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Department of Economics, Management and Quantitative Methods

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Essays on the Interactions between Credit Default Swap and Bond Markets

Ph.D. in Economics
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Preface

This Ph.D. thesis consists of three research papers which are the result of my studies at the Lombardy Advanced School of Economic Research, University of Milan. Part of this work was accomplished during my visiting period at the Market Operations Analysis division at the European Central Bank.

The three papers, each one contained in a separate chapter, are linked by one common subject: the Credit Default Swaps (CDSs), and the consequences that their trading has, in broad terms, on the financial system.

This preface introduces the content of the three chapters and briefly explains how the research questions have been addressed.

A CDS is a derivative on the credit risk of a reference entity which, similarly to an insurance contract, enables the transfer of risk from one subject to another. As such, the main benefit is that a CDS allows to hedge credit risk exposure. Simultaneously, it also opens the possibility to speculate on the credit risk of financial, corporate or sovereign entities. Actually, CDSs are not regulated as insurance instruments. The possibility of buying uncovered CDSs is one of the reasons which contributed to the exponential growth of the market in the first half of the last decade. As a matter of fact, just before the onset of the global financial crisis in 2007, the CDS market reached the peak of approximately USD 58 trillion. This contributed in raising concerns on the potential destabilising role of such derivatives; additionally, speculation in CDSs has been blamed to be one of the root causes of the global financial crisis by both policy makers and popular press. A heated debate on welfare implications of trading in CDSs has accordingly risen in academia.

The final objective of this work is to contribute to this debate, and to shed light on the potential negative consequences that speculation in CDSs may have on financial stability.

The first chapter of this thesis provides an introduction to the topic and surveys some segments of the rapidly growing literature on CDSs. Specifically, after discussing the features of CDSs and the relative welfare implications, it reviews: (i) some theoretical studies about CDS pricing, (ii) empirical literature exploring the determinants of CDS spreads, and (iii) papers which study the relationship between CDSs and underlying bonds. The aim of the first chapter is twofold: to highlight gaps in the existing literature, and to identify room for further research. In this context, we identify three main existing gaps which underpin the studies presented in Chapters 2 and 3.

The second chapter analyses the mechanism of price discovery between banks' CDSs and underlying bonds, also to assess the impact that the leading role of the CDS market has on the determinants of bank's credit risk. As highlighted in Chapter 1, existing literature on this topic concentrates mainly on CDSs written on corporate or sovereign entities, resulting in a lack of analysis on the financial sector. The study in Chapter 2 aims to close this gap by focusing on CDS-bond price discovery process for 32 European banks.

In theory, the CDS-bond price discovery should first happen on the bond market, and arbitrage relationship should keep bond and CDS spreads at par. However, as in practice the bond-CDS basis is not equal to zero, this mechanism may be reversed, resulting in a leading role of the CDS market in price discovery process. This aspect assumes critical importance as speculation on the derivative market may affect the refinancing cost of a targeted entity.

The first result of Chapter 2 shows that, for a sub-sample of banks, the CDS market leads the price discovery process. In order to further explore the implications of such findings, a second analysis empirically decomposes banks' CDS spreads variation according to various credit risk drivers. It is shown that, for those banks for which CDSs lead bonds, there exists a significantly stronger impact of the home country's sovereign risk on banks' credit default swaps. One innovative aspect of this second analysis is that we make use of confidential data obtained during my visiting at the European Central Bank.

Another gap in the existing literature identified in Chapter 1 is the lack of studies which evaluate the role of CDSs in the context of portfolio optimization. The last chapter aims to bridge this gap.

Specifically, the study in Chapter 3, which is jointly written with Francesco Menoncin and forthcoming on the Journal of Financial Services Research, derives a closed-form solution to the optimal investment problem for an agent maximising the expected utility of his or her final wealth. It is assumed that the agent can invest in a frictionless and complete financial market where a riskless asset, a defaultable bond, and a CDS written on the bond are listed. The model is calibrated to market data of six European countries. Thus, the investment strategy can be considered that of a financial institution which can sell or buy credit risk protection on a sovereign entity.

The results of the numerical analysis show that the investor always speculates on the credit risk of the reference entity by optimally short selling (or issuing) CDSs in a quantity which is larger, in absolute value, than that of bonds optimally purchased. The magnitude of this speculative strategy is positively linked to the risk of the underlying sovereign entity.

The three chapters, although not being conclusive on the potential destabilising role of CDSs, highlight existing weaknesses that may pose threats to the stability of financial markets. Further research is needed to assess the deep implications of credit default swaps trading on financial markets.

Giuseppe Ambrosini
Milan, November 2016

Chapter 1

Credit default swaps: a literature survey

GIUSEPPE AMBROSINI *

Abstract

Since the issuance of the first credit default swap (CDS) contract in 1994, trading volume in CDS has been growing exponentially, transforming a niche market into a multi-trillion-dollar market. The subprime crisis in the US and the sovereign crisis in the EU contributed to raise concerns on the potential destabilising role of such an instrument. Thus, also the academic literature started to hotly debate the benefits and costs that trading in CDSs has on both the financial market and the real economy. The aim of this study is to review some segments of the literature on CDSs, and to identify existing gaps and room for further research. After discussing the features of CDSs and the relative welfare implications, we survey theoretical studies which model CDS pricing, empirical literature exploring the determinants of CDS spreads, and papers which study the relationship between CDSs and their underlying assets.

Keywords Credit Default Swap

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1.1 Introduction

A credit default swap (CDS) is a derivative contract on credit risk which, similarly to insurance, allows the transfer of the default risk of a reference entity from one subject (the so-called "protection buyer") to another (the so-called "protection seller"). The first CDS contract was developed in late 1994 by J.P. Morgan and since then the market experienced an enormous growth, reaching the peak of approximately USD 58 trillion in 2007.

The sub-prime crisis in the US and the sovereign crisis in the EU have focused the world's attention on CDSs. The exponential growth of the market, together with its opacity and its low level of regulation, contributed in raising concerns about the destabilising effects of CDS trading on the economy. It has been often argued that speculation in credit risk has exacerbated the effects of the crisis. As a result, in recent years, a number of critiques and calls for a more effective regulation of the CDS market have been made from both popular press and policy makers.

Similarly to any other type of financial derivatives, credit default swaps have both social benefits and social costs for the overall economy. On the one hand, they allow investors to take short positions on credit risk, improving diversification, and facilitating the hedging of credit exposures. On the other hand, it is often argued that they can give rise to market manipulations and are potentially destabilizing because of the high leverage they pose.

Also in the academic literature credit default swaps became a very hot and controversial topic. Research in CDS was initially focused in modelling the price of such instruments and it has rapidly expanded into a wide research field in financial economics. As pointed out by Augustin et al. (2016), at the end of 2015, over more than one thousand working papers posted on the Social Science Research Network are directly related to the economic role of CDSs, or involve CDSs as a research tool. This large number reflects the different findings and often asymmetrical conclusions of existing studies. On the one hand it is argued that credit default swap trading implies more efficient risk sharing and better pricing of credit risk (Duffee and Zhou, 2001; Ashcraft and Santos, 2009; Thompson, 2010), more efficient risk management and capital allocation (Stulz, 2010; Norden et al., 2014), lower borrowing costs and better liquidity in the market (Massa and Zhang, 2012; Salomao, 2015). While on the other hand, it has been shown that trading in CDSs reduce monitoring incentives for banks (Parlour and Winton, 2013; Arping, 2014), pose moral hazard and create adverse selection (Duffee and Zhou, 2001; Bolton and Oehmke, 2011), increase counterparty, systemic and contagion risks (Allen and Carletti, 2006; Delatte et al., 2012; Stephens and Thompson, 2014; Chiarella et al., 2015; Fostel et al., 2015).

This study assesses the state-of-the-art of existing literature in credit default swaps. We aim to highlight some contradictions and weaknesses arising from existing studies, to identify existing gaps and room for potential improvements.

The remainder of this chapter is organised as follows. Section 1.2 reviews the definition of credit default swap, the features of the market and the implication on welfare. Section 1.3 surveys the theoretical literature which models CDS pricing. Section 1.4 analyses the literature which empirically studies the determinants of CDS spreads. Section 1.5 focuses on the studies which explore the relationship between CDS and underlying bond. Finally, Section 1.6 concludes.

1.2 Overview of credit default swaps

1.2.1 Definition and features

A credit default swap is a bilateral derivative contract on the credit risk of a reference entity. Functionally, a CDS is equivalent to an insurance contract: an investor (i.e. the protection buyer) can transfer the credit risk associated with the reference entity (e.g. a corporation, a bank, or a Government) to another investor (i.e. the protection seller) by paying a periodic premium, called CDS spread. In the case the reference entity fails to meet its obligations before the maturity of the CDS, the protection seller is then obliged to pay a given notional amount to the buyer of protection. More specifically, a CDS agreement usually includes information about a specific class of the reference entity's capital structure, references to a particular amount of insured debt, and defines the features of the periodic payments.

The failure of the reference entity to pay its obligations is one of the credit events that trigger the payments of the notional amount; other events are bankruptcy, obligation default, acceleration, repudiation and restructuring. Generally, the occurrence of a credit event must be documented by public notice. The contract may specify the form of settlement, which can be either cash or physical. In the first case, the payment following the credit event has to be equal to the difference between the notional amount insured and the recovery rate upon default (i.e. the loss given default, LGD). Instead, when the contract specifies that the settlement of insurance is to be made in physical form, the payment to the protection buyer equals the face value of the underlying bond; in exchange, the claimant must transfer the reference obligation specified in the contractual agreement to the protection seller.

Credit default swaps, or more generally credit derivatives, emerged in response to two long-standing problems in banking: diversification and hedging of credit risk. Before the introduction of credit derivatives, hedging credit risk was seldom feasible, as shorting bonds requires the seller to borrow the asset, and diversification was practically difficult to achieve, as the statistical properties of credit risk requires, for achieving an efficiently diversified loan portfolio, a number of entities significantly larger than an equity or bond portfolio.

CDSs provide solutions to both problems: (i) they allow investors to take a short position on credit risk, without having to seek consent of the reference entity, and (ii) they can reduce, or set to zero, lenders' exposure to certain entities, thus improving their credit portfolio diversification. As a consequence, CDSs facilitate risk sharing and allow market participants to price credit risk more accurately, by both increasing liquidity in the economy and reducing frictions in the credit market (Ashcraft and Santos, 2009).

Beside these benefits, like any derivative instrument, credit default swaps carry also some costs for the economy. One main feature of CDSs is that they are traded over-the-counter (OTC) and, consequently, their market is opaque and may allow for market distortions and manipulations. Furthermore, CDSs are not regulated as insurance contracts and, consequently, investors can take short positions in credit default swaps, without owning the underlying bonds the derivative is written on. As discussed in the next subsections, these features of CDSs raised a number of critiques, calling for a more effective regulation on the market from both academic literature and press.

1.2.2 Market structure and evolution

CDSs are a relatively recent financial innovation. As described in Tett (2009), the first CDS was issued in late 1994 by J.P. Morgan. The author describes how, in June 1994, Exxon needed a USD 4.8 billion loan to cover potential damages resulting from the 1989 Exxon Valdez oil spill. J.P. Morgan was reluctant in turning down the oil company; however, a USD 4.8 billion loan to Exxon would have tied up significant reserves of the bank to provide for the risk of the loans. To overcome this issue, J.P. Morgan developed a way to transfer the credit risk associated with the loan to a third party: they granted the credit line to Exxon and sold to the European Bank of Reconstruction and Development (EBRD) the credit risk associated with the loan. EBRD took on the risk that Exxon might go into default and, in exchange, J.P. Morgan paid the EBRD a fee on a yearly basis.

The International Swaps and Derivatives Association (ISDA) has played a significant role in the development of the CDS market since its inception, by providing guidance on the legal and institutional details of CDS contracts and by deliberating over issues involving credit events, CDS auctions and other related matters. In 2002, ISDA updated its Master Agreement,[1] which provides OTC counterparties with standardized contracts, and includes certain legal and credit provisions as a basis for negotiating credit derivatives. In 2003, the Association published the Credit Derivatives Definitions, which was revised and updated in 2014,[2] and provides further guidance for credit derivative agreements.

Since the first CDS contract, and up to the onset of the financial crisis, the market has been experiencing an exponential growth. The Bank of International Settlements (BIS) undertakes semi-annual voluntary surveys on CDSs, covering the major dealers within the G10 countries and Switzerland. The BIS periodically publishes the survey results, including the notional amount outstanding and gross market values for traded CDSs. As can be seen from Figure 1.1, from the end of 2005 to the onset of the financial crisis in 2007, the total gross notional of CDSs outstanding experienced a three-digit growth rates, reaching a peak of approximately USD 58 trillion.

The size of the CDS market dropped considerably after the 2007-08 crisis, in particular after the Lehman Brothers' default. At the end of June 2015, the gross notional

[1] See http://www.isda.org/publications/isdamasteragrmnt.aspx
[2] See http://www2.isda.org/asset-classes/credit-derivatives/2014-isda-credit-derivatives-definitions/

amount was close to USD 15 trillion. Major determinants of the drop of CDS market size have been regulators' concerns about central clearing and counterparty risk, which led to significant portfolio compressions.

Gross notional amounts outstanding provide a measure of market size. However, such amounts are not those truly at risk. A more accurate measure of the scale of credit risk transfer in the market can be obtained by the gross market value.[3] As depicted in Figure 1.1, the gross market value reached the peak of USD 5.6 trillion at the end of 2008 and then declined, like the notional amount, to the value of USD 453 billion in June 2015. The overall transfer of credit risk, despite reflecting a still sizeable market, is less astonishing than the overall size of the CDS market in gross notional amount.

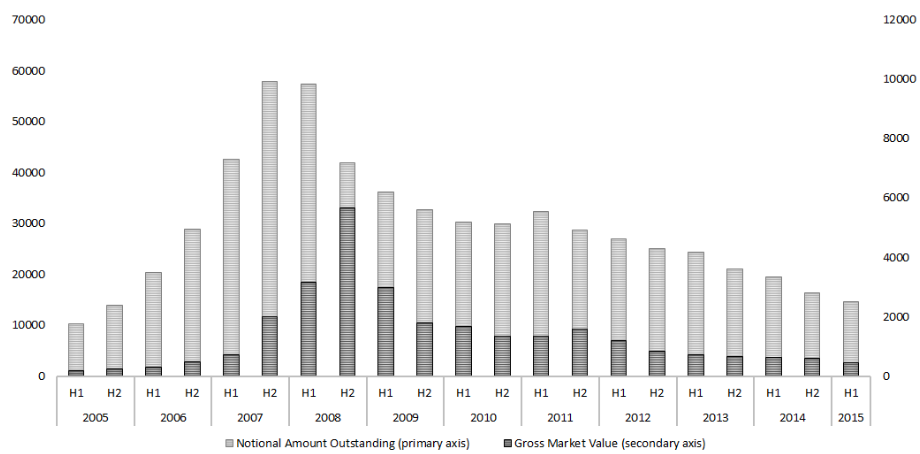

Figure 1.1: Global Gross Notional Amount and Gross Market Value of the CDS Market – This figure presents the evolution of the global gross notional amount outstanding (on the left vertical axis) and global gross market value (on the right vertical axis) in the CDS market. Values are in billions of US dollars; the data is based on the bi-annual survey conducted by the Bank for International Settlements (www.bis.org).

The rapid growth of the market, the rise to a chain of linked exposures between dealers of CDSs and the excessive speculation in credit risk are among the reasons why

[3]The BIS defines the gross market value as "the sums of the absolute values of all open contracts with either positive or negative replacement values evaluated at market prices prevailing on the reporting date" (BIS, 2015).

trading in credit default swaps is accused of having contributed to worsen both the subprime crisis in US and the sovereign crisis in EU. The implications on welfare, the role in the crisis and the related literature on this topic are further assessed in Subsection 1.2.4.

1.2.3 Insurance

A CDS is analogous to an insurance contract: the buyer pays a periodic premium and, in return, receives a sum of money if, before the expiration, one of the events specified in the contract occurs. It is then easy to argue that the moral hazard implied by CDSs is similar to that implied by traditional insurance. However, CDSs are not regulated as insurance contracts and, among market participants, the so-called "Potts opinion" is widely accepted. This is an opinion which the lawyer Robin Potts gave to ISDA in 1997 and in which it was concluded that credit derivatives should not be characterised as insurance contracts (see, Schmaltz and Thivaios, 2014 and the cited literature supporting the argument that CDSs are not insurance). However, the economic equivalence between the two instruments, fuelled the discussion in favour of regulating CDSs as insurance contracts (see, e.g. Kimball-Stanley, 2008; Saunders, 2010; Juurikkala, 2011; and Juurikkala, 2012).

Since CDSs are not subject to the insurance contract regulation, they cannot be invalidated if buyers have no interest in the insured entity. Thus, it is possible to buy credit protection on a reference entity, even without having any material interest which may be adversely affected by a credit event. In other words, investors can take speculative positions on the market via the so-called naked, or uncovered, CDS.

The exponential growth of the market before the onset of the 2007-08 US financial crisis and the rise of CDS spreads during the sovereign financial crisis in Europe, inflamed the discussion on the destabilising role of naked CDSs both in academic literature and in the press. The main argument against uncovered CDSs can be summarised as follows: distress on financial market increases the incentives to buy credit protection via CDSs, for both hedging and speculative purposes. While purchases in CDSs for hedging can only be limited to the exposure of the investor to a given entity, there is not a theoretical limit to the number of speculative positions that an investor can take via naked CDSs. This may negatively affect the cost of funding for troubled entities. Additionally, given that the few dealers of CDSs act both as sellers and buyers of protection, the result is a chain between institutions which poses systemic and domino effect risks.

In the wake of these critiques, and following the uncontrolled rise of sovereign bond yields in Europe, in November 2012 the European Council approved a ban of uncovered position in sovereign CDSs in the European Union.[4] The main objective of the EU regulatory intervention was to reduce the magnitude of speculation on sovereign CDSs. Nevertheless, Juurikkala (2012), among others, recalls that many aspects may prevent this regulation from being effective, namely, the over-the-counter nature of CDSs, the worldwide framework of financial markets and the absence of a similar rule in the US.

With the aim to contribute to this literature and to fuel the discussion on the role of speculation in CDSs, in Chapter 3, we assess the role of speculation in an optimal portfolio with CDS for an agent maximising the expected utility of his or her final wealth.

1.2.4 Welfare implication

CDSs have been widely blamed to be among the main causes of the US sub-prime crisis, as well as the Euro-zone sovereign debt crisis. In the former instance, CDSs were mainly criticised for their role in the burst of the housing bubble; in the latter case, commentators argue that speculation in CDSs has accelerated the rise of sovereign bonds yields. A detailed discussion of the role of CDSs in the sub-prime crisis can be found in Stulz (2010), while Fostel et al. (2015) explain how CDSs contributed to the burst of the bubble. Empirical evidence of speculation in CDSs on the sovereign crisis has been reported, for instance, by Delatte et al. (2012) and Chiarella et al. (2015).

The main argument in favour of a CDS ban is that the use of such contracts creates moral hazard: in distressed situations, unhedged creditors would favour and support bankruptcy over a renegotiations of the debt, especially for sovereign entities (see, e.g. Bolton and Oehmke, 2011). On the other side, CDSs, like any other derivative, increase welfare and market efficiency by facilitating risk-sharing among investors, by improving price discovery, and by making the allocation of capital more efficient (Stulz, 2010).

In the academic literature there are different views and opposite empirical results of the impact of CDS trading on welfare and market efficiency. For example, there are

[4]Regulation (EU) No 236/2012 of the European Parliament and of the Council of 14 March 2012 on short selling and certain aspects of credit default swaps.

studies which find that CDSs: (i) reduce monitoring incentives for banks (Parlour and Winton, 2013; Arping, 2014), (ii) create adverse selection in the bank-debtor relationship (Duffee and Zhou, 2001), (iii) exacerbate counterparty risk (Stephens and Thompson, 2014), (iv) increase systemic and contagion risks (Allen and Carletti, 2006). At the same time, there are also studies which argue that CDS trading implies: (i) more efficient risk sharing (Duffee and Zhou, 2001; Thompson, 2010), (ii) improved risk management (Norden et al., 2014), (iii) lower borrowing costs (Salomao, 2015), (iv) better pricing of credit risk and less frictions in credit markets (Ashcraft and Santos, 2009) and (v) better liquidity on the bond market (Massa and Zhang, 2012).

The largely mixed conclusion on the effect of CDS trading does not allow to draw a final assessment on the impact on welfare of such derivative instruments. Mixed and opposite results may also be a consequence of the low quality of data available on credit default swaps. The low level of regulation and the opacity of the market limit the public availability of data and only few researchers have the privilege to access to high quality data. A regulatory intervention aimed also to improve the quality and granularity of data, may have a positive impact on the quality of the research and, consequently, lead to a clearer picture on the welfare implication of the CDS trading.

The goal of Chapters 2 and 3 is to contribute to this literature and to additionally fuel the discussion on the implication of CDS trading on the economy.

1.3 Modelling

One of the first works on the CDS pricing can be found in Duffie (1999). In that paper, a simple arbitrage-free pricing model is presented by setting the CDS equal to the value of a portfolio containing a default free and a defaultable floating-rate bond. The author shows that a protection buyer's cash flows can be replicated by purchasing the default-free bond, at price $G(t_0, T)$, and simultaneously taking a short position on a defaultable bond, at price $B(t_0, T)$, both issued in t_0 at par-value and with the same maturity T. Thus, the protection buyer receives the interest rate $r(t)$ from the default-free bond and pays the interest rate $\delta_B(t)$ on the defaultable bond. In the case a credit event occurs before maturity, the protection buyer liquidates the portfolio at the coupon date immediately following the event, obtaining the par-value from the default-free bond and paying the market value of the defaultable bond, $B(\tau,\tau)$. The pay-off of this portfolio is consequently equal to that of a CDS on the same defaultable bond, whose spread is called $\delta_X(t)$.

