A guided inquiry based teaching and learning sequence on oscillations based on experiments and data-logging techniques

Marco Giliberti[1], Enrico Rigon[2], Marco Stellato[1], Marina Tamborini[3]

[1] Dipartimento di Fisica, Università degli Studi di Milano, Milano, Italy

[2] ISIS “Daniele Crispi”, Busto Arsizio, Varese, Italy

[3]Liceo Scientifico “P. Bottoni”, Milano, Italy

Abstract

We present here a teaching and learning sequence on oscillations entirely based on experiments and data logging techniques. The sequence has been proposed to three different groups of students during curricular and extracurricular lessons. The purpose of this paper is to discuss a way to introduce upper secondary school students to complicated topics, such as those of coupled oscillations, avoiding the use of too much mathematics and calculus, but with an intense use of data logging techniques.

Keywords: oscillations, harmonic motion, coupled oscillators, normal modes, data logging, video analysis

Introduction

In the Italian school students face the topic *oscillations* between the 11th and 12th grade, that is between the third and the fourth year of upper secondary school, as an introduction to the wider topic of *waves*. Generally, in teaching practice, only short time is devoted to harmonic motion, rarely coupled oscillators are treated and almost never normal modes of oscillation are presented. Moreover harmonic and coupled oscillations are rarely supported by experiments in lab activities. Not only in Italian school, but also in the literature it is difficult to find out teaching paths on normal modes for secondary school with a detailed analysis of disciplinary knots and learning problems. As harmonic oscillations and normal modes of oscillations have a great importance for the understanding of many fundamental topics such as acoustic and optics and, moreover, they are fundamental for the approach to modern physics [1], we present here a guided, inquiry based sequence on oscillations, together with some preliminary results coming from two experimentations. The path we have developed has been tested on three different groups of students. It is entirely based on an experimental approach using two different data logging and data analysis systems: the commercial Vernier Logger Pro system [2] and the Tracker video analysis free software [3].

The context

The path on oscillation has been proposed to three different groups of students: two classes of 30 students each, during curricular lessons, and a group of 40 students during extracurricular activities in the framework of “Milan open labs” of PLS (Scientific Degrees Plan). PLS is an Italian national project that the Ministry of Education has created to promote the collaboration between upper secondary school and University in order to increase the interest of young students for science [4]. The curricular classes were composed of students attending the third year of scientific oriented high school. They had only a relatively poor mathematical background (little trigonometry, second degree equations and no calculus) and they had not previously studied waves.

A pre-test and a final questionnaire have been given to students. The final test has been administered five weeks after the end of the sequence, to verify medium term effectiveness. Due to the preliminary nature of the study, only qualitative research methods have been used to analyse the data [5].

The teaching and learning sequence

The teaching and learning sequence is based on a number of experiments. The experiments are supported by the use of data logging [2,3], video analysis [3] and applet simulations [6,7]. For the sake of brevity we describe here only the most significant: *the vertical mass-spring oscillator, the bouncing disk, the rotating disk, the coupled pendulums* and *the Shive wave machine (many coupled torsional pendulums).* Each topic is introduced starting from a brainstorming in the form of interview where the teacher/interviewer tries to understand students' individual conceptions as it is foreseen in teaching experiment design [8]. All these experiments are meant to introduce the harmonic oscillations and, through harmonic oscillations, the normal modes of oscillation of complex systems. In our path the harmonic oscillation is seen as a privileged type of oscillation among all the periodic oscillations [1,9,10,11]. It allows describing the motion of almost all oscillating systems provided you comply with some constraints [10]. In this context the harmonic motion is introduced from the dynamic point of view as the motion a body performs when it is subject to a restoring force [10]. That is, a force whose graph lies between the second and the fourth quadrant of the diagram force vs displacement, passing through the origin of the axes and that is differentiable in the origin. All these forces can be approximated to their tangent line in the origin provided the amplitude of oscillation is small enough. So the central point is that any body subject to a restoring force, for small amplitude of oscillation, is governed by forces of the kind $F = -kx$ which generate harmonic motion.

