         |
| $\rho_g$          | beta               | 0.500 | 0.2000  | 0.0836                 | 0.0145           | 0.1478  |
| $\rho_\mu$        | beta               | 0.500 | 0.2000  | 0.9546                 | 0.9259           | 0.9841  |
| $\rho_x$          | beta               | 0.500 | 0.2000  | 0.8767                 | 0.8364           | 0.9157  |
| $\rho_p$          | beta               | 0.500 | 0.2000  | 0.9156                 | 0.8678           | 0.9700  |
| $\rho_w$          | beta               | 0.500 | 0.2000  | 0.2081                 | 0.0953           | 0.3196  |
| $\varphi$         | norm               | 2.000 | 0.7500  | 0.2746                 | 0.2492           | 0.2997  |
| $\Phi_B^b$        | norm               | 0.090 | 0.2500  | 0.6201                 | 0.5414           | 0.6964  |
| $\Phi_B^s$        | norm               | 0.090 | 0.2500  | -0.1159                | -0.1319          | -0.0995 |
| $\lambda$         | beta               | 0.300 | 0.0500  | 0.2446                 | 0.2344           | 0.2592  |
| $\phi_\pi$        | norm               | 1.500 | 0.7500  | 1.6663                 | 1.1197           | 2.1792  |
| $\phi_y$          | norm               | 0.100 | 0.0500  | -0.0625                | -0.0970          | -0.0276 |
| $\phi_R$          | beta               | 0.600 | 0.2000  | 0.7459                 | 0.5614           | 0.9560  |
| $\phi_{\Delta y}$ | norm               | 0.063 | 0.0500  | 0.0956                 | 0.0301           | 0.1577  |
| $\alpha$          | beta               | 0.750 | 0.3000  | 0.5423                 | 0.4491           | 0.6414  |
| $\varepsilon^g$   | invg               | 0.100 | 2.0000  | 0.6540                 | 0.5923           | 0.7150  |
| $\varepsilon^\mu$ | invg               | 0.100 | 2.0000  | 0.8291                 | 0.7439           | 0.9101  |
| $\varepsilon^x$   | invg               | 0.100 | 2.0000  | 1.1383                 | 1.0307           | 1.2428  |
| $\varepsilon^p$   | invg               | 0.100 | 2.0000  | 0.3631                 | 0.2922           | 0.4306  |
| $\varepsilon^w$   | invg               | 0.100 | 2.0000  | 0.8138                 | 0.7157           | 0.9058  |
| Log data density  |                    |       |         | -827.5                 |                  |         |

Table 2: Estimated parameters

to the fiscal rule on saver's labor income is negative,  $\Phi_B^s = -0.13$ . The inverse of Frisch elasticity  $\varphi$  has a posterior mean of 0.28. Overall, it appears that the data are quite informative on the behavioral parameters, as indicated by the lower variance of the posterior distribution relative to the prior distribution.

## 2.1 Model dynamics

Table 3 shows an analysis of the variance decomposition which allows us to understand the importance of each shock in determining the volatility of the main macro variables. The mark up shock is the most important source of volatility

|   | $\varepsilon^g$ | $\varepsilon^x$ | $\varepsilon^\mu$ | $\varepsilon^p$ | $\varepsilon^w$ |
|---|-----------------|-----------------|-------------------|-----------------|-----------------|
| Y | 1.71            | 34.30           | 62.24             | 0.01            | 1.74            |
| C | 1.03            | 34.54           | 62.67             | 0.01            | 1.75            |
| i | 0.02            | 10.18           | 87.57             | 2.21            | 0.03            |
| w | 0.83            | 32.10           | 66.34             | 0.00            | 0.73            |
| N | 2.42            | 6.91            | 88.19             | 0.02            | 2.46            |

Table 3: Variance Decomposition (in percent)