Table 1.1: Simple Arbitrage Pricing Model – This table shows the payoff of both the arbitrage portfolio and a credit default swap written on the same defaultable bond, in a simple three period model. $G(t_0,T)$ is the price of a default-free bond at time t_0 whose maturity is T; $B(t_0,T)$ is the price, in t_0, of a defaultable bond whose maturity is T; $r(t)$ is the riskless interest rate at time t; $\delta_B(t)$ is the interest rate paid by the defaultable bond at time t; $\delta_X(t)$ the CDS spread at time t; and τ is the default time.

	Arbitrage portfolio	*Credit default swap*
t_0	$B(t_0,T)-G(t_0,T)$	0
t	$r(t)-\delta_B(t)$	$-\delta_X(t)$
$\tau \leq T$	$G(T,T)-B(\tau,\tau)$	$1-B(\tau,\tau)$
$\tau > T$	$G(T,T)-B(T,T)$	0

Assuming a simple three period model and that the default of the reference entity can occur in $\tau > t$, the arbitrage-free pricing model proposed by Duffie (1999) can be summarised as in Table 1.1.

It follows that, in absence of arbitrage, the CDS spread must be equal to the spread over the risk-free rate of the underlying bond issued by a reference entity:

$$\delta_X(t)=\delta_B(t)-r(t). \tag{1.1}$$

As we will discuss in Section 1.5, such arbitrage relationship, in reality, can be seen only as an approximation. Actually, several frictions prevent the two spreads from being equal when the bond and the CDS are written on the same reference entity and have the same maturity. In order to departure from this arbitrage-free relationship, a number of more advanced CDS pricing frameworks have been developed borrowing from the literature on credit risk. The most commonly used approaches to model CDSs can be grouped in structural models and reduced-form models. The main difference between the two approaches lies in the model specification of the default time: while structural models assume that default occurs when an exogenously modelled asset value crosses a given threshold, in reduced-form models the default time is specified using an exogenous intensity process. Default can be predicted in the first case, while it is a random event in the second case.

The structural approach to credit risk pricing is influenced by the pricing framework developed in Black and Scholes (1973) and Merton (1974). These models assume that the value of a firm's assets evolves randomly over time and the default occurs when the firm's asset value falls below a given threshold. Typically, asset value is modelled by a stochastic process such as a geometric Brownian motion and credit spreads are determined by firm's characteristics, such as leverage and asset volatility, and market conditions, such as interest rates. Through these models, it is then possible to estimate either the risk-neutral default probability or the credit spread on the debt of the firm. However, despite the fact that the structural approach is widely used in credit risk modelling, several studies find that, empirically, they do not accurately explain the magnitude of credit spread (see Section 1.4 for more details).

Reduced form models assume that the default time follows a Poisson process and occurs randomly based on an underlying probability distribution. Despite this approach has proven to be more successful in practical terms (Jarrow and Protter, 2012), it does not formulate any economic argument about the reasons of default. In the context of CDSs pricing, reduced-form models are used to value the premium (fixed) leg and the protection (floating) leg of the swap. The premium leg is defined as a series of payments made until the earlier between the contract maturity and a contingent credit event, while the protection leg is the contingent payment made upon occurrence of the credit event, if it happens before the CDS maturity. It is then possible to quantify the CDS spread by considering that, at inception, the value of the swap must be zero, that is, the expected present value of the premium leg must be equal to the expected present value of the protection leg. This will be analysed in greater details in Subsection 1.3.1.

A common view is that the two approaches for modelling credit risk are competing and disconnected (see, e.g. Duffie, 2002; Bielecki and Rutkowski, 2004; or Lando, 2004) and there is a debate in the professional and academic literature as to which class of models is best in terms of default prediction and hedging performance (see Anderson and Sundaresan, 2000; Jarrow et al., 2003; Eom et al., 2004; Ericsson and Reneby, 2005). In this context, Jarrow and Protter (2012) show that the reduced form model can be obtained from structural model if a smaller, incomplete, information set is used. Specifically, the authors argue that structural models assume complete knowledge of a very detailed information set, akin to that held by the firm's managers; while reduced form models assume knowledge of a less detailed information set, akin to that observed by the market. Their main conclusion is that the structural models can be transformed into the reduced form models as the information set changes and becomes less refined, shrinking from the information available to the firm's management to that available to the market.

1.3.1 A focus on reduced form models

Reduced form models describe default by means of an exogenous jump process and were originally introduced by Jarrow and Turnbull (1995). In the basic framework, the default time τ is defined as the first jump time of a Poisson process with intensity rate $\lambda(t)$. The (risk neutral) probability of default within time $[t, t+dt)$ conditional to non-earlier default is then characterised by

$$Pr[\tau < t + dt \mid \tau \geq t] = \lambda(t)dt. \tag{1.2}$$

It can be shown that the survival probability to time T, conditional on survival to the valuation time t, $Q(t,T)$, is given by

$$Q(t,T) = e^{-\int_t^T \lambda(s)ds}. \tag{1.3}$$

Following this approach, the premium (fixed) leg of a CDS ($P(s,T)$) can be expressed, under the risk-neutral probability measure $\mathbb{Q}$, as

$$P(s,T) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T \delta e^{-\int_t^s r(u)+\lambda(u)du} ds\right], \tag{1.4}$$

in which $r(t)$ is the riskless interest rate and δ is the constant spread of the CDS which is continuously paid. Similarly, assuming that, in case of default, the bond holder will recover a fraction $(1-w)$ of the par value of the bond, the protection (floating) leg of a CDS ($PR(s,T)$), can be expressed as

$$PR(s,T) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T (1-w(s))\lambda(s)e^{-\int_t^s r(u)+\lambda(u)du}ds\right]. \tag{1.5}$$

Setting the two legs equal, the CDS spread is given by

$$\delta = \frac{\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T (1-w(s))\lambda(s)e^{-\int_t^s r(u)+\lambda(u)du}ds\right]}{\mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T e^{-\int_t^s r(u)+\lambda(u)du}ds\right]}. \tag{1.6}$$

In this way, the CDS spread can be seen as a weighted average of risk-adjusted expected losses.

Default intensity and risk-free rate

Reduced form models are particularly suitable for modelling credit spreads and allow easy calibration of CDS spreads. Consequently, this family of models is commonly used to infer implied default probabilities from market quotes. In this context, following the pricing framework proposed by Duffie and Singleton (1997), the riskless rate and the default intensity are usually calibrated by assuming their independence. This simplifying assumption makes the discount factor $D(t,T) = e^{-\int_t^T r(s)ds}$ independent of the default time τ (see, e.g. Hull and White, 2000; Longstaff et al., 2005; Brigo and Alfonsi, 2005; Houweling and Vorst, 2005; or Schneider et al., 2011 among others).

The dynamic interactions between interest rates and credit spreads have important implications for credit risk modelling (Chen et al., 2013). While the independence assumption has significant computational benefits, it is often considered to be unlikely. Early studies have often identified a negative relationship between credit spreads and short-term interest rates (Duffee, 1998). Thus, in a number of works (e.g. Feldhütter and Lando, 2008; Frühwirth et al., 2010; or Driessen, 2005) a negative loading of the instantaneous interest rate is directly incorporated into the credit-spread specification.

As argued by Hull and White (2000), the independence assumption can be justified by an offset of the effects of interest rates and default probabilities acting in opposite directions. For instance, high default probabilities may have two effects: (i) increase rates at which future payoffs are discounted, and (ii) reduce market values for bonds issued by the reference entity. As a consequence, the first effect reduces the CDS spread, while the second acts in the opposite way.

In Chapter 3, when performing the numerical simulation of our optimal portfolio model with CDS, we set the covariance between the default intensity and risk-free rate equal to zero. In this way we assume that the two variables follow two independent stochastic processes.[5]

1.3.2 Optimal portfolio choices with defaultable assets

Structural and reduced form models have been used by many researchers to model default in the context of portfolio optimisation problems. Following the work of Merton (1969), who initiated the use of stochastic dynamic programming techniques to solve continuous-time optimal portfolio problems, there has been significant interest in continuous-time maximisation problems. Initially, the literature was focused on markets consisting of default-free securities. More recently, a number of studies developed optimal portfolio models with securities subject to default. For instance Korn and Kraft (2003) study, in a Merton structural default framework, optimal portfolio problems with defaultable assets. In a similar framework, Kraft and Steffensen (2006) extended the analysis by defining the default as the first passage time of an economic state variable below a given threshold.

There is an extensive literature which models default event in the context of portfolio optimisation using reduced form approach. One of the first studies is Walder (2002), who studies the optimal portfolio problem for an agent that can invest in a treasury bond and a portfolio of corporate zero-coupon bonds. One major weakness of that study is that, to obtain tractable results, the author assumes a zero recovery rate. Bielecki and Jang (2006) derive optimal investment strategies for a constant relative risk aversion (CRRA) investor, who can allocate the wealth among a defaultable bond, a risk-free bank account and a stock. The authors explicitly include the recovery rate in the model. In a similar framework, Bo et al. (2010) study an infinite horizon portfolio optimisation problem, in which an agent can dynamically choose a consumption rate and allocate his/her wealth

[5]The model developed in Chapter 3 allows the default intensity and the risk-free rate to be correlated. We make the assumption of independence between the two variables for the sake of simplifying the simulations of the model.

in a market where three financial securities are listed: a defaultable perpetual bond, a default-free risky asset, and a money market account.

Another branch of the literature, which is closely related to decisions in presence of credit default swaps, relates to portfolio choices with mortality contingent claims. In this context, and similarly to the firm's default process in reduced form models, the force of mortality of individuals is modelled as a Poisson process with stochastic intensity. There is an extensive literature on this topic and a number of studies successfully model optimal consumption and portfolio choices in presence of mortality risk. One of the first paper which deals with such optimisation problem is Dahl (2004). That paper investigates the risk management for insurance companies by developing a model in which the mortality intensity follows a stochastic process, and in which some traded assets depend on the development of such process. Following that paper, a number of studies analyse optimal portfolio choices in similar frameworks (see, e.g. Battocchio et al., 2007, Menoncin, 2009, Menoncin and Regis, 2015).

Despite the extensive literature on optimal portfolio choices with defaultable securities or mortality risk, to the best of our knowledge, the only paper studying optimal asset allocation with CDSs is Bo and Capponi (2016). In that paper, the authors include credit derivatives in a dynamic portfolio optimization framework and develop a contagion credit risk model with interacting default intensities. Precisely, they consider a portfolio consisting of two CDSs, with the main goal to assess the impact of default contagion on the optimal CDS strategy.

In Chapter 3 we contribute to the very limited literature on optimal portfolio choices with CDSs. In particular, we derive the optimal investment strategy for an agent with hyperbolic absolute risk aversion (HARA) who can invest in a risk-free asset, a defaultable bond, and a CDS written on the bond.

1.4 The determinants of credit default swap spreads

CDS spreads should reflect market expectations about the financial health of a reference entity, and, accordingly, allow to infer the implicit default probability. This section reviews the empirical literature that explores the determinants of credit risk and analyses the variables that are used to decompose spread variations.

There is a significant literature that empirically analyses the factors that determine credit risk using spreads as a proxy, in different periods and with different type of reference entities. For instance, Collin-Dufresn et al. (2001), Longstaff et al. (2005) and Bedendo and Colla (2015) focus on corporate CDSs; Pan and Singleton (2008), Longstaff et al. (2011) and Dieckmann and Plank (2012) use sovereign CDSs; Annaert et al. (2013) and Hasan et al. (2015) are two of the few studies which analyses the determinants of CDSs written on banks. Those studies decompose the explained part of CDS spreads changes according to various risk drivers.

Regardless the type of underlying reference entity, structural models represent the starting point for the selection of the variables explaining CDS premiums. In Merton (1974), for instance, the credit spread of a company should be primarily determined by the asset volatility of the firm's balance sheet, the financial leverage and the risk-free term structure. However, it has been shown that variables defined in a structural credit risk model explain only a limited part of credit spread variations. For example, Eom et al. (2004) empirically test five structural models of corporate bond pricing and find that the predicted spreads are not accurate. Similar results are found also in Chen et al. (2006), where, using CDS spreads to estimate and compare models, the authors find that structural models either overestimate or underestimate actual price levels. Similarly, Ericsson et al. (2009) investigate the linear relationship between theoretical determinants of default risk and default swap spread and find that a minimal set of theoretical determinants of default risk are consistent with theory and are significant both statistically and economically. Additionally, González-Hermosillo (2008) show that the impact of proxy variables is strongly time-varying on CDS spreads.

To improve the fit of structural models, many authors have added variables that capture market conditions. It has been documented that credit spreads and default probabilities vary through the business cycle. Additionally, it is necessary to distinguish between different types of reference entities. In fact, variables that are found to be explanatory for non-financial companies, may lose their relevance when applied to financial entities (e.g. Raunig and Scheicher, 2009 and Grammatikos and Vermeulen, 2012).

In the next subsections we group explanatory variables into two sets: (i) factors which are specific to the reference entity, directly affecting its default probability, and (ii) factors that are representative of market conditions, affecting indirectly the entity's probability of default.

1.4.1 Reference entity specific variables

In this set, one of the most commonly used variable is the stock return of the reference entity. As explained, among others, by Collin-Dufresn et al. (2001), stock returns can be seen as a proxy for financial leverage. In fact, if stock returns are positive, leverage measured in market values will decrease, leading to lower credit spreads. Additionally, stock returns can capture also other factors, as they reflect the entity future profitability: positive returns indicate positive future prospects and, consequently, lower default risk. As a result, there exists a negative relationship between stock returns and credit spreads.

Another variable taken into consideration by the existing literature is the idiosyncratic volatility: an increase in asset volatility should positively affect the probability to hit the default threshold. Many asset volatility measures are used in the literature. For instance, Annaert et al. (2013) measure volatility as standard deviation of daily stock returns over different rolling windows (7 days, 25 days and 50 days). Similarly, in Bedendo and Colla (2015) the volatility is computed as the standard deviation of stock returns over a 180-day rolling window. In the context of credit risk determinants, the best way to measure volatility would be to use intraday stock volatility, as it would be able to capture the daily fluctuations of stock prices; however, such a measure is rarely available from standard data providers.

Longstaff et al. (2005) argue that bond illiquidity is priced into credit spreads, and, in fact, few studies include, in their specifications, proxies for the liquidity of the reference entity. However, it is often underlined the difficulty in finding an adequate proxy, as individual liquidity is hardly observable on the market. For example, Annaert et al. (2013) measure individual liquidity as the difference between CDS bid and ask quotes. They argue that the bid-ask spread is mostly correlated with liquidity proxies such as data on trades. We believe that CDS bid-ask spreads are indeed a measure of the CDS liquidity, but cannot be considered as an adequate proxy of the liquidity position of the reference entity. Departing from this reasoning, in the next chapter, we develop an empirical analysis to assess the impact of global and specific variables on banks' credit spreads. In our specification we include confidential data on the recourse to Eurosystem refinancing operations as a proxy of banks' individual liquidity. We believe that this measure, which however could not be used for non-financial credit spreads, can reflect adequately the liquidity position of a financial institution.

1.4.2 Global variables

One main issue arising from the basic specification proposed by structural models is that model residuals contain common variation. As pointed out by Collin-Dufresn et al. (2001), the reason behind the common variation in the residuals is the omission of explanatory variables, which are likely not firm-specific. The solution used in the literature to overcome this issue is the inclusion of variables that capture general market and economic conditions about equity, fixed income, and currency markets (see, e.g. Annaert et al., 2013 or Bedendo and Colla, 2015).

One widely used global measure is the risk-free interest rate. Its inclusion follows the Merton (1974) model in which the risk-free rate is the drift in the risk neutral world; the higher it is, the less likely default becomes. Hence, there should exist a negative relationship between risk-free and credit spreads. This confirms the results of Duffee (1998) and the later studies which assume a negative relationship between the two factors as described in Section 1.3.

A second, widely used, global factor is the general business climate: improvements in business conditions decrease probabilities of default and increase recovery rates. The business climate is frequently captured by variables which reflect the evolution of stock, fixed income and currency markets. For instance, some of the variables which are used to capture general business climate are: (i) stock market indices returns, (ii) stock market volatility indices, (iii) government bond indices, (iv) corporate bonds indices, or (v) exchange rates.

More recently, few studies underlined the importance of the role of the sovereign risk on credit spreads both for non-financial (Bai and Wei, 2012) and financial institutions (Avino and Cotter, 2014). The rationale behind this is straightforward: the financial distress of a government is likely to affect negatively the corporate sector, as the government would transfer the debt burden onto the financial sector. To the best of our knowledge, Bedendo and Colla (2015) is one of the first studies which includes sovereign CDS as a determinant of credit spreads. The authors find that an increase in sovereign risk is associated with an increase in the credit risk of the reference entity.

The statistical significance of sovereign risks on financial and non-financial credit spreads fuels the discussion on the empirical relevance of risk measures on CDS spreads. The recent financial crisis has shown how an increase in global risk could also affect credit spreads of entities which are not directly involved in a crisis through contagion and spillover effects. To the best of our knowledge, however, no existing studies consider wider risk measures as global relevant factors for credit spreads. In the empirical analysis presented in the next chapter, we further analyse the role of global risk by including measures of systemic risk as determinant of a bank's credit risk spreads.

1.5 Relationship between credit default swaps and bonds

As shown in equation 1.1, there should exist a parity relationship between CDS and bond spreads, and accordingly the so-called CDS basis should be equal to zero (see, e.g. Choudhry, 2006). However, a number of reasons can explain the actual existence of a spreads differential: the cheaper to delivery option embed in the CDS contract,[6] for example, can lead to a negative basis (i.e. CDS spread larger than bond spread), while the counterparty risk of the protection seller or the exclusion of accrued interests in the CDS payment triggered by the credit event can lead to a positive basis (i.e. bond spread larger than CDS spread).

The existence of a spread basis has contributed to the development of two branches of the empirical literature on CDSs. The next subsections review the literature which analyses: (i) reasons and causes behind the CDS basis, and (ii) the mechanism of incorporation of new information into bond and CDS spreads.

1.5.1 Parity relationship and spread basis

The drivers of the CDS-bond basis are investigated by a number of studies. Longstaff et al. (2005) is one of the first studies providing evidence relating to the basis, using

[6] Cheaper to delivery option refers to the possibility for the protection buyer, after the credit event, to choose between a basket of deliverable bonds in the case of physical delivery.

CDS data for 68 financial and non-financial firms, mainly listed on US and UK markets, from March 2001 to October 2002. Assuming that credit default swap spreads reflect a measure of default risk, the authors use the CDS basis as a proxy for the non-default component. They conclude that the basis is strongly related to the illiquidity of the bond market.

Nashikkar et al. (2011) analyse the CDS basis over the period from July 2002 to June 2006, for 1,167 firms. The authors investigate the non-default component of bond spreads, by running regressions of the basis on measures of bond liquidity and other factors. Their main finding is that both latent liquidity and CDS bid-ask spread have significant explanatory power for the basis.

While early studies find the basis to be positive prior to 2007, Fontana (2011) shows that, during the 2007-09 crisis, the difference between bond and CDS, on the same reference entity, turned negative. Consequently, a number of studies investigate the drivers of the CDS-bond basis during and after the US subprime crisis. In this context, Gârleanu and Pedersen (2011) develop an asset pricing model, in which the funding constraints can give rise to price differences between two financial instruments with identical cash flows but different margin requirements. They empirically test the model predictions for the basis using CDS and bond data over the 2005-2009 period. The authors conclude that the time-series variation in the basis is closely related to the shadow cost of capital. This supports the theories which argue that the basis became negative due to deleveraging activity of financial institutions: strong selling activities on the bond market may have decreased bond prices, driving the basis in negative domain. However, this theory has been challenged by Choi and Shachar (2014). Using data from the Federal Reserve Bank of New York on dealers' aggregate bond inventories, they argue that, after Lehman Brothers' default, dealers were "providing liquidity" by purchasing corporate bonds from hedge funds. They thus conclude that, while dealers were "leaning against the basis," their activity was insufficient to close the basis gap.

Bai and Collin-Dufresne (2011) investigate the negative and persistent basis during crisis and post-crisis periods, using a sample of 487 firms with single-name CDS. To explain the non-zero basis between CDSs and bonds, they identify a set of proxies for trading frictions, such as liquidity, funding cost, and counterpart risk. Subsequently, the authors find that, while during the crisis their proxies explain the non-zero basis, most of the factors lose their explanatory power after the crisis.

In summary, the majority of empirical studies find that the basis is mainly driven by the different degree of liquidity in the two markets. Generally speaking, several factors underpin a greater liquidity of the CDS market: (i) CDS contracts can be sold in arbitrarily large amounts, while bonds are limited in supply; (ii) naked credit default swaps allow the possibility of taking short positions in bonds, without trading on the bond market; (iii) on a given borrower, the bond market is significantly more fragmented than the CDS market, as it is made up of a number of issuances, usually characterised by different rates; (iv) bonds are often purchased as part of a buy and hold strategy, reducing the liquidity.

1.5.2 Leading process in price discovery

The existence of the CDS basis raises the issue of which market has the leading role in price discovery process.[7] Intuitively, the bond market should be the first to incorporate the new information and derivative instruments should then adjust to its movement. If this were not the case, trading in CDSs would be able to affect the price of bonds or, in other words, the financing costs of corporate, financial or sovereign entities. Consequently, speculation activity in CDSs could have serious effects on the stability of financial markets.

The price discovery process has been initially analysed by estimating a vector error correction model (VECM) as in Hasbrouck (1995). If we model two dependent market one of which has a leading role in the price discovery process, then they will converge to their equilibria with different speeds. Specifically, the adjustment of the leading market price on the follower market price will be slower. Such a mechanism can be described by a VECM, in which the error correction coefficients provide a measure for the intensities of the price adjustments. In order to estimate VECMs, the series of bond and CDS spreads should be non-stationary and cointegrated.

Following this approach, the price discovery process between bond and CDS markets has been analysed by a number of authors and in different frameworks. The discussion

[7] The price discovery on financial markets has been defined as the efficient and timely incorporation of information into market prices (Lehmann, 2002).

was initially focused on corporate CDSs. One of the first works in this field is Blanco et al. (2005). On a sample of 33 investment grade firms (17 from Europe and 16 from USA) between January 2001 and June 2002, the authors find evidence that the CDS market leads the determination of the price of credit risk over the bond market. They argue that price discovery occurs in the CDS market as it is the most convenient location for the trading of credit risk, due to structural factors. Zhu (2006) performs a similar analysis on 24 investment grade entities (8 banks and 16 corporate companies), over the period between 1999 and 2002, both in the US and the EU. The author shows that derivatives tend to move ahead of bonds and the CDS market plays a key role in improving the efficiency of the price discovery mechanism for corporate credit risk.