The vertical mass-spring oscillator and *the bouncing disk* are designed to investigate different types of periodic oscillations and to identify the characteristics of the harmonic motion. *The rotating* disk is used to solve graphically the equation of harmonic motion [10]. The *coupled pendulums* are designed to study the properties of simple systems of coupled oscillators and to introduce the normal modes of oscillation for such systems, with the aim of showing how every oscillation of a complex system can be seen as a superposition of simple harmonic oscillations at fixed frequencies: those of the normal modes of the system. *The Shive wave machine*, with so many coupled torsional pendulums, is designed to study the normal modes of a discrete but more complex system of oscillators. This may help the transition to a continuous system such as the vibrating string and the comprehension of stationary waves as normal modes of the string (instead of the more usual superposition of travelling waves).

The experiments

The vertical mass-spring oscillator and *the bouncing disk (Figure 1).*

The vertical mass-spring oscillator is a typical example of harmonic oscillator while *the bouncing disk* is an example of periodic but non-harmonic oscillator.

The mass-spring oscillator consists of a mass appended at the bottom of a vertical spring (see Figure 1). The vertical configuration avoids the problem of the friction with surfaces. The mass is chosen so as to have a stable vertical oscillation, that is: the system has an almost linear behaviour and there is no coupling between the vertical spring mode and the transverse pendulum mode. Nonetheless, the same care must be taken in choosing the mass, because the spring mode and the pendulum mode do indeed become resonant when the spring oscillation frequency doubles that of the pendulum [12,13].

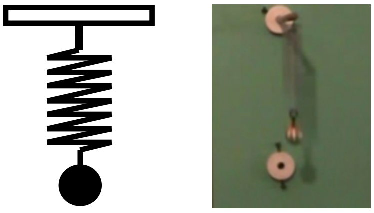
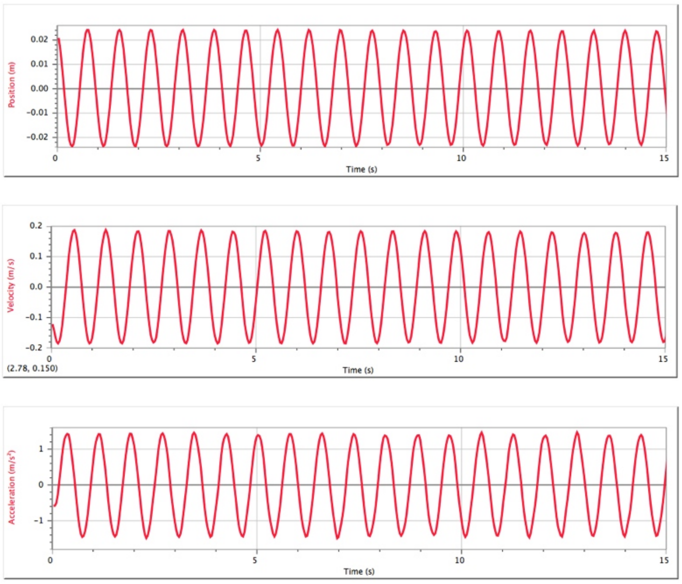


Figure 1. The vertical mass-spring system

The bouncing disk consists of a disk moving on an air table, so to reduce friction, and bouncing between two elastic edges (see Figure 2).

Figure 2. The bouncing disk on an air table, it is well visible the target object (black ball)

