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ERHARD WEIGEL AND HIS INFLUENCE ON LEIBNIZ'S PHILOSOPHY OF MATHEMATICS

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Introduction

This work is the result of a research project on Gottfried Wilhelm Leibniz's philosophy of mathematics and the role of Erhard Weigel in its development. Erhard Weigel (1625 – 1699) was Leibniz's teacher of mathematics during Leibniz's early years. He started his academic career as an astronomer, studying at the University of Leipzig at the time of Leibniz's birth and began teaching mathematics in Jena in 1653. His studies however included a much wider range of topics than the ones strictly related to his profession, from the philosophy of nature to metaphysics, ethics and law. While Leibniz is widely known as one of the most brilliant minds of the 17th century, Weigel is little known, even among scholars: it is universally acknowledged that in 1663 the young Leibniz spent a semester at the University of Jena under Weigel's guidance, but this fact is not considered particularly relevant with regards to the development of Leibniz's mathematics, mostly because a significant turning point on this subject happens in the following years, during Leibniz's stay in Paris. Even more, in Weigel's writings is not contained at first glance any decisive reference on topics close to Leibniz's philosophy of mathematics, such as the foundation of the infinitesimal calculus.

However, the publication of Leibniz's writings in recent years shed some light on the relationship with Weigel, highlighting Leibniz's interest in Weigel's works until the end of his life. Lately, some major studies on this topic began to emerge, but a complete account of Weigel's influence is still missing.

In this work, through a reconstruction that goes beyond Weigel's generally accepted influence before Leibniz's Parisian stay, I will argue that some major achievements of Leibniz's philosophy of mathematics were conceived thanks to a constant reference to his former teacher. The ambiguity of the term 'philosophy of mathematics' expresses in an appropriate way the wide range of topics in which this exchange of ideas proved to be effective: from the metaphysical foundation of arithmetics and geometry to the use of mathematics in a metaphysical description of the world. The final outcome of this research will be showing that Leibniz arrived in Paris with the precise purpose of developing in mathematical terms some universal principles, such as the principle of contradiction and the principle by which the whole is greater than its part, endorsed thanks to Weigel's decisive influence between 1663 and 1671. The superior mathematical knowledge achieved in Paris will change Leibniz's original research project dramatically, but another exchange with Weigel around 1679 will renew his interest in the foundational approach on mathematics so much, that from that moment on, Weigel will become a constant reference for Leibniz in this topic, exemplified by the adoption of Weigel's mathematical notation at the end of Leibniz's life.

Weigel's influence then could be ideally divided in two waves of reception, represented respectively by the first and the second part of the present work. The first part does not deal directly with Leibniz’s foundational approach in mathematics, but with the adoption of those principles that at a later time will be used for this specific purpose. This adoption happens between 1663 and 1671 and entails Leibniz’s involvement with Jena’s cultural movement that saw in Weigel one of its most important figure. In this context, Weigel's metaphysical interpretation of Hobbes' philosophy, together with his syncretistic efforts in combining it with some ideas taken from Aristotle and the Scholastics, will constitute the ideal background in which Leibniz's philosophy of nature emerges. Leibniz's idea of primary matter and its role in the conciliation between Aristotle and the moderns, his growing interest in Hobbes' *conatus*, and his derivative knowledge of Galilei, Boyle and many others will be explained by highlighting Weigel's importance. After a general premise on Leibniz's correspondence of those years, the analysis of the dissertations developed by academics under Weigel's guidance will reveal how Weigel's influence on Leibniz was not limited to the semester spent in Jena in 1663, because Leibniz's *Hypothesis physica nova*, written around 1671, could be seen as an ideal development of those dissertations. Conjecturing then a progressive adoption of Weigel's ideas that saw its peak in 1671, a reasonable explanation of how Leibniz adopted some principles taken from Scholastics, while openly criticizing them, could be given. These principles will constitute at a later time the essence of Leibniz's foundation of mathematics. Moreover, although abandoning many of the physical theories developed before Paris, Leibniz will re-elaborate some reflections on the geometry of space taken from those years, introducing them in the wider context of his foundational attempts.

The second part deals with Leibniz's further development of those principles endorsed before Paris in a coherent account that allows the definition of mathematical objects, such as numbers. Aside from the topic of Weigel's influence, it could be seen also as a tentative explanation of Leibniz's claim of deriving the whole of mathematics from the principle of contradiction. Since the role of the principle of contradiction in Leibniz is often underestimated, in the first chapter is presented a general premise on Leibniz's use of it in many heterogeneous topics, such as modal logic, the demonstration of God's existence, the theory of perception and, naturally, the foundation of mathematics. By highlighting the importance of this principle, in the following chapter I will show its connection with the principle by which the whole is greater than its part, relying mostly on Leibniz's explanation of the foundation of mathematics given in his last works on this subject. These universal principles will in turn be connected with Leibniz's concept of homogeneity, founded on the distinction between quality and quantity. This is I believe the most important concept related to Leibniz's philosophy of mathematics and it will be associated with Weigel's decisive influence, making the hypothesis of interpreting Leibniz's efforts as the re-elaboration of Weigel's intuitions a reasonable conjecture.

Having achieved a better insight on the fundamental concepts involved, in the last chapter of the second part I will finally introduce Weigel's influence from an historical point of view. Homogeneity, the difference between quality and quantity and the idea that motion has a precise role in geometry, which Leibniz seems to possess already before his mathematical studies in Paris, will be once again connected to the cultural movement born in Jena in those years. The definition of number given by Weigel in his *Idea matheseos universae* will reveal a close resemblance with the one given by Leibniz, since both are based on the same idea of homogeneity and on the comparison of line segments by means of ratios and proportions. Leibniz's renewed interest for these topics in 1679, related also to the development of his *Analysis situs*, correspond in fact to his first letter to Weigel. From that moment on, during the second wave of reception, Leibniz will extensively read Weigel's works akin to his project of founding mathematics, from the works already read in the early years to the newer ones, like Weigel's 1693 *Philosophia mathematica*. This work is particularly relevant with regards to Weigel's influence, because it introduces the mathematical symbols for 'greater' and 'less' that will be later adopted by Leibniz as an homage to Weigel's achievements, but with the precise purpose of extending them to the mathematics of the infinite. Finally, I will also show that the *Initia rerum mathematicarum metaphysica*, Leibniz's 1714 work originally taken as the best example of his mature theory, was conceived by him in the context of Weigel's school in Jena, even if his former teacher had already passed away at that time.

The third part of the present work is relatively independent from the topics sketched above, because it revolves around the introduction of mathematical concepts in the metaphysical explanation of reality, exemplified by Weigel's and Leibniz's development of binary arithmetics. It is conceived as a new insight on the problem of determining the possible influence of Weigel on Leibniz's dyadic. This subject is debated since the beginning of the 20th century, often with scarce results, depended mostly on an inaccurate knowledge of Weigel's works and on Leibniz's hesitation in openly admitting his debt to Weigel. The reconstruction of Leibniz's discovery of binary arithmetics will show instead how it was developed from the very beginning thanks to a peculiar composition of mathematical achievements and metaphysical concepts that suggest a connection with Weigel's interest in the Pythagorean tradition. Leibniz takes from Weigel not only the idea of counting using a different base from that of the base ten model, but also the idea of connecting this mathematical outcome with the metaphysical distinction between unity and nothingness. The expression of the whole world through these two concepts will stress some major problems on the composition of the continuum that in the end will dissuade Leibniz from connecting binary arithmetics to the other parts of his philosophy, despite his claims about its usefulness and its metaphysical relevance. Even more, this final outcome reveals Leibniz's theological concerns in associating God and numbers, offering a possible explanation of his carefulness in debating about Weigel with his correspondents.

In conclusion, the research in all these fields will reveal Leibniz's huge debt to Weigel and his tradition. However, it will be also clear that, as much as in some specific topics there is indeed a stunning resemblance between the two, Leibniz's novelty consists in his innovative combination of Weigel's principles and general ideas, with a rational strictness that was unknown to his contemporaries. Leibniz's unprecedented rationalism turned the convoluted ideas and daring claims of his former teacher in one of the most advanced mathematical theories of the 17th century.

From a methodological point of view, the present work focuses, as much as possible, on new insights or relatively new insights on Leibniz's philosophy, resorting to traditional and widely known interpretations only when needed in the context of Weigel's reception. Being this a study on the influence of an author over another one, I relied extensively on quotes from Weigel’s and Leibniz’s works, presented in their original language, in order to show the adoption of specific terms or constructions. Some of the writings analysed are unpublished or particularly difficult to find, meaning that their exposition in the original form is also needed for a fair confrontation on the subject. Since many of the dissertations or works examined are not correctly numbered in their original editions, I have given, when needed, a contextual reference based on their chapter divisions, or other internal criteria.

PART I: Philosophy of Nature and the Adoption of Universal Principles

1.1 General Remarks on the Purpose of This Part

The secondary literature on Leibniz’s early years agree in principle on the intricacies posed by his position: identifying distinctive influences on his philosophy has proven to be challenging, due to the large amount of authors quoted and read during the first ten years of his production. Evidently, some major influences can indeed be traced, Scholastics or modern and ancient atomists to name a few[1], but what we gain by this, as major ideas emerge in Leibniz’s writings, is lost when the uniformity of these positions does not allow the conferment of original accounts to specific authors. Perhaps Thomas Hobbes is one of the few authors that can be genuinely listed among Leibniz’s direct influences without effort, not only because he is openly quoted from the start, but also because his theories were so unique and recent that no manual or specimen would have successfully adopted them. Yet, I will soon show that even Hobbes' reception depended ultimately on a version of his philosophy mutuated by a cultural tradition that was adopting and modifying his works for their purposes. The very difference between newer and older theories then seems at stake here: during the early years, Leibniz’s knowledge seems very derivative and manuals adopted by him are often responsible for this lack of focus on a single author. If we turn then to the history of ideas, moving from the impact of specific authors to the transmission of precise concepts, a study on the young Leibniz becomes even more challenging, if only because a philosopher’s position is by no means a collection of previous statements on the same matter, combined in order to produce a new interpretation.

Knowing these difficulties, presenting Weigel as a major influence on Leibniz’s rationalism in these years seems somehow misleading, even more if, as it will be argued

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[1] There are of course too many works that deal with Leibniz’s reception of Hobbes, Democritus, Gassendi, Alsted, Bisterfeld and the scholars. For more information on the young Leibniz’s influences related to the topic of this section see Mercer (2002a, 27-49), who openly recognize Weigel’s influence, much like Piro (1990) already did. A relatively new collection of essays on this topic can be found in Kulsted (ed. 2009), whereas a systematic historical approach can be found in Antognazza (2009). Much less are instead the writings on Leibniz’s early atomism, for example Wilson (1994) Arthur (2003) and Mormino (2012, 111-142).

in this section, the key factor that determines it is the adoption of universal principles, especially the principle of contradiction. The adoption of this principle is in fact quite common among the Scholastics: Thomas Aquinas adopted it himself and distinctive traces of its use can be found in Boethius, determining its presence in Suárez and several other authors of this tradition[2]. However, the fact that Leibniz read about this principle in these authors, perhaps even before his first contact with Weigel's works, does not necessarily mean that he had found it suitable for its emerging philosophy. This hypothesis seems to be confirmed by the fact that Leibniz was not always akin to Scholasticism, due to his adoption of the new physics, as the famous quote on his difficult choice between the old and the new vision of the world shows[3]. From these premises, an apparent inconsistency follows: after years of fascination for the new ideas and a severe judgment on the terminological confusion of the Scholastics on the matter of what will be later called theodicy[4], in the 1672's *Confessio philosophi* Leibniz abruptly adopts a Scholastic principle, the principle of contradiction, to solve the same problems. Consequently, a comprehensive study on the development of this principle seems necessary: it is true that Leibniz already turned to some of the Scholastics' ideas before the *Confessio philosophi*, but a naïve adoption of this principle was almost impossible for a man extremely aware of the great innovation that was taking place in the philosophy of nature, such as Leibniz. What I believe is often underestimated is the impact of the new physics: adopting, refuting, maintaining or changing the old theories was surely a matter that had to be analysed depending on the specific subject, such as the problem of individuation or modality, but it is hard to deny that the urgency of the debate depended ultimately on the fact that in other and more relevant topics of that time the old theories were severely criticised. These criticisms, bearing a whole new interpretation of the world destined to affect more than few philosophers, are perhaps more important than the

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[2] The genesis and development of this concept will be examined in the last chapter of the present part.

[3] I'm referring here to the famous quote in Leibniz's 1697 letter to Thomas Burnett: "La plus part de mes sentimens ont esté enfin arrestés apres une deliberation de 20 ans. Car j'ay commencé bien jeune à mediter: et je n'avois pas encor 15 ans quand je me promenois des journées entieres dans un bois pour prendre parti entre Aristote et Democrite. Cependant j'ay changé et rechangé sur des nouvelles lumiéres. Et ce n'est que depuis environ 12 ans que je me trouve satisfait, et que je suis arrivé à des demostrations sur ces matieres qui n'en paroissent point capables. Cependant de la maniere que je m'y prends, ces demostrations peuvent estre sensibles comme celles des nombres quoyque le sujet passe l'imagination" (AA I-14, 224). I believe however that there was never a time when Leibniz had modern ideas completely deprived of scholastics' influences, so much that the famous episode could be seen more like a rhetorical tool used by Leibniz to impress his correspondent.

[4] For more information on the topic of predetermination see Mormino (2005 and 2009).

account of a specific teacher or manual for a perceptive mind such that of Leibniz, who will later contribute himself to the scientific revolution. For these reasons, the use of contradiction by Leibniz can't be seen as a mere return to Scholastics, if only because it should be consistent with his entire philosophy at that time.

It is in this line of thought that Weigel's influence becomes relevant: rather than emphasising his impact to the detriment of other authors, something that even the young Leibniz wouldn't probably admit, I will argue that his main contribution was teaching a critical-aware approach to some major ideas taken from the Scholastics, such as the principle of contradiction, and others taken from the moderns, especially Hobbes. The purpose of this section is hence showing how the first wave of Weigel's reception has to be considered on one hand, as if Weigel was only one among several authors that contributed to Leibniz's formation and, on the other hand, as if he played an important role in altering some fundamental Scholastic ideas, making them suitable for a new interpretation in the light of modern physics. A small contribution in the eyes of the young Leibniz can be considered an important achievement, turning instead to the whole development of Leibniz's philosophy, since it affects the very core of his rationalism in the following years, as it will be shown.

At first, this purpose may appear far from that of studying Leibniz's philosophy of mathematics, but it will hint instead to a very strong consistency between physics, geometry, logic and even law in Leibniz's early years. So much that, as an outcome of this research, it will show that Leibniz's very first purpose was also solving a problem on the ground of physics, rather than one pertaining only the so-called traditional and metaphysical philosophy. This consequence seems obvious on a general basis, but proving it on the assumption of a relation between physics, mathematics and the principle of contradiction becomes challenging. The result is a possible line of thought that connects Leibniz's general attitude during these years to his later remarks on the possibility of founding the whole of mathematics on the principle of contradiction[5]. What is even more relevant in fact is that the reinstatement of the principle of contradiction is only the most prominent result of Weigel's influence, bringing along a distinctive set of

[5] The reference is to the famous letter to Clarke, dated 1715: "Le grand fondement des Mathématiques, est le principe de la contradiction, ou de l'identité, c'est-à-dire, qu'une énonciation ne sauroit être vraie et fausse en même tems; et qu'ainsi A est A, et ne sauroit être non A. Et ce seul Principe suffit pour démonter toute l'Arithmétique et toute la Géométrie, c'est-à-dire, tous les Principes Mathématiques" (GP VII, 355).

ideas and problems that Leibniz showed to be aware of at an early stage, but that he decided to develop and face during a much longer period of time than that of their early adoption.

This entire section will be focused on explaining how these general ideas are connected to Weigel's specific influence. Through the analysis of Leibniz's correspondence between 1663 and 1671, in the next chapter I will show not only that a connection with Weigel is possible, but also how it developed in time.

In the following chapters, starting from the philosophy of nature, I will examine how Weigel's school influenced Leibniz on the topics of motion, primary matter, quantification and geometry. Starting from the analysis of the *Disputatio metaphysica de principio individui*'s corollaries, I will deal with the physical problem of the *penetratio dimensionum*, a perfect example of the incompatibility between the qualitative approach of the Aristotelian physics and the quantitative approach of the new one. In this context, avoiding contradiction was considered the primary requisite of a well-grounded theory, explaining Leibniz's efforts, who at first reintroduces the impossibility of penetration in a new fashion taken from the new physics and then finally adopts the principle of contradiction as an ontological tool developed to reunite the opposing theses.

Having sketched the context in which a connection between physics and logic is possible, through an in-depth analysis of Weigel's *Analysis aristotelica ex Euclide restituta* and other writings belonging to Weigel's school we will be able to determine the extent of Weigel's influence: the exact same conclusions proposed by Weigel, i.e. Aristotelian key concepts saved from the severe criticism towards Scholastics, are contained in what we established as the final stage of this period, that is Leibniz's *Hypothesis physica nova* and his coeval writings. Having achieved new insights on this topic, it will be possible to follow Leibniz's progressive adoption of various principles, especially in logic and geometry of space, between 1663 and 1672. In the last chapter of this part, the outcome of this research will be that at the beginning of the Parisian period Leibniz was well aware of the connections between his metaphysics and mathematics, so much that his mathematical studies in Paris, from that time on, will be focused on topics that he could have already decided to develop at an earlier stage. The final conclusion to this part will allow me to explain in the following one in what sense from a philosophical point of view Leibniz's ontological use of logical laws affects his idea of space in geometry and arithmetics: through the analysis of apparently unrelated topics we witness the birth of Leibniz's concept of homogeneity, a key element of his philosophy of mathematics.

1.2 Weigel's School and its Pivotal Role on the Young Leibniz's Philosophy: A Study on the 1663–1671 Correspondence

The Weigel-Schule and its objectives In order to show the importance of the principle of contradiction and its connection with the philosophy of nature, a thorough study on the time of its adoption in Leibniz seems at least legit. By no means Weigel played a secondary role in the process of adopting this concept, tainted otherwise by decisive, yet elusive, Scholastics' influences, but some preliminary distinctions should be pointed out.

First of all, we should refrain from emphasising the importance of contradiction in Leibniz's early years in the light of its decisive influence at a later stage: by doing so, we would ignore the relevance of the context in which this principle arose and, consequently, we wouldn't be able to explain for what reason it seems related to so many topics in Leibniz's philosophy. On the contrary, Weigel constitutes a decisive influence especially because the principle of contradiction is only one among those topics in which Leibniz seems close to his views. Nevertheless, all the other topics involved are precisely the ones that will be connected to the principle in Leibniz's production after the 1670s, meaning that there is a way through Weigel to retrace this common background. Considering this a mere coincidence would be far too unwise: it is as if a general analysis on the fundamental principles in Leibniz could be retraced in Weigel's works, following a more specific path. In the *Analysis aristotelica ex Euclide restituta* the reference to the principle of contradiction per se is of course present, but the references to the connection between identity and contradiction, the importance of the *reductio ad absurdum* in the light of syllogistic, the use of contradiction in the definition of modal concepts, the foundation of mathematics on the principle by which the whole is greater than its part, a constant reference to the Euclidean background and a possible connection between mathematics and a kind of *ars combinatoria* are also present. All these topics are the ones that in the following chapters will allow a coherent reconstruction of the relationship between the principle of contradiction and Leibniz's *analysis situs*.

However, referring to Weigel's *Analysis aristotelica* only would not be sufficient for understanding the dynamics behind Leibniz's reception: my main aim for this part would be that of showing how important traces of Weigel's influence could be found from the *Disputatio metaphysica de principio individui* to the *Hypothesis physica nova* and then maintained to a certain extent even later, regardless the evident changes in Leibniz's philosophy. What I'm suggesting here is that Weigel's influence, much like the topic of contradiction, should be seen in the light of a progressive adoption, i.e. something that evolves in time and adapts to Leibniz's newer views, rather than something that obeys to the simple law of reading once, evaluating and then discarding or adopting. Leibniz in fact extensively read Weigel throughout his life and there are times and topics in which Leibniz is openly against Weigel's views, but there is no denying that the evolution of Leibniz's philosophy led him to the revaluation of his early reception of Weigel's ideas, introduced however in a different context. The difference between the first and the second wave of reception then is the broader background in which the former is integrated: during the first wave, Weigel's ideas are undoubtedly present in other authors, but it was he the one who played a pivotal role for Leibniz, because in Weigel these ideas are present altogether and they are presented in an appropriate way, different from that of naïve Scholastics and ready for the new physics.

This is what I will argue as far as the purely theoretical confrontation between the two authors goes, but founding the importance of Weigel's influence only on an ideal connection would be an utter mistake in the context of Leibniz's early years, because similar theories presented by different authors are alternately adopted and discarded by him, so much that connections in such a way could be ideally found for any author. Thankfully, Weigel's influence possesses two unique and relevant features that sets it apart: (1) priority in space and time and (2) being the result of a school of thought that shared common interests and topics, founded by Weigel himself.

The priority in space and time, although it will be proved once more by the analysis of Leibniz's correspondence, is a truth universally acknowledge: no one questions the fact that Leibniz spent time in Jena with Weigel in 1663[6]. I believe however that the importance of this aspect is often underestimated. The various reconstructions of Leibniz's early years indulge often on the idea of a young genius, driven by an insatiable curiosity fostered by the goal of mastering the entire knowledge of his times. Even if this description is undoubtedly true, it suggests that the variety of authors chosen by Leibniz rests ultimately on a peculiar trait of his character, giving to these choices a degree of freedom and awareness that is hard to prove without resorting to mere conjectures. In this

[6] See for example Moll (1973, 33).

interpretation, the various influences stand on the same ground and Leibniz's wit, corroborated by the correspondences with his works, is responsible for the inclination towards a specific author. However, if it is possible to prove instead that many of these authors are somehow connected to Weigel's influence, his presence next to Leibniz since 1663 would gain a much more profound meaning: we could argue then that Weigel introduced some of these authors to Leibniz and that Leibniz developed his own position from there, aiming to achieve cultural relevance in the specific context in which he lived. I believe that this hypothesis is much more reasonable for a young and talented student in need of acknowledgment. By identifying a connection between Weigel and other authors, I do not mean that Weigel could be hidden behind all these authors as the true reference for Leibniz, which would be against the true purpose of a serious study on Leibniz's sources. Rather, I argue that talking about Hobbes, Descartes, Gassendi, Jungius and Aristotle in a certain way is the result of a cultural environment that Weigel himself contributed to create.

In this regard, the idea of a Weigelian school or milieu helps us in understanding the development of Leibniz's philosophy. The main characters of this cultural influence were Weigel himself, together with many of his students[7], like Samuel Pufendorf, Friedrich Nitzsch and a young Johann Christoph Sturm, but also people close to Weigel, like Johann Andreas Bose, Abdias Trew or Hermann Conring. Not to mention the syncretistic approach that Leibniz already learned from Jakob Thomasius and Johann Adam Scherzer[8]. The multiple goals of this cultural program that stretched from Jena and Leipzig's areas were as follows:

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[7] A list of Weigel's students and their enrolment in the University of Jena is found in Herbst (2016, 345-365).

[8] Scherzer and Weigel were very different in their approach to syncretism: Schrezer introduced Jewish theology and the Kabbalah in his idea of clarifying the confused state of the seventeenth century philosophy, tending towards a mysticism that was unknown to Weigel. Weigel's syncretism was much more inclined towards rationalism and, consequently, it gained quickly Leibniz's favour. Although often confusing, Weigel's works never abandon the idea of a precise and rational exposition of a connection between all aspects of human knowledge. Testament to this idea is for example the conclusion of Weigel's *Disputatio statica de aestimatione gravium*: "Astrologicae vanitates, quales sunt erectorum thematum judicia, praedicationes rerum futurarum & c. Matheseos partem non faciunt" (Weigel-Scheidlin 1665, *Corollaria*, 6). This is the reason why I do not believe in a priority of Scherzer over Weigel, but that does not mean that they were not sharing the same goals in their cultural programs. On the connection between Italian renaissance, present in Scherzer, and the Jesuits' revaluation of Kabbalah see for example Altmann (1987, 173). On the common interests shared by Scherzer and Weigel see Mercer (2002b, 239).

- Reconciling the new physics endorsed by men such as Gassendi, Hobbes and Descartes with some intuitions belonging to the older physics, by highlighting that a proper interpretation of Aristotle's philosophy would reject some of the Scholastics' ideas that were clearly against the newer philosophers.

- Achieving a rational theory of morality and jurisprudence by extending the universal principles used in metaphysics and mathematics to these fields, thus reconciling the excellent but somehow dangerous ideas developed by Hobbes to those of authors such as Grotius.

- Adopting Euclid and Aristotle as paradigmatic examples of rational proofing, both in mathematics and logic, giving birth to a logic based on the idea of mathematical combination and a syllogistic that strives for unification in the fashion of the Euclidean method.

- Founding a society of German cultured men able to compete with the other European academies of sciences.

Analysing the influence of this program is in my opinion the key to understand the problem of Leibniz's progressive adoption of universal principles in an ever changing background such that of his early years. This problem, although being present throughout the whole history of Weigel and Leibniz's relationship, is of course particularly relevant in the transition between the *Disputatio metaphysica de principio individui* and the *Hypothesis physica nova*. If we accept however the existence of such program, we can clearly see for example that the young Leibniz's initial adhesion to atomism and its rejection in the following years is not necessarily inconsistent with a constant Weigelian influence: in this syncretistic background, both positions were accepted and used. Although rejecting a purely atomistic view for example, we will show that Weigel compares, in an Aristotelian context, the combinatorial ability of a universal logical language inspired by Hobbes to the collisions between atoms described by Democritus. In the letter to Thomasius dated April 1669, Leibniz will state in the same fashion: "Mihi enim neque vacuum neque plenum necessarium esse, utroque modo rerum natura explicari posse videtur"[9]. The bold claims of Weigel and other authors close to him show that they were already past any possible concern regarding theological matters, although aiming ultimately for the complete reconciliation[10].

The basis for this reconciliatory project is a peculiar interpretation of Aristotle's *Apodeixis* as a tool applicable in all sorts of fields[11]. Talking about jurisprudence then was not that different from talking about logic, metaphysics, morality or mathematics, since the method and the structure of the universal principles adopted was one and the same. I will argue then that Leibniz's philosophical shift is the realisation of a research project endorsed by him in the light of Weigel's syncretistic school, on the ground of physics, ethics and logic. From Leibniz's initial reception in 1663 it will reach its peak in the famous letter to Thomasius dated April 1669, with the *Hypothesis physica nova* of the following years as the most prominent outcome of this reception, cherished by Weigel himself. In order to do so, I will turn now to the analysis of Leibniz's correspondence in those years.

From Leibniz's Perspective The very first letter of the critical edition of Leibniz's philosophical correspondence contains also the first reference to Weigel. It is a letter to Thomasius dated September 1663, meaning that at that time Leibniz already discussed his *Disputatio metaphysica de principio individui* in June. He was enrolled for the summer semester at Weigel's University of Jena, but he was about to come back to Leipzig the next month. In this letter we read:

> Unus mihi Dominus Pufendorfius notus est, qui tamen sua *elementa jurisprudentia[e]* ex Weigelii nostri Ethica Euclidea manuscripta dicitur fere tota efformasse (AA II-1, 5).

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[9] AA (II-1, 25).

[10] An approach that was different from that of Leibniz, much more concerned about the reception of his theological ideas in the society of his time. This is the reason why, from one side, Weigel's influence will be denied by Leibniz in the following years and, from the other side, the tension between the ideas belonging to the Weigelian school and the traditional ones, opposed to any form of syncretism, will lead to Leibniz's original solution concerning theodicy.

[11] See Thomas Behme's introduction in Pufendorf (2009, IX).

This reference is extremely relevant for several reasons. I won't overinterpret this "Weigelii nostri", but both the best and the worst-case scenario would lead to interesting results. It could be a mere geographical reference, although as we saw such references shouldn't be underestimated: between 1647 and 1650 in fact, Weigel studied at the University of Leipzig and obtained the qualification there with a dissertation entitled *Dissertatio Metaphysica Prior (De Existentia) und Posterior (De Modo Existentiae, qui dicitur Duratio)*. Only in 1653 he will start his career as a professor in Jena. This means that even if Leibniz here is implying with Thomasius only a common geographical background, that background is still the one that gave birth to Weigel's philosophy.
Giving more importance to Leibniz's words instead would be possible by outlying that Leibniz and Thomasius here seem accustomed to a Weigelian reference: the Euclidean ethics mentioned could probably be the one contained in the *Analysis aristotelica ex Euclide restituta*. This book, published in 1658, is the most important work needed to retrace Weigel's influence on Leibniz's adoption of universal and physical principles. As we already sketched, it contains many ideas consistent with the cultural program previously mentioned. The crucial point here is understanding if Leibniz was influenced by this book only after his arrival in Jena or before that, already in his *Disputatio metaphysica*. I believe that the latter interpretation is the most reasonable[12]: not only in the *Disputatio metaphysica*, as it will be shown in the next chapters, we find concepts possibly taken from Weigel's *Analysis*, but also the fact that Weigel studied in Leipzig suggests that Thomasius'circle itself could have introduced Leibniz to this work. This should not sound surprising, if we think for example that around 1669, Thomasius' own son, Christian Thomasius, although studying in Leipzig and not in Jena was greatly influenced by Pufendorf, Weigel's closest disciple[13].

If Leibniz's early adoption of Weigel's work is plausible, then we could think that the semester spent in Jena was favoured by Leibniz with the precise purpose of studying under Weigel. This interpretation is supported by the fact that by the time of the 1663 letter to Thomasius, Leibniz joined in Jena an academic society, the *Societas quaerentium*, led by Weigel himself and composed by students and teachers, with the

[12] For a different account on the same topic see Piro (2005, 85). Weigel's supposed platonism, related to the topic of identifying his influence before Jena, will be discussed in the following chapters.
[13] See the *Institutiones Iurisprudentiae Divinae Libri III in Quibus Fundamenta Iuris Naturalis Secundum Hypotheses Illustris Pufendorffii Perspicue Demonstrantur* (Thomasius 1994).

purpose of studying and comparing works taken both from the old and new tradition[14]. Even more, it is not surprising that Weigel was the president of this society: by the time of its foundation, Weigel had already been director of the *Philosophischen Fakultät* in 1656 and dean of the whole university the following year. Weigel's institutional prestige and reputation is important in understanding his role in the cultural movement born in Jena, because it suggests that he had not a secondary part in its development. At the same time, the publication of his controversial *Analysis* in 1658 lead to a conflict with his university. Only in 1675 Weigel will be elected once again dean of the university and regain the favour of his community. It follows that, by the time of Leibniz's studies in Jena, Weigel was indeed an important cultural figure, but at the same time, he was not at the peak of his career, meaning that Leibniz's choice was not driven only by Weigel's prestige and it involved a degree of risk.

Weigel's institutional relevance was complemented by his theoretical influence: it is Leibniz himself that in the letter to Thomasius previously quoted establishes not only a connection between Weigel and Pufendorf, but also Weigel's direct influence on his pupil's *Elementorum iurisprudentiae universalis libri duo*. This idea of a complete dependence of Pufendorf's work was further developed by Leibniz in the following years, since he is credited to be one of the first men who recognized how the new appendix to this book, published in 1669, was probably written by Weigel and not by Pufendorf[15]. As Leibniz recalls in the *Nouveaux essais sur l'entendement humain*:

> Feu M. Erhard Weigel Mathematicien de Jena en Thuringe inventa ingenieusement des figures, qui representoient des choses morales. Et lors que feu M. Samuel de Puffendorf, qui estoit son disciple, publia ses *Elemens de la Jurisprudence Universelle* assés conformes aux pensées de M. Weigelius, on y adjouta dans l'Edition de Jena la *Sphere morale* de ce Mathematicien (AA VI-6, 385).

[14] This affiliation is already documented in Aiton (1985, 32) and in every other book concerning Leibniz's life, for example Antognazza (2009, 59).

[15] The appendix is indeed a precise description of Weigel's moral sphere, plus there is also a quote from Weigel's 1652 *Dissertatio Metaphysica Prior (De Existentia) und Posterior (De Modo Existentiae, qui dicitur Duratio)* written during his stay in Leipzig (Pufendorf 1669, 516).

Since the *Elementorum iurisprudentiae universalis libri duo* is the work in which Pufendorf is concerned the most with the project of reconciling Hobbes' theories with those of Grotius and other philosophers, it suggests that both Weigel and Pufendorf were behind Leibniz's ever growing interest in Hobbes' philosophy and in his project of harmonizing it with Aristotle[16]. If the *Elementorum*'s appendix was really written by Weigel as Leibniz believed, it also shows that Weigel was active in the field of law and ethics in 1669, which is consistent with our purpose of showing that Weigel's influence shouldn't be restricted to Leibniz's 1663 semester in Jena, but it caused its most profound consequences around the time of the 1669 letter to Thomasius and the following years. On a side note, by analysing these writings we would already find a peculiar contamination between universal principles taken from logic, their application in law and ethics and their description through mathematical models that rested on the exactness of the Euclidean method.

Moving on from Leibniz's 1663 letter, between 1664 and 1667 we witness few direct references to Weigel in Leibniz's correspondence. This does not mean however that Weigel was not present in Leibniz's mind: he is quoted, again about law and ethics, in the XV *quaestio* of the 1664 *Specimen quaestionum philosophicarum ex iure collectarum*, in the 1666 *De casibus perplexis* and in the 1667 *Nova Methodus*[17]. Above all, he also is present in the form of his disciple Johann Christoph Sturm in the *Dissertatio de arte combinatoria*. Again, it is Leibniz himself, in an important passage of his *Essais de Théodicée*, that suggests a very close relationship between the young Sturm's works and Weigel's, like he did with Pufendorf:

> Feu M. Sturmius, Mathematicien célèbre à Altorf, étant en Hollande dans sa jeunesse, y fit imprimer un petit livre sous le titre d' *Euclides catholicus*, où il tâcha de donner des règles exactes et générales dans des matières non

[16] For example, the fact that in the same 1663 letter to Thomasius Hobbes is exstensively quoted just before the reference to Weigel and Pufendorf, is not a mere coincidence: "Demum vivus undique aut Hobbes aut Hobbesianus elucet. Cum enim utilitatem aequi matrem habeat, igitur prout illi velificabitur jus omne, stabit cadetque: cum cuilibet principum absolutum det imperium, sola suspicio principi ad supplicia jus dabit: demum quia a civili lege omnis justitia propullulat, necessario obligatio omnis ac foederum servandorum necessitas ruet inter civitates. Haec in Hobbesio saepe reprehendentem V.E. audivi. Quare cepit me quaedam de Hobbesio a V.E. quod ejus pace fiat, sciscitandi cupiditas, quis ille, an adhuc superstes, an Antagonistam na[c]tus, an habuerit, qui in jure naturae illustrando paria fecerint, meliora aut aequalia, si subtilitatem spectes" (AA II-1, 5).

[17] A reconstruction of these quotes if found in Moll (1978, 65-67).

> mathématiques, encouragé à cela par feu M. Erhard Weigel, qui avoit été son précepteur. Dans ce livre, il transfère aux semblables ce qu'Euclide avoit dit des égaux (GP VI, 245).

Since Leibniz here is prising Sturm's early works, he is probably referring to his 1660 *Aristoteles Mathematicus* and his 1661 *Universalia euclidea*, both known by Leibniz at the time of his *Dissertatio de arte combinatoria*. In these works, there are many interesting passages about Aristotle's syllogistic and about Euclid's influence that will be very useful once we will deal with the problem of the foundation of mathematics and the *analysis situs*.

Around 1668, encouraged by his achievements in those years, the Weigelian project of reconciling the old and the new became a priority for Leibniz, especially on the ground of physics. This interpretation is consistent with Leibniz's private thoughts on this matter, but also with his ambitions in the society of that time. His mind was set on this project, as a letter to Thomasius dated September 1668 shows:

> Satis ostendit Raey in clave philosophiae naturalis, tenebras Aristotelis a scholastico fumo esse, Aristotelem ipsum Galilaeo, Bacono, Gassendo, Hobbesio, Cartesio, Digbaeo mire conformari. Quid enim aliud Aristoteli materia prima est, quam iners moles sine motu, et per consequens, si omnia plena sunt, sine figura? (AA II-1, 18).

Here we find all the elements that constitute Weigel's cultural influence: the distinction between the real Aristotle and his calculatedly-perverted version given by the Scholastics, the importance of the new philosophers, like Galileo, Gassendi, Hobbes and Descartes and a focus on the physical problem of motion and matter. Leibniz quotes Johannes de Raey, a philosopher that with his *Clavis philosophiae naturalis, seu introductio ad naturae contemplationem Aristotelico-Cartesiana*, as the title suggests, had a similar project in mind. We will soon see that once again it is Leibniz himself that points towards a priority of Weigel's influence over Raey's[18].

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[18] However, this does not mean that Leibniz could not have found in de Raey similar ideas on the universal principles that supported Weigel's influence: "Aristoteles i.poster. Anal. cap. 10. *Axiomata* vocat, *quae non tantum per sese vera esse, sed vera etiam videri debent*. Quod cave ne ita intelligas, quasi necessum sit id

Meanwhile in 1668, Leibniz was also concerned with the cultural relevance of his efforts. As it is widely known, he wanted to gain the favour of Peter Lambeck for the foundation of his biannual review *Nucleus Librarius Semestralis*, modelled after the *Journal des Sçavans*[19]. This idea was consistent with the aspirations of the cultural environment based in Jena, set on competing with the rest of Europe. Testament to his commitment to the syncretistic cause, the 1668 letter to Lambeck shows some interesting information on Leibniz's sources:

> Nam etsi nullus sit in mundo motus prorsus rectus, erunt tamen demonstarationes Galilaei verissimae, et etsi nemo hactenus lineam Quadratricem accurate duxerit, nihil hoc officiet demonstrationibus de ea Procli, Pappi et Clavii. Similiter, etsi servi nulli sint […] ex jure naturali est ab ICtis Romanis maximam partem absolute demonstratum […]. Idem sensit Ioach. Hopperus, sensit Hugo Grotius, sentit et Herm. Conringius, cujus de instituto nostro literas habemus; quos tamen nonnulli sibi ad contemnendam Romanam Iurisprudentiam falso duces putant (AA I-1, 14).

Once again, law becomes one of the main topics in which Aristotle's *Apodeixis* is displayed, but the comparison with Galilei is also important per se, because it shows that the new interpretation in the light of Euclid was perhaps supported by his works and those of Proclus and Clavius. This reference is relevant for our purposes because it outlines that the Euclid read in the Weigelian school was the one harmonized with his most metaphysical and foundational interpretations.

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omne, quod communis notion seu axioma est, ita ab omnibus ac singulis percipi, ut a nemine ignorari, vel negari possit. Quanquam enim cunctis obvium sit, totum esse majus sua parte, factum infectum fieri non posse, similiaque, quibus nullae praeconceptae repugnant opiniones, sed quae vel quotidianis sensuum experimentis confirmantur, vel saepe nobis occurrunt ac perpenduntur; id tamen locum non habet in quam plurimis aliis, quae non minus quam ista per se nota sunt atque pro communibus notionibus haberi debent. Ut enim oculus non videt quae praesentia illi non sunt, ita mens nostra innumeras nescit communes notiones, de quibus cogitandi nulla datur occasio, cujusmodi sunt pleraque eorum quae sensus fugiunt, nec non quam plurima Geometrarum principia" (Raey 1654, 37). On de Raey and physics see Ruestow (2012, 61), but also Mercer (2002a, 107).

[19] See Antognazza (2009, 97). Leibniz was supported by Boineburg: "Is est Godofredus Gulielmus Leibnütius Iuris Doctor, juvenis et labori assuetus, et qui profecto interioribus ac amoenioribus literis, adeoque omni eruditioni quaquaversum exporrigit se, philosophiae et scientiis pene omnibus magna felicitate et animi contentione incubuerit exemplo plane raro et perinsigni" (AA I-1, 8).

Despite the fact that Thomasius was not completely convinced by Leibniz's ideas[20], in 1669 he witnessed Leibniz's most ambitious desire to turn these inclinations in an actual theory. In the famous letter dated April 1669, among the by now well-known distinctions between Aristotle, Scholastics and the newer philosophers, a new category is introduced, that is the one composed by those who tried to achieve this reconciliation, in which obviously Leibniz puts himself in. This passage, which is worth quoting in its entirety, reads:

> Neque vero vereor, ut, quae dixi hactenus, ex Raeo descripta, aut hujus auctoritatem me sequi amplius putes. Dudum talia a me cogitata sunt, antequam de Raeo vel audivi. Legi Raeum quidem, sed ita, ut nunc eorum, quae disserit, vix recorder. Neque vero Raeus conciliatorum inter Aristotelem et recentiores primus solusque est. Primus Scaliger mihi viam stravisse videtur; nostris temporibus Kenelmus Digbaeus et ejus assecla Thomas Anglus, ille in libro de animae immortalitate, hic in institutionibus peripateticis, idem longe ante Raeum ex professo egere. Nec abludunt tum Abdias Trew, tum inprimis Erhardus Weigelius (AA II-1, 28-29).

Erhard Weigel then is both one of the most important and one of the closest 'ancestors' of Leibniz, concerning the topic of reconciling Aristotle. Knowing that Weigel introduced Leibniz to these topics back in 1663 we can now safely say that Weigel's influence lasted for several years, making the hypothesis of a progressive adoption a reasonable assumption. Moreover, as it happened with Pufendorf and Sturm, the priority of Weigel's influence is also confirmed by the reference to de Raey and Leibniz's claim that he was into these kind of thoughts even before he read de Raey's works. However, the difference between 1663 and 1669 is that now Leibniz actually read de Raey, and many more: he was now ready for his own solution to this long-standing problem, symbolized in part by the *Hypothesis physica nova*. In order to understand how this work should be seen in the light of Weigel's cultural influence we will turn now to some references in Weigel's writings of those years and in Leibniz's correspondence between 1670 and 1672.

[20] See for example his letter dated October 1668 (AA II-1, 21).

From Weigel's Perspective By analysing Leibniz's correspondence and his works, we can safely assume that Weigel had a role in his attempt to save some principles of Aristotle's philosophy and that this attempt was a pressing priority in 1669. However, if we depend only on Leibniz's quotes, ignoring the fact that he spent time in Jena in the *Societas quaerentium*, we could still believe that Weigel's influence has to be treated in the same way of any other author that Leibniz read during those years. A look at Weigel's coeval works will help us in understanding that the context in which this influence arises was much closer to Leibniz than many others were.

As I already pointed out, the *Analysis aristotelica ex Euclide restituta* is indeed a very important work on this topic, but the fact that it was published in 1658 could lead to the assumption that between 1669 and 1671 his importance was already fading in Jena. On the contrary, 1669 marks the start of one of the most important and prolific period of Weigel's philosophy: from that year on, he will publish in Jena many dissertations under his tutoring and many original works, like the *Idea matheseos universae* (1669), the *Tetractys summum tum arithmeticae tum philosophiae discursivae compendium, artis mangnae sciendi genuina radix* (1673), and the *Universi corporis Pansophici caput summum* (1673), among others. Above all, he also republished the *Analysis aristotelica* in 1671, under the name *Idea totius encyclopaediae mathematico-philosophicae*, meaning that the topics of this book were still debated. Since these years are also crucial for Leibniz's *Hypothesis physica nova*, could it be that this work, and Leibniz philosophy in general, developed in this dynamic cultural context in which Weigel was so active? Once again, Leibniz's correspondence proves itself to be very useful on this matter, as the letter to Hermann Conring dated April 1670 shows:

> Audio et Cl. V. Erhardum Weigelium, libello de aestimatione mox prodituro, plurima ex jure nostro, quae ad quantitates gradusque voluntatis, scientiae, diligentiae, malitiae, poenae, damni, certitudinis, praesumtionis, probabilitatis, aliorumque, quorum in re morali crebra mentio est, delibaturum (AA, II-1, p. 70).

Giving the variety of topics outlined by Leibniz and the specific language used, the reference is probably to Weigel's *Universi corporis pansophici caput summum*, except that this work was published three years after this letter, meaning that Leibniz was aware of its existence during the time of its draft. It surely shows that Leibniz was in contact with Weigel's milieu in those years, but not only that, because in the *Universi corporis pansophici* Leibniz's *Theoria motus concreti* is quoted by Weigel, as he refers to the "Bullulae, quales Orbis eruditi Senatui Anglicano tum Gallico nuper exhibuit honoratissimus meus Leibnizius, sagacissimi vir ingenii, in Theoria motus"[21]. It shows that Weigel was also aware of the political and cultural intent behind Leibniz's *Hypothesis*, besides praising its content, as he will remark once more in another work published in 1674[22]. If we combine these data, i.e. the fact that the *Hypothesis* was known and prised by Weigel and the fact that the same work in which it is quoted was known by Leibniz three years before its publication, during the draft of the *Hypothesis*, we could suppose a common intent and a reasonable confrontation.

This idea is supported by Leibniz's correspondence between 1669 and 1672, as it shows that the contacts with the Weigelian circle were intensifying. Both in 1669 and in 1670 for example, Leibniz will write to Johann Andreas Bose, historian and scholar in Jena close to Weigel, asking him to greet his former teacher[23]. This reference is important in the light of the previous letter to Conring, because it suggests that Bose, Weigel and Conring were related, as other references show[24]. It seems natural then that Leibniz was speaking freely about Weigel to Conring and, above all, that in the following years Conring will become one of the best suited correspondent for Leibniz, in order to discuss about topics very close to Weigel's philosophy, such as the universal principles, logical reasoning and the modal demonstration of God's existence[25]. About the use of the principle of contradiction in modal logic, the first letter to Bose shows another important

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[21] Weigel (1673c, 39). The same reference can be found in Moll (1982, 60).
[22] The very last sentence of Weigel's Corollaria in his 1674 *Pendulum ex tetracy deductum...sistit* is: "Speciosa est hypothesis Leibnüziana, quae bullulis pleraque Phaenomena Corporum salvare docet" (Weigel 1674, Corollaria, 8).
[23] "Nunc tamen, cum ad Amplmos Viros Strauchium et Weigelium vestros, scribendi mihi causa aut occasio esset, Te insalutatum praeterire piaculo proximum duxi" (AA I-1, 77) and "P.S. AmplUM Weigelium, si qua occasio detur, meis verbis salutari etiam atque etiam rogo" (AA I-1, 93).
[24] Unfortunately for Bose, but luckily for our purpose, the best way to show the relation between Bose, Weigel and Coring is probably by pointing out that Weigel and Conring wrote together in 1674 two elegies in the *oratio funebris* for Bose's death, the *Frommer Christen Heimfahrt aus dem Thränen-Thal dieser Welt in das himmlische Vaterland […] daselbst* (Weigel-Conring 1674). On Weigel and Conring see Dreitzel (1983, 157) and in general Stolleis (1983).
[25] See for example a letter dated 1678 (AA II-1, 578), consistent with Leibniz's renewed interest in Weigelian ideas and with Weigel's second wave of reception. On the principle of contradiction and its connection with identity see the letter dated March 1678 (AA II-1, 602).

reference, since Weigel is quoted together with Johann Strauch: Weigel and Strauch were in fact teachers of Magnus Wedderkopf and, as it will be shown, Leibniz's important letter to him, dated 1671[26], contains one of the most relevant hints in order to prove that Leibniz was influenced by Weigel in the adoption of the principle of contradiction.