Subsequently, researchers moved their focus on sovereign CDSs. Conclusions of studies on emerging markets are similar to those on corporate CDSs. One of the first papers that focuses on sovereign entities is Ammer and Cai (2011). The authors perform an empirical study on 9 emerging markets and they find that sovereign CDSs often move ahead of bonds. Their main conclusion is that the relatively more liquid market tends to lead the other. Similarly, Aktug et al. (2012) evaluate the dynamic relationship between the two markets over the period between 2001 and 2007, across 30 emerging countries. In this case the results suggest that the CDS market has a leading role in 37% of the cases. Also Hassan et al. (2015) perform price discovery analysis on emerging markets. Specifically, the authors analyse the bond-CDS relationship on seven sovereign entities and conclude that, in 71% of the sample, the bond market leads the process by adjusting to new credit information before the CDS market.

Following the turbulence on Euro-zone countries, academic attention shifted on sovereign CDSs written on European countries. Delatte et al. (2012) analyse the price discovery process in developed member States of the European Union. The main conclusion is that the bond market plays a dominant role only in the core-European countries and during calm periods. However, the higher the distress, the more the CDS market dominates the information transmission mechanism. The authors argue that CDSs became a bear-market instrument to speculate against the deteriorating conditions of sovereign countries. Similarly, Coudert and Gex (2013) analyse the price discovery process between CDS and bond spreads of 17 financial and 18 sovereign entities during the 2007-2010 period. They conclude that for financial entities the bond market incorporates the new information after the CDS market, while for sovereign entities the bond market is shown to lead the CDS market only for low-yield countries.

All the above cited studies, although focusing their analyses on different type of entities or on different sample periods, find evidence of a leading role of the CDS market for a subset of the analysed sample. However, the causes and reasons of this empirical evidence, which are pointed out by the different authors, are not homogeneous and often not supported by further analysis. Despite the large number of studies in this field, we can point out two main issues that we believe are worth further research. Firstly, the literature analysing the price discovery process for financial entities is scarce. Secondly, and most importantly, there are no studies, to the best of our knowledge, which try to evaluate the impact of the leading role of the CDS market on the determinants of credit risk. In the next chapter, we develop empirical analysis to try to close these gaps in the literature.

1.6 Conclusion

CDSs are a recent financial innovation and, as such, the academic literature on this topic is still evolving and growing. In recent years, an increasing number of researchers shifted their focus on credit default swaps, also driven by the need of investigating the role of these derivatives in the global financial crisis.

In this work we provide an overview of the existing literature about CDSs. More precisely, three main areas of research, and their interactions, are reviewed: (i) theoretical works that model CDS price, (ii) empirical studies which analyse the determinants of CDS spreads, and (iii) empirical studies which assess the relationship between CDSs and underlying bonds.

The results of this survey can be summarised as follows. In relation to the literature that models CDS, we argue that, while the conceptual foundations of pricing are well established, the role of CDSs in an optimal portfolio needs to be assessed. Secondly, in the context of CDS spread determinants we discuss that, despite the significant number of papers analysing this topic, there is still room for improvement as the variables which have been identified by existing literature do not fully explain CDS spread variations. Thirdly, concerning the price discovery process, we identify the absence of studies which empirically explore the consequences of a leading role of the CDS market as a relevant gap in the literature.

The studies reviewed in this work have a common link which we believe translates into a major question which has not been addressed yet by the literature. While there is a substantial evidence that CDS trading makes markets more efficient, there is a significant number of papers which claim the opposite. Further research should aim to assess the impact that trading in CDSs has on welfare and market efficiency.

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Chapter 2

The implications of the leading role of CDS on banks' credit risk determinants

GIUSEPPE AMBROSINI *

Abstract

This study performs two empirical analyses to address (i) whether credit default swaps (CDS) written on European banks lead the price of their underlying bonds and (ii) the implications of the leading role of the CDS market on the determinants of banks' credit risk. The results of the first analysis show that the CDS market leads bonds' prices for nearly half of the banks in our sample. To assess the impact of this phenomenon on banks' credit risk, the sample is split into two groups, and the explained part of CDS spread changes is decomposed according to various risk drivers. It is shown that the home country sovereign risk has a different impact on the two groups, revealing a stronger transaction mechanism of risk when the CDS market is in the lead.

Keywords Credit Default Swap · Price Discovery · Credit Risk · Bank

JEL classification G14 · G15 · G21

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2.1 Introduction

The subprime crisis in the US and the sovereign debt crisis in the EU have raised concerns on the role of credit default swap (CDS) trading on financial market and the related implication on financial stability. A CDS is a derivative contract on credit risk, which is functionally equal to an insurance contract: a (protection) buyer acquires protection against the default risk of a reference entity, by paying a periodic premium (spread) to a (protection) seller. Nonetheless, such derivatives are not regulated as insurance contracts, and buyers of CDSs are not required to hold any bond issued by the reference entity the CDS is written on. This results in the possibility of speculating on the default risk of corporate, financial, or sovereign entities, via so-called naked, or uncovered, CDSs.

It has been argued that trading activity on the CDS market has been able to exacerbate the effects of a financial crisis (see e.g. Delatte et al., 2012; Stephens and Thompson, 2014; Chiarella et al., 2015; and Fostel et al., 2015). Turbulences on financial markets increase the incentives for investors to enter into CDS contracts, by "betting" on the default of troubled reference entities. The higher demand in CDS protection raises CDS spreads. Such increase, in return, could affect the yields of underlying bonds, resulting in higher refinancing costs for a reference entity. Speculation in CDSs could then generate a vicious circle, therefore exacerbating the effects of a financial crisis.

The reason behind this mechanism is the arbitrage relationships which should exist between the spread of a bond, with respect to the risk-free rate, and that of a CDS written on the same bond and with the same maturity. As the price determinants of the two instruments are the same, the two spreads should always be at par. However, in reality this arbitrage relationship rarely holds, as there often exists a spread basis. This raises the key issue of which market firstly incorporates the new information, or, in other words, which of the two instruments leads the price discovery process (PDP).

The aim of this study is to analyse the interactions between bonds and CDSs written on European banks, and to explore the implications of a leading role of the CDS market in the process of price discovery. In this context, two main branches of the literature in CDS are directly related to this study: papers which study the PDP between bonds and CDSs, and studies which decompose the explained part of CDS spreads changes according to various risk drivers.

The price discovery process between bond and CDS markets has been recently analysed by a number of authors, in different frameworks and for different underlying entities (see, e.g. Blanco et al., 2005; Zhu, 2006; Ammer and Cai, 2011; Coudert and Gex, 2013; or Delatte et al., 2012). The empirical literature on this topic is quite wide and the conclusions are, to some extent, similar: CDSs have the lead in price discovery process for some subsets of the analysed samples. However, we can point out two main weaknesses of existing literature: (i) there is very little empirical work on CDSs written on banks and (ii) existing studies do not extend their analysis to the understanding of the implication of a leading role of the CDS market.

In this chapter, we analyse the PDP between bonds and CDSs for 32 European banks, between 1 January 2011 and 31 December 2013. We require that banks have the following characteristics: (i) are headquartered in Europe, (ii) have access, directly or indirectly, to refinancing operations within the European Central Bank (ECB), and (iii) have outstanding debt instruments that are priced on European markets. The reasons behind these requirements and the selection of the sample period will be clarified in the next sections. The results of this analysis show that the CDS market has a dominant role on the bond market for 15 banks.

In order to deepen the implications of these results, we perform a second analysis to assess the behaviour of credit risk determinants when the CDS market is in the lead. To do so, we split our sample into two subgroups on the basis of the outcome of the PDP analysis and we decompose the explained part of CDS spreads changes according to various risk drivers. A number of authors empirically analysed the factors that determine credit risk using CDS spreads as a proxy (see Annaert et al. (2013) and the cited literature). We improve the specifications proposed by the existing studies by adding, as explanatory variables, confidential data owned by the ECB. The results show that, when the CDS market has the leading role in price discovery process, there exists stronger transmission mechanisms of credit risk from sovereign entities to banks.

The remainder of this chapter is organised as follows: Section 2.2 presents the price discovery analysis, Section 2.3 presents the determinants of credit risk and explores the impact of the leading role of CDS on credit risk determinants, and Section 2.4 concludes.

2.2 Price discovery analysis

Price discovery on financial markets has been defined as the efficient and timely incorporation of information into market prices (Lehmann, 2002).

The main function of a CDS is the transfer of credit risk of a reference entity from a protection buyer to a protection seller. In brief, a CDS can be seen as an insurance contract on the notional value of a bond and, in case of default of the bond issuer, the derivative pays back the underlying face value or the loss given default (LGD), depending on the type of settlement specified in the contract. In theory, a portfolio with a CDS and the underlying bond eliminates all the risks associated with the default of the reference entity and, hence, it should have a return equal to the risk free rate. This implies that there should exist a parity relationship between the CDS premium and the spread of the underlying bond on the risk free rate (Duffie, 1999). Arbitrage activity should reduce any price differences between the two markets over time. However, in reality, bond spreads and CDS premia are not at parity for several reasons, such as the cheaper to delivery option embedded in CDS contracts, short selling constraints on bond markets, accrued interest, and counterparty risk.[1]

The existence of a spread basis raises the issue of which of the two markets has the leading role in price discovery process. Intuitively, the bond market should incorporate first the new information and derivatives, by definition, should adjust to its movement. However, if the process is reversed and the new information is firstly incorporated by the derivative market, then trading in CDSs would influence bond prices. This would imply that the financing cost of firms, financial institutions or governments would be set by an over-the-counter derivative market, characterised by low levels of transparency and regulation. Considering also that market participants can speculate on the risk of default by entering into naked CDSs, then the leading role of the CDS market may have potential negative consequences on financial stability.

For these reasons, the price discovery process between bond and CDS has become a relevant research topic, which has been recently analysed by a number of studies. For example, Blanco et al. (2005) is one of the first papers exploring the relationship between

[1]For a more detailed explanation of the parity relationship between bond and CDS spreads, the reader is invited to refer to Sections 1.3 and 1.5 of this thesis.

the two markets in the context of price discovery. In that study, on a sample of 33 investment grade firms (17 from Europe and 16 from USA) between January 2001 and June 2002, the authors find evidence of a leading role of the CDS market on bond prices. It is argued that price discovery occurs in the CDS market as it is the most convenient location for the trading of credit risk, due to structural factors.

Evidence of a leading role of CDSs is found also on sovereign entities. Ammer and Cai (2011), for instance, analyse the discovery process on 9 emerging markets between 2001 and 2005 and find that sovereign CDSs often move ahead of bonds. The main conclusion of that study is that the relatively more liquid market tends to lead the other. Delatte et al. (2012) perform a similar analysis on developed members of the European Union. The authors show that the bond market plays a dominant role only in the core-European countries and during calm periods. However, during distressed periods the CDS market dominates the information transmission mechanism. The main conclusion is that, during the EU sovereign crisis, credit default swaps became a bear-market instrument to speculate against the deteriorating conditions of sovereign countries.

In this section we investigate the price discovery process between CDS and underlying bond spreads within the European banking sector.

2.2.1 Data description

Our analysis focuses on European banks on which CDS quotes are available and liquid, and which have issued Euro-denominated bonds. The analysis includes 32 banks headquartered in the main European countries and covers the period between 1 January 2011 and 31 December 2013. We select this period, first of all, to be in line with existing studies which consider, on average, a sample period between one to four years. Secondly, we aim to include both a period characterised by high volatility on the CDS market and a period of relative financial stability. In this regard, high volatility on the CDS market has been observed over the periods between August 2011 and July 2012 and between April and August 2013.[2] Finally, the range selection takes also into consideration the availability of data used for the analysis carried in Section 2.3.

[2]We proxy the volatility on the CDS market by computing the weekly standard deviation of the iTraxx index on European banks.

To perform the price discovery analysis, we use Euro-denominated daily CDSs data. We obtain, from Datastream, CDS quotes for senior unsecured debt on a 5-year maturity. Accordingly, we need bond with the same maturity and of the same currency. Given that, in practice, it is nearly impossible to find debt instruments that have maturities that exactly match those of CDS, we need to estimate the implicit 5-year spot rate for each bank using bonds of different maturities.

To achieve high degree of consistency, we carry out the following procedure:

1. For each bank, we search in Datastream and Bloomberg for zero coupon bonds (ZCBs) with maturities from 3 months to 15 years. For banks with a sufficient number of listed ZCBs and with a complete structure of maturities, we then estimate the 5-year yield using the Nielson-Siegel model (Nelson and Siegel, 1987). Unfortunately, only few banks have a sufficient number of ZCBs.

2. To estimate the 5-year yield for those banks without enough securities in the market, we then search for a ZCB having time to maturity between 3 and 5 years at the beginning of our sample period, and another ZCB with more than 8 years to maturity at the beginning of the sample period. For the banks with two ZCBs with these characteristics, we interpolate the two yields in order to obtain the 5-year spot rate.

3. For banks without enough ZCBs on the market, we estimate a 5-year yield to maturity by interpolating the yield of available securities. We exclude from our search floating-rate securities and bonds that have embedded options, step-up coupons or any special feature that would result in different pricing. To ensure a sufficient level of liquidity, we exclude bonds for which the trading volume has been equal to zero in at least one day within the sample period. In this way, we are able to obtain 5-year yields that are highly correlated with CDS spreads.

As a risk free rate, we use the 5-year spot rate of German bonds obtained from Datastream.

Table 2.1 gives an overview of the banks that are included in our study, together with their country of residence and the estimation method for the 5-year rate. Our sample includes credit institutions headquartered in 12 European countries. Overall, out of 32 banks in our sample, 7 are resident outside the Euro-area.

Table 2.2 shows the descriptive statistics of ZCBs and CDS spreads for the reference entities in our sample. Bond spreads are calculated as the difference between the estimated 5-year rate and the free risk rate. As shown in the table, the rates obtained depict high correlations with CDS spreads.

2.2.2 Methodology

The econometric discussion used in this section borrows from the existing literature on CDS and bond markets (see, e.g. Blanco et al., 2005). The majority of existing studies use a vector error correction model (VECM) to examine the individual adjustment process towards the long run cointegration relationship.

Considering two related markets and assuming that one of them has the leading role in price discovery process, it is then plausible to assume that the price is firstly determined on the leading market, and that the other market will subsequently converge towards to equilibrium. As a consequence, the two markets will adjust to equilibrium with different speeds. Specifically, the adjustment of the leading market price on the follower market price will be slower. This mechanism can be described by a VECM, in which the intensities of the price adjustments are measured by the error correction coefficients.

Financial theory predicts that CDS premium and bond rate move together since they are both determined by the same underlying factors. Thus, there should exist a long-run relationship between the two spreads for the same borrower and maturity, driven by arbitrage principles (i.e. the two series should be cointegrated). This relationship can be expressed through the following equation:

$$CDS_{it} = \mu_i + \alpha_1 BS_{it} + z_{it}, \tag{2.1}$$

in which CDS and BS are respectively the CDS and the bond spreads for the i^{th} bank, at time t; μ denotes the bank specific intercept; z is the vector of errors; α_1 is the long run equilibrium coefficient. If there were no arbitrage opportunities in the market (i.e. it is perfectly efficient) then α_1 should be 1 and μ_i should be 0.

Table 2.1: Bank list and yield estimation method – This table lists the banks included in our analysis. For each bank, we also report the home country and the estimation method we use to obtain the 5-year yield

#	Bank	Country	Yield Estimation Method
1	Banca Monte dei Paschi di Siena	Italy	Yield to maturity interpolation
2	Banco Comercial Português	Portugal	Yield to maturity interpolation
3	Banco Popolare	Italy	Yield to maturity interpolation
4	Banco Popular Español S.A.	Spain	Yield to maturity interpolation
5	Banco Sabadell S.A.	Spain	Yield to maturity interpolation
6	Banco Santander S.A.	Spain	Yield to maturity interpolation
7	Bank of Ireland	Ireland	Yield to maturity interpolation
8	Bankia	Spain	Yield to maturity interpolation
9	Barclays Bank	UK	Yield to maturity interpolation
10	BBVA	Spain	Yield to maturity interpolation
11	BNL Spa	Italy	Yield to maturity interpolation
12	BNP Paribas	France	Nelson-Siegel Model
13	Commerzbank AG	Germany	Yield to maturity interpolation
14	Crédit Agricole	France	Yield to maturity interpolation
15	Crédit Mutuel CIC	France	Yield to maturity interpolation
16	Credit Suisse	Switzerland	Yield to maturity interpolation
17	Deutsche Bank AG	Germany	Spot rate interpolation
18	Dexia Credit Local	France	Yield to maturity interpolation
19	Erste Group Bank AG	Austria	Yield to maturity interpolation
20	Fortis Bank SA	Belgium	Spot rate interpolation
21	HSBC Bank	UK	Yield to maturity interpolation
22	ING Bank	Netherlands	Yield to maturity interpolation
23	Intesa San Paolo Spa	Italy	Nelson-Siegel Model
24	Lloyds Bank	UK	Spot rate interpolation
25	Mediobanca Spa	Italy	Yield to maturity interpolation
26	Natixis	France	Spot rate interpolation
27	Nordea Bank AB	Sweden	Yield to maturity interpolation
28	Société Générale SA	France	Yield to maturity interpolation
29	The Royal Bank of Scotland	UK	Yield to maturity interpolation
30	UBS	UK	Yield to maturity interpolation
31	Unicredit Spa	Italy	Spot rate interpolation
32	Unione di Banche Italiane	Italy	Yield to maturity interpolation

Table 2.2: Descriptive Statistics – This table shows summary statistics of the daily bond and CDS spreads. For each bank, we depict mean, standard deviation, maximum and minimum values, in basis points, for both series. Bond spreads are obtained as the difference between the estimated 5-year bond rate and the 5-year German yield. Last column represents the correlation between the two spreads

Bank	**Estimated Bond Spread**				**Credit Default Swaps**				**Corr**
	Mean	**Sd**	**Max**	**Min**	**Mean**	**Sd**	**Max**	**Min**	
Banca Monte dei Paschi di Siena	544.44	152.68	863.88	241.22	509.94	167.30	906.43	178.09	0.8341
Banco Comercial Português	758.33	269.82	1324.28	368.54	859.15	395.51	1925.5	353.08	0.8524
Banco Popolare	564.13	182.23	1163.85	303.77	501.27	162.26	941.06	183.30	0.8912
Banco Popular Español S.A.	446.63	92.48	634.76	258.20	564.51	189.81	953.66	222.83	0.8346
Banco Sabadell S.A.	657.08	208.56	1253.30	361.01	506.65	178.85	837.90	195.59	0.7583
Banco Santander S.A.	347.52	98.75	552.17	145.73	293.79	78.08	506.66	122.1	0.8527
Bank of Ireland	1760.11	1069.83	4467.30	676.12	726.50	358.66	1476.67	214.86	0.8732
Bankia	573.01	144.50	883.89	327.27	719.18	270.21	1574.35	220	0.7309
Barclays Bank	406.24	146.38	756.90	209.92	161.77	44.10	277.75	91.28	0.8902
BBVA	430.18	117.38	595.40	205.37	299.73	81.17	515.12	117.98	0.5095
BNL	265.82	43.58	393.19	203.14	169.15	64.88	371.05	82.82	0.8537
BNP Paribas	254.70	98.01	507.67	139.19	171.20	65.28	367.23	80.84	0.9024
Commerzbank AG	235.15	73.73	422.09	123.81	205.87	59.33	379.61	110.05	0.7836
Crédit Agricole	368.48	147.20	696.55	144.19	207.19	68.94	411.06	103.26	0.9051
Crédit Mutuel CIC	368.84	129.60	703.96	205.89	182.48	61.59	356.13	98.53	0.8914
Credit Suisse	214.82	25.06	268.22	162.59	121.08	35.58	212.75	67.07	0.7895
Deutsche Bank AG	235.09	56.07	377.96	149.77	138.85	45.48	327.60	80.76	0.8071
Dexia Credit Local	773.31	404.74	1617.33	275.45	546.01	217.58	977.45	248.89	0.8804
Erste Group Bank AG	165.38	36.17	243.03	88.71	197.51	75.57	425.16	109.33	0.8448
Fortis Bank SA	181.14	53.34	307.71	69.10	147.26	25.52	227.5	95.5	0.8425
HSBC Bank	199.37	44.50	272.52	112.77	104.36	26.74	183.49	62.62	0.6272
ING Bank	135.08	46.35	226.44	53.71	124.98	62.01	277.93	54.35	0.7090
Intesa San Paolo Spa	351.16	122.34	705.85	129.05	310.59	111.48	607.88	126.45	0.9000
Lloyds Bank	259.13	99.94	478.52	69.14	217.96	79.13	398.72	99.09	0.9416
Mediobanca Spa	374.36	123.69	756.71	166.24	283.68	109.80	597.55	124.48	0.9400
Natixis	300.94	26.60	359.74	245.57	191.30	47.72	295.32	102.45	0.7513
Nordea Bank AB	275.56	87.43	522.79	162.63	102.70	38.94	200.37	53.53	0.9655
Société Générale SA	408.17	136.81	713.26	203.32	219.63	84.16	436.41	99.93	0.8932
The Royal Bank of Scotland	324.08	136.17	666.17	155.05	235.37	74.21	417.91	114.51	0.9353
UBS	209.55	56.96	327.40	129.70	129.63	45.26	246.24	64.64	0.8488
Unicredit Spa	377.62	146.15	835.65	168.70	352.86	125.57	711.72	149.19	0.8571
Unione di Banche Italiane	467.01	254.54	1557.55	113.94	329.04	114.28	661.02	158.81	0.8035

According to Equation (2.1), the efficient price follows a random walk process with equilibrium given by $z_{it} = 0$. If the two spreads are cointegrated, then at least one of the two series adjusts back to equilibrium in case of short-run deviations (Engle and Granger, 1987).