At the very beginning some experiments, concerning periodic oscillations, have been shown to the students: a pendulum, a ball bouncing vertically on the floor, a slinky oscillating vertically, a rod tilting on a flat pivot, a ball running back and forth along a semi-circular rail and many other real periodic oscillations. A brainstorming on what students saw followed. Then the first task for students has been to describe and categorize the previous oscillating systems that they observed by the naked eye. The students have been asked to group the oscillating systems according to some properties they decided by themselves. In a second moment students, divided into small groups of three-four, have been asked to analyse the forces acting on oscillators. They had to provide some qualitative graphics to be discussed inside each student's group, among different groups and with the teacher. This guided procedure, allowed the students to make a new categorization based on the analysis of the forces acting on each oscillator, thus giving the hint to define as harmonic oscillations those that are driven by a restoring force. At this point the students were ready to perform a quantitative analysis of the vertical mass-spring motion and of the bouncing disk motion, via two different data logging techniques. The goal, in first instance, was to verify that only the motion of the oscillator driven by a restoring force is harmonic like. In a second instance students could analyse the data and the graphics provided by the data logging to fix the properties of the harmonic motion. The students analysed the motion of the vertical mass-spring via the sonar detection and Logger Pro analysis while they studied the bouncing disk, after filming by smartphones the experiment, via the video analysis software Tracker. In Figure 3 are reported the graphs for the mass-spring provided by the logger pro: a) displacement vs time; b) velocity vs time and c) acceleration vs time.



a)

b)

c)

Figure 3. The mass-spring system: a) displacement, b) velocity, c) acceleration vs time

Students were able to see that the motion law is sinusoidal-like. In fact position and velocity as functions of time have the same sinus-like shape, but they are shifted of a quarter of a period. Furthermore, the acceleration vs time graph is still a sinus-like function and results, at each time, opposite to the displacement one according to $F = -kx$ law stating harmonic motion. Moreover, the Logger Pro provides also the acceleration versus position diagram (Figure 4) which results in a straight line lying in the second and fourth quadrant and passing through the origin of the axes. Using position vs time diagram (Figure 3a), students could verify the important property of harmonic motion that the frequency of the oscillation is amplitude independent, that is it is fixed by the parameters of the system. In fact the amplitude of oscillation registered by the sonar decreases with time due to the air friction.

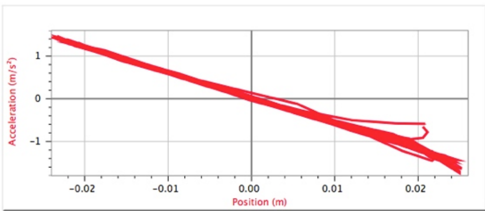


Figure 4. Acceleration vs position for the mass-spring oscillator

In this case the restoring force no longer depends only on position, but also on velocity. This situation has not yet been faced by students. Nonetheless, from an experimental point of view, for small amplitudes and for not too large time intervals, the damping is very small so that we can neglect the dissipative contribution and consider the force as being dependent only on position thus giving a precise sense to measurements of the period. Obviously, waiting a long enough time, the amplitude of oscillations decreases and the damping becomes evident. One can thus perform a new measurement of the period of our motion in a new situation when the amplitude has diminished, but always remaining in the approximation of friction-less motion. The students could measure the period (and consequently the frequency) of the oscillation directly in different sections of the diagram with different amplitudes and verify it is constant. The Logger Pro provides another powerful tool to confirm that the frequency of the harmonic oscillation is fixed: the FFT (Fast Fourier Transform). The FFT of the motion waveform results a sharp line (Figure 5) at the same frequency the students found directly by measuring the period on the diagram. Of course our students did not posses yet the mathematical background for understanding how FFT works. They just knew it is a tool, a kind of button to push, that is able to find all the frequencies present in a waveform. To make this clear to students we showed them, with a simulation, the complicated waveform resulting from the sum of two (and three) sinusoidal function with different frequency. Then applying the FFT to the waveform we obtained the frequencies we mixed.

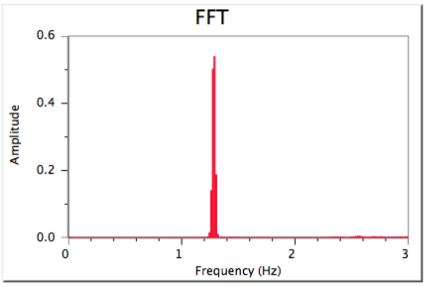


Figure 5. The Fast Fourier Transform of the waveform obtained for the mass-spring oscillator

After this, the students analysed the motion of the bouncing disk. This requires the use of Tracker because it is difficult to target, by sonar, the motion of an object which can have two motion components. As shown in Figure 2, it is necessary to mark the tracked object by a well contrasted target. The Tracker software can provide the same diagram as the Logger Pro. This time the analysis of the diagrams as the ones of Figures 3 and 4, clearly shows that the motion is no more governed by a restoring force and it is no longer harmonic as in the previous case. See Figure 6: a) position vs time, b) velocity vs time and c) acceleration vs time. In Figure 7 it is reported the diagram of acceleration vs position that clearly does not represent a restoring force.