About Leibniz's *Hypothesis* instead, the most important reference is Leibniz's second attempt to gain Peter Lambeck's favour. In a letter dated August 1671, Leibniz writes:

> Addam potius nonnulla, de aliis quibusdam curis meis quibus Iuris studium interstinguo, suetus mutatione laborum uti pro quiete. Ac inprimis rerum mathematicarum ac naturalium perquisitione delector valde, tum quod deprehendi non raro incognita hactenus, generi tamen humano profutura solent. Ea fini novis libris experimentisque inviglio, cum egregiis passim viris, in Italia RR. PP. Kirchero et Lana, in Germania Gerickio, Conringio, Reinh. Blumio, Ludolpho, Strauchio, Thomasio, Weigelio [...], commercium literarum colo, a quorum plerisque literas nec exiguas nec raras, subinde accipio. Excogitavi quin etiam Hypothesin quandam physicam novam, quae ex unico quodam universali motu in globo nostro supposito, nec Thyconicis nec Copernicus aspernando pleraque naturae phaenimena singulari simplicitate repetit (AA I-1, 62-63).

Conring, Strauch, Thomasius and Weigel then are here quoted as German representatives of a tradition that helped in the draft of the *Hypothesis physica nova*. Given the connection with the revaluation of Hobbes' philosophy that will be shown, this influence is consistent with Leibniz's interest in his *De corpore*.

It is widely known however, that the *Hypothesis* was also influenced by Leibniz's notes on Huygens' theory appeared in 1669 on the *Philosophical Transactions*[27]. Whether Weigel suggested Huygens' works to Leibniz or not is hard to determine[28], but what we

[26] AA (II-1, 117-118).
[27] AA (VI-2, 157).
[28] It is nevertheless possible, given the fact that Weigel invested some efforts on the topic of the center of gravity between 1663 and 1665 and in the study of pendula. Huygens is quoted by Weigel in his *Pendulum ex tetracy deductum...sistit*: "pro quo maximas Inventori, sive is *Hugenius* fuerit, alias etiam de se solida Philosophia tam bene meritus, sive alius in societatem Inventionis aut applicationis admissus, habemus gratias" (Weigel 1674, Definit. VI, Cap II).

know for sure is that Weigel admired Huygens, so much that he will visit him in Holland at the end of his life and speak with him about their common disciple[29]. It is also known that Huygens offered to Leibniz a much better mathematical and physical knowledge at a later time, but Leibniz's eagerness does not justify the fact that starting from his stay in Paris the relationship with Weigel fades in a scarce number of references, until we witness Leibniz's renewed interest around 1679: something probably happened between the two, so much that Leibniz's attitude in his correspondence about Weigel after 1671 changes significantly, showing a coolness and carefulness never seen before.

These then are only some of the references that prove how, between 1669 and 1671, Leibniz was deeply involved in his cultural background, in which Weigel played a primary role. Many more could be shown[30], but I believe these are sufficient in order to

[29] In a letter dated February 1691 Leibniz will talk to Huygens about Weigel for the first time: "Je seray bien aise de voir un jour ce qu'on a imprimé en France de la part de l'Academie Royale, sur tout ce qu'il y a de vous. Je me souviens d'avoir aussi remarqué autres fois des voyes de demonstrer la regle de l'equilibre differentes de celle d'Archimede. Mons. Römer me parla aussi d'une sienne, et un Professeur de Jena nommé Weigelius en a aussi donné. Mais j'ay sur tout envie de voir un jour vôtre maniere, sçachant que vous avés coustume de donner quelque chose d'elegant" (Huygens 1891, X, 15-16). Few months later, Huygens letter to Leibniz annouces Weigel's arrival: "Avant hier me vint voir icy le Sr. Weigelius, professeur à Jena, qui m'entretint de ses grands desseins pour l'avancement des sciences et qui paroit extremement satisfait de certaines demonstrations qu'il pretend avoir de l'existence de Dieu et de la Providence. Je l'iray voir à la Haye, où il dit avoir un coussin rempli de ressorts et autres curiositez qu'il veut me montrer. Il dit qu'il a l'honneur de vous connoitre, depuis le temps que vous estudiez en mathematiques sous luy. J'aimerois bien mieux voir icy son disciple" (Huygens 1891, X, 141-142). At the end of his life then, Weigel was still concerned with the project of creating a German academy of sciences. The strange inventions reported by Huygens are typical of Weigel's character, very interested in the creation of new machineries. The demonstration of God's existence quoted instead is the same about which Leibniz will be interested after his Parisian stay.

[30] I will offer here some examples. The reason why in these letters Weigel is not quoted as many times as one would imagine is that the references to his works were widely known by Leibniz's correspondents. On the contrary, when Leibniz had to write to people outside the Weigelian circle, he often used his name. For example, a part from this second attempt to gain Peter Lambeck's favour, he writes to Johann Leyser in 1671 about his projects. Leyser's answer is interesting, as we read: "De Tuis tractatibus afferas quosdam velim aut mittas, et quid praeterea in animo habeas significes. Si posses ab E. Obtinere harmoniam religionis, aut Academiarum et scholarum Reformationem Universalem et uniformem, multum mereberis de bono publico. Quicquid ego alibi hac in parte potero, nullus intermittam. Weigelii opuscula hic sunt admodum rara, ut et Beckeri" (AA I-1, 159). Once again Leibniz's syncretistic approach is connected here with that of Weigel. Even more, the letter shows that Leibniz suggested Weigel's works to Leyser, but given the lack of availability reported by Leyser, it also shows that Leibniz's access to Weigel's writings depended on a privileged position. About Leibniz's interest in Leyser see Laerke (2009, 161). There are then many other references to the newer philosophers and the syncretists, like the 1670 letter to Spener related to the preface of the 1553 book of Nizolius, which contains Thomasius'1669 letter: "Addidi epistolam ad doctissimum quemdam virum aliquando a me scriptam de Aristotele recentioribus (id est Gassendo, Chartesio, Hobbio, Digbio aliisque id genus philosophiae restauratoribus) reconciliabili. Qui Syncretismus mihi et rectior theologico et tutior videtur" (AA II-1, 56). Leibniz will write again to Spener about Weigel in a 1700 letter, renewing his early desire for a new academy of sciences based on the model of his former teacher, already passed away at that time: "es ist aber nicht wohl anders als durch eine guthe erziehung dazu zu gelangen. Daher zuwündschen, daß des seel. Herrn Weigelii, Herrn Franckens, und ander wohlgesinneten Leute vorhaben und vorschläge vollzogen auch wo nothig verbeßert, und so gefaßet werden

argue that Leibniz's activity intensifies in these years, thanks also to Boineburg's efforts. However, the *Weigel-Schule* was active even before Leibniz's relationship with Boineburg and the way in which he writes to his correspondents that were active around Jena suggests a confrontation that had its roots many years before. In this regard then, understanding from Weigel's perspective what was so special about Leibniz's *Hypothesis*, worth many appreciations, could help us in understanding at the same time what was the common background between this work and Weigel's.

If this reconstruction of Leibniz's correspondence between 1663 and 1671 is by no means complete, since many other threads could be followed, many other equally important authors could be examined, it was useful nonetheless for the specific purpose of this work, because it outlines three important facts: there is a connection between Weigel and Leibniz's *Hypothesis* in physics, a connection between Pufendorf and Weigel in Leibniz's ethical efforts and a connection between Weigel and Sturm in Leibniz's logic. Following Leibniz himself then, who in the *Specimen quaestionum* quotes the Weigelian classification of the sciences in natural, moral and notional[31], in the next chapters I will argue that during these years Weigel's importance intensifies as much as Leibniz's contacts with his school and his efforts on these topics. What I want to show is that universal principles, particularly the principle of contradiction and the definition of necessity, were debated in Weigel's school in every field above-mentioned.

I will now analyse Weigel's influence starting from physics, reserving the notional-logical analysis for the next part of the present work.

mögen, daß man zugleich den nuzbaren Zweck und der Leute, so in der Welt etwas zusagen, beifall erhalte ohne welchen alle guthe absehen nicht als wündsche zu bleiben pflegen" (AA I-18, 704). The reference to Spener is important because in another letter (December 1670, AA II-1, 115) Leibniz connects Grotius to Hobbes and Euclid, praising his demonstrative method. Inside the cultural circle of Jena instead, Leibniz's correspondence with Friedrich Nitzsch is also worth remembering. In Nitzsch's letter dated December 1670 there is again a connection between Weigel and Sturm (AA II-1, 118) and an interesting debate on a problem regarding optics. Leibniz's conclusion will be that "A Weigelio fortasse et structuram discere possumus" (AA II-1, 143), meaning that a confrontation with Weigel looked feasible to Leibniz in 1671. Another important connection is that with Daniel Crafft, examined by Moll (1982).

[31] Wegiel takes this classification from the Stoics, as a quote from one of the dissertations written under his supervision, the *Theses philosophico-mathematicae*, shows: "Stoica Philosophiae distinctio in naturalem, moralem & notionalem totam quidem Philosophiam non exhaurit, non tamen penitus est improbanda" (Weigel-Vinhold 1671, 2). The tension between this general distinction and Weigel's need of organizing the human knowledge following the general principles of his philosophy was giving birth in those years to the *Universi corporis Pansophici caput summum*.

1.3 Physical Necessity, Matter and Geometry

Starting Point and Development of Leibniz's Physics In the secondary literature, the *Disputatio metaphysica de principio individui*'s *corollaria* are often taken as the best example showing the starting point of Leibniz's philosophy. Since some of these corollaries were already traced back to Weigel, I will start as well by analysing the first five:

> I. Materia habet de se actum Entitativum.
> II. Non omnino improbabile est materiam et quantitatem esse realiter idem.
> III. Essentiae rerum sunt sicut numeri.
> IV. Essentiae rerum non sunt aeternae nisi ut sunt in DEO.
> V. Possibilis est penetratio dimensionum (AA VI-1, 18).

Usually, the third and the fourth corollaries are the ones associated with Weigel, since the very same expression used by Leibniz in the third one could be found in Weigel's *Analysis aristotelica* and in many other writings, but I will reserve my analysis of this sentences for the next chapters, since they are better suited for explaining Weigel's influence on the adoption of universal principles in metaphysics. What I would like to point out in this chapter regarding physics instead is that, if there were some kind of influence on Leibniz at this point, that should be also associated with the first two corollaries. The first one simply ascribes to matter its own existence, while the second one associates matter with quantity[32]. They hint then to a revaluation of the general concept of matter that later in those years will be associated with the corresponding revaluation of the Aristotelian concept of primary matter, inserted however in a context consistent with the new physics. I will show that this syncretistic operation was possible thanks to a reflection on the concept of quantity and its relation to the concept of matter that has its roots in Weigel's writings.

However, the idea of an early influence of Weigel on Leibniz, before his stay in Jena, is somehow discredited by the fifth corollary: allowing the possibility for the penetration

[32] Di Bella (2005a, 30) argues that we also find in these corollaries a tension between a particularist claim, a holistic view and a combinatorial view. He recognises that the first two corollaries contain echoes of those developments in Scholastics aiming at a reconciliation with the new science.

of dimensions is in fact a statement apparently against Weigel's philosophy, as in some of his writings the opposite idea is maintained[33]. My conjecture, compatible with the idea of a progressive adoption, is that at that time Weigel's influence was indeed present, but only because Leibniz already read the *Analysis aristotelica*, without any decisive and direct contact with the Weigelian milieu. This distinction explains in my opinion some inconsistencies among Leibniz's early works and the peculiar syncretistic solution that he will follow in the end. I will show that in Jena Leibniz could have learned, thanks to Hobbes' influence mutuated by Weigel, a new way of reconciling Gassendi's possibility for the penetration of dimensions with the opposite solution.

Any statement concerning the penetration of dimensions should be seen as related to physics, because the debate at that time revolved around whether or not accepting the principle by which the possibility of penetration is denied as a property that should inhere to a body. This topic was very important for the natural philosophers of the 17th century[34]: in a mechanical world where everything is described by means of bodies interacting with each other through collisions, accepting or rejecting the idea that the existence of dimensions prevents the penetration of a body into another one could have led to a completely different opinion regarding the structure of reality, the existence of void and the way in which one could apply the mathematical model to physics.

Cartesians and Scholastics were following this principle, because they needed it in order to justify the very existence of their bodies, thus explaining how they could be pushed and pulled by means of external forces, or how they couldn't be described through an atomistic account. The Cartesians for example, defining a body using only extension, were not allowed to admit any other principle that inhered to them, except extension itself. Without allowing void or atoms, this principle was extremely important because, although the bodies' different hardness could have been explained by other means, absolute hardness was not explainable otherwise. The explanation of the possibility of a coherent interaction between bodies then was based on an argument taken from simple reasoning: if two bodies were to penetrate, that would have led to the superposition of two or more dimensions, exploiting the mechanism and leading to infinite regress[35].

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[33] Another corollary of Weigel's *Disputatio statica de aestimatione gravium* reads: "Penetrationem corporum dari, absurdum est, non item dimensionum" (Weigel-Scheidlin 1665, *Corollaria* 4).

[34] For an in-depth analysis of this problem see Mormino (2012, 143-165).

[35] Arguing for a complete homogeneity between corporeal substance, extension and quantity, Descartes is openly against Gassendi: "Sintque etiam nonnulli adeo subtiles, ut substantiam corporis ab ejusdem

While Scholastics shared the same principle, but for a slightly different reason given their qualitative approach to physics, atomists instead were not following it, because the importance of dimensions was clearly related to a geometrically bound account for reality that they were not forced into adopting. Absolute hardness was given to atoms by an intrinsic propriety, antitypy, so that the penetration of dimensions was conceivable, despite the fact that bodies still retained their impenetrability, thanks to the antitypy of their constituent atoms. While in the Cartesian fashion extension as the fundamental property of a body grants the absolute homogeneity between reality and geometry, a body in the atomistic fashion could indeed be cut, for example, by a geometric shape without generating contradictions, because that division would stand only on a hypothetical ground.

Given these premises, we can clearly understand why Weigel favoured the principle and why some interpreters have reduced Weigel to Descartes: if his aim was reconciling the old and the new, Descartes' philosophy was better suited for this task, at least on the ground of physics. It shared with Aristotle's physics some principles, like the negation of the void, that were easily adaptable: the penetration of dimension was one of those. Moreover, grounding a property like impenetrability on a purely geometrical account opened the possibility for a geometrical description of the world that was ontologically relevant, rather than the mere outcome of a hypothetical model applied to the actual world. This last consequence could have been particularly cherished by Weigel, because apparently it granted that homogeneity between the world and the human mind on which his *mathesis universalis* was based. I will show however that, despite his conciliating claims, Weigel's physics was far from being completely identified in that of Descartes, because it was conceived in the light of Hobbes' philosophy of nature.

In any case, allowing the possibility of the dimensions' penetration, Leibniz's *Disputatio* was undoubtedly influenced on this specific topic by Gassendi's atomism[36],

quantitate, atque ipsam quantitatem ab extensionem distinguant" (Descartes 1963, VIII, 42). The possibility of such distinction in fact allows the penetration of dimensions.

[36] Gassendi openly argues against the negation of penetrations' dimension in the second book of his *Syntagma Philosophicum*'s physics: "Excipiendum est nihil esse corporeum, nihil, quale solent Substantiam, Accidensve intelligere; at esse tamen aliquid, quale Locum, seu mavis Spatium, Intervallum […] Dimensionem intelligere licet" (Gassendi 1964, 184). It is clear then that the refutation dimensions' penetration is connected with the refutation of void's existence: if we admit something that could be filled and yet it is conceivable through a spatial description, then we are already admitting the penetration of dimensions.

but Gassendi was by no means akin to syncretism, as his aversion to Scholastics is widely known[37]. At first glance then it seems that Leibniz is lost in the enigma of his early atomism[38]: accepting the penetration of dimensions while admitting antitypy is consistent with his atomism, but at the same time it is not consistent with the most popular syncretistic way of reconciling Aristotle with the new physics. The idea of a naïve revaluation of Scholastics is not sufficient in order to argue that Leibniz changed his mind after 1663. Given Leibniz's claims in the 1669 letter to Thomasius, this opposition indeed grounds Leibniz's many statements against Scholastics, but it also shows that there must be another way by which he harmonised Aristotle with the new physics. This way is best summarized by a passage from Thomasius' letter: "Materia prima est ipsa massa, in qua nihil aliud quam extensio et ἀντιτυπία, seu impenetrabilitas; extensionem a spatio habet, quod replet"[39]. What we find here is the use of a concept (primary matter) taken from Aristotle and the use of both the atomists' antitypy and the Cartesians' extension. Even more, the superposition between space and extension recalls an important passage from Hobbes' *De corpore*: "*Corpus est quicquid non dependens a nostra cogitatione cum spatii parte aliqua coincidit vel coextenditur*"[40]. In other words, the fact that Leibniz allowed antitypy and atoms was not excluding other fundamental concepts, like primary matter, extension or space from a feasible physical model. Rather, they were deprived of their foundational role while preserved as intrinsic properties of one body. Explaining the peculiar way in which bodies intrinsically maintain these properties and its consequences is the purpose of this chapter.

The first step in this direction is questioning the absolute incompatibility between atomism and the principle that negates the penetration of dimensions. Even by a purely theoretical analysis of this problem in fact, it is reasonable to argue that the absolute incompatibility is valid only for the Cartesians' side: the impossibility of dimensions' penetration is necessary in a world where nothing else should account for impenetrability, while in the atomistic account it is indeed unnecessary, yet conceivable and possible. One

[37] In the *Exercitationes Paradoxicae Adversus Aristoteleos* for example we read: "Verumtamen, ut liquido constat, excogitata illa non sunt a viris Aristoteleis: qui Aristotele contenti nihil aliud venantur in Scholis, quam suas illas frivolas, inutilesque modalitates" (Gassendi 1959, 67).
[38] This is how Arthur (2003, 183) describes the difficulty in understanding Leibniz's apparently incoherent shifts of his early years.
[39] AA (II-1, 26).
[40] Hobbes (1999, 83).

cold then imagine an atom having antitypy as a property insofar as extension. This less travelled way was the one endorsed by some of Aristotle's commentators, like John Philoponus[41] or Hasdrai Crescas[42], as they attribute extension and the impossibility of dimensional penetration to bodies characterized by antitypy. Conceiving bodies in this fashion represents a coherent alternative to the opposition between Scholastics and atomists, because the two properties are still assumed individually, yet they happen to be co-existent in the same body at the same time.

Was this hypothesis available to Leibniz between 1663 and 1671? In my opinion, a connection could be found by revaluating the importance of Francis Bacon on Leibniz's early philosophy. There is no need to prove that an actual influence happened, since Leibniz had always been glad to share its importance in his early years[43], but it has always been interpreted like a generic contribution to Leibniz's scientific method that I find unconvincing if taken alone. However, in Bacon's *Novum Organum* we read:

> Motus Primus sit Motus *Antitypiae* materiae, quae inest in singulis portionibus ejus; per quem plane annihilari non vult: ita ut nullum incendium, nullum pondus aut depressio, nulla violentia, nulla denique aetas aut diuturnitas temporis possit redigere aliquam vel minimam portionem materiae in nihilum; quin illa et sit aliquid, et loci aliquid occupet, et se (in qualicunque necessitate ponatur) vel formam mutando vel locum liberet, vel (si non detur copia) ut est subsistat; neque unquam res eo deveniat, ut aut nihil sit, aut nullibi. Quem Motum Schola (quae semper fere et denominat et definit res potius per effectus et incommoda quam per causas interiores) vel denotat per illud axioma, quod *Duo corpora non possint esse in uno loco*; vel vocat motum *Ne fiat penetratio dimensionum*. Neque hujus motus exempla proponi consentaneum est: inest enim omni corpori (Bacon 2004, II-XLVIII, 1).

[41] Philoponus is quoted by Weigel in his *Analysis aristotelica* (Weigel 1658, 190)
[42] In his essay, Mormino (2012, 151n) quotes for example Philoponus (1888, 567).
[43] See for example the famous letter to Foucher, dated 1675: "Bacon et Gassendi me sont tombé les premiers entre les mains, leur style familier et aisé estoit plus conforme à un homme qui veut tout lire; il est vray que j'ay jetté souvent les yeux sur Galilée et des Cartes; mais comme je ne suis Geometre que dépuis peu, j'estois bien tost rebuté de leur maniere d'écrire, qui avoit besoin d'une forte meditation" (AA II-1, 389).

Bacon basically identifies antitypy with the Scholastics' negation of the *penetratio dimensionum*. His awareness in doing so sway us from interpreting this as a simple mistake: he is perfectly conscious of the difference between the two accounts, since atomists' antitypy is accurately interpreted through *causas interiores*. While the Scholastics reasoning is faulty, yet he identifies their principle with antitypy. This tension[44] is the same found between Leibniz's 1663 *Disputatio* and his 1669 letter to Thomasius. It suggests a way to interpret Leibniz's readmission of extension as a property related to primary matter, without sacrificing his aversion for Scholastics or arguing for a naïve adhesion to Descartes' physics that never happened. The principle against penetration and extension in fact are two separate things, the former being a property of the latter, but the superposition between antitypy and this principle suggests the co-existence of antitypy and extension.

Leibniz's conception of bodies during these years then is similar to what I would call a layered model, because antitypy, matter, extension and space pertain altogether to a single body at the same time, but they are on the other hand conceivably distinct and operationally different. This conception then is clearly taken from Hobbes, but, since Hobbes was not admitting antitypy, it gives rise to some interesting questions: how are these properties connected between themselves? Is there a priority of a property over another? Is for example antitypy prior to extension in the definition of primary matter? Having now sketched the problem, in the next section I will at last introduce Weigel's influence, in order to explain Leibniz's adoption of Hobbes' vision, his idea of primary matter and the role of extension in its definition.

Tendency and Experimentalism in the Study of Gravity Between 1663 and 1669 Leibniz's considerations on physics led him to the revaluation of Aristotle's primary matter.

[44] Bacon argues that the Scholastics' negation of dimensions' penetration could be useful on paper in a study on collisions, but it is not sufficient, accepting their theory as a whole, for a complete explanation on motion, especially given that hardness conceived in such a way justifies the beginning of motion, but not the fact that it lasts in time: "Similiter, sit natura inquisita Motus Missilium, veluti spiculorum, sagittarum, globulorum, per aërem. Hunc motum Schola (more suo) valde negligenter expedit; satis habens, si eum nomine motus violenti a naturali (quem vocant) distinguat; et quod ad primam percussionem sive impulsionem attinet, per illud, (quod duo corpora non possint esse in uno loco, ne fiat penetratio dimensionum,) sibi satisfaciat; et de processu continuato istius motus nihil curet" (Bacon 2004, II-XL, 3-4). In order to explain the continuation of motion for a projectile, a choice must be made between the idea that it is the air, moving itself around the object like water against the hull of a boat, that moves the projectile, or the idea that the projectile's parts are moving away from the impact. Scholastics ignore completely this issue, yet Bacon here is still tryng to reconcile their thesis with a quantitative approach.

However, as I showed in the previous chapter, those years are also the ones in which he came into contact and intensified his relationship with the Weigelian school. Having joined the *Societas quaerentium* in Jena, we also know that Leibniz read there some contemporary authors, although we don't know exactly their names. There is a way nonetheless by which we could better understand what was Jena's cultural advancement, and that is the analysis of the dissertations published in those years under Weigel's tutoring. In regard to physics, I will now analyse three of them: the *Exercitatio Philosophica De Quantitate Motus Gravium* by Georg Samuel Dörffel, the *Disputatio Statica De Aestimatione Gravium* by Johann Andreas Scheidlin and the *Theses Philosophico-Mathematicae* by Christian Andreas Vinhold. I believe that these works, especially those written by Dörffel and Scheidlin, are extremely important documentary evidence of what was Leibniz's knowledge in physics before Paris. Dörffel, astronomer whose acquaintance with Leibniz is documented[45], enrolled at the University of Jena in 1662, but his *Exercitatio* was published in 1663, the same year in which Leibniz enrolled. Scheidlin's *Disputatio* was published in 1665, but his enrolment in Jena is dated May 1663, just two months before Leibniz's enrolment. Since Vinhold's *Theses Philosophico-Mathematicae* were published in 1671[46], these writing cover the same period of time of Weigel's influence on Leibniz. It is also important to remind that very often in those years and contexts these dissertations were written with the help of the supervisor or, in some cases, by the supervisor alone. This is the case for example for another dissertation written under Weigel that Leibniz himself considered as the genuine work of his former teacher[47]. Despite the problem of identifying the true author of these dissertations, their affinity with Weigel's philosophy is indisputable, as suggested by the authors' constant reference to Weigel's works. They show a very advanced approach to the *status quaestionis* of early modern physics that was nowhere near the *scholastico fumo* of other traditional universities.

[45] In the margin of a December 1670 letter to Leibniz written by Friedrich Nitzsch, the previously quoted scholar interested in optics, we find Leibniz's note: "Lentes Titelii quomodo fabricatae, quid praestent. Ott.Roberval. Dorffel" (AA II-1, 117).

[46] The information about the enrolment of these students is found in Herbst (2016, 345-365). For Dörffel see also Pfitzner (1996, 119).

[47] In the 1679 letter to Weigel and its reply (AA II-1, 745), both Leibniz and Weigel refer to *De supputatione multitudinis a nullitate per unitates finitas in infinitum collineantis ad deum* as a work written by Weigel himself, although it is the defense of Caspar Büssing, enrolled at the University of Jena in 1677 (Herbst 2016, 347).

Dörffel and Scheidlin's dissertations, as their titles suggest, focus more on the problem of defining gravity. Both display the same development: at first a general definition is given, then the various accounts on gravity are analysed and in the end a solution is suggested. By displaying the various theories then, these works offered to Leibniz a starting point for his reflections on physics. After a brief preface on the different branches in which science is divided, Scheidlin's *Disputatio* begins with a basic definition of a heavy body: "Gravia dicimus corpora, quae sua natura deorsum tendunt"[48]. At first glance, judging by this idea of an intrinsic tendency downwards and by the subsequent reference to a point "ad quod gravia sublunaria naturaliter tendunt"[49], it appears as if Scheidlin is still influenced by the Aristotelean and Scholastic background. However, this definition is used on the topic of determining the centre of gravity and it refers only to an aspect of the entire topic: Scheidlin considers the idea of determining the centre for three different entities, i.e. shape, gravity and another entity that he calls *tendentia* (tendency). Each of these have their own definition of centre[50], suggesting a distinction between the purely geometrical centre, determined between objects defined through extension and the centre of bodies determined in the actual world, where their description cannot be based on extension alone, thus needing also the concept of *tendentia* previously introduced. The result is summarised by a corollary of Vinhold's *Theses Philosophico-Mathematicae*: "Centrum gravitates, centrum figurae & centrum tendentiae licet sint distinctissima, possunt tamen penitus coincidere"[51]. In the *Disputatio Statica* then there is already a distinction similar to that of Leibniz, where some proprieties of physical bodies cannot be described by extension alone, yet they happen to be in the same place. Going back to the first definition of a heavy body then, the reference to the earth as the field of action for bodies conceived through tendency should not be mistaken as a claim that supports the old physics. Given the fact that it is related to the specific topic of gravity in fact, it only states that every object seen on earth seems to show this intimate tendency, but the reason and origin of this tendency is yet to be determined. For now, Scheidlin is still

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[48] Weigel-Scheidlin (1665, Sectio Prima).
[49] Ibid.
[50] The passage reads: "*Figura, Gravitatis & Tendentia*, nobis considerande veniunt". After that, Scheidlin defines every centre: "*Centrum figura seu magnitudinis est istud punctum, quod respectu extensionis est medium* […] *Centrum gravitatis est istud punctum, vel intra vel extra figuram ita situm* […] *Centrum tendentiae est illud in globo terraquaëreo punctum, ad quod gravia sublunaria naturaliter tendunt*" (Ibid.).
[51] Weigel-Vinhold (1671, 84).

dealing with phenomena as they appear and his reasoning will indeed develop in a different way from that of tradition. He starts by extending the concept of gravity to any object that belongs to earth[52] and, in doing so, references to the scientific background of Weigel's school emerge.

The first author mentioned is of course Weigel: he proved that the atmosphere possesses weight. This proof shows that air is subject to gravity inasmuch as any other object, rejecting the theory that ascribed to air a qualitatively different and opposite property, i.e. lightness (*levitas*), conceived not as a relational property that expresses the lesser weight between different objects, but as the positive property of opposing that gravity that inheres intrinsically to a body. This passage is clearly against Aristotle's definition of *levitas* that pertains air and fire in *De caelo*. Scheidlin exemplifies the contradiction contained in this positive concept through the image of a vase filled with air, arguing that many experiments have already proved how this situation cannot be successfully explained through the *levitas-gravitas* model. The evidences here mentioned[53] are the ones taken from Gaspar Schott's important works, containing all sorts of experiments developed in those years. In the passage of Schott's *Magia universalis naturae et artis* quoted by Scheidlin are found many authors often mentioned in the writings of the Weigelian school, such as Marin Mersenne, Galileo Galilei, Giovanni Battista Riccioli and, above all, Otto von Guericke. The passage from Schott's *Magia* quoted by Scheidlin reads:

> Sed omnibus hisce praxibus aëris gravitatcm pondere deprehendendi, nulla est certior, & luculentior, adde etiam & ingeniosior, quam illa quam fuse descripsimus in fine Mechanicae nostrae citata in Experimento novo Magdeburgico, excogitara a Viro Amplissimo Ottone Gericke, Magdeburgensis Urbis Consule, & faurore meo singulari. In aliis enim omnibus ponderatur aër non in suo statu narurali, sed violento constitutus, nimirum aut vehementer compressus, aut vehementer rarefactus; in hoc vero

[52] "Omnes Globi terraquaërei partes graves sunt" (Weigel-Scheidlin 1665, Sectio Prima, Observatio). The use of the word "terraquaërei" instead of "terraquei" hints at the argument explained in the following passage.
[53] Ibid.

aêr in statu suo naturali constitutus ad libram examinatur (Schott 1658, III, 324).

In the passage above, the idea of a superiority of Guericke's experiment based on a sort of esthetical criteria by which air is presented in his natural state is rather interesting: it suggests that the best experiment is the one that mimics in the most accurate way the ideal situation in which a body is influenced as little as possible by external disturbances.

The fact that Schott favours Guericke's experiment is obvious, since his 1657 *Mechanica hydraulico-pneumaica* contains the first description of Guericke's famous air pump[54] and even the fact that the Weigelian school knew these experiments do not sound surprising, since Schott studied in Leipzig and Jena. Nonetheless, explaining the attitude of Weigel and his pupils towards experimentalism is important, because it displays their scientific method and their philosophical premises.

Another author quoted by Scheidlin in fact is Robert Boyle, famous for his experiments on the air pump and for his confrontation with Hobbes[55], besides being mentioned by Leibniz in the *Confessio naturae contra atheistas*[56]. These references suggest that Weigel's school could have endorsed pure experimentalism and it certainly explains their unresolved attitude towards the existence of void, shared with Leibniz in those years and based on the fact that experiments were not completely proving its existence or its refutation. They also show that Leibniz's knowledge about these topics in those years was probably very derivative and tied to Weigel's background[57]. It is no coincidence then that, as a result of this influence, in Leibniz's 1671 letter to Duke Johann Friedrich, Guericke is quoted just before Leibniz's mention of his *Hypothesis physica nova*, or the fact that Leibniz corresponded with Guericke on his experiments between 1671 and 1672[58].

These new discoveries then were indeed debated, but the hypothesis of a school based on pure experimentalism is not acceptable in its entirety, due to Weigel's idea of a perfect

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[54] See Schott (1657). On the importance of experiments in early modern physics see Dear (1995).
[55] An account of their confrontation is found in Shapin (1985).
[56] AA (VI-1, 490). It is very likely then that Leibniz was introduced to Boyle's advancements by Weigel.
[57] In the letter to Thomasius dated April 1669 for example we read: "Pro vacuo pugnant Gilbertus, Gassendus, Gerickius; pro pleno, Cartesius, Digbaeus, Thomas Anglus, Clerk in libro *de plenitudine mundi*. Pro possibilitate utriusque Thomas Hobbes, et Robertus Boylius" (AA II-1, 25). In a letter to Oldenburg dated 1671 then, Leibniz refers again to Guericke and his experiments: "Ingeniosissimi Gerickii Magdeburgenses Meditationes atque experimenta his nundinis in publico expectamus" (AA II-1, 146).
[58] AA (II-1, 173, 257).

correspondence between data gathered through experiments and their explanation by means of propositions, syllogisms and the rational relationship between them. Echoes of this Aristotelean and Euclidean heritage are found in Scheidlin, where he states that "*Veritas autem data propositionis* non experimentis solum confirmari, sed & rationibus, quod a *Mathematicis* dudum factum est, demonstrari potest"[59]. Compatible with this idea, Weigel's *Analysis aristotelica* thoroughly explains the relationship between data and reasoning. In the very first chapter of this work, while the definition of a general demonstration is founded on syllogisms, so that "Est autem *Demonstrare* (ut universim dicamus) uti penes omnes Mathematicos, ita cumprimis penes Euclidem nihil aliud, quam *rerum propositarum certitudinem necessariam e certis principiis tanquam suis causis a parte rei necessariis indubitatio cognoscendam syllogisitce deducere*"[60], a scientific and demonstrative proposition exhibits also a correspondence with a material aspect that inheres to objects *a parte rei*:

> Verum autem cum Aristotele nostro dico Syllogismum non tantum formaliter & notionaliter, sed praeprimis hic materialiter, eum, cujus omnes termini, tam simplices quam complexi, praecisa etiam mentis operatione aut notionali suppositione, in rerum natura revera cohaerent, ut unum ab altero, sicut dicitur, dependeat, sive mutua illa cohaesio & dependentia sit necessaria, qui syllogismus dicitur scientificus & demonstrativus (Weigel 1658, 23).

The scientific syllogism entails then a correspondence between nature and universal reasoning, but also between the notional and logical constructions of our Mind among themselves, in a way that resembles that of Leibniz's 1677 *Dialogus*[61]. It is not an unexpected outcome, since the combinatory logic proposed by Weigel was conceived for an implementation in the actual world. It follows that observation must be complemented with logical reasoning: a well-founded theory does not depend completely on gathering data through elaborate experiments.

The connection between Weigel's method and Guericke could explain Leibniz's dissatisfaction over Gericke's corporeal version of the World Soul. This hypothesis is

[59] Weigel-Scheidlin (1665, Sectio Secunda, Propositio I).
[60] Weigel (1658, 23).
[61] AA (VI-4, 24).

considered faulty, despite the usefulness of experiments: "Und ob mein Hochg. H. gleich mit einem schöhnen experiment solche virtutes mundanas beweiset, so sind sie doch damit nicht ercläret, denn es eben so tunckel bleibt, wohehr sowohl *in globo illo ex mineralibus composito*, als *in mundo* solche virtutes entstehen"[62]. No beautiful experiment could prove itself useful, if the theories developed contain something obscure. Leibniz's reasoning here was sometimes associated with his emerging need of a metaphysical explanation of phenomena that was not compatible with a purely corporeal World Soul[63], but the fact that he once again criticise Scholastics just before the passage above quoted advocate against a naïve revaluation of traditional metaphysics. I believe that Weigel's reference explains in a reasonable way this balance between rational reasoning and aversion for tradition. Rather interesting in this regard is the fact that in Scheidlin's *Disputatio* Aristotle is quoted among the ones who have proposed experiments in order to prove that air is subject to gravity, just before Scheidlin's mention of Galilei[64]. It is a contradictory outcome given the origin of the problem in Aristotle's *De caelo*, because the end result of this section of the *Disputatio* is that "*nullam levitatem naturaliter inesse corporibus*"[65], which is in any case against a qualitative approach to physics. While openly against some passages of his works, Aristotle's association with the modern philosophy of nature and his separation from the Scholastics' background was a necessary step towards Weigel's revaluation of his works on logic and, consequently, towards the aversion for any obscure reasoning.

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[62] AA (II-1, 239-240).
[63] Mercer (2002a, 278-279) argued that this passage could be interpreted in this way. Closer to Mercer's idea is this passage from Leibniz's *Confessio naturae contra atheistas*: "Ac principio philosophis, *Democriti* et *Epicuri* resuscitatoribus, quos *Robertus de Boyle* corpuscolares non inepte appellat, ut *Galileo, Bacono, Gassendo, Cartesio, Hobbesio, Digbaeo* facile condescendo assensus sum, in reddendis corporalium Phaenomenorum rationibus neque ad Deum, neque aliam quamcunque rem, formamque aut qualitatem incorporalem sine necessitate confugiendum esse […] omnia quoad ejus fieri possit, ex natura corporis, primisque ejus qualitatibus: Magnitudine, Figura et Motu deducenda esse. Sed quid si demonstrem, ne harum quidem primarum qualitatum originem in natura corporis reperiri posse? Tum vero fatebuntur, ut spero, naturalistae nostri, corpora sibi non sufficere nec sine principio incorporeo subsistere posse" (AA VI-1. 480-490). No one denies Leibniz's need for such principle. The focus however couldn't be on the existence of this principle, but on the way in which it is connected with the principles of a mechanical account of the world. In this regard, even if these mechanical principles are not self-sufficient they are still considered useful and relevant, as suggested by the *Theoria motus abstracti*. It seems then that the principle suggested is nothing more than God conceived as the prime mover, closer to Leibniz's reflection in the following years.
[64] "Laudatus *Vir Nobilissimus Dn. Otto Gerike*, Patricius & Reip. Magdeburgensis Consul gravissimus, quaeque a *Rev. & Excellentiss. Schotto* peculiari libro in lucem emissa. Aliorum Auctorum experimenta idem Schottus […] congessit, inter quos non praetereundus I. *Aristoteles*". Weigel-Scheidlin (1665, Sectio Secunda, Propositio I).
[65] Ibid.

Moving on from the topic of experimentalism, another reference in Scheidlin's work which cannot and must not go unnoticed is that of Galileo Galilei, because Jena's cultural ambience was greatly influenced by his works. He is quoted in every work above mentioned and, as a result of his relevance, he is the only one to whom is dedicated an entire section of Dörffel's *Exercitatio Philosophica De Quantitate Motus Gravium*. In the *Observatio II* is found in fact an exposition of Gaileo's famous rule for falling bodies by which the distance increases as the square of the time. Although Gassendi's *Epistola De Motu impresso a Motore translato*, quoted by Dörffel, is certainly responsible for the initial reception of Galileo in Jena's circle, the references to Galileo's *Dialogo sopra i due massimi sistemi del mondo*, to Giovanni Riccioli and Niccolò Cabeo and the specific language adopted by Dörffel suggest an increasing interest that cannot be explained resorting to derivative sources alone[66]. The importance of Galileo in Jena's circle will be decisive in the third part of the present work, because it explains once again the relationship between Leibniz's writings around 1671 and his 1679 letter to Weigel. Specifically, the 1672 *Accessio ad Arithmeticam Infinitorum*, written for Jean Gallois and quoted by Leibniz in his letter to Weigel, exhibits some reflections on Galileo's infinite number[67] that I believe are fundamental in order to understand the development of binary arithmetics between Weigel and Leibniz.

[66] Dörffel exemplifies Galileo's rule: "*Spacia trasmissa esse inter se ut Quadrata Temporum*, id quod commodious per exemplum declarabo. Sit e.g. lapis, qui tempore 1. minuti horarii descendat ex altitudine 100. passuum : Jam in alio tempore v.g. 3. min. idem lapis juxta hanc proportionem descendet per spacium (non 300. sed) 900. passuum. Namq; ut se habet 1. (Quadratus numerus prioris Temporis, se 1. min) ad 9. (h.e. quadratum alterius temporis, sc. 3. min. Tertria enim 9. constituunt) ; Ita etiam 100. (Sc. passus spacii, quod priori Tempori respondet) sese habent ad 900. (passus, quos lapis ille altero tempore 3. min cadendo absolvet). Alias haec proportion, in respect ad tempora aequalia continua ejusdem motus, aliter enunciator, dum motus talis fieri dicitur *secundum numeros impares ab unitate*, vel juxta *Progressionem continuam per impares ab unitate*" (Weigel-Dörffel 1663, Observatio II). The specific expression "*impares ab unitate*" could be taken from Gassendi's letter, where he paraphrases: "Caeterum, duo quaedam praemittenda sunt, quae inter alia bene multa magno Galileo debentur. Unum; corpus suopte decidens motu ea ratione accelerari, ut temporibus aequalibus maiora semper spatia pervadat, iuxta proportionem, quam habent numeri impares inter se, initio sumpto ab unitate" (Gassendi 1964, 440), but also from Galileo's Latin translation of his *Dialogo*, entitled *Systema cosmicum*: "Demonstrat, accelerationem motus recti gravium fieri secundum numeros impares *ab unitate*, hoc est, signatis quibuscunque & quantiscunque placucrit temporibus aeqpuibus, si in primo tempore mobile quietem relinquens transierit tale spatium, exempli causa unius ulnae, in secundo tempore transibit tres ulnas, in tertio quinque, in quarto septem,& ita consequenter, secundum numeros impares succedentes, quod in universum idem est ac si dicamus, quod spatia transmissa a mobili quietem relinquente, habeant inter sese proportionem duplicatam illius quam habent tempora, quibus ista spatia dimensi sumus : vel si mavis, quod spatia transmissa, sint inter se, sicuti quadrata temporum". (Galilei 1640, 163).

[67] AA (II-1, 348).

Focusing on physics instead and having examined these authors quoted both by Scheidlin and Dörffel, we can clearly see that Weigel's school knew the new physics in its entirety. The definition of Weigel's *tendentia* as something that intrinsically pertains a body considered not only in its extensional description then could be amended avoiding the interpretation of the word "tendency" as the legacy of a qualitatively described physics. In Dörffel's *Exercitatio* is found the best physical explanation of the concept of tendency. He divides motion in three different constituents or modes: motion conceived as impetus, motion conceived as something that acquires a different direction and motion conceived strictly speaking as the act of changing location continuously. Through the first definition we can understand what was the real aim of Weigel's school:

> I. *Impetus* sive *Motio* i.e. conatus ille exeundi loco suo. Specialiter in Gravibus appellatur *Gravitatio, Tendentia naturalis*, vel etiam *Gravitas*, & comparative potius *Pondus*. Habet enim hic Modus gradus suos aestimativos, neq; indivisibili consistit (Weigel-Dörffel 1663, §2).

The word tendency associated with the word *conatus* then suggests the adoption of the concept of tendency described by Descartes in the second law of his *Principia*[68]. If I believe that this reference is correct in regard to the adoption of the term, I think however that the first definition of motion given by Dörffel entails, from the point of view of the explanation of phenomena, Hobbes' *conatus* rather than Descartes'. I'm arguing here that Weigel and his followers were including Hobbes in their syncretistic projects not as a mere name in the list of the new philosophers that needed an interpretation through Aristotle, but as the main character of these reconciliatory efforts. Natural tendency in fact is the exemplification on the topic of gravity of the general definition of *impetus*, i.e. Hobbes' *conatus*. This interpretation is suggested by no other than Leibniz: in his 1689 *Phoranomus*, recalling his early years, he effectively describes and connects many topics of the present chapter:

[68] "Unamquamque partem materiae, seorsim spectatam, non tendere unquamut secundum ullas lineas obliquas pergat moveri, sed tantum modo secundum rectas" (Descartes 1963, VIII, 55). Descartes' use of tendency involves always a reference to the idea of moving throughadirection or space, thus it is closer to the usual meaning of the word.

> Cum igitur solam imaginationis jurisdictionem in rebus materialibus adhuc agnoscerem, in ea eram sententiam nullam in corporibus inertiam naturalem intelligi posse, et in vacuo aut campo libero corpus quiescens quantulicunque alterius velocitatem accipere debere […] Ego igitur nihil aliud concipiendo in *materia* quam extensionem et impenetrabilitatem, vel uno verbo impletionem spatii, et in motu nihil aliud intelligendo quam mutatione spatii, videbam corpus motum ab eodem quiescente singulis momentis eo saltem differre quod corpus in motu positum semper habet *conatum* quondam, seu (ut verbo Erhardi Weigelii insignis in Saxonia Mathematici utar) *tendentiam,* hoc est initium pergendi (PHO, 788-790).

In Leibniz's reconstruction of his early years then, not only the important identification between Hobbes' *conatus* and Weigel's *tendentia*, but also his indecision on the existence of void, his superposition between antitypy and extension and his strictly mechanistic account for bodies encourage us in hypothesizing Weigel's school behind Leibniz's *Hypothesis physica nova*[69]. This is in my opinion a decisive turning point in the interpretation of Weigel's impact on Leibniz: an interpretation that were to insist only on Weigel's Platonism and Aristotelianism, considering his philosophy as a simple expression of some kind of German Neoplatonism would completely miss the point of his syncretism.

Several evidences of this connection between Hobbes and Weigel are spread among Leibniz, Weigel and his disciples' writings. First of all, as Leibniz suggested, tendency is not a concept used only by Weigel's scholars. Weigel himself uses it in the *Corporis pansophici pantologia*, a section of his *Universi corporis Pansophici caput summum* published independently in 1673: "*Motus* spectatus in se dici potest Vel *primus,* ut (1) *Conatus, Nisus, Motio*, quae est tendentia aliquorum, licet nulla fiat progressio […] Vel *secundus* diciturque (3) *Promotio,* quae est de loco in locum progressio"[70]. There is a distinction here between *conatus* happened in an instant and the actual progression of a body in a certain direction that is impossible to ascribe completely to an interpretation of tendency based on Descartes. The definition of tendency described here then becomes all

[69] Leibniz's *Hypothesis* is in fact mentioned in the omitted passage of the previous quote: "quemadmodum pluribus in libello exposui, quod juvenis edideram" (PHO, 788).
[70] Weigel (1673c, 120).

the more important, reminding that in the previous chapter I showed how Leibniz refers to the draft of Weigel's *Universi corporis Pansophici* three years before its actual publication, in the same year of the *Hypothesis*' draft. Moreover, Leibniz's 1683 notes on this work display that he perceived this passage as one of the most important in Weigel's physics[71], feeling the need of going back to his roots before venturing in a new directions. Weigel's tendency then influenced Leibniz two times: at first during Leibniz's early years it presented Hobbes' *conatus* as the fundamental principle for a precise explanation of physics, while later, once Leibniz attained a new vision on physics, it was reintroduced in the new context by Leibniz himself, adapting its meaning to his aim. Leibniz in fact uses for example the word *tendentia* in the definition of primitive forces as internal tendencies[72]. The background is completely different, but the word remained and it was adopted by Leibniz with the same meaning in mind.