The contribution of price discovery can then be assessed by investigating the adjustment process to the long-run equilibrium. To do so, we estimate the following VECMs:

$$\Delta CDS_{it} = \lambda_1(z_{it-1}) + \sum_{j=1}^{p} \beta_{1j} \Delta CDS_{it-j} + \sum_{j=1}^{p} \delta_{1j} \Delta BS_{it-j} + \varepsilon_{1it}, \tag{2.2}$$

$$\Delta BS_{it} = \lambda_2(z_{it-1}) + \sum_{j=1}^{p} \beta_{2j} \Delta CDS_{it-j} + \sum_{j=1}^{p} \delta_{2j} \Delta BS_{it-j} + \varepsilon_{2it}, \tag{2.3}$$

in which λ_1 and λ_2 are the error correction coefficients, or speed of adjustment coefficients to the long-run relationship, of the CDS premiums and bond spreads, respectively, β_{1j}, β_{2j} and δ_{1j}, δ_{2j} are the short-term effects, and ε_{1it} and ε_{2it} are i.i.d. shocks.

The contribution of price discovery depends on the relative values of λ_1 and λ_2. Given our cointegration relationship, λ_1 must be significant and negative for the CDS market to adjust on the bond market to incorporate information. On the other hand, λ_2 must be significant and positive for the bond market to adjust on the CDS market. Thus, if λ_1 is significant and negative and λ_2 is not significant, then the bond market has the leading role in price discovery. If λ_2 is significant and positive and λ_1 is not significant, then the CDS market is the dominant market. In the case that both coefficients are significant, then the price discovery process occurs in both markets. In the latter situation, the dominant market in the process has the lower adjustment speed in absolute values. In other words, if the adjustment speed of CDS is lower than that of bonds ($|\lambda_1| < |\lambda_2|$), the CDS market has a dominant role in price discovery, thus, leading the bond market.

2.2.3 Empirical analysis and results

To perform the analysis, we follow a three step procedure. First, we check the stationarity of the two series via Augmented Dickey-Fuller (ADF) tests. If the variables are non-stationary, then we perform Johansen Cointegration tests to examine whether there is a long run relationship between the two markets. If the series are cointegrated, then a VECM is appropriate to check for the price discovery mechanism as the third and final step. If variables are not cointegrated, VECM is not a valid approach. Instead, one can perform Granger Causality tests using first differences. This test allows to assess whether there exists a pattern of shifts in one series preceding the other.

The results of the Augmented Dickey-Fuller tests (not reported) show that all bond and CDS series are non-stationary at 5 per cent significance level. Table 2.3 shows the results of the cointegration analysis. For 26 out of 32 banks, there exists a long run equilibrium relationship between bond and CDS spreads at 5 per cent significance level.

Given the results of cointegration, we estimate VECMs for 26 banks, while for the other 6 banks we perform Granger Causality tests. Results are presented in Tables 2.4 and 2.5.

Vector Error Correction Models show that for 12 banks the price discovery process occurs first on the CDS market. When performing Granger Causality tests for the 6 non-cointegrated banks, the bond market Granger-causes the CDS market in 3 cases, while in the other 4 cases, the CDS market Granger-causes the bond market. Therefore we can conclude that price discovery occurs in the bond market for 2 banks, it occurs in the CDS market in the 3 other cases and it occurs in both markets for the remaining bank.

This means that the market of derivatives has a leading role on bond spreads for 15 banks, implying that, for nearly 50% of the sample, the price discovery process is, to some extent, inconsistent with financial theory.

In order to find a possible link between these 15 banks, which may be able to justify the the leading role of the CDS market, we analysed and compared a number of banks' specific characteristics. Among other things, we explored descriptive statistics of CDS and bond spreads, we analysed business models, geographical location, and also balance sheet data of the banks in our sample. However, we are not able to similarities which highlight differences between the two subsamples and may justify disparities in price discovery process.

Table 2.3: Cointegration Analysis Results – This table shows the result of the cointegration analysis. Second and third columns present Johansen trace test statistics for the number of cointegrating relationship between CDS and bond spreads for each bank, under the null hypothesis that the two series are not cointegrated. The fourth column contains the number of lags in the underlying vector autoregression and last column shows whether the two series are cointegrated or not. A constant is included in the long-term relation, and the number of lags in the underlying vector autoregression is optimized using the Akaikes Information Criterion for each entity

Bank	**# of Cointegrating vectors**		**# of**	**Coint**
	None	**At most 1**	**Lags**	
Banca Monte dei Paschi di Siena	9.986	4.079	2	N
Banco Comercial Português	8.353	0.183	2	N
Banco Popolare	12.979	0.714	1	N
Banco Popular Español S.A.	**15.888**	0.661	4	Y
Banco Sabadell S.A.	**15.698**	0.775	1	Y
Banco Santander S.A.	**35.463**	0.116	2	Y
Bank of Ireland	**16.062**	0.409	1	Y
Bankia	4.8388	0.430	1	N
Barclays Bank	**18.109**	0.820	3	Y
BBVA	**27.591**	3.647	4	Y
BNL	**35.452**	2.525	1	Y
BNP Paribas	**27.320**	0.776	4	Y
Commerzbank AG	**19.007**	0.406	2	Y
Crédit Agricole	**26.209**	0.921	2	Y
Crédit Mutuel CIC	**17.081**	0.277	2	Y
Credit Suisse	**19.782**	1.181	2	Y
Deutsche Bank AG	**24.505**	1.122	2	Y
Dexia Credit Local	**16.578**	1.853	5	Y
Erste Group Bank AG	**20.094**	0.782	2	Y
Fortis Bank SA	**29.137**	0.019	3	Y
HSBC Bank	12.408	0.359	2	N
ING Bank	5.896	0.667	2	N
Intesa San Paolo Spa	**21.862**	2.152	3	Y
Lloyds Bank	**22.061**	0.055	2	Y
Mediobanca Spa	**23.775**	1.202	2	Y
Natixis	**22.841**	1.670	2	Y
Nordea Bank AB	**15.805**	0.591	3	Y
Société Générale SA	**47.236**	1.939	4	Y
The Royal Bank of Scotland	**17.696**	0.385	3	Y
UBS	**16.238**	0.184	2	Y
Unicredit Spa	**15.453**	0.998	2	Y
Unione di Banche Italiane	**17.857**	2.258	2	Y
5% Critical Value (cointegrating rank)	**15.41**	**3.76**		

Table 2.4: Price Discovery Analysis, Vector Error Correction Models results – This table shows the result of the Vector Error Correction Models for each bank. Column 2 and 3 report the value of the estimated error correction coefficients of the CDS premiums and bond spreads, respectively. Column 4 shows the leading market

Bank	λ_1	λ_2	**Leading Mkt**
Banca Monte dei Paschi di Siena		*Not Cointegrated*	
Banco Comercial Português		*Not Cointegrated*	
Banco Popolare		*Not Cointegrated*	
Banco Popular Español S.A.	-0.00325	0.0128^{***}	*CDS*
Banco Sabadell S.A.	0.00448	0.0245^{***}	*CDS*
Banco Santander S.A.	-0.0317^{***}	0.0175^{***}	*Bond*
Bank of Ireland	-0.0027	0.0737^{***}	*CDS*
Bankia		*Not Cointegrated*	
Barclays Bank	-0.0351^{***}	0.0105	*Bond*
BBVA	-0.00687	0.0431^{***}	*CDS*
BNL	-0.00981	0.0329^{***}	*CDS*
BNP Paribas	-0.0185	0.0288^{***}	*CDS*
Commerzbank AG	-0.0371^{***}	0.00475	*Bond*
Crédit Agricole	-0.0352^{***}	0.0340^{***}	*Bond*
Crédit Mutuel CIC	-0.0429^{***}	0.00451	*Bond*
Credit Suisse	-0.0278^{***}	0.00226	*Bond*
Deutsche Bank AG	-0.0366^{***}	0.0198^{**}	*Bond*
Dexia Credit Local	-0.0151^{***}	0.0103	*Bond*
Erste Group Bank AG	-0.00746	0.0209^{***}	*CDS*
Fortis Bank SA	-0.0341^{***}	0.0358^{***}	*CDS*
HSBC Bank		*Not Cointegrated*	
ING Bank		*Not Cointegrated*	
Intesa San Paolo Spa	-0.0257^{**}	0.0301^{***}	*CDS*
Lloyds Bank	-0.0391^{***}	0.0103	*Bond*
Mediobanca Spa	-0.04554^{***}	0.0030343	*Bond*
Natixis	-0.0200^{***}	0.000357	*Bond*
Nordea Bank AB	-0.0162	0.0497^{***}	*CDS*
Société Générale SA	-0.0144	0.108^{***}	*CDS*
The Royal Bank of Scotland	-0.0349^{***}	0.00907	*Bond*
UBS	-0.0292^{***}	-0.00181	*Bond*
Unicredit Spa	-0.0198^{***}	0.0156^{**}	*Bond*
Unione di Banche Italiane	-0.0147^{***}	0.0595^{***}	*CDS*

${}^{*}p < 0.1$; ${}^{**}p < 0.05$; ${}^{***}p < 0.01$

This empirical result may have relevant implications in terms of financial stability. As CDS contracts are not regulated as insurance, in periods of financial distress speculators are free to satisfy their increasing incentives by buying naked CDSs. As a consequence, a price discovery process that is structurally determined on the derivative market would imply that trading activities in CDSs are systematically able to influence the borrowing costs of financial institutions. This aspect assumes great importance when considering the over-the-counter nature of the CDS market and its low levels of transparency and regulation.

Previous studies which find similar evidence (e.g. Ammer and Cai, 2011, Delatte et al., 2012 or Coudert and Gex, 2013) argue that when economic conditions deteriorate speculation activities on the CDS market increase, resulting in strong inflows of liquidity on the derivative market. As a consequence, sellers of protection issue a larger number of CDS contracts which results in a switch of the leadership in price discovery process. However, such studies do not provide empirical evidence for this argument.

Table 2.5: Price Discovery Analysis, Granger Causality Test Results – This table shows the result of the Granger Causality Tests performed only for the non-cointegrated banks

Bank	H_0: *Bond spread does not cause CDS spread*		H_0: *CDS spread does not cause Bond spread*		**Lead Mkt**
	F-Stat	**p-value**	**F-Stat**	**p-value**	
Banca Monte dei Paschi di Siena	1.94	0.1010	5.8193	0	*CDS*
Banco Comercial Português	1.42	0.2428	5.37	0.0048	*CDS*
Banco Popolare	4.203	0.0153	14.201	0	*Both*
Bankia	1.76	0.1731	5.0844	0.0065	*CDS*
HSBC Bank	2.32	0.0991	0.2181	0.8041	*Bond*
ING Bank	9.18	0.0001	2.05	0.1298	*Bond*

Approximately half of our sample period is characterised by relative financial stability and low volatility on credit markets. For this reason, we cannot link our findings only to financial instability, as did in existing literature. We believe that further evidence, to understand better the dynamics behind the PDP, may be found by exploring the behaviour of credit risk's determinants when the CDS market is in the lead. In order to take a step further with respect to existing studies, in the next section we perform a second empirical analysis to investigate whether the leading role of CDSs is, to some extent, reflected in the behaviour of the determinants of banks' credit risk.

2.3 Implications on credit risk determinants

The aim of this section is to analyse the determinants of banks' credit risk, to assess whether they provide further evidence of the leading role of credit default swaps.

To capture the impact of the leading role of the CDS market on credit risk's determinants, our sample of banks is separated into two sub-groups on the basis of the results of the price discovery analysis carried in the previous section. The first group contains those banks for which the bond market has the lead on CDSs; the second group contains those banks for which the derivatives lead their underlying assets.

There is a significant literature that empirically analyses the factors that determine credit risk using CDS spreads as a proxy, on different sample periods and different kind of reference entities. For instance, Collin-Dufresn et al. (2001), Longstaff et al. (2005) and Bedendo and Colla (2015) focus their analysis on corporate CDSs; Pan and Singleton (2008), Longstaff et al. (2011) and Dieckmann and Plank (2012) use sovereign CDSs; Annaert et al. (2013) are among the few who study the determinants of CDSs written on banks. Those studies decompose the explained part of CDS spreads changes according to various risk determinants.

In this section we follow and complement the methodology used by the above papers. One of the main limits identified in the existing literature is the inability of the empirical specification to capture the liquidity position of the entities under analysis, as it is hard to find adequate proxies for liquidity. We propose a solution to this issue by including, among the regressors, data on the banks' recourse to central bank liquidity. The main feature of this data is that they proxy short and medium term liquidity needs of a bank.

2.3.1 Explanatory variables

To perform this analysis, we include in our specification bank's specific variables, that potentially affect directly the credit risk of a bank, and global variables, that capture market conditions and systemic risk. Table 2.6 lists the variables used in this analysis.

Table 2.6: Control Variables – This table lists the variables used in the model specification, their descriptions, the type, and their sources

Variable	Description	Type	Source
Sovereign CDS	Log-change in the (bank's home-country) sovereign credit default swap spreads	Global/Country	Datastream
Stoxx Glob 18	Change in the STOXX Global 1800 index	Global	Bloomberg
Systemic Risk	Change in the Systemic Risk Indicator	Global	ECB Statistical Data Warehouse
Risk Free	Change in the 5-year German bund	Global	Datastream
Exchange rate	Change in the USD-EUR exchange rate	Global	Bloomberg
Stock Return	Log-change in the stock price	Bank Specific	Datastream
MRO	Change in the recourse to MROs	Bank Specific	ECB Confidential
LTRO	Change in the recourse to LTROs	Bank Specific	ECB Confidential

Global and risk factors

As in Annaert et al. (2013), we take into consideration variables that reflect global equity and fixed income markets conditions. In order to capture the business climate, we use the Stoxx Global 1800 index returns and we expect a negative relation between this index and credit spreads: the improvement in global markets' climate decreases default probabilities. A second global factor taken into consideration is the risk free rate. Since we use the five-year CDS spread as dependent variable, we measure the risk-free rate using the 5 year German spot rate.[3] In the Merton (1974) model, the riskless interest rate constitutes the drift in the risk neutral world. The higher it is, the less likely default becomes. This can also be explained in macroeconomic terms: an increase in the risk free interest rate can be linked to economic growth and to improvements in macroeconomic conditions, hence it should reduce default probabilities. For this reason, a negative relationship with credit spreads is expected. In addition, currency market variations are captured by including changes in the US Dollar-Euro exchange rate.

One innovation of our analysis is to include, in our specification, a variable that captures global risk. Specifically, we include the Systemic Risk Measure produced by the European Central Bank. The ECB defines this indicator as measurement of joint default risk of Euro area large and complex banking groups (ECB, 2012). This variable is expected to have a positive relationship with credit spreads.

Finally, among the global factors, we include the sovereign risk of the bank's home country. Given the key role that banks play in the actual structure of the financial system and the efforts made by governments during the crisis to avoid default of big financial institution, we believe that sovereign risks play a key role among the determinants of banks' credit spreads. This is confirmed by the conclusion of Avino and Cotter (2014), in which the existence of high interconnections between banks and sovereigns credit default swaps is shown. Also in this case, a positive relationship between the two variables is expected.

Idiosyncratic factors

The first idiosyncratic variable used to explain credit risk is the stock return. Stock prices evolution reflect profitability and, at the same time, future prospects of a bank. Positive stock returns reflect better overall expectations, thus we expect a negative relationship between returns and credit spreads.

Another innovation of this analysis is that we include, in the specification, the recourse to central bank liquidity. We believe that changes in the recourse operations with the central bank could be considered as an adequate proxy of the internal liquidity position of a bank. In fact, under a situation of liquidity drain, a bank will increase its

[3] We also use 1-year and 10-year rates for robustness checks, obtaining similar results

recourse to central bank operations. Specifically, we include the following ECB liquidity providing operations: main refinancing operations (MROs), lending operations carried by the Eurosystem with a maturity of seven days, and longer term refinancing operations (LTROs), lending operations with a maturity above three months.[4] We expect a positive relationship between credit spreads and these two variables. One important characteristic of this data is that the European Central Bank treats it as confidential and, as such, it is not available to the public.

Our sample of banks includes also institutions that have their headquarters outside the Eurozone. However, all the banks in our sample have access, directly or indirectly, to Eurosystem refinancing operations. In order to make our analysis as consistent as possible, we use refinancing operation data at group level on an aggregate basis for those banks which operate in the Eurozone only through subsidiaries.

2.3.2 Empirical analysis and results

Estimation Technique

Given the selection of variables we estimate the following ordinary least squares regression:

$$\begin{aligned}\Delta\log(CDS_{it}) =& \alpha_i + \gamma\Delta\log(StockP_{it}) \\ & + \lambda\Delta O_{it} + \eta\Delta X_t + \beta\Delta\log(SovCDS_{jt}) + \varepsilon_{it}\end{aligned}, \tag{2.4}$$

in which i is the subscript identifying the bank, t indicates the time period, and j indicates the residence country of the bank. $\Delta\log(CDS_{it})$ is the log change in CDS spread; $\Delta\log(StockP_{it})$ is the log change in the stock price; ΔO_{it} is the change in the outstanding refinancing credit within the ECB; ΔX_t is the change in the global factors which captures any fixed effects across time; while α_i is bank's i fixed effect. However, in the result reported, we will restrict the intercept term to be constant across banks, since the F-test does not reject this restriction at any significance level.

Like in the previous section, the data span from the period starting from 1 January

[4] We do not make any distinction between standard and non-standard LTROs. As a consequence, this variable includes: standard three months LTROs, regularly carried by the ECB, and the non-standard three years LTROs launched by the ECB in December 2011 and February 2012.

2011 to 31 December 2013. Due to a different frequency of some of the explanatory variables, in this section we use monthly instead of daily data. This results in 1112 (unbalanced) panel observations.

Regression results

Explorative univariate regressions (not reported) show that all variables included in our specification are empirically linked to banks' CDS spread changes.

The results are presented in Table 2.7 and are in line with our conjectures and with existing literature. The first column presents the outcome of our estimates for the full sample of banks. Over the sample period, explanatory variables are able to explain up to 66% of the variation in CDS spread changes. All global and idiosyncratic factors carry the right sign and are statistically significant. As in Bedendo and Colla (2015) we find empirical evidence of spillover effects of risk form sovereign to sample entities. Increase in sovereign risk explains positive variations of banks' credit risk. The estimated effects of stock returns, risk free return and market return are consistent with Annaert et al. (2013): we find that an increase in all these variables has a negative effect on credit default swap spreads variations.

Considering the explanatory variables that have been used for the first time by this study, the results of the regressions are consistent with our expectations. The relationship between credit spreads and the Systemic Risk Indicator is positive and significant. The ECB providing operations coefficients (for both MROs and LTROs) are significant and positive. We find empirical evidence that changes in the recourse to refinancing operations have a positive effect on CDS spreads. This effect is stronger for the recourse to liquidity with shorter maturity. Consistent with our initial conjectures, more frequent access to refinancing operations reflects worsening credit condition perceptions, especially in the short term.

The main objective of this second analysis is to investigate whether the leading role of credit default swaps is also linked to a different impact of risk determinants on banks' credit risk. Columns 2 and 3 of Table 2.7 present the empirical results for the banks grouped on the basis of the price discovery analysis carried in Section 2.2.

Table 2.7: Credit Risk Determinants Analysis Results – This table shows the results of pooled regressions (no fixed effects), robust standard errors are reported in parentheses. Column (I) represents the result for the full sample of banks; column (II) shows the results for the first group of banks, for which the bond market has the leading role in price discovery process; column (III) shows the results for the second group of banks, for which the CDS market has the leading role in price discovery process

Variable	**(I)**	**(II)**	**(III)**
Sovereign CDS	0.1963^{***}	0.109^{***}	0.2983^{***}
	(0.0234)	(0.0327)	(0.0339)
Stock Return	-0.0540^{**}	-0.0473	-0.0516^{*}
	(0.0212)	(0.0330)	(0.0276)
MRO	0.005^{**}	0.0075^{*}	0.0041^{*}
	(0.002)	(0.0039)	(0.0023)
LTRO	0.0037^{***}	0.0039^{**}	0.0034^{**}
	(0.0011)	(0.0018)	(.00013)
Stoxx Glob 18	-0.0013^{***}	-0.0013^{**}	-0.0013^{**}
	(0.0004)	(0.0006)	(0.0006)
Systemic Risk	0.0354^{***}	0.0381^{***}	0.0317^{***}
	(0.0022)	(0.0028)	(0.0032)
Risk Free	-0.0648^{***}	-0.0591^{***}	-0.0681^{***}
	(0.0146)	(0.0205)	(0.0206)
Exchange Rate	-0.4250^{**}	-0.4680^{**}	-0.3510
	(0.1688)	(0.2111)	(0.2681)
N	1112	595	517
R^2	0.661	0.636	0.698

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Regressions results for the first group of banks are similar to those obtained for the whole sample. All the variables have the same signs; nevertheless, the coefficient of stock returns loses its significance. Similar conclusions can be drawn for the second group of banks. However, in this case, it is the coefficient of exchange rate that loses the significance. For both regressions we obtain adjusted R-squared similar to the one obtained for the full sample.

The main empirical difference between the two groups concerns the magnitude of the impact of sovereign risk on banks' credit spreads. When the CDS market has the lead in price discovery, the estimated coefficient of the changes in sovereign CDSs is more than twice of the same coefficient when the bond market is in the lead. Precisely, our estimates indicate that a 1% increase in sovereign spreads translates into an increase in bank spreads of 0.11% for the first group, and an increase of 0.3% for the second group. The difference between the two estimates is statistically different than zero. This suggests a different risk transfer mechanism from sovereign to financial entities when prices of banks' bonds are led by credit default swaps. Specifically, the different impact of sovereign risk on CDS spreads is stronger for those banks for which the credit default swaps market has the leading role in price discovery process.

This asymmetric impact of sovereign risk within the two groups, also in the light of the findings of existing studies, could suggest the existence of a threat to financial stability posed by CDS trading. As seen in the previous section, existing studies argue that speculation in CDS can affects the prices of sovereign bonds. If this is the case and, at the same time, (i) the CDS market is able to lead prices of bonds issued by some banks and (ii) the sovereign risk has a stronger impact on the credit risk of these banks, then speculation activities in CDS, on sovereign and financial entities, could trigger a dangerous vicious circle. A circle which could exacerbate financial distress and possibly lead, in an extreme scenario, to the default of financial institutions.

