Microscopic study of the isoscalar giant monopole resonance in Cd, Sn, and Pb isotopes

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(Received 22 June 2012; revised manuscript received 14 October 2012; published 26 November 2012)

The isoscalar giant monopole resonance (ISGMR) in Cd, Sn, and Pb isotopes has been studied within the self-consistent Skyrme Hartree-Fock + BCS and quasiparticle random phase approximation (QRPA). Three Skyrme parameter sets are used in the calculations (i.e., SLy5, SkM*, and SkP) since they are characterized by different values of the compression modulus in symmetric nuclear matter; namely, $K_\infty = 230$, 217, and 202 MeV, respectively. We also investigate the effect of different types of pairing forces on the ISGMR in Cd, Sn, and Pb isotopes. The various calculated energies and the strength distributions of the ISGMR are compared with available experimental data. We find that SkP underestimates the various energies for all isotopes due to its low value of the nuclear matter incompressibility; namely, $K_\infty = 202$ MeV. However, it can give a better description on the constrained energies for Cd isotopes and a reasonable peak energy for some nuclei. On the other hand, the SLy5 parameter set, supplemented by an appropriate pairing interaction, gives a reasonable description of the scaling energies in Cd and Sn isotopes and a good centroid energy in Pb isotopes. A better description of ISGMR in Cd and Sn isotopes is achieved by the SkM* interaction, which has a somewhat softer value of the nuclear incompressibility.

DOI: 10.1103/PhysRevC.86.054313 PACS number(s): 21.60.Jz, 21.65.Cd, 21.65.Mn, 24.30.Cz

I. INTRODUCTION

The compression modulus associated with the nuclear many-body systems plays an important role in the description of the structure of finite nuclei in the dynamics of heavy-ion collisions and in the physics of neutron stars and core-collapse supernovae [1,2]. For more than thirty years, much effort has been spent to deduce the value of the nuclear matter incompressibility both theoretically and experimentally. The measurements of the compression modes, such as the isoscalar giant monopole resonance (ISGMR) in finite heavy nuclei, have been the best tool so far to determine the value of the nuclear matter incompressibility K_∞. The analysis has mostly, but not only, been based on the distribution of the ISGMR strength in ^{208}Pb. The main results are reviewed in Ref. [3]. The extracted values of K_∞ are somewhat model dependent, but the accepted value from ^{208}Pb is 240 ± 20 MeV. In general, mean-field models, either nonrelativistic or relativistic, have been used to extract the value of the nuclear matter incompressibility. Widely used nonrelativistic Skyrme models give a value for the nuclear matter incompressibility around 230 MeV. It had been claimed that relativistic mean-field models are characterized by larger values of the nuclear matter incompressibility (around 250 MeV). However, using new fitted Skyrme forces with different density dependence, the authors of Ref. [4] pointed out that forces with $K_\infty = 250$ MeV can also reproduce the ISGMR experimental data of ^{208}Pb very well. So the residual model dependence in the extracted value of K_∞ is attributed to the fact that the distribution of ISGMR in ^{208}Pb is also sensitive to the density dependence of the symmetry energy [4–6].

Recently, the distribution of ISGMR strength in Cd, Sn, and Pb isotopes has been measured at the Research Center for Nuclear Physics (RCNP) at Osaka University [7–10]. These data raise a further question on the nuclear matter incompressibility. Namely, the Skyrme effective interactions with $K_\infty \sim 230$ MeV and the relativistic mean-field (RMF) models having $K_\infty \sim 270$ MeV can reproduce the experimental ISGMR distribution in ^{90}Zr and ^{208}Pb very well. However, the same models overestimate the centroid energies of the ISGMR in Sn isotopes [8,11–14]. This discrepancy shows that the observed ISGMR in Sn isotopes is softer than those in ^{90}Zr and ^{208}Pb, and this might be related to our incomplete understanding of surface, asymmetry, and pairing contributions to the incompressibility of finite nuclei.

Skyrme models have been used in Ref. [15] to investigate the correlation between the asymmetry contribution to the incompressibility K_τ and K_∞, and to validate the extraction of K_τ from the Sn data. Models having K_τ and K_∞ compatible with the Sn data and built within the RMF framework are shown to significantly underestimate the the distribution of strength in ^{208}Pb [16]. By calculating the ISGMR in nuclei with large neutron excess, the authors of Ref. [17] concluded that the incompressibility of neutron-rich matter is still an important open problem. It has also been pointed out that superfluidity may have a sizable effect on the incompressibility of nuclear matter and finite nuclei [18,19]. This conclusion has been drawn by exploiting constrained Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB) calculations [4,20,21] in order to determine the inverse energy-weighted sum rule m_{-1}, and by using this together with the energy-weighted sum rule m_1 to define the ISGMR centroid as

TABLE I. Pairing strength V_0 for various types of pairing interactions defined in Eq. (1), in units of MeV fm^3. For details see text.

	Volume	Surface	Mixed
		^{112}Cd	
SLy5	261	738	388
SkM*	230	675	342
SkP	215	692	328
		^{120}Sn	
SLy5	218	645	325
SkM*	255	725	381
SkP	213	688	328
		^{204}Pb	
SLy5	265	875	409
SkM*	255	863	392
SkP	211	771	335

$E_{\mathrm{ISGMR}} = \sqrt{m_1/m_{-1}}$. It should be stressed that the effect of pairing on the ISGMR, within the self-consistent quasiparticle random phase approximation (QRPA) on top of HFB, has been highlighted in Refs. [11,22], where it has been shown that the inclusion of pairing reduces, to some extent, the discrepancy between the values of K_∞ extracted from ^{208}Pb and Sn isotopes data, respectively.

A different type of calculation was performed in Ref. [23] to describe the ISGMR strength distribution in Sn isotopes and ^{208}Pb. The theoretical models used are the QRPA and the quasiparticle time-blocking approximation (QTBA), that includes quasiparticle-phonon coupling. Also in this case, a satisfactory description of ^{208}Pb and Sn isotopes at the same time has not been achieved. This calculation is not a fully self-consistent one.