Figure 2: Prior and Posterior Distributions

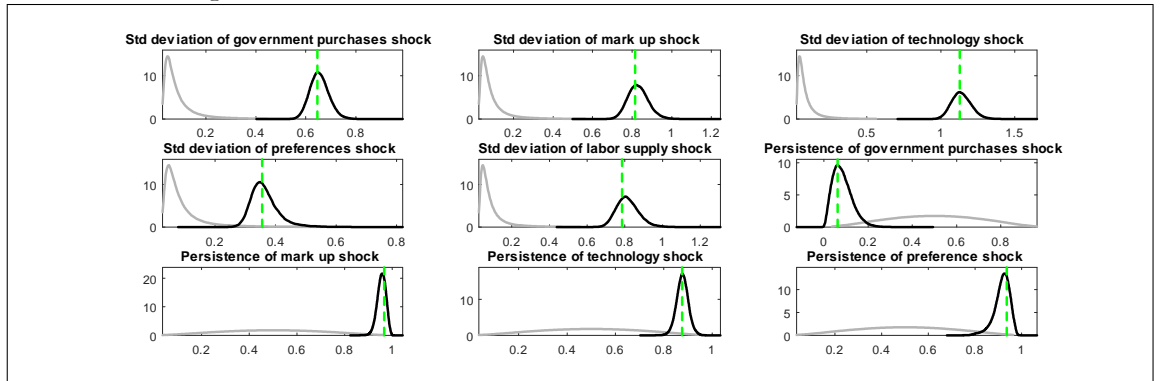


Figure 3: Prior and Posterior Distributions

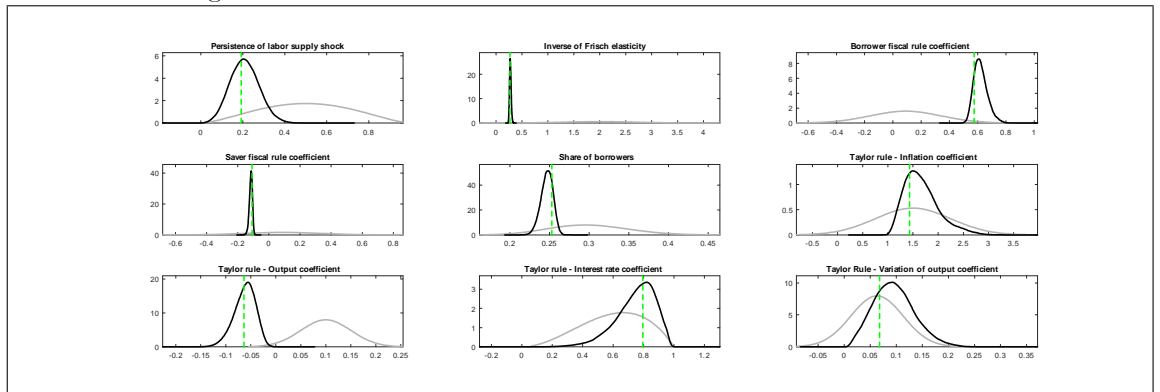
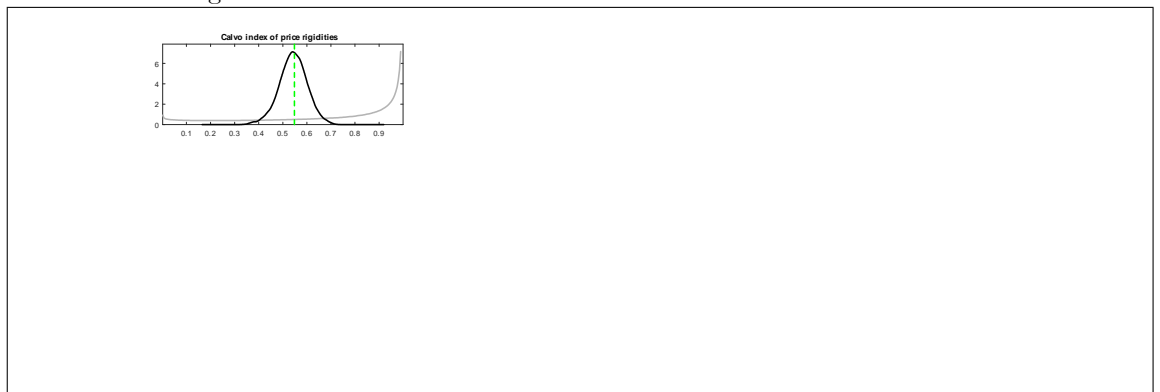


Figure 4: Prior and Posterior Distributions



for all variables, being the first shock in order of importance for employment, accounting for 88% of its volatility, for the nominal interest rate (88%), for real wage (66%), for consumption (63%) and output (62%).

Turning to the technology shock, it is worth noticing that it also plays an important role for the volatility of all variables: consumption (35%), output (34%), real wage (32%), the nominal interest rate (10%) and employment (7%).