Robustness check

In order to check the robustness of the stronger impact of sovereign risk on CDS spreads changes when the derivative has a leading role in price discovery, we perform two robustness tests that examine different samples of the data or use alternative data definitions. Table 2.8 reports the results of the tests.

Table 2.8: Credit Risk Determinants Robustneess check – This table shows the results of pooled regressions (no fixed effects), robust standard errors are reported in parentheses. Column (I) and (II) represent the result of the regressions with the addition of the change in the MSCI country indices. Column (I) represent the first group of banks; column (II) the second group of banks. Column (III) and (IV) represent the results of the regressions excluding the period of financial distress. Column (III) represent the first group of banks; column (IV) the second group of banks

Variable	**(I)**	**(II)**	**(III)**	**(IV)**
Sovereign CDS	0.0925^{***}	0.2481^{***}	0.0628^{*}	0.1516^{***}
	(0.0319)	(0.0323)	(0.0370)	(0.0482)
Stock Return	-0.0135	-0.0359	0.0146	-0.0326
	(0.0308)	(0.0286)	(0.0401)	(0.0374)
MRO	0.0084^{**}	0.0040^{*}	0.0042	0.0071^{*}
	(0.0040)	(0.0022)	(0.0074)	(0.0042)
LTRO	0.0035^{**}	0.0031^{**}	-0.0002	0.0054^{*}
	(0.0015)	(0.0012)	(0.0021)	(0.0030)
Stoxx Glob 18	0.0015^{**}	0.0001	-0.0032^{***}	-0.0041^{***}
	(0.0007)	(0.0007)	(0.0009)	(0.0010)
Systemic Risk	0.0312^{***}	0.0263^{***}	0.0618^{***}	0.0513^{***}
	(0.0029)	(0.0032)	(0.0038)	(0.0049)
Risk Free	-0.0539^{***}	-0.0599^{***}	-0.0761^{***}	-0.1196^{***}
	(0.0188)	(0.0199)	(0.0222)	(0.0268)
Exchange Rate	-0.1397	-0.2046	-0.1537	-0.0440
	(0.2040)	(0.2514)	(0.2551)	(0.3550)
MSCI Country	-0.0007^{***}	-0.0008^{***}		
	(0.0001)	(0.0002)		
N	595	517	323	285
R^2	0.665	0.715	0.606	0.553

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

One possible concern relates to the characteristics of the sovereign CDSs and to the fact that the country of residence may influence the outcome of the estimates. In order to check the robustness against this, we include, as additional variable, the changes in the MSCI index of a bank's home country. This variable, which measure the stock market performance in a given region, allows us to control for country specific characteristics that may influence banks' credit risk, but are not captured by the home country sovereign risk. The results, reported in columns 1 and 2, show that coefficients on sovereign CDSs are essentially unchanged and remain highly statistically significant, for both groups of banks.

A second concern is that the results might be affected by the sovereign debt crisis. Thus, we estimate the regression only after mid-2012. The results, presented in columns 3 and 4, remain qualitatively unchanged.

2.4 Conclusion

In this work we assess the price discovery process between bonds and CDSs written on European banks, and we explore the implications of a leading role of the CDS market on banks' credit risk determinants.

Whereas there is substantial literature on bond-CDS price discovery, empirical studies focusing on the banking sector are scarce and, to the best of our knowledge, there is no literature that empirically explores the implications of a leading role of the derivative market.

We concentrate our analysis on a sample of 32 European banks, over the period between 2011 and 2013. Our main findings can be summarised as follows: (i) for 15 banks, the CDS market has the leading role in price discovery process on the underlying bond market; (ii) for this subgroup of banks, when analysing the determinants of credit risk, a stronger impact of sovereign risk on banks' credit risk is observed.

Our findings suggest that trading activities in CDSs can influence the financing cost of some banks. As pointed out by previous studies, this empirical evidence can be driven by multiple reasons, such as global conditions of the financial system, idiosyncratic characteristics of a bank, or specific features of CDS contracts. Our study, in contrast to existing literature, explores the implications of this evidence on credit risk's determinants.

We show that, when the derivative market has the leading role in price discovery, the home country sovereign risk has a significantly stronger impact on banks' CDS spreads, while all other risk determinants do not change their magnitude.

Our results, when read together with the conclusions of previous studies on sovereign CDSs price discovery process, could indicate the risk of a dangerous vicious circle which might have its inception in CDSs trading. If CDSs are able to influence the financing cost of some banks and, at the same time, the sovereign risk has stronger impact on their credit risk, then it may be the case that trading in CDS could put financial institutions under serious distress not necessary driven by worsening fundamentals. This may occur in particular if the price discovery process takes place on the derivative market also for sovereign entities, as empirically shown by existing literature.

Our study does not provide evidence on the mechanism of the overall phenomenon, nor can our results be generalised to other contexts or periods. We cannot neither conclude that CDSs lead underlying bonds because of the stronger transaction mechanism or, to the contrary, that the stronger effect of sovereign risk on banks' credit risk is due to the price discovery process. Nevertheless, our study provides further empirical evidence of a potential market weakness that may have several implications on the credit risk of financial entities. Given the potential harmful implications of this evidence, we believe that further analysis is needed to understand whether naked CDSs are the reason behind the leading role of the derivative market.

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Chapter 3

Optimal portfolios with credit default swaps

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Abstract

Using a continuous-time, stochastic, and dynamic framework, this study derives a closed-form solution for the optimal investment problem for an agent with hyperbolic absolute risk aversion preferences for maximising the expected utility of his or her final wealth. The agent invests in a frictionless, complete market in which a riskless asset, a (defaultable) bond, and a credit default swap written on the bond are listed. The model is calibrated to market data of six European countries and assesses the behaviour of an investor exposed to different levels of sovereign risk. A numerical analysis shows that it is optimal to issue credit default swaps in a larger quantity than that of bonds, which are optimally purchased. This speculative strategy is more aggressive in countries characterised by higher sovereign risk. This result is confirmed when the investor is endowed with a different level of risk aversion.

Keywords Credit Default Swap · Optimal Dynamic Programming · Hyperbolic Absolute Risk Aversion

JEL classification G11 · G20 · G12

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3.1 Introduction

During the past 15 years, the market of credit default swaps (CDSs) has become one of the largest segments of derivatives markets, reaching its peak at the beginning of the global financial crisis. At the end of June 2008, the total notional amount of outstanding CDS contracts was USD 57.325 trillion.[1]

CDSs are bilateral derivative contracts under which an investor can buy protection against the credit risk of a reference entity by paying a periodic premium to the seller. This feature makes CDS functionally equivalent to insurance contracts. However, the main difference with respect to an insurance contract is the possibility of buying a CDS without owning the underlying asset (i.e. buying the derivative in the so-called uncovered or "naked" form).

The strong growth of the market and the possibility of buying uncovered CDSs has raised concerns regarding their use. One of the main arguments is that speculation on CDSs exacerbated the recent European sovereign crisis, driving CDS premia of some (distressed) countries to record highs and, consequently, influencing their cost of funding (Haugh et al., 2009, Sgherri and Zoli, 2009 and Fontana and Scheicher, 2016).

The main practical consequence of these concerns has been the adoption, by the European Council, of a regulation aiming to ban any person or legal entity in the EU from entering into naked, or uncovered, CDS on sovereign debt. The regulation entered into force in November 2012.[2] The main objective of this intervention was to reduce the magnitude of speculation on sovereign CDSs. However, a number of weaknesses have been highlighted. Juurikkala (2012), for instance, lists three issues that threaten the effectiveness of the ban: (i) the over-the-counter nature of CDS contracts; (ii) the worldwide framework of financial markets; and (iii) the absence of a similar rule in the US.

In this work, we study the optimal asset allocation for an agent who wants to maximise the expected utility of his or her final wealth by investing in a riskless asset, a (defaultable) bond and a CDS written on the bond. This study aims to understand whether

[1] Bank for International Settlements, OTC derivatives market activity in the first half of 2008, available at: http://www.bis.org/publ/otc_hy0811.pdf.

[2] Regulation (EU) No 236/2012 of the European Parliament and of the Council of 14 March 2012 on short selling and certain aspects of CDSs.

it is optimal for such an agent to invest in CDSs.

Investment decisions in the presence of CDSs are closely related to portfolio choices with mortality contingent claims. Actually, the default process of a firm can be modelled as the force of mortality of an individual or a population. There is extensive literature that explores this subject and successfully models the force of mortality by making use of well-known results about stochastic processes (see, e.g., Dahl, 2004; Biffis, 2005; Menoncin, 2008; Menoncin, 2009). However, literature on optimal investment choices with CDSs is scarce.

In this study we use a framework similar to that presented in Menoncin and Regis (2015), but we apply it to the case of an investment in CDS. We provide a closed-form solution to the problem of an agent endowed with a general hyperbolic absolute risk aversion (HARA) class of preferences. In addition, we calibrate the model on market data of six European countries, to assess the behaviour of an investor exposed to different degrees of sovereign risk. In this way, the investment strategy can be considered that of a financial institution that can sell or buy credit risk protection on a sovereign entity.

The results of the calibration show that (i) it is optimal to invest in CDSs and, specifically, to sell credit protection instead of buying it, and (ii) speculation in CDSs plays a crucial role in investment strategy. In the calibrated optimal portfolio, the investor always issues more CDSs than the bonds held in the portfolio. In addition, the magnitude of speculation is directly linked with the underlying risk of the reference entity: the higher is the sovereign risk, the stronger is the speculation. This result is robust to changes in the investor's risk aversion.

Few studies empirically analyse the role of CDS trading on the European sovereign market and their conclusions are, to some extent, related to the results of our calibrated model. For instance, Delatte et al. (2012) highlight the link between the underlying risk of the reference entity and the magnitude of the speculation in CDSs. In assessing the potential influence of the growing CDS market on the borrowing cost of sovereign states during the European sovereign crisis, the authors conclude that speculation is a significant driver of activity in the CDS market during distress periods. A second empirical study that considers the role of speculation on the CDS market is Chiarella et al. (2015). In that study, a heterogeneous agent model is estimated to address whether the recent movements in European sovereign credit spreads are driven by weakened fundamentals or momentum trading behaviour. The authors conclude that, for troubled peripheral European countries, momentum or non-fundamental trading played a dominant role in increasing their sovereign CDS spreads beyond the levels justified by weakening fundamentals.

In our calibrated model, one consequence of the speculation strategy of the investor is an increase in liquidity on the CDS market. The market becomes more liquid as the investor optimally issues more and more CDSs. The role of liquidity has been investigated by a number of studies. For instance, Badaoui et al. (2013) argue that the surge in CDS spreads observed during the sovereign crisis was mainly due to a rise in market liquidity rather than an increase in default intensity. Aizenman et al. (2013) and Dewachter et al. (2015) analyse the role of macroeconomic fundamentals on CDS spreads in the Euro area. Both studies show how fundamentals are not able to fully explain changes in the sovereign risk of peripheral countries and show that liquidity effects and market overreaction play a dominant role during distress periods. Similarly, in assessing the sovereign credit and liquidity spread interactions over the recent periods of crisis, Calice et al. (2013) find that for several countries, including Greece, Ireland, and Portugal, the liquidity of the sovereign CDS market had a relevant influence on sovereign bond credit spreads. The authors argue that a sovereign debt market failure for several Eurozone countries was prevented by coordinated EU action.

The rest of this chapter is structured as follows. Section 3.2 presents the model set-up, while in Section 3.3, investors' preferences are described and the portfolio optimisation problem is solved in closed form. In Section 3.4, a calibration of the model, based on data of six European countries, is presented. Finally, Section 3.5 concludes.

3.2 The model set-up

3.2.1 State variables

On a continuously open and frictionless financial market over the time set $[t_0, +\infty[$, the economic framework is described by a set of s state variables $z(t) \in \mathbb{R}^s$ which solve the following (matrix) stochastic differential equation:

$$\underset{s\times 1}{dz\,(t)} = \underset{s\times 1}{\mu_z\,(t,z)}dt + \underset{s\times n}{\Omega\,(t,z)'}\underset{n\times 1}{dW\,(t)}, \tag{3.1}$$

where $z\,(t_0)$ is a deterministic vector that defines the initial state of the system, $W\,(t)$ is a vector of n independent Wiener processes,[3] and the prime denotes transposition. The usual properties for guaranteeing the existence of a strong solution to equation (3.1) are assumed to hold.

3.2.2 Financial market

In the financial market n risky assets are traded. Their prices $S\,(t) \in \mathbb{R}^n_+$ solve the (matrix) stochastic differential equation

$$\underset{n\times 1}{I_S^{-1}}\underset{n\times 1}{dS\,(t)} = \underset{n\times 1}{\mu\,(t,z)}dt + \underset{n\times n}{\Sigma\,(t,z)'}\underset{n\times 1}{dW\,(t)}, \tag{3.2}$$

where I_S is a diagonal matrix containing the elements of vector $S\,(t)$. The initial asset prices $S\,(t_0)$ are deterministic. Finally, a riskless asset exists, whose price $G\,(t) \in \mathbb{R}_+$ solves the ordinary differential equation

$$G\,(t)^{-1}\,dG\,(t) = r\,(t,z)\,dt, \tag{3.3}$$

where $r\,(t,z) \in \mathbb{R}_+$ is the instantaneously riskless interest rate. We assume $G\,(t_0) = 1$, i.e. the riskless asset is the *numéraire* of the economy. The financial market is assumed to be arbitrage free and complete. In other words, a unique vector of market prices of risk $\xi\,(t,z) \in \mathbb{R}^n$ exists, such that $\Sigma\,(t,z)'\,\xi\,(t,z) = \mu\,(t,z) - r\,(t,z)\,\mathbf{1}$, where $\mathbf{1}$ is a vector of ones (i.e. $\exists!\ \Sigma\,(t,z)^{-1}$).

Girsanov's theorem allows us to switch from the historical ($\mathbb{P}$) to the risk-neutral probability $\mathbb{Q}$ by using $dW^{\mathbb{Q}}\,(t) = \xi\,(t,z)\,dt + dW\,(t)$. The value in t_0 of any cash flow

[3] The case with dependent Wiener processes can be easily obtained through Cholesky's decomposition.

$\Xi(t)$ available at time t can be written as

$$\begin{aligned}\Xi(t_0) &= \mathbb{E}_{t_0}^{\mathbb{Q}}\left[\Xi(t)\frac{G(t_0)}{G(t)}\right] = \mathbb{E}_{t_0}^{\mathbb{Q}}\left[\Xi(t)\,e^{-\int_{t_0}^{t} r(u,z)du}\right] \\ &= \mathbb{E}_{t_0}\left[\Xi(t)\,m(t_0,t)\,e^{-\int_{t_0}^{t} r(u,z)du}\right],\end{aligned}$$

where $\mathbb{E}_{t_0}[\bullet]$ and $\mathbb{E}_{t_0}^{\mathbb{Q}}[\bullet]$ are the expected value operators under the historical ($\mathbb{P}$) and the risk neutral ($\mathbb{Q}$) probabilities respectively, conditional on the information set at time t_0, and the martingale $m(t_0,t)$, such that $m(t_0,t_0)=1$, solves $m(t_0,t)^{-1}\,dm(t_0,t) = -\xi(t,z)\,dW(t)$.

3.2.3 Credit risk market

Among the risky assets, whose prices are gathered in vector $S(t)$ in equation (3.2), we assume there is a derivative written on the credit risk of an underlying asset. This credit risk is measured by a default intensity $\lambda(t,z)$, which is one of the state variables, whose values are gathered in $z(t)$. It is easy to demonstrate (e.g. Duffie and Singleton, 2003) that the value in t_0 of an asset paying $\Xi(t)$ monetary units in t and whose issuer may go bankrupt with intensity $\lambda(t,z)$ is given by

$$\mathbb{E}_{t_0}^{\mathbb{Q}}\left[\Xi(t)\,e^{-\int_{t_0}^{t} r(u,z)+\lambda(u,z)du}\right],$$

where $\mathbb{E}_{t_0}^{\mathbb{Q}}[\bullet]$ is the expected value under so-called risk neutral probability, given the information set available at time t_0. Furthermore, the value of cash flow available at the default time τ (we call this $\Xi(\tau)$) is given by

$$\mathbb{E}_{t_0}^{\mathbb{Q}}\left[\int_{t_0}^{\infty}\Xi(s)\,\lambda(s)\,e^{-\int_{t_0}^{s} r(u,z)+\lambda(u,z)du}ds\right],$$

where we assume that the default time is defined on the interval $[t_0,+\infty[$ (see Lando, 1998).

In this study, we analyse the case of a CDS written on a bond. The CDS is a derivative in which the protection buyer pays a spread at fixed dates, while the protection seller engages to pay the loss given default (LGD) on a certain reference entity if it goes bankrupt before the expiration of the derivative. The value of the CDS is presented in Subsection 4.1.1.

3.3 Investor's maximisation problem

3.3.1 Investor's wealth

The investor holds $\theta_S(t) \in \mathbb{R}^n$ units of the risky assets and $\theta_G(t) \in \mathbb{R}$ units of the riskless asset. Thus, at any instant in time, the total value of the investor's assets (i.e. his or her financial wealth) $R(t)$ is given by the static budget constraint

$$R(t) = \theta_S(t)' S(t) + \theta_G(t) G(t), \tag{3.4}$$

whose differential is the dynamic budget constraint

$$dR(t) = \theta_S(t)' dS(t) + \theta_G(t) dG(t) + \underbrace{d\theta_S(t)' (S(t) + dS(t)) + d\theta_G(t) G(t)}_{dR_a(t)}. \tag{3.5}$$

The first two components on the right hand side of equation (3.5) account for the changes in prices. The $dR_a(t)$ component, which accounts for the dynamic adjustment of the portfolio allocation, must take into account the intensity of default between t and $t + dt$, which is given by $\lambda(t, z) dt$. Thus, the investor's wealth dynamics are

$$dR(t) = \theta_S(t)' dS(t) + \theta_G(t) dG(t) + \lambda(t, z) R(t) dt. \tag{3.6}$$

Once the static budget constraint (3.4) and the asset differentials (3.2) and (3.3) are suitably taken into account, $dR(t)$ becomes

$$\begin{aligned} dR(t) &= \big(R(t)(r(t,z) + \lambda(t,z)) + \theta_S(t)' I_S (\mu(t,z) - r(t,z)1)\big) dt \\ &\quad + \theta_S(t)' I_S \Sigma(t,z)' dW(t). \end{aligned} \tag{3.7}$$

3.3.2 Investor's preferences and objective

The investor obtains utility from the wealth at the end of the financial horizon

$$U_R(R(T)) = \frac{(R(T) - R_m)^{1-\delta}}{1-\delta},$$

where $\delta > 1$ and the constant R_m can be interpreted as the minimum subsistence value of final wealth. This utility belongs to the HARA family. In fact, the Arrow-Pratt absolute risk aversion index is $\delta/(R(T) - R_m)$. Accordingly, the higher R_m, the higher the risk aversion: an agent who has to guarantee a higher minimum level of final wealth will choose a safer investment. The case of constant relative risk aversion preferences is obtained with $R_m = 0$.

The investor chooses $\theta_S(t)$ which maximises the expected utility of final wealth if the credit event has not occurred yet:

$$\max_{\theta_S(t)} \mathbb{E}_{t_0}\left[\frac{(R(T) - R_m)^{1-\delta}}{1-\delta} e^{-\int_{t_0}^{T} \rho(u,z)+\lambda(u,z)du}\right], \tag{3.8}$$

where $\rho(t, z)$ is a possibly stochastic subjective discount rate. The budget constraint equalises the initial wealth to the expected present value of the final wealth under the risk neutral probability:

$$R(t_0) = \mathbb{E}_{t_0}^{\mathbb{Q}}\left[R(T)\, e^{-\int_{t_0}^{T} r(u,z)+\lambda(u,z)du}\right]. \tag{3.9}$$

3.3.3 The optimal portfolio

Problem (3.8) under the constraint (3.9) can be solved either through dynamic programming (via the so-called Hamilton-Jacobi-Bellman equation) or through the so-called martingale approach. This latter method is viable in our framework because of market completeness.

Proposition 1. *The optimal portfolio-solving problem (3.8) is*

$$\begin{aligned} I_S \theta_S^* (t) \quad = \quad & \frac{R(t) - H(t,z)}{\delta} \Sigma(t,z)^{-1} \xi(t,z) \\ & + \frac{R(t) - H(t,z)}{F(t,z)} \Sigma(t,z)^{-1} \Omega(t,z) \frac{\partial F(t,z)}{\partial z} \\ & + \Sigma(t,z)^{-1} \Omega(t,z) \frac{\partial H(t,z)}{\partial z}, \end{aligned} \tag{3.10}$$

where

$$H(t,z) = \mathbb{E}_t^{\mathbb{Q}} \left[R_m e^{-\int_{t_0}^T r(u,z) + \lambda(u,z) du} \right], \tag{3.11}$$

$$F(t,z) = \mathbb{E}_t^{\mathbb{Q}_\delta} \left[e^{-\int_t^T \left(\frac{\delta - 1}{\delta} r(u,z) + \frac{1}{\delta} \rho(u,z) + \lambda(u,z) + \frac{1}{2} \frac{1}{\delta} \frac{\delta - 1}{\delta} \xi(u,z)' \xi(u,z) \right) du} \right], \tag{3.12}$$

$$dW(t)^{\mathbb{Q}_\delta} = \frac{\delta - 1}{\delta} \xi(t,z) dt + dW(t). \tag{3.13}$$

Proof. See Appendix 3.A. □

In the solution, we used the new probability measure $\mathbb{Q}_\delta$ defined in equation (3.13). It has two relevant properties: (i) for a log-utility agent, that is $\delta = 1$, the probability $\mathbb{Q}_\delta$ coincides with the historical probability; (ii) when the agent is infinitely risk averse, that is $\delta \to +\infty$, the probability $\mathbb{Q}_\delta$ coincides with $\mathbb{Q}$. In fact, we can think of the Wiener processes under $\mathbb{Q}_\delta$ as a weighted mean of the Wiener processes under the risk neutral and the historical probabilities:

$$dW(t)^{\mathbb{Q}_\delta} = \left(1 - \frac{1}{\delta}\right) dW(t)^{\mathbb{Q}} + \frac{1}{\delta} dW(t). \tag{3.14}$$

Some important properties of the optimal portfolio are worth highlighting.