Since Leibniz in the *Phoranomus* describes the consequence of the action of tendency as a beginning in direction (*initium pergendi*), it shows that Weigel did not possess a misconceived version of Hobbes' *conatus*, regardless his intention of harmonizing it with Aristotle. Maintaining Aristotle was not a task that needed in his mind a denial of that modern physics based on the study of collisions between bodies. The consequences of this attempt are shown in Dörffel's *Exercitatio*. I have already pointed out that its chapter on this topic begins dividing motion in three aspects or modes. If the first one is tendency as it was described before, the second one is the one called *dispositio motus*, which is no other than direction obtained by the impact of a body into another one[73]. By combining momentum and direction Dörffel explains the principles of his *Phoronomia*[74], so that tendency is truly conceived as something that happens in an instant. In doing so however, Weigel's school follows Hobbes rather than Leibniz, as the passage on the definition of

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[71] Leibniz's notes on this passage are as follows: "Actio naturalis infinita Dei, finita actio communis dicitur Motus: actus entis in potentia quatenus in potentia estque illocalis ut tempus physicum id est fluxus durativus in rebus durantibus; vel localis Motus. Motus vel primus, ut conatus nisus motio" (AA VI-4, 1197).

[72] GP (II, 275). A reconstruction of Leibniz's use of tendency, without however its foundation in Weigel, is found in a paper written by Rutherford in Goldenbaum (2008, 255).

[73] "*Dispositio motus*, quae est directio Mobilis certum terminum […] Dependet autem determinatio ista in motu impresso primo a nuda superficie Corporis moventis & implentis, postmodum manente eodem impetus communicato pro diverso resistentis opposite diversimode potest variari" (Weigel-Dörffel 1663, §2).

[74] "Certum est vel ex primis Phoronomiae principiis, *Mobile quodcunq. dum movendo in obvium aliud corpus resistens impingit, quanto vehementiorem tunc edit impulsum, tanto majorem impetum, tanto etiam celeriorem motum habere*" (Weigel-Dörffel 1663, Observatio II, §1)

motion that negates its nature as an indivisible showed[75]. Leibniz's solution in his *Theoria motus abstracti* is unique then and it is the result of the efforts of a young talented student, made to impress the cultural circle in which he belonged.

The third mode of motion in Dörffel's *Exercitatio* introduces instead Aristotle's concept of a motion that happens through contraries: "*Motus* stricte dictus sive *Promotio* sive *Latio*, actualis nimirum & continua loci permutatio, dum Mobile per spatium tractum aiquem longitudinarium continuo tempore describit. Quo spectat, quam Aristoteles tradit […] explicatio, quod sit […] mutatio contrarietatis, quando secundum locum accidit"[76]. In this third definition of motion Dörffel quotes the fourth chapter of the first book of Aristotle's *De Generatione et Corruptione*. Much like, as Aristotle argues, a man who was musical becomes unmusical, yet remains a man[77], so a body, moving through space, remains that body, even if some accidents turn in their opposites. Aristotle's alteration, which is not generation or corruption, is reintroduced as a sort of principle of identity in space during motion. It is as if this Aristotelian concept is taken because it explains in an accessible way what we actually see when we look at a body moving, but the precise explanation of the event is entrusted to the decomposition in *conatus*, direction and collisions. Aside from maintaining the identity of a body with itself then, the concept of alteration in a subject has no other use, so that motion explained through contrariety does not hinder the definition of motion taken from the new physics.

[75] This important passage of Hobbes' *De corpore* in fact rules out indivisibles: "Primo definiemus *Conatum esse motum per spatium et tempus minus quam quod datur*, id est, *determinatur sive expositione vel numero assignatur*, id est, *per punctum* et *in instant*. Ad cujus definitionis explcationem meminisse oportet per punctum non intelligi id quod quantitatem nullam habet sive quod nulla ratione potest divide (nihil enim est ejusmodi in rerum natura), sed id cujus quantitas non consideratur, hoc est, cujus neque quantitas neque pars ulla inter demonstrandum computatur, ita ut punctum non habeatur pro indivisibili, sed pro indiviso, sicut etiam *instans* sumendum est pro tempore indiviso, non pro indivisibili" (Hobbes 1999, 155). It's clear however that Leibniz's solution was in some way suggested by Hobbes himself by pointing out this possible outcome.

[76] Weigel-Dörffel (1663, §2).

[77] The passage quoted from Aristotle's *De Generatione et Corruptione* reads: "If, however, in such cases, any property (being one of a pair of contraries) persists, in the thing that has come-to-be, the same as it was in the thing which has passed-away – if, e.g., when water comes-to-be out of air, both are transparent or cold – the second thing, into which the first changes, must not be a property of this. Otherwise the change will be alteration. Suppose, e.g., that the musical man passed-away and an unmusical man came-to-be, and that *the man* persists as something identical. Now, if musicalness (and unmusicalness) had not been in itself a property of the man, these changes would have been a coming-to-be of unmusicalness and a passing-away of musicalness; but in fact a property of the persistent thing. (Hence these are properties of the man, and of *musical man* and *unmusical man,* there is a passing-away and a coming-to-be.) Consequently such changes are alteration" (Aristotle 1984, 14-15, 319b24-319b32).

As a result in fact, just after defining the third way in which motion is conceived, we find the true meaning of the word "natural" used in the expression "natural tendency" about bodies subject to gravity, which we have found in both Dörffel and Scheidlin: "Sic gravia, quando remotis solummodo abstaculis locum mutant, motum per Lineam directionis, quantum permittitur, excercent, & ille motus specialiter vocatur *Naturalis*"[78]. Therefore, Natural motion regarding bodies subject to gravity is tendency considered in a body without any other external forces preventing it to move in a straight line. It is indeed very different from something that pertains intrinsically one body, being closer to a study on inertia, rather than a qualitatively described motion. Having explained Weigel's school advancements in the philosophy of nature, I will show now their metaphysical premises and their relation with the concept of a physical body.

Physical Necessity in Weigel's School Dörffel's *Exercitatio* addresses the problem of describing motion in an ideal situation, in order to obtain Galileo's rules of free fall. Scheidlin's *Disputatio* instead, faces the problem of weight from a different perspective, because it doesn't start with general rules, but leaves the origin of tendency in bodies as a matter that needs to be determined at a later time. By doing so, Scheidlin moves ahead from the topic of air's weight and describes the different theories used to account for the origin of tendency. His reasoning then is not as clear as Dörffel's, because he tends to present the different theories as if they stand on the same ground, but it is nonetheless useful, because it gives us the possibility of understanding the position of Weigel's school in the contemporary debate. The problem is presented as follows: is gravity a principle that inheres things that possess weight, or is it the consequence of an attraction that has its origin at the centre of the earth?[79] At first Scheidlin introduces the topic, pointing out that phenomena as they appear are not sufficient in order to prove the correctness of a theory over another one[80]. Conclusions deduced by obscure principles cannot be evident,

[78] Weigel-Dörffel (1663, §2).

[79] "Gravitatis principium aut in ipsa re gravi, aut in meditullio terrae quaerendum" (Weigel-Scheidlin 1665, Sectio Prima).

[80] Later in the same passage Scheidlin exemplifies this by quoting the different accounts of Gassendi and Descartes on the origin of light: "In Opticis idem fieri solet, cum enim & illic incertum hucusque sit, quo pacto lucis radii se habeant, ut occursu variorum corporum flecti frangique possint, ad hypotheses duas Opticarum subtilitatum Magistri descenderunt, quibus quaecunque de luce per experientiam habemus, explicare solent: quarum *unam Celebrissimus Gassendus* hodie recoxit, qui Solem revera aliquas substantiolas, quas vocat *radios substantiales*, emittere statuit : altera *ingeniosissimi Renati des Cartes est*, qui materiale nihil e corpore lucido ad oculos usque manare, sed subtilem, quae continuo intercedit,

thus they cannot produce knowledge in its proper meaning. Scheidlin quotes here the famous letter to Picot, published in the form of the preface of Descartes' *Principia* in 1647. There[81], Descartes argues that, even if we witness bodies with a natural tendency towards the centre of the earth, this is not an argument that necessarily favours gravity as something that inheres to bodies intrinsically. The passage quoted is relevant for our purposes, because in Descartes' letter it occurs just before another passage on how Aristotle was misinterpreted by his followers. In other words, this letter could be one of the work that inspired Weigel's syncretism, even more because at the end of that preface we find a classification of sciences similar to that of Weigel.

The two different accounts on gravity proposed by Scheidlin then are presented with Descartes' reasoning in mind. The first one argues that "in spissitudinis telluris centro esse quendam per analogiam sic dictum magnetem, qui, effluviis mediantibus, quaqua versum seu sphaerico emissis vique aliqua attractrice praeditis, corpora ad se trahat, adliciatque"[82]. When Scheidlin writes about the attraction caused by the earth as something that resembles a magnet, he is not considering a sort of innate or occult quality, but a mechanical system that through the interaction of particles draws the objects towards the centre of the earth. His reasoning is, to a certain extent, similar to that of Descartes and in fact he comes to the same conclusions: an attraction that do not pertain intrinsically to objects explains the fact, shown by Weigel in the *Exercitatio* as he quotes[83], that they increase their speed while approaching the centre of earth. The other account instead does not even explain the difference in weight between objects. Both these conclusions are taken from the fourth part of Descartes' *Principia.* Scheidlin in fact argues that weight depends also from the shape of a body. If the body is shaped like a porous material, then a sort of lighter matter, which he calls "*materiae celestis*" like Descartes did, will permeate the body, making it lighter, while a compact body will approach the earth earlier

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materiam ejus vice fungi adserit" (Ibid.). Gassendi was not completely rejected by Weigel's school and it is often quoted. It is true that Descartes is generally favored, but at the same time we can't find in their writings many passages openly against atomism. A corpuscular theory for example is sometimes adopted, like in the explanation of gravity.

[81] Descartes (1963, IX, 1-20).

[82] Weigel-Scheidlin (1665, Sectio Prima).

[83] "Quod & peculiari Disputatione de *quantitate motus gravium* sub *Ampliss. atq. Excellentiss.* Dn. PRAESIDIS moderamine habita clarissime ostensum est" (Ibidem.). Dörffel here is not even mentioned, suggesting once again the importance of the teacher over his students in the draft of their dissertations.

for the opposite reason[84]. Even if this argument is taken directly from Descartes, it is nonetheless rather confusing, because it is as if Scheidlin wants to combine the idea of a magnet at the centre of the earth with Descartes' reasoning that addressed gravity to the action of matter filling the above space previously occupied by the heavy body. Scheidlin identifies in fact the celestial matter with the magnet's *effluvia*, a thesis that was not completely accepted even in Weigel's school[85]. The most interesting part of the *Disputatio* however, follows from these premises. If Scheidlin's reasoning for now was proceeding according to a very classical and Cartesian way of understanding phenomena, in the following passage he introduces an unexpected turning point which is worth reporting in its entirety:

> *Alii* recensitis hanc etiam superaddunt probabilitatem: Si gravitas, inquiunt, naturaliter inesset v.g. lapidi, adeoque principium primum motus suum intra gremium continerent gravia, tunc semper deorsum moverentur, quia proprietates & adfectiones naturales stricte sic dictae omni, soli & semper convenient: at corpora gravia remoto omni impedimento non semprer deorsum moventur. Si enim DEUS per potentiam suam absolutam totum hoc Universi systema, unico relicto gravi, annihilaret, tunc istud grave tantum abest ut moveretur deorsum, ut potius indifferens ad omnem motum relinqueretur. Hinc, dicunt, sequi probabiliter maxime, corpora gravia non in

[84] Descartes writes: "Atque ita gravitas cujulque corporis terreftris non proprie efficitur ab omni materia coelesti illud circumfluente, fed praecise tantum ab ea ipsius parte, quae, si corpus istud descendat, in ejus locum immediate ascendit, ac proinde quae est illi magnitudine plane aequalis. Sit, exempli caussa, B corpus terrestre in medio aëre exsistens, & constans pluribus particulis tertii elementi, quam moles aëris ipsi aequalis, ac proinde pauciores vel angustiores habens poros, in quibus materia coelestis contineatur: manifestum est, si hoc corpus B versus I descendat, molem aëris ei aequalem in ejus locum ascensuram. Et quia in sta mole aëris, plus materiae coelestis quam in eo continetur, manifestum etiam est, in ipsa esse vim ad illud deprimendum" (Descartes 1963, VIII, 213-214). Strictly speaking then, one body is not heavier because it is less permeated by celestial matter – that would be a theory similar to the *levitas* theory previously described –, but because, being less permeated, the difference with the quantity of celestial matter that tends to move away from the centre of the earth and that fills the same volume in the space left above by the compact body makes that body heavier.

[85] In Vinhold's *Theses* we read: "Magis probabile videtur principium motus gravium esse in psis partibus, quam in centro terrae". At first glance this statement could be misinterpreted as a revaluation of Aristotle's account, but Vinhold's *Theses* were written in 1671, around the time of Leibniz's *Theoria motus concreti*. In this regard, Leibniz's work represents an advancement in Weigel's school conception of gravity, because its existence is proved referring to moving parts that inhere to a body, while maintaining a purely mechanical account.

sese habere primum motus principium (Weigel-Scheidlin 1665, Sectio Prima).

Some others then use an argument based on pure possibility in order to argue that a qualitative account of gravity is incorrect. If in fact gravity is a quality, then it inheres the body in every possible situation. However, if this situation were to be that of a space where every possible obstacle is removed, where nothing exists except the body, then following the law of inertia, thinking of a different direction in which the body moves wouldn't be contradictory, whereas the qualitative approach would force us to choose a specific direction. It is obvious here, taking the word "annihilaret" as the most important evidence, that the author mentioned is no other than Hobbes, specifically a section of his *De corpore* that inspired Leibniz as well[86]. Once again his influence emerges as the most important one, even if Scheidlin is not directly suggesting in this chapter the superiority of his proof[87]. I believe however that the definition of tendency previously explained favours an interpretation that argues for a priority of Hobbes' proof: ultimately in fact, Hobbes' proof is the last one presented before the end of the section. These evidences suggest in the end that Weigel's *conatus* is not that of Descartes, despite the fact that Weigel uses it in the same context in which Descartes used it, that is the topic of gravity. Weigel's tendency in fact is conceived as something separated from direction, allowing the possibility of explaining motion in a way similar to that of Hobbes.

[86] In *De corpore*'s chapter VIII we read: "Quod quiescit, semper quiescere intelligitur, nisi sit aliud aliquod corpus praeter ipsum, quo supposito quiescere amplius non possit. Supponamus enim corpus aliquod finitum existere et quiescere, ita ut reliquum omne spatium intelligatur vacuum. Si jam corpus illud coeperit moveri, movebitur sane per aliquam viam. Quoniam igitur, quicquid in ipso corpore erat, disponebat ipsum ad quietem, ratio, quare movetur per hanc viam, est extra ipsum; similiter si per aliam viam quamcunque motum esset, ratio quoque motus per illam viam esset extra ipsum. Cum autem suppositum sit extra ipsum nihil esse, ratio motus per unam viam eadem esset quae ratio motus per omnem aliam viam; ergo aeque motum esset per omnes vias simul, quod est impossibile. Similiter, quod movetur, semper moveri intelligitur, nisi aliud sit extra ipsum, propter quod quiescit. Nam si supponamus nihil extra esse, nulla ratio erit, quare nunc quiescere debeat potius quam alio tempore; itaque motus ejus in omni simu temporis puncto desineret" (Hobbes 1999, 91). Hobbes' argument is slightly different from that of Scheidlin, but only because Hobbes considers the two possibilities (motion and rest), giving an account for both and because Scheidlin considers instead a specific motion, the one caused by gravity. Both however use the *annihilatio* in order to consider the possibility of moving, arguing for a conclusion that rests on a hypothetical situation.

[87] After presenting the various accounts, the passage ends with a rather obscure "evidentiae tamen physicae tantis per indulgentibus videtur ac si prior de terrae centro sentential plus probabilis, atque altera habeat. *Edocti meliora sequemur*". If we take for granted that the qualitative approach is discarded, still it does not clearly favour Descartes' or Hobbes' account.

On a side note, it is rather interesting how in Weigel's works and in the dissertations prepared under his guidance many authors are mentioned, many are even celebrated, like Descartes, yet one of the most influential of them is almost never mentioned directly. It surely shows that Weigel's impact on Leibniz needs a rethinking starting from a wider perspective, plus the very interpretation of Weigel's philosophy needs to distance itself from the easy path that leads to Aristotle, Scholastics and Descartes only. A reasonable conjecture on why Hobbes' influence in Weigel's writings concerning the philosophy of nature could be traced only by analysing the specific terms belonging to his philosophy and not through direct quotes is that, while on the subject of law, ethics and politics Hobbes' impact was too big to be hidden or dismissed, its adoption in physics was considered too dangerous and needed a superior degree of carefulness. Judging by how Weigel was condemned more than one time for his theses during his life, we already know that his carefulness was never rewarded. From Leibniz's perspective, this is perhaps the reason why to the departure from Weigel and Hobbes, recalled in the *Phoranomus*, corresponds Leibniz's increasing carefulness in talking with and about Weigel with his correspondents in the following years.

At this point, having explained the huge debt of Weigel's school with Hobbes, one could argue that if in the end Weigel's impact resolves into Leibniz's increasing interest in Hobbes, Leibniz's genius and his culture soon led him to Hobbes' primary sources, so that the introduction of Weigel's influence becomes unnecessary. However, I believe that this interpretation is inaccurate for several reasons: Weigel's priority in space and time, as defined in the previous chapter, should suggest the opposite. Even more, my purpose is not that of granting to Weigel an exclusive preference over other authors, but explaining, at least for this part of the present work, the aim of Leibniz's *Hypothesis physica nova*. Leibniz's solution based upon Hobbes is better explained considering the context in which that solution was accepted and required. The crucial point then is rather understanding if Weigel's reception of Hobbes could have influenced Leibniz's reception of the same author. I will argue now that Weigel offered to Leibniz a version of Hobbes' philosophy founded in a heavy metaphysical background that was unknown to Hobbes. In a general sense, this outcome is obvious and depends on the fact that Weigel revaluates Aristotle and Euclid in an ontological and metaphysical sense far from Hobbes' reasoning, but something relevant could be said also for the topics of the present chapter.

Another wording of the same problem would be that of questioning if the references seen in Weigel's dissertations are based on general metaphysical principles taken from Weigel's *Analysis aristotelica ex Euclide restituta*. There is indeed a path that leads from Scheidlin and Dörffel's works to the universal principles described in Weigel's work, showing a decisive reinterpretation of Hobbes' philosophy. If we take for example the last passage quoted from Scheidlin's *Disputatio*, we can clearly see that Hobbes' *annihilatio mundi* was reinterpreted as a situation that could be conceived thanks to God's *potentia absoluta*. Leibniz's take on God's *potentia absoluta* was already debated[88], because the difference between Leibniz's solution and that offered in the medieval background in which the topic developed, above all in Scotus, leads to a form of intellectualism apparently close to that of Scholastics: while in Scotus the absolute power of God would be in some way incapacitated, assuming that God's will should always be determined by his intellect, in the post-Ockhamist view, as Piro[89] defines it, the opposite account was defended. Regarding Leibniz's philosophy, the result is that even the *potentia absoluta* should be affected by what Leibniz considers as the essence of intellectualism or rational reasoning, that is the principle of contradiction. It follows that what God *could* do is always what it is *possible* to do and, being possibility defined as something non-contradictory, what God could do is always non-contradictory. It is a decisive restraint to God's power that implies the problem of determinism faced by Leibniz in his *Confessio philosophi*: counterfactuals are always imagined as coherent causal chains where contradiction does not take place, much like free will is denied because it would introduce an unexplainable discontinuity between cause and effect. This is of course the final outcome of Leibniz's early years found in the *Confessio*, whereas a genetic approach shows many changes and developments of these ideas before 1672. In the next chapter I will argue that the adoption of the principle of contradiction in such a way is also the last result of Weigel's progressive influence, started in 1663 and ended in 1671, but I believe that the considerations on this topic from a physical point of view are essential as well.

Scheidlin's combination between annihilation and God's absolute power shows very interesting results: if we do not consider the topic of God's absolute power per se and we

[88] See for example Mondadori (1989).
[89] Piro (2002, 29-30).

accept the intellectualist interpretation shared by Weigel’s school and Leibniz, then absolute power could be used in physics in order to create deterministically controlled alternate hypotheses. In the example of the body subject to gravity, God could remove some of the requisites that made that body behave in a certain manner, i.e. the other objects in space, preventing a study of its action when it depends only on one *conatus*, but that does not change the fact that (1) the remained body still behaves following the exact laws of motion given before God’s annihilation and (2) if the other objects were to be reintroduced, they would act in the same way as before, following the same rules. This perspective implies a strong determinism, yet its reasoning is based on contingent situations that could be otherwise and not on absolutely necessitated circumstances. Philosophy of nature and theology are then connected in such a way that they become mutually useful. Schott’s study of air’s behaviour adopted by Weigel’s school is also consistent with this approach, since the experiment, following Schott's appreciation, recreates a situation in which air is influenced as little as possible by external forces.

In the third chapter of Weigel’s *Analysis aristotelica*, entitled *On Necessity*, is found the theoretical foundation of Scheidlin’s argument and of the general idea of using pure possibility to infer actual properties. After having introduced absolute necessity, defined like in Leibniz and in the Sholastics as that by which the opposite implies a contradiction, Weigel introduces another form of necessity, called restricted (*restricte*) necessity:

> Restricte autem & secundum ordinarium naturae cursum quod verum, certum & necessarium est effatum scibile, aeternum quidem dici quoq; solet, sed saltem concessive quasi per mentis anticipationem. Interim tantam adhuc habet necessitatem, ut, nisi per extraordinariam Dei potentiam, quam in naturalibus rarissime, & saltem ubi miraculo fuerit opus, experimur, aliter se habere non posit (Weigel 1658, 20-21)

Propositions characterised by restricted necessity are necessary indeed, plus they follow the regular course of nature, if not thanks to God’s absolute power. Things could be different from how they are, but that does not prevent Weigel to attribute them necessity in a deterministic fashion. Weigel clearly ascribes this form of necessity to objects analysed on the ground of physics: “hoc modo necessaria sunt pleraeque Propositiones

Physicae, itemque multae, quae in explixatiori tractatu, quem mathesin mixtam dicimus, Philosophiae Naturali debentur"[90]. The introduction of God's absolute power then seems to be allowed by Weigel for the sole purpose of justifying, in the eyes of Jena's faculty of theology, things such as miracles and other Faith's dogmas, rather than truly believing that God could really have an impact on nature's course different from that founded on logical coherence. The exemplification of God's action on nature in fact is rather revealing and it confirms that, when it is applied as a tool, God's absolute power is used only to hypothesize alternative yet coherent situations:

> Ita certe Propositio est: *Luna plena in alterutro nodorum constituta, aut a propiore nodo secundum longitudinem infra 10. gradus distans, patitur eclipsin*; quippe secundum ordinarium naturae cursum maxime necessaria […] Absolute tamen & immutabiliter necessaria non est; si enim Deus altius eveheret Lunam, aut ad Solem propius admoveret Terram […] tunc licet in ipsis nodis consisteret Luna plena, Eclipsin tamen non pateretur (Weigel 1658, 21).

Physical necessity is different from absolute necessity, because God could still move the celestial bodies in order to achieve a different outcome on phenomena like full moon or eclipse. Nevertheless, God could change reality only acting upon entities exactly determined by their causes: the fact that an eclipse does not occur anymore is still the result of the determinate relationship between the celestial bodies involved, even if they changed their place thanks to God's will. Weigel specifies that the truth by which the moon is positioned in such a way is something that expresses the natural course of the world from its very beginning, suggesting that phenomena should be interpreted as causal chains.

In Weigel's definition of physical necessity, the use of the term "*concessive*" related to the use of the "*anticipatio mentis*" is particularly important: restricted necessity applies to knowable propositions, i.e. coherently based proposition that could be true or false, but sometimes, Weigel argues, they are called necessary in a concessive way, as if our mind had mistakenly identified absolute necessity with restricted necessity. The "*anticipatio*

[90] Weigel (1658, 20-21).

mentis" in fact is a negative term taken from Bacon[91] that expresses the way in which men jump to conclusion from simple observations, without adopting a rigorous method. Weigel however is not arguing that some men mistake these truths as necessary truth – the context being that of the definition of restricted necessity, just after the definition of absolute necessity, suggests that such interpretation would be wrong – rather they are mistaking absolute necessity with restricted necessity, as if they were one and the same. It is not necessity in itself to be at stake here, but the possibility of specifying the difference between determinism and necessitism.

While the fact that Weigel uses the word concessive is interesting, because the use is very similar to Leibniz's idea of necessity *ex hypothesi*[92], the reference to Bacon is also interesting per se, especially concerning physics. It could be that the revaluation of Bacon's superposition between the negation of dimensions' penetration and antitypy that I analysed in the previous section is related in some way to Weigel's physical necessity. As a matter of fact, another section of Weigel's *Anaylisis aristotelica* suggests a possible connection. In the chapter entitled "*De Natura causarum Demonstrativarum & ratione causandi*", Weigel tries to define a specific cause, demonstrative cause considered from the point of view of the object, comparing it to another cause, called *causa emanationis*, a term usually adopted in theology in order to explain the relationship between Father, Son and Creatures. From a theological point of view, the problem was that of explaining how the existence of creatures follows directly from the existence of God, without altering God's perfection. Weigel argues that the *causa emanationis* is commonly defined as something like an "*actu concessivo*"[93], a concessive act, because the existence of creatures is based on God's existence. Weigel however adds that the *causa emanationis* coincide with a complex cause, because it is based on necessarily related conditions that happen to exist at the same time in the same place. In an interesting turning point, exemplifying the relationship between emanation and complex cause, Weigel shifts from a theological reasoning to a reasoning concerning the philosophy of nature:

> Licet enim, quando v.g. *Materia* dicitur emanandi causa respectu *magnitudinis*, h.e. modi continuitatis aestimativi, puta tripedalitatis, &c.

[91] Specifically, in the first book of Bacon's *Novum Organum* (Bacon 2004, I, 26).
[92] AA (VI-4, 1547).
[93] Weigel (1658, 129).

> utrumque (causa & causatum) ut simplex quippiam compendii gratia (more veterum) dicatur; quoad rem tamen, addito subjecto quod subintelligebatur (*corpore naturali*) complexum quippiam est, quo *Corpus naturale cum materia, & hac porro cum magnitudine, necessario complicata* supponuntur, ex quo, tanquam causa complexa, necessario resultat causatum complexum, *Corpus naturale cum magnitudine quoq. necessario complicatum esse* (Weigel 1658, 129).

If we consider matter in respect to magnitude alone, we can safely say that matter is the *causa emanationis* of magnitude, being magnitude a mode of matter's continuity that allows its quantification. They are necessarily related, in a way that justifies to some extent the use of a single term to address both, even if one is the cause of the other. They are however conceptually distinct entities: once we add natural body as a substratum, this distinction becomes relevant, because it describes different aspects of the same body. At the same time, even if they are distinct entities they are necessarily related, even when they inhere to a single body, like fire and heat. This way of describing physical bodies has in Weigel its metaphysical outcome, expressed just after the passage above quoted:

> Causa demonstrativa […] non certo quodam productionis actu quatenus talis causa est, sed nuda saltem simulpositione & socia quadam sed necessario concatenata exhibitione, quae, si placet, uno verbo *Con-necessitatio* dici potest, quod haec causa dum ipsa ponitur *Con-* i.e. secum, esse faciat & a parte rei *necessitet* poni causatum (Weigel 1658, 130).

Weigel's *Con-necessitatio* (necessity-with) is the metaphysical tool developed to explain the superposition of necessarily related properties in a single natural body. It is both "position at the same time" and "necessarily concatenated exhibition" of properties. I believe it is quite clear that Weigel's approach is very similar to that found in Bacon, Hobbes and Leibniz. The same superposition of magnitude and matter is found in Leibniz's 1669 letter to Thomasius and it allows the possibility of adopting antitypy. Weigel help us in understanding how this is possible: while (1) bodies belong to a physical description of the world, thus they are subject to physical necessity and (2) physical necessity is a form of determinism that allows contingency, (3) in the same body there are, so to say, *con-necessitated* properties. Matter, extension and antitypy could be conceived independently, yet they happen to necessarily inhere to the same body. Weigel however specifies that this explanatory model is valid only *a parte rei*, because from a genetic and metaphysical point of view it would add a form of discontinuity that cannot be taken as the final account of reality. Another objective of Weigel's school then will be that of retracing the way that from a supposed metaphysical unity reaches the realm of physics and the layered model of defining bodies.

In conclusion, Weigel's school at the time of Leibniz's enrolment in Jena and in the following years exhibits a level of complexity and advancement in natural sciences that I believe it was often underestimated. Galileo, Schott and Hobbes helped in defining Weigel's concept of tendency, while the reintroduction of Aristotelian and Scholastics principles helped in giving birth to the description of natural bodies through physical necessity and *Con-necessitas*. All that is left is understanding how the physical properties that pertain one body are connected also from a genetic point of view.

Primary Matter and Geometry of Space Assuming that the 1669 letter to Thomasius represents the beginning of the peak of Weigel's influence on Leibniz, I will start by analysing this letter in the light of what I previously showed on physics regarding Weigel's school. It is widely known that in those years Leibniz wanted to harmonise Aristotle with the moderns with the objective of unifying a vision of the natural world through magnitude, figure and motion with the existence of God as the prime mover. The fact that in modern terms a body has no principle of motion in itself was considered by Leibniz quite fitting for these purposes for obvious reasons, since God would act in this situation as the entity that causes motion. In doing so however, Leibniz makes some distinctions among the moderns: it is true that they all adopt the magnitude-figure-motion model, but Descartes did that only on paper, because in the process of applying this method he added some ideas that were arbitrarily taken for granted, rather than necessarily derived from the model endorsed[94]. This outcome in Leibniz's early

[94] "Cartesianos vero eos tantum appello, qui Cartesii principia sequuntur, ex quo numero magni illi viri Verulamius, Gassendus, Hobbius, Digbaeus, Cornelius ab Hoghelande etc. prorsus eximi debent, quos vulgus Cartesianis confundit, cum tamen vel Cartesio aequales vel etiam superiores aetate et ingenio fuerint, me fateor nihil minus quam Cartesianum esse. Regulam illam omnibus istis philosophiae Restauratoribus communem teneo, nihil explicandum in corporibus, nisi per magnitudinem, figuram et motum. In Cartesio

philosophy is quite interesting, because Descartes, among the new philosophers, is probably the one that resorts the most to the use of God in the foundation of the physical world. Leibniz instead clearly favours Hobbes in this regard, but this is consistent with the huge impact of Hobbes' philosophy on Weigel's physics that was previously shown. Two facts support a connection between Weigel, Hobbes and Leibniz: Leibniz's indecision with respect to void, based on the reference to the same authors found in Weigel's school dissertations[95], and his argument against motion as a property that inheres to one body. In regard to this last topic we witness the closest resemblance to the argument exposed in Scheidlin's *Disputatio*:

> Et Aristoteles ut dixi pro certo habet corpus nullum in se solo principium motus habere, et hoc unico argumento Huic objectioni dupliciter respondes; primum, hoc argumentum nihil posse apud Epicurum, qui suis atomis largiatur per se motum deorsum. Fateor, apud eum nihil posse hoc argumentum, nisi ei praedemonstretur, hoc ipsum absurdum et impossibile esse, quod corpus habeat motum a se ipso, quod et jam tum Cicero ni fallor in libris de natura Deorum facit, eleganter Epicurum irridens, quod quiddam sine causa et ratione in suis Hypothesibus hoc modo introducat. Nam in rerum natura nihil esse deorsum, sed quoad nos, neque igitur causam, cur corpus aliquod in hanc potius quam illam plagam moveatur. Epicuro igitur neganti quicquid movetur ab alio extra se moveri facile occurremus et laborantem existentiae Dei certitudinem vindicabimus (AA II-1, 34).

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ejus methodi tantum propositum amo; nam cum in rem praesentem ventum est, ab illa severitate prorsus remisit, et ad Hypotheses quasdam miras ex abrupto delapsus est" (AA II-1, 24-25).

[95] As I already recalled in this passage of Leibniz's letter to Thomasius where the possibility of harmonizing Aristotle, the reference to Guericke and Boyle and the idea that for Hobbes and Leibniz the existence of void and its negation are both acceptable theories are found: "Quare dicere non vereor plura me probare in libris Aristotelis […] quam in meditationibus Cartesii; tantum abest, ut Cartesianus sim. Imo ausim addere totos illos octo libros, salva philosophia reformata ferri posse. Qua ratione illis ipso facto occurretur, quae tu, Vir clarissime de Aristotele irreconciliabili disputas. Quae Aristoteles enim de materia, forma, privatione, natura, loco, infinito, tempore, motu, ratiocinatur, pleraque certa et demonstrata sunt, hoc uno fere demto, quae de impossibilitate vacui, et motus in vacuo asserit. Mihi enim neque vacuum neque plenum necessarium esse, utroque modo rerum natura explicari posse videtur. Pro vacuo pugnant Gilbertus, Gassendus, Gerickius; pro pleno, Cartesius, Digbaeus, Thomas Anglus, Clerk in libro de plenitudine mundi. Pro possibilitate utriusque Thomas Hobbes, et Robertus Boylius. Et fateor, difficulter quidem, posse tamen sine vacuo rerum rarefactiones explicari". (AA II-1, 25).

According to Leibniz, Epicurus admitted atoms that have motion towards a specific direction (downwards), making his philosophy unsuitable for the ascension to the prime mover. The confutation of this hypothesis rests in Leibniz as much as in Weigel on conceiving an alternate situation in which bodies are not subject of any other force, arguing that there is no apparent reason why that body should move downwards and not in any other direction. As I already noticed, a very similar argument is found in Hobbes' *De corpore*, but some details of Leibniz's argument suggest that it was nonetheless adopted from Weigel. Weigel and Leibniz's version in fact develops in a slightly different way: their conclusion is that in the ideal situation the direction of one body's movement is indifferent, while in Hobbes' version the reason why a specific direction would be unacceptable, if not ascribed to the action of an external body, is that admitting this means allowing one body to have the possibility in itself to actually move in all directions, which is contradictory. It is the same argument, but one is based on the idea that there is no reason for one body to do so, while the other is based on the idea that, if we accept that there is a reason, then that body could do so in all directions, all at once and at the same time.

Another similarity with Weigel's take on the problem is the direction chosen: while in Hobbes the argument is presented for any possible direction, both in Leibniz and Weigel's school the bodies analysed move downwards. At first glance, this result does not seem particularly relevant, but the common direction could suggest that Epicurus' atoms and bodies characterized by gravity have a common background in Weigel's school. In this regard, Dörffel's *Exercitatio* contains an important passage that connects bodies affected to gravity to atoms and to the prime mover, ultimate goal of Leibniz's letter. Dörffel argues that regarding the principle of the motion of objects subject to gravity, there are two fundamental and opposite theories, i.e. the one that maintains an internal principle and the one that maintains an external one. Predictably, Dörffel writes that Peripatetics support the first one, adding that this natural tendency was given by God in his role of prime mover since the very beginning[96]. However, he also suggests that the same theory

[96] "De ipso Motus hujus Gravium principio quaedam praemittamus notissimum est opinionum bivium, dum principium illud vel *internum* ipsis Corporib. vel *externu* constituunt Autores. Ex *illis* primo nominandi sunt *Peripatetici* […] ab immobili Motore, DEO T. O. M. primitus pro radicali proprietate ad finem sibi praestitutum concreatam" (Weigel-Dörffel 1663, Sectio III, §3). It is rather interesting that here the first theory is ascribed to Peripatetics, without a direct mention of Aristotle or a reference to his works, very

is implicitly held by the atomists: “Fovent vero hanc sententiam tacite, qui *Epicuraeorum* veterum, *Democriti* imprimis, doctrinam (quam interpolavit hodie *P. Gassendus*) sequuntur”[97]. It is very likely then that once in Jena, the young Leibniz, already following Gassendi on atoms as we saw in the *Disputatio metaphysica*’s corollaries, was encouraged by Weigel to separate the usefulness of antitypy from the contradictory thesis of an intrinsic tendency, favouring instead Hobbes’ solution.

If the context of Leibniz’s letter to Thomasius is highly influenced by Jena’s cultural background, then a confrontation with Weigel also on the topic of primary matter is justified. I already sketched Leibniz's definition of primary matter: it is characterised by extension and antitypy. Antitypy is the requisite for motion, because impenetrability makes a coherent study of collisions possible, while extension is a property given to matter simply because it fills space. There is a distinction in Leibniz then between extension attributed to matter and space: space is prior to matter, since matter fills an already existing space, and it is the origin of matter's extension, because being extended and being in space are one and the same thing[98]. It follows that from the point of view of the object space, extension, impenetrability and matter are in the same place, yet conceivably different, but from a genetic perspective space and extension precede matter. Matter then, strictly speaking, does not possess extension intrinsically, because extension is a consequence of its truly unique property, that is the possibility of filling a space. The term “*impletionem*”, used by Leibniz in his *Phoranomus* recalling the early years, exemplifies in fact the union of the terms extension and impenetrability and it has the same meaning of the verb “*replet*” used in the definition of primary matter given in the letter to Thomasius.

The same idea on matter is expressed by Weigel in his *Analysis aristotelica*. The concept of matter is analysed in the third section, in the chapter entitled *De Philosophia Mathematica*, between the chapter on first philosophy and the one on natural philosophy. Interestingly then, primary matter is placed on a middle ground between pure metaphysics and physics. In this chapter Weigel describes the first two *summa genera*, that are space and matter. Space is also called intelligible matter, while matter strictly speaking is also

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common elsewhere. The author that argues for an external principle of gravity instead will be Descartes. Later in the passage it is also relevant Dörffel’s quote of Zabarella’s account.

97 Ibid.

98 "Extensum autem esse nihil aliud est, quam esse in spatio" (AA II-1, 36).

called sensible matter[99]. Space is at rest and it is defined as *extensiva capacitate*, capability of extension, while sensible matter is in motion and it is defined as *extensiva repletione* of space[100], that is the act of filling space. In order to describe the relation between space and sensible matter Weigel adopts the tools previously introduced: from the point of view of the object there is a superposition between matter and space, while from the genetic point of view both space and sensible matter emanate properties, extension and continuity regarding space and discernibility regarding sensible matter, that are, so to speak, bound to their very existence[101]. The outcome of this distinction in Weigel bears an important consequence:

> Illa [space, intelligible matter] porro pro conceptu suae naturae proprio sibi vendicat *Capacitatem* s. receptivitatem alius absque sui disruptione, quam Philosophi dicunt penetrationem dimensionum ; haec [sensible matter] *Repletivitatem* sibi servat propriam, & a receptivitate alius de sua substantia absque partium disruptione nimirum quantum abhorret, atque ita penetrationem suorum corporum, quantum in se est, non admittit (Weigel 1658, 186).

The negation of dimensions' penetration found in the *Theses philosophico-mathematicae* of Weigel's school then has to be reinterpreted as a shortened version of a much more complex theory: space defined as capacity in fact allows the penetration of dimensions. The interesting outcome however is that the explanation of the possibility of penetration is not based on a sheer reflection on the nature of geometry, but it is connected with the other principle on the ground of physics. If in fact space has to be filled, then allowing the penetration of its dimensions is only natural, because the very idea of filling implies

[99] "Prius illud (ut ad rem proprius accedamus) Spatium latini dicunt, Aristoteles materiam intelligibilem vocat, eo quod mente tantum non sensu percipiatur : posterius hoc Materiam absolute vocant Philosophi, sed distinctionis gratia materiam sensibilem dixerunt veteres" (Weigel 1658, 185).
[100] "Illa materia, ceu diximus, *immobilis* est, & *extensiva capacitate* nude sic concepta constituitur; haec uti dictum, *mobilis* est, & in *extensiva repletione* spatii sibi respondentis, si praecise spectetur, consistit" (Weigel 1658, 185-186).
[101] "Utraq; materia semper conjunctam sibi habet, aut affectionis s. proprietatis instar ex naturae comunis primo conceptu (*extensione*, *continuitate*) velut emanare facit *divisibilitatem* […] sed haec, nempe sensibilis, eam accedente formali suae naturae conditione (*mobilitate*) in *discerptibilitatem*, s. *dissociabilitatem* commutat, a qua prior illa prorsus aliena est" (Weigel 1658, 186).

the penetration. If this possibility was not given, intelligible matter and sensible matter would basically coincide. This outcome however does not rule out completely the penetration's negation, because that is reintroduced thanks to sensible matter. Weigel's *Repletivitatem*, that is no other than Leibniz's *Impletionem*, grants this possibility, because the act of filling brings coextension. It follows that, even if sensible matter can be separated, it happens to be separated at the same time and in the same place for the same objects. Even when these object are conceived in space through geometry then, being tied to sensible bodies that never penetrate, also their shapes won't ever happen to penetrate. There is a difference however between Leibniz and Weigel, because Leibniz feels the need of adding antitypy, while antitypy is never mentioned by Weigel. This is probably the result of Leibniz's reflection on the inconsistencies of Weigel's argument: the two principles, space and sensible matter, are presented in Weigel as if they are already determined in such a way, but this is not a feasible explanation for their difference, because, if the dimensions' penetration is admitted unlike Descartes, they are both described through extension while possessing opposite properties. In other words, we still need a principle that states the impossibility of penetration, that is antitypy. Perhaps interpreting Weigel's *Repletivitatem* as antitypy, or as something that has more or less the same function, is possible, but in this occasion and in these years Leibniz felt the need of adding the term antitypy adopted from Gassendi to effectively stress the difference between space and matter.

The other major difference between space and sensible matter found in Weigel is that space is conceived at rest and sensible matter is constantly in motion. Regarding motion alone, Weigel is very close to Leibniz's attempt of introducing God as the prime mover: intelligible matter is at rest but, Weigel specifies, it is not moving something else while being still, otherwise it would be the prime mover. Sensible matter on the other hand is constantly in motion, but movement was given to it by something else in motion, forcing sensible matter to do the same and transmit motion[102]. In this line of thought then, the exclusion of a function similar to that of the prime mover for both principles prepares for God's necessary existence.

[102] "Illa [intelligible matter] ut de se immobilis est, ita movere quippiam ipsa non potest (solus enim Deus immobilis est motor) […] Haec [sensible matter], ut ipsa mobilis est, ita motu recepto stantes alioquin sui partes alias simul movere potest" (Weigel 1658, 186).

A part from the role of God, the distinction between motion and rest is interesting in itself. Weigel's argument is based on a strong dichotomy between space and sensible matter that influences this topic as well: some properties that belong to space will belong to a world conceived in stillness, while other properties pertaining sensible matter will be conceivable only in motion. As a result, space and stillness involve geometry as a whole, i.e. lines, shapes quantity and quality:

> Illa [space], si abstracte spectetur, de se indeterminata est, sed certis a parte rei semper determinatur formis, mente seorsim conceptibilibus, *Linea, Superficie, Corpore* (puta mathematico) certisque stipatur modis, ut sunt, *quantitates,* v.g. tripedalitas, & *qualitates*, v.g. *curvitas, rectitude, parallelism., angularitas, figuratio, congruential* quibus in plures species dispescitur immobilis haec & orta Substantia (Weigel 1658, 186).

This is the first time in which we witness a distinctive connection between the problems faced in physics by Weigel and Leibniz and some fundamental concepts of their *analysis situs*. Quality and quantity are extremely important regarding this topic and I will not fail to analyse them in the next part of the present work, but for now some remarks could still be made: quality and quantity belongs to a description of a world at rest and they are entities conceivable by our minds. Given the fact that Weigel adopts the principle of contradiction in a way similar to that of Leibniz, so that conceivability, possibility and being non contradictory are one and the same thing, this is the first hint to a possible way of reconciling the foundational attitude of Leibniz's *analysis situs* with his claims on the possibility of deriving the whole of mathematics on the principle of contradiction.

Undoubtedly, quantity and quality belong to a vision of the world that sets itself apart from the description that involve real bodies, i.e. hard bodies having the ability of filling space. It follows that moving bodies have their own properties, leading us to the very first definition of primary matter found in Weigel:

> Haec [sensible matter] eodem modo de se indeterminata est (materia prima dicta) sed spatio coextensa, preaterquam quod formas illas quasi genericas & fundamentales participet, iisque a parte rei nunquam non similiter

> determinetur, etiam superaddita propriae naturae conditione (*mobilitate*) plures adhuc admittit differentias quasi specificas, quibus in mille formas ulteriores abit Substantia mobilis (Weigel 1658, 186-187).

Even if sensible matter shares spatial properties because of its coextension with space, its unique characteristic is that of transforming through motion. Primary matter then is no other than sensitive matter considered as something undetermined, that is only for its property of filling space without taking into account motion. The relationship between primary matter and motion is not always clear in Weigel, but there is no denying that its definition is the same used by Leibniz: "praecise spectate (materiam primam vocavit Aristoteles) […] consistit in *extensione repletiva mobili*, h.e. in substantiali habentia partium extra partes, quibus sibi respondens & aequale spatium replet"[103] In this regard, intelligible matter and sensitive matter are almost the same, because depriving sensitive matter of motion means conceiving it only as a continuum coextended with space. It is the ideal starting point of what by means of motion and transformation will be the world that we perceive, full of different shapes.

The same reasoning is present in Leibniz's letter to Thomasius. I will examine now some quotes of this letter in order to show the strong resemblance between the Leibniz's and Weigel's theory. Leibniz contemplates two distinct aspects in the description of the world, that are primary matter conceived without motion and bodies described when motion becomes one of its properties. Primary matter fills the world being at rest, meaning that the ideal starting point at the beginning of the world is a homogeneous situation:

> Haec jam massa continua mundum replens, dum omnes ejus partes quiescunt, materia prima est, ex qua omnia per motum fiunt, et in quam per quietem resolvuntur. Est enim in ea mera homogeneitas, nulla diversitas nisi per motum (AA II-1, 26).