Results from the analysis of the variance decomposition show that the mark up shock and the technology shock are important drivers of volatility. Since we are interested in analyzing the dynamics of the estimated model in response to these two shocks, together with the fiscal stimulus, we also show the impulse response functions of the estimated model. 5 shows the Bayesian impulse response functions of the model to a positive government purchases shock. As the public expenditure shock hits the economy, agents - in particular, savers - internalize the government budget constraint through the fiscal rules, distortive taxes on their labor incomes and public debt holdings by savers and so recognize that an increase in public expenditure today effectively implies a tax on themselves, today or in the future. Hence, as the government purchases shock hits the economy, on one side employment increases but on the other side households sell their holdings of government debt in the short run, and it leads to a temporary increase in the size of the corresponding central bank holdings above and beyond the newly issued debt required to finance the fiscal stimulus. The money supply increases and it leads to a nominal rate decrease. The drop of real wage is typical element of the sticky-price environment, while the decrease on impact of overall consumption is mainly explained by the large share of savers we estimate in our model; as they internalize the government budget constraint through their public debt holdings, they decrease their consumption, as we can observe from the overall effect on consumption.

In Figure 6 we present the Bayesian IRFs to a positive shock to mark up. It leads unambiguously to an overall contraction. The sticky price environment explains why real wages decrease on impact, bringing down consumption, and consequently output and employment. The Taylor rule, as monetary authority response, leads to the decrease of nominal interest rate.

Finally, in Figure 7 we present the Bayesian IRFs to a positive technology shock. The IRFs of this kind of shock compensate, almost completely, the IRFs of the mark up shock. In fact, a positive technology shock brings to an overall expansion in all variables (including the rise in nominal interest rate due to the Taylor rule), except for employment, which obviously decrease after an increase of productivity.

## 2.2 Variance and historical growth decompositions

We conclude our exercise with the analysis of the historical contribution of each shock to the dynamics of three variables: output, borrowers consumption and savers consumption.