- The function $H(t,z)$ is the expected value, under $\mathbb{Q}$, of the minimum final wealth R_m appropriately discounted for both financial and credit risk.
- The function $F(t,z)$ is the expected value (under the preference-adjusted measure $\mathbb{Q}_\delta$) of discount factors taking into account both the financial risk and the credit risk and, thus, it can be thought of as a "global" discount factor.

- We remark that the difference $R(t) - H(t,z)$ is relevant for computing the optimal portfolio, which depends also on the sensitivities of $H(t,z)$ and $F(t,z)$ with respect to the state variables z.

- We identify three components in the demand for the risky assets: (i) a speculative component, related to the risk premium ξ, (ii) a hedging component against the fluctuations of the global discount factor $F(t,z)$, and (iii) a hedging component against the fluctuations of the expected imbalance to finance minimum wealth $H(t,z)$.

- The last two components depend on: (i) the risk aversion of the individual, (ii) the variance-covariance matrix of the state variables, and (iii) the sensitivities of $F(t,z)$ and $H(t,z)$ with respect to changes in the state variables.

3.4 A portfolio with bond and CDS

3.4.1 Two state variable – Two asset case

We take into account a framework with two independent state variables

$$\begin{bmatrix} dz_1(t) \\ dz_2(t) \end{bmatrix} = \begin{bmatrix} \mu_{z_1} \\ \mu_{z_2} \end{bmatrix} dt + \underbrace{\begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix}}_{\Omega'} \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix},$$

and two financial assets which are derivatives on these variables:

$$\begin{bmatrix} \frac{dS_1(t)}{S_1(t)} \\ \frac{dS_2(t)}{S_2(t)} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} dt + \underbrace{\begin{bmatrix} \eta_{S_1,z_1}\omega_1 & \eta_{S_1,z_2}\omega_2 \\ \eta_{S_2,z_1}\omega_1 & \eta_{S_2,z_2}\omega_2 \end{bmatrix}}_{\Sigma'} \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix}.$$

where $\eta_{S_i,z_j} \equiv \frac{\partial S_i}{\partial z_j}\frac{1}{S_i}$ are the semi-elasticities. Thus,

$$\Sigma^{-1} = \begin{bmatrix} \eta_{S_1,z_1}\omega_1 & \eta_{S_2,z_1}\omega_1 \\ \eta_{S_1,z_2}\omega_2 & \eta_{S_2,z_2}\omega_2 \end{bmatrix}^{-1} = \frac{1}{\left(\eta_{S_1,z_1}\eta_{S_2,z_2} - \eta_{S_1,z_2}\eta_{S_2,z_1}\right)\omega_1\omega_2} \times \begin{bmatrix} \eta_{S_2,z_2}\omega_2 & -\eta_{S_2,z_1}\omega_1 \\ -\eta_{S_1,z_2}\omega_2 & \eta_{S_1,z_1}\omega_1 \end{bmatrix},$$

$$\Sigma^{-1}\Omega = \frac{1}{\eta_{S_1,z_1}\eta_{S_2,z_2} - \eta_{S_1,z_2}\eta_{S_2,z_1}} \begin{bmatrix} \eta_{S_2,z_2} & -\eta_{S_2,z_1} \\ -\eta_{S_1,z_2} & \eta_{S_1,z_1} \end{bmatrix}.$$

Finally, with $\xi = \begin{bmatrix} \xi_1 & \xi_2 \end{bmatrix}'$, the optimal portfolio is given by

$$\begin{aligned} \begin{bmatrix} S_1\theta_{S_1} \\ S_2\theta_{S_2} \end{bmatrix} = & \frac{R(t) - H(t,z)}{\delta\left(\eta_{S_1,z_1}\eta_{S_2,z_2} - \eta_{S_1,z_2}\eta_{S_2,z_1}\right)\omega_1\omega_2} \begin{bmatrix} \eta_{S_2,z_2}\omega_2\xi_1 - \eta_{S_2,z_1}\omega_1\xi_2 \\ -\eta_{S_1,z_2}\omega_2\xi_1 + \eta_{S_1,z_1}\omega_1\xi_2 \end{bmatrix} \\ & + \frac{R(t) - H(t,z)}{\eta_{S_1,z_1}\eta_{S_2,z_2} - \eta_{S_1,z_2}\eta_{S_2,z_1}} \begin{bmatrix} \eta_{S_2,z_2}\eta_{F,z_1} - \eta_{S_2,z_1}\eta_{F,z_2} \\ -\eta_{S_1,z_2}\eta_{F,z_1} + \eta_{S_1,z_1}\eta_{F,z_2} \end{bmatrix} \\ & + \frac{H(t,z)}{\eta_{S_1,z_1}\eta_{S_2,z_2} - \eta_{S_1,z_2}\eta_{S_2,z_1}} \begin{bmatrix} \eta_{S_2,z_2}\eta_{H,z_1} - \eta_{S_2,z_1}\eta_{H,z_2} \\ -\eta_{S_1,z_2}\eta_{H,z_1} + \eta_{S_1,z_1}\eta_{H,z_2} \end{bmatrix}. \end{aligned} \tag{3.15}$$

3.4.2 The state variables

We take into account a setting where there are two state variables: the instantaneously riskless interest rate $r(t)$ and the default intensity $\lambda(t)$ (i.e. $z(t) = \begin{bmatrix} r(t) & \lambda(t) \end{bmatrix}'$). Furthermore, the two state variables are assumed to be independent and follow a mean reverting square root process:

$$\begin{bmatrix} dr(t) \\ d\lambda(t) \end{bmatrix} = \begin{bmatrix} a_r\left(b_r - r(t)\right) \\ a_\lambda\left(b_\lambda - \lambda(t)\right) \end{bmatrix} dt + \begin{bmatrix} \sigma_r\sqrt{r(t)} & 0 \\ 0 & \sigma_\lambda\sqrt{\lambda(t)} \end{bmatrix} \begin{bmatrix} dW_r(t) \\ dW_\lambda(t) \end{bmatrix}, \tag{3.16}$$

where $r(t_0)$ and $\lambda(t_0)$ are both known and for $i \in \{r, \lambda\}$, $a_i > 0$ is the (constant) strength of the mean reversion effect, $b_i > 0$ is the (constant) long term mean which the process reverts towards. Here, we assume that $2a_ib_i \geq \sigma_i^2$ so that both $r(t)$ and $\lambda(t)$ are always positive.

In this section we will use the following result.

Proposition 2. *If the stochastic variable $X(t)$ follows the process*

$$\begin{aligned} dX(t) &= a(b - X(t))\,dt + \sigma\sqrt{X(t)}dW(t)\,, \\ X(t_0) &= X_0, \end{aligned}$$

then

$$\begin{aligned} V(t,T) &= \mathbb{E}_t\left[(1-\chi+\chi X(T))\,e^{-\int_t^T X(s)ds}\right] \qquad (3.17) \\ &= \frac{1-\chi+\chi ab\int_t^T e^{-\int_s^T\left(a+C(u;a,\sigma,T)\sigma^2\right)du}ds + \chi e^{-\int_t^T\left(a+C(u;a,\sigma,T)\sigma^2\right)du}X(t)}{e^{ab\int_t^T C(u;a,\sigma,T)du+C(t;a,\sigma,T)X(t)}}, \end{aligned}$$

where

$$C(t;a,\sigma,T) = 2\frac{1-e^{-\sqrt{a^2+2\sigma^2}(T-t)}}{\sqrt{a^2+2\sigma^2}+a+\left(\sqrt{a^2+2\sigma^2}-a\right)e^{-\sqrt{a^2+2\sigma^2}(T-t)}}. \qquad (3.18)$$

Proof. See Appendix 3.B. □

In order to keep the statistical properties of (3.16) unchanged when switching between probabilities (either $\mathbb{Q}$ or $\mathbb{Q}_\delta$), we assume that the market prices of risk for $W_r(t)$ and $W_\lambda(t)$ are given by

$$\xi_r(t) = \phi_r\sqrt{r(t)}, \quad \xi_\lambda(t) = \phi_\lambda\sqrt{\lambda(t)},$$

where ϕ_r and ϕ_λ are constant. Under these assumptions, (3.16) can be rewritten under both $\mathbb{Q}$ and $\mathbb{Q}_\delta$ just by changing the mean reverting strength and the long term mean as follows (with $i \in \{r, \lambda\}$)

$$a_i^{\mathbb{Q}} \equiv a_i + \sigma_i\phi_i, \quad b_i^{\mathbb{Q}} \equiv \frac{a_ib_i}{a_i+\sigma_i\phi_i},$$

$$a_i^{\mathbb{Q}_\delta} \equiv a_i + \sigma_i \frac{\delta - 1}{\delta}\phi_i, \quad b_i^{\mathbb{Q}_\delta} \equiv \frac{a_i b_i}{a_i + \sigma_i \frac{\delta-1}{\delta}\phi_i}.$$

Given these results, the function $H(t, z)$ in (3.11) can be simplified as follows:

$$\begin{aligned} H(t,z) &= R_m \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_{t_0}^{T} r(u,z)du} \right] \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_{t_0}^{T} \lambda(u,z)du} \right] \\ &= R_m e^{-a_r^{\mathbb{Q}} b_r^{\mathbb{Q}} \int_t^T C\left(u; a_r^{\mathbb{Q}}, \sigma_r, T\right)du - C\left(t; a_r^{\mathbb{Q}}, \sigma_r, T\right) r(t)} \\ &\quad \times e^{-a_\lambda^{\mathbb{Q}} b_\lambda^{\mathbb{Q}} \int_t^T C\left(u; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, T\right)du - C\left(t; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, T\right) \lambda(t)}, \end{aligned}$$

and, accordingly, the vector $\frac{\partial H(t,z)}{\partial z(t)}$ has the following form:

$$\frac{\partial H(t,z)}{\partial z(t)} = \left[\begin{array}{c} -\frac{\partial H(t,z)}{\partial r(t)} C\left(t; a_r^{\mathbb{Q}}, \sigma_r, T\right) \\ -\frac{\partial H(t,z)}{\partial \lambda(t)} C\left(t; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, T\right) \end{array} \right].$$

In the same way, the function $F(t, z)$ in (3.12) can be written as

$$\begin{aligned} F(t,z) =& e^{-\frac{\rho}{\delta}(T-t)} \mathbb{E}_t^{\mathbb{Q}_\delta} \left[e^{-\frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right)\int_t^T r(u)du} \right] \mathbb{E}_t^{\mathbb{Q}_\delta} \left[e^{-\left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right)\int_t^T \lambda(u)du} \right] \\ =& e^{-\frac{\rho}{\delta}(T-t)} e^{-\frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right) a_r^{\mathbb{Q}_\delta} b_r^{\mathbb{Q}_\delta} \int_t^T C\left(u; \frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right) a_r^{\mathbb{Q}_\delta}, \frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right)\sigma_r, T\right)du} \\ & \times e^{-C\left(t; \frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right) a_r^{\mathbb{Q}_\delta}, \frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right)\sigma_r, T\right) r(t)} \\ & \times e^{-\left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right) a_\lambda^{\mathbb{Q}_\delta} b_\lambda^{\mathbb{Q}_\delta} \int_t^T C\left(u; \left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right) a_\lambda^{\mathbb{Q}_\delta}, \left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right)\sigma_\lambda, T\right)du} \\ & \times e^{-C\left(t; \left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right) a_\lambda^{\mathbb{Q}_\delta}, \left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right)\sigma_\lambda, T\right)\lambda(t)}, \end{aligned}$$

from which the value of the partial derivatives $\frac{\partial F(t,z)}{\partial z(t)} \frac{1}{F(t,z)}$ as follows:

$$\eta_{F,r} = \frac{\partial F(t,z)}{\partial r(t)} \frac{1}{F(t,z)} = -C\left(t; \frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right) a_r^{\mathbb{Q}_\delta}, \frac{\delta-1}{\delta}\left(1+\frac{1}{2}\frac{1}{\delta}\phi_r^2\right)\sigma_r, T\right),$$

$$\eta_{F,\lambda} = \frac{\partial F(t,z)}{\partial \lambda(t)} \frac{1}{F(t,z)} = -C\left(t; \left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right) a_\lambda^{\mathbb{Q}_\delta}, \left(1+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\phi_\lambda^2\right)\sigma_\lambda, T\right).$$

In the following subsection, we define the prices of the assets listed in the financial market.

3.4.3 Financial assets

On the financial market three assets are listed:

- the riskless asset whose price $G(t)$ solves (3.3);
- a defaultable constant time-to-maturity (T_B) bond which pays a constant coupon δ_B, whose price is

$$B(t) = 1 + \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{t+T_B} (\delta_B - r(s) - (1-w)\lambda(s)) e^{-\int_t^s r(u)+\lambda(u)du} ds \right], \tag{3.19}$$

  where w is the (constant) recovery rate of the bond issuer; and
- a constant time-to-maturity (T_X) CDS written on the bond $B(t,T)$; the mark-to-market value of this CDS is

$$X(t) = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{t+T_X} ((1-w)\lambda(s) - \delta_X) e^{-\int_t^s r(u)+\lambda(u)du} ds \right], \tag{3.20}$$

  where δ_X is the (constant) spread which is paid periodically.

Because of the independence between $r(t)$ and $\lambda(t)$, the values of the bond and the CDS can be simplified as follows

$$\begin{aligned} B(t) =& \delta_B \int_t^{t+T_B} \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^s r(u)du} \right] \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^s \lambda(u)du} \right] ds \\ &+ w \int_t^{t+T_B} \mathbb{E}_t^{\mathbb{Q}} \left[\lambda(s) e^{-\int_t^s \lambda(u)du} \right] \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^s r(u)du} \right] ds \\ &+ \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^{t+T_B} r(u)du} \right] \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^{t+T_B} \lambda(u)du} \right], \end{aligned}$$

$$\begin{aligned} X(t) = (1-w) &\int_t^{t+T_X} \mathbb{E}_t^{\mathbb{Q}} \left[\lambda(s) e^{-\int_t^s \lambda(u)du} \right] \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^s r(u)du} \right] ds \\ - \delta_C &\int_t^{t+T_X} \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^s r(u)du} \right] \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^s \lambda(u)du} \right] ds, \end{aligned}$$

where, all the expected values can be computed in closed form as shown in Proposition 1. In this framework the bond whose price is $B(t)$ can be considered a derivative on the interest rate $r(t)$ and, in the same way, the CDS whose price is $X(t)$ can be considered a derivative on the default intensity $\lambda(t)$. Accordingly, the volatility matrix $\Sigma(t,z)$ is given by the following terms

$$\Sigma(t,z) = \begin{bmatrix} \frac{1}{B(t)}\frac{\partial B(t)}{\partial r(t)} & \frac{1}{B(t)}\frac{\partial B(t)}{\partial \lambda(t)} \\ \frac{1}{X(t)}\frac{\partial X(t)}{\partial \lambda(t)} & \frac{1}{X(t)}\frac{\partial X(t)}{\partial \lambda(t)} \end{bmatrix}.$$

Some simple algebra allows as to conclude that the derivatives we seek can be written as follows:

$$\begin{aligned}\frac{\partial B(t)}{\partial r(t)} =& -\delta_B \int_t^{t+T_B} C\left(t; a_r^{\mathbb{Q}}, \sigma_r, s\right) \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s \lambda(u)du}\right] ds \\ & - w \int_t^{t+T_B} C\left(t; a_r^{\mathbb{Q}}, \sigma_r, s\right) \mathbb{E}_t^{\mathbb{Q}}\left[\lambda(s) e^{-\int_t^s \lambda(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right] ds \\ & - C\left(t; a_r^{\mathbb{Q}}, \sigma_r, t+T_B\right) \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+T_B} r(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+T_B} \lambda(u)du}\right],\end{aligned}$$

$$\begin{aligned}\frac{\partial B(t)}{\partial \lambda(t)} =& -\delta_B \int_t^{t+T_B} C\left(t; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, s\right) \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s \lambda(u)du}\right] ds \\ & - w \int_t^{t+T_B} C\left(t; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, s\right) \mathbb{E}_t^{\mathbb{Q}}\left[\lambda(s) e^{-\int_t^s \lambda(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right] ds \\ & + w \int_t^{t+T_B} \frac{e^{-\int_t^s \left(a_\lambda^{\mathbb{Q}} + C\left(u; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, s\right)\sigma_\lambda^2\right)du}}{e^{a_\lambda^{\mathbb{Q}} b_\lambda^{\mathbb{Q}} \int_t^s C\left(u; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, s\right)du + C\left(t; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, s\right)\lambda(t)}} \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right] ds \\ & - C\left(t; a_\lambda^{\mathbb{Q}}, \sigma_\lambda, t+T_B\right) \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+T_B} r(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+T_B} \lambda(u)du}\right],\end{aligned}$$

$$\begin{aligned}\frac{\partial X(t)}{\partial r(t)} =& -(1-w) \int_t^{t+T_X} C\left(t; a_r^{\mathbb{Q}}, \sigma_r, s\right) \mathbb{E}_t^{\mathbb{Q}}\left[\lambda(s) e^{-\int_t^s \lambda(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right] ds \\ & + \delta_C \int_t^{t+T_X} C\left(t; a_r^{\mathbb{Q}}, \sigma_r, s\right) \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right] \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s \lambda(u)du}\right] ds,\end{aligned}$$

$$\begin{aligned}\frac{\partial X(t)}{\partial \lambda(t)} = &-(1-w)\int_t^{t+T_X} C\left(t;a_\lambda^{\mathbb{Q}},\sigma_\lambda,s\right)\mathbb{E}_t^{\mathbb{Q}}\left[\lambda(s)\,e^{-\int_t^s \lambda(u)du}\right]\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right]ds\\ &+(1-w)\int_t^{t+T_X}\frac{e^{-\int_t^s\left(a_\lambda^{\mathbb{Q}}+C\left(u;a_\lambda^{\mathbb{Q}},\sigma_\lambda,s\right)\sigma_\lambda^2\right)du}}{e^{a_\lambda^{\mathbb{Q}}b_\lambda^{\mathbb{Q}}\int_t^s C\left(u;a_\lambda^{\mathbb{Q}},\sigma_\lambda,s\right)du+C\left(t;a_\lambda^{\mathbb{Q}},\sigma_\lambda,s\right)\lambda(t)}}\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right]ds\\ &+\delta_C\int_t^{t+T_X} C\left(t;a_\lambda^{\mathbb{Q}},\sigma_\lambda,s\right)\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s r(u)du}\right]\mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^s \lambda(u)du}\right]ds,\end{aligned}$$

where we note that the terms $C(t;\bullet,\bullet,t+T_B)$ do not depend on t and, in fact, $C(t;\bullet,\bullet,t+T_B) = C(0;\bullet,\bullet,T_B)$.

3.4.4 Calibration

In this subsection, we compute the optimal portfolio for an investor and show how it reacts to changes in both the levels of risk aversion and underlying risk. Accordingly, we set four scenarios, whose parameters are estimated from market data of France, Germany, Ireland, Italy, Portugal, and Spain.

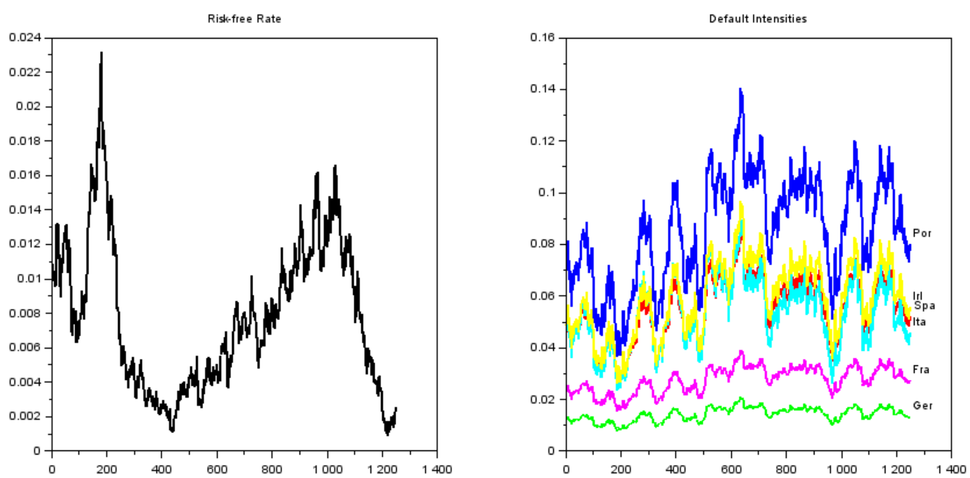

Figure 3.1: Calibrated Path of State Variables – This figure shows the result of the calibration of the risk-free interest rate (chart on the left) and the default intensities (chart on the right) as in eq. 3.16. The underlying Wiener processes are independent, however for the country specific default intensities one common process is used

Calibrations of state variables in equation (3.16) are common to all scenarios, all the data are collected with a daily frequency, and parameters are estimated via maximum likelihood estimations, where ordinary least squares estimates are used as starting point of the optimisation. To obtain default intensities parameters we first infer default intensities by bootstrapping the default probability curve from listed CDS spreads as in Hull and White (2000). We use CDS daily quotes of each country from 21 July 2008 to 31 December 2014. Riskfree interest rate parameters are obtained using the daily return on the 3-month German Bund, from 18 November 2002 to 7 November 2011. The selected periods reflect the longest series available from Thompson Reuters Eikon; we remove the last 3 years from the 3-month Bund series due to negative rates. Estimates of the risk-free interest rate parameters are reported in Table 3.1.