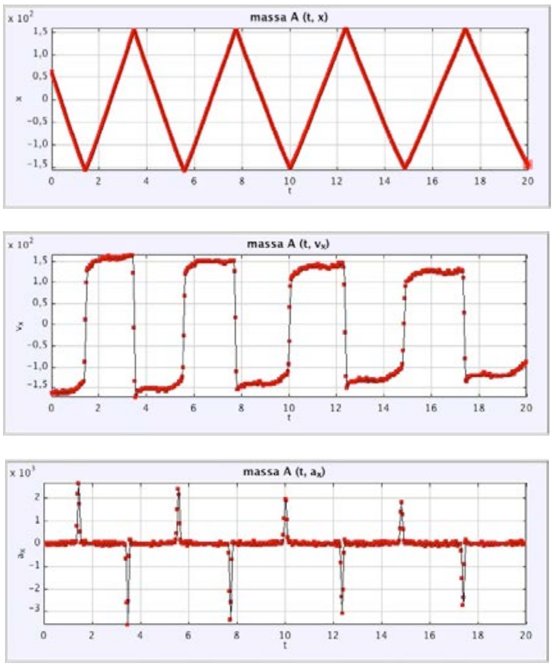


Figure 6. The bouncing disk of: a) displacement, b) velocity, c) acceleration vs time

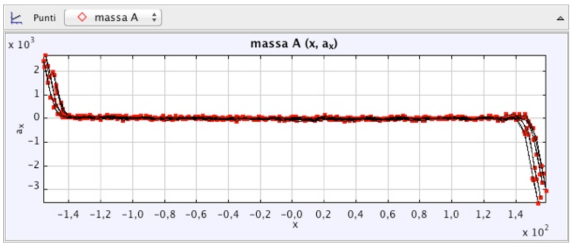


Figure 7. The acceleration versus position for the bouncing disk

The rotating disk

Once the definition of harmonic motion, as the one ruled by the dynamical law $F = -kx$, has been given, one has to face the problem of finding a way to integrate the differential equation $a = -k'x$ to obtain x as a function of t (with students that have no calculus background). Our strategy has been to use the projection on a diameter of a point-mass moving in circular motion. In fact, in this way it is easy to observe that the projection of the acceleration is given by $a = -k'x$ and that the projected velocity and position have a sinusoidal dependence on time.

Most of the Italian text-books define harmonic motion just as the projection of a circular motion over a diameter in a cinematic perspective. We, on the contrary, have chosen a very different dynamical approach and use circular motion only as a device to integrate a differential equation.

Moreover this is quite simple to obtain tracking the motion of a target dot on a rotating disk.

The coupled pendulums

The system consists of two to five physical pendulums coupled by identical springs [1,9,11,14]. These experiments (and the following one), together with the data logging techniques, turn out to be particularly useful because they allow: *i)* to easily introduce some particular (a student said "spectacular") motion configurations of the entire system: the *normal modes; ii)* to recognize that when such a complex system oscillates in one of its normal modes, there is no energy exchange between the single parts (oscillators) of the system; *iii)* to see that every casual motion configuration of the system is simply a superposition of its normal modes.

Each pendulum consists of a plastic disc stuck to the terminal part of a metal rod. Quantitative measurements are taken by using Vernier Logger Pro and Tracker video analysis as well. Tracker is more suitable when there are more than two coupled pendulums because it allows tracking simultaneously any number of bodies while Logger pro is limited to two bodies at once. The setup used for this experiment is shown in Figure 9.