The result is one of the most important ideas that Leibniz took from Weigel: heterogeneity founded in homogeneity by means of motion. Ideally, tracing back motion following lines and shapes generated from it, we could reach that starting situation in which motion is

[103] Weigel (1658, 193). Later in the same passage extension is introduced "uti materia cum spatio coincidit",

not inhering bodies and matter is truly considered as a primary and homogenous entity. In the same year of Leibniz's letter to Thomasius, Weigel writes in his *Idea Matheseos Universae*:

> *Materia prima*, tanquam subjectum ultimum, in quod omnia materialia, si continuo trasformata fuerint, ultimo resolvintur […] Hinc ut illa, nempe *materia prima*, subjectum trasformatorium ultimum est; ita spatium subjectum transformatorium ultimum dici potest: ut illa praecise fine forma suum sistit actum entitativum; ita spatium praecise fine contento (*Vacuum*[104] alias dicunt) puram ostendit essentiae suae rationem (Weigel 1669, 41).

Although very similar, Leibniz slightly modifies Weigel's argument: it looks as if Weigel conceived in the end a real superposition between space and primary matter, when the latter is considered at rest, while Leibniz seems to maintain a distinction, founded probably on the addition of antitypy, something that implies a degree of reality sufficient in order to maintain the distinction between the two concepts. However, even in Leibniz's writings we find a tension between primary matter constantly in motion and primary matter identified with space. For example, in a fragment entitled *De materia prima*, written between 1670 and 1671, Leibniz writes "His ego jam adjungo, materiam primam si quiescat esse nihil", but just after this statement he adds that "Omnia esse plena, quia materia prima et spatium idem est"[105].

The view adopted by Leibniz entails also Weigel's difference between a sort of framed version of the world geometrically describable in its stillness, and the world in motion subject to transformations. As Leibniz specifies in his letter to Thomasius in fact, motion sets things apart, while rest grants homogeneity:

> Nec obstat, quod generatio fit in instanti, motus est successivus, nam generatio non est motus, sed finis motus, jam motus finis est in instanti, nam figura aliqua ultimo demum instanti motus producitur seu generatur, uti circulus extremo demum momento circumgyrationis producitur. Ex his etiam

[104] This is a perfect example of Weigel's syncretism: he is not accepting void here, yet he accepts and adopts the similarities between different views.
[105] AA (VI-2, 280).

> patet, cur forma substantialis consistat in indivisibili, nec recipiat magis aut minus. Etsi enim circulus circulo sit major, non tamen est circulus altero magis circulus, nam circuli essentia consistit in aequalitate linearum a centro ad circumferentiam ductarum, jam aequalitas consistit in indivisibili, nec recipit magis aut minus (AA II-1,29).

Only when motion stops, or the world is considered without motion, shapes are generated. In explaining this, Leibniz shows that his metaphysics of space resembles that of Weigel, because shapes are conceived as qualitatively similar entities – no circle is more or a better circle than another one – and quantity is obtained through confrontation. Particularly interesting in fact is the connection between substantial forms, indivisibles and geometrical figures, because it favours the idea that figures and space can be considered as proper substances[106]. The way in which a substantial form is indivisible is the same in which figures possess some of their characteristics. In this regard, Leibniz's idea that one figure taken in itself cannot be augmented, yet it retains some intrinsic properties, preludes the distinction between quality and quantity. This conception of geometry explains some apparently obscure ideas found in Weigel's school, for example that "Linea non constituitur ex punctis, neque superficies ex lineis, neque corpus ex superficiebus[107]" or "Etiamsi plura puncta, plures lineas, pluresque superficies conjungas ; nulla linea, nulla superficies, nullum corpus exinde componetur"[108]. The relation between a point and a line in fact cannot be explained by a mere sum of parts, but needs motion in order to gain continuity. Later in 1679 Leibniz will remind the same idea in his first letter to Weigel: "quemadmodum spatium ex punctis compositum intelligi non potest, ne quidem numero infinitis, ita nec tempus videtur componi ex instantibus"[109].

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[106] "Figuram esse substantiam, aut potius spatium esse substantiam, figuram esse quiddam substantiale, probaverim quia omnis scientia sit de substantia, Geometria autem quin scientia sit negari non possit" (AA II-1, 30).

[107] Weigel-Vinhold (1671, 53)

[108] Weigel-Scheidlin (1665, Corollaria, I).

[109] AA (II-1, 748). In this regard, Leibniz also adopts Weigel's distinction between continuity emanated from intellective matter and discontinuity emanated from sensible matter, turning it in continuity as a property of primary matter: "Quantitatem quoque habet materia, sed interminatam, ut vocant Averroistae, seu indefinitam, dum enim continua est, in partes secta non est, ergo nec termini in ea actu dantur: extensio tamen, seu quantitas in ea datur: non de extrinsecis mundi seu totius massae, sed intrinsecis partium terminis loquor" (AA II-1, 26). In the same way, discontinuity follows matter in motion: "Sin vero ab initio continua est, necesse est, ut formae oriantur per motum (nam de annihilatione certarum partium ad vacuitates in materia procurandas, quia supra naturam est, non loquor) quia a motu divisio, a divisione termini partium, a terminis partium figurae earum, a figura formae, ergo a motu formae" (AA II-1,27).

However, the fact that there is a distinction between the creation and the development of geometrical figures does not mean that there is no connection between the geometrical world and the world in motion, because motion allows the creation of different shapes or figures, as Leibniz argues against Scholastics[110]:

> Sed si rem cogitemus accuratius, apparebit demonstrare eam ex causis. Demonstrat enim figuras ex motu: ex motu puncti fit linea, ex motu lineae superficies, ex motu superficiei corpus. Ex motu rectae super recta oritur rectilineum. Ex motu rectae circa punctum immotum oritur circulus, etc. Constructiones igitur figurarum sunt motus; jam ex constructionibus affectiones de figuris demonstrantur. Ergo ex motu, et per consequens a priori, et ex causa. Geometria igitur vera scientia est. Ergo non invito Aristotele subjectum ejus, nempe spatium, substantia erit. Neque vero adeo absurdum est, Geometriam agere de forma substantiali corporum (AA II-1, 30-31).

This idea of a geometrical creation through motion and it nature *a priori* is once again taken from Weigel's *Analysis aristotelica*. In this work Weigel makes the classical distinction between *Natura naturans,* God as an absolute, eternal, independent being, and *Natura naturata*, that is everything that depends in some way from God's existence. *Natura naturata* is further divided in order to account for two kind of dependency: a logical, *a priori* dependency and the one instead caused by the physical necessity that I examined in the previous section. In the first kind of subordination is found a similar approach to that of Leibniz:

> [The first kind of *Natura naturata*] purum & immutabilem, quem a divino Numine participavit, in causando semper exercet actum, immediate sub Deo consituta, quails est rerum metaphysicarum & geometricarum, Idearum puta, Numerorum & Figurarum, quarum ortus, generations & causations simplices puro quodam & velut continuato emanationis ab ipso Divino Numine

[110] "Et vero tam abjecte de Mathematicis scholastici primum senserunt, omni conatu id agentes, ut ex perfectarum scientiarum numero Mathesin excluderent: eo praecipue argumento, quod non semper ex causis demonstret" (AA II-1, 30-31)

> celebrantur actu (Linea formali fluxu puncta : Conus aeterna Trianguli conversione : Sphaera perpetua radiorum ex eodem puncto quaquaversum infinitorum egressione : Novenarius ternario in se formaliter ducto & c) (Weigel 1658, 147).

In this passage, not only the idea of creation through motion in geometry, but the very idea that geometry, mathematics in general and metaphysics have a foundation *a priori* in God is defended. Weigel defines them in almost every work as *notitiae nobiscum natae*, suggesting the homogeneity between the human mind and the mind of God, at least regarding metaphysics and mathematics. The use of the word "emanation" in Weigel's terms suggests also a form of dependency of God from these entities that is very similar to what we find in Leibniz. This is one of the most important ideas in fact that Leibniz will adopt even when, on the ground of physics, his views will differ greatly from those of Weigel. Nevertheless, through physics these ideas concerning mathematics and metaphysics were successfully passed down to Leibniz in the early years.

On a side note, the peculiar way in which substantial forms are considered and applied to the definition *a priori* of geometrical entities and Leibniz's claims against Scholastics suggest that arguing for a young Leibniz already in possession of some ideas of his mature theory should be carefully considered: it seems that at this point we are far from substantial forms as conceived by Leibniz in his mature philosophy.

In conclusion, in the distinction between situations subject to motion and situations subject to rest lies then one of the most important result of Weigel's influence in Leibniz's philosophy of mathematics, hence the need of analysing Weigel's influence on his philosophy of nature. From his reflections on physics, Leibniz takes the idea of transformation as a tool used to connect homogeneity and heterogeneity. If the physical outcome is heterogeneity founded in homogeneity by means of motion, the mathematical outcome will be homogeneity founded in heterogeneity by means of transformation: the possibility of comparing homogeneous entities, like those found in geometry, is the starting point of Leibniz's attempts on the foundation of mathematics.

The Final Outcome of Weigel's Influence in Physics At the beginning of this chapter I argued that the first two corollaries of Leibniz's *Disputatio metaphysica* show the first documented influence of Weigel on Leibniz, while the writings around the time of the *Hypothesis physica nova* represent its peak, at least concerning the philosophy of nature. Now that many themes present in Leibniz were connected with Weigel and his school, I believe that showing this progressive adoption becomes easy.

Regarding the first two corollaries – "Materia habet de se actum Entitativum" and "Non omnino improbabile est materiam et quantitatem esse realiter idem" –, while the first one is present in Weigel but it could be argued ideally for any author, since it simple states that matter has some form of existence, the second one is closer to Weigel's views. Just before the definition of primary matter as *extensione repletiva mobili* in fact, Weigel identifies two universal principles, ideas and numbers, that correspond in natural objects to the distinction between form and matter[111]. If, as I will argue in the next chapter, the following corollary on the essence of things as numbers can be easily connected with Weigel, then the idea of matter as quantity, shared by Leibniz and Weigel, gains a deeper meaning, because it becomes a way of conceiving essences as numbers and allowing their combinatorial description. Against Descartes, that as I showed ridiculed all these unnecessary differences, there is a precise distinction between quantity in itself, quantity ascribed to matter and magnitude. Weigel argues for a metaphysical priority of quantity, because things to which quantity is ascribed possess it only as an extrinsic property. It does not follow however that quantity is a mere accidental property of things: on the contrary, things are imperfect because they do not exhibit quantity in its essence. This distinction leads to the metaphysical distinctions found in Weigel: if metaphysics is the science of being in itself, arithmetics is the science of beings considered as numbers, while geometry is the science of quantity applied to extension, at rest and in motion[112]. The metaphysical reinterpretation of Hobbes' need of differentiating from Descartes, allowing a layered model in physics, bears then an important consequence in logic, because it allows the relational foundation and description of entities taken as universals (universal characteristics), quantified entities (foundation of arithmetics) and entities defined through quantification in space (*analysis situs*). This will be in Leibniz the final

[111] "*In Ente* duo distinguntur *principia* quasi radicalia, formale unum ex qui resultant *Idee* ; alterum materiale quod comitantur *Numeri* ; sic *Res naturales* duobus absolvuntur principiis radicalibus h.e. constitutivis, quorum unum *Materia* dicitur ; alterum *Forma* vocatur" (Weigel 1658, 193). After the definition of primary matter, forms will be described by Weigel as ways by which we could conceive secondary qualities in natural bodies.

[112] Weigel (1658, 146).

outcome of that initial reception found in his *Disputatio*. Later during his early years, Leibniz will adopt these distinctions on quantity in one of the *Theoria motus abstracti*'s draft: "Extensio est Quantitas sumta cum positione partium (quo differt a Numero, qui est quantitas sine position (seu suppositione existentiae) partium, seu extensio est quantitas relata ad sensum; Numerus est quantitas relata ad intellectum)"[113].

With respect to the *Hypothesis physica nova*, I will add few remarks in order to show that this work could be seen as belonging to the same tradition of Weigel's dissertations. As I argued, Hobbes' *conatus* is adopted in the *Theoria motus abstracti* thanks to Weigel's idea of tendency, but at this point Leibniz is already well aware of the origin of this concept, favouring the reference to Hobbes. In the *Theoria motus concreti* however, many topics and ideas that we have already found in Weigel's school are present In Leibniz's analysis of what he calls "globo nostro Terr-aqu-aéreo[114]": the need of a foundation that begins with homogeneity and derives heterogeneity through motion[115], the demonstration of air's heaviness and the reference to experimentalists[116], the application of Hobbes' physics to a model that explains gravity in a mechanical fashion and a reflection on the direction of bodies subject to gravity[117], the superposition between extension and other properties and the idea of filling space[118] and many others.

Consistent with this background is also Leibniz's 1671 *Summa hypotheseos physicae novae*, where the reference to Weigel's school is very clear with respect to Galileo:

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[113] AA (VI-2, 167).

[114] AA (VI-2, 226).

[115] "Supponatur autem globus terrestris initio fuisse totus homogeneus, atque ita neque tam rarus, ut aer est, naeque tam crassus, ut terra est sed […] naturae ad aquam accedentis" (AA VI-2, 224). But also: "Supponantur initio Globus Solaris, Globus Terrestris, et spatium intermedium, massa, quod ad Hypothesin nostram attinet, quiescente, quam aetherem vocabimus, quantum satis est […] plenum" (AA VI-2, 223).

[116] "Aër nil nisi aqua subtilis est: Aërem enim in eo ab aethere distinguo, quod aër est gravis aether circulatione sua causa gravitatis" (AA VI-2, 224). A reference to Boyle is also present: "Ex doctissimi Boylii at aliorum observationibus supponamus, aërem esse aqua millies leviorem" (AA VI-2, 233).

[117] "Gravitas oritur ex circulatione aetheris circa terram, in terra, per terram" (AA VI-2, 227). "Cum ostensum sit in abstracta motus Theoria, pleraque repercussionum phaenomena non oriri ex liquidis motus notionibus, sed habere longe alias ab oeconomia et motu systematis insensibili causas, quemadmodum gravitas, attractio […] speciatim vero baculus aquam ideo secum commovet, quia ea ei gravitate sua atque intestino motu innititur; quod de aethere dici non potest [...] cum liquida nostra jam tum, etiam remoto baculo, sint in perpetuo motu" (AA VI-2, 224). "Contra naturalem gravitatem sursum levatarum" (AA VI-2, 235). "Cum enim turbet circulationem, expellentur; non sursum […] (superficies sphaericae crescunt in duplicata ratione diametrorum, non in eadem cum diametris ratione; ac proinde sectionum quoque in idem corpus agentium inaequalitas major evenit) ergo deorsum; id est, descendent" (AA VI-2, 228).

[118] "Triplex constructio est: Geometrica, id est imaginaria, sed exacta; Mechanica, id est realis, sed non exacta; et Physica, id est realis et exacta" (AA VI-2, 270). As in Weigel's dissertations tendency is attributed to the last division, later in this passage Leibniz connects Physics to the concept of *Nisus*, term that, as we saw, was used in Weigel's *Pantologia* as a substitute for tendency. As for the idea of filling: "Ut recte docuit cum Cartesio Hobbius, eandem molem plus minusve spatii implere non posse" (AA VI-2, 247)

> Primus de motuum compositionibus digne philosophatus est Galilaeus […] Gravitas est conatus corporis alicuius quantum sentiri potest spontaneus versus centrum terrae, eum autem ab aetheris Turbantia removere conantis pressione oriri […] Si conatus novus esset semper aequalis primo, acceleraretur motus in ea ratione, quae est numerorum quadratorum deinceps ab unitate, ut demonstartum est a Galilaeo (AA VI-2, 333-349).

As I've shown, the ideas that motion is a composition and gravity is a conatus that let the body accelerate following the Galilean rule *impares ab unitate* were debated extensively in Weigel's school, suggesting again Leibniz derivative knowledge in this regard.

In conclusion, Leibniz's *bullae*[119], introduced in the *Hypothesis physica nova*, could be seen as the perfect realization of Weigel's syncretistic efforts. They are not atoms, because they are created through a mechanical process, yet they maintain the structure of atoms. They are born from a homogeneous context, yet they explain gravity through a mechanical account[120]. Recalling then the appreciation of Leibniz's *Hypothesis* found in Weigel's *Pantologia* it does not sound surprising that Leibniz's *bullae* are conceived in Weigel's work as the last form of corporeal bodies, the first one being primary matter[121].

Many of these views will be abandoned by Leibniz, but they were nonetheless fundamental for Leibniz's adoption of Weigel's philosophy of mathematics. The idea of mathematical homogeneity however is combined in Leibniz's foundational efforts with the importance of the principle of contradiction. I will show now how its adoption depended also on Weigel and how some connections could be made with his advancements in physics.

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[119] "Hae jam bullae sunt semina rerum, stamina specierum, receptacula aetheris, corporum basis, consistentiae causa et fundamentum tantae varietatis, quantam in rebus, tanti impetus, quantum in motibus admiramur" (AA VI-2, 223)

[120] As Bussotti (2015, 79) explains, while air is considered as a heavy body, Leibniz still adopts here a concept of *levitas*, applied however to aether. This is not a claim completely against Weigel's tradition, because the writings concerning air's heaviness argues against *levitas* ascribed to this particular element. A further act of syncretism, reintroducing the Aristotelian *levitas* would have been cherished by Weigel as well. The fact that this peculiar quality is ascribed to aether is nonetheless interesting: if we combine this fact with aether's property of permeating bodies and with the idea of homogeneity that is somehow connected to aether, we could argue that this element is in Leibniz's mind very close to his idea of primary matter. Although evidences are scarce in order to prove a specific connection, perhaps aether was conceived as the first actual and physical entity in motion, coming into being just after that ideal situation in which primary matter consists in the simple act of filling space.

[121] Weigel (1673c, 39, 40).

1.4 Modal Logic and Existential Coherence

The State of Contradiction Before Leibniz and Weigel's Contribution to Its Adoption In my analysis of Leibniz's correspondence between 1663 and 1671 I hypothesized Leibniz's estrangement towards Jena's cultural circle after 1671, deriving it from the fact that contacts with Weigel's acquaintances begin to soften. It is true that some of them, like Conring, will remain, but we do not find many direct references to Weigel. This outcome is not surprising and it does not refute the standard interpretation that nowadays ascribes to the Parisian stay a turning point in the development of Leibniz's philosophy, mostly based on his mathematical studies. Aside from mathematics however, Leibniz's development is also evident in almost every other topic, the *Confessio philosophi* being a relevant example on the ground of modal logic and the theological problem of predestination. In this work, we find many theories that recall Leibniz's mature writings, from the idea of harmony, the problem of evil and reality conceived as *series rerum* to the analysis of hypothetical syllogisms and the problem of individuation. Many of these ideas are not completely equivalent to their mature counterparts, e.g. the identity of indiscernibles still allows the difference *solo numero* or the series of things are not expressed through possible worlds, but the similarities are undoubtedly more than the ones found in Leibniz's earlier writings before Paris.

While identifying then the Parisian stay with a moment of discontinuity in the development of Leibniz's thought is important and reasonable, I believe that understanding what kind of concepts were taken from Leibniz's reflection before Paris and in which way they survived and adapted to the new background is important as well. There is no denying in fact that some fundamental ideas were adopted by Leibniz just before Paris. In this regard, his *Confessio philosophi* becomes extremely relevant, because, being a work written at the beginning of Leibniz's stay between 1672 and 1673, it could be seen as the ideal bridge between the two periods. More specifically, my aim would be that of following the adoption of the three essential ideas that will constitute at a later time Leibniz's foundational approach to mathematics: the principle of contradiction, the principle by which the whole is always greater than its parts and the concept of homogeneity founded in the distinction and combination between quality and quantity.

As for the last idea, I already highlighted the importance of Weigel's cultural influence on the ground of physics. The distinction between motion and rest in the description of primary matter could be seen as an important contribution in the adoption of homogeneity: if matter is described through quantity and motion, it follows that the world conceived at rest possesses different properties from those derived by these two concepts. Since the properties of objects considered at rest are connected to geometrical entities that are averse from a description based on quantity alone, they could be seen as possessing intrinsic qualities that do not follow from quantification or motion. On the other hand, quantification could be identified with matter, but its concept is separated from that of matter and constitutes a superior metaphysical level that does not derive from physics alone. In the next part of the present work I will show that these distinctions have a precise correspondence, both in Weigel and in Leibniz, with respect to mathematics. However, their adoption is also relevant for the present part, because it represents the perfect example of concepts endorsed before Paris but used in a different context and, specifically in this case, throughout Leibniz's life. Identifying these references is not simple, because the strict distinction between rest and motion is clearly against Leibniz's further development of his law of continuity and his idea that rest could be interpreted as a specific case of motion, yet we had to ignore the final outcome in the specific topic in order to cast some light on Leibniz's early years and identify a relevant consequence for his mature theory on a different topic.

Things are much easier regarding the principle by which the whole is greater than its parts: Leibniz clearly adopts it from Hobbes, even if I tried to point out that Hobbes' influence should be inserted in a context, such that of Weigel's school, where his ideas had a decisive metaphysical development that involved principles taken from other philosophers. Leibniz's openly states many times[122] that Hobbes recognized the importance of the principle of the whole, plus this is the first principle, among these three, that is expressly connected from the very beginning of Leibniz's reflections with foundational attempts in mathematics. While in the *Theoria motus abstracti* we find that

[122] "Totum esse maius parte, primus demonstravit Hobbius, fundamentum Scientiae de Quantitate. Nihil esse sine ratione, ego quod sciam primus demonstravi fundamentum scientiarum de mente et motu" (AA VI-2, 48).

the whole of geometry could be derived from this principle[123], the *Confessio philosophi* expresses the same idea and it extends it to arithmetics: "enim *totum esse majus parte*, Arithmeticae et Geometriae, scientiarum de quantitate, principium est"[124]. Physics and morality instead are founded on the principle of sufficient reason, one of the few principles that Leibniz recognises as his genuine discovery.

About the novelty of Leibniz's position, I believe that the unique development of his philosophy of mathematics does not depend on the original adoption of these principles, but in how he combined them in order to obtain a completely different theory. Upon further inspection, the way in which these principles are connected by Leibniz will reveal their hierarchy, suggesting a priority of the principle of contradiction. This principle in fact is the only one needed in order to prove *a priori* the principle by which the whole is always greater than its parts. Homogeneity comes at a later time, as a tool developed to build from few entities every mathematical object. It is in the development of this structured theory that Leibniz emerges as a true and original thinker and probably one of the first men that offered a feasible account of a logical foundation of arithmetics, not only on paper with bold claims, but developing a concrete method never seen before.

Given these premises, the adoption of the principle of contradiction becomes a crucial topic, but understanding the dynamics behind this process is not as simple as for Hobbes' principle of the whole. A simplified approach on this matter would easily dismiss the problem by pointing out that, starting from Paris, Leibniz revaluates the principle of contradiction, simply because he discovers his usefulness in explaining some problems pertaining contingency and the existence of evil in a world created by God. He in fact introduces it in the definition of modal concepts, such as possibility, contingency, necessity and impossibility: "*Necessarium* ergo illud vocabo, cuius oppositum implicat contradictionem, seu intelligi clare non potest"[125]. From this definition Leibniz derives contingency as something that is not necessary, possibility as something non-contradictory and impossibility as something that is not possible. The law of contradiction then is used as a rule in order to evaluate any possible modal statement. Even more, it

[123] "Non facile alioquin in tota Geometria aut phoronomica occurrit: cum ergo caetera omnia pendeant ex principio illo, totum esse majus parte, quaeque alia sola additione et subtractione absolvenda Euclides praefixit Elementis […]" (AA VI-2, 268).
[124] AA (VI-3, 118).
[125] AA (VI-3, 126).

establishes a form of homogeneity between things that are possible in themselves, i.e. that follow the principle of contradiction, and the possibility for us to conceive them.

This definition of modal operators then leads to a consequent revaluation of Scholastics, because it is universally acknowledge that the use of the principle of contradiction in order to define modal concepts in this way derives mainly from Suárez's interpretation of Thomas Aquinas[126]: the definition of possibility as something non contradictory, or necessity as something for which its opposite leads to contradiction in fact was very common and universally accepted at the time of Leibniz's *Confessio*.

This theory however cannot be ascribed directly to Aristotle, even if Scholastics tried to do so in order to benefit his authority, because he more often conceived modality using a statistical account, i.e. believing that necessity is ascribable to any state of affairs that is always true, impossibility to any state of affairs that is always false and possibility to any state of affairs that is at times true and false[127]. It follows that Aristotle's approach is against any possible connection between the idea of contradiction used in modal logic and the same idea used in order to define the universal principles in the foundation of mathematics: in a statistical account there is no distinction between something that happens to be always true and something that is always true because from its intrinsic structure follows that the opposite would be contradictory. Even if it is always true that the sun always sets it seems to us that this necessity possesses a degree of certainty that is different from that of other statements on purely logical or mathematical entities. Such account then does not allow a clear distinction derived from the idea of contingency and this outcome reduces the importance of something like mathematical entities that could be conceived as necessary, even if the whole world would disappear, placing them instead on the same ground of any entity of that world.

The fact that before Paris Weigel is always connected with Leibniz's revaluation of Aristotle in the light of the moderns and with a form of controversy against Scholastics,

[126] In Thomas Aquinas' *Summa* for example, he uses them in order to define God's omnipotence: "Relinquitur igitur quod Deus dicatur omnipotens, quia potest omnia possibilia absolute, quod est alter modus dicendi possibile. Dicitur autem aliquid possibile vel impossibile absolute, ex habitudine terminorum, possibile quidem, quia praedicatum non repugnat subiecto, ut Socratem sedere; impossibile vero absolute, quia praedicatum repugnat subiecto, ut homine esse asinum [...] Hoc igitur repugnat rationi possibilis absoluti, quod subditur divinae omnipotentiae, quod implicat in se esse et non esse simul [...] quaecumque igitur contradictionem non implicant, sub illis possibilibus continentur, respectu quorum dicitur Deus omnipotens" (Thomas 1989, 4, 327).

[127] On this topic in Aristotle see Crivelli (2004, 60). For the evolution of the problem of future contingents and divine foreknowledge see Craig (1988).

plus the fact that in Paris we witness the definitive revaluation of the Scholastics' concept of modality, would lead to the conclusion then that in Paris one of the most important signs of Leibniz's discontinuity with his past concerns his adoption of modality. However, as I will show, Leibniz's adoption of the principle of contradiction in the definition of possibility happens before Paris and it is connected with a harsh critic of the Scolastics' terminology in this specific matter. From this fact an apparent contradiction emerges: if we admit the Scholastics' influence on Leibniz on this topic before Paris we have to deal with an image of Leibniz highly influenced by Hobbes and Gassendi, often arguing against Scholastics, which is not compatible with a naïve adoption of Scholastic principles, while resorting to a later influence during the Parisian period would completely ignore the fact that the principle of contradiction was used before that time. Ignoring the Scholastics' influence is not possible as well, because the way in which Suárez deals with the problem of necessity and the way in which he connects it to universal principles in mathematics is too similar to that of Leibniz to be easily dismissed.

Another possible solution would be that of arguing for an autonomous revaluation of the principle before Paris, maintaining a kind of distance from the Scholastics' background and at the same time Leibniz's awareness on the origin of the principle from that background. This is clearly the most reasonable hypothesis, but it does not follow very well from the premises of Leibniz's early years, thus it needs to be thoughtfully analysed. The reason of this inconsistency is that for example Gassendi and Hobbes have not only a definition of necessity that is not founded on the principle of contradiction but also a distinct aversion to that definition used in Schools. In his *Syntagma Philosophicum*, Gassendi is very clear on the definition of necessity: "Necessitatem nihil esse aliud […] quam lationem, percussionem, repercussionem materia, hoc est, atomorum, quae materia sunt rerum"[128]. He is distinctly interpreting necessity as a consequence of truths derived from physics, dismissing an approach that defines at first the absolute concept of necessity in logical terms and then eventually applies it to the physical world. For these reasons, Gassendi questions the Scholastics' concept of modality derived from their peculiar interpretation of Aristotle's *De Interpretatione*: if necessity or possibility are modes applied to propositions, then there is no sufficient reason in order to restrict the number of modes to those traditionally ascribed to modal logic, that are necessity, impossibility,

[128] Gassendi (1964, 834).

possibility and contingency[129]. If any possible adverb or even word could be a mode, then a definition of necessity based on this distinction, while apparently placed at a higher metaphysical level than that of Gassendi, is actually not as effective as resorting to atoms and collisions.

Hobbes draws the same conclusions from different premises. In his *De corpore*, he exhibits a statistical account of modality, close to that of Aristotle, interpreted without Scholastics' influences. For examples Hobbes defines impossibility as follows: "Porro quod neque est neque fuit neque erit, neque esse potest, nomen tamen habebit, hoc ipsum scilicet *quod neque est, neque fuit*, etc. vel brevius hoc *impossibile*"[130]. Later in this work, the statistical account is complemented by a theory that connects modality with the possibility of predication:

> [Propositio] *Necessaria* est, quando nulla res concipi potest sive fingi ullo tempore, cujus nomen sit *subjectum*, quin ejusdem nomen sit etiam *praedicatum*. Ut *Homo est animal* necessaria propositio est, quia quocunque tempore supponimus rei alicui convenire nomen *homo*, eidem rei conveniet quoque nomen *animal*. *Contingens* vero est, quae modo vera, modo falsa esse potest ut *Omnis corvus est niger*; hodie quidem contingere potest, ut sit vera, alio tempore, ut sit falsa. Rursus in omni propositione necessaria *praedicatum* vel aequivalet subjecto ut in hac *Homo est animal rationale*, vel pars aequivalentis est ut in hac *Homo est animal* (Hobbes 1999, 37).

This is by far one of the most relevant differences between Hobbes and Leibniz in logic: while the connection between subject and predicate hints at a possible resemblance, the statistical account openly negates it. Hobbes' reference to a temporal dimension that

[129] "Jam in libris DE INTERPRETATIONE, insignis est prae caeteris insufficientia illius partitionis *Modorum*, ex quibus propositiones *Modales* appellatae sunt. Illos nempe, ex vulgari etiam Interpretatione, limitat ad quatuor, *Necessarium, Impossibile, Possibile, Contingens*. Quaeso vero qua ratione assignare plures non liceat? Si *modus* est, qui modificat propositionem, hoc est, indicat quomodo praedicatum insit subjecto: an-non omnia adjectiva pari jure possunt esse modi? Certe ut illa propositio dicitur Modalis, *Necessarium* est hominem esse animal, itemque illa, *Contingens* est Socratem sedere; ita profecto et istae Modales dicendae sunt, *Honestum* est hominem esse virtutis studiosum; *Utile* est hominem esse ad laborem impigrum; *Justum* est filium esse Patri obsequentem: *Pulchrum* est pro patria mori; *Bonum* est nos hic esse, &c. Quae potior enim ratio est, dum serio adverteris? Praetereo non illi solum sumptis adverbialiter, sed adverbiis etiam caeteris id muneris convenire, ut *modum significationis addant*, ac proinde propositiones Modales efficiant" (Gassendi 1964, 117).
[130] Hobbes (1999, 22).

shows how a man would always be an animal is decisive, because it reveals that he was not conceiving a clear distinction between necessary entities and entities that are true for any given time. Putting it in Weigel's terms, it is as if even in logic Hobbes allowed that peculiar necessity ascribed to physics. In that context, *con-necessitas*, that is the idea shared by Weigel and Hobbes that for some entities if one entity exists then it automatically brings together another entity, was the strongest form of necessity. Fire and heat in fact are connected in a way that resembles the connection between a proposition and its truth – a necessary proposition is a proposition connected with truth for any given time –, but this is not a complete account on necessity, because absolute necessity is not explainable from these premises. This does not mean however that Hobbes was not a necessitist, because his determinism was way stronger than that of the Scholastics. It simply means that the deterministic relationship between cause and effect had a stronger role than the ideal and logical principles in arguing for the necessity of things.

This incompleteness in the explanation of necessity could be seen as one of the few cases where Scholastics possessed a more advanced theory on the ground of logic, but it was possible only by endorsing the principle of contradiction as a truly universal principle applicable also to real objects. According to Hobbes this is impossible, because of his *plusquam nominalis* approach: necessity makes sense only in a context made of words and propositions, but predicating it about real entities is impossible[131]. The result is a predictable reinterpretation of those principle that were founding Scholastics' modality:

> Nomen autem *positivum* et *negativum* contradictoria inter se sunt, ita ut ejusdem rei nomina ambo esse non possint. Praeterea contradictoriorum nominum alterum quidem cujuslibet rei nomen est. Quicquid enim est, vel homo est vel non-homo, album vel non album, et sic de caeteris. Quod quidem manifestius est, quam ut probari aut explicari amplius debeat. Nam qui hoc sic enuntiant *Idem non potest esse et non esse*, obscure: qui vero sic *Quicquid est, vel est vel non est,* etiam absurde et ridicule loquuntur. Hujus axiomatis certitudo, nimirum *Duorum nominum contradictoriorum alterum cujuslibet rei nomen esse, alterum non esse*, principium est et fundamentum omnis

[131] "Necessariae itaque propositiones illae sunt quae sempiternae veritatis sunt. Hinc quoque manifestum est veritatem non rebus sed orationibus adhaerere, veritates enim aliquae aeternae sunt; semper verum erit *Si homo, tum animal*; ut autem homo, aut animal in aeternum existat necesse non est" (Hobbes 1999, 37).

> ratiocinationis, id est, omnis Philosophiae; itaque accurate enuntiari debuit, ut omnibus per se clara et perspicua esset, sicut revera est, nisi iis qui longos de hac re sermones apud Metaphysicos legentes, ubi nihil vulgare dici putant, id quod intelligunt, intelligere se nesciunt (Hobbes 1999, 23).

Hobbes here quotes almost directly Suárez's third *Disputatio*[132] and establishes a precise distinction between his principle and that found in Schools, even if they share a foundational meaning. We could go as far as saying that Hobbes' principle resembles that of contradiction, because a choice must be made between a name and its negation, but there is nonetheless an important logical difference between saying "if something is a name of one entity, then its contradictory is not" and saying "something and its contradictory cannot exist at the same time and in the same regard". The former principle, the one endorsed by Hobbes, does not take the problem of avoiding coexistence between contradictory properties as a basic premise for every possible reasoning. It is a consequence of something given, so to say, *ex hypothesi*, rather than a principle of intrinsic coherence. For example, from the fact that one thing could always be named "man" it follows that it cannot be named "non-man", but this happens given that one thing was named in such a way: since I could conceive otherwise, it means that Hobbes' solution is similar to the statistical account found in modality. It shares with the statistical modality the problem of defining pure possibility, i.e. something that is possible even if it is always false in the actual world, or in this case something that is a name even it is not a name for any given thing. In Hobbes' definition of impossibility recalled before in fact, he is forced to add "nomen tamen habebit" just after the statistical definition, otherwise we would not be allowed to talk about impossible things in the first place. However, sharing Aristotle's approach, Hobbes shares also its problems: if "impossible" is an artificial name given to something that never happened and won't ever happen, then that name is a name given to a mere process happening in our minds. It follows that the only difference between impossible things is not in those things but in the different way in which we decided to name the same faulty reasoning. This account then does not explain the intuitive idea that a contingent impossibility is impossible in a different way than pure contradiction.

[132] Suárez (1965, Disp. III, §5).

Besides, since Hobbes' principle follows from the existence of the name already given independently, it does not provide a form of coherence for the existence of that name. On the other hand, the Scholastics' principle is different because it suggests an ontological consequence that derives directly from the coming into being of any possible thing: it does not need a choice between "man" and "non-man" with respect to one thing, but it states that whenever "man", "non-man" and even "thing" come into being, that automatically shows that they are coherent entities, because they follow the principle of contradiction. We could say that the beginning of Hobbes' principle – *Duorum nominum contradictoriorum* – already implies in Scholastics terms the principle of contradiction, otherwise it could have not been even expressed.

The distinction between these principles is extremely relevant for Leibniz's foundation of mathematics, because it shows that, even if Leibniz himself ascribes to Hobbes the idea of founding mathematics on the principle by which the whole is greater than its parts, that principle needed to be perfected by introducing the principle of contradiction as its general premise. This process of reconciling the two principles will last for Leibniz's entire life and it is the main reason why Leibniz feels the need of developing foundational tools in mathematics.

Given the significant difference between Gassendi and Hobbes' account on necessity and that of Scholastics then, from a genetic point of view it is hard to argue that Leibniz, while following the new philosophers almost completely on the ground of physics, was totally ignoring them on the ground of modal logic. In Leibniz's *Von der Allmacht und Allwissenheit Gottes und der Freiheit des Menschen*, written between 1670 and 1671, this tension and the impossibility for a naïve revaluation of Scholastics are evident. Leibniz starts by criticizing "der Schul-Lehrer Ausflüchte" on the matter of predestination. They introduced many different definitions and names in order to account for predetermination – *Necessitas* is among them – but according to Leibniz, without resorting to simple and accessible terms those names caused only confusion[133]. Later on, the criticisms against Scholastics continue: "wenn man die Schul-Lehrer *de radice possibilitatis* wie sie es nennen […] fragen wird, wird man so wunderliche und so verwirrte dinge hören, daß man Gott danken wird wenn sie aufhören"[134]. Interestingly enough, Leibniz then ascribes both

[133] AA (VI-1, 538).
[134] AA (VI-1, 540).

the statistical account of modality[135] and the one based on the principle of contradiction to Scholastics and men in general. Here, Leibniz gives the first definition of possibility: “Ist also möglich was sich deutlich ohne Verwirrung und Wiedersprechen gegen sich selbst erklären lasset”[136]. This definition then, was given by Leibniz in a highly polemical context, yet he immediately uses it in order to prove that the proposition: "That which God foresees will happen, that is: it is not possible that is not going to happen" contains an implicit distinction between something that is only conceivable as happening and something that is necessarily happening. The use of contradiction in the definition of possibility leads to the identification of the famous *sophisma pigrum*.

I will now try to explain this paradoxical tension between Leibniz's criticisms against Scholastics and his use of their principle by conjecturing that from a theoretical point of view the idea was taken from Schools, but from a genetic and biographical point of view it was perhaps adopted from Weigel's cultural background.

As for the theoretical part, I will follow Suárez, since he is widely known by Leibniz and his writings exhibit some of the most interesting insights on this topic. The first disputation immediately introduces the topic of universal principles, especially the principle of contradiction: "haec scientia [metaphysics] est perfectissima sapientia naturalis; ergo considerat de rebus et causis primis et universalissimis, et de primis principiis generalissimis, quae Deum ipsum comprehendunt, ut: *Quodlibet est vel non est*"[137]. Very interesting in this regard is the fact that Suárez quotes Proclus' comment on Euclid's *Elements* on the relationship between metaphysics and mathematics[138]. It shows that the interest towards a metaphysical approach to the *Elements* was shared by Scholastics and Weigel. The difference is that in Weigel's philosophy this topic is so important that it affects the very core of his metaphysics, while in Suárez the relationship between mathematics and metaphysics, although undoubtedly present, is somehow weakened by the fact that the principle of contradiction is indeed one of the first and most

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[135] Ibid. In this passage, Leibniz establishes a difference between what men do and what they say and think. The statistical account of modality is related to what they do, because instead of being connected with pure possibility, it needs that something actually happened, in order to judge its modal significance from its truth-values.
[136] AA (VI-1, 540).
[137] Suárez (1965, Disp. I, §19, 76).
[138] "Secundum munus praecipuum quod huic scientiae tribuitur, est prima principia confirmare ac defendere [...] et Proclus, li. 1 Comment. In Euclid., cap. 4, ubi etiam mathematicis scientiis ait metaphysicam suppeditare principia" (Suárez, Disp. I, 4, §15, 160).

important principles, but it is also unquestionable, to the point that any possible analysis of it had to be abandoned, taking it for granted. This is evident for example in Suárez's claims that the principle of contradiction cannot be analysed through final or efficient causes, yet it is in this very moment that Suárez connects it with the principle by which the whole is greater than its parts:

> Nam imprimis, quod attinet ad causam efficientem, haec non habet locum in universalissimis principiis constantibus ex terminis communibus Deo et creaturis; nam sicut respectu Dei nulla potest dari causa efficiens, ita nec respectu illorum principiorum, quae in Deo ipso veritatem habent, ut est illud: *Quodlibet est, vel non est*, et, *Impossibile est aliquid de eodem affirmare et negare*. Atque hinc etiam constat, haec principia non posse per causam finalem demonstrari, quia causa finalis est in ordine ad affectionem et operationem, et ideo quae abstrahunt ab efficiente, abstrahunt etiam a fine. [...] ut, verbi gratia, *omne totum esse majus sua parte*, verum est, omni efficientia seclusa, et sic caeteris (Suárez, Disp. I, 4, §21).

From a foundational point of view Suárez is correct in depriving these principles of a connection with an efficient cause, but it seems as if this outcome rules out any further analysis. In any case, I believe that this is a very relevant quote, because it suggests a possible connection between those two principles that Leibniz tried to reunite at a later time. Yet, we do not witness in Suárez the same attention on the reduction of every possible analytical statement to contradiction. On the contrary, while Leibniz will not be concerned identifying the principle of contradiction with identity, Suárez is careful in doing so, proposing instead always a priority of a positive expression over the same expression formulated in negative terms[139]. This outcome affects the principle of contradiction as well, so that unlike Leibniz, Suárez cannot easily identify identity and contradiction. A possible connection then between the principle of contradiction and the

[139] "Omnis autem negatio in priori aliqua affirmatione fundatur; ergo datur aliud principium prius illo, in quo fundetur: quale erit, vel hoc, *Necesse est idem esse, vel non esse*; vel hoc, *Unum contradictorium necessario aliud destruit*" (Suárez, Disp. III, 3, §1). This is the same passage quoted by Hobbes in his *De corpore*. The last definition resembles closely the principle adopted by Hobbes, but it is inserted in a more complex and, I would say, refined context. Suárez was undoubtedly more concerned than Hobbes in the subtle differences among these principles, while Hobbes saw them as unnecessary and confusing.

principle by which the whole is greater than its parts is indeed present, but Leibniz unifying approach from an analytical point of view was unparalleled, causing Leibniz's focus on the relationship between logic and mathematics. Echoes of Leibniz's dissatisfaction towards a foundational approach that does not question the priority of one principles over another are found in Leibniz's *Demonstratio propositionum primarum*, written between 1671 and 1672[140].

These Scholastics ideas then, even if in need of some refinement, represented an important premise to Leibniz's philosophy of logic, but how they were adopted by him while openly despising Schools still needs to be explained. If we accept the idea that Leibniz was thinking about adopting the principle of contradiction in modal logic at the time of his *Von der Allmacht*, around 1670 and 1671, then we could safely say that this adoption happens at the peak of Weigel's influence, as shown before. It is true that many of the ideas that Leibniz could have possibly taken from Weigel and his school are indeed very derivative, at times taken from Hobbes, Descartes, Aristotle and more. It should be clear by now however, that the very fact that those ideas are present in Weigel is relevant with regards to Leibniz's early years. If Leibniz, as proved by his 1669 letter to Thomasius, was sharing with Weigel the project of reconciling Aristotle with the new philosophers, then any idea found in Weigel was legitimated in taking part to the syncretistic project simply by its presence in Weigel's works: it is relatively important that these ideas are objectively suitable for a reconciliation, if they were chosen by Weigel for a specific purpose. Any syncretistic project implies a degree of discrepancy with the original ideas endorsed, as I already pointed out with respect to physics. For example, Aristotle's idea of motion through contraries was reinterpreted in a way that was compatible with the definitions of motion taken from Hobbes and Descartes, even if their similarity is highly implausible. In the same fashion, to Hobbes' *conatus* was given the name tendency, probably taken from Descartes, and it was introduced in a context where thinking about God's *potentia absoluta* was reasonable.

[140] "Rationis sunt Propositionis illae ex solis ideis, vel quod idem est definitionibus conjunctis orientes, sensui originem non debentes, ac proinde hypotheticae, necessariae. Aeternae, ut Geometricae, Arithmeticae, Phoronomicae abstractae omnes: ita totum esse maius parte, Nihil esse sine ratione, Circulos esse ut quadrata diametrorum, Numeros impares esse differentias quadratorum, quae omni talia sunt, ut ex sola accurata distinctaque expositione, id est definitionibus pendeant. Idem Aristoteles vidit, idem Lullius, ambo Viri magni. At sunt qui putant, esse quaedam Axiomata per se nota, haec esse in demonstrando definitionibus adjungenda" (AA VI-2, 479-480).

The principle of contradiction could have followed the same fate: reintroduced by Weigel as a genuine product of Aristotle and Euclid's philosophy, it could have gained in this way its freedom from the Scholastics' background. This interpretation would explain both Leibniz's aversion for Schools and his use of their principle, but for now it is a mere conjecture that needs to be proved. The principle of contradiction is indeed used by Weigel in his *Analysis aristotelica* in the definition of absolute necessity:

> Necessarium autem quod h.l. dicitur est vel *absolute* tale, cujus oppositum involvit contradictionem; vel *restricte*, cujus oppositum non involvit contradictionem : Illud praeterquam quod verum, certum & necessarium dicatur, aeternitatis primario sortitur denominationem, & *Ens aeternum*, velut quoddam Divini Numinis consectarium, quod divinam veritatem, certitudinem & necessitatem ab aeterno consequatur, nec ullo modo mutari queat, sicut Deus est immutabilis, dici solet. Et talem veritatem, certitudinem & necessitatem habent propositiones demonstrativae, quae in Metaphysicis & Logicis […] itemque in Arithemticis & Geometria, disciplinis sane divinis, occurrunt (Weigel 1658, 20).

The first thing that I would like to remark is that this passage is found in the chapter entitled *De Necessitate*, just before the definition of that restricted necessity that I defined in the previous chapter as a physical necessity. It is relevant per se, because the definition of physical necessity involved all sorts of concepts derived from the interpretation of Hobbes' physics that Leibniz adopted, meaning that Weigel's syncretistic efforts were aiming for a reconciliation between the absolute and logical necessity found in Schools and that of the modern philosophy of nature. This account was not found in any of the positions previously analysed, if not in Leibniz himself. Furthermore, the foundation of necessity on the idea of contradiction is connected in Weigel with metaphysics, logic, arithmetics and geometry, that are the very same fields in which Leibniz adopts it. It is closer to Leibniz's approach than that of Hobbes, because there the foundation was based on the principle of the whole but the idea of contradiction was replaced with a different principle, and it is closer than that of Suárez, because there the reductionist approach was not adopted, given that we still need to prove that the principle of the whole is used also by Weigel.

Another interesting feature of Weigel's definition of absolute necessity is the way in which necessary truths are connected with God. There is a logical dependence that preludes to one of the most famous traits of Leibniz's philosophy: on one hand, logical dependence makes these truths something that even God is forced to respect and, on the other hand, it shows that God and men, in this regard, think alike. Being God an infinite mind, restricted necessity becomes fundamentally different for men since they cannot master the infinite chain of causes that lead to an event, but in absolute necessity there is room for arguing a kind of homogeneity between God and men. It is not a coincidence in fact that conceivability is based on possibility, interpreted as the lack of contradiction: God and men conceive possible entities in the exact same fashion.