In this work, we employ the Skyrme QRPA approach on top of HF-Bardeen-Cooper-Schrieffer (HF-BCS) to study the ISGMR strength distribution in Cd, Sn, and Pb isotopes. This method allows systematic and fully self-consistent calculations for the ISGMR. All terms of the interaction, including the one-body and two-body spin-orbit and Coulomb parts, are included when the ground-state mean-field and the residual interaction are evaluated. Three Skyrme parameter sets, SLy5 [24], SkM* [25], and SkP [26] are used in the calculations. These Skyrme interactions display different values for the nuclear matter incompressibility. We also compare the effect of volume, surface, and mixed pairing interactions on the ISGMR properties in Cd, Sn, and Pb isotopes. This paper is organized as follows: we will briefly report the main features of our Skyrme HF-BCS plus QRPA model in Sec. II. The results for ISGMR in Cd, Sn, and Pb isotopes are discussed and compared with available experimental data in Sec. III. Section IV is devoted to the summary and discusses the perspectives for future work.

II. THEORETICAL MODELS

The Skyrme interaction is quite successful in the description of nuclear properties both of ground states and excited states. As mentioned in the previous section, we will use the QRPA approach on top of HF-BCS for our theoretical investigation.

First we solve the Skyrme HF-BCS equation for the ground state in coordinate space. The radial mesh on which the equations are solved extends up to 18 fm, and the mesh size is 0.1 fm. For all nuclei under study, this radial mesh is large enough so that the results are stable. The pairing correlations are generated by a density-dependent zero-range force,

$$V_{\mathrm{pair}}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left[1 - \eta \left(\frac{\rho(\mathbf{r})}{\rho_0}\right)\right] \delta(\mathbf{r}_1 - \mathbf{r}_2), \qquad (1)$$

where $\rho(\mathbf{r})$ is the particle density in coordinate space and $\rho_0 = 0.16$ fm^{-3} is the nuclear saturation density. The value of η is taken as 0, 0.5, or 1 for the volume, mixed, or surface pairing interactions, respectively. The pairing window (i.e., the states that are taken into account for the solution of the BCS equations) includes five unoccupied orbitals above the last occupied level in the HF approximation. In this space, the pairing strength V_0 is fixed by fitting the pairing gap extracted from experimental data of odd-even mass difference by using the five-point formula. The values of V_0 that we have obtained are displayed in Table I. They reproduce well the gap associated with one typical nucleus for each isotope chain; in particular, we have chosen to reproduce the empirical pairing gaps of ^{112}Cd ($\Delta_n = 1.334$ MeV), ^{120}Sn ($\Delta_n = 1.321$ MeV),

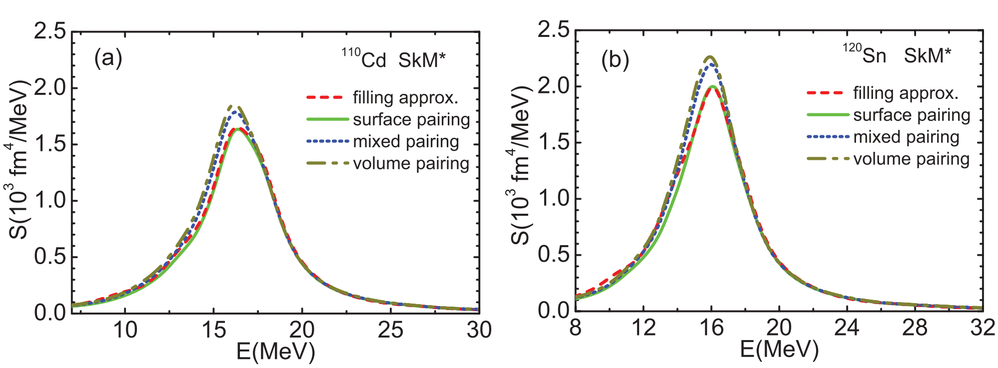

FIG. 1. (Color online) The ISGMR strength distribution in (a) ^{110}Cd and (b) ^{120}Sn, calculated by using either the filling approximation, the volume, the surface, or the mixed pairing forces. The SkM* force is adopted in the *p-h* channel.