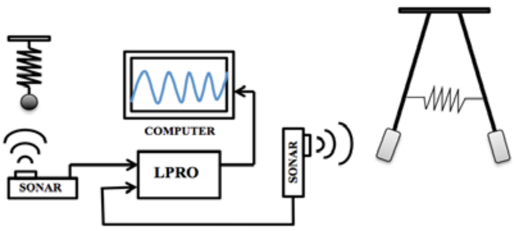


Figure 9. The experimental setup with sonar motion detection e Logger Pro software

As a first step, the students were asked to try and guess to imagine some “special ways of movement” of the system of two and three coupled pendulums. Surprisingly most students were able to predict which are the two normal modes of the two coupled pendulums. On the contrary, most students found it difficult to predict which are the normal modes higher than the second one for more complex systems (three to five pendulums). To get through the difficulty of predicting the motion configuration of a given normal mode, we proposed, as a very useful strategy, an analogy with stationary waves on a string (see Figure 10):

n coupled oscillators are represented by *n* equally spaced points on a string

the n^{th} normal mode configuration of the oscillators is recognizable by n^{th} stationary waves on the string, as Figure 10 clearly shows.

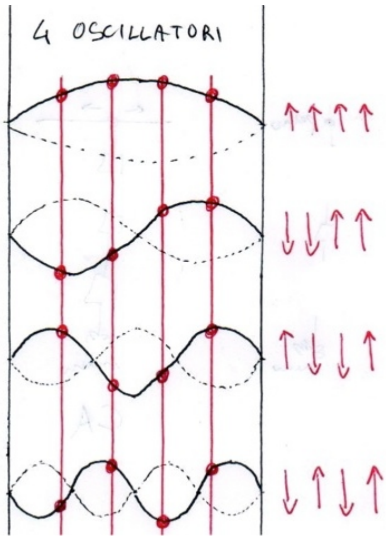


Figure 10. The sketch a student made to use the analogy with stationary waves to predict the shape of the four normal modes of a system with four coupled oscillators

In the case of mass-spring oscillators, this graphic analogy allowed students not only to predict the motion configuration of each normal mode but also to have a hint of the relative amplitude of oscillators in that mode. The further step has been to let the students “play” with the two-coupled pendulums, trying with different initial conditions. They easily realized that if one starts with a normal mode, the system continues moving that way and, besides, that looking at just one oscillator, while hiding all the others, one can’t understand whether it is coupled or not. This happens because there is no energy transfer (some students used the expression: “the pendulums do not exchange motion to each other”). The further step has been to perform a quantitative analysis via the data logging. Students tried to put into motion the three coupled pendulums (Figure 11) in the first, the second and the third normal mode and obtained the respective frequencies via the FFT (Figure 12). Then they put into motion the system in many randomly chosen different ways. From the analysis of the waveform, in both cases of normal mode and random motion configuration, the students could see that when the system oscillates in one of its normal mode, each of its parts (pendulums in this case) oscillates with harmonic motion at the same frequency and with a fixed phase relation with the others. The amplitude of oscillation of each pendulums doesn’t change, except for friction with the air, to indicate that there is no energy exchange between parts of the system. In addition, the higher the mode is, the higher is the frequency.

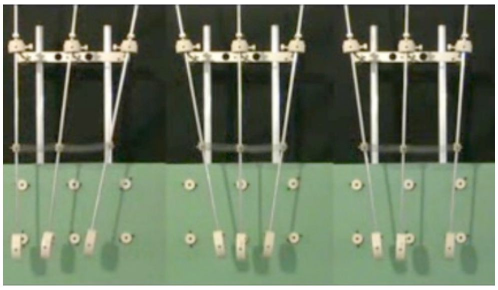


Figure 11. The three coupled pendulums. From left to right: first mode, second mode and third mode configuration.