On a general basis then, a connection between Weigel's and Leibniz's account is possible, so we could wonder if Leibniz's writings around 1671 suggest this interpretation. The first step in this direction however is reconsidering once again Leibniz's 1663 *Disputatio*. I already outlined that even the third and fourth corollaries – "Essentiae rerum sunt sicut numeri" and "Essentiae rerum non sunt aeternae nisi ut sunt in DEO" – could have been adopted by Leibniz following Weigel. Some could argue that conceiving the essence of things as numbers is something that does not need any kind of influence in order to be imagined by a brilliant mind such that of Leibniz, because it is a tempting general premise for a philosopher that firmly believes in the effectiveness of his combinatorial approach. However, given the background in which Leibniz was integrated that I sketched in the previous chapter, it is worth noting that this idea was extremely important for Weigel and his school. Similar arguments or even the exact same words of Leibniz's *Dissertatio* are found in several works that I already analysed. In Weigel's *Analysis aristotelica* to begin with, Leibniz could have found the basis for his corollary: "Non immerito *Scientia Numerorum* h.e. Entium quatenus Numeri sunt, definiri solent"[141]. However, the use of the term "Ens" could be misleading for the reader not accustomed with Weigel's philosophy. Weigel in fact uses the word "Ens" in order to talk about both material entities and formal entities: "Ex formali vero (quod est Essentia, seu

[141] Weigel (1658, 184).

id quo Ens est) resultant formae rerum […] quas divinus Plato vocavit *Ideas*"[142]. Entities, Ideas, Essences and Numbers are for Weigel the same thing, although considered under different aspects: "Entities" is the general name used for both the material and the formal aspect, "Ideas" and "Essences" constitute the formal part, while "Numbers" constitute the material part. The combination of ideas in God could be described as the combination of numbers, following a decisive Platonic influence. Recalling the distinction between matter and form connected with the distinction between numbers and ideas of the previous chapter, we could see that something that resembles Weigel's theory is found in four of the seven corollaries of Leibniz's *Disputatio*.

Even if this interpretation would be quite remarkable per se, ascribing Leibniz's corollaries to Weigel's influence is not necessarily relevant with regards to the adoption of contradiction in modal logic. Rather, it should be noted that Leibniz's contact with Weigel's school surely intensified the importance of the relationship between essences and numbers in the following years: "Rerum Essentias sicut Numeros esse, quod Scientissimi Veterum olim monuerunt, hinc clare perspicere licet"[143] is a passage found in Weigel's *Pantologia* available to Leibniz around 1670 and "Essentiae vero rerum sint sicut numeri"[144] is found in Vinhold's 1671 *Theses*. In the same year, Leibniz's letter to Magnus Wedderkopf connects this idea with the concept of pure possibility and its intrinsic relation with God:

> Quae ergo intellectus divini? harmonia rerum. Quae harmoniae rerum? nihil. Per exemplum quod ea ratio est 2 ad 4 quae 4 ad 8, ejus reddi ratio nulla potest, ne ex voluntate quidem divina. Pendet hoc ex ipsa Essentia seu Idea rerum. Essentiae enim rerum sunt sicut numeri, continentque ipsam Entium possibilitatem quam Deus non facit, sed existentiam: cum potius illae ipsae possibilitates seu Ideae rerum coincidant cum ipso Deo. Cum autem Deus sit mens perfectissima, impossibile est ipsum non affici harmonia perfectissima, atque ita ab ipsa rerum idealitate ad optimum necessitari (AA II-1, 186).

[142] Weigel (1658, 182).
[143] Weigel (1673c, 147).
[144] Weigel-Vinhold (1671, 46).

Since at that time, as I previously pointed out, Wedderkopf was studying under Weigel, even if we accept that the idea found in Leibniz's *Dissertatio* was developed independently, it is highly improbable that in this letter Leibniz is not referring to Weigel's theory. Other than the independent nature of possibilities from God's will and the mutual dependence between God and ideas, Leibniz's reference to the *ratio* between numbers suggests once again that he was reinterpreting Weigel's philosophy: as it will be shown in the next part of the present work in fact, the ideas of ratio and proportion constitute Weigel's most important heritage found in Leibniz's foundation of mathematics.

While Leibniz original contribution was that of applying these principles to the problem of predestination, it is regarding the use of universal principles and their relationship with mathematics instead that Weigel becomes extremely important for the development of his philosophy. In the *Analysis aristotelica* the principle of contradiction and the principle of the whole are connected in a way that was much closer to that of Leibniz than the accounts found in Hobbes or in Suárez. The following passage of Weigel's *Analysis* is worth quoting in its entirety, because it clearly shows that the core idea of Leibniz's philosophy of mathematics was developed in the early years thanks to the decisive influence of Weigel:

> *Rationem* […] hoc loco dicimus facultatem hominis, residuam imaginis divinae scintillulam, aut si secundum exercitium spectanda sit, actum homini cogenitum, quo veritates primas, tanquam ultima scientiaru principia demonstrativa etiam absque singularium perceptione ex nobis metipsis intelligimus & ex iis alia deducimus. Ita si in Metaphysicis, *Impossibile esse idem simul esse & non esse* : *Quodlibet esse vel non esse* & quae ex Metaphysicis una cum praecipuarum rerum demonstrationibus ad Mathesin transfiere : *Totum esse sua parte majus. Si aequalibus addas aequalia tota esse aequalia*, & quamplurima alia […] *Axiomata* vocamus. Sunt igitur *Axiomata* stricte loquendo nihil aliud quam veritates primae […] quae Deus ter Opt. Max. in natura quasi fundamenti loco primo statuit & a parte rei (non imaginatione hominum) aeterna esse iussit […] Et has ipsas veritates intellectus noster singulari Divini Numinis indultu non tantum ex semetipso

perfectissime semper novit, sed & primo prout in se sunt directissime cognoscit (Weigel 1658, 107-108).

For the first time the principle of contradiction and the principle by which the whole is greater than its parts are not only named under the list of universal principles, but also explicitly connected, establishing a correspondence between metaphysics and mathematics. It is the closest version of the same argument found in Leibniz.

We can also clearly see that the inspiration for the homogeneity between God and men was given by Leibniz in a context highly influenced by Platonism. Weigel's passage above quoted in fact significantly ends with a reference to Plato's reminiscence[145]. Weigel also refers to the relationship between the disciple and his teacher and to the fact that the young student "primis & nobiscum natis principiis, adeoque puero notissimis, facili via continuo progrediuntur"[146] would achieve knowledge, a clear allusion to Plato's *Meno*. The idea of possessing universal principles from the moment we were born is found also in Weigel's 1679 *De supputatione* and it will highly influence Leibniz in the following years.

Weigel's approach then could appear confusing because of his need of unifying different theories, but I believe that underestimating his importance in the adoption of universal principles would lead us to an interpretation that relies on a preconceived idea of the history of philosophy, where Leibniz follows directly from Hobbes and Scholastics without taking into account the environment in which he lived. The process of identifying the direct influences on specific accounts is something that Leibniz did in his early years in order to appeal to a wider audience without referring to unknown authors, but it is not something that we can do while analysing those years, otherwise some connections and distinctions among universal principles would be irremediably lost.

Existential Coherence Between Physics and Law As a final remark for this first part, I would like to suggest a possible connection between the principle of contradiction and

[145] "Unde fieri contingit ut intellectus, quicquid nudo rationis usu ex iis principiis deductum jam distincte perpendit, recordando saltem, seu per nudam *Reminiscentiam,* admonitione facta se cognoscere sibi plane habeat persuasum. Quod ut divino prorsus ingenio perspicacissimus Plato dudum agnovit ita demonstrationes Disciplinarum rationalium, puta metaphysicas, arithmeticas, geometricas, & ex parte quoque ethicas, ex illis primis veritatibus Euclide methodo conscriptas […] penitius aliquanto inspiciamus, sane non adeo absurdam hanc esse de *Reminiscentia* sententiam re ipsa comperiemus" (Weigel 1658, 108).
[146] Weigel (1658, 104).

Leibniz's philosophy of nature. If the reference to Weigel's influence helped in explaining how Leibniz's aversion for Scholastics does not exclude the adoption of their modal concepts, a conciliation on the ground of physics is still missing. It is clear that Suárez, Weigel and Leibniz's idea of the principle of contradiction is slightly different from its expression nowadays, because at times it combines the law of the excluded middle, the law of contradiction and the principle of bivalence, but it is from this original union that one of the most important consequence emerges: the principle of contradiction has also an ontological relevance. Given that every possible entity is a non-contradictory entity, it follows that any existent thing, being also possible, is also non-contradictory. It is highly improbable that this consequence was not affecting Leibniz's philosophy of nature, but how could it be reconciled with Leibniz's anti-scholastic views on physics? A possible solution is given by conjecturing that the ontological use of the principle of contradiction found in the *Confessio* was suggested to Leibniz by his reflection on the principle that negates the dimensions' penetration. A complete analysis of this problem involves the reference to many authors and many passages of Leibniz's writings that would take us far from the purpose of this work, specifically related to Weigel. However, I will offer some remarks connected to this author and to the concepts analysed in the previous chapters.

As it was shown, the principle that negates the dimensions' penetration was not fully adopted by Leibniz in his early years, but it was reintroduced as a contingent consequence of his layered model. Matter fills space, so that at this point the penetration of dimensions is already allowed, but it fills it uniformly, so that to every ideal partition of matter corresponds an ideal partition of space. Assuming Leibniz and Weigel's determinism, there is never a time when the penetration of dimensions happens, if not for the first yet homogenous penetration. Given however that a form of penetration is allowed, something else was needed in order to grant absolute hardness to bodies colliding in space, that is in Leibniz's theory the role of anitypy. Following this interpretation, in 1676 Leibniz was still unsure about the possibility for extension alone to account for hardness, as a passage of Leibniz's notes on Descartes shows[147], but this does not change the fact that, even if in a contingent yet deterministic fashion, at the time of the adoption of universal principles in 1671 a form of the scholastics' negation of penetration was given to every physical body in Leibniz's philosophy of nature. I believe that this is a relevant consequence,

[147] AA (VI-3, 215). This passage was already analysed by Mormino (2012, 156)

because the Scholastics negation is founded on metaphysical and logical basis that were not present in the atomistic idea of antitypy. In this regard in fact, Leibniz clearly follows Hobbes, who was able to express a principle of existential coherence without relying to antitypy. In Leibniz's *Specimen demonstrationum De Natura Rerum Corporearum ex phaenomenis*, dated 1671, is found in fact a solution very similar to that of Hobbes' *De corpore*[148]:

> Duo corpora in eodem loco esse non posse.
> Idem corpus in pluribus locis esse non posse.
> Idem corpus diversis temporibus inaequalia spatia implere non posse.
> Idem spatium diversis temporibus inaequalia corpora capere non posse. (AA VI-2, 308).

These consequences are not against the Scholastics' principle as much as they are not against antitypy, but they show a logical structure that is similar to that of the ontological version of the principle of contradiction. If existing and being in one place are related, adding the principle of contradiction would negate the possibility of being in the same place for any other entity. The reasoning would be the following: (1) everything that exists is possible, (2) every possible thing is non-contradictory, (3) every non-contradictory thing cannot be its negation at the same time and in the same regard, but (4) one body exist, thus (5) it cannot be its negation at the same time and in the same regard. Since the negation of one body is any other possible entity, all the other bodies included, it will follow that two bodies cannot be in the same place, given the existence of one body. In the ontological use of the principle of contradiction then there is a logical outcome that could be easily retraced in the Scholastics' negation of the dimensions' penetration, because in the atomistic account the same result is obtained through a simple quality pertaining objects and it does not follow from an intrinsic logical impossibility related to the very existence of any given object.

How much this aspect of the Scholastics' principle influenced Leibniz is hard to determine, but it is indisputable that logical and existential coherence were sought by Leibniz in many fields where antitypy was not allowed. Remarkable in this regard is the

[148] Hobbes (1999, 86).

problem of two people possessing the same thing, found in law[149]. In the fourth question of Leibniz's *Specimen quaestionum philosophicarum ex iure collectarum* the problem is solved using an analogy taken from physics: "Unde patet elegans inter possessionem et positionem seu situm corporum in loco Analogia. […] Et colligerentur omnia gravia circa centrum in unum punctum, si enim duo esse in eodem loco possunt, quidni plura, quidni omnia?"[150]. In this work Leibniz is still influenced by Gassendi, but the problem has a distinctive Scholastic origin. A paradigmatic example is the comment on the same topic found in the *Digest* of Justinian. Here we read that "plures eandem rem in solidum possidere non possunt. Contra naturam est, ut cum aliquid teneam, tu quoque id tenere videaris"[151]. In his comment to this passage, Denis Godefroy, jurist quoted by Leibniz in the *Specimen*[152], will use a familiar concept in order to explain the impossibility of possessing the same object:

> Duo corpora in uno loco esse non possunt. Corpora se occupare et simul esse, natura non permittit. Dimensiones nequeunt se invicem subire. Sic dimensio suum locum occupat, ut obstaculo sit, ne quaevis alia dimensio, et quodvis aliud corpus, pariter unaque eum ea eundem locum occupare ac replere possit. Penetratio dimensionum natura est impossibilis: nam si duarum potest esse substantiarum penetratio, et trium et quatuor et infinitarum esse poterit (Godefroy 1583, III, §5).

The *penetratio dimensionum* is connected here with the idea of *replere* and it was surely taken into account by Leibniz.

It could be then that Pufendorf's influence on Leibniz in law, that is Weigel's influence, could have helped Leibniz in recognizing the logical and existential coherence connected with the Scolastics' principle found in Godefroy. Following in fact the application of Aristotle's *Apodeixis* and Leibniz's reference to Weigel in the *Specimen* about the distinction between natural, moral and notional truths, the importance of Weigel's *Analysis aristotelica* emerges also in the topic of law. Again, it is the *Analysis*

[149] For a detailed analysis see the introduction found in Artosi (2013).
[150] Artosi (2013, 55).
[151] Godefroy (1583, III, §5).
[152] Artosi (2013, 61).

aristotelica's chapter on necessity that casts some light on the matter, because there, just after the introduction of absolute necessity and restricted necessity, Weigel propose another kind of necessity, called necessity *ex impositione*:

> *Necessarium impositivum morale* est, quod in genere morum & in vitae humanae statu non potest aliter se habere, licet absolute sapius ex arbitrio dependeat. Ita necessariae sunt Propositiones demonstrationum Juridicarum, quatenus e principiis impositivis tenquam primis, h.e. e statutis hominum & legibus positivis qua talibus deducuntur (Weigel 1658, 22).

Although being fundamentally different from absolute necessity, by introducing the concept of imposition, Weigel was able to transfer the structure founded on contradiction to the necessity applied in law. Even if in this field absolute truth does not follow directly and independently from any necessary statement, the common structure grants the possibility of giving mathematical explanations that otherwise would be impossible to achieve. As Vinhold in his 1671 *Theses* writes "Doctrina proportionis Arithmeticae & Geometricae usum suum habet in Philosophia morali". The extension of the theory of proportions to law and ethics resembles Leibniz's way of explaining possibility of ideas as numbers found in the letter to Wedderkopf. It is no coincidence either that Leibniz writes to Conring in 1670, adopting the method of mathematical proportions[153] and quoting Euclid and the *Digest*. Conring will reply by quoting Hobbes and Aristotle's "apodicticae artis"[154]. In this context then, it seems natural that the idea of necessity founded in contradiction, thus the idea of possibility as something non-contradictory, was associated with the problem of existential coherence, specifically with the problem of the dimensions' penetration in law.

Ultimately, aside from law, the main aim of Weigel's school was reintroducing some ideas taken from Aristotle and saving them from the Scholastics' influence. The idea of existential coherence connected with the principle on the penetration of dimensions is clearly a genuine product of Aristotle's philosophy, especially on physics. In his *Physics* in fact, Aristotle argues that nothing can be inside itself. The reason is that otherwise two

[153] "Illic proportio arithmetica, hic geometrica" (AA II-1, 47).
[154] AA (II-1, 53).

things would be at the same time and in the same place, violating the principle of contradiction[155]. In Aristotle's philosophy in fact, coming into being is expressly related to contradiction:

> Change from non-subject to subject, the relation being that of contradiction, is coming to be […] Those which take the form of becoming and perishing, that is to say those which imply a relation of contradiction, are not motions […] One kind of change, then, being change in a relation of contradiction, where a thing has changed from not-being to being it has left not-being. Therefore, it will be in being; for everything must either be or not be (Aristotle 1984, 388-397).

I believe that this is the fundamental idea behind the ontological use of the principle of contradiction and, since it is related to Weigel's school, the fact that Leibniz, starting from the *Confessio*, adopts a model of modality that involves ascribing non-contradiction to existing entities suggests a possible connection that needs further investigations. While openly against the negation of dimensions' penetration then, Leibniz could have learned from this principle the need of a coherent premise for every existing object, favouring at a later time the ontological use of the principle of contradiction.

Conclusion of the First Part By retracing Weigel's influence between 1663 and 1671, Weigel's important contribution to the young Leibniz's philosophy emerges. While developing a physical theory that was inspired by the metaphysical reinterpretation of Hobbes' philosophy, at the beginning of the Parisian stay Leibniz fully adopted those principles that will be fundamental for his foundation of mathematics at a later time: the principle of contradiction, the principle by which the whole is greater than its parts and the principle of homogeneity. Leibniz will constantly refer to these principles throughout is life, so that, even if it is true that strictly speaking these principles cannot be considered as pure mathematical knowledge, their influence on the development of Leibniz's mathematics becomes relevant. After developing the mathematics of the infinite in Paris

[155] "The place cannot *be* body; for if it were there would be two bodies in the same place […] Nor is it possible for a thing to be in itself even accidentally; for two things would be at the same time in the same thing" (Aristotle 1984, 354-348).

in fact, Leibniz aim will be that of reconciling his new discoveries with these foundational principles, adopted from Jena's cultural background.

PART II: Contradiction and Homogeneity

2.1 Leibniz’s Idea of Contradiction and Its Development After 1671

After an in-depth analysis of Weigel’s first reception, this part of the present work is dedicated to the use of Leibniz's universal principles after 1671. In the next chapter I will argue that Leibniz's *analysis situs* is the final outcome on the ground of geometry of a general idea that involves the use of homogeneity in order to define relational properties between mathematical objects. Having highlighted the importance of Leibniz's concept of homogeneity in mathematics, in the following chapter I will show how this concept was developed, from an historical point of view, through Leibniz's constant relationship with Weigel.

Before focusing on Leibniz's foundational approach however, some general remarks on Leibniz’s idea of contradiction are needed. This topic is in fact often underestimated by the secondary literature: even if a role is usually granted to the principle of contradiction throughout Leibniz’s works in very specific contexts, a complete and satisfying reconstruction of its adoption and development is still missing. This task is obviously worth more than a single introductory chapter, but the lack of a persuasive study force us to present at least some remarks on this matter, since our interpretation of Leibniz’s rationalism in the light of Weigel is based on the assumption of a stronger meaning for this principle than the one usually acknowledged.

One could in fact ask in what sense the principle of contradiction contributed to Leibniz’s rationalism. The easiest answer would be that, being a universal and logical principle, it could be considered as the general premise for every possible logical assumption. This is obviously true, since the principle affirms that contradictory statements cannot both be true in the same regard and at the same time. Applying the principle to any possible statement, we can easily infer that violating it would inevitably lead to a possible inconsistency in a given theory. Focusing on the domain of logic then, the importance of this principle clearly emerges, but at the same time it wouldn’t seem as effective as hoped to be: it is indeed a general premise but also nothing more than a general premise. If for example a distinctive feature of Leibniz’s rationalism is determinism[156], how avoiding contradiction could affect this topic, when some possibilities are not determined in this world and yet perfectly conceivable? As I have shown, contradiction and conceivability are connected in Leibniz, but the concept of possibility founded in the principle of contradiction does not distinguish between pure possibility and the possibility of actual entities. It seems as if something should be added to offer a convincing account of determinism, even if actual entities are already logically determined by the law of contradiction. For these reasons, compossibility and the principle of sufficient reason are usually introduced as truly distinctive features of Leibniz's rationalism. Following this interpretation, the principle of contradiction is a simple rational premise and this is precisely why its presence in several writings belonging to different periods[157] is not considered a threat to the nowadays consolidated general assumption that Leibniz's philosophy evolved with significant differences in time.

I believe however that dealing with the problem of contradiction in Leibniz from the point of view of a rigid distinction between logic, metaphysics and physics ultimately fails to give an adequate representation of Leibniz's aim. The principle of contradiction is in fact often used by Leibniz in contexts that are different from that of logic, or rather, that we wouldn't call logical contexts in our perception[158]. As a starting point, it is possible to list the major achievements of Leibniz's thought where the principle of contradiction plays a fundamental role:

[156] I'm indebted to Piro (2002) and Mormino (2005a) for this interpretation of Leibniz's determinism in the light of Hobbes.

[157] Even if in the previous chapter I argued for the adoption of the principle of contradiction in modal logic around 1671, it does not mean that Leibniz had not used it before that time in different contexts. In 1666 he already recognised the importance of this principle in his *Disputatio arithmetica de complexionibus*, introducing it as the first logical corollary of the demonstration of God's existence: "Duae sunt propositiones primae, una principium omnium theorematum seu propositionum necessarium : Quod est (tale) id est seu non est (tale) vel contra ; altera omnium observationum seu propositionum contingentium : Aliquid existit" (AA VI-1, 228). Its importance will be stressed by Leibniz until the very end of his life, as the quote of the letter to Clarke showed, or as we could witness in the *Monadologie*: "Nos raisonnements sont fondés sur deux grands principes, celui de la contradiction en vertu duquel nous jugeons faux ce qui en enveloppe, et vrai ce qui est opposé ou contradictoire au faux" (Monadologie, §31).

[158] The fact that Leibniz's use of contradiction is unusual for our standards does not necessarily mean that it belongs to a sort of pre-logical era where the difference between ontology and pure logic was not completely clear. For example, I have shown that the difference between the principle of contradiction and the principle of excluded middle was known by Leibniz, making the use of these two principles as one a specific choice that was justified by the ontological consequences of the principle applied in physics.

- The principle of contradiction is essential in Leibniz's unique modal demonstration of God's existence, or rather, it is considered by Leibniz the key difference that allows the completion of Spinoza and Descartes' demonstration of God's existence.

- As shown, it is the foundation of Leibniz's modal logic, since the definitions of possibility and necessity depend on it, not only from a logical point of view, but also from an ontological prospective.

- It has a major role in Leibniz's physics, because the early modern notion of body poses several challenges about the possibility for different entities to occupy the same space.

- It is the only principle in Leibniz's theory of perception that resides in the human mind without being derived from senses and it is also the only one that grants coherence to our experience.

- It is considered the negative manifestation of the concept of identity.

- From a logical point of view, it identifies the *reductio ad absurdum*, a fundamental tool used by Leibniz to establish a connection between different syllogisms and to prove some of his most important demonstrations in arithmetics and geometry.

- It is the only principle used in the demonstration that the whole is bigger than its part.

- It is the only principle needed for the foundation of arithmetics and geometry.

Given the variety of topics involved, it's clear that the importance of this principle should not be underestimated: its heterogeneous use is at least problematic, because it poses a threat to the classical distinctions between God and reality and between reality and mathematical objects in Leibniz's philosophy. However, one could argue that, even admitting its presence in every topic listed above, its role is not central as it may seem, but a brief analysis of every part of Leibniz’s philosophy involved will be sufficient to prove the opposite, or at least to cast some doubts on this interpretation.

Leibniz’s modal proof of God’s existence is based on the assumption that Descartes and Spinoza’s proofs shouldn’t be completely rejected, but rather perfected. The exact word used by Leibniz is “defectueuse”[159], a word that could be misleading in respect to our purpose: it seems as something must be added to a theory that is already well-grounded. The nature of this completion however entails questioning the possibility of a concept, such that of God, in order to prove its necessity. It is clear then why Leibniz calls this proof a modal proof, since the concepts of possibility and necessity are taken directly from the realm of modal logic. In other words, if the concept of God is conceived as a sum of perfections, or rather if every perfection describes an aspect of the concept of God, even before taking into account existence, it is required on one hand that this specific perfection is possible and on the other hand that it is compossible with every other perfection attributed to God. The outcome of Leibniz’s attempt is very peculiar, because God is conceived as the only being for which from its possibility follows directly its necessary existence. Even if the analysis of this achievement needs an appropriate research that would sway us from our specific purpose, what is already clear is that this completion is not achieved when the concept of God is already founded: questioning the possibility is the first and not the last step in Leibniz’s modal demonstration of God’s existence. It follows that, if possibility and necessity are founded using the principle of contradiction, as we already mentioned about the *Confessio philosophi*, hence God must follow the principle of contradiction in order for it to be possible and necessary. The only way to deny this is to argue that Leibniz’s conception of possibility at the time when he developed this proof changed from that of the *Confessio*, having abandoned the foundation on the principle of contradiction, but this hypothesis is not consistent for

[159] It’s a word used by Leibniz in a 1686 letter to Simon Foucher: “la demonstration de l’Existence de Dieu, inventée par Anselme, et renouvellée par des Cartes est defectueuse. *Quicquid ex definitione Entis perfectissimi sequitur, id ei attribui potest. Atqui ex definitione entis perfectissimi seu maximi sequitur existentia, nam Existentia est ex numero perfectionum, seu ut loquitur Anselmus, majus est existere quam non existere. Ergo Ens perfectissimum existit. Respondeo: Ita sane sequitur, modo ponatur id esse possibile*” (AA II-2, 92-93). Even if in this passage possibility as non-contradiction is not introduced, Leibniz extensively uses it in the same letter, mainly to establish a distinction between a *definition reelle* and *nominale*.

example with what we read in an important work written by Leibniz for Henning Huthmann in 1678, where Leibniz describes his demonstration as a “fastigium doctrinae Modalium”[160], where contradiction is openly associated with modal concepts: “aut Ens necessarium implicare contradictionem, sive non esse possibile: vel si possibile est, conclusio contradictoria de eo fieri non potest”[161].

Moving then to the analysis of the purely modal use of the principle of contradiction, I already argued that Leibniz used it in the very definition of possibility and necessity in the *Confessio*: possible is defined as something which is not contradictory, whereas necessary is defined as something of which the opposite involves contradiction. It is obvious here that the central role of contradiction is not at stake. Rather, the same problem that emerged about the proof of God’s existence could concern modality, since we are not sure that Leibniz opted to follow the same definitions during the entire evolution of his philosophy. Unlike God’s existence, an evolution of Leibniz’s philosophy seems likely here to be the case, because Leibniz is often celebrated as the first philosophers conceiving modality through possible worlds. This popular interpretation[162], although recognising that he never expressly states it this way, argues that Leibniz *defines* modal

[160] AA (II-1, 587).

[161] Here the whole argument is in fact a tribute to modality conceived through contradiction, but it is also connected to an idea of necessity that is directly derived from geometry, establishing a distinction with the simpler proofs of Spinoza and Descartes: “Spinosa ita ratiocinatur post Cartesium. Idem est dicere aliquid in rei alicujus natura sive conceptu contineri, ac dicere id ipsum de ea re esse verum (quemadmodum in Trianguli conceptu continetur, seu ex essentia ejus sequitur ejus angulos tres esse aequales rectis duobus). Atqui existentia necessaria in Dei conceptu eodem modo continetur. Ergo verum est de Deo dicere necessariam existentiam in eo esse, seu ipsum existere. Huic ratiocinationi aliisque similibus opponi potest: propositiones illas omnes esse conditionales, nam dicere in trianguli natura vel conceptu involvi tres angulos aequales duobus rectis; nihil aliud est dicere, quam si existat triangulum, tunc ipsum hanc proprietatem habere; ita eodem modo, etsi concedatur de Dei conceptu esse existentiam necessariam, tamen inde colligetur tantum, si existat Deus, tunc ipsum hanc proprietatem (necessariae existentiae) habere, sive si Deus existat, eum necessario existere. Nostra vero ratiocinatio hanc difficultatem non recipit, sed probat aliquid majus, nempe Deum si modo possibilis sit, necessario existere actu” (AA II-1, 591). This passage is particularly interesting because it can be seen as a further step in connecting the ontological use of contradiction to its use in the foundation of arithmetics that will happen in Leibniz at a later time, through Weigel’s influence: existence is for God a quality in the sense sketched by Leibniz in mathematics and not a mere propriety arbitrarily added to an object. Here it seems that Leibniz is differentiating God from a mathematical concept, but upon further inspection Leibniz does not deny the idea of a quality pertaining the object. Rather, he says that this peculiar object from which we should draw our propriety (necessary existence) is not “God”, but “God as a possible being”. It really shows that Leibniz was conceiving non-contradiction as the primary premise of his argument. The connection between contradiction and the modal proof of God’s existence is present in many writings, for example in the famous *Quod Ens Perfectissimum existit*, written for Spinoza in 1676 (AA II-1, 428). For a reconstruction of the confrontation between Leibniz and Spinoza on this topic see Pasini (2005). Several remarks on the proof can be found in Di Bella (2005b), whereas for a contemporary approach to Leibniz’s proof see Griffin (2013, 34-82).

[162] See for example Mondadori (1973), Wilson (2000), Rescher (2013, 1-44) and Mugnai (2013).

operators using possible worlds: possible would be that which is true in at least one world, whereas necessary would be that which is true in every possible world, in the same fashion of contemporary modal logic. Whether this interpretation is valid or not, we can safely assume that the modal definitions through the principle of contradiction are at least consistent with Leibniz's late reflections on modality, because he still openly uses them in the *Essais de Théodicée*. Even more so, since the *Théodicée* is a work conceived for publication and to appeal to a variety of different positions: there is more consistency between the use of contradiction and Leibniz's unpublished writings, like the *Confessio philosophi*, than between them and the use of possible worlds[163]. A different matter would be that of determining if possible worlds overtook contradiction at a foundational level[164], but its presence and use is sufficient to prove that contradiction still has a central role in modal logic and everything else that derives from it in the matter of predestination for a period of time that surpasses the fundamental evolution of Leibniz's philosophy through the concept of monad.

With regards to the central role of the principle of contradiction in Leibniz's theory of perception, it again derives from his adoption in modal logic. More precisely, Leibniz refers to the ontological consequences of this endorsement, following an obvious conclusion already highlighted about his philosophy of nature: if something exists, it is possible, but something that is possible is non-contradictory, thus everything that exists is non-contradictory. In the *Nouveaux essais sur l'entendement humain*, this simple conclusion is the basis for every coherent experience. This coherence is possible because there is a peculiar correspondence between our mind and the ontological structure of

[163] The distinction endorsed in the *Confessio* is found also in the *Essais de Théodicée*: "La Verité necessaire est celle don't le contraire est impossible ou implique contradiction. Or cette verité, qui porte que j'ecriray demain, n'est point de cette nature, elle n'est donc point necessaire. Mais suppose que Dieu la prevoye, il est necessaire qu'elle arrive; c'est à dire la consequence est necessaire, savoir qu'elle existe, puisqu'elle a ètè prevue, car Dieu est infeillible: c'est ce qu'on appelle une necessité hypothetique. Mais ce n'est pas de cette necessité don't il s'agit iey: c'est une necessité absolue qu'on demande, pour pouvoir dire qu'une action est necessaire, qu'elle n'est point contingente, qu'elle n'est point l'effect d'un choix libre" (GP VI, 123-124).

[164] In my opinion this is not the case: following a contemporary approach, the possible-worlds model would ultimately derive its truth from the truth of single propositions in specific worlds, but Leibniz, following an intuition that he already had in the *Confessio philosophi* is not akin to what we would call today a statistical definition of modality. Given the important connection between possibility and conceivability that highlights the need of an intrinsic logical coherence, Leibniz perhaps would interpret the contemporary possible-world model as a statistical-Aristotelian model, only applied to a wider field, i.e. entire worlds. On a foundational level, the principle of contradiction as a distinctive propriety of possibility defines instead in Leibniz a realm of conceivable things, regardless their presence in one or several worlds. A different problem is that of compossibility: here Leibniz's concept of a whole world and its relations is much more effective and similar to the contemporary account.

reality[165], since the principle of contradiction is the only primitive principle of the human mind used to distinguish truth from falsity, as Leibniz writes in the preparatory works for the *Nouveaux essais*:

> Mon opinion est donc, qu'on ne doit rien prendre pour principe primitif, si non les experiences, et l'axiome de l'identicité, ou (qui est la même chose) le principe de la contradiction; qui est primitif, puisqu'autrement il n'y auroit point de la difference entre la verité et la fausseté (AA VI-6, 4-5)[166].

The fact that every possible experience is a non-contradictory experience of something non-contradictory in itself becomes extremely relevant in an indefinitely divisible world, such that of Leibniz. In this perspective, an error in our perception derives from the impossibility for our mind to master infinity, rather than the occurrence of a real contradiction, happened at some point in the relation between the subject and the object[167]. Once again, it seems as if the philosophical context in which this topic is displayed depends on the peculiar constitution of the world conceived by Leibniz.

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[165] An important passage from the 1677 *Dialogus* shows that, althought Leibniz's approach could be labelled as nominalist, the possibility of the generalization through names is founded on the impossibility for the human mind to master infinity, that is on the idea that we perceive as incomplete objects that are composed by non-contradictory indefinitely actual parts. In this way, although perception is at fault, the possibility of finding truth is grounded by the same rational order: "Sed hoc tamen animadverto, si characteres ad ratiocinandum adhiberi possint, in illis aliquem esse situm complexum ordinem, qui rebus convenit, si non in singulis vocibus (quamquam et hoc melius foret) saltem in earum conjunctione et flexu, et hunc ordinem, variatum quidem in omnibus linguis, quodammodo respondere […] etsi characteres sint arbitrarii, eorum tamen usus et connexio habet quiddam, quod non est arbitrarium" (AA VI-4, 24).

[166] Here we find again the use of the principle as a substitute for identity. Moreover, there are several reasons to believe that the *Quelques remarques* and the *Nouveaux essais* were developed by Leibniz with Weigel and, in general, the reinstatement of scholastics' ideas in mind: in AA (VI-6, 5), just after this quote, Leibniz refers to Euclid, together with Apollonius and Proclus, as the perfect example of how we should develop our theory based on principles and axioms. In AA (VI-6, 9) we read how Leibniz conceives the reinstatement of scholastics' ideas as a work of interpretation of the same ideas in a different context, with an interesting parallelism between Leibniz's work and that of the Italian Accademia della Crusca, a society born in Florence in 1583 in order to preserve the purity of the Italian language.

[167] It is Leibniz's take on the Cartesian problem of confused knowledge. In the *Meditationes de Cognitione, Veritate et Ideis* we read that knowledge is "Confusa, cum scilicet non possum notas ad rem ab aliisdiscernendam sufficientes separatim enumerare, licet res illa tales notas atque requisita revera habeat, in quae notio ejus resolvi possit: ita colores, odores, sapores…ideao nec caeco explicare possumus, quid sit rubrum, nec aliis declarare talia possumus, nisi eos in rem praesentem ducendo, atque ut idem videant" (AA VI-4A, 587). Here we find again the difference between a *definition reelle* and *nominale*. The real definition is the one the infers the possibility, i.e. non-contradiction, of one thing and again it is considered by Leibniz a distinctive feature of his nominalism that saves us from Hobbes' position (AA VI-4A, 589). The problem of a defective knowledge has to be found then in our impossibility of describing a single object with a finite set of propriety, rather than a defectiveness of the object in itself. The very same ideas are expressed in the

Being the specific purpose of this part, the central importance of contradiction in the aspects of logic and mathematics previously listed will be explained in the next chapter, but the outcome of our research will show again a consistency with other aspects of Leibniz's thought: here, a peculiar concept of unity arises from Leibniz's reflections on logical coherence. The concept of a non-contradictory unity then leads the way from logic and mathematics to physics, as previously shown.

If the remarks in all these fields are valid, we can clearly see that there is only one last possible objection against a serious consideration of the principle of contradiction and it is again related to the evolution of Leibniz's philosophy in time: it could be indeed true that everything reported in the above list is affirmed by Leibniz at a certain point, but this doesn't necessarily mean that every aspect exposed should be taken at the same time. Given the renowned instability of Leibniz's position in time, it could be that some of the aspect listed are not always valid and compossible, even more if we consider the famous turning point in Leibniz's physics happened at the end of the seventeenth century. The evolution in the consideration of the principle however follows in Leibniz a very natural flow: its definitive adoption in modal logic happens around 1671, then, having considered also its ontological consequences, it was only natural to question the coherence of that peculiar object called God, thus we witness the development of his proof at the end of the 1670s[168]. In the 1680s contradiction is applied to the problem of confused knowledge and in the 1700s, although Leibniz turned to the concept of monad, everything contradiction-related, from modality, to ontology, theory of perception and God's existence remains unscathed by the great change, all of this while in logic and mathematics contradiction steadily becomes more and more important at a foundational level, culminating with Leibniz's claims in his 1715 letter to Clarke. We could say then that the topic of contradiction is a case of continuity in an extremely dynamic context. This is precisely why its study is important and challenging at the same time: it means that every change happened in Leibniz's philosophy should be consistent with the principles derived from

Nouveaux essais (AA VI-6, 86, 376, 379-80). On the topic of confused knowledge different interpretations are those of Wilson (1999) and Puryear (2005).

[168] We are taking Huthmann's 1678 letter as a final word on Leibniz's proof of God's existence, but from that time on this achievement is maintained and celebrated by Leibniz: "Car il faut bien que s'il y a une réalité dans les essences ou possibilités, ou bien dans les vérités éternelles, cette réalité soit fondée en quelque chose d'existant et d'actuel ; et par conséquent dans l'existence de l'Être nécessaire, dans lequel l'essence renferme l'existence, ou dans lequel il suffit d'être possible pour être actuel (§ 184-189, 335)" (Monadologie, §44).

that of contradiction, at least on a general basis. This consequence becomes quite problematic, considering the interpretations generally accepted nowadays, because the adherence to the principle of contradiction puts several important constraints to the evolution of Leibniz's thought: if we take for example the evolution in physics and the reinstatement of substantial forms, it is indeed true that this change in Leibniz's theory lead to a departure from the mechanists' belief that everything in the world can be described using the concept of body and its movement in space, but at the same time the results of this departure, being conceivable and real, must follow the same non-contradictory rule followed by bodies and other concepts involved in the modern physics. If this is the common consistent background and, on a side note, if this is also the reason why Descartes' important mistake has been found by Leibniz using the very same concepts used by Descartes and not conceiving a radically different approach[169], then by no means the reintroduction of substantial forms should be interpreted as a way to escape from the law of contradiction, reintroducing free will or other unexplainable causes that are not coherent with Leibniz's determinism. Rather, interpretations that focus on Leibniz's peculiar determinism, or I would say metaphysical mechanism[170], will be easier to explain in the light of his constant use of contradiction.

This outcome is connected with another renown interpretation challenged by the revaluation of contradiction, that is the one that recognise a fundamental ontological distinction between real unity and phenomenal unity. There is no denying that this important distinction arises in Leibniz's philosophy[171], even more so at a later stage, but the real focus should be on what is the criteria used for establishing it. We already briefly sketched how the principle of contradiction is considered by Leibniz another side of identity and unity and, in this specific case, it would be useful to understand the relation between this kind of unity, a non-contradictory unity, and the real and phenomenal unities. Specifically, are both real unities and phenomenal unities non-contradictory

[169] In other words, Leibniz still uses the modern conception of bodies and their relations, studied through the laws of collision to achieve insights on his new theory.
[170] If it is true that for Leibniz nothing in the world is contradictory, then from the refutation of atoms as real beings does not follow the refutation of determinism or the refutation of a mechanical way to describe the world. Our impossibility of understanding it completely derives from our impossibility to master infinity, but the power of names is that of reuniting groups of infinite and yet coherent parts in finite and coherent relations. This is the reason why Leibniz still believes in the project of an *ars combinatoria*, at least on paper, even after abandoning the atomistic model.
[171] I'm obviously referring here to the *Discours de métaphysique* and the other writings of that time.

unities? As for the latter, it seems that this is the case: I already conjectured that on the ground of physics the reason why Leibniz adopts contradiction is to preserve some qualities that pertain to atoms and other conceptions of bodies, without sacrificing the possibility of indefinitely divisible entities. We are all familiar however with Leibniz's later assumption that real unities are also the only true unities. After this important discovery, in several writings Leibniz is inclined to say that the phenomenal unity is, consequently, a false unity, thus establishing a difference between the world as we see it and his metaphysics, in a platonic fashion. Following the same path then, one could argue that the ontological validity of contradiction is maintained only at a phenomenal unity level, because it is the level that originally adopted contradiction when this was in Leibniz's mind the one and only world. The real unity would be then a peculiar kind of entity, saved from the principles and rules that were valid before this important change in Leibniz's philosophy. This interpretation could be refuted, pointing out that the connection between contradiction and phenomenal unity is not the same that exists between extension and the phenomenal world: extension is responsible for the fact that every possible phenomenal unity can be divided in two or more phenomenal unities and there is no denying that every unity involved is also a non-contradictory unity, but it does not follow from these premises that there can't be non-contradictory unities that are not bounded to the concept of extension. This is consistent with what I argued in the previous part: while logical coherence may have been adopted for physical entities because of a property pertaining to extension, that is the negation of dimensions' penetration, it was later identified by Leibniz as a separate concept related to conceivability. Since this outcome is maintained in the *Théodicée* and in the *Monadologie*, then even monads must be considered as conceivable unities.

In Leibniz's late thought this is also the case for the concept of unity in arithmetics: the *Initia rerum mathematicarum metaphysica*, being one of the most intriguing exposition of Leibniz's foundational approach to mathematics, will show that numbers, i.e. unities conceived as a mathematical objects, are nothing else but non-contradictory and well-ordered unities, even if they do not belong to reality in the same way of the phenomenal unities.

It follows that non-contradiction is not related directly and only to extension: if the non-contradictory unity was conceived by Leibniz as a shaping tool, i.e. something that generates extensional partitions between different entities, then avoiding contradiction would have put constraints to the type of unities that we are allowed to consider in the real world. This process would have lead us to admit infinitely small and homogeneous non-contradictory unities, similar to those of atoms, forcing us to treat them as imaginary tools or exposing them to Leibniz's severe criticisms on similar entities. In other words, in this interpretation non-contradiction would be used in the classical Scholastic fashion of a quality pertaining extended bodies, that is the negation of penetrations' dimension, but Leibniz's approach is different because, being possible, everything in the real world is non-contradictory, regardless of the arbitrary shape considered: the ontological value of the principle grants the propriety of a body of being coherent within itself and with the rest of the world, thus granting for it the possibility of occupying a space, but the logical and immaterial structure of the principle leaves the problem of determining what is that makes that body occupy that space to the realm of physics.

Leibniz's arbitrary and yet non-contradictory unity is perhaps non-intuitive, compared with our belief in a world composed by separated entities, but it is consistent with his repudiation of the void and with his late remarks on the phenomenal appearance of the actual world. This "unshaped coherence" is the great potential hidden in the principle of contradiction that Leibniz already saw at the beginning of the 1670s and that he finally developed at the end of his life, as a bridge between reality and mathematics. Following this approach, numbers would be defined as non-contradictory and unshaped unities, connected, through well-ordered and well-founded relations, whereas geometry would be conceived as a middle ground, taking the propriety of having a shape or extension and at the same time following the rigid well-ordered and well-founded construction of arithmetics. In this context, the idea that the whole is necessarily bigger than its parts, founded again on the principle of contradiction, becomes the cornerstone for every possible coherence, both in arithmetical and geometrical terms. The fact that the finitist approach to arithmetics is stronger in Leibniz at the end of his life, after the introduction of the concept of monad, is not a mere coincidence then, because it shows Leibniz's purpose of identifying a non-extensional ground where contradiction is still valid, making it consistent with the assumption that the few things that can be safely conceived as necessary belongs to mathematics and with the indication of the principle of contradiction among the two great principles introduced in the *Monadologie*[172].

As a final remark, we could now re-think Leibniz's rationalism in the light of the principle of contradiction: if it is true that it has a central role in the ontological development of Leibniz's concept of unity, then perhaps it is not a mere coincidence that the evolution of the principle of sufficient reason from the form "nothing happens without a reason" to the form "nothing happens without a reason for it to be so and not otherwise" takes place at the same time as the adoption of contradiction. In this subtle change an important evolution of Leibniz's philosophy is hidden, because, other than its usual extensional feature derived from Hobbes' influence, the principle of sufficient reason gains through the principle of contradiction an intensional meaning, more akin to Leibniz's metaphysics of that time[173]. The perfected principle in fact entails two different meanings: on one hand, everything that happens has a reason, meaning that there is a specific chain of causes that leads to that event, hence the extensional meaning. On the other hand though, from the fact that there is always a reason why something is so and not otherwise it follows that something is always as it is and not otherwise. This last meaning perhaps is derived from the coeval adoption of principle of contradiction in modal logic, because the requisites for not being otherwise are not only the determined causes that precede the analysed event in time: that specific event must also be non-contradictory in itself or Leibniz's whole determinism would be at stake. If we admit for example that something is coloured in a certain way and at the same time and in the same regard it is not coloured in that certain way, then we admit the possibility of an incoherent and instant transition that would surpass and rewrite the chain of causes previously considered[174]. Having however added the principle of contradiction, Leibniz's determinism is saved, because a contradiction won't ever arise, not only in the *series rerum*, but also in the single event considered as a whole and in every possible and infinite

[172] See note 157 of this chapter.

[173] An account of the development of Leibniz's idea of sufficient reason can be found in Piro (2002) and Dascal (2008).

[174] In other words, taking the opposite route previously outlined, we could say that if something exists as it is and not otherwise, then it must be *possible* for it to be so and not otherwise, i.e. non-contradictory. In this case the principle is important not only for the coherence of something with itself, but also for the coherence of something against the possibility for it of being something completely different. It highlights one of the most important feature of Leibniz's principle: it tells us something not only about one thing, but also about all the other possible things in this world, i.e. that they are not that thing.

partition of that same event. Again, starting from the 1670s, the concepts of unity, identity and contradiction take a leading role, rewriting some fundamental parts of Leibniz’s rationalism, until we witness a definite inversion in the priority of the principles, between contradiction and sufficient reason, as Leibniz states in the margin of the work against the Socinian Christoph Stegmann:

> Aliquoties notavi bina summa esse principia omnis cognitionis nostrae a prioribus rerum deductae: principium contradictionis, ne nobiscum ipsis pugnemus, et principium rationis, seu ne quid unquam sine ratione sufficiente evenire judicemus. Ex principio contradictionis oritur (Jolley, 179).