Table 3.1: Riskfree interest rate parameters estimated using the daily return of the 3-month German Bund (from 18 November 2002 to 7 November 2011)

a_r	0.3033606
b_r	0.0106621
σ_r	0.0800704

To calibrate $X(t)$ and $B(t)$, we assume that the CDS and defaultable bond have the same constant maturity, equal to five years ($T_X = T_B = 5$). The constant spread (δ_X) is estimated as the average of the listed 5-year CDS spreads for the selected reference entities, while for the constant coupon of the bond (δ_B), we calculate the average of coupons of fixed rate bonds of 5-year maturity issued by each selected sovereign country from November 2002 to December 2014.[4]

To estimate ϕ_r and ϕ_λ, we numerically solve the following system for each country

[4] In order to calculate the average constant coupon, we seek 5-year fixed-rate bonds issued by the six countries from November 2002 to December 2014.

$$\begin{cases} \mathbb{E}_t\left[d \ln B\left(t\right)\right] = & Mdt, \\ \mathbb{E}_t\left[X(t)\right] = & z, \end{cases}$$

where M is the estimated average return of the 5-year sovereign bond over the period 21 July 2008 to 31 December 2014 and z is the estimated price of a 5-year CDS with recovery rate of 40% and notional value of 1. We infer CDS prices by using averages of spreads and bootstrapped default intensities. Estimated parameters by country are reported in Table 3.2.

Table 3.2: Country specific parameters estimated using daily 5-year credit default swap spreads and daily 5-year bond returns (from 21 July 2008 to 31 December 2014)

	France	Germany	Ireland	Italy	Portugal	Spain
a_λ	0.61336	0.81642	0.83057	1.46744	0.80642	0.84172
b_λ	0.02304	0.01209	0.04887	0.04623	0.06977	0.04601
σ_λ	0.07113	0.05665	0.15064	0.16705	0.18453	0.13337
δ_B	0.02895	0.0238	0.045	0.0349	0.0392	0.0368
δ_X	0.0081	0.0043	0.0308	0.0214	0.0424	0.02178
$\mathbb{E}_t\left[X\left(t\right)\right]$	0.05723	0.03237	0.12674	0.11722	0.16033	0.11613
$\frac{1}{dt}\mathbb{E}_t\left[d \ln B\left(t\right)\right]$	0.01805	0.0142	0.0407	0.0338	0.0614	0.0349
ϕ_r	-3.47994	-3.83582	-0.14240	-1.27866	-0.06139	-0.57855
ϕ_λ	-4.65392	-7.24681	-4.50507	-4.91423	-3.48991	-4.21003

Figure 3.1 shows the evolution of calibrated risk-free rates and default intensities by country; the underlying Wiener process is common to all sovereign countries. The period selected for the estimation of default parameters also captures the European sovereign debt crisis. This can be shown in calibrated intensities: countries that have been

affected more severely by the crisis (i.e. Italy, Ireland, Portugal, and Spain) show default intensities that are constantly above those of countries that have been affected less seriously (i.e. France and Germany).

Base scenario

In our base scenario, we consider an investor whose preferences are described by the following parameters:

- initial wealth R_0 equal to 100;
- moderately risk adverse, with $\delta = 2.5$;
- desired wealth at the end of the period $R_m = 120$;
- subjective discount rate $\rho = 0.01$;
- horizon of 5 years ($T = 5$).

Finally, we assume the recovery rate constant at $w = 0.4$.

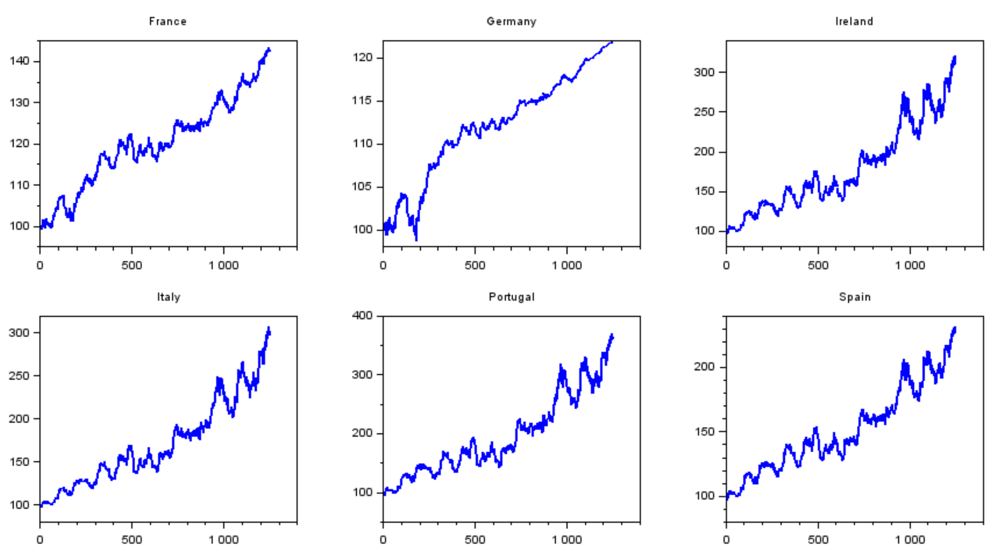

Figure 3.2: Wealth Evolution in the Base Scenario – This figure shows the evolution of investor's wealth resulting from the optimal portfolio, in the Base Scenario, for each calibrated sovereign

Figure 3.2 shows the evolution of the investor's wealth for each country and Figure 3.3 shows the optimal asset allocation. The underlying sovereign risk plays a clear role: the higher is the risk, the higher are both the average and final investor's wealth. Less risky countries, such as Germany and France, allow the investor to reach final wealth that is notably lower than the wealth obtained with an investment in riskier countries. In the same way, the underlying risk is positively correlated with the volatility of investor's wealth.

We stress that in most of the simulations, the portfolio shares do not show sharp movements and, accordingly, they do not imply overly expensive transaction costs. Actually, the smooth behaviour of portfolio shares could be approximated suitably by a piecewise function that allows keeping the portfolio unchanged for a given period of time.

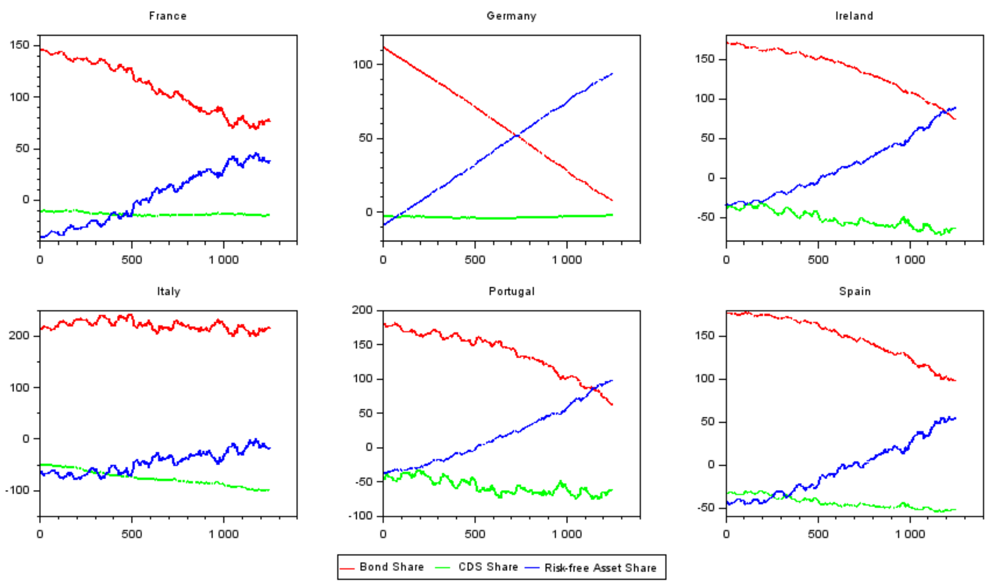

Figure 3.3: Optimal Portfolio Shares in the Base Scenario – This figure shows, for each calibrated country, the composition of optimal portfolios in percentage terms, under the Base Scenario. The red line represents the evolution of bond share, the green line the evolution of credit default swap share and the blue line is the risk-free asset share

Optimal portfolios are composed of long positions in the defaultable bonds and short positions in the CDSs: in other words, the investor issues the CDSs to speculate on the credit risk of the reference entity. The share of wealth invested in the bonds decreases over time, and this reduction is compensated by an increase in the risk-free asset share. The investment in the CDSs remains relatively stable during the 5-year period for less risky countries, but shows a slight decrease for riskier countries. The only exception to this behaviour is Italy, where the share of the bonds remains stable on average and the slight increase in the risk-free asset is compensated by a small reduction in the CDSs.

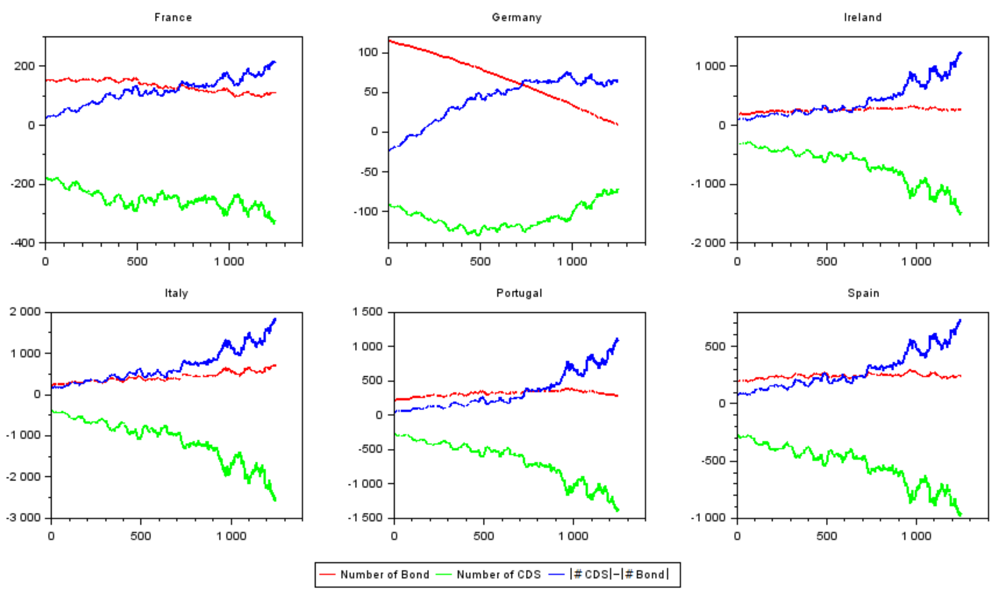

Figure 3.4: Number of Contracts held in the Optimal Portfolio under the Base Scenario – This figure shows the evolution of the number of bond and CDS contracts held in the optimal portfolio for each calibrated country, under the Base Scenario. The blue line is the difference between CDS and bond contracts in absolute values

The magnitude of credit risk speculation is directly linked to the sovereign risk: the higher is the risk, the higher is the amount of credit default swaps issued by the investor.

In addition, in this case, the only exception is the portfolio calibrated on Italian data, which is characterised by the highest percentage of bonds and by the strongest issuance of CDSs. This result suggests that the Italian sovereign risk measured in the financial market is more than compensated for by the return, at least according to the risk aversion of the representative agent we consider.

The average percentage of wealth invested in CDSs ranges during the 5-year horizon from -3% in the case of Germany, to -75% in the case of Italy; while the percentage of defaultable bonds ranges from an average of 60% for Germany to an average of 220% for Italy.

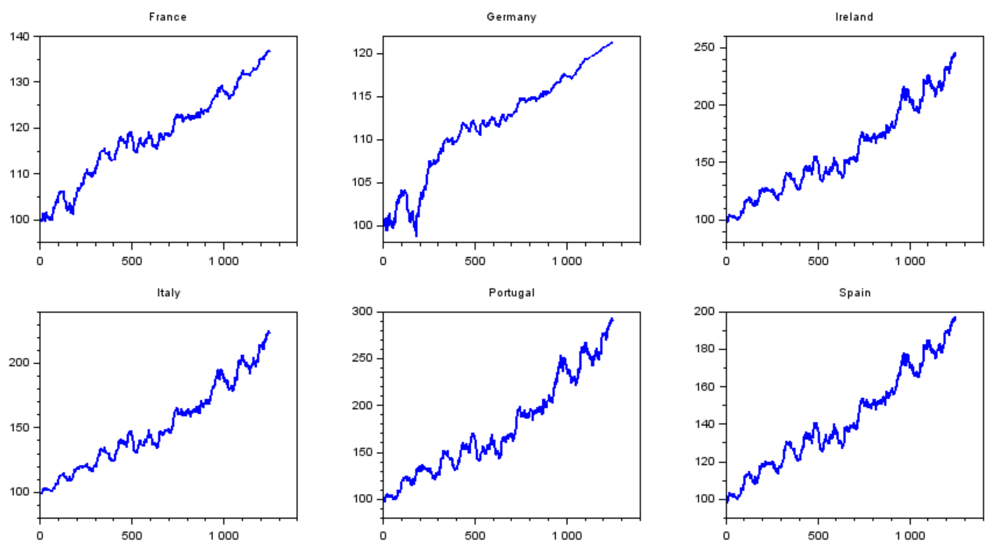

Figure 3.5: Wealth Evolution in the Higher Risk Aversion Scenario – This figure shows the evolution of investor's wealth resulting from the optimal portfolio, when δ is set to 4.5, for each calibrated sovereign

The investment strategy as a percentage of wealth does not allow capturing the effective role of speculation. The investor significantly speculates on the credit risk of the reference entity by issuing more CDS contracts than bonds held in the optimal portfolio. Figure 3.4 shows the evolution of the number of contracts for each calibration. The difference, in absolute terms, between CDS and bond contracts held in the portfolio increases over time; it is a minimum in the case of Germany and a maximum in the case of Italy. The number of defaultable bonds held in the portfolios at the end of the period is largely outweighed by the number of CDSs issued by the investor. This is also the case at the beginning of the period, with the only exception being the German portfolio, in which defaultable bonds overweight CDSs for the first 195 simulated days.

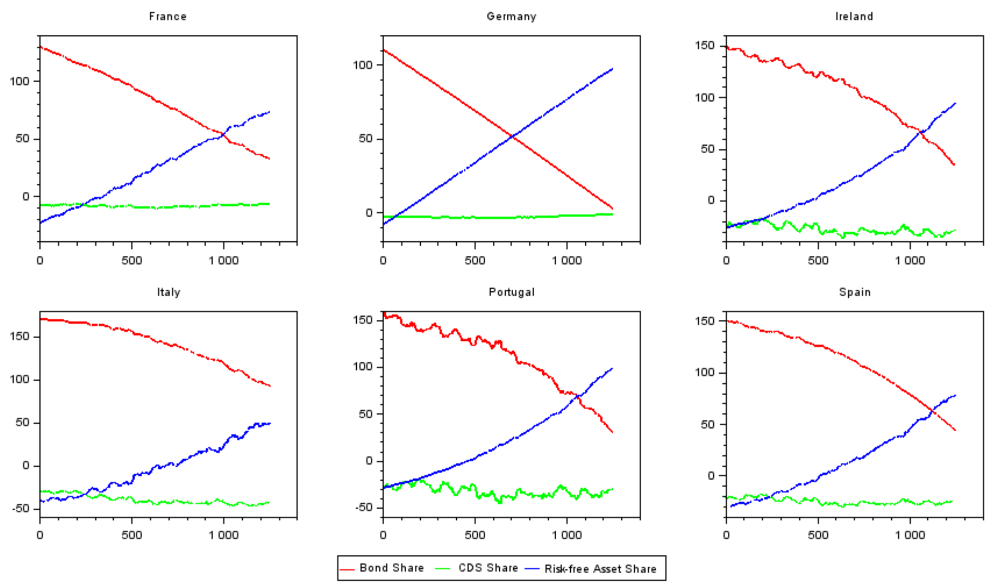

Figure 3.6: Optimal Portfolio Shares in the Higher Risk Aversion Scenario – This figure shows, for each calibrated country, the composition of optimal portfolios in percentage terms, when δ is set to 4.5. The red line represents the evolution of bond share, the green line the evolution of credit default swap share and the blue line is the risk-free asset share

The main rationale for short selling the CDS is that it provides positive cash flows, (coinciding with the spread δ_X) that can be invested in high-return assets in order to accumulate sufficiently high wealth over time both to face the credit risk and to obtain a positive return. Furthermore, if the investor wants to accumulate enough wealth, the CDSs necessarily must be overweighted with respect to the bonds.

Higher risk aversion scenario

We recalibrate the model with δ set to 4.5 to take into account a higher degree of risk aversion.

Figure 3.5 shows the evolution of the investor's wealth for the six optimal portfolios: higher risk aversion determines a lower average in the wealth growth rate but also a lower value in its volatility.

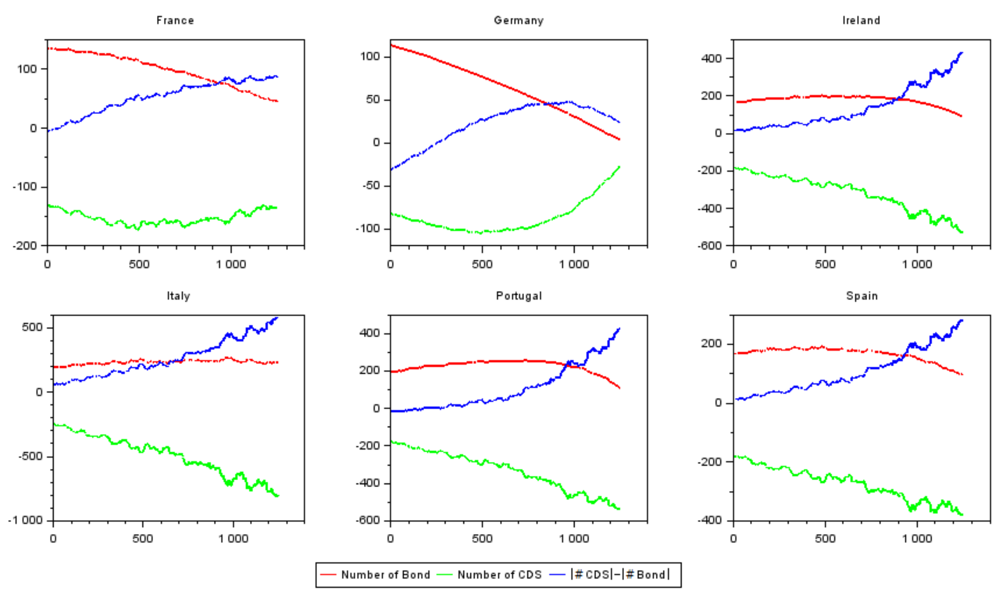

Figure 3.7: Number of Contracts held in the Optimal Portfolio under the Higher Risk Aversion Scenario – This figure shows the evolution of the number of bond and CDS contracts held in the optimal portfolio for each calibrated country, when δ is set to 4.5. The blue line is the difference between CDS and bond contracts in absolute values

As in the base scenario, the investor takes long positions in defaultable bonds and issues CDSs, with the share invested in bonds decreasing over time and compensated for by an increase in risk-free assets, as shown in Figure 3.6. Higher risk aversion results in larger investment in risk-free assets and in a stronger substitution effect, particularly for low-risk countries. Portfolios calibrated on German and French data exhibit a decrease in bonds and an increase in risk-free asset that are nearly linear and constant over time.

The investor speculates on credit risk by issuing more CDS contracts, on average, than defaultable bonds purchased. The magnitude of speculation is positively correlated with the underlying risk of the reference entity. As shown in Figure 3.7, the difference in the number of contracts is lower than in the base scenario and initially negative for three countries. Specifically, the initial portfolio composition includes more defaultable bonds than CDSs for Germany, France, and Portugal and this strategy lasts for 249, 63, and 180 days, respectively. This less aggressive behaviour of the agent reflects the higher risk aversion.

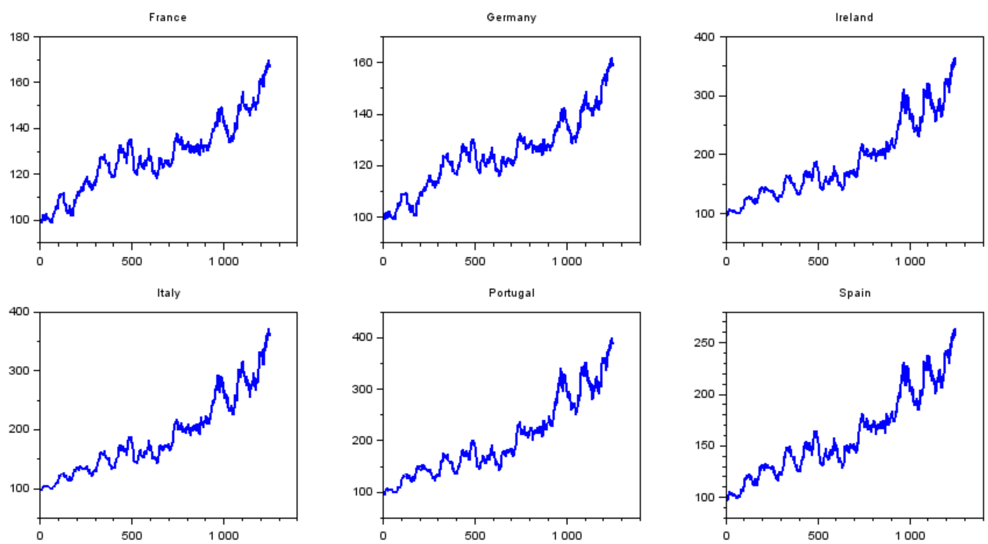

Figure 3.8: Wealth Evolution in the Lower Target of Final Wealth Scenario – This figure shows the evolution of investor's wealth resulting from the optimal portfolio, when R_m is assumed to be equal to 100, for each calibrated sovereign

Lower target of final wealth scenario

Under this scenario, we recalibrate the model with $R_m = 100$, which coincides with the level of the initial wealth, that is the investor does not want to suffer any loss.

A lower value of R_m entails higher willingness to take risks and this is reflected in a larger level of wealth obtained at the end of the horizon. As shown in Figure 3.8, the investor's wealth depicts, in comparison with previous scenarios, a higher level of volatility over the 5-year period for all countries.