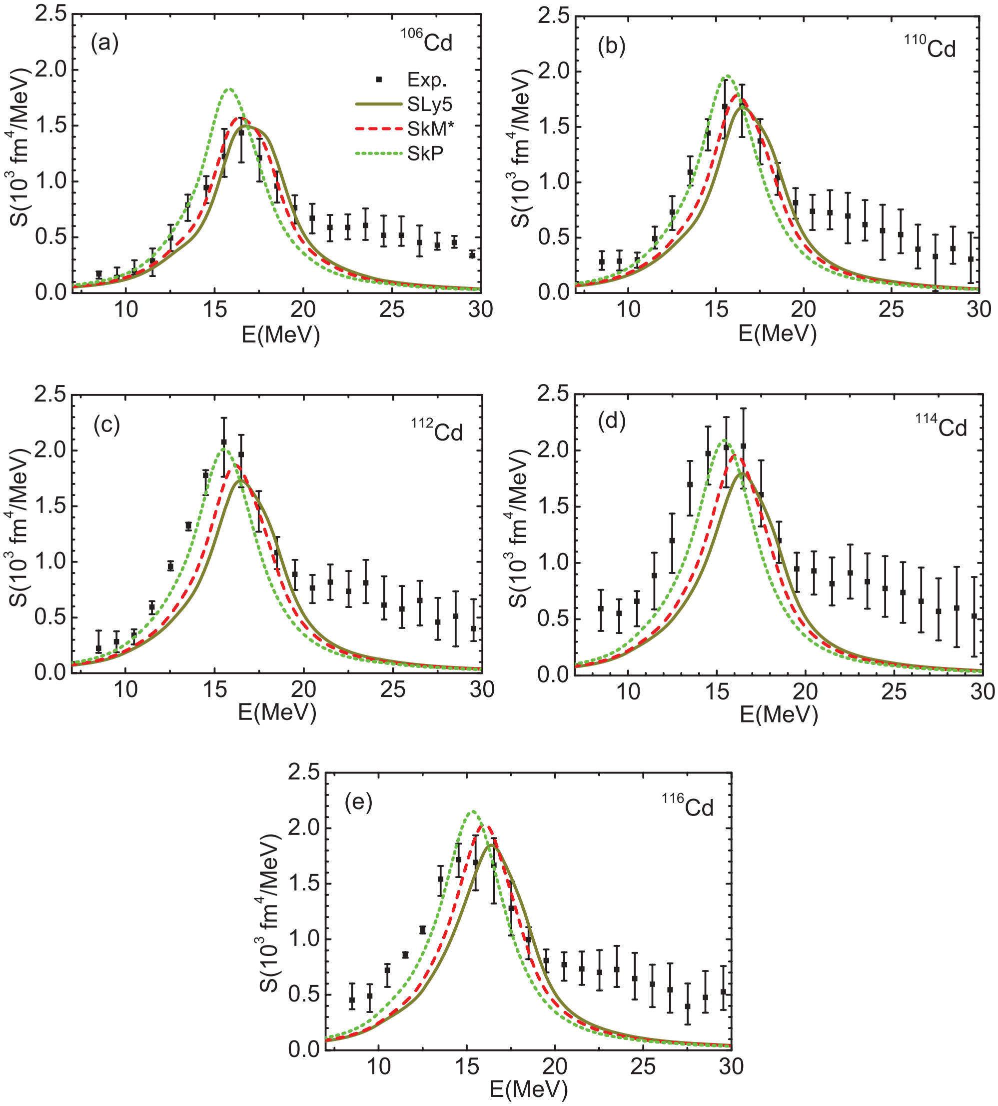

FIG. 2. (Color online) The calculated ISGMR strength distribution in $^{106-116}$Cd are compared with the experimental data obtained at RCNP [7]. The SLy5 (solid line), SkM* (dashed line), and SkP (dotted line) forces are adopted in the calculations whose results are shown here, together with the mixed pairing interaction.

and ^{204}Pb ($\Delta_n = 0.841$ MeV). For Sn and Pb isotopes, there is only neutron pairing because these nuclei have closed proton shells associated with $Z = 50$ and 82, respectively. For Cd isotopes, the proton pairing also exists in principle because the proton number 48 is not a magic number. However, we have found numerically that the effect of proton pairing in Cd isotopes on the ISGMR strength distribution is very small (of the order of tenths of keV). So in our calculation we use the filling approximation for the proton $1g_{9/2}$ state in Cd isotopes. The pairing strengths V_0 obtained by fitting the empirical gaps are different for each Skyrme parameter set and for each pairing model.

We now provide few details about the QRPA calculations. The single-particle continuum is discretized by setting the nuclei in a spherical box of radius equal to 18 fm. For every value of the quantum numbers (l, j) associated with the single-particle states, we include in the QRPA model space the unoccupied states up to the maximum number of nodes given by $n_{\max} = n_{\text{last}} + 12$, where n_{last} is the number of nodes of the last occupied state with a given (l, j). The convergence of the calculated results is checked by looking at the results for the energy and the strength of the ISGMR. The QRPA matrix equation having good angular momentum and parity J^π is given by

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = \hbar\omega_n \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix}, \tag{2}$$

where $\hbar\omega_n$ is the energy of the nth QRPA state and X^n, Y^n are the corresponding forward and backward amplitudes, respectively. The explicit forms of the matrices A and B are given elsewhere [27–30]. The p-h matrix elements are derived from the Skyrme energy density functional including all the

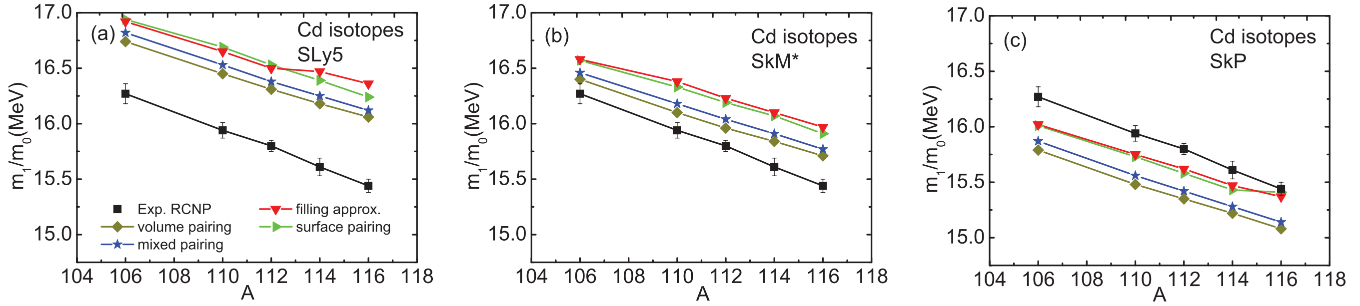

FIG. 3. (Color online) Calculated ISGMR centroid energies in the even-even $^{106-116}$Cd isotopes are compared with the experimental data obtained from Ref. [7]. The forces (a) SLy5, (b) SkM*, and (c) SkP are adopted in the present calculations together with either the filling approximation, the volume, the surface, or the mixed pairing interactions, respectively.

terms such as the two-body spin-orbit and two-body Coulomb interactions. We should mention that several previous works devoted to the study of the ISGMR strength in Sn isotopes are not fully self-consistent [15,23] since the spin-orbit interaction is not taken into account in the residual interaction. In Ref. [31], the authors have discussed the self-consistency violation in HF-RPA calculations for nuclear giant resonances: they have shown, for example, that the two-body spin-orbit interaction gives a slight repulsive contribution to the ISGMR strength in light nuclei whereas, for medium and heavy nuclei, it produces an attractive effect on the ISGMR strength, so that the centroid energies are pushed downward by about 0.6 MeV. Meanwhile the two-body Coulomb interaction gives a small repulsive contribution to the ISGMR strength; for example it can shift the strength to a higher-energy region by about 300 keV in Sn isotopes.

TABLE II. Calculated ISGMR constrained energies ($E_{\rm con}$), centroid energies ($E_{\rm cen}$), scaling energies ($E_{\rm s}$), and peak energies in even-even $^{106-116}$Cd isotopes are compared with the experimental data. The theoretical results are obtained in the interval between 10.5 and 20.5 MeV by using the SkP, SkM*, and SLy5 parameter sets together with the mixed pairing interaction. The experimental data are from Ref. [7]. The values in parenthesis are the differences between the theoretical values and the experimental data. Units are MeV.