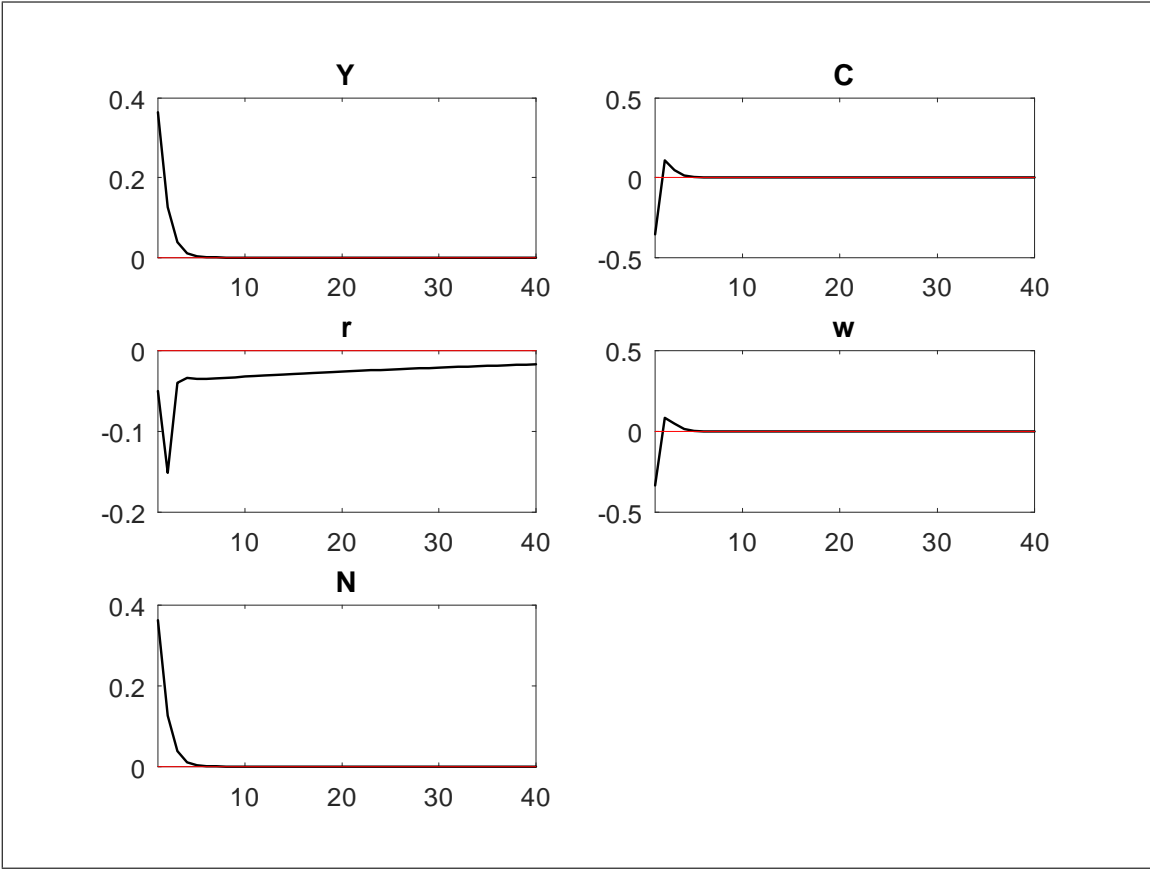


Figure 5: Bayes IRFs of the model to the government purchases shock

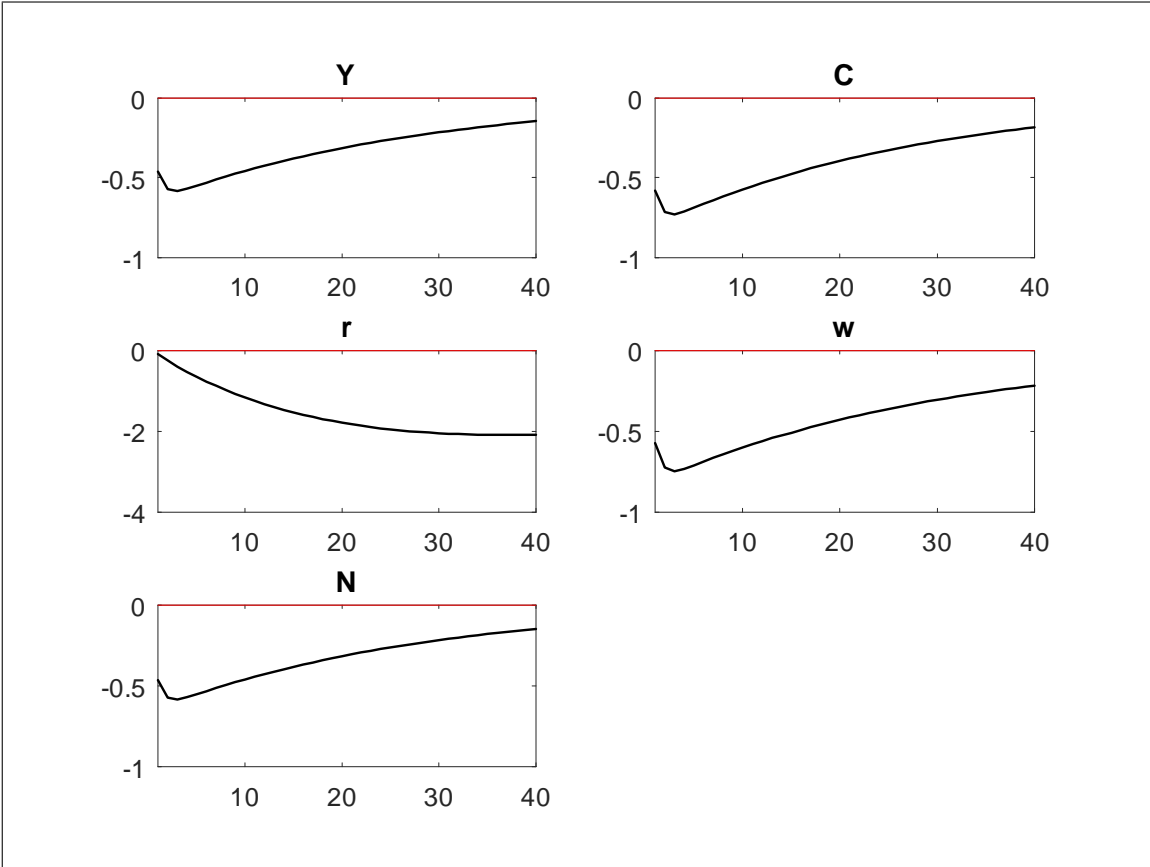


Figure 6: Bayes IRFs of the model to the mark up shock

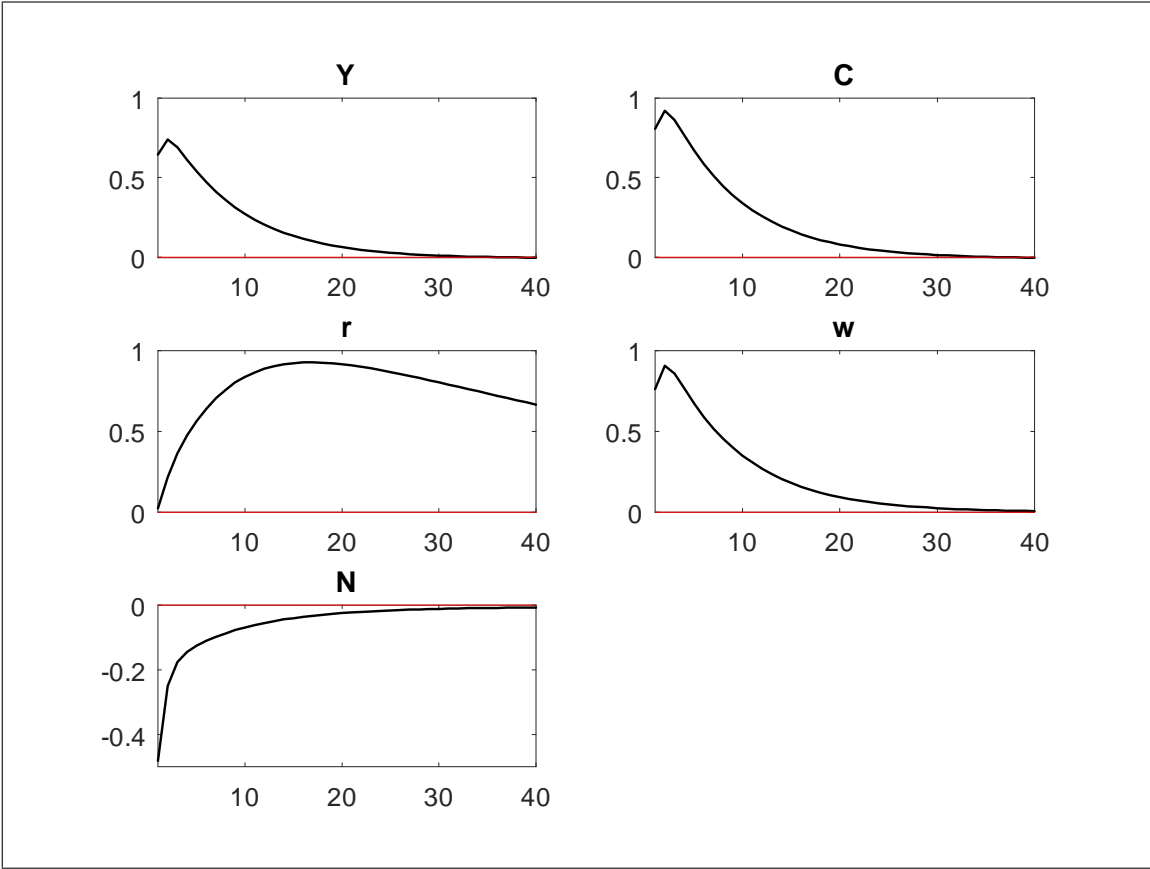


Figure 7: Bayes IRFs of the model to the technology shock



Figure 8: Historical decomposition of GDP growth

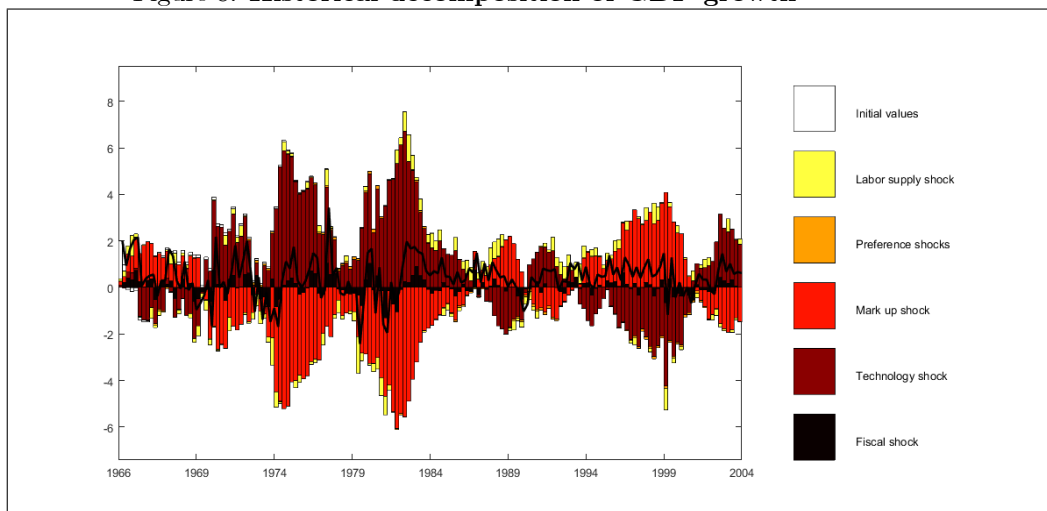


Table 3 reports the variance decomposition for some key variables. The mark up and technology shocks cause about 97% of output and consumption volatility. Shocks to the labor supply account for about 2% of output and consumption volatility. Preference shocks contribute to 2% of interest rate volatility, but have no role otherwise. The most striking result is the irrelevance of fiscal shocks.