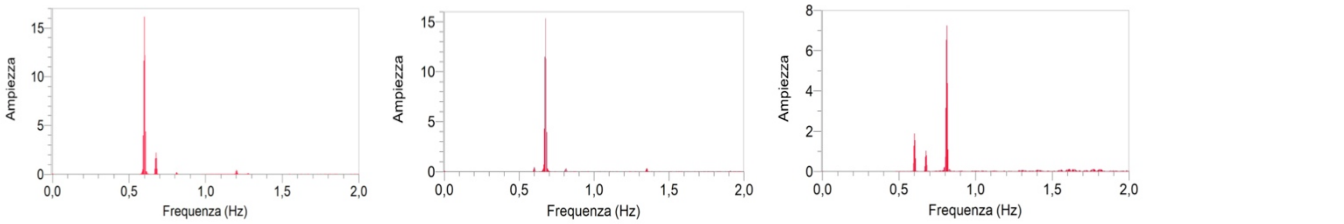


Figure 12. The three coupled pendulums. From left to right: the frequencies of the first, the second and the third normal modes.

On the other hand, if the system is put into motion randomly, we can see that there is energy exchange between the pendulums. In fact the motion waveform of each pendulum clearly presents the beat phenomenon and the amplitude of oscillation varies in time. The more relevant didactic issue here is that, if we perform the FFT of each pendulum waveform, we obtain exactly the same frequencies of the normal modes previously measured (see Figure 13). Each frequency peak, given by the FFT, has, in general, a different amplitude according to the way the normal modes superimpose, depending on the initial conditions. This allowed to show to the students that all the oscillations of the system are a linear combination of its normal modes.

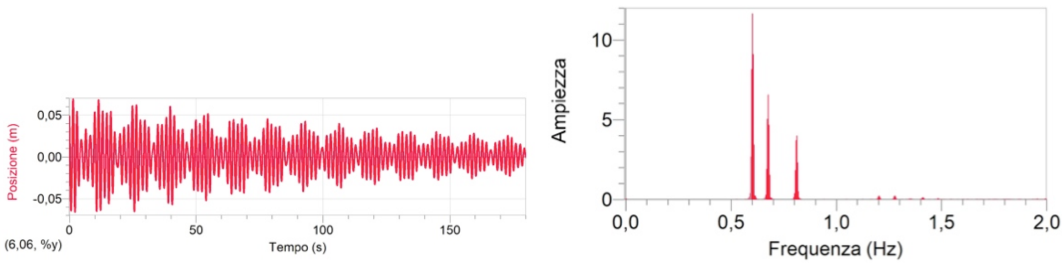


Figure 13. On the left: the waveform of a system of three coupled pendulums excited randomly, with the typical beats. On the right: the frequency of the normal modes superimposed.

In Figure 14 is shown a system of five coupled pendulums together with the motion waveforms of each pendulum. It is also shown the FFT some students performed for one of these waveforms with the frequencies of the five normal modes mixing.