	Expt.	SkP	SkM*	SLy5
		$E_{\rm con} = \sqrt{m_1/m_{-1}}$		
^{106}Cd	16.06 ± 0.05	15.81 (−0.25)	16.40 (0.34)	16.75 (0.69)
^{110}Cd	15.72 ± 0.05	15.50 (−0.22)	16.11 (0.39)	16.46 (0.74)
^{112}Cd	15.59 ± 0.05	15.35 (−0.24)	15.97 (0.38)	16.29 (0.70)
^{114}Cd	15.37 ± 0.08	15.21 (−0.16)	15.84 (0.47)	16.16 (0.79)
^{116}Cd	15.19 ± 0.06	15.06 (−0.13)	15.70 (0.51)	16.02 (0.82)
		$E_{\rm cen} = m_1/m_0$		
^{106}Cd	16.27 ± 0.09	15.87 (−0.40)	16.46 (0.19)	16.82 (0.55)
^{110}Cd	15.94 ± 0.07	15.56 (−0.38)	16.18 (0.24)	16.54 (0.60)
^{112}Cd	15.80 ± 0.05	15.42 (−0.38)	16.05 (0.25)	16.39 (0.59)
^{114}Cd	15.61 ± 0.08	15.28 (−0.33)	15.91 (0.30)	16.25 (0.64)
^{116}Cd	15.44 ± 0.06	15.14 (−0.30)	15.78 (0.34)	16.12 (0.68)
		$E_{\rm s} = \sqrt{m_3/m_1}$		
^{106}Cd	16.83 ± 0.09	16.02 (−0.81)	16.63 (−0.20)	16.99 (0.16)
^{110}Cd	16.53 ± 0.08	15.74 (−0.79)	16.37 (−0.16)	16.74 (0.21)
^{112}Cd	16.38 ± 0.06	15.62 (−0.76)	16.24 (−0.14)	16.61 (0.23)
^{114}Cd	16.27 ± 0.09	15.49 (−0.78)	16.11 (−0.16)	16.49 (0.22)
^{116}Cd	16.14 ± 0.07	15.35 (−0.79)	15.97 (−0.17)	16.37 (0.23)
		$E_{\rm peak}$		
^{106}Cd	16.50 ± 0.19	15.80 (−0.70)	16.40 (−0.10)	16.70 (0.20)
^{110}Cd	16.09 ± 0.15	15.60 (−0.49)	16.20 (0.11)	16.50 (0.41)
^{112}Cd	15.72 ± 0.10	15.50 (−0.22)	16.20 (0.48)	16.50 (0.78)
^{114}Cd	15.59 ± 0.20	15.40 (−0.19)	16.10 (0.51)	16.40 (0.81)
^{116}Cd	15.43 ± 0.12	15.30 (−0.13)	16.00 (0.57)	16.40 (0.97)

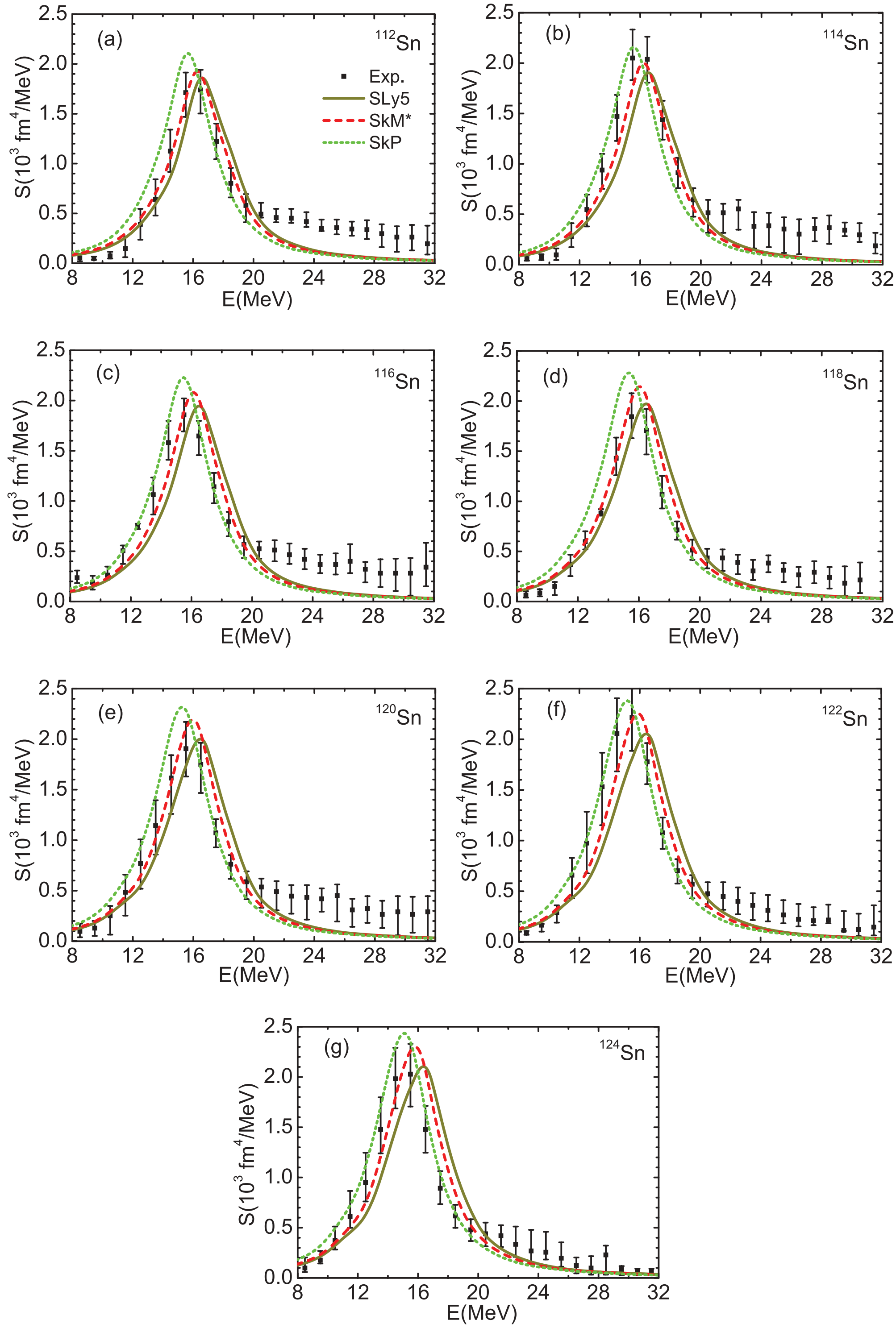

FIG. 4. (Color online) The calculated ISGMR strength distributions in $^{112-124}$Sn are compared with the experimental data from Refs. [8,9]. The SLy5 (solid line), SkM* (dashed line), and SkP (dotted line) forces are adopted in the calculations whose results are shown here, together with the mixed pairing interaction.