The analysis of GDP growth historical decomposition allows to identify the specific contributions of policy and non-policy shocks over the sample period (8). It is interesting to compare the main sources of the various recession over this period. It is quite clear that, at all horizons, productivity and mark-ups are the most important drivers of GDP. While the recessions of the mid 1990s and the beginning of the new millennium are driven mainly by technology shocks, the recessions of 1970s and 1980s are due primarily to positive mark-up shocks (associated with the oil crisis). The other shocks explain only a minor fraction of the total variation in GDP.

Figure 9 depicts the historical contribution of the different types of shocks to borrowers' consumption over the sample period. It is interesting to notice that Figure 8 and Figure 9 are very similar, also if the volatility of borrower consumption is smaller than GDP volatility.

On the contrary, Figure 10 shows a scenario completely different. The volatility of the variable is the smallest with respect to output and borrowers' consumption. And recessions of the mid 1880s and the beginning of the new millennium are driven not only by technology shock but also by a new important driver of savers' consumption: the preference shock.

However, the most striking result is the irrelevance of fiscal shock.

Figure 9: **Historical decomposition of borrowers' consumption**

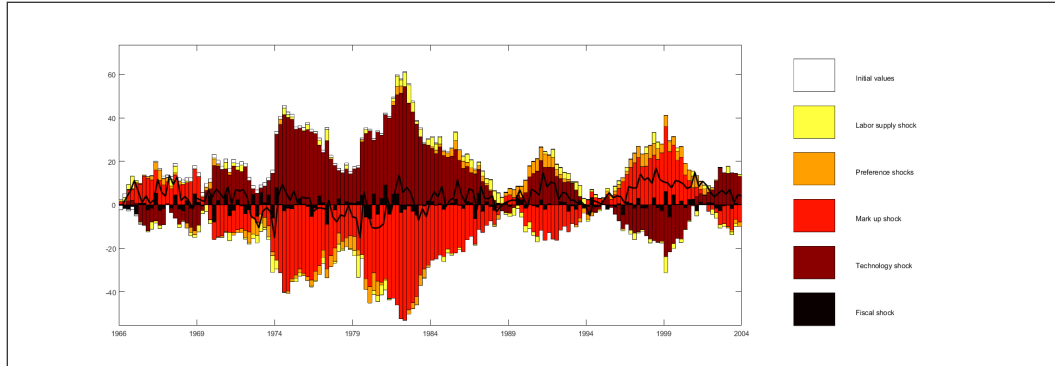
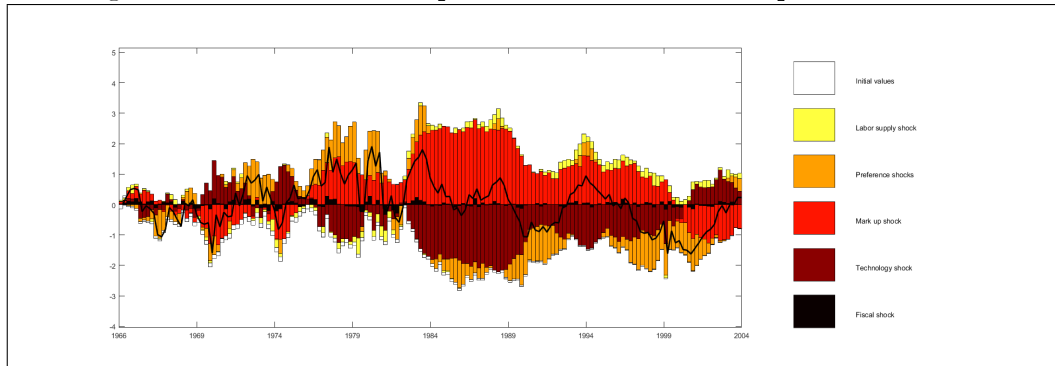