In conclusion, giving an exhaustive account of Leibniz’s idea of contradiction is not the purpose of this chapter, if only because the variety of topics involved and the amount of Leibniz’s writings considered in time require a wider and more appropriate space, but these few remarks should be sufficient to recognise at least a missing link in the interpretations of Leibniz’s philosophy. If the principle of contradiction is so important in all these fields as it seems to be, underestimating Leibniz's claim that it is also the foundation of the whole of mathematics would be inappropriate. However, finding in Leibniz a specific theory that derives mathematics from contradiction is not easy. In the next chapter I will argue that the closest idea found in Leibniz's writings is his demonstration of the principle by which the whole is greater than its parts and its use as a tool in the confrontation and superposition of mathematical objects.

After 1671, Leibniz develops his own take on Weigel's syncretism. While in the adoption and connection of these principles the relationship with Weigel had a fundamental role, this passionate desire of reducing logical and metaphysical statements to simpler and simpler terms is a distinctive feature of Leibniz's rationalism that he was able to develop with a proficiency and effectiveness that was unknown to Weigel.

2.2 Leibniz's Attempt to Define Numbers

A Tentative Explanation of Leibniz's Foundational Approach In this chapter I will try to give an adequate account of Leibniz's foundation of mathematics based on the *Initia rerum mathematicarum metaphysica*, a work written after 1714 that represents in my opinion Leibniz's most important attempt in defining mathematical objects. This chapter then has two main objectives: (1) trying to explain Leibniz's belief in the possibility of founding the whole of mathematics on the principle of contradiction and (2) highlighting the importance of the concept of homogeneity in every part of Leibniz's philosophy of mathematics.

The first objective is not directly connected with Weigel's influence after 1671, but it could be seen as the final outcome of Leibniz's adoption of the universal principles before Paris, re-elaborated together with a growing focus on the principle of contradiction, as shown in the previous chapter. This objective is the most challenging, because apparently in the *Initia rerum*, while containing many statements connected with the philosophy of mathematics, there is no explicit mention of the principle of contradiction.

The second goal is that of revaluating the concept of homogeneity and free it from its necessary connection with the concept of *situs*: while situational analysis seems to describe a wider range of geometrical objects, I will argue that from the point of view of defining objects, homogeneity allows both a more precise definition and its possibility of being used outside the geometrical framework. Homogeneity will highlight some of the most important concepts related to Weigel and his influence on Leibniz's philosophy of mathematics, such those of quantity, quality, equality, ratio, and proportion.

In order to evaluate the final outcome of Weigel's influence then an explanation of these concepts is much needed. For these reasons, this chapter should be seen as a purely theoretical premise, without any presumption of being an exhaustive historical reconstruction[175]: Leibniz's position on these topics evolves in time and even in the last writings some inconsistencies emerge, as it will be shown for the *Initia rerum*. However, I will take this work as the final outcome of a general attitude towards mathematics that involves the use of the concepts above mentioned.

[175] For an adequate reconstruction of the evolution of Leibniz's *analysis situs* see De Risi (2007).

Combining the two objectives, a contradiction emerges: if our aim is that of understanding the difference between arithmetic and geometry, in order to prove that homogeneity is independent from geometry, we have to deal in the *Initia rerum* with a definition of number given by Leibniz in a context heavily contaminated by the use of geometry and by its terminology, so much that it appears as if the foundations of arithmetic itself is based completely on geometry. As it will be shown in detail in the next section in fact, Leibniz defines number as “id quod homogeneum est Unitati, seu quod se habet ad Unitatem, ut recta ad rectam”[176]. This assumption is somehow contradictory if confronted with what Leibniz said in the letter to Clarke dated 1715, where geometry and arithmetics stood on the same ground with regards to the principle of contradiction, because in this definition the reference to straight lines could be interpreted as a necessary reference to geometry. Judging by Leibniz's claim in the letter to Clarke instead, logic seems to have a place that is somehow above arithmetic and geometry from a metaphysical point of view. The dependence from straight lines found in the *Initia rerum* could invalidate these relationships. However, since the *Initia rerum* were written by Leibniz after 1714, we should find at least a kind of consistency with the idea expressed in the letter to Clarke in the same years. In the *Initia rerum*, the closest correspondence with this claim is the following:

> Notandum est etiam, totam doctrinam Algebraicam esse applicationem ad quantitates Artis Combinatoriae, seu doctrinae de Formis abstractae animo, quae est Characteristica in universum, et ad Metaphysicam pertinet (GM VII, 24).

It is no coincidence that this statement appears just after Leibniz’s definition of number. What emerges from this passage is that algebra derives from an application of something, which is the science of abstracted forms that belongs to metaphysics, to something else, which is described as quantity. In this sense, we are not far from the relationships between the different areas of mathematics that we would expect from Leibniz, given their origin from Weigel's philosophy sketched in the previous part: unities considered without quantity in their coexistence define logic and metaphysics, quantity considered per se

[176] GM (VII, 24).

defines arithmetics and quantity applied to extension defines geometry. Our first task then would be that of understanding the true role of geometry in Leibniz's foundations of arithmetics and why he describes this process as a metaphysical foundation. What I would like to suggest on this topic is that Leibniz saw the *Initia rerum* as a last effort on the realisation of a *Mathesis Universalis*, the dream of a superior science that describes itself as an ideal bridge between geometry and arithmetic. Being an application of the universal characteristic to something else, there is no identification between *Characteristica Universalis* and *Mathesis Universalis*, since the former has a wider range of use in Leibniz's view[177]. As such, *Mathesis Universalis* is not a purely formal theory, however it contains, as it will be shown, a decisive implementation of formal properties. A possible confirmation of this interpretation is contained in the *Matheseos Universalis Pars Prior*, found in the seventh volume of Gerhardt's *Mathematische Schriften*:

> Hinc etiam prodit ignorata hactenus vel neglecta sub-ordinatio Algebrae ad artem Combinatoriam, seu Algebrae Speciosae as Speciosam generalem, seu scientiae de formulis quantitatem significantibus ad doctrinam de formulis, seu ordinis, similitudinis, relationis etc. expressionibus in universum, vel scientiae generalis de quantitate ad scientiam generalem de qualitate, ut adeo speciosa nostra Mathematica nihil aliud sit quam specimen illustre Artis Combinatoriae (GM VII, 61).

Since this passage revolves around the idea of the application of quality to quantity and the subordination of algebra to the science of forms, we can safely assume that the *Initia rerum* are Leibniz's last attempt on the foundational theory of mathematics sketched here, following the idea of a *Mathesis Universalis*, because the concept involved are the same. This last work on the metaphysical origin of mathematics is therefore an ideal starting point in order to understand Leibniz's mature theory: here, concepts like quality, quantity,

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[177] That of Crapulli (1969) is still an excellent book on the origin of the term, especially because it highlights the importance not only of the words used, but also of the amount of work done around Proclus' commentary on Euclid's Elements. It shows that a reconstruction of the influence of this term in Leibniz can't be focused only on the obvious reference to Descartes' *Regulae ad directionem ingenii*: the debate on a *Mathesis universalis* was advanced and widely known, so much that a reference to Proclus' commentary is found in Suárez, as shown in the previous part.

order, similarity and relation are in fact thoroughly defined. Therefore, it's now time to move to a detailed analysis of these concepts and their relation.

Leibniz's Definition of Number In the *Initia rerum*, before the definition of number, Leibniz introduces several concepts that will be fundamental for his main aim. However, following Leibniz's approach, undoubtedly rigorous, is difficult, because it requires a deep knowledge of all the concepts involved and their relation. Therefore, in this exposition of Leibniz's theory, I will start from the definition of number and then gradually introduce the other concepts involved, following a logical scheme, more than the order in which they are presented.

Leibniz's definition of number, quoted in the previous section, is here presented in its entirety:

> Numerum in genere integrum, fractum, rationalem, surdum, odinalium, trascendentem generali notione definiri posse, ut sit id quod homogeneum est Unitati, seu quod se habet ad Unitatem, ut recta ad rectam. Manifestum est etiam, si Ratio a ad b consideretur ut numerus qui sit ad Unitatem, ut recta a ad rectam b, fore Rationem ipsam homogeneam Unitati; Unitatem autem repraesentare Rationem aequalitatis (GM VII, 24).

Number then is defined as that which is homogenous to unity and it is immediately connected with the concept of homogeneity and with that of *ratio*, exemplified by the comparison between straight lines.

As a preliminary remark, I would like to point out the basic consequence of this definition: every number is something which bears some kind of relation to unity. If every number must be defined through a confrontation with unity, unity becomes the cornerstone of Leibniz's foundation of arithmetic. Other than unity however, this notion relies on a peculiar relation between unity and numbers and on a reference to geometrical objects that is worth analysing. From this definition two possible interpretations seem to arise, depending on how we decide to interpret that "ut", used in "ut recta ad rectam": if Leibniz meant a proper connection, then numbers are defined through geometrical entities, otherwise it is just the exemplification of a similar relationship. I will now try to endorse the first possibility and show that ultimately both possibilities are not as distant as they seem at first glance.

Homogeneity In the next section I will discuss the use of the term "recta" and the problems related to this choice, but for now, I would like to focus on the peculiar relation that a number must bear in order to be defined: homogeneity. This relation is essential to understand if the foundation is based on a purely geometrical definition or not. At first glance in fact, the definition of number seems to rely completely on the geometrical notion of a straight line, at least in its exemplification, but the relation of homogeneity adds a new level of complexity, which should not be underestimated. Leibniz defines homogeneity in this way:

> Homogenea sunt quibus dari possunt aequalia similia inter se. Sunto A et B, et possit sumi L aequale ipsi A, et M aequale ipsi B sic ut L et M sint similia, tunc A et B appellabuntur Homogenea. Hinc etiam dicere soleo, Homogenea esse quae per transformationem sibi reddi possunt similia, ut curva rectae. Nempe si A transformetur in aequale sibi L, potest fieri simile ipsi B vel ipsi M, in quod transformari ponitur B (GM VII, 19).

Here, two definitions of homogeneity are hidden: the first one is what I would call the general definition of homogeneity, which involves the introduction of two new fundamental relations, equality and similarity, applied to four different entities — A, B, L and M — following a certain scheme or rule. Equality is applied between A and L, and between B and M, whereas similarity is applied between L and M. The other definition is what I would call the purely geometrical definition, which derives from the geometrical exemplification expressed by Leibniz between curves and straight lines. In this case, the scheme of the general definition is not completely respected, because the fourth entity, that is M, is taken as unnecessary. A bears the same equality relation to L, but in this case Leibniz admits the possibility of a direct connection between L and B through similarity. This is legitimate, given the geometrical exemplification, because the transformation happens in a geometrical context in which B could be already quantified and suitable for the similarity relation[178]. I believe that this is a relevant distinction, because it means that in Leibniz's geometrical example B and L are ontologically closer than the same entities described in the general definition. Understanding which homogeneity relation is applied to the definition of number is therefore necessary, because it will also show the ontological connection between the concept of number and pure geometry.

If we now apply the general definition of homogeneity to the definition of number previously given and taken as if the straight lines are not used only as an exemplification, the result is the following: a number (B) is something homogeneous to unity (A), i.e. unity (A) is equal to a straight line (L), the given number (B) is equal to another straight line (M), while these straight lines (L and M) are similar to each other. It is possible to apply the general definition because in the quoted passage Leibniz openly uses four entities, two numbers and two straight lines. However, it doesn't necessarily mean that in this case the purely geometrical definition can't be applied: since the purely geometrical definition is a special case of the general one, it could still be that the straight line L, related to unity, is also similar to the given number B. Even if this new definition of number seems more detailed than the original one then, it is now evident that the definitions of equality and similarity are needed for a complete understanding of Leibniz's aim.

Leibniz defines equality as the relation between things that have the same quantity. Quantity then is defined in this way:

> Quantitas seu Magnitudo est, quod in rebus sola compraesentia (seu perceptione simultanea) cognosci potest. Sic non potest cognosci, quid sit pes, quid ulna, nisi actu habeamus aliquid tanquam mensuram, quod deinde aliis applicari possit (GM VII, 18).

Until now, we have considered Leibniz's foundation of arithmetic without questioning its coherence, but after introducing this definition, it is possible to explain a passage of the original definition of number that could be considered incoherent at first glance: if Leibniz's aim is that of defining a number and his first step in this direction is conceiving it as something equal to something else, that is to say, something that has the same

[178] The connection between homogeneity and transformation in a geometrical sense and its role in Leibniz's calculus is analysed in Pasini (1993, 26-29).

quantity of something else, then a contradiction arises, because the concept of quantity, related to numbers, is contained in the definition of number itself. However, Leibniz's definition of quantity is not that of a simple numerical description, but it is based on the idea of a simultaneous compresence. In this regard, stating something as "A measures 6 meters" is completely meaningless, until A is introduced in a peculiar set of relations, in which an order between different objects is established. The introduction of similarity in a context where different things are compared is then necessary, otherwise equality would be a mere statement on the identity of two things, in the form of "A is B":

> Neque adeo pes ulla definitione satis explicari potest, nempe quae non rursus aliquid tale involvat. Nam etsi pedem dicamus esse duodecim pollicum, eadem est de pollice quaestio, nec majorem inde lucem acquirimus, nec dici potest, pollicis an pedis notio sit natura prior, cum in arbitrio existat utrum pro basi sumere velimus (GM VII, 19).

The difference between a numerical definition of quantity and Leibniz's definition is essential, because it shows that Leibniz had a significant idea of a foundation of arithmetic and not a mere exposition of the concept of number in geometrical terms: the problem of defining a number, without relying to a numerical reference or to the idea of counting, is in fact very frequent in writings concerning this topic, as it will be in the foundational attempts of the 20th century. This is remarkable per se, because it places Leibniz far ahead of his times.

This passage also confirms that an adequate definition is the one that involves similar things, otherwise it would be completely arbitrary. Similarity is defined through quality, instead of quantity:

> Qualitas autem est, quod in rebus cognosci potest cum singulatim observantur, neque opus est compraesentia. Talia sunt attributa quae explicantur definitione aut per varias modificationes quas involvunt […] Similia sunt ejusdem qualitatis. Hinc si duo similia sunt diversa, non nisi per compraesentiam distingui possunt" (GM VII, 19).

Quality is then something that pertains to an object, without requiring compresence. It could be described as a set of properties that intrinsically pertain to something. In the case of the straight line for example a quality would be that of proceeding interminably in both directions, or the idea of a length with no width. These properties pertain to a straight line in a way that would make us unable to distinguish another straight line from the one considered, if they are taken as the only object existing in our space. It follows that these qualities are maintained for any given part of the object:

> Rectam esse inter sua extrema aequabilem. Neque enim aliquid assumitur, unde reddi possit ratio varietatis. Itaque oportet, ut unus locus puncti in ea moti ab altero discerni non possit seposito respectu ad extrema. Hinc et pars rectae recta est, itaque intus ubique sibi similis est, nec duae partes discerni possunt inter se, cum suis extremis discerni non possint (GM VII, 26).

In other words, similar objects are those that cannot be distinguished if not considered in a reference system. It also follows that some properties instead derive from compresence: talking about the slope of a straight line for example has no meaning if only that line is conceived, without establishing the coordinate system.

It is possible now to apply these distinctions again to the original definition of number, obtaining in this way the most detailed description of Leibniz's original passage, considering the relations involved: a number is something that can be known through a relation of homogeneity to unity, that is to say, conceiving both unity and the given number as straight lines in a simultaneous compresence. These lines have the same intrinsic properties and they are co-present with each other. Having exposed every relation involved, it is finally evident that the homogeneity between a given number and unity is not the purely geometrical homogeneity, as it was defined before. If this was the case, "L" would be similar to "B": the straight line equal to unity would be similar to the number defined. However, if being similar means having the same quality, by no means the intrinsic properties of a straight line are the same of those of a mere number. In Leibniz's terms, this would mean that a number and a straight line would be completely undistinguishable if taken as single entities, which is obviously not the case. It's clear now that Leibniz's foundation is not a purely geometrical foundation: even if we accept that the geometrical exemplification in the definition is a fundamental exemplification, unity and the other numbers are *treated* as geometrical entities but they are not *defined* as geometrical entities. In other words, geometry is used as a tool, an extremely useful and powerful tool indeed, but it doesn't express the essence of numbers: in Leibniz's definition, unity and the other numbers stand untouched by the geometrical qualities, they are just assigned to objects having geometrical properties, through the quantitative relation.

At first glance, the difference between a geometrical foundation and a foundation that uses geometry as a tool could be seen as a trivial distinction, but it bears an important consequence: ideally, equivalence and similarity can be used in the definition of number without a direct reference to geometry. It would be as admitting that quality and quantity can be found in non-geometrical contexts. According to Leibniz, this consequence seems at least legitimate: the exemplification used in the definition of quantity is consistent with this assumption, because it highlights how the notion of a measure is arbitrary, if taken alone, while the definition of quality relies on the idea of intrinsic proprieties, i.e. something legitimately conceivable in other contexts where other proprieties are given. Describing these new contexts as "every context in which the existence of qualities is admitted" is perhaps a metaphysical assumption too big to master, especially in Leibniz's mature philosophy, where some important differences between what we would call reality and the realm of mathematics are stressed. However, if this extension is limited to mathematics, there are contexts other than geometry where the definition and identification of qualities is possible. A step in this direction is done by Leibniz in this passage of the *Initia rerum*, which is worth quoting in its entirety because it directly follows the quote on algebra as an application to quantity of the science of abstracted forms, discussed in the previous section:

> Productum multiplicatione $a + b + c +$ *etc.* per $l + m + n +$ *etc.* nihil aliud est quam summa omnium binionum ex diversi ordinis literis, et productum ex tribus ordinibus invicem ductis, $a + b + c +$ *etc.* in $l + m + n +$ *etc.* in $s + t + v +$ *etc.* fore summam omnium ternionum ex diversi ordinis literis; et ex aliis operationibus aliae prodeunt formae. Hinc in calculo non tentum lex homogeneorum, sed et justitiae utiliter observatur, ut quae eodem modo se

> habent in datis vel assumtis, etiam eodem modo se habeant in quaesitis vel provenientibus, et qua commode licet inter operandum eodem modo tractentur; et generaliter judicandum est, datis ordinate procedentibus etiam quaesita procedere ordinate (GM VII, 24-25).

Here, the science of the abstracted forms of calculations is applied to similar relations, established between generic, yet distinct, numbers. A part from the law of justice, Leibniz admits that also the law of homogeneous entities is respected in this case, which means that quantity and quality are used in a non-geometrical context, hence the possibility of finding correspondences in similar relations.

Finally, it is possible to understand why in several statements about the connection between logical-metaphysical principles and principles belonging to mathematics, Leibniz considers universal characteristics above arithmetic and geometry: in this framework, a general notion is applied to two specific field with a certain degree of autonomy. These fields share with their origin the concept of homogeneity: in defining objects there must be homogeneity between characters or letters – taken as non-numerically-quantified entities –, homogeneity between numbers, and homogeneity between different geometrical objects. It follows that the real essence of Leibniz's foundational method is the distinction between quantity and quality and the way in which these concepts are applied in order to achieve a set of coherently related entities. If this is possible in non-geometrical terms, as shown, it means that the choice of the straight lines in the definition of number is in a sense arbitrary. Numbers in fact could be homogenous within themselves, so that a relation with other homogeneous entities, such as straight lines, is not necessary. However, I believe that this interpretation does not fully grasp Leibniz's intent with regards to numbers because, if numbers are already homogenous between themselves before their definition, then Leibniz's use of homogeneity in the definition would seem unnecessary.

There surely is a tension in the definition of number: homogeneity involves equality, equality involves quantity, quantity is given by compresence, but compresence is also the way in which the similar entities taken as equal to the original entities find which is greater than the other. It seems then that compresence is used both for defining quantity and for specifying quality, but it should be noted that the compresence used by Leibniz in the definition of quantity previously given is different from that used among entities having the same qualities. As I've shown, Leibniz argues that compresence in quantity is arbitrary, while in its use among similar entities it states something that is not arbitrary. I will try and give an exemplification of this process and I will use line segments in order to simplify the reasoning: we want to define the numbers 3 and 7 by establishing their homogeneity with unity, that is 1. The first step is identifying something equal to unity, i.e. arbitrarily taking something, a line segment, and stating that unity is assigned to it by means of compresence. The length of this line segment is not important, otherwise we would define quantity with quantity. The next step is establishing equality between 3, 7 and two line segments that are qualitatively similar to the line segment used for unity. Since these line segments are similar, by means of their compresence we can now see which one is greater than the other. Comparing the segments assigned to 3 and 7 to the one assigned to unity we discover that they are both greater than unity, but we also discover that the line segment related to 3 is greater than the one related to 7. We have now our *ordo*: 1, 7, 3. I have switched 7 and 3 with respect to the universally accepted idea so that I could show that order here is given only by the confrontation among similar entities. "3" and "7" at the beginning of our reasoning are mere names, they acquire their meaning only at a later time through homogeneity. This is the difference between the arbitrary compresence of equality used to assign line segments to names and the compresence applied to similar entities used in the confrontation between those line segments. It follows that conceiving from the beginning similarity between numbers would dismiss the complexity of Leibniz's idea of transformation that is found in the very definition of homogeneity and the arbitrary trait of equality defined without quality. Numbers are not immediately *unitates,* as Hobbes would say: they need to be defined through qualitatively similar objects in order to achieve that status. These objects however are not necessarily taken from geometry, if quality and quantity exist also outside geometry.

A possible outcome then would be proving that the choice of geometrical entities for the exemplification in the definition of number is the best one, or at least one of the best, in respect to Leibniz’s foundational purpose. In order to show this possibility, a thorough analysis of the geometrical entities chosen by Leibniz is needed.

Straight Lines and Line Segments In Leibniz's definition of number the qualitatively similar entities chosen to establish a relation between numbers and unity are straight lines. Until now, I deliberately postponed the analysis of these entities. However, the notion of a straight line poses a problem about its very nature: is Leibniz referring to the notion of a straight line as something that proceeds interminably in both directions, or is he referring to the notion of a line segment, i.e. something that is conceived as a finite part of a straight line? The origin of this problem lies in an incoherent use of this term by Leibniz in the *Initia rerum*: in the definition of number, Leibniz uses the word *recta* and, according to another passage[179], he makes a distinction between a straight line, called *recta* as well, and a general line, called *linea*. From this passage it seems that Leibniz is referring to a straight line, but in a third passage[180], very close to the one previously quoted, Leibniz writes about *rectae* that have some kind of finite quantity related to them, as in a comparison between line segments. It would be wise not to underestimate this problem, because straight lines and line segments could have, in Leibniz's terms, different qualities, so that, ideally, one could be more adequate than the other as a geometrical tool used in the definition of number.

In Leibniz's definition of number, a specific term is used to define the relation between the given lines, that is *Ratio*[181]. Now, this term is also used in another section of the *Initia rerum*:

> Sed omnium Relationum simplicissima est, quae dicitur Ratio vel Proportio, eaque est Relatio duarum quantitatum homogenearum, quae ex ipsis solis oritur sine tertio homogeneo assumto. Veluti si sit y ad x ut numerus ad unitatem seu $y = nx$, quo casu x positis abscissis, y ordinatis, locus est recta, locus inquam seu Linea quam ordinatae terminantur (GM VII, 23).

[179] "Recta, quae est linea intus sibi similis" (GM VII, 21), which can be adapted both to a straight line and a line segment. However: "Ex duobus punctis prosultat aliquid novi, nempe punctum quodvis sui ad ea situs unicum, horumque omnium locus, id est recta quae per duo puncta proposita transit" (GM VII, 21). The verb '*transit*' used here suggests that Leibniz is conceiving here a straight line.

[180] "Sint datae duae rectae, quae inter se comparentur utcunque. Verb. Gr. detrahatur minor ex majore" (GM VII, 23).

[181] See Leibniz (GM VII, 24).

Leibniz uses here the equation of a straight line in the form $y = nx$, where the y-intercept is equal to 0, to describe the same relation used in the definition of number, that is *Ratio*. This is also the only passage in which a reference to the previously discussed definition of number is given, because “y is to x as any number is to unity”, in the same way in which a number is to unity as a straight line to a straight line. However, in this case, if we replace x and y with unity and a given number, then n, i.e. slope, would be equal to the ratio between these numbers. In other words, it would seem appropriate to identify numbers with line segments more than straight lines, because they are equivalent to scalars, that is to say, the line segments generated by the projection of a point belonging to a straight line to the x axis and the y axis. Yet, in the definition of number, Leibniz adds a new depth to this idea, because he writes that even the ratio itself is homogeneous to unity[182]. Being a quotient, even a ratio is a number and every number can be expressed as a ratio, but in this context a ratio is also the slope of a straight line with a y-intercept equal to 0. As much as every number can be seen as a ratio, every line segment can be seen as a straight line having a slope equal to that ratio and a y-intercept equal to 0. This interpretation is confirmed by Leibniz himself: "Ex his sequitur, lineas similes esse in ratione rectarum Homologarum"[183]. Line segments are indeed similar to each other. In the end, the incoherent use of the term *recta* to identify both straight lines and line segments hides Leibniz’s belief that they can be seen as having the same function. I believe that he was aware of this problem and I would like to suggest that perhaps Leibniz left this terminological incoherence because he didn’t want to openly admit that a finite geometrical entity can be expressed through an infinite one, in the context in which the *Initia rerum* were meant to be published[184]. In the next chapter I will show the importance of the relationship between finite and the infinite in the development of Leibniz's foundational attempts.

Putting aside the mere terminological problem however, considering line segments as slopes of straight lines having the y-intercept equal to 0 highlights some important features of Leibniz’s foundations of arithmetic. If the succession of different numbers is given through a description of the relation they have with unity and the other numbers,

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[182] “Manifestum est etiam, si Ratio a ad b consideretur ut numerus qui sit ad Unitatem, ut recta a ad rectam b, fore Rationem ipsam homogeneam Unitati” (GM VII, 24).
[183] GM (VII, 24).
[184] The *Initia rerum* were probably meant to be published in the *Acta Eruditorum*: as the opening suggests with Leibniz’s reference to Wolff, this work was not conceived by Leibniz only for a personal use.

comparing line segments to see which one is greater than the other could lead to the false opinion that a comparison between numerical quantities is still involved, even if it is not the case. In the representation of every number as a straight line that differs only in having a different slope instead, it is clear that this is not the case: every number is, so to speak, a unity, because every infinite straight line is similar to that of unity, undistinguishable if not taken in a context in which the different slopes are perceivable, that is to say by means of compresence. At the same time, every number is unique, because it is defined through the infinite relations with the other numbers. The infinite relations are however expressed following a specific order that gives continuity to the definition. In this sense, any number can be conceived following this order step by step. This way of defining objects is no other than what Leibniz called *Situs*:

> Situs est coexistentiae modus. Itaque non tantum quantitatem, sed et qualitatem involvit (GM VII, 18).

> Situs quaedam coexistendi relatio est inter plura, eaque cognoscitur per alia coexistentia, intermedia, id est quae ad priora simpliciorem habent coexistendi relationem (GM VII, 25).

In this case the situational analysis would be used so that order between qualitatively similar entities could be given. In geometrical terms then, homogeneity could be seen as a stricter way of determining *situs*, because it relies only on the comparison between similar entities.

Contradiction and the Principle of the Whole Setting aside for now the problem of defining situs, from the geometrical representation of numbers through specific straight lines it also follows the most important consequence: the straight line that represents unity has the form $y = x$, that is identity: It means that every numerical identity — $3 = 3$, $8 = 8$, and so on — can be conceived as a point that belongs to the straight line which represents unity, or as the ratio of this line[185]. In Leibniz's eyes this should have been considered a great achievement, and it is perhaps one of the most convincing explanation, at least in a

[185] "Unitatem autem repraesentare Rationem aequalitatis" (GM VII, 24).

geometrical form, of the idea expressed in the correspondence with Clark about the possibility of deriving both arithmetic and geometry from identity, i.e. the principle of contradiction. Another proof of this theory rests in the analysis of the key aspects of Leibniz's foundations of arithmetic: unity and coexistence are the cornerstones of Leibniz's theory and they are both founded using the principle of identity or contradiction. As it is for unity in fact, coexistence is contradictory if it doesn't follow a specific rule: the whole is greater than a part. Comparing similar coexisting objects in fact wouldn't be enough to establish a precise order, if a principle of coherence is not given. By highlighting the role of comparison and coexistence we can finally grasp the importance of the principle of the whole, taken from Hobbes and Weigel in Leibniz's early years, and we can understand why Leibniz states many times that it is related with the foundation of mathematics. In the *Initia rerum* Leibniz gives a demonstration of this principle:

> Totum est majus parte […] Res etiam Syllogismo exponi potest, cujus Major proposition est definitio, Minor propositio est identica:
> Quicquid ipsius A parti aequale est, id ipso A minus est, ex definitione,
> B est aequale parti ipsius A, nempe sibi, ex hypothesi,
> ergo B est minus A.
> Unde videmus demonstrations ultimum resolve in duo indemonstrabilia: Definitiones seu ideas, et propositiones primitivas, nempe identicas, quails haec est B est B, unumquodque sibi ipsi aequale est, aliaeque hujusmodi infinitae (GM VII, 20).

I believe that the most important part of the demonstration is the *propositio identica*, that is that "nempe sibi" in the minor proposition: B = B. At first glance this demonstration seems quite confusing, because here the first definition seems to already embed the whole principle. In Leibniz's demonstration, being smaller in general is defined as that which is equal to a part of A. B is a part of A, but this is not sufficient in order to prove that B is smaller than A: if that was the case the demonstration would be hidden in the definition given. However, the criteria for being smaller than A is not "being a part of A", but "being *equal* to a part of A". In fact, being a part of something is not the requisite needed in order to be smaller than that thing, since I could always conceive another entity that is smaller than A, yet not being one of its parts. It follows that the requisite for the part B in order to be smaller is being equal to a part of A. Equality is not a property given in the first definition, but it follows naturally from the very existence of B, because everything that is, is also equal to itself. Since B is equal to itself, it is also equal to a part of A, thus B is smaller than A.

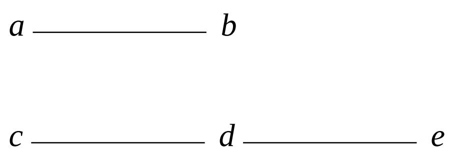
The identity of something with itself is then the cornerstone of the demonstration by which the whole is greater than the part and it is the only notion needed. If the principle of the whole, as shown, is the principle that grants coherence in the confrontation of homogenous entities and this confrontation allows the definition of numbers and such, then I believe that the identity used in the demonstration is the closest exemplification of what Leibniz meant in the letter to Clarke about deriving the whole of mathematics from identity. In my opinion, it is also quite clear from the demonstration why identity is considered the positive expression of the principle of contradiction: being equal to oneself means showing from the very existence the fact of not being something else, in the ontological sense of the principle of contradiction debated in the previous part. The identity of something to itself shows an intrinsic consistency and coherence that, once compared with another entity, which is considered equal to the former and at the same time a part of something else, exhibits the impossibility of being something else than what it is. In the demonstration used by Leibniz then there is a fundamental, yet poorly explained, difference between B taken as equal to a part of A and B taken as a part of A. This is the reason why B = B is the most important part of Leibniz's proof, because it proves through the principle of contradiction the equality between the two definitions of B.

Another way of explaining these relationships is that of geometrical terms: being equal to itself in fact could be conceived as the property of a line segment, contained in another line segment, of being equal to another line segment that is not contained in the greater line segment. Equality would be given by the superposition of the two segments. The line segment which is contained in the greater one would be B defined as a part of A, whereas the external line segment would be B defined as equal to that B taken as a part. This explanation is allowed, because in the *Initia rerum* equality has a precise definition, as shown. If being equal means having the same quantity it could be that through the confrontation of homogenous line segments Leibniz wants to define, in a rigorous way, the concept of identity.

Consistent with this idea, in the *Demonstratio propositionum primarium*, dated 1671, it is pretty clear that Leibniz was conceiving the proof already during that time as something that involved the confrontation with a third line segment equal to a part of another segment:

Totum *cde* est maius parte *de*

> Definitio: Maius est cuius pars alteri toti aequalis est
> […] Ex hac definitione maius minusque aestimant homines universi; duas enim res datas congruentes sibi aut saltem parallelas collocant, ut *ab* et *cde*, ita enim apparet *cde* esse maius, seu aliquid aequale ipsi *ab* nempe *cd* et aliquid praeterea *de* habere (AA VI-2, 482).



After this passage, the demonstration is similar to that found in the *Initia rerum*. The reference is clearly to Hobbes' *De corpore*[186], but, given the figure above, there is a significant use of line segments in order to explain the relation of equality. Identity, contradiction, equality and the principle of the whole are all related in Leibniz's mind and this outcome explains Leibniz's efforts throughout his life in all the fields in which these concepts are involved, from 1671 to 1716. It is clear that before leaving for Paris, Leibniz had not already developed a definitive conception of equality like that of the *Initia rerum*, but the basic idea was already there and it involved the distinction between quantity and quality.

[186] "*Totum esse majus sua parte* hic demonstrabimus […] Definitur *majus* esse, cujus pars est aequalis alteri toti. Si jam ponatur totum aliquod A et pars ejus B: quoniam totum B est aequale sibi ipsi et pars totius A est ipsum B, erit pars ipsius A aequalis toti B; quare, per definitionem *majoris*, A est majus quam B; quod erat probandum" (Hobbes 1999, 93.94).

The evolution of Leibniz's philosophy of mathematics, consistent with the evolution of the principle of contradiction examined in the previous chapter, corresponds to his acknowledgment of contradiction as a tool that enrich the concept of identity through the idea that an identity of something with itself expresses information also about what that thing is not.

One of the most important examples of Leibniz's efforts in connecting identity, contradiction, the principle of the whole and concepts taken from his *analysis situs* is a letter to Conring, dated March 1678. The three different drafts of the same part are testament of Leibniz's struggle. In the first formulation, written in the margin of the draft and then cancelled, is found identity, equality and the principle of the whole:

> Axiomata mihi videntur propositiones quae vel non possunt vel quas non necesse est demonstrari. Demonstrari non possunt illae, quae sunt identicae, verbi gratia: *A est A*, *unumquodque sibi ipsi aequale est*, et similes infinitae (tot enim dantur quot termini) quas qui negat eo ipso frustra ratiocinari sese ac disputare fatetur, sublata enim erit inquisitio veritatis si idem verum et falsum esse possit. Demonstrari necesse non est alias quae ab auditoribus omnibus sine difficultate admittuntur, ut *totum esse majus parte*; quae etsi formaliter identicae non sint, facile tamen ad identicas reduci id est demonstrari possunt (AA II-2, 599).

Then, in the following draft, we find identity, the principle of contradiction, equality and similarity, but the principle by which the whole is greater than its part is missing:

> Res ergo omnis redit ad definitiones et axiomata. Ex axiomatibus autem ego illa tantum per se nota, seu indemonstrabilia esse arbitror, quae sunt identica, verbi gratia *A est A* vel *A non est non A* vel *unaquaque res talis est qualis est, unaquaque res sibi ipsi similis, sibi ipsi aequalis est*. Reliqua omnia ope definitionum ad identica reduci id est demonstrari possunt. Idque Scholastici omnes confirmant, qui axiomatum veritatem patere ajunt intellectis terminis; id est posse ipsa facili negotio nec longa definitionum serie demonstrari, sive

ad identica reduci; sive quod idem est semper ostendi posse, quod contrarium implicet contradictionem (AA II-2, 599).

Finally, the final draft revolves around contradiction and identity only:

> Necessario inquam, id est ut contrarium implicet contradictionem, qui est verus atque unicus character impossibilitatis. Porro ut impossibili respondet necessarium, ita propositioni contradictionem implicanti respondet identica ; nam ut primum impossibile in propositionibus est haec : *A non est A*, ita primum necessarium in propositionibus est haec: *A est A* (AA II-2, 602).

From the impossibility of contradiction then, identity follows directly. Many other passages[187] in Leibniz's writings are dedicated to establishing the right connection between these concept and the *Initia rerum* could be seen as the final effort in this direction.

Analysis Situs and Homogeneity The fact that the definition of identity needs equality, plus the fact that Leibniz seems at times to derive the equality between the part and its equal from the superposition between line segments could suggest that in the end the concept of identity is founded on the concept of *situs*, because *situs* is a mode of

[187] I will show here some examples of the connections between these concepts from an historical perspective. In 1679, in the *Calculus ratiocinator*, similarity and homogeneity are predicated about the whole and the part; "id quo plura homogenea sunt, seu similia et cujus modificatione ipsa differunt, v.g. spatium figuris, materia corporibus, tempus horis, motus suis partibus. Pars et totum similia sunt. Omnes partes simul non differunt a toto" (AA VI-4A, 278). Again in a coeval writing we read: "Basis Homogeneorum est, quod in toto et partibus per omnia simile est", adding that "Talis basis homogeneorum est etiam materia corporum communis", a claim close to Leibniz's reflections around 1671. (AA VI-4A, 310). In 1680 we find the definition of number, connected with homogeneity and ratio: "Numerus est totum, quod resolvi potest in partes unitatis. Itaque ipsa unitas est Numerus, et pars unitatis etiam est Numerus. Eodem res redit si dicamus Numerum esse Homogeneum Unitati […] Huc Numerum habens, rationem habens. Ratio est eadem, cum est similitudo in magnitudinum comparatione. Numerus est Homogeneum unitatis" (AA VI-4A, 390). In the 1680 *Specimen ratiocinationum mathematicarum*, equality is defined through homogeneity, just after the introduction of the principle of the whole: "Totum est majus sua parte. Corollar. Duo homogenea quorum unum altero nec majus nec minus est, aequalia sunt" (AA VI-4A, 418). In the draft of a letter to Arnauld, dated 1687, whole and part are connected to homogeneity: "qu'on n'a pas communément une notion assez distincte du tout et de la partie, qui dans le fonds n'est autre chose qu'un réquisit immédiat du tout, et en quelque façon homogène" (AA II-2, 251). Interestingly enough, this remark is written in the context of a reflection on the difference between substance and matter. Being a primary requisite, it shows its application in any sort of field. Other relevant examples are found in AA (VI-4A, 381, 383, 392, 421), AA (VI-4C, 1987) and E (176).

coexistence. I believe however that in the *Initia rerum* this is not the case: while *situs* seems to be determined by a general relation of coexistence, homogeneity seems to be a form of coexistence that differs from that of *situs* because it is rigorously defined between objects that share the same qualities. There is a fundamental difference between the description of geometrical space through situational analysis and the definition founded in homogeneity: if two things cannot be superpositioned, then they do not possess the same *situs*, but it does not follow that they cannot be described in the same environment through the concept of *situs*, even if they are heterogeneous between themselves. The equality between line segments in the demonstration of the principle by which the whole is greater than its part instead is based on the superposition of two similarly described entities, that is two homogenous line segments. It follows that from a descriptional point of view, *situs* embeds a wider range of possibilities, whereas from a foundational point of view homogeneity is the most important concept.

In the *Initia rerum* in fact there is a clear connection between homogeneity and the concepts of whole and part: "Quod inest homogeneum, Pars appellatur, et cui inest appellatur Totum, seu pars est ingrediens homogeneum"[188]. The same difference between a rigorous approach found in homogeneity and a generic approach is in fact translated in the very act of confronting entities: "Aestimatio magnitudinum duplex est, imperfecta et perfecta ; imperfecta, cum aliquid majus minusve altero dicimus, quamvis non sint homogenea, nec habeant proportionem inter se"[189]. This passage suggest that proportion is the equivalent of the application of homogeneity. Later on, this suggestion is fully endorsed in a passage already quoted:

> Omnium Relationum simplicissima est, quae dicitur Ratio vel Proportio, eaque est Relatio duarum quantitatum homogenearum, quae ex ipsis solis oritur sine tertio homogeneo assumto (GM VII, 23).

Once again we are back to the relations given by ratios and proportions, the ones used in the definition of number. There is undoubtedly a shift in the definition of homogeneity: at times it possesses the distinctive feature of bringing transformation in order to compare

[188] GM (VII, 19).
[189] GM (VII, 22).

similar entities, but in this case being homogenous means being already capable of confrontation through similarity, without resorting to a third entity. This last definition of homogeneity is closer to the one generally accepted and it explains Leibniz's claims that a straight line is homogeneous with its part. This tension testifies Leibniz's efforts in connecting the finite with the infinite, a task that only Leibniz could do in those years. At the same time, it shows that Leibniz's definition of number is an original extension of a traditional one, because it is based on ratios and proportions, in the same fashion in which, as I will show in the next chapter, Euclid, Descartes and Weigel conceived it.

From the analysis of Leibniz's foundation of arithmetics in the *Initia rerum* however, we gained a better understanding of those concepts that in the following historical reconstruction will be essential in order to testify Weigel's influence on Leibniz in this topic: homogeneity, quantity, quality, ratio and proportion. These concepts led Weigel to the definition of numbers through the superposition of line segments that Leibniz will adopt at a later time, with the precise purpose of perfecting it, introducing in this topic the science of the infinite.

2.3 Ratio, Homogeneity and Situs Between Weigel and Leibniz

1663 – 1679: Weigel's Definitions of Ratio and Proportion Now that we possess some of the basic concepts of Leibniz's philosophy of mathematics, interpreting some of his passages from an historical perspective will be easier.

In his excellent book on Leibniz's *analysis situs*[190], De Risi identifies, among others, three decisive moments in the history of this concept: one of the first legitimate references found in Leibniz in 1671, the turning point in its development in 1679 and its final outcome connected with the *Initia rerum* at the end of Leibniz's life. It seems quite interesting that these moments correspond to those that define Leibniz's relationship with Weigel: 1671, as I've shown, represents the peak of Weigel's influence in the early years, 1679 is the year of Leibniz's first letter to Weigel and, as I will show, the *Initia rerum* were conceived by Leibniz in the context of Weigel's school. Without overinterpreting this coincidence, it suggests a possible relationship that should not be underestimated.

The first reference highlighted is Leibniz's 1671 preparatory work to the *Elementa de Mente et Corpore*:

> Geometria scribenda est sine motu, solo situ, vel loco seu distantia. Est enim recta situs puncti ad punctum. Caetera omnia rectarum compositiones. Hanc sequitur doctrina de productionibus vel linearum per motus, vel figurarum per sections. Ultima doctrina est productio motuum per motus. Ubi non de figura, sed vi et effectu (AA VI-2, 282).

While this quote undoubtedly shows that Leibniz possessed a precise, even if not refined, concept of *situs* before his mathematical studies in Paris, it should be clear by now that this remark is not sufficient in order to argue a complete originality of this concept in his mathematics. Leibniz's reasoning in fact is very similar to that explained in the previous part, where a distinct difference between reality at rest and in motion was established in a geometrical description of the physical world. Combined however with what I presented in the previous chapter, this outcome shows another important consequence: if the world of geometry is the world at rest and it involves the notion of quality, the fact that Leibniz

[190] De Risi (2007, 41-116).

introduces now the concept of *situs*, concept that relies on the distinction between quality and quantity, could not be a mere coincidence. It could be that at that time Leibniz already possessed the distinction between these two concepts, even more if we connect this statement with Leibniz's advancements in physics in those years. Primary matter in fact was conceived at rest with a specific property, that was homogeneity, another concept that later on will be defined by Leibniz through the distinction between quantity and quality. The last part of the previous quote seems to confirm this hypothesis, because on one hand the idea of motion as something that needs to be introduced at a later time remains also in Leibniz's mature writings on geometry[191], and on the other hand that last motion explained through "vi et effectu" clearly reveals the physics of collisions endorsed by Leibniz before Paris.

The possibility then that homogeneity, quality, quantity, similarity and *situs* were adopted by Leibniz thanks to Weigel's influence is perfectly reasonable and it could be confirmed by the analysis of many passages. If, given the explanation of Weigel's role in the studies on the philosophy of nature made in Jena, we tend to believe this was the case for homogeneity, quality and quantity, the concept of *situs* seems relatively new in the background of Weigel's influence, but it should be noted that it was instead extensively used. The concept of *situs* in fact is present in almost every work written by Weigel and, even if the terminology is not always rigorous[192], it involves the same idea of coexistence that is found in Leibniz's works on geometry. As I will soon show, Weigel's *Corporis pansophici pantologia* contains several definitions of *situs* that were known for sure by Leibniz around 1683, since notes on that book belonging to that period were found. However, even if I recalled several times that this work was probably read by Leibniz in 1670, the reference to Weigel's *Analysis aristotelica* is sufficient in order to establish a strong resemblance.

First of all, there surely is a connection between these geometrical properties and Weigel's physics. The superposition between space and primary matter granted it, as stated in the *Analysis aristotelica*:

[191] "Motus est mutation situs […] Via est locus continuus successivus rei mobilis" (GPVII, 20). "Est autem in percipiendi transitu quidam ordo, dum ab uno ad aliud per alia transitur. Atque hoc via dici potest […] ita via puncti erit linea" (GP VII, 26).
[192] Sometimes it is called *Status*, like in Weigel (1679, §21).

> Et primo quidem, quoniam Materia sensibilis a Spatio, tanquam intelligibili materia, cui partes suas coextensas habet, omnes Magnitudinis species & differentias, puta Longitudines, Latitudines, Altitudines, Profunditates, Situs & ortas inde divisiones & fîgurationes, participat (Weigel 1658, 195).

As much as space transmits geometrical properties to matter, conversely Weigel's philosophy of nature passed down to Leibniz, much more concerned about physical matters at that time, some fundamental concepts in geometry that he will re-elaborate at a later time. Another interesting use of the word *situs* in the *Analysis aristotelica*[193] connects this concept with the difference between quality and quantity:

> Corpora mundana sic in Spatio mundano hinc inde disseminata primo omnium aggreditur Physica Specialis, eorumque mundanas affetiones & proprietates, puta Situs & Motus, qualitates (Eclipses, figuras) quantitates (distantias, amplitudines, tempora) & c. pervestigat (Weigel 1658, 202).

It is true that this distinction is made on the ground of physics, but given the previous quote we can safely assume that its origin is in the geometrical properties that space gives to matter.

The most striking resemblance about the concept of *situs* however is found in the *Physcae Pansophicae*, a dissertation written by Johannes Wülfer in 1673, again under Weigel's tutoring in Jena[194]. The concept of *situs* is applied to bodies, but once again the context allows the connection with geometry, thanks to the theory of superposition between space and primary matter. In fact, after having introduced the geometrical principles, in the *axiomata propria* Wülfer introduces *situs*: "*Corpora certum inter se habent Situm*. Et hoc dicimus stare vel consistere, locari. *Corpora variis modis situm mutant*. Et hoc dicimus moveri". The result will be: "Motus est mutatio Situs"[195], which is the same definition found in Leibniz's *Initia rerum*[196]. This was Wülfer's use of the

[193] Actually, there are some other interesting uses. For example *situs* was used in measuraments: "Species Geodaesiae est *Gnomonica* h.e. Anchinoea determinandi distantias horarias in quovis subjecto magno, cujuscunque sit situs & figurae" (Weigel 1658, 240).
[194] See Herbst (2016, 364).
[195] Weigel-Wülfer (1673b, 76).
[196] GM (VII, 20).