After solving the QRPA equations, various moments of the strength distributions can be obtained by means of the equation

$$m_k = \int E^k S(E) dE, \tag{3}$$

where $S(E) = \sum_n |\langle 0|\hat{F}|n\rangle|^2 \delta(E - E_n)$ is the strength function associated with the monopole operator

$$\hat{F} = \sum_i r_i^2. \tag{4}$$

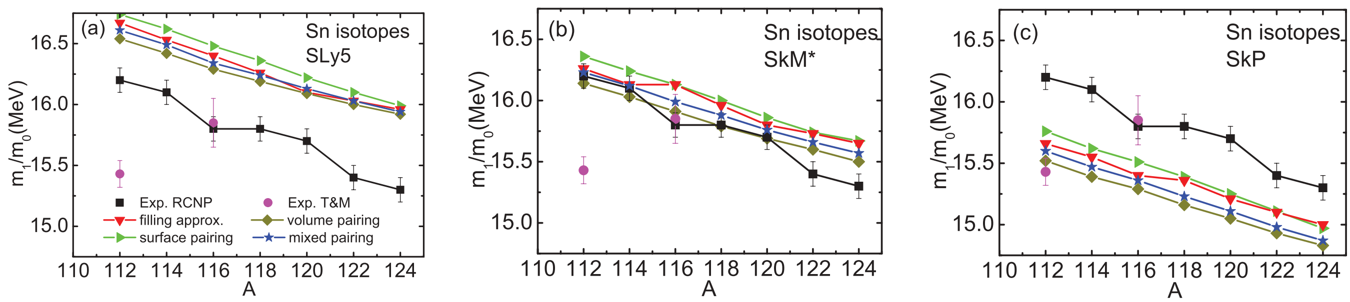

FIG. 5. (Color online) The calculated ISGMR centroid energies in the even-even $^{112-124}$Sn isotopes are compared with the experimental data from Refs. [8,9]. The forces (a) SLy5, (b) SkM*, and (c) SkP are adopted in the present calculations together with either the filling approximation, the volume, the surface, or the mixed pairing interactions, respectively.

The constrained energy E_{con}, the centroid energy E_{cen}, and the scaling energy E_s of the resonance are then defined as

$$E_{con} = \sqrt{\frac{m_1}{m_{-1}}}, \quad E_{cen} = \frac{m_1}{m_0}, \quad E_s = \sqrt{\frac{m_3}{m_1}}, \qquad (5)$$

respectively.

III. ISOSCALAR GIANT MONOPOLE RESONANCE RESULTS AND DISCUSSION

In order to check how much the pairing correlations affect the ISGMR strength distributions, we show in Fig. 1 the ISGMR strength distributions in ^{110}Cd and ^{120}Sn calculated by using different pairing models. The effective force SkM* is adopted in the particle-hole channel. For the pairing channel, we take the surface, mixed, and volume pairing interactions and compare the results with those obtained within RPA (no pairing) and the filling approximation. We find that the surface pairing and the filling approximation give almost identical results. On the other hand, the volume and the mixed pairing predict slightly lower peak energies although the difference is rather small. The ISGMR centroid energy in ^{110}Cd (^{120}Sn) are 16.38 (15.80), 16.33 (15.86), 16.10 (15.69), 16.18 (15.76) MeV, when calculated by employing either the filling approximation, the surface, the volume, or the mixed pairing interactions, respectively. The maximum difference due to different pairing models is 280 (110) keV in ^{110}Cd (^{120}Sn).

A. Cd Isotopes

Very recently, measurements of the ISGMR strength distributions in Cd isotopes were performed at RCNP, Osaka University [7]. In keeping with the fact that it is difficult to reproduce equally well Pb and Sn isotopes with a unique Skyrme force, we would like to see what are the results that these forces provide for the strength distributions in Cd isotopes. To address this question, we have performed the calculations for the ISGMR strength in the Cd isotopes with the three different aforementioned Skyrme interactions together with various pairing models. Figure 2 displays the QRPA results for the ISGMR strength distribution in $^{106-116}$Cd, calculated by using the SkP (dotted line), SkM* (dashed line), and SLy5 (solid line) interactions, respectively. The pairing force adopted in Fig. 2 is the mixed pairing interaction. For all nuclei, the strength distributions are concentrated in a single peak around 16 MeV, which exhausts almost all the energy weighted sum rule. However, the location of the peak found with each Skyrme interaction is slightly different. The SkP interaction predicts lowest peaks while the SLy5 interaction gives peaks at the highest energies. The peaks obtained by using the SkM* interaction stays in the middle between the other two cases. As is known from previous studies, the relative position of the peaks is governed by the nuclear matter incompressibility associated with each effective interaction.