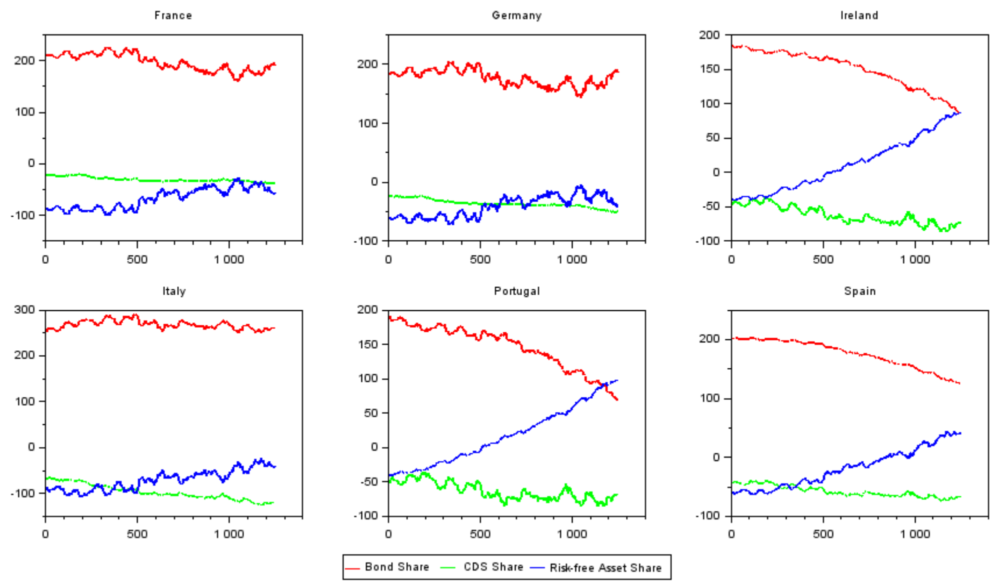

Figure 3.9: Optimal Portfolio Shares in the Lower Target of Final Wealth Scenario – This figure shows, for each calibrated country, the composition of optimal portfolios in percentage terms, when $R_m$is assumed to be equal to 100. The red line represents the evolution of bond share, the green line the evolution of credit default swap share and the blue line is the risk-free asset share

By analysing the composition of optimal portfolios (Figures 3.9 and 3.10), the evolution of the investment strategy for high-risk countries is very similar to that of the base scenario. The investor takes long positions in defaultable bonds and issues CDSs; however, for low-risk countries, two main differences arise: (i) the share of wealth invested in bonds and CDSs at the beginning of the horizon is larger, and (ii) the overtime substitution effect between risk-free assets and bonds is nearly null. This is a consequence of more aggressive speculation on credit risk, which results in a notably larger difference between issued CDSs and bonds held in the portfolio.

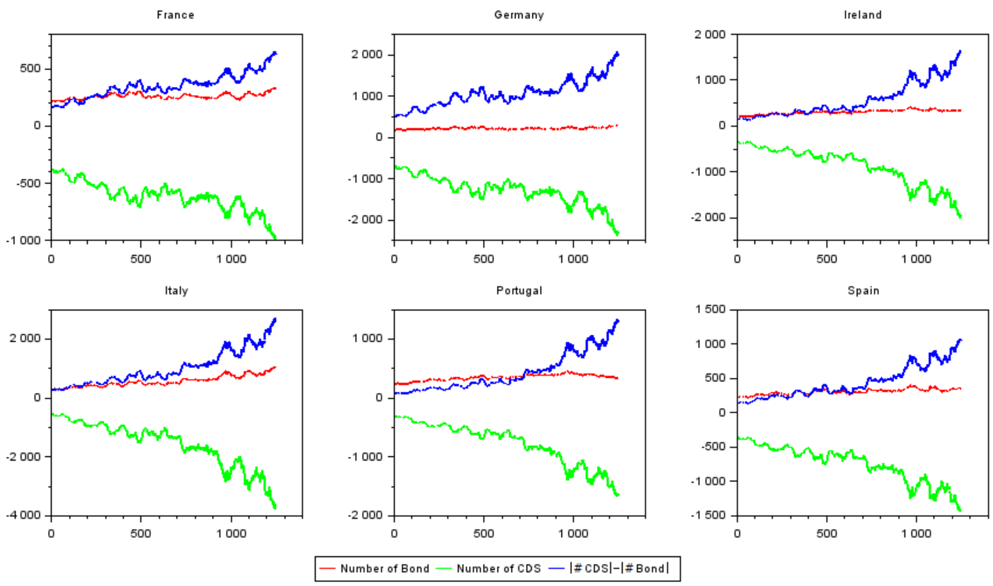

Figure 3.10: Number of Contracts held in the Optimal Portfolio under the Lower Target of Final Wealth Scenario – This figure shows the evolution of the number of bond and CDS contracts held in the optimal portfolio for each calibrated country, when R_m is assumed to be equal to 100. The blue line is the difference between CDS and bond contracts in absolute values

Lower risk aversion scenario

In order to examine the optimal portfolio for an investor with a higher risk appetite, we recalibrate the model with δ set to 1.5 and R_m to 100.

Figure 3.11 shows the evolution of the investor's wealth: lower risk aversion results in larger average values of final wealth with higher volatility. As in previous scenarios, portfolios written on riskier countries entail a larger level of wealth for the investor than safer countries. However, over the 5-year horizon, the volatility increases mainly for low-risk countries: wealth obtained from the German portfolio depicts an increase in standard deviation of 200%, while for the Portuguese portfolio, the increase is 35%.

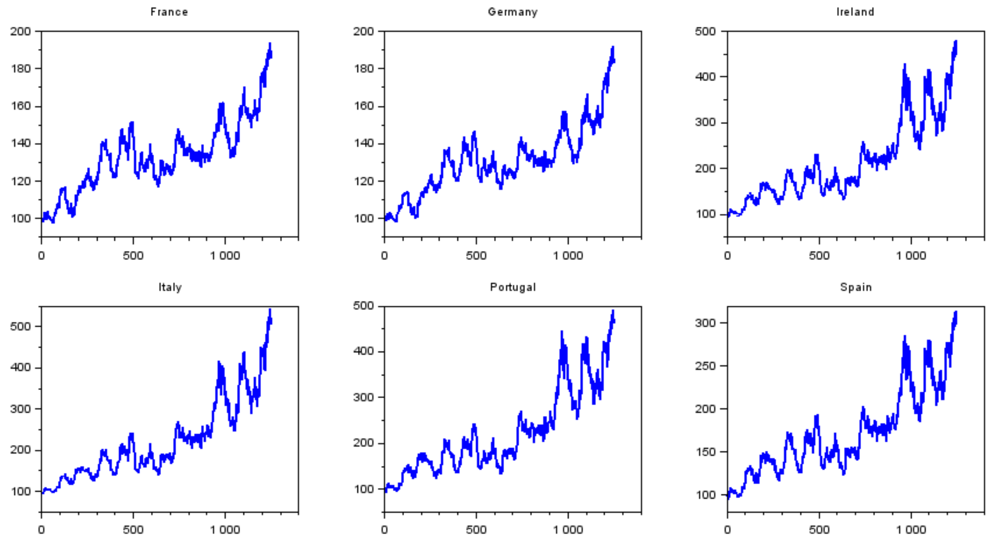

Figure 3.11: Wealth Evolution in the Lower Risk Aversion Scenario – This figure shows the evolution of investor's wealth resulting from the optimal portfolio, when δ and R_m are set respectively to 1.5 and 100, for each calibrated sovereign

As shown in Figure 3.12, the evolution of portfolio shares is similar to other scenarios, with short selling of both CDSs and risk-free asset and long positions in bonds, while speculation on credit risk is stronger, mainly for France and Germany. As depicted in Figure 3.13, the direct link between underlying risk and magnitude of the speculation does not seem to hold as in previous scenarios: the largest number of CDSs are issued in German and Italian portfolios.

3.5 Conclusion

This study computes the optimal portfolio for an investor who maximises the expected utility of his or her final wealth. The agent invests in a complete market in which a riskless asset, a defaultable bond and a CDS written on the bond are listed.

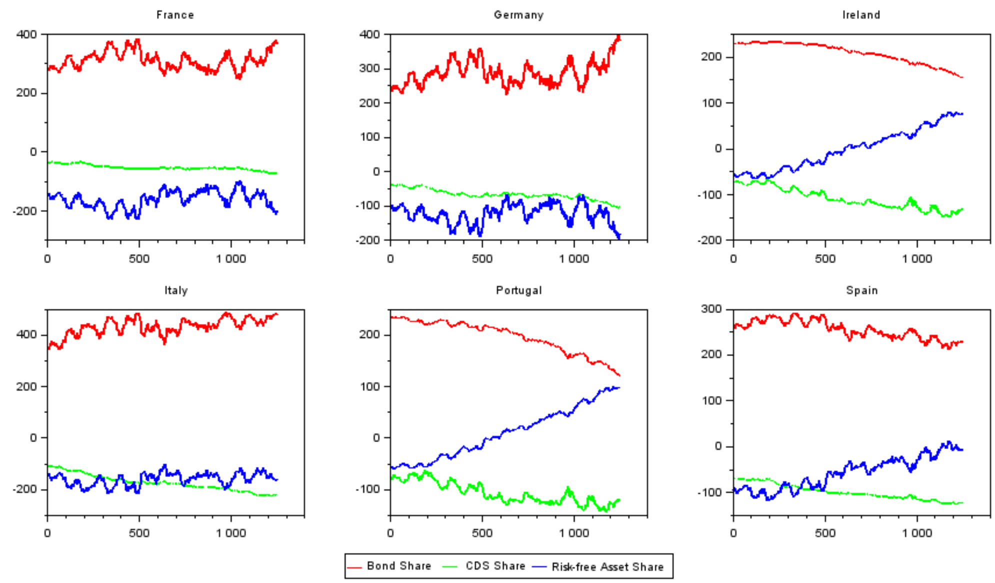

Figure 3.12: Optimal Portfolio Shares in the Lower Risk Aversion Scenario – This figure shows, for each calibrated country, the composition of optimal portfolios in percentage terms, when δ and R_m are set respectively to 1.5 and 100. The red line represents the evolution of bond share, the green line the evolution of credit default swap share and the blue line is the risk-free asset share

Once a closed-form solution of the problem is found, we calibrate our model to market data of six European countries in order to assess the behaviour of an investor exposed to different levels of underlying risk. We find that it is always optimal to issue more CDSs than bonds that are optimally purchased. The number of CDS contracts optimally issued is higher for countries characterised by higher sovereign risk. This result is obtained even when the investor is endowed with a different level of risk aversion. However, when both risk aversion and the final wealth target are lowered, such a relationship no longer holds.

Our results suggest that when it is possible to buy CDSs in the so-called "naked" form, financial institutions have incentives to speculate on the risk of sovereign entities. This may add support to the notion that speculation in CDSs exacerbated the recent European sovereign crisis. Consequently, regulatory intervention would be required to increase the effectiveness of the EU ban on uncovered CDSs.

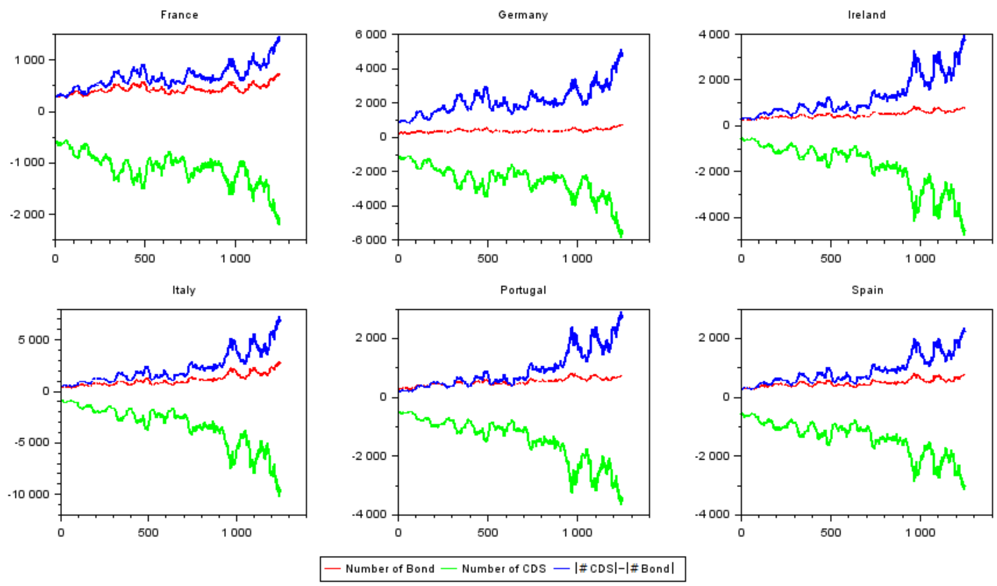

Figure 3.13: Number of Contracts held in the Optimal Portfolio under the Lower Risk Aversion Scenario – This figure shows the evolution of the number of bond and CDS contracts held in the optimal portfolio for each calibrated country, when δ and R_m are set respectively to 1.5 and 100. The blue line is the difference between CDS and bond contracts in absolute values

The numerical application presented in this study assumes that the riskless interest rate and the default intensity are independent. An interesting extension of this study would be to analyse the optimal investment strategy when these two state variables are not independent.

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Appendix 3.A Proof of proposition 1

We solve problem (3.8) following the martingale approach. Its Lagrangian function under constraint (3.9) is:

$$\mathcal{L} = \mathbb{E}_{t_0}\left[\frac{(R(T)-R_m)^{1-\delta}}{1-\delta}e^{-\int_{t_0}^{T}\rho(u)+\lambda(u)du}\right]+\kappa\left(R(t_0)-\mathbb{E}_{t_0}\left[R(T)\,m(t_0,T)\,e^{-\int_{t_0}^{T}r(u)+\lambda(u)du}\right]\right), \tag{3.21}$$

where the functional dependencies on z have been omitted for the sake of simplicity, κ is the (constant) Lagrangian multiplier, and all the expected values have been written under the historical probability. The first order condition on final wealth is

$$\frac{\partial\mathcal{L}}{\partial R(T)} = \mathbb{E}_{t_0}\left[(R(T)-R_m)^{-\delta}e^{-\int_{t_0}^{T}\rho(u)+\lambda(u)du}-\kappa m(t_0,T)\,e^{-\int_{t_0}^{T}r(u)+\lambda(u)du}\right]=0, \tag{3.22}$$

and the optimal final wealth is

$$R^*(T) = R_m+\left(\kappa m(t_0,T)\,e^{-\int_{t_0}^{T}r(u)du}e^{\int_{t_0}^{T}\rho(u)du}\right)^{-\frac{1}{\delta}}. \tag{3.23}$$

When the constraint is rewritten at time t (instead of t_0) as follows

$$R(t) = \mathbb{E}_t\left[R(T)\,m(t,T)\,e^{-\int_t^T r(u)+\lambda(u)du}\right], \tag{3.24}$$

and the optimal final wealth is substituted in it, we obtain the following expression:

$$R(t) = \left(\kappa m(t_0,t)\,\frac{e^{-\int_{t_0}^{t}r(u)du}}{e^{-\int_{t_0}^{t}\rho(u)du}}\right)^{-\frac{1}{\delta}}F(t,z)+H(t,z) \tag{3.25}$$

where

$$H(t,z) = \mathbb{E}_t^{\mathbb{Q}}\left[R_m e^{-\int_t^T r(u)+\lambda(u)du}\right], \tag{3.26}$$

$$F(t,z) = \mathbb{E}_t\left[m(t,T)^{1-\frac{1}{\delta}}\,e^{-\int_t^T\left(\frac{\delta-1}{\delta}r(u)+\frac{1}{\delta}\rho(u)+\lambda(u)\right)du}\right]. \tag{3.27}$$

While $m(t,T)^{1-\frac{1}{\delta}}$ is not a martingale, $m(t,T)^{1-\frac{1}{\delta}}\, e^{\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\int_t^T \xi(s)'\xi(s)ds}$ is:

$$\left(m(t,T)^{1-\frac{1}{\delta}}\, e^{\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\int_t^T \xi(s)'\xi(s)ds}\right)^{-1} d\left(m(t,T)^{1-\frac{1}{\delta}}\, e^{\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\int_t^T \xi(s)'\xi(s)ds}\right) = -\frac{\delta-1}{\delta}\xi(T)\, dW(T)\,. \tag{3.28}$$

Accordingly, we define the new probability

$$dW(t)^{\mathbb{Q}_\delta} = \frac{\delta-1}{\delta}\xi(t)\, dt + dW(t)\,, \tag{3.29}$$

and write

$$F(t) = \mathbb{E}_t^{\mathbb{Q}_\delta}\left[e^{-\int_t^T\left(\frac{\delta-1}{\delta}r(u,z)+\frac{1}{\delta}\rho(u,z)+\lambda(u,z)+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\xi(u,z)'\xi(u,z)\right)du}\right]. \tag{3.30}$$

The differential of (3.25), through Ito's lemma, is (the drift term is neglected since it is immaterial to replication):

$$\begin{aligned} dR(t) &= (\ldots)\, dt + \frac{1}{\delta}\left(\kappa m(t_0,t)\, e^{-\int_{t_0}^t r(u)du} e^{\int_{t_0}^t \rho(u)du}\right)^{-\frac{1}{\delta}} F(t,z)\,\xi(t,z)'\, dW(t) \\ &\quad + \left(\kappa m(t_0,t)\, e^{-\int_{t_0}^t r(u)du} e^{\int_{t_0}^t \rho(u)du}\right)^{-\frac{1}{\delta}} F_z(t,z)'\,\Omega(t,z)'\, dW(t) \\ &\quad + H_z(t,z)'\,\Omega(t,z)'\, dW(t)\,, \end{aligned} \tag{3.31}$$

where the subscripts on $F(t,z)$ and $H(t,z)$ indicate partial derivatives. Once the following relationship

$$\frac{R(t)-H(t,z)}{F(t,z)} = \left(\kappa m(t_0,t)\, e^{-\int_{t_0}^t r(u,z)du} e^{\int_{t_0}^t \rho(u,z)du}\right)^{-\frac{1}{\delta}}, \tag{3.32}$$

is suitably taken into account, the differential equation becomes

$$dR(t) = (\ldots)\, dt + \left(\frac{R(t)-H(t,z)}{\delta}\xi(t,z)' + \frac{R(t)-H(t,z)}{F(t,z)}F_z(t,z)'\,\Omega(t,z)' + H_z(t,z)'\,\Omega(t,z)'\right) dW(t)\,. \tag{3.33}$$

When $\Sigma(t,z)\, I_S\theta_S(t)$ is set equal to the diffusion term of (3.33), the optimal portfolio in Proposition 1 is found.

Appendix 3.B Computation of $V(t,T)$

If the stochastic variable $y(t)$ follows the process

$$dy(t) = a(b - y(t))\,dt + \sigma\sqrt{y(t)}dW(t),$$
$$y(t_0) = y_0,$$

then the expected value

$$V(t,T) = \mathbb{E}_t\left[(1 - \chi + \chi y(T))\,e^{-\int_t^T y(s)ds}\right],$$

must solve the partial differential equation

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial y}a(b-y) + \frac{1}{2}\frac{\partial^2 V}{\partial y^2}\sigma^2 y = yV,$$

with the boundary condition

$$V(T,T) = 1 - \chi + \chi y(T),$$

where the parameter χ can take either value 1 or value 0. Now we use the guess function

$$V(t,y) = (E(t) + F(t)\,y)\,e^{-A(t)-C(t)y},$$

where the function A, C, E, and F must be computed in order to solve the previous differential equation. The boundary condition translates into the following conditions:

$$E(T) = 1 - \chi,$$
$$F(T) = \chi,$$
$$A(T) = 0,$$
$$C(T) = 0.$$

Once the partial derivatives of V are substituted into the differential equation we obtain[5]

$$
\begin{aligned}
0 = & \left(\frac{\partial E}{\partial t} + \frac{\partial F}{\partial t} y \right) + (E + Fy) \left(-\frac{\partial A}{\partial t} - \frac{\partial C}{\partial t} y \right) \\
& + (F - (E + Fy)\, C)\, a\, (b - y) \\
& + \frac{1}{2} \left(-2CF + (E + Fy)\, C^2 \right) \sigma^2 y - y\, (E + Fy)\,,
\end{aligned}
$$

which is an ordinary differential equation in A, C, E, and F. Since this equation must hold for any value of y then we can split it into three ordinary differential equations as follows

$$
\begin{cases}
0 = \frac{\partial E}{\partial t} + Fab - E\,(A_t + Cab)\,, \\
0 = \frac{\partial F}{\partial t} - F\,(A_t + Cab) - Fa - CF\sigma^2, \\
0 = -\frac{\partial C}{\partial t} + aC + \frac{1}{2} C^2 \sigma^2 - 1.
\end{cases}
\tag{3.34}
$$

We immediately see that the value of function $C\,(t)$ can be computed from the third equation. With the suitable boundary condition the only solution of the differential equation for $C\,(t)$ is given by

$$
C\,(t) = 2 \frac{1 - e^{-\sqrt{a^2 + 2\sigma^2}(T-t)}}{\sqrt{a^2 + 2\sigma^2} + a + \left(\sqrt{a^2 + 2\sigma^2} - a \right) e^{-\sqrt{a^2 + 2\sigma^2}(T-t)}}.
$$

The values of all the other functions can be written as functions of $C\,(t)$. Now, if we wanted to compute just the probability to be solvable, then we would have $E = 1$ and $F = 0$ with the function A accordingly solving

$$
0 = \frac{\partial A}{\partial t} + Cab,
$$

with the boundary condition $A\,(T) = 0$. The only solution of this equation is

$$
A\,(t) = ab \int_t^T C\,(s)\, ds.
$$

[5]For the sake of simplicity, we have omitted the functional dependencies.

Given this value for $A(t)$, the two first equations of system (3.34) become

$$0 = \frac{\partial E(t)}{\partial t} + F(t)\, ab,$$
$$0 = \frac{\partial F(t)}{\partial t} - F(t)\left(a + C(t)\,\sigma^2\right).$$

We now compute the value of F from the second equation by obtaining

$$F(t) = \chi e^{-\int_t^T \left(a + C(s)\sigma^2\right) ds},$$

and the value of E can then be computed from the first equation

$$E(t) = 1 - \chi + ab \int_t^T F(s)\, ds.$$

Finally, we can write

$$V(t,T) = \left(1 - \chi + \chi ab \int_t^T e^{-\int_s^T \left(a + C(u)\sigma^2\right) du} ds + \chi e^{-\int_t^T \left(a + C(u)\sigma^2\right) du} y(t)\right) \times e^{-ab \int_t^T C(u) du - C(t) y(t)}.$$

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