Figure 10: **Historical decomposition of savers' consumption**



### 3 Conclusions

We presented the results of the estimation of the model with heterogeneous agents and distortive tax. We considered a model hit by different shocks. Our results convey a key message: a debt-financed fiscal stimulus played no role in determining the US area business cycle. Another important result is the main role played by productivity and mark-up shocks, as the most important drivers of key macroeconomic variables of the model. The preference shock adds to these two ones in explaining the volatility of savers' consumption.

### 4 References

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### A Log-linear model

1. **Saver Budget constraint:**  $C_s \hat{C}_{s,t} + b^H \hat{b}_t^H + M_s \hat{M}_{s,t} + \Omega V \hat{V}_t = \frac{b^H}{\pi} i \hat{w}_{t-1} + \left[ \left( \frac{1+i}{\pi} b^H \right) \left( \hat{b}_{t-1}^H - \hat{\pi}_t \right) \right] + \frac{i A_s}{\pi} \hat{w}_{t-1} - \frac{1+i}{\pi} A_s \hat{\pi}_t + \left[ \frac{M_s}{\pi} \left( \hat{M}_{s,t-1} - \hat{\pi}_t \right) \right] + \left[ w N_s (1 - \tau_s) \left( \hat{w}_t + \hat{N}_{s,t} \right) \right] - w \tau_s N_s \hat{r}_{s,t} + \Omega V \hat{V}_t + \Omega P \hat{P}_t$
2. **Saver Euler equation for bond:**  $\frac{i}{1+i} \hat{w}_t + \hat{C}_{s,t} - \hat{C}_{s,t+1} - \hat{\pi}_{t+1} + \hat{\varepsilon}_{t+1}^P - \hat{\varepsilon}_t^P = 0$
3. **Saver Euler equation for share holdings:**  $V = \hat{\varepsilon}_{t+1}^P - \hat{\varepsilon}_t^P + \hat{C}_{s,t} - \hat{C}_{s,t+1} + \left[ \left( \frac{1}{V+P} \right) \left( V \hat{V}_{t+1} + P \hat{P}_{t+1} \right) \right]$
4. **Saver labor supply:**  $\varphi \hat{N}_{s,t} + \hat{C}_{s,t} = \hat{w}_t - \frac{\tau_s}{1-\tau_s} \hat{r}_{s,t} + \hat{\varepsilon}_t^\omega$
5. **Saver money demand:**  $\hat{M}_{s,t} = \hat{C}_{s,t} - \frac{\hat{w}_t}{1+i}$
6. **Borrower Euler equation for bond:**  $\frac{\hat{C}_{b,t}}{C_b} = \beta_b \frac{1+i}{\pi C_b} \left( \hat{\pi}_{t+1} + \hat{C}_{b,t+1} \right) - \beta_b \frac{i}{\pi C_b} \hat{w}_t - \hat{\phi}_t + \hat{\varepsilon}_{t+1}^P - \hat{\varepsilon}_t^P$
7. **Borrower labor supply:**  $\varphi \hat{N}_{b,t} + \hat{C}_{b,t} = \hat{w}_t - \frac{\tau_b}{1-\tau_b} \hat{r}_{b,t} + \hat{\varepsilon}_t^\omega$