In Fig. 3 we show the theoretical ISGMR centroid energies in $^{106-116}$Cd obtained by the QRPA calculations. The results in Figs. 3(a), 3(b), and 3(c) are obtained by using the SLy5, SkM*, and SkP Skyrme interactions, respectively, together with various pairing interactions. Figure 3(a) shows the ISGMR centroid energies obtained by the SLy5 interaction. The calculated results are about 700 keV systematically larger than the experimental data, which can be seen clearly from the figure. For the case of the SkM* interaction whose results are shown in Fig. 3(b), the conclusion is different from that obtained for the SLy5 interaction, due to the lower value of the nuclear matter incompressibility (we remind the reader that $K_\infty = 217$ MeV for SkM*, and that $K_\infty = 230$ MeV for SLy5). Although the theoretical results are still larger than the experimental data, compared with the case of SLy5 the gap between theoretical and experimental results is much reduced. With the volume pairing, SkM* reproduces the ISGMR centroid energies of $^{106-112}$Cd within 160 keV, for $^{114-116}$Cd the difference is slightly larger; about 270 keV. Finally, it should be expected that the results obtained with the SkP parameter set underestimate the experimental ISGMR centroid energies in $^{106-116}$Cd [cf. Fig. 3(c)], due to the lower value of the nuclear matter incompressibility associated with this parameter set; namely $K_\infty = 201$ MeV. Also in this case the results depend on the choice of the pairing force: the filling approximation and the surface pairing interaction provide a good reproduction of the experimental centroid energies (the difference is within 260 keV) and when volume and mixed pairing interactions are adopted, the results tend to worsen.

Whereas we can clearly confirm from the present results that the value of the nuclear matter incompressibility does play a key role in dictating the location of the ISGMR centroid

TABLE III. Calculated ISGMR constrained energies ($E_{con} = \sqrt{m_1/m_{-1}}$), centroid energies ($E_{cen} = m_1/m_0$), scaling energies ($E_s = \sqrt{m_3/m_1}$), and peak energies in even-even $^{112-124}$Sn isotopes are compared with the experimental data. The theoretical results are obtained in the interval between 10.5 and 20.5 MeV by using the SkP, SkM*, and SLy5 parameter sets together with the mixed pairing interaction. The experimental data are taken from Refs. [8,9,32,33]. The values in parentheses are the difference between the theoretical values and the experimental data. Units are MeV.

	Expt.	SkP	SkM*	SLy5
		$E_{con} = \sqrt{m_1/m_{-1}}$		
^{112}Sn	16.1 ± 0.1	15.55 (−0.55)	16.18 (0.08)	16.55 (0.45)
	$15.23^{+0.10}_{-0.10}$			
^{114}Sn	15.9 ± 0.1	15.42 (−0.48)	16.06 (0.16)	16.42 (0.52)
^{116}Sn	15.7 ± 0.1	15.31 (−0.39)	15.94 (0.24)	16.29 (0.59)
^{118}Sn	15.6 ± 0.1	15.17 (−0.43)	15.82 (0.22)	16.17 (0.57)
^{120}Sn	15.5 ± 0.1	15.05 (−0.45)	15.71 (0.21)	16.05 (0.55)
^{122}Sn	15.2 ± 0.1	14.92 (−0.28)	15.60 (0.40)	15.94 (0.74)
^{124}Sn	15.1 ± 0.1	14.80 (−0.3)	15.49 (0.39)	15.85 (0.75)
	$14.33^{+0.17}_{-0.14}$			
		$E_{cen} = m_1/m_0$		
^{112}Sn	16.2 ± 0.1	15.60 (−0.6)	16.23 (0.03)	16.61 (0.41)
	$15.43^{+0.11}_{-0.10}$			
^{114}Sn	16.1 ± 0.1	15.27 (−0.83)	16.12 (0.02)	16.49 (0.39)
^{116}Sn	15.8 ± 0.1	15.36 (−0.44)	16.00 (0.20)	16.36 (0.56)
	$15.85^{+0.20}_{-0.20}$			
^{118}Sn	15.8 ± 0.1	15.23 (−0.57)	15.88 (0.08)	16.25 (0.45)
^{120}Sn	15.7 ± 0.1	15.11 (−0.59)	15.78 (0.08)	16.13 (0.43)
^{122}Sn	15.4 ± 0.1	14.99 (−0.41)	15.67 (0.27)	16.03 (0.63)
^{124}Sn	15.3 ± 0.1	14.87 (−0.43)	15.57 (0.27)	15.95 (0.65)
	$14.50^{+0.14}_{-0.14}$			
		$E_s = \sqrt{m_3/m_1}$		
^{112}Sn	16.7 ± 0.2	15.74 (−0.96)	16.38 (−0.32)	16.77 (0.07)
	$16.05^{+0.26}_{-0.14}$			
^{114}Sn	16.5 ± 0.2	15.62 (−0.88)	16.18 (−0.32)	16.66 (0.16)
^{116}Sn	16.3 ± 0.2	15.53 (−0.77)	16.16 (−0.14)	16.54 (0.24)
^{118}Sn	16.3 ± 0.1	15.40 (−0.9)	16.05 (−0.25)	16.45 (0.15)
^{120}Sn	16.2 ± 0.2	15.29 (−0.91)	15.96 (−0.24)	16.35 (0.15)
^{122}Sn	15.9 ± 0.2	15.18 (−0.71)	15.86 (−0.04)	16.26 (0.36)
^{124}Sn	15.8 ± 0.1	15.07 (−0.73)	15.76 (−0.04)	16.18 (0.38)
	$14.96^{+0.10}_{-0.11}$			
		E_{peak}		
^{112}Sn	16.1 ± 0.1	15.70 (−0.40)	16.30 (0.20)	16.60 (0.50)
^{114}Sn	15.9 ± 0.1	15.60 (−0.30)	16.20 (0.30)	16.50 (0.60)
^{116}Sn	15.8 ± 0.1	15.50 (−0.30)	16.10 (0.30)	16.50 (0.70)
^{118}Sn	15.6 ± 0.1	15.30 (−0.30)	16.10 (0.50)	16.50 (0.90)
^{120}Sn	15.4 ± 0.2	15.30 (−0.10)	16.00 (0.60)	16.50 (1.10)
^{122}Sn	15.0 ± 0.2	15.20 (0.20)	15.90 (0.90)	16.40 (1.40)
^{124}Sn	14.8 ± 0.2	15.10 (0.30)	15.80 (1.00)	16.40 (1.60)

energy, it is also true that the pairing interaction lowers the energy of the ISGMR to some extent, typically a few hundred keV. This qualitative conclusion is the same that was first found in Ref. [11]. Thus, the pairing interaction cannot be neglected if one aims to reproduce not only the ISGMR centroid energies in Cd isotopes, but also, more generally, in other open-shell nuclei. It should also be noticed that the slope of the isotope dependence of the ISGMR centroid energy is rather well reproduced by all the three interactions, while the absolute values are much more sensitive to the choice of the Skyrme parameter set and of the pairing force.