"Situatio, quam juxta GEOMETRIAM, spatii scientiam, applicatione saltem immobilium ad mobilia definimus"[197].

Weigel's influence on these matters was quite predictable, given that in Leibniz's writings, *analysis situs* is often associated with the efforts in clarifying and demonstrating parts of Euclid's *Elements*. Weigel's *Analysis aristotelica* was conceived as an explanation of Aristotle's philosophy in the light of Euclid. For that time, thinking that Euclid came before Aristotle and that the Stagirite was greatly inspired by his work was common, mostly because there was a confusion between Euclid of Alexandria, the author of the *Elements* lived after Aristotle, and Euclid of Megara, a Socratic philosopher lived before Aristotle. Although being a clear mistake, it caused an interesting evolution in philosophy, especially among Weigel and some other authors already quoted, like Abdias Trew: it allowed the possibility of interpreting philosophy through mathematics. The Aristotle inspired by the *Elements* was the one that, while arguing in philosophy the necessity of universal principles, was referring to the geometrical properties found in Euclid's work as the best example of universal truths. This interpretation could be seen as another connection between the logical and universal principles found in Aristotle and the foundational attempts found in mathematics, because it inevitably led to Weigel's re-evaluation of the *reductio ad absurdum*, seen as the application in mathematical terms of the principle of contradiction[198].

An interesting passage of Weigel's *Analysis* about geometry is the one that explains the nature of some of Euclid's postulates:

[197] Weigel-Wülfer (1673b, 28).

[198] "Demonstratio totaliter, dixerim terminaliter spectata, non ostensiva, sed *indirecta, per impossibile* vel *ad absurdum* ducens dicitur; tum ipsa *Hypothesis* praedicato contradictoria sui *destructiva* vocatur, licet *astructiva* sit absurdi attributi per ostensivam demonstrationem collecti. Vicissim si falsum ab Adversario positum fuerit Effatum, v.g. *diametrum esse commensurabilem costae*: Analyticus commensurabilitatem assumit esse, sed ex ea quippiam absurdi colligit ostensive, nempe *parem numerum aequalem esse impari*, lib. I. prior. 35. & consequenter diametrum non esse commensurabilem per impossibile demonstrat. Ut igitur in ostensiva demonstratione Hypothesis semper est astructiva; ita in ea, quae ducit ad impossibile, destructiva dici potest terminaliter, esto quod astructiva sit absurdi consequentis. §6. Ex quo colligitur, non tantum ex falsis principiis, (puta suppositivis & hypotheticis) non ut ex causis, sed ut ex occasione data, verum demonstrari (h.e. veri demonstrationem saltem indirectam inchoari) atque ita quod res sit, ex eo quod non sit (h.e. quod alias absurdum accideret si non esset) per deductionem ad impossibile ostendi posse" (Weigel 1658, 100). The use of the word impossible and contradiction would be sufficient, but there is an even clearer remark: "*Hypothesis destructiva* […] per indirectum (adhibiro illo primo principio, quod *impossibile sit idem simul esse & non esse*)" (Weigel 1658, 86).

> Posterioris Classis Postulata sunt illa geometrica, quae, licet de potentia, vel potius abstracte de esse nominali, loquantur quatenus ad ipsam adhibentur demonstratione, actualem tamen existentiam a parte rei semper includunt, atque ita semper positiva sunt (Weigel 1658, 99).

Weigel then introduces the first three postulates of Euclid's *Elements*. He focuses his attention on the first one, specifically on the possibility of drawing a straight line that connects two points. With regards to these geometrical construction, Weigel derives then this interesting conclusion:

> *Ducere* enim *lineas* in demonstrationibus geometricis non est *facere lineas*, sed hic praesentes, & datis punctis natura jam interjectas, vel ad datam plagam actu semper excurrentes, saltem designare, & designate distincte concipere. Unde cum in demonstrando inter principia perfectiva nonnunqua allegamus *Structuram*, *Constructionem*, non cogitandum est, ipsam velut ex arbitrio nostro factam effatum non nisi contingenter probare, quin potius rei veritatem ab ipsa natura hoc ipso dependere sciendum est. Res enim geometricae […] non sunt a nobis, sed immobiles & ingenerabiles sunt (Weigel 1658, 99-100).

Weigel's general dissatisfaction over considering postulates without offering an adequate demonstration, affected Leibniz since the very beginning of his studies. Besides, geometry defined at rest reminds to that stillness that was a peculiar trait of Leibniz's geometry in the 1669 letter to Thomasius. Particularly interesting is the fact that Weigel uses in this passage the word "interjectas" about the possibility of identifying a straight line, because it is the same term that Leibniz uses in his 1671 *Specimen demonstrationum de natura rerum corporearum ex phaenomenis*[199], coeval with the demonstration of the principle by which the whole is bigger than its part that I highlighted in the previous chapter. Although then Leibniz himself admits in a famous passage[200] that at that time he

[199] AA (VI-2, 307). Again, this reference is present in De Risi (2007, 46).

[200] "Cela m'a encor empeché de lire avec soin les livres de Geometrie ; et j'ose bien avouer, que je n'ay pas encor pu obtenir de moy de lire Euclide autrement qu'on n'a coustume de lire les histoires. J'ay reconnu par l'experience que cette methode en general est bonne; mais j'ay bien reconnu neantmoins qu'il y a des auteurs qu'il en faut excepter. Comme sont parmy les anciens philosophes Platon et Aristote, et des nostres Galilée, et des Cartes" (AA II-1, 389).

had not already carefully read Euclid's *Elements*, it does not follow that he was not able to grasp through Weigel some of the notions contained in that work. This outcome was only natural, given the reference to Hobbes' principle of the whole that was elaborated from Euclid's *Elements*.

Judging by the general context in which the concept of *situs* emerges, the reformation of Euclid's *Elements*, and the debate on the role of homogeneity, a closer connection with Weigel on the distinction between quality and quantity seems possible. The last obstacle is a letter, dated 1677, sent by Leibniz to Gallois:

> Après avoir bien cherché, j'ay trouvé que deux choses sont parfaitement semblables, lors qu'on ne les sçauroit discerner que *per compraesentiam*, par exemple deux cercles inegaux de même matiere ne se sçauroient discerner qu'en les voyant ensembles, car alors on voit bien que l'un est plus grand que l'autre […] Cette proposition est aussi importante en Metaphysique et même en Geometrie et en Analyse, que celle du tout plus grand que sa partie. Et neantmoins personne que je sçache l'a enoncée. On demontre par là aisement le theoreme des triangles semblables, qui semble si naturel, et qu'Euclide demonstre par tant de circuits (AA II-1, 568-569).

This quote is not particularly relevant now for the theories displayed, because we are already accustomed to the use of coexistence, its connection with the principle of the whole, or the possibility of proving geometrical demonstration, but its importance derives from that claim – "Et neantmoins personne que je sçache l'a enoncée" – that seems to exclude any possibility for a prior influence. We could simply suppose that Leibniz were lying in that moment, but thankfully we don't need to resort to any conjecture, because it is Leibniz himself that, at a later time, will correct this statement in his *Théodicée*, by recognizing Weigel's influence. In a wider context, this reference, already given in the first part, is particularly meaningful:

> La consequence de la quantité à la qualité ne va pas tousjours bien, non plus que celle qu'on tire des egaux aux semblables. Car les egaux sont ceux dont la quantité est la même, et les semblables sont ceux qui ne different point

> selon les qualités. Feu M. Sturmius […] il tâcha de donner des regles exactes et generales dans des matieres non mathematiques, encouragé à cela par feu M. Erhard Weigel […] *Si similibus addas similia, tota sunt similia*; mais il fallut tant de limitations pour excuser cette regle nouvelle, qu'il auroit eté mieux, à mon avis, de l'enoncer d'abord avec restriction, en disant, *Si similibus similia addas similiter, tota sunt similia*. Aussi les Geometres ont souvent coutume de demander *non tantum similia, sed similiter posita* (GP VI, 245).

The reference to the help of Erhard Weigel and to a young Sturm date Leibniz's knowledge of Sturm's work to the early years. Unsurprisingly in fact, Sturm is quoted in Leibniz's 1666 *Dissertatio de arte combinatoria*[201]. Perhaps, the claims of originality contained in Leibniz's 1677 letter could be explained by the difference highlighted in the *Théodicée*, because Leibniz was developing a stricter definitional tool than that of Sturm. Nonetheless, the reference to themes belonging to the *analysis situs* already in 1671 is reasonably explained now by Leibniz's acquaintance with Surm's and Weigel's idea of similarity.

The importance of the year 1669 in Weigel's reception and the connection between geometry and primary matter lead to what I believe is Weigel's most important work on the philosophy of mathematics, the *Idea matheseos universae*. This small essay was published in 1669 and it was already quoted about Leibniz's philosophy of nature, because it contained a definition of primary matter very similar to that of Leibniz, especially highlighting the idea of a continuous transformation that ultimately leads to homogeneity. It contains a tentative foundation of the concept of number that relies on the definition of quantity through similarity and homogeneity by means of comparison between entities, both geometrical and arithmetical. Since it is a work that extends the concept of quantity to every possible field, typical of Weigel's approach, it contains several definition of quantity. The first definition, formal quantity, already entails the terminology that we will find at a later time in Leibniz: "Est igitur *Quantitas* formaliter & abstracte spectata nihil aliud, quam *determinata ratio qualitatis*, aut purius, *talitatis* seu formalitatis"[202]. In this

[201] AA (VI-1, 186).
[202] Weigel (1669, 3).

first definition, some of the concepts found in Leibniz's *Initia rerum* are already present: the need of using the concept of ratio in order to define entities, the specification that this ratio must be between qualitatively similar objects and a concept of quality that seems to embed the idea of intrinsic formal properties. The definition of the *respective* quantity connects the former definition with the idea of a comparison between something greater and something smaller and with the whole and the part:

> *Respective* definimus […] quantitatem, comparatione objecti cum sibi simili, prius tamen cognito, aut saltem supposito, quatenus illa comparatio in terminis generalibus subsistit, ut si, ad quaestionem quanta sit res? Dicamus eam esse *majorem* vel *minorem* alia quadam sibi simili, nobisque cognita […] ita τό *finitum* & τό *infinitum, totum, pars, omne, quoddam, majus, minus* termini quantitativi sunt (Weigel 1699, 12).

The comparison happens between things that are similar between themselves, even in the infinite. Another definition given to the *respective* quantity is that which expresses "*rationem aequalitatis aut inaequalitatis*"[203], so that another concept seen later in Leibniz is found. However, the most astonishing resemblance with Leibniz's theory is in the definition of the concept of *ratio*, which is worth quoting in its entirety:

> *Ratio* est *quantitas primo respective concepta*, h.e. valor unius termini in mensura alterius terminis similis & homogenei, priusque cogniti, vel saltem suppositi & assumpti. Ut si quaeratur, quam longa sit linea A ———— B ? Respondeaturque per linea pedalem C ——— D tanquam per mensuram, dicaturque, linea A B esse v.g. *quadrupedalem*. Ubi quantitas respective concepta est quadruplicitas, seu quadruplum, ratio nempe sive valor quem habet linea A B ad lineam C D. *Proportio* vero est *quantitas bis respective concepta* h.e. ratio per similem rationem, quam habent alii duo termini, expressa: Ut si ad quaestionem, quam longa sit linea A ———— B? supposita linea C ——— D tanquam ejus mensura, assumptisque simul duobus aliis terminis similiter se habentibus v.g. semuncia & drachma

[203] Weigel (1669, 3).

> respondeatur: lineam A B ad C D praecise tantam esse, quanta est semuncia ad drachmam (Weigel 1669, 14-15).

This definition of quantity by means of comparison between homogeneous enetities is almost identical to that of Leibniz. It even embeds in the end the idea that four heterogeneous entities could be defined by establishing a proportion between pairs of two homogeneous entities, much like numbers and straight lines. Naturally, this was not a theory uniquely adopted by Weigel[204], but the similarities and the context in which it was developed suggest that Weigel should be considered Leibniz's major influence in its adoption. Much like Leibniz distinguishes the arithmetical definition from the geometrical one, thanks to his abstract science of relations, so Weigel allows the independent arithmetical definition:

> Ratio differentialis sive Arithmetica est, quae similitudinem determinat per differentiam termini respective definiendi, quam habet ad terminum definientem, tanquam ad mensuram. Ita 6. arithmetice definiuntur per 2. si dicatur: Senarium esse 4. unitatibus majorem binario: ita 8. definiuntur per 8. si dicatur, nihilo majorem aut minorem esse octonarium illum, fors notum, hoc octonario forsan ignoto. (Nam & inter aequales intercedit arithmetica Ratio) (Weigel 1669, 14-15).

The effort was that of defining by means of comparison, without resorting to quantification in the commonly accepted sense. If this is Weigel's general mathematical and metaphysical principle that governs all things, it is possible to understand now the extension of the concept of ratio and proportion to ethics and law found in his school. Being at the core of Weigel's metaphysics, it is highly improbable that before the Parisian stay Leibniz never came across this idea and the definition of all the properties that it involved. Above all, because the concept of homogeneity here defined through similarity was also used by Weigel in that physical account of the world that had such an important impact on the young Leibniz's philosophy. This common origin of Leibniz's and Weigel's

[204] The most common references are Descartes' *Regulae ad directionem ingenii* (Descartes 1963, X, 11), Cavalieri's *Geometria* (Cavalieri 1653) and unity interpreted as the infinite number in Galileo (Galilei 1929, VIII, 69). However, particularly important for Weigel is Kepler's *Chilias logarithmorum* (Kepler 1624, 7).

theories will help us in understanding the development of their relationship in the following years.

1679 – 1683: Leibniz's First Letter to Weigel In September 1679, Leibniz sends his first letter to Weigel. Since it is an important letter in order to understand the impact of Weigel on Leibniz's development of binary arithmetics, I will reserve its complete analysis to the next part of the present work. However, some interesting insights could be achieved also on the topic of this part. After a reference to his 1672 *Accessio ad Arithmeticam Infinitorum*[205] in fact, in this letter Leibniz presents to Weigel some of his studies on arithmetic progressions. It is particularly interesting, not only because the numbers chosen for the progressions are the so called figurate numbers, taken from the Pythagorean tradition, but also because, just after having introduced the different series, Leibniz analyses the same numbers, taken however as fractions having one as a numerator. It was not something unique for Leibniz, given that the same argument is present in a fragment dated 1675[206], or judging by Leibniz's 1672 *Accessio*, where this topic is clearly connected with Pascal, but now that I have established the relevance of the concept of ratio in Weigel's philosophy it could be hypothesized that Leibniz's choice of analysing the series as fractions depended also on Weigel's influence. In this regard, I will argue that the superior mathematical knowledge achieved in Paris set Leibniz on the project of extending to the infinite the usefulness of Weigel's concept of ratio.

As a starting point, we could ask ourselves if Leibniz's ideas on the number series were not completely rejected by Weigel's school. Predictably, this is not the case, as one of Vinhold's *Theses* show[207] and, above all, as Weigel's *Analysis aristotelica* show:

> Nam sicut *Proportio* duplex est, *simplex* nempe, quae est identitas duarum tantum rationum, qualis est, quam 2. habet ad 4. & 4. ad 8. (uti enim quaternarius binarii ; ita & octonarius quaternarii duplum est, adeoque eadem est utrobique ratio) & *continuata*, quae est identitas plurium rationum continua serie geometrice progredientium v.g. 2 ad 4. 4. ad 8. 8 ad 16. 16 ad

[205] "Notavi nimirum primus atque etiam demonstravi theorema sequens quod nulli inter arithmetica hactenus inventa cedere visum est amicis" (AA II-1, 745).
[206] AA (VII-3, 676).
[207] "Facilius partier & jucundius est computare per fractos quam per integros" (Weigel-Vinhold 1671, 216).

32. & ita in infinitum: [...] in se contineat ejusdem speciei proportiones [...] Et sicut in additionibus, unde syllogismus spectata hac analogia nomen est mutuatus, si plures fuerint addendi termini, una addito est & una summa, licet numeris in quotcunque classes distinctis plures summae particulares constitui possint, quae postea seorsim additae nihilominus eandem exhibeant summam communem, ita & hic (Weigel 1658, 149–150)[208].

Here, Weigel's concept of proportion is applied to a series of fractions, without rejecting the idea of the infinite. Certainly, Leibniz was aware of Weigel's interest in the concept of ratio and he presented accordingly in 1679 a reasoning that could have made manifest to his former teacher his improvements, and superiority, in mathematics:

Nimirum summa hujus seriei infinitae $\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}$ etc. est finita nempe $\frac{1}{1}$ quemadmodum facile si desideres demonstrare possum […] At summa seriei hujus infinitae $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}\ \frac{1}{5}$ etc. Est $\frac{1}{0}$ quae quantitas est infinita, major scilicet quovis numero assignabili, quemadmodum etiam demonstrare possum. Interim multo imo infinities minor est quam summa seriei hujus $\frac{1}{1}\ \frac{1}{1}\ \frac{1}{1}\ \frac{1}{1}$ etc. Vides itaque inter illud infinitum ordinarium, quod in omnium unitatum collectione consistit, et inter finitum, nempe unitatem, dari aliquid intermedium nempe $\frac{1}{0}$ quod est summa fractionum omnium possibilium numericarum, unitatem pro numeratore habentium, simul sumtarum (AA II-1B, 745).

The result of the first series is a finite number, while the result of the last series is the regular infinite. Between the two however there is another series that is still infinite: this result is Leibniz's first attempt, with regards to the relationship with Weigel, in applying the concept of ratio without worrying about the impossibility of understanding the infinite. Weigel's answer however will be very disappointing: "Infinitum enim definiri contradictionem implicat, hinc ipsum medium vel quasi nempe $\frac{1}{0}$ i.e. unum nihil,

[208] This passage is already quoted in Bullynck (2013).

indefinitum est"[209]. Leibniz reaction to Weigel's answer, that was basically dismissing the problem, is annotating on this reply the demonstration that the series $\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}$ is equal to 1[210]. Unfortunately, there is no trace of the much more problematic demonstration, that is $\frac{1}{2}\,\frac{1}{3}\,\frac{1}{4}\,\frac{1}{5}$ is $\frac{1}{0}$, even if Leibniz states in his previous letter that it can be demonstrated. I believe however that Leibniz was relying more on his intuition than on a rigorous demonstration for this series. Nevertheless, the exchange with Weigel was considered fruitful by Leibniz in this regard, as a letter to Clüver of the following year shows[211]. From that moment, Leibniz's consideration for Weigel diminishes from a theoretical point of view, although in those years we witness a renewed interest in sharing some projects for the advancement of the society of that time[212].

A part from the adoption of the concept of ratio in the early years, another reason why Leibniz's 1679 letter could be seen as a prosecution in the infinite of Weigel's aim is that a fundamental work, that is *De supputatione multitudinis*, was sent by Weigel to Leibniz before Leibniz's first letter. In this work Weigel offers a demonstration of the rules of proportions and defines once again ratio using the concept of homogeneity: "*Arithmetica Ratio* dicitur *Valor duorum finitorum*, eat. Homogeneorum inter se, *nomine* multitudinis *puro definitus*"[213].

The complete revaluation of Weigel's philosophy however happens shortly thereafter, around 1683, thanks to Leibniz's notes on several of Weigel's works. The most important one is the *Corporis panophici pantologia*, because, judging from the length of the notes, Leibniz's spent most of his time on the definitions of *situs* there exposed[214]. Many of these definitions would be sufficient to prove Leibniz's debt towards Weigel, but the geometrical one is particularly relevant. In the first section, Weigel analyses the *classis situum purorum* and with respect to the correlative *situs* he identifies four different categories: conjunction, disjunction, convergence and divergence. In this framework,

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209 AA (II-1, 762).

210 Strictly speaking, Leibniz tries to demonstrate that $\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}$ is equal to 2. (AA II-1, 765).

211 "Circa serierum summas multa inveni quae magni usus esse expertus sum" (AA II-1, 800).

212 For Leibniz's hesitation see the letter to Christian Philip dated March 1681 (AA II-1, 814), while for the projects of reforming the calendar and founding a new school see the correspondence with Vincent Placcius and De Bauval (AA II-3, 375, 378)

213 Weigel (1679, Exemplum I).

214 "Relatio est modus respectivus quo unum ad alterum se refert. Status aut habitudo rei est modus terminativus quo quippiam quocunque modo constitutum est et se habet. Situs est modus terminativus, quo quodque certa ratione se sistit in consortio et complexu caeterorum omnium" (AA VI-4B, 1191).

Weigel introduces the concept of *coincidentia* as a type of *situs*. It could be partial coincidence or total coincidence, and the exemplification is the superposition of two triangles. The opposite type is *disjunctio*, that is distance, conceived as the study of the situational relation between points, points and lines or surfaces, lines and lines or surfaces, and surfaces and surfaces[215].

Once again the concepts of situation, homogeneity and ratio were all present in the work of Leibniz's former teacher. It could be that Leibniz was not completely aware of the debt he owed to Jena's cultural movement, but in time he eventually realised it. However, the challenging ideas expressed in Weigel's writings were not complemented by an appropriate mathematical knowledge. It is thanks to this tension that in the following years Weigel's legacy will become for Leibniz something that needed to be perfected.

1683 – 1716: The Acknowledgment of Weigel's Importance After 1683, Leibniz's desire of a confrontation to prove Weigel his exceptional knowledge in mathematics weakens: while still finding in Leibniz's writings honest criticisms about Weigel's philosophy, especially on the demonstration of God's existence, Leibniz generally shows a benevolent attitude towards Weigel. After the disappointment of Weigel's 1679 reply a process of re-evaluation of the role of his former teacher was taking place. The first hint in this direction was, as I have shown, Leibniz's effort in reading and annotating some works written by Weigel that he already read when he was younger. Later, in Leibniz's 1689 *Phoranomus*, Weigel is remembered as the professor that shared with Leibniz his philosophy of nature in the early years. Leibniz's need of rethinking Weigel's philosophy is evident in an unpublished manuscript, dated around the time of the *Phoranomus* and entitled *Inventa Weigelii doctrinalia*. It is a list of Weigel's achievements in every field of the human knowledge, from philosophy to ethics and education. Judging by the writing style, the manuscript was probably written for personal use and it shows a quite impressive knowledge of Weigel's accomplishments that is difficult to retrace judging only by Weigel's references in Leibniz's texts. The use of numbered lists is also quite revealing, because it suggests the idea of organizing something that was qualitatively relevant and

[215] Weigel (1673c, 69). Even if it is extremely relevant, the passage is presented in the text in a schematic way that is difficult to report in its original state without causing confusion.

at the same time expressed in a confuse way, as Leibniz often remarks about Weigel in his correspondence. Predictably, in the list of the philosophical achievements the concept of ratio and proportion are present[216].

In 1690 the plan of perfecting Weigel consciously emerges in Leibniz's *Animadversiones ad Weigelium*:

> Scientia de quantitate in universum vel de aestimatione, ut vocat celeberrimus Weigelius, mihi pro dimida tantum parte tradita videtur. Exstat enim ea tantum pars quae finitas quantitates versat; sed restabat matheseos generalis pars sublimior, ipsa scilicet Scientia infiniti saepe ad finitas ipsas investigandas necessaria, quam fortasse primus analyticis praeceptis adornavi, novo etiam calculi genere proposito, quem nuncegregii viri passim adhibent (GRUA, 148).

Although Leibniz believed that Weigel neglected the science of the infinite, he tried until the end of his life to introduce him to that science. In the same year of the *Animadversiones* in fact, Leibniz writes to Weigel, trying to stress the connection between his mathematics and the use of ratios that Weigel promoted[217], with very little success.

After the death of Weigel, Leibniz will eventually achieve this conciliation. In 1710 he published in the *Miscellanea Berolinensia*, the journal of the Berlin Academy, a *Monitum De Characterinus Algebraicis*, i.e. an essay on the mathematical symbols used at that time. In this essay he adopts from Weigel's *Philosophia mathematica*, published in 1693, the mathematical symbols for 'greater' and 'less'[218]. I believe that this is relevant for several reasons. Having explained the importance of the idea of comparison, connected with the principle by which the whole is greater than the part and with the concept of

[216] LH 38 Bl., 178.

[217] "Quidem circulatione Harmonica hoc est tali ut distantiis a sole sumtis in progressione Arithmetica velocitates circulandi sint in progressione Harmonica (quod fit si velocitates sint distantiis reciproce proportionales), tunc planetam moveri secundum legem a Keplero ex observationibus ductam ut scilicet areae orbitae ex sole radiis eductis abscissae sint temporibus proportionales. Puto etiam Te specimina vidisse novae cujusdam analyseos a me inventae, cujus ope circa indivisibilia (hoc est infinite seu incomparabiliter parva) et infinita ratiocinor perinde ut in quantitatibus ordinariis. Eaque ratione lineas transcendentes (ut appello) quas Cartesius sua sua Geometria et Analysi excluserat, calculi legibus subjicio" (AA II-2, 347).

[218] GM (V, 158). This reference is also found in some histories of the mathematical notation, such as Cajori (1928, II, 484).

homogeneity, we can hardly believe that this was an accidental homage: we could say that from a certain perspective they are the most important mathematical symbols for Leibniz. Quite revealing in this regard is that the symbol for 'greater' is no other than a line segment parallel to a smaller line segment ( ) , while 'less' is represented conversely. It suggests the idea that determining what is greater or less involves the comparison between two line segments, that is the coexistence of two homogeneous entities, in the exact same way in which the demonstration of the principle of the whole was exemplified by Leibniz in 1671.

Clearly, this may seem a simple conjecture, but the exam of these writings reveals a closer connection. In Weigel's *Philosophia mathematica* those symbols are introduced while defining two specific concepts: *Totum* and *Pars*[219]. On the other hand, the very structure of Leibniz's *Monitum* suggests that he was trying that completion of the finite with the infinite that Weigel's account needed. At first he introduces the common operations, like sum and such, then he defines the fundamental concepts of equality, greater and less and in the end he introduces proportionality, similarity and congruence[220]. These are the principles needed for the finite, but his exposition does not end there: "Quas exposuimus Notae, ad Analysin commune pertinent, seu ad scientiam Finiti, sed novae adjecta sunt Notae, per detectam nuper *Scientiam infiniti*, seu *Analysin infinitesimalem*". The ideal completion of the finite with the infinite suggests a possible connection that is confirmed by the following essay contained in the *Miscellanea Berolinensia*, written again by Leibniz, the *Symbolismus memorabilis calculi Algebraici & Infinitesimalis in comparatione potentiarum & differentiam ; & de Lege Homogeneorum Trascendentali*. As the title suggests, this comparison will lead Leibniz to a further adoption of homogeneity[221], since this is indeed the text in which Leibniz introduces the famous transcendental law of homogeneity. The concept originally taken from Weigel in 1710 was adapting to both the infinitesimal calculus and the *analysis situs* from a foundational perspective.

It seems reasonable then that after four years, an important work such as the *Initia rerum* was conceived in the same background. It is widely known that the *Initia* were written in response to Wolff's *Elementa matheseos universae*, published in 1713.

[219] Weigel (1963, 135).
[220] GM (V, 158-159).
[221] GM (V, 165).

However, Wolff was acquainted with Jena's University: although Weigel was already passed away, he was considered a "Schüler im indirekten Sinne"[222], because in 1702 he studied in Jena under Hamberger and Hebenstreit, Weigel's scholars. On a side note, the influence of Weigel could be the reason why Wolff's interpretation of Leibniz's philosophy relies heavily on the principle of contradiction.

With regards to our purpose instead, this connection suggests that the *Initia rerum* were not written as an immediate reaction to an isolated read. It could be that Leibniz during that time was in contact once again with Weigel's school. Given that in 1710 Leibniz publishes the *Théodicée*, where Weigel and Sturm are quoted, while adopting Weigel's mathematical notation, a connection with Georg Albrecht Hamberger, Weigel's and Sturm's scholar and Wolff's teacher, seems reasonable.

In conclusion, there is a precise path that from an historical point of view leads to those general ideas about Leibniz's philosophy of mathematics that I analysed in the previous chapter through the *Initia rerum mathematicarum metaphysica*. Since the beginning and throughout his life, Leibniz had the chance of sharing his ideas with Weigel, even after the death of his teacher.

[222] Herbst (2016, 365).

PART III: Binary Arithmetics

3.1 A Tentative Reconstruction of Weigel's Influence

Preliminary Remarks In this part I will argue that Weigel’s *De supputatione multitudinis a nullitate per unitates finitas in infinitum collineantis ad deum* is one of the most important works that influenced Leibniz’s development of binary arithmetics. This work, published in 1679, the same year of Leibniz’s *De progressione dyadica,* contains some fundamental ideas adopted by Leibniz both in mathematics and metaphysics. The controversial theories expressed in these writings will also help in understanding why Leibniz tried to hide Weigel’s influence between 1671 and 1710. The origin of binary arithmetics is a quite independent topic from the ones analysed in the previous parts, mainly for two reasons: the first one is that philosophy of mathematics, as it will be described here, does not concern the use of metaphysical concepts, like those of homogeneity and quality, in order to achieve a better insight on the foundation of mathematics, but it concerns the application of mathematics to metaphysical concepts, for example by describing real entities through the use of numbers. The second one is that, given that past interpretations tend to dismiss Weigel's role on this topic, the analysis will be focused more on the possibility of the transmission of Weigel's ideas to Leibniz, than on their affinity, which is evident per se. Hence, the problem has to be analysed also from the point of view of the history of science, especially evaluating past interpretations of Weigel's influence on Leibniz with regards to binary arithmetics.

In recent years, Leibniz’s previously unpublished writings have cast a new light on his relationship with Weigel, from a mere influence during the early years to a confrontation that lasted at least until Weigel’s death in 1699 and beyond. Some interpreters focused on Weigel’s influence in specific topics, such as the endorsement of Aristotelian ideas, the mathematization of reality or logical reasoning, but a complete reconstruction of the relationship and confrontation between Weigel and Leibniz is still missing: this fact led to the underestimation of Weigel's importance in the topic of binary arithmetics.

Moreover, the idea of dismissing Weigel’s importance at a later time in Leibniz's life is perhaps influenced by Leibniz himself, who in a letter to Christian Philipp dated March 1681 shares this opinion on him:

> Mons. Weigelius a beaucoup d'esprit sans doute ; mais souvent il est peu intelligible, et il semble qu'il n'a pas tousjours des pensées bien nettes. Je voudrois qu'il s'appliquât plus tost à nous donner quantité de belles observations, qu'il a pû faire en practiquant les mecaniques, que de s'amuser à des raisonnemens generaux, où il me semble qu'il se perd quelques fois. Non obstant tout cela je ne laisse pas de l'estimer beaucoup ; et de reconnoistre qu'il se trouve beaucoup de bonnes pensées dans tous ses écrits (AA II-1, 815).

This is a perfect example of Leibniz's general and ambiguous attitude towards Weigel: on one hand he prises some of Weigel's ideas and on the other hand he acts as if he is judging them from a distance, without recognising explicitly their direct influence on his philosophy. As Leibniz remarked, being a philosopher who establishes a perfect connection between metaphysics, ontology, physics and human knowledge, Weigel could appear indeed confusing, especially since his style somehow tries to express all these relationships at the same time, but this difficulty shouldn't sway us from our purpose of identifying the exact extent of his influence in the development of the dyadic.

I believe that the outcome of this confrontation could prove itself extremely useful for understanding some intricacies that were always related to binary arithmetics in Leibniz: its relation with the other parts of Leibniz's philosophy and their development in time, or its enigmatic use both as a mathematical tool and a metaphysical tool. The outcome of this analysis will be that Leibniz has not developed binary arithmetics as a mere mathematical tool and then applied it to metaphysics only at a later time, because since the very beginning he was influenced by Weigel's *De supputatione*. This work already embeds some of the most important features of Leibniz's binary arithmetics, both from the mathematical and the metaphysical point of view. It also helps us in understanding why and how the topic of binary arithmetics could be related to the topic of *analysis situs*, which, as I previously argued, saw a fundamental evolution in the same year in which Leibniz developed his binary system.

As a starting point then, after having analysed the *status quaestionis* on this topic, I will outline the correspondence between Weigel and Leibniz in 1679, in order to point out the importance of Weigel’s *De supputatione* in the light of a Pythagorean influence and a possible connection between the ideas expressed in the letters and binary arithmetics. In the second chapter, I will offer a brief introduction on Leibniz’s binary arithmetics, focusing on some important concepts spread throughout his writings, and a thorough analysis of Weigel’s *De supputatione*. The exact correspondence between Leibniz’s and Weigel’s ideas will lead us to recognize Weigel as one of the main influences in Leibniz’s development of his arithmetics. Ultimately, the analysis of these topics will help us express a reasonable hypothesis on why Leibniz tried to hide Weigel’s influence during his life.

Weigel's Reception in 1679 and the Debate on Leibniz's Originality The topic of a supposed influence of Weigel on Leibniz’s development of binary arithmetics is rather old and dates back to Couturat’s *La logique de Leibniz*. In this book Couturat argues with an impressive intuition, given the availability of primary sources at that time, that Leibniz was influenced by Weigel’s *Tetractys*, a work published in 1673 which explains a way of counting in a base-four system, instead of the usual base-ten one. Couturat’s reasoning is simple: since the first writing on binary arithmetics in Leibniz, *De progressione dyadica*, is dated 15 March 1679, it could be that Leibniz took the idea of changing the base and then applied it to his base-two system. This interpretation is justified by a kind of accusation formulated by Johann Bernoulli in a letter dated 11 April 1701 in which he outlines the similarities between the two systems, more than Couturat’s actual research on Weigel[223]. In a letter dated 20 April 1701, Leibniz replies that, even if the similarities could be perceived, he started his reflections on these topics many years before his confrontation with Weigel’s work. On a side note, he also adds that his system is much more useful than the one explained by Weigel, because there is no real reason for a human

[223] In the *Logique* we read : “son Arithmétique dyadique ou sa numération binaire. Il importe de donner un peu plus de détails sur celle-ci. On a vu que Leibniz avait été amené à cette invention par la recherche d'une notation aussi claire et aussi adéquate que possible pour les nombres. Elle lui avait été probablement suggérée par la *Tetractys* de son ancien maître Weigel, publiée en 1673. Leibniz n'approuvait pas ce système de numération a base 4, qui n'avait aucune raison d'être“ and again in the related footnote: “Pourtant Leibniz prétendait plus tard avoir inventé sa Dyadique avant la *Tetractys* de Weigel. Peut-être sa mémoire le trompait-elle, ou s'exagérait-il son originalité; peut-être aussi l'idée première lui avait-elle été suggérée, non par le livre, mais par l'enseignement de son maitre : *Lettre à Jean Bernoulli*, 29 avril 1701 : ‘Molitus hoc sum ante multos annos, etiam antequam quicquam constaret de Tetracty illa nuper ressuscitata’ (Math, III, B, 2)“ (Couturat 1901, 473). As for Bernoulli and Leibniz’s correspondence on this topic see the forthcoming AA (III-8).

being to change his way of counting from a base-ten model to a base-four one, whereas the base-two model follows the idea of simplicity, since only the number 1 and the number 0 are used[224]. At the same time, Leibniz's system shows in his opinion a better way to express some proprieties that pertain numbers in general and their progression. Despite Leibniz's efforts in claiming the originality of his theory, it seems that the idea of a decisive influence by Weigel was shared between Bernoulli's brothers, as a letter from Jakob Bernoulli dated 28 February 1705 shows: "De mysterio Arithmeticae Tuae Dyadicae (quam video esse supplementum Tetractys Weigelianae) nihil adhuc mihi innotuerat"[225].

The topic of understanding Weigel's influence then seems to be reduced to whether believe or not in Leibniz when he says that he was not aware of Weigel's writings at the time of the birth of binary arithmetics. Another take on this problem is that of Gaston Grua, who argues that Couturat mistook Weigel's *Aretologistica* with the *Tetractys*: "La *Tetractys* classe les êtres par quatre. L'invention du calcul binaire le 15 mars 1679 ne lui doit rien, malgré COUT. Op. 278, qui a confundu cet ouvrage avec le premier exposé de la numération quaternaire, en 1687, en appendice à l'*Aretologistica*"[226]. Grua bases his assumptions on a passage from Leibniz's *Animadversiones ad Weigelium*:

> Atque hoc nunc quidem ad *Speculum* Viennese breviter notare placuit; praesertim cum nondum antea mihi fuerit lectus hic liber, non magis quam alter Aretologisticus, qui longius etiam sese in res metaphysicas diffundit […]. Quod tetractycam arithmeticen attinet, arbitror in praxi si quid mutandum esset potius duodecimalem vel sedecimalem fore adhibendam pro decimali; quo majoris enim numeri progressio adhibentur (dummodo tabulae Pythagoricae fundamentales memoria teneantur) eo expeditior est calculus […] puto non tantum tertactycam decimali esse praeferendam; sed et ipsi

[224] Strictly speaking, in the dyadic 1 and 0 are digits and not numbers. Numbers are those composed by sequences of 0 and 1. However, I will keep using the word number because I believe that the fact that 1 and 0 are also numbers is very relevant with regards to what Weigel and Leibniz were thinking. Those were not mere digits: 0 was the number generally associated with nothingness and 1 was the number used by Galilei in his reflection on the infinite number and by Weigel in his way of conceiving God and the world. The excitement that often shine through Leibniz while talking about binary arithmetics derives also from the fact that the digits used in the definition were those exact numbers.
[225] AA (III, 96).
[226] See GRUA (I, 330).

> teractycae rursus praeferendam esse dyadicam, quae omnium perfectissima est (FOUCHER2, 164-166).

Both Couturat's and Grua's theories are somehow defective for different reasons. We cannot prove with Couturat that Leibniz read the *Tetractys* before 1679, because Leibniz's first references to this work are notes taken in 1683, after the birth of the calculus[227]. On the other hand, Grua's assumption that Weigel's first writing on a base-four model is the *Aretologistica*, dated 1687 i.e. after Leibniz's *De progressione dyadica*, is based on a mistake[228]: around 1673 Weigel published not one, but two different works, one entitled *Tetractyn tetracty pythagoreae correspondentem*, and the other one entitled *Tetractys Summum tum Arithmeticae tum Philosophiae discursivae Compendium*. If it is true that the former work, the one that was probably verified by Grua, deals only with the general ideas related to the *tetractys*, the latter contains a base-four model very similar to that of the 1687's *Aretologistica*[229].

The reconstruction of these interpretations however leads us to an important result: even if it is highly doubtful, we could still believe Leibniz when he declares to Bernoulli that he was not influenced by Weigel's *Tetractys*, because we could still believe that he was referring only to the specific work entitled *Tetractys* and we could still assume that Leibniz's notes on it dated 1683 correspond also to the first moment in which he actually read it, but the same thing cannot be said for the reference to Weigel in the *Animadversiones ad Weigelium*, because in this work Leibniz deals with Weigel's base-four system in general. The *Animadversiones* are in fact dated 1690 and at that time Leibniz already read the *Tetractys* and several other works by Weigel, included his *De supputatione*, as we will soon prove. In other words, it seems somehow suspicious that in the *Animadversiones* Leibniz, after so many reads on Weigel, feels the need of quoting

[227] See AA (VI-4B, 1162). As shown, around 1683 Leibniz reads and annotates several writings published by Weigel in 1673: the *Tetractys, summum tum arithmeticae tum philosophiae discursivae compendium artis magnae sciendi genuina radix* and the *Tetractyn Pythagoreae corrspondentem ut PRIMUM disceptationum suarum specimen ulteriori curiosorum industria exponit Societas Pythagorea,* but also the *Methodus discendi nov-antiqua,* the *Universi corporis Pansophici caput summum a rebus naturalibus moralibus et notionalibus denominativo simul et aestimativo gradu cognoscendis abstractum* and the *Corporis pansophici pantologia*. At that time then, Leibniz read both writings on the *tetractys*.

[228] This mistake is also made in other reconstructions of the history of binary arithmetics, for example in Glaser (1981). Another take against Weigel's influence on the birth of binary arithmetics is found in Knecht (1981, 28).

[229] See Weigel (1673a, 15-24). I'm not quoting here passages from this work because there is contained nothing more than the exemplification of the calculus.

the *Aretologistica*, an obscure and confusing work in German, published many years after the first writings on this topic, while he was perfectly aware of Weigel's earlier and more important essays written in Latin. It is as if Leibniz wanted to divert the attention from the similarities between him and Weigel. I believe that this is a key factor in determining the extent of Weigel influence on this topic, because it suggests that perhaps there are more similarities than the ones generally recognised. The older interpretations are based in fact only on a supposed influence on the idea of changing the base system and on the operations derived from this change, but I would like to argue that Weigel's influence is much deeper and it has its roots also in the metaphysical background related to binary arithmetics.

The first step in this direction is the analysis of Leibniz's and Weigel's correspondence. It seems that everything revolves around year 1679, the fated year of Leibniz's *De progressione dyadica*, and it is not a coincidence that the first letter to Weigel was sent by Leibniz in the same year, in September. This letter starts with an extremely useful information for our purposes:

> Dissertationem tuam *de supputatione* legi non sine magna animi voluptate et quod eam mittere voluisti gratias ago. Quanquam enim nonnulla non satis assequerer, multa tamen notavi praeclara et profunda. Eaque occasione Tibi proponam observationem meam quae ad institutum tuum (tractas enim ut in titulo habes *de supputatione multitudinis a nullitate per unitates finitas in infinitum collineantis ad Deum*) pertinere nonnihil videtur (AA II-1, 745).

From this passage we can infer that in September 1679 Leibniz already read Weigel's *De supputatione*, but also that we don't know the beginning of this correspondence, that is Weigel's first letter, or something similar, in which he attached his work. Given that this work contains many similarities with Leibniz's writings on binary arithmetics, determining if Leibniz received it before the 15th of March 1679 could also determine if Leibniz was directly influenced by Weigel on this topic in that year. Unfortunately, there is no way to retrieve this information, but further observations are needed.

Weigel's *De supputatione* could have been published before the 15th of March 1679, plus it wouldn't be the first time in which Leibniz is aware of a work written by Weigel before its actual publication, even more considering that at that time Weigel already had a high opinion of his disciple[230]. Also, Leibniz's letter seems at least unusual: it starts with this reference to Weigel's work but, given that few months before Leibniz already developed his binary arithmetics, one would think that the observations Leibniz wants to make are going to be a brief exposition of this theory, in order to celebrate the affinity with his teacher's ideas. Surprisingly, there is no mention of his binary arithmetics in this letter, replaced instead with the exposition of peculiar proprieties pertaining some number series that I already analysed. Besides, this is not a letter with no philosophical or mathematical content, since, aside from the aforementioned number series, there are also references to Leibniz's 1672 *Accessio ad Arithmeticam Infinitorum* and to his recently developed *characteristica geometrica*[231].

By no means the observation on number series is not consistent with the topics of the *De supputatione*, as Leibniz himself writes, because the number series chosen are built through a continuous addition of unities in order to generate the series of natural numbers, the series of triangular numbers and the series of pyramidal numbers. These series are similar to the Pythagorean ideas expressed in Weigel's *De supputatione*, because they suggest the possibility of expressing geometrical dimensions through the repetition of a simple unity. However, if this was Leibniz's aim, the absence of a theory, such that of binary arithmetics, representing so well the idea of a constitution of the world through unity and zero could not be a mere coincidence.

The analysis of the confrontation between Weigel and Leibniz then suggests that a study on the ideas and terminology used in their 1679's writings is needed in order to compensate for the lack of information on the reception of Weigel's *De supputatione*.

[230] I'm referring here to the already quoted letter to Hermann Conring, dated 1670 (AA II-1, 70). As I already argued. the reference is probably to Weigel's *Universi corporis pansophici caput summum*, a work that will be published three years after this letter. It is particularly relevant for two reasons: it is a work in which some important ideas related to the *Tetractys* are already present and it is one of the work analysed by Leibniz in 1683, casting a lot of doubts on the hypothesis that those notes could testify that those works, *Tetractys* included, were read by Leibniz for the first time in 1683. About Weigel's appreciation for Leibniz, the very last sentence of Weigel's *Corollaria* in his *Pendulum ex tetracy deductum...sistit* already highlighted is a sufficient proof. (Weigel 1674, Corollaria, 8)

[231] AA (II-1B, 745n, 747n).

3.2 The Role of Weigel's *De supputatione* and Leibniz's Theological Concerns

De organo sive arte magna cogitandi in the light of Weigel's De supputatione Retracing the influences of other authors in Leibniz's four-page manuscript *De progressione dyadic*a is not an easy task: Leibniz presents here nothing more than the operations needed in his binary calculus (addition, subtraction and the like) and the hypothesis for a machine built on these principles. Perhaps, the only hint we could find in this work is a reference to the multiplication table, called table of Pythagoras, that here is quoted two times[232]. It tells us that one of the main topics involved in the development of the calculus is the possibility of simplifying operations or completely avoiding the use of such tables. This topic is consistent with what Leibniz writes in the *Animadversiones ad Weigelium*, in which he points out, quoting again the table of Pythagoras, how the use of Weigel's four-base model does not help much in this effort, as I've already shown.

Thankfully, Leibniz's *De organo sive arte magna cogitandi*, written around the same time of his *progressione dyadica*, gives us a better understanding of Weigel's influence. In this work Leibniz associates for the first time his binary calculus to the metaphysical relationship between unity and nothingness:

> Fieri potest, ut non nisi unicum sit quod per se concipitur, nimirum Deus ipse, et praeterea nihilum seu privatio, quod admirabili similitudine declarabo. Numeros vulgo explicamus per progressionem decadicam, ita ut cum ad decem pervenimus, rursus ab unitate incipiamus, quam commode id factum sit nunc non disputo; illud interea ostendam, potuisse ejus loco adhiberi progressionem dyadicam, ut statim ubi ad binarium pervenimus rursus ab unitate incipiamus […] Immensos hujus progressionis usus nunc non attingo: illud suffecerit annotare quam mirabili ratione hoc modo omnes numeri per unitatem et nihilum exprimantur (AA VI-4A, 158).

This relationship, displayed at such an early stage in the history of binary arithmetics, proves that since its beginning Leibniz develops the mathematical achievements together with the metaphysical ones. Following the platonic tradition, Leibniz conceives

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[232] See LH 35, 3b 2 Bl., 1, 3.

nothingness as non-existence, as a tool that helps shaping the world, but what he adds is that the role of nothingness resembles the role of the mathematical zero, whereas the role of existence, both God and creature's kind of existence, resembles the role of the mathematical unity. It follows that binary arithmetics is also somehow connected to the idea of an essential limitation pertaining creatures[233], to the problem of differentiating the unity expressed by God from that of such creatures and to the true nature of nothingness, both as an absolute concept and as something that can be conceived only together with something else.