8. **Borrower money demand:**  $\hat{M}_{b,t} = \hat{C}_{b,t} - \frac{\hat{i}_t}{1+i}$
9. **Production function:**  $\hat{Y}_t = \hat{x}_t + \hat{N}_t$
10. **Labor demand:**  $mc_t = \hat{w}_t - \hat{x}_t + \hat{\mu}_t$
11. **Phillips curve:**  $(2\pi - 1)\pi\hat{\pi}_t = \beta_s [\pi^2 - \pi] [\hat{C}_{s,t} - \hat{C}_{s,t+1}] + \beta_s \pi (2\pi - 1)\hat{\pi}_{t+1} + \frac{Y}{\theta} (\epsilon - 1) mc_t$
12. **Aggregated real profits:**  $\hat{P}_t = \frac{1}{P} [Y\hat{Y}_t - wN(\hat{w}_t + \hat{N}_t)] - \frac{Y(\epsilon-1)\alpha}{(1-\alpha)(1-\alpha\beta)}\pi(\pi-1)\hat{\pi}_t$
13. **Consolidated budget constraint:**  $b^H \hat{b}_t^H + [(\lambda\tau_b w N_b)(\hat{w}_t + \hat{N}_{b,t} + \hat{\tau}_{b,t})] + [(1-\lambda)\tau_s w N_s(\hat{w}_t + \hat{N}_{s,t} + \hat{\tau}_{s,t})] + M\hat{M}_t - \frac{M}{\pi}(\hat{M}_{t-1} - \hat{\pi}_t) = \frac{b^H}{\pi}\hat{i}_{t-1} + \frac{1+i}{\pi}b^H(\hat{b}_{t-1}^H - \hat{\pi}_t) + G\hat{G}_t$
14. **Aggregate consumption:**  $C\hat{C}_t = \lambda C_b \hat{C}_{b,t} + (1-\lambda)C_s \hat{C}_{s,t}$
15. **Aggregate money:**  $M\hat{M}_t = \lambda M_b \hat{M}_{b,t} + (1-\lambda)M_s \hat{M}_{s,t}$
16. **Aggregate labor:**  $N\hat{N}_t = \lambda N_b \hat{N}_{b,t} + (1-\lambda)N_s \hat{N}_{s,t}$
17. **Resource constraint:**  $Y\hat{Y}_t = C\hat{C}_t + G\hat{G}_t + \frac{Y(\epsilon-1)\alpha}{(1-\alpha)(1-\alpha\beta)}\pi(\pi-1)\hat{\pi}_t$
18. **Borrower tax process:**  $\frac{\tau_b}{Y}\hat{\tau}_{b,t} = \Phi_b^B \frac{b^H}{Y}\hat{b}_t^H$
19. **Saver tax process:**  $\frac{\tau_s}{Y}\hat{\tau}_{s,t} = \Phi_s^B \frac{b^H}{Y}\hat{b}_t^H$
20. **Taylor rule:**  $\frac{i}{1+i}\hat{i}_t = \phi_r \frac{i}{1+i}\hat{i}_{t-1} + (1-\phi_r)(\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t) + \phi_{\Delta_y}(\hat{Y}_t - \hat{Y}_{t-1})$

#### SHOCKS

1. **Government expenditure shock:**  $\frac{G}{Y}\hat{G}_t = \rho_g \frac{G}{Y}\hat{G}_{t-1} + \varepsilon_t^g$
2. **Mark up shock:**  $\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_t^\mu$
3. **Technology shock:**  $\hat{x}_t = \rho_x \hat{x}_{t-1} + \varepsilon_t^x$
4. **Preference shock:**  $\hat{\varepsilon}_t^P = \rho_p \hat{\varepsilon}_{t-1}^P + u_t^p$
5. **Labor supply shock:**  $\hat{\varepsilon}_t^\omega = \hat{\varepsilon}_{t-1}^\omega + u_t^\omega$

### **Acknowledgments**

I am grateful to my supervisor Lorenza Rossi. Her ideas and support have been fundamental for the realization of this work. I am indebted to Alice Albonico and Carla La Croce for precious advice and suggestions. I would like to thank Alessandro Missale for his support in many occasions.

I don't think I would have been able to go through 4 years without the psychological and financial support of my family. But I'd like to thank also my grandfather Dario who was always proud of my studies. I am also grateful to my friends Camilla, Federica, Irene and Marianna, but also to my colleagues Clara Capelli, Daniele Biancardi and Kinzica Soldano who shared this long journey with me. Finally, I would like to thank Alessio to have been present everyday, during my long path of studies, since the last year of High School.