The various kinds of centroid energies and the calculated peak energies are shown in Table II. The results that we report here are obtained with the mixed pairing interaction for each Skyrme parameter set. From Table II we can see that the SkP interaction underestimates systematically the various energies of Cd isotopes and the predicted constrained energies close to the experimental data; also the experimental peak energies

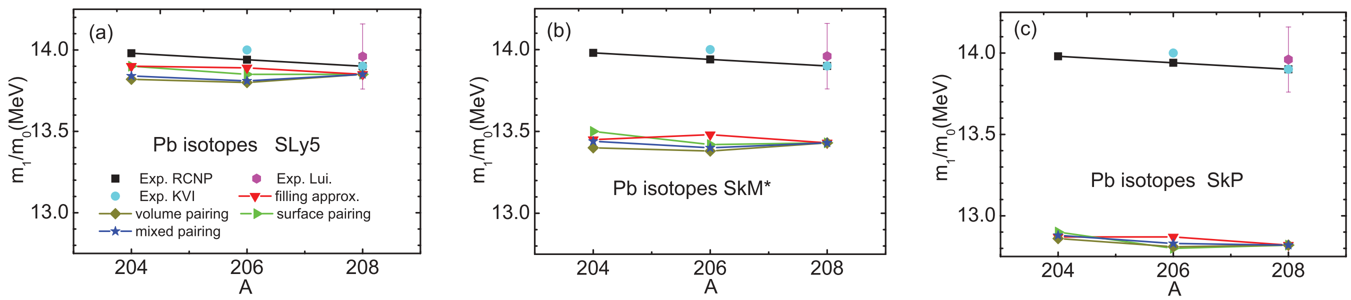

FIG. 6. (Color online) Calculated ISGMR centroid energies in the even-even $^{204-208}$Pb isotopes are compared with the experimental data obtained from Refs. [10,32,33]. The forces (a) SLy5, (b) SkM*, and (c) SkP are adopted in the present calculations together with either the filling approximation, the volume, the surface, or the mixed pairing interactions, respectively.

are well reproduced in the mass region of 112 to 116. With the mixed pairing force, SkM* can give a better description of the centroid energies $E_{\rm cen} = m_1/m_0$ and the scaling energies $E_{\rm s} = \sqrt{m_3/m_1}$ for all nuclei, while a slight overestimation of the constrained energies $E_{\rm con} = \sqrt{m_1/m_{-1}}$ is produced. SkM* gives a very good prediction of the peak energies for ^{106,110}Cd. In general, the SLy5 interaction overestimates all the energies defined in the text and the peak energies. But it can provide better results for the scaling energies; the differences are within 230 keV and are displayed in Table II.

It is not completely clear from the experimental point of view whether the constrained, centroid, or scaling energies are more suitable to be compared with the experimental data. However, from what we have just concluded, it can be stated that the reasonable values of nuclear incompressibility that can be extracted from the present Cd data are in between the values of SkM* and SkP. This number is slightly smaller than the one extracted from ^{208}Pb.

B. Sn Isotopes

Figure 4 shows the calculated ISGMR strength distributions in even-even $^{112-124}$Sn together with the experimental data taken from Refs. [8,9]. The results obtained with the Skyrme sets SLy5 (solid line), SkM* (dashed line), SkP (dotted line), and with the mixed pairing interaction are displayed. All three Skyrme interactions give a single peak around 16 MeV. The interaction dependence of the peak energy is qualitatively the same as in the case of Cd isotopes; namely, the SkP result is the lowest, the SkM* result lies in the middle, and the SLy5 result is found at the highest energy—in agreement with the associated values of the incompressibility.

The mass-number dependence of the calculated ISGMR centroid energies in the Sn isotopes is shown in Fig. 5 for the case of the SLy5, SkM*, and SkP sets with the various pairing interactions. The results obtained without pairing, by using the filling approximation, are also shown in the same figure. It should be noticed that in some cases the experimental data are not consistent with each other. However, at the same time it is quite clear that the predicted centroid energies obtained using the SLy5 and SkP interactions are not in agreement with the experimental data: the SLy5 results, even when the pairing effects are taken into account, overestimate the experimental data while the SkP results underestimate them. In the case of the SkM* interaction, the predictions are much improved compared to the case of SLy5 and SkP. The calculations with the volume and mixed pairing forces (and the filling approximation) give better predictions for ^{112,114}Sn. The results provided by the volume pairing force in ^{116,118,120}Sn are very close to the experimental data, while they slightly overestimate the experimental findings in ^{122,124}Sn.

There is some difference between the results presented here and those already discussed in Ref. [11]. We have included here the contribution of the two-body spin-orbit interaction which provides an attractive effect and lowers the RPA and QRPA ISGMR energies. However, in order to be able to perform many systematic calculations, we have chosen in the current work the QRPA on top of HF-BCS instead of HFB. Pairing effects are slightly different in the two approaches, so the discussion of this section on the results obtained with either volume, surface, or mixed pairing forces, is not exactly the same as in Ref. [11]. Despite these differences, we have to stress that the overall qualitative conclusion that the Sn data are rather consistent with the value of nuclear incompressibility associated with SkM* once the pairing is taken into account is the same as in Ref. [11].

This conclusion can be reinforced by the values reported in Table III, which are the various calculated energies obtained by means of the QRPA calculations performed with the mixed pairing interaction. The interaction SkP underestimates all energies defined in the text with respect to experiment but gives better peak energies for $^{114-124}$Sn, whereas SLy5 overestimates all the energies. The interaction SkM* gives better results for both the constrained energies, the centroid energies, and the scaling energies and predicts better peak energies for $^{112-116}$Sn.

Thus, the discrepancy between the values of the nuclear incompressibility extracted either from Sn or Pb data still remains to some extent a puzzle; however, the pairing effects need to be taken into account and reduce this discrepancy to only ≈5%.