After this work, evidences of this reasoning are spread all over Leibniz's production about binary arithmetics. In the 1695 *Dialogue effectif sur la liberté de l'homme et l'origine du mal*, after having introduced the concept of nothingness, to the question on how nothingness is capable of entering in the composition of things, Leibniz replies: "vous savez pourtant comment dans l'Arithmétique les zero joints aux unités, font des nombres differens comme 10, 100, 1000 […] et il e nest de même de toutes les autres choses, car ells sont bornées ou imparfaites par le principe de la Negation ou du Neant qu'elles renferment"[234]. Here we have both the use of unities and zero in a composition and the idea of a priority of unity over zero, which resembles closely the priority of existence over non-existence. Three years after this dialogue, in a letter to Schulenburg dated 29 March 1698, the bond with essential limitation is even stronger:

> Nimirum fines seu limites sunt de Essentia Creaturarum, limites autem sunt aliquid privativum, consistuntque in negatione progressus ulterioris. Interim fatendum est, creaturam, postquam jam valorem a Deo nacta est, qualisque in sensus incurrit, aliquid etiam positivum continere, seu aliquid habere ultra fines neque adeo in meros limites seu indivisibilia posse resolve […] Atque haec est origo rerum ex Deo et nihilo, positivo et privativo, perfectione et imperfectione, valore et limitibus, activo et passivo, forma (id est entelechia, nisu, vigore) et materia seu mole, per se torpente nisi quod resistentiam habet. Illustravi ista nonnihil origine numerorum ex 0 et 1 a me observata, quae

[233] On this topic see Fichant (1998, 85-119) and Mormino (2005, 115-140).
[234] GRUA (364).

pulcherrimum est emblema perpetuae rerum creationis ex nihilo, dependentiaeque a Deo (AA II-3B, 426-427).

The same reasoning is also present in the famous *Explication de l'arithmetique binaire*, written in 1703, were Leibniz adds that

> Le calcul par deux, c'est à-dire par 0 et par 1, en récompense de sa longueur, est le plus fondamental pour la science, et donne de nouvelles découvertes, qui se trouvent utiles ensuite, même pour la pratique des nombres, et surtout pour la Géométrie, dont la raison est que les nombres étant reduits aux plus simples principes, comme 0 et 1, il paroit partout un ordre merveilleux (GM VII, 225).

These quotes show that the metaphysical background of Leibniz's binary arithmetics was consistent throughout his life. This is relevant, since the first work on this topic, *De organo sive arte magna cogitandi*, was written in 1679: it means that these metaphysical assumptions were maintained despite the well-known change in Leibniz's philosophy happened in the 1680's. The previously quoted letter to Schulenburg is perhaps the best evidence of this consistency, because the philosophical achievements of binary arithmetics are expressed together with Leibniz's new discoveries on the nature of substances[235].

Regarding our purpose of determining a possible influence of Weigel's *De supputatione* on Leibniz's *De organo*, the reference to Leibniz's writings after 1679 was needed in order to point out how the comparison with unity, God and nothingness was considered by Leibniz a distinctive feature of his base-two model, especially in the confrontation with Weigel's. The same reasoning in fact is found in the *Animadversiones ad Weigelium*[236], although here expressed in order to state the superiority of Leibniz's system from that of Weigel. It is as if Leibniz himself wanted to divert the attention from the metaphysical background to the idea of changing the base of counting. In a way, he achieved this result, since both Couturat and Grua's reconstruction of the birth of the

[235] AA (II-3B, 427).
[236] FOUCHER2 (166).

binary calculus are based on the assumption that, if there was some kind of influence by Weigel, it had to be only in the mathematical aspect. This theory should be rejected judging by what I believe is, in reference to the confrontation between Weigel and Leibniz on this topic, the most important quote from Weigel's *De supputatione*:

> Principium nempe finitatis, ipsiusque multitudinis & ordinis est, quod hinc, etiam in computatione primum, ubique praesupponitur. Estque conceptu suo vel *purum*, quod est NULLITAS, τό *Nihil*, purae computationis (*Additionis* , *Subctrationis*) principium; vel *modale* seu mensurativum, quod est τό *Semel* , aut simplum, modalis computationis (*Multiplicationis*, *Divisionis*) principium : Quatuor haec (*principium*, bina *data*, *productum* seu τό Facit) exacta proportione sui generis progrediuntur, dum in omni computo, sicut se principium ad datorum unum habet (Weigel 1679, part I, § 29)[237].

In order to understand this quote, a look at the very beginning of *De supputatione* is needed. Here, some universal rules are set:

> I. Omnia quae realiter (i.e. actu) sunt, singularia sunt.
> II. Omnes Actiones reales circa singularia sunt.
> III. Omia singularia finita Valorem in se complectuntur *Pondere, Mensura*, *Numero*, sed & *Ordine*, certum; inter se certa *Ratione* certaque *Proportione* definita sunt.
> IV. Omnes agendo circa res occupati Supputant (Weigel 1679, Praefatio).

According to Weigel, everything in this world can be conceived as a unity. This possibility deprives every object of their specific proprieties, but at the same time it makes them homogeneous one another, that is suitable for a mathematical description, as it was shown in the application of the concept of ratio. The first quote then shows that every finite object conceived this way is a single entity composed of nothingness and unity. In Weigel's philosophy in fact, the word *Semel* stands for unity or God, because it is a

[237] I'm quoting this work by using the reference to its parts and chapters. Weigel's *De supputatione* in fact was published with no page numbers until part III, making the reference to a specific page confusing for the reader.

reference to the most important operation related to divinity, that is multiplication. It is extremely relevant because, unlike a generic platonic reference to non-existence, these universal principles are associated with mathematical operations. The idea is that God represents pure unity, while creatures represent compositions of unity and nothingness, that is zero. The expression of God's infinity is multiplication, because the multiplication 1 x 1, while it could be executed indefinitely, still gives as a result the pure unity. God then embeds the whole world, that is the product of all its finite unities, from nothingness to pure infinity, as the complete title of *De supputatione* suggests. It follows that addition and subtraction are considered in a negative way, thus associated with nothingness: addition and subtraction force things to come into existence as separate things, because they free them from the logic of unities' multiplications. Given these premises, Weigel adds a fundamental consequence:

> Sicut autem primum omnium Veritatum principium […] *est Veritas* infinita, DEUS; Objectivum autem NULLITAS qua puram; UNITAS finita (simplicissime punctum) qua modalem, finitorum rationem; ita TETRACTYS ab utroque principio, per quaternitates propotionum, illic purarum, hic modalium (Weigel 1679, part I, §27).

The *tetractys* then was chosen by Weigel because it shares through proportions a relationship with both unity and nothingness. It is important to remember that, despite the impossibility of proving that Leibniz read the *Tetractys* at this stage, Weigel had already published this work in 1673. Therefore, in 1679's *De supputatione*, while establishing a relation between the Pythagorean *tetractys* and the metaphysical concepts of unity and nothingness, Weigel establishes also a relationship between his base-four arithmetics and these philosophical ideas: "rationes numerorum & ordinum, earumque progressionem quadrordinalem (Tetractyn) secudum quam DEUS in gratiam humanae Mentis omnes Essentias tanta varietatis pulcritudine concinnavit, supputando penetremus"[238]. It follows that there is no way of understanding Weigel's influence on Leibniz as a mere suggestion on changing numbering's base model. Weigel was extremely close to binary arithmetics as it was conceived by Leibniz, because every element of it was already present in his

[238] Weigel (1679, part III, § 20).

works, so much that Leibniz's efforts could be seen as a prosecution of his teacher's work, as Bernulli suggested.

Leibniz's image of the world's creation through unity and nothingness and its mathematical expression then were already present in Weigel, but a possible criticism to this interpretation would be that of arguing for a simple Pythagorean influence, rather than a specific influence by Weigel's Pythagoreanism. Even accepting this criticism, we could remark that before 1679 Leibniz quoted Pythagoras mainly in geometrical writings, for obvious reasons, and together with Plato in writings concerning metempsychosis, but only after April 1679 he is also credited for his metaphysical theories on numbers. The most famous quote is that of *De numeris characteristicis ad linguam universalem constituendam*, again in 1679:

> Vetus verbum est, Deum omnia pondere, mensura, numero fecisse. Sunt autem quae ponderari non possunt, scilicet quae vim ac potentiam nullam habent; sunt etiam quae carent partibus ac proinde mensuram non recipiunt. Sed nihil est quod numerum non patiatur. Itaque numerus quasi figura quaedam metaphysica est, et Arithmetica est quaedam Statica Universi, qua rerum gradus explorantur. Jam inde a Pythagora persuasi fuerunt homines, maxima in numeris mysteria latere. Et Pythagoram credibile est, ut alia multa, ita hanc quoque opinionem ex Oriente attulisse in Graeciam (AA VI-4A, 263).

Almost every information displayed here could be retraced in Weigel's *De supputatione*, from the famous partition in "*pondere, mensura, numero*"[239] to the less famous use of the metaphysical number, making him at least the major influence in the adoption of Pythagorean theories[240].

Particularly interesting is the way in which Leibniz defines arithmetics, related to the description of the world as a *Statica Universi*. This term suggests the idea that arithmetics could offer us a kind of static description of the world, but this idea is not consistent with the basic notion we have about arithmetics: if this description involves the use of

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[239] A part from the already quoted preface, see also Weigel (1679, part I, § 36 and part III, § 22).
[240] "Mirum non est, quod TETRACTYS a Pythagoraeis adeo celebrata" (Weigel 1679, part I, § 27).

arithmetical operations, it cannot be based solely on them, because the world is shaped in a different way than that of numbers. Both are well-ordered systems, but the relationship between things in our world need some kind of spatial reference that numbers per se don't need. This order is expressed in Weigel by the notion of *Status*. Again, the reference to Weigel could help in explaining Leibniz's theory. As we previously sketched, in Weigel's philosophy every entity is conceived as a unity, making it suitable for mathematical operations. However, the existence of our world and the possibility of knowing it is not based solely on arithmetics, but on the study of the relational proprieties and positions of such unities:

> Cujus & totius, & cujusque partis, ut numeri partialis, unitates ORDINE *certo*, simul ac numerantur, etiam disponuntur a DEO, tum *simul*, & Ordo dicitur *Status*; tum secundum prius & posterius, & Ordo dicitur *Motus* (Weigel 1679, part II, §17).

For Weigel, *Status* is the set of relationships between unities that give birth to the world in an instant, while *Motus* is the connection of these instantaneous descriptions of the world in time. In other words, they are Weigel's equivalent of space and time, as he himself writes in his first reply to Leibniz's 1679 letter:

> Spatium (sc. ubicativum, i.e. rerum juxta se mutuo simul existentium non-repugnantia loco nihili,rerumque concepta) et Tempus in abstracto (tanquam Spatium quandicativum,i.e. rerum omnium, ut unius copiae, secundum prius et posterius existentium i.e. repetitarum, non-repugnantia loco nihili rerumque concepta) tanto magis analoga sunt inter se, quanto praecisius utrumque dicit potentiam perceptibilis positionis, illud simultaneae hoc successivae (AA II-1B, 762).

This reference could explain how binary arithmetics fits in Leibniz's renewed interest in logic[241] and *analysis situs* around 1679 that I analysed in the previous part. It surely shows Weigel's estrangement towards his theories on space in 1671.

Another important resemblance between Weigel's *De supputatione* and Leibniz's *De organo* that points in this direction is found at the starting point of the reasoning that leads to the binary system in *De organo*: for a theory that claims the possibility of describing the whole world through a peculiar way of expressing numbers and their operations it seems unusual that the first step in this direction would be that of analysing the relationship between the world and the human mind. We would be inclined to think in fact that such description of the world would be a sort of objective description, since it's based on purely mathematical assumptions. In this work however, before the introduction of the binary system, Leibniz reflects on the idea of conceivability:

> Maximum Menti Remedium est si inveniri possint cogitationes paucae, ex quibus exurgant ordine cogitationes aliae infinitae. Quemadmodum ex paucis numeris ab unitate usque ad denarium sumtis caeteri omnes numeri ordine derivari possunt. Quicquid cogitatur a nobis aut per se concipitur, aut alterius conceptum involvit. Quicquid in alterius conceptu involvitur id rursus vel per se concipitur vel alterius conceptum involvit […] Tametsi infinita sint quae concipiuntur, possibile tamen est pauca esse quae per se concipiuntur. Nam per paucorum combinationem infinita componi possunt (AA VI-4A, 157-158).

Many ideas expressed in this brief passage date back to the ideas found in Leibniz's early years previously exposed, although here they will be later connected with the binary system. The first idea expressed is a minimalistic approach to universal principles: the relationship between the number of universal principles chosen and the number of things that they are able to explain should always aim for the smallest number of principles and

[241] Leibniz kept the connection between logic and binary arithmetics also in later writings. For example in the *Explication de l'arithmetique binaire*, after introducing his calculus, we read: "Cependant je ne sçai s'il y a jamais eu dans l'écriture Chinoise un avantage approchant de celui qui doit être nécessairement dans une Caractéristique que je projette. C'est que tout raisonnement qu'on peut tirer des notions, pourroit être tiré de leurs Caractères par une manière de calcul, qui seroit un des plus importans mo yens d'aider l'esprit humain" (GM VII, 227).

the highest number of things explained by them, which is consistent with Leibniz's reductionism regarding the principle of contradiction. For the purpose of this part, it is sufficient to say that in *De organo* this minimalistic yet fertile approach is connected with numbers and their relationships. A passage of Weigel's *De supputatione* outlines a similar approach on the universal principles that we already found in his *Analysis aristotelica*:

> Directorii vero f. Normae moralis, naturaliter & ordinarie ducentis, officio funguntur Notitia primae nobiscum nata, quae dicuntur Axiomata (v.g. Semel unum est unum etiamsi sit infinitum : Finis rei [terminus & limes rei] nihil est prater cogitationem : Finitorum autem Bis unum sunt duo : Totum sua parte majus est) Weigel (1679, part I, § 11).

For Weigel, the universal principles residing in the human mind are the ones that allow the foundation of arithmetics, such as the idea of multiplication, the idea of addition and the idea that the whole is bigger than its part. At first it wouldn't seem as if Leibniz's reasoning in *De organo* has any kind of reference to universal principles, but we should remember that for Leibniz, at least after 1672's *Confessio philosophi*, conceivability involves non-contradiction, since in that work he establishes a bond between the concept of possibility and the act of conceiving, founding both on the principle of contradiction. If conceivability is presented through the principle of contradiction, then Leibniz's reasoning is not that far from his mathematics as it may seem: we should be able to recognize by now that these principles are the ones that Leibniz uses in his foundation of arithmetics, but the novelty here resides on Leibniz's focus on the fact that they reside in the human mind, consistent with further developments that will lead to the *Nouveaux essais*.

Weigel's *De supputatione* starts in the same way of Leibniz's *De organo*, establishing complete homogeneity between the world and the human mind:

> Unusquisque nostrum, etiamsi solus sit, *simul* ac de semetipso cogitat (dum secum habitat) Semetipsum illico familiariter agnoscit: idque (1) *Ratione* STATUS […] (2) *Ratione* MOTUS […] ex Identitate cogitationis suae

> successivae clarissime deducit, se posterius in essendo praesentem, *eundem esse qui*, prius in essendo praesens, *erat* (Weigel 1679, partI, §1).

Here Weigel argues, following probably Descartes' method, that, even if we were completely deprived of our experiences, in our minds is contained everything we need not only to understand but also to express the whole world. The very simple act of perceiving unity and its constant perception in time give birth to the subjective representation of *Status* and *Motus*, that is space and time. This is possible because of the universal principles residing in human minds:

> Notitias illas, ut Mentis alias tenebricolae luculas (scintillulas) principia, rationes, & causas Mentaliter operandi primas; una cum immediatis experientiis, quisque nostrum simpliciter & indubie Semper novit, intime perspectas habet, intelligit, atque sapit (Weigel, 1679, part I, § 13).

The idea of the spark recalls the platonic influence found in the *Analysis aristotelica* about reminiscence. Unity, addition, multiplication and the principle for which the whole is bigger than its part, that is *Ordo*, are the same elements that we already saw in the objective description of the world. Once corroborated by experiences, these principles show the homogeneity between men and the world:

> Axiomatum, ut generalium principiorum, suscitabula sunt *Experientiae*, tum *immediata*, Mentis ipsius per se v. g. *Mentem hominis* in *Ubi* per corpus suum & ipsius Mundi *definito contineri* […] tum *mediata*, Mentis per sensus, h.e. per oblationem ad adjuncto sibi corpore factam, obsevabiles, v.g. *Res Mundi circa Nos vario Status & Motus variabilis ordine disponi* (Weigel 1679, part I, §12).

There are several similarities between these passages and Leibniz's idea of truth expressed through relations, both in understanding truth for things that already exist, like in 1677's *Dialogus*, or in discovering new ones, as in the *ars inveniendi*[242]. The homogeneity between the universal principles instead will be crucial in Leibniz's *Nouveaux essais*.

As a final remark for this section, I would like to add that the first quote of Weigel's *De supputatione* here presented could again suggest that Leibniz read this work around the time of his *De organo*: in Weigel, the two universal principles are introduced as "τό *Nihil*" and "τό *Semel*". We could say that the use of the ancient Greek's definite article is a distinctive trait of Weigel's writing, used in every work ever published. He uses Latin in metaphysical essays, while adding German in order to express examples or specimina and Greek for the most important principles of his philosophy. The definite article is his way of promoting a concept to a universal principle, much like his use of italics or small capitals. This peculiar way of writing is not present in Leibniz in the years before 1679, but it suddenly appears for the first time in April and vanishes around the end of that summer[243], that is the period of time between Leibniz's *De organo* and his first letter to Weigel. Leibniz adopted this trait and I believe that this could be an interesting hint on the exact moment in which Leibniz received Weigel's work, because it is based on a specific pattern, and not on a generic use of ancient Greek, very frequent in these kind of writings.

Conclusion: God and Numbers After having analysed Weigel's influence on binary arithmetics, the remaining task is understanding why Leibniz tried to hide it throughout his life. The obvious conclusion seems to be that Weigel's theory was too similar to that of Leibniz and admitting its influence would have led to a charge of plagiarism, but I believe that in Leibniz's attitude there is more than just fear. In 1679, after his stay in Paris, Leibniz achieved a superior mathematical knowledge, unknown to Weigel, that affects their different ways of conceiving arithmetics. While in Weigel's *De supputatione*

[242] Weigel has his own *ars inveniendi*, called at times *deductio productionalis* or *inventio*: "Exserit autem se computus per SCISCITATIONEM & INVENTIONEM […] *qua ratio latens inter ipsas rationes illi conjugatas quaeritur & inventa producitur*. Μανθάνω enim, latine *sciscitare*, non est receptas sententias discere, quaerentique verbaliter, i.e. interroganti, recitare […] sed *ignotas Veritates cum judicio rimari, tandemque* si Veritatum Autor annuat *producere*: non ex nihilo, sed *ex datis* positisque certis veritatibus & rationibus" (Weigel, 1679, part I, § 22).

[243] The first use is a "το *non-fortunatum*" in the *Calculus consequentarum*, written in April 1679 (AA VI-4A, 223). Then we find it in *De negatione* (Summer 1679, AA VI-4A, 300) and in the *Potest aliqua notio esse alia generalior ut tamen non sit simplicior* (Spring - Summer 1679, AA VI-4A, 303).

the relationship between the finite and the infinite is standard — God is the expression of the infinite, creatures are the expression of the finite — Leibniz had to deal with his recent discoveries on the infinitesimal calculus. If both binary arithmetics and the infinitesimal calculus were to be applied to an indefinitely divisible world, the problem of dealing with infinity arises in a much more complicated way than that of Weigel, as suggested by Leibniz's development of Weigel's homogeneity in the previous part. In the same fashion, it could be that Leibniz believed that his binary arithmetic expressed a better image of the world than Weigel's calculus. This belief probably explains why Leibniz's first letter to Weigel, instead of reporting the analogies with his calculus, deals mainly with the problem of infinity.

Having analysed Weigel's influence however, a better understanding of Leibniz's aim is possible. He believes that binary arithmetics is an appropriate expression of an infinite world because, as we saw in Weigel, its description is based on the study of relational proprieties, more than arithmetic operations. In this regard, the binary system is nothing more than a way of labelling things, much like names or characters. Its advantages are that instead of names, it uses only two digits, connected in a way that allows at the same time the possibility of naming things indefinitely and the creation of names that are always distinguishable between themselves. However, the fact that order is derived solely from the notion of *Status* is also the most important limit of Leibniz's binary arithmetics: the arithmetical operations lose a distinctive role, meaning that a connection between the wonderful proprieties that Leibniz saw studying the binary series in artihmetics are not coherently related to the use of the binary system in metaphysics. This difference explains why Leibniz refers often to the idea of an analogy between binary aithmetics and the world, rather than an exact expression. Despite these difficulties, Leibniz was probably afraid that the reference to Weigel would have diverted the attention from what he believed was a better explanation of how binary calculus is useful in dealing with an infinite reality.

Another reason why Leibniz was not inclined in recognising Weigel's influence is the main metaphysical consequence of a purely mathematical description of the world: if unity is what describes the entire world, then the difference between God and creatures is at stake. If for Weigel the main difference is only that "*infinitam* dari *Mentem,* a qua

Mentes finitae Mensuram capiant”[244], it follows that the universal principles are shared between God and his creations. The result is not only that God as the *Computator* acts in the same way of creatures, only with an infinite mind, but also that the finite is contained in the infinite through the use of multiplication: “ENS REALE vel *infinitum* est & PRIMUM, DEUS, in quo vivimus & movemur”[245]. Judging by this quote, it comes as no suprise that in the same year Weigel was forced to retract one of his works by the faculty of theology. Leibniz was interested in this topic and in Weigel’s demonstration of God’s existence[246] and maybe this awareness explains the need of expressing a criticism to Spinoza in Leibniz’s letter to Schulenburg about binary arithmetics[247].

Focusing on the demonstration of God's existence, Leibniz's interest on Weigel's version predictably dates back to the 1670's, the same period in which Leibniz was busy developing his modal demonstration. The interest continues also in 1679, since Leibniz was corresponding with Adolf Hansen about Weigel for more than a year[248]. As much as Leibniz was interested in the metaphysical implications of his binary arithmetics during the following years, so he was about Weigel's demonstration of God's existence, especially given the relation between God and creatures that it implied. The first sentence of Leibniz's *Inventa Weigelii doctrinalia* states: "Demonstratio Mathematica existentiae Dei supponit existentiam rerum non esse continuam, et ita esse perpetuam recreationem"[249]. Around the same time, in 1685, Leibniz writes to Ludwig von Seckendorff, mentioning Weigel, while commenting a work of Villiers[250]:

[244] Weigel (1679, part I, § 10).
[245] Weigel (1679, part III, § 26).
[246] See Di Bella (2005°, 260-261).
[247] AA (II-3B, 427).
[248] Hansen wrote to Leibniz about Weigel for the first time in a letter dated Febrary 1678, without knowing that Leibniz had been one of his disciples: "Ie voudrois bien savoir si l'invention du charoit qui se peut changer en tente et en batteau n'est pas de MR Weichelius, qui est Professeur à Iena, j'en ai vû à Gotha de son invention, il est homme d'un grand genie, et peut être que vous ne serez pas faché d'entrer en commerce avec lui, il est fort curieux, et m'a montré souvent de tres bonnes choses" (AA I-1, 320). Then again in December 1678: "Ie vous prie de mander quelque chose de MR Weigelius, il a de tres-belles inventions et je suis faché de n'en avoir pas profité dans le temps que j'avois la commodoté de le voir tous les jours, il n'est pas envieux da sa science" (AA I-1, 389). In April 1679 Hansen mention Weigel's demonstration of God's existence: "I'ai relu toutes vos lettres avec la plus grande satisfaction du monde [...] Ie voudrois que vous m'apprissiez un peu, s'il vous plait, ce que c'est devenu la nouvelle demonstration de l'existence de Dieu de M. Weigelius, je n'en ai rien apris depuis long temps" (AA I-1, 457), before Leibniz's letter to Weigel.
[249] LH 38 Bl., 178.
[250] See Villiers (1685).

> Aliqua video inspergi quae a Weigelii nostri placitis non abhorrent et a me aliquando expensa sunt. Putat enim nihil manere idem, sed omnia esse in perpetuo fluxu excepto Deo. Weigelii demonstrationem existentiae Dei, quam Germanica pariter ac latina lingua aliquot abhinc annis amicis communicavit, vidisse te arbitror. Mihi valde consideratu digna videtur, tametsi nonnulla supplenda videri monuerim, quorum adhuc demonstrationem desiderabam. Optima mihi haec videtur via si ostendi posset omnia accidentia nihil aliud esse quam modos substantiam concipiendi, ita enim demonstratum esset quod illi ponunt, substantias ipsas revera non durare, si rem ad summam ακριβειαν exigas, sed alias atque alias prioribus similes vel si mavis easdem a Deo continue creari, Deus autem necessario eximi deberet, alioqui si omnia continuo perirent, ne ipso quidem excepto, nihil esset quod ea restitueret, nam ubi semel nihil esset in toto universo, in aeternum nihil maneret (AA II-1, 874).

It is clear that Leibniz was very intrigued by the idea of the continuous creation. If however that was conceived in mathematical terms, there was little room for a true differentiation between God and creatures. On a side note, the words "omnia esse in perpetuo fluxu excepto Deo" recall the distinctions found in primary matter in Leibniz's early years. There is a significant difference however: at that time something that was at rest, yet not identifiable with God existed, primary matter considered at rest or space, while here everything is in motion, except God. Although then we could find some connections there is an evolution both in Leibniz's conception of physics, as it is widely known, but also in Weigel's philosophy. A sort of mathematical relationship between entities and their creation in fact could be found also in the *Analysis aristotelica*[251], but Weigel was not influenced at that time by the Pythagorean tradition as much as he will be after 1673. This is the reason why Weigel's influence on Leibniz should be taken into account in the early years, even if there are few direct references to the Pythagorean tradition, plus it explains convincingly what happens around 1679: when Weigel's

[251] "Ex entis materiali […] fluunt *Numeri, quibus, discretim spectando, quodvis est vel Ens h.e. unum seorsim: vel conjunctim Ens & Ens h. e. unum & unum (quod commoditatis gratia dicimus duo) vel Ens & Ens & Ens.h.e. unum & unum & unum (tria) vel Ens & Ens & Ens.& Ens. h. e. unum & unum & unum & unum (4.) & sic in infinitum*" (Weigel 1658, 182).

philosophy witnessed a fundamental evolution, so did Leibniz's philosophy by accepting some Pythagorean elements in his binary arithmetics.

Leibniz himself however explains clearly in the *Théodicée* his cautious approach. At first Leibniz reports Weigel's demonstration together with his critique on the composition of reality[252]. Leibniz admits a modified version of Weigel's argument by specifying that a degree of freedom is needed, otherwise creatures would lose their freedom:

> Cette action de Dieu est libre. Car si c'étoit une emanation necessaire, comme celle des proprietés du cercle, qui coulent de son essence, il faudroit dire que Dieu a produit d'abord la creature necessairement, ou bien, il faudroit faire voir comment en la creant une fois, il s'est impose la necessité de la conserver (GP VI, 343).

Leibniz's cautiousness displayed here surely depends from the context in which the *Théodicée* is written, but it also hides an honest criticism on the continuum problem. I believe that the world conceived as a combination of independent unities leads necessarily to the idea of a world of points that derive time from a series of instants, as Leibniz remarks. This is probably the reason why Leibniz always writes about binary arithmetics without any references to the other developments of his philosophy of mathematics. If Weigel had no problem in extending his philosophy of homogeneity and ratio to this realm as well, Leibniz obviously had a better understanding of the problems related to infinity, even if the extension of homogeneity to the infinite was taken from Weigel

[252] "Feu Monsieur Erhard Weigel, Mathématicien et Philosophe célèbre à Jéna, connu par son *Analysis Euclidea*, sa Philosophie mathématique […] communiquait à ses amis une certaine démonstration de l'existence de Dieu, qui revenait en effet à cette création continuée. Et comme il avait coutume de faire des parallèles entre compter et raisonner, témoin sa Morale Arithmétique raisonnée (*rechenschaftliche Sittenlehre*) il disait que le fondement de la démonstration était ce commencement de la table pythagorique, une fois un est un. Ces unités répétées étaient les moments de l'existence des choses, dont chacun dépendait de Dieu, qui ressuscite, pour ainsi dire, toutes les choses hors de lui, à chaque moment. Et comme elles tombent à chaque moment, il leur faut toujours quelqu'un qui les ressuscite, qui ne saurait être autre que Dieu. Mais on aurait besoin d'une preuve plus exacte pour appeler cela une démonstration. Il faudrait prouver que la créature sort toujours du néant, et y retombe d'abord ; et particulièrement il faut faire voir que le privilège de durer plus d'un moment par sa nature, est attaché au seul être nécessaire. Les difficultés sur la composition du Continuum entrent aussi dans cette matière. Car ce définie paraît résoudre le temps en moments : au lieu que d'autres regardent les moments et les points comme de simples modalités du continu, c'est-à-dire comme des extrémités des parties qu'on y peut assigner, et non pas comme des parties constitutives. Ce n'est pas le lieu ici d'entrer dans ce labyrinthe" (GP VI, 343).

himself. In this regard, it is relevant that the exemplification of necessity in the passage above rests on the emanation, term found in Weigel as I showed, of intrinsic properties of a circle, i.e. qualities: it is as if Leibniz is stating that he would follow Weigel in mathematics, while casting some doubts on the mathematization of metaphysics done through the concepts of unity and nothingness.

It follows that this other side of Weigel's and Leibniz's philosophy of mathematics, probably because, resting on the distinction between categorematic and syncategorematic infinite, conceives the mathematical operations as something that needs to be applied to numbers, had a different origin from that of the other mathematical concepts. It is no coincidence then that in Leibniz's 1679 letter to Weigel there is a reference to the *Accessio ad Arithmeticam Infinitorum* and to the infinite number. Clearly, Galilei's influence on Jena's cultural movement that I sketched in the first part of the present work is responsible for Weigel's and Leibniz's growing interest in the properties that one number could exhibit. The *Accessio* already in 1672 shows a reflection on Galileo's infinite number:

> Galilaeus in dial. Mechan. 1[253]. Infinitum Numerum comparat unitati […] Quid ergo? attributa aequalis, majoris, minoris, non habere locum in infinito. Et subjicit si ullus sit numerus infinitus, eum esse unitatem, in ea enim esse illud necessarium requisitum numeri omnium unitatum infiniti, ut tot sint in ea radices, quot quadrati et cubi […] Ego vero ajo: si ullus sit iste numerus infinitus, eum esse zero, seu Nullam, vel quod idem est dicere, Numerum istum infinitum esse nullum, seu = 0. […] Cum ergo in numero isto infinito tot sint Numeri pares, quot numeri pares et impares simul, seu quot numeri simpliciter, sequitur in Numero isto infinito fallere Axioma illud: totum esse majus parte at Axioma illud fallere impossibile est, seu quod idem est, Axioma illud nunquam, ac non nisi in Nullo seu Nihilo fallit, Ergo Numerus infinitus est impossibilis, non unum, non totum, sed Nihil. Ergo Numerus infinitus = 0 (AA II-1, 349).

[253] The reference is to Galileo's *Discorsi e dimostrazioni matematiche, intorno a due nuove scienze attenenti alla mecanica e i movimenti locali* in Galilei (1929, VIII, 78-83).

The idea of an infinite number invalidates the principle by which the whole is greater than its part, that is the very foundation of Weigel's and Leibniz's mathematics of ratios. However, the reflections on its impossibility led Leibniz to his idea of nothingness. This could be the reason why in Leibniz's *Accessio* we find the studies on the number series that we will find in his 1679 letter to Weigel, because these two opposite solutions, while remaining opposite, were conceived at the same time[254]. Given that in the same year of the *Accessio* Weigel invented his base four calculus[255], it is highly probable that they were both influenced by the Galilean tradition in Weigel's school. The application of the concept of infinite number to the concept of God and Weigel's idea of counting in a different base led then Leibniz to binary arithmetics, but the original sin of being born from an impossible concept always set binary arithmetics apart from Leibniz's other reflections on the philosophy of mathematics, even more so, since it entailed a dangerous idea of God.

In conclusion, despite these possible explanations of his cautiousness, Leibniz was interested in Weigel’s philosophy throughout his whole life. It is now clear that since its beginnings Leibniz was influenced by Weigel on many of the topics highlighted: from the terminology adopted and the reference to the Pythagorean tradition to the use of a different base-model and its connection with unity and nothingness by means of arithmetical operations. In 1679 this confrontation gave birth to binary arithmetics.

[254] As it is known, in this regard the importance of Leibniz's studies on Pascal is also relevant, especially for the number series. See Pascal (1665).
[255] Although dated 1673 in fact, versions of Weigel's *Tetractys* are found already in 1672.

Conclusion

The analysis of Weigel's influence on Leibniz proved that a new reconstruction of the evolution of Leibniz's mathematical knowledge during his life was much needed. Leibniz himself warns us that his knowledge before Paris was very derivative, but this remark is not decisive, if the theories encountered in Jena were not just a mere glimmer of those developed by the original authors, but syncretistic accounts that already contained a degree of rigour. Even before his stay in Paris in fact, Leibniz was able to grasp some fundamental concepts that will be essential in his later reflection on mathematics. Besides, once achieved a superior understanding on the same matter, instead of completely dismissing his past ideas, he firmly decided to make them suitable for his new mathematics.

By no means Weigel played a secondary role in this quest, both in the adoption of those ideas before Paris and in reminding Leibniz throughout his life their metaphysical and mathematical relevance. During the early years, Weigel built that cultural environment where Leibniz found, among others, philosophers very dear to him, such as Gassendi and Hobbes, conceived in a syncretistic effort that was eliminating the evident contradiction posed by their combination with Scholastics' ideas. The result is a new take on Leibniz's peculiar attitude in those years: instead of reserving the original reformulation of these ideas to Leibniz's character only, a reasonable confrontation with the context in which Leibniz lived is now possible with new insights. It is indeed true that many other syncretists were quoted and read by Leibniz, but none of them were in that moment closer than Weigel and, above all, none of them tried as much as Weigel did the daring attempt of prioritising Hobbes over any other author, Descartes included, in every possible field, from logic to the philosophy of nature and ethics. This outcome is all the more important in the light of the authors chosen by Weigel among the ancient philosophers: in order to conciliate Aristotle with the moderns a stronger estrangement towards the Scholastics was needed and it caused the happiest of circumstances, that is the possibility of revaluating Aristotle's logic and his logical ontology while maintaining and highlighting the connections with Hobbes' logic. The fact that Weigel erroneously conceived Aristotle in the light of Euclid, resulted in a very useful mistake, because the rigorous mathematical demonstrations became the litmus test of any possible reasoning, Scholastics' reasoning included. In this regard, Hobbes' constant reference to Euclid was cherished as a fitting alternative, while the Scholastics' principles were maintained only in their most general form. Nonetheless, while this estrangement towards Scholastics was valid on paper, it depended ultimately on Weigel's will, so much that many of their ideas were reintroduced by him in the form of Aristotle's original ideas. The complexity of these relationships fits nicely with the complexity of Leibniz's position in those years, explaining some contradictions that were considered particularly puzzling.

On the other hand, Plato's reminiscence was endorsed by Weigel in a mathematical and foundational context in order to grant the homogeneity between God, reality and the human mind for those principles that were taken from Aristotle and Hobbes. Setting aside Leibniz's derivative knowledge in those years, Plato's *Meno*, quoted by Weigel, represents probably the original ancestor of Leibniz's idea of founding the whole of mathematics on a single principle. Not to mention the consequences of conceiving essences as numbers in the mind of God, an older version of Leibniz's idea of conceivability as a tool used to express sets of coherently related possibilities.

Another relevant outcome of this research is on the ground of physics: the analysis of the dissertations written under Weigel's guidance showed the huge debt of Leibniz's *Hypothesis physica nova* to Jena's circle. Interpreting Leibniz's *Hypothesis* as nothing more than another dissertation of those years is surely inaccurate, but the reference to the same authors, the use of the same examples and the adoption of the same objectives is certainly surprising. Since Leibniz's *Hypothesis* was immediately praised by Weigel, an honour reserved for very few in his writings, we can safely assume that it was conceived with a much more important task in mind, consistent with the project of founding an academic society in Germany. It also follows however that the year 1671 represents the peak of Weigel's influence, meaning that it is no coincidence that in the same year we find one of the most relevant passages on Leibniz's conception of geometry. As it was shown, motion and rest could be interpreted as concepts that, while evolving in time during Leibniz's life, caused in that very moment the transition of the difference between quality and quantity from physics to mathematics.

The importance of the concept of ratio emerges from this distinction, because a ratio is possible only between homogeneous entities and homogeneity is defined through quality and quantity. Leibniz's definition of number is based on these reflections and it resembles closely that of Weigel. It cannot be argued either that these concepts have a secondary role in the philosophy of Leibniz's teacher, because it was widely known by Leibniz from the very beginning that Weigel's metaphysics was based on the application of ratios and proportions to every possible field of knowledge, as Leibniz himself admits and remarks.

Given also Weigel's interest on numbers series expressed through ratios, Leibniz's 1679 letter could be seen as an attempt to impress him by displaying his recent achievements in the mathematics of the infinite. This attempt was not particularly appreciated, or even understood by Weigel, but that did not stop Leibniz's growing interest in the works of his former professor. The more Leibniz was taking notes on Weigel's works about homogeneity, quality, ratios and *situs*, the more he was motivated in his foundational attempts, even if they were already reaching a degree of complexity that was unknown to Weigel. Following this path, Leibniz probably realised that the ideas imparted by Weigel in his early years had a significant impact on him and the thought of completing Weigel's philosophy of mathematics with his original contribution about the infinite began to cross his mind, until the definite acknowledgment around 1710. Even if already passed away, Weigel inspired Leibniz one last time in the form of a disciple of his school, Christian Wolff. This last exchange gave birth to one of the most interesting works on the philosophy of mathematics, the *Initia rerum mathematicarum metaphysica*.

As much as the texts considered prove this reconstruction, Leibniz's attitude towards Weigel seems at times against a serious consideration of his influence. During his early years, Leibniz appreciation for his teacher is evident, for example in the 1669 letter to Thomasius, but after 1671 a series of sparse and convoluted references begin to replace Leibniz's initial enthusiasm, at least until 1680. Leibniz's apparently incoherent attitude is in part justified by the fact that some of the letters in the correspondence with Weigel are missing, judging by the reference to unknown topics in the letters already published. However, another possible conjecture is that Leibniz was afraid of being associated with Weigel on theological matters. This interpretation is consistent with Weigel's conflicts with the theological faculty of Jena, began even before Leibniz's arrival in that city in 1663 and continued throughout his life. It is also consistent with Leibniz's concerns expressed in his *Essais de Théodicée* and with his cautiousness on the ground of the metaphysical interpretation of binary arithmetics. Besides, it is quite clear that Leibniz was not very satisfied by the association with Weigel on these topics, done by Bernoulli. Binary arithmetics was the only topic in which Leibniz felt the need of openly defending himself, as if he was accused of plagiarism. He was probably concerned about the possibility for his contemporaries of understanding how the idea of counting in a base two model was innovative not only because of the change in the base system, but also because of the specific base system chosen. Yet, it does not seem that in other topics Leibniz is willing to admit Weigel's influence, hinting only to his correspondents to the great thoughts contained in his works. Particularly interesting in this regard is the fact that even while corresponding with Weigel himself, not only on their late academic projects, but specifically on mathematical problems, Leibniz never refers explicitly to the fact that in the same years he was extensively reading his works.

Influenced by this attitude, together with the theoretical achievements summarised above, one would be inclined to think that Leibniz was trying to deceive his contemporaries, but this inclination is probably due to the specificity of this research. From a wider perspective, it is clear that Leibniz had nothing to fear from Weigel: even in the specific field of mathematics, Weigel's influence was related more to those concepts that were an ideal bridge between philosophy and mathematics, rather than those concerning purely mathematical discoveries, except for binary arithmetics.

However, it is probably the ambiguous origin of these ideas that led Leibniz to his foundation of mathematics. The numerous claims on Weigel's confusion, always associated with prises, suggest that in Leibniz's mind Weigel's ideas did not need to be rejected, but perfected through a rationalist account. This is I believe one of the most stunning features of Leibniz's rationalism: the ability of discarding unnecessary results, combining the ones left and obtaining a completely new theory. As much as during his life Leibniz was extremely concerned by the political and theological consequences of his actions, in his private writings he exhibits a silent and steady intolerance for any superfluous reasoning. As much as he was interested in all sorts of topics with his widely known and impressive culture, he strictly admitted only the smallest number of universal principles in his thought, resorting to the introduction of a new one only when it was inevitable. Consequently, the process of creation becomes an art of combinations that goes beyond the limits of the former theories endorsed and cuts their unnecessary contradictions.

This attitude, that I would call a metaphysical minimalism or a methodological reductionism, is particularly evident in Leibniz's use of Weigel's ideas. In the early years, Leibniz immediately recognises the origin of Weigel's tendency in Hobbes' *conatus* and adopts it in its purest form, while modifying it for his needs. He is intrigued by Weigel's syncretistic project, but he allows only those principles that were necessary for the explanation of reality, i.e. the principle of sufficient reason and the principle by which the whole is greater than the part. Once endorsed the principle of contradiction in 1671, he seriously takes the project of his former teacher of deriving necessary truths from the smallest number of principles with an unprecedented rigour. Since the principle of contradiction is the very essence of necessity, he prioritizes it in the deduction of the other principles, by recognising its connection with identity and placing it at the core of the demonstration of the principle of the whole. Since the latter was the only principle needed for the foundation of geometry, this possibility was inherited by the principle of contradiction: what was a mere idea in Weigel's account became, thanks to Leibniz, a structured and complex theory of logic that had the capability of defining mathematical objects. As a result, the concepts of homogeneity, quality and quantity witness a continuous elaboration in rigorous definitions. In the same fashion, once Leibniz discovers a superior mathematics than that of Weigel he forces himself in finding a comprehensive theory that would bring homogeneity between the finite and the infinite. Finally, while developing binary arithmetics, he turns Weigel's base four model in a base two model, because Weigel's choice was an unnecessary homage to the Pythagorean tradition that was not appropriate with respect to the purpose of the calculus. Weigel's subtle mysticism was completely rejected. At the same time, because of his concerns on the possibility of harmonizing binary arithmetics with the rest of his mathematics, Leibniz carefully set dyadic apart from his consistent achievements in other fields, in the name of his rational coherence.

It is in this struggle for the most rational theory that Leibniz emerges as a true and unique genius. In this light, Weigel's influence was the spark that led to one of the most interesting theories on the philosophy of mathematics. By means of transformation, Leibniz brought homogeneity to the heterogeneous framework of Weigel's philosophy.

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Leibniz's Works

An Explanatory Note on Leibniz's Quotations With regards to references such as the secondary literature and other authors, I generally used the Chicago-style citation in the form author-date. However, Leibniz's publishing history, with so many editions spread in multiple volumes, is not completely suited for this style. For these reasons and for Leibniz only, I opted for the name of the edition instead of the author's name, followed by the volume number and the page number when needed. The abbreviations used for the different editions are as follows:

AA — Leibniz, Gottfried W. 1923 –. *Sämtliche Schriften und Briefe*, Darmstadt-Leipzig-Berlin: Akademie der Wissenschaften (Akademieausgabe). Collected in 8 series, each one divided in several volumes. Series I: *Allgemeiner, politischer und historischer Briefwechsel*. Series II: *Philosophischer Briefwechsel*. Series III: *Mathematischer, naturwissenschaftlicher und technischer Briefwechsel*. Series IV: *Politische Schriften*. Series V (publication planned): *Historische und sprachwissenschaftliche Schriften*. Series VI: *Philosophische Schriften*. Series VII: *Mathematische Schriften*. Series VIII: *Naturwissenschaftliche, medizinische und technische Schriften*.

C — Leibniz, Gottfried W. 1903. *Opuscules et fragments inédits, extraits des manuscrits de la Bibliothèque Royale de Hanovre*. Edited by Louis Couturat. Paris: Presses Universitaires de France.

D — Leibniz, Gottfried W. 1989. *Opera omnia. Nunc primum collecta, in classes distribute, praefationibus et indicibus exornata, studio Ludovici Dutens*. Edited by Louis Dutens. Hildesheim-Zürich-New York: Georg Olms Verlag. Collected in 6 volumes. Reprinted edition of: Leibniz, Gottfried W. 1768. *Gothofredi Guillelmi Leibnitii, S. Caesar. Majestatis*

Consiliarii, & S. Reg. Majest. Britanniarum a Consiliis Justitiae intimis, nec non a scribenda Historia, Opera omnia, nunc primum collecta, in classes distribute, praefationibus et indicibus exornata, studio Ludovici Dutens. Geneva.

E Leibniz, Gottfried W. 1995. *La caractéristique géométrique*. Edited by J. Echeverría, M. Parmentier. Paris: Vrin.

FICHANT Leibniz, Gottfried W. 1994. La *réforme de la dynamique*. De corporum concursu (1678) *et autres textes inédits*. Edited by Michel Fichant. Paris: Vrin.

FOUCHER1 Leibniz, Gottfried W. 1975. *Lettres et opuscules inédits, précédés d'une introduction par A. Foucher de Careil*. Hildesheim-New York: Georg Olms Verlag.

FOUCHER2 Leibniz, Gottfried W. 1971. *Nouvelles lettres et opuscules inédits, précédés d'une introduction par A. Foucher de Careil*. Hildesheim-New York: Georg Olms Verlag.

GM Leibniz, Gottfried W. 1971. *Leibnizes mathematische Schriften*. Edited by Carl I. Gerhardt. Hildesheim-New York: Georg Olms Verlag. Collected in 7 volumes. Reprinted edition of: Leibniz, Gottfried W. 1849-1863. *Leibnizes mathematische Schriften*. Edited by Carl I. Gerhardt. Berlin-Halle.

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GRUA Leibniz, Gottfried W. 1957. *Textes inédits d'après les manuscrits de la Bibliothèque Provinciale de Hanovre*. Edited by Gaston Grua. Paris: Presses Universitaires de France. Collected in 2 volumes.

GUHR Leibniz, Gottfried W. 1966. *Deutsche Schriften*. Edited by Gottschalk E. Guhrauer. Hildesheim: Georg Olms Verlag. Collected in 2 volumes.

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