C. Pb Isotopes

Measurements of the ISGMR strength distributions in the even-even nuclei ^{204,206}Pb have been done recently at RCNP, Osaka University [10]. Several measurements of the ISGMR strength distribution in ^{208}Pb had been performed already in the past; for example, at KVI (Netherlands) and Texas A&M University (USA). One of the motivations to perform the new

TABLE IV. Calculated ISGMR constrained energies ($E_{\rm con} = \sqrt{m_1/m_{-1}}$), centroid energies ($E_{\rm cen} = m_1/m_0$), and scaling energies ($E_s = \sqrt{m_3/m_1}$) in even-even ^{204,206,208}Pb are compared with the experimental data. The theoretical results are obtained in the interval between 10 and 20 MeV by using the SkP, SkM*, and SLy5 parameter sets, together with the mixed pairing interaction. The experimental data are taken from Refs. [10,32]. The values in parentheses are the difference between the theoretical values and the experimental data. Units are MeV.

	Expt.	SkP	SkM*	SLy5
		$E_{\rm con} = \sqrt{m_1/m_{-1}}$		
^{204}Pb		12.84	13.40	13.80
^{206}Pb		12.79	13.36	13.76
^{208}Pb		12.70	13.29	13.71
		$E_{\rm cen} = m_1/m_0$		
^{204}Pb	13.98	12.88 (−1.10)	13.44 (−0.54)	13.84 (−0.14)
^{206}Pb	13.94	12.83 (−1.11)	13.40 (−0.54)	13.81 (−0.13)
^{208}Pb	13.96 ± 0.2	12.74 (−1.22)	13.34 (−0.62)	13.85 (−0.11)
		$E_s = \sqrt{m_3/m_1}$		
^{204}Pb		13.01	13.56	13.98
^{206}Pb		12.97	13.54	13.97
^{208}Pb		12.88	13.49	13.93

measurement on Pb isotopes at RCNP has been the study of a conjecture; that is, the possible appearance in the ISGMR energies of the so-called mutually enhanced magicity (MEM) effect proposed by Lunney and Zeldes in the context of the mass systematics [34,35]. According to this conjecture, the ISGMR energy in double closed shell nuclei might be higher if compared with the systematic values in its neighboring open-shell isotopes [19]. Experimentally, the peak energies that have been obtained for the $^{204-208}$Pb isotopes [10] are 13.98, 13.94, and 13.90 MeV, respectively. From these experimental results, we can not infer any kind of MEM effect in Pb isotopes.

In Fig. 6 and Table IV we show our theoretical results for the ISGMR in Pb isotopes. The centroid energies calculated by using different Skyrme sets and pairing forces are displayed in Fig. 6. We can see that both SkM* and SkP underestimate the experimental data, although the results obtained with SkM* are better. The Skyrme force SLy5, having an associated value of incompressibility $K_\infty = 230$ MeV, reproduces very well the centroid energies in Pb isotopes. The same conclusion can be drawn from the results shown in Table IV. It has been known for some time that this value of K_∞ can be extracted from the ^{208}Pb data, if the density dependence of the force is the one that characterizes most of the recent Skyrme forces [4]. Indeed, from the present results one can see that the effect of pairing in the Pb isotopes is rather small. Finally, there is no evidence for any MEM effect in our theoretical calculations, which is also confirmed by the experimental results obtained at RCNP.

IV. SUMMARY

In summary, we have studied systematically the isoscalar giant monopole resonance (ISGMR) in Cd, Sn, and Pb isotopes within the self-consistent Skyrme HF + BCS and quasiparticle random phase approximation (QRPA). Three different Skyrme parameter sets are used in the present calculations; namely, SLy5, SkM*, and SkP. They are chosen since they are characterized by different values of the nuclear incompressibility, $K_\infty = 230$, 217, and 202 MeV, respectively. To study the role of the pairing correlations, we choose three types of pairing interactions (i.e., the so-called volume, surface, and mixed pairing forces). For the sake of comparison, we also produce RPA results (without pairing) within the filling approximation.

The various kinds of energies and the detailed strength distributions of the ISGMR in Cd, Sn, and Pb isotopes are compared with the available experimental data. We have found that the pairing correlations always decrease the peak energies of the ISGMR because of the attractive character of the particle-particle force in the 0^+ channel. The typical size of this effect is several hundred keV.

From the present study, we find that the SkP interaction underestimates the various energies defined in the text in all the studied isotopes due to its low value of nuclear matter incompressibility; however, it gives reasonable peak energies for some isotopes. The SLy5 parameter set (having an associated incompressibility of 230 MeV) gives a reasonable description of the ISGMR in Pb isotopes, whereas a better overall description in the case of Cd and Sn isotopes is achieved by using the force SkM* (characterized by incompressibility of 217 MeV). We have also found that the change of the ISGMR energies by pairing correlations in ^{204,206}Pb is quite small. The results for the Pb isotopes suggest that both theoretically and experimentally there is no evidence for the so-called MEM effect.

Pairing helps in reducing the discrepancy between the values of the nuclear incompressibility extracted either from Pb or Sn data. We also found that Cd data do not introduce further problems as they seem to be rather consistent with Sn data. A small discrepancy of about 5% in K_∞ between the conclusions drawn from Pb and Sn data remains and may deserve further investigation. Since the size of this discrepancy depends on the pairing force and the model to treat its effects, one should better analyze whether we can constrain the attractive particle-particle matrix elements that appear in the QRPA calculations.

Pairing forces are usually constrained only by means of ground-state properties, and at present it is unclear if volume or surface pairing interactions should be preferred. Along a different line, the present results may be interpreted by saying that the role of surface and surface-symmetry contributions to the nuclear incompressibility are still not precisely fixed.

ACKNOWLEDGMENTS

We would like to thank M. Fujiwara, Y.-W. Lui, and U. Garg for stimulating discussions and for providing us the experimental data prior to publication. This work is partially supported by the Japanese Ministry of Education, Culture, Sports, Science and Technology by a Grant-in-Aid for Scientific Research under the program number (C(2))20540277. L. C. acknowledges the support of the National Science Foundation of China under Grants No. 10875150 and No. 11175216. The support of the Italian Research Project "Many-body theory of nuclear systems and implications on the physics of neutron stars" (PRIN 2008) is also acknowledged. This research was supported in part by the Project of Knowledge Innovation Program (PKIP) of the Chinese Academy of Sciences, Grant No. KJCX2-EW-N01.

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