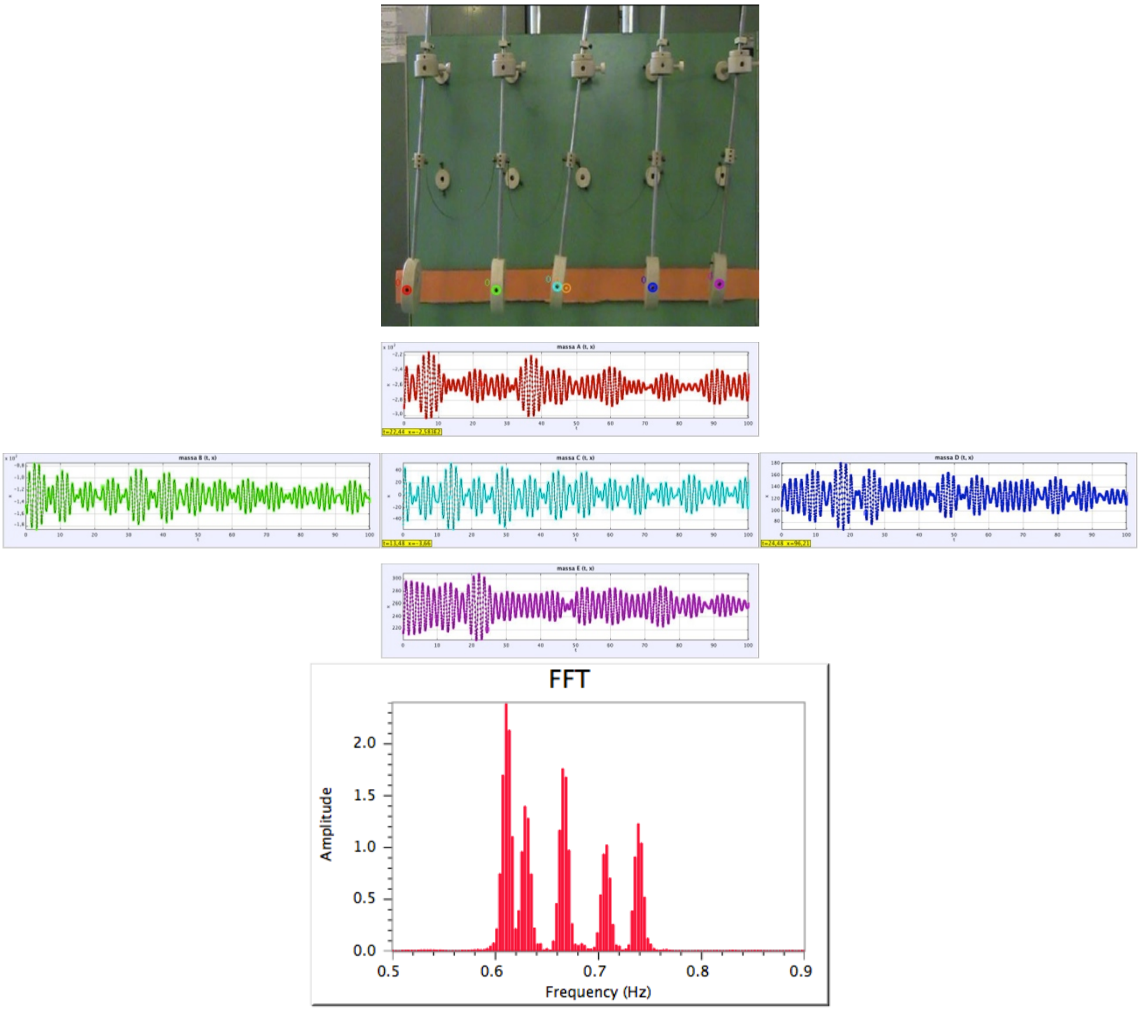


Figure 14. Five coupled pendulums. From top to bottom: the system, the waveform of each pendulum and the FFT related to one of the waveforms.

These modes superimpose to give the motion of each pendulum. Moreover, when the system is excited randomly, the motion of each pendulum, being a linear combination of harmonic motions (the normal modes) is no more harmonic and generally neither periodic. In this case Tracker allows to plot the motion waveform of the centre of mass of the system which appear to be harmonic. See Figure 15.

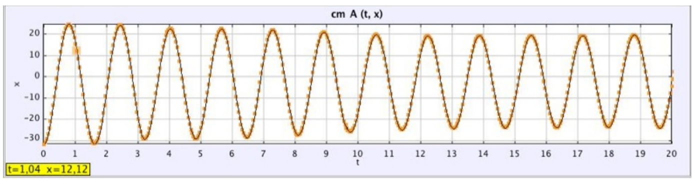


Figure 15. Five coupled pendulums: the waveform of the motion of the centre of mass

The Shive wave machine

The Shive machine is a system of many torsional pendulums, as in Figure 16. In our case we reduced the system to 18 pendulums to have them spaced enough. This was required for better data logging. In fact the sonar detector can’t distinguish between two objects if they are too close. This experiments turns out to be of didactic interest because it can facilitate the conceptual transition from the discrete to the continuous case (for instance the vibrating string). In Figure17 it is reported the motion waveform obtained by a group of students tracking the complicated motion of one pendulum. This data collection has been performed with the sonar and the Logger Pro software, but comparable results have been obtained with Tracker as well. The FFT, as depicted in Figure 17, shows the frequencies of all the eighteen normal modes of the system. In this case it results evident that the first four normal modes are those that mostly contribute to the motion of the tracked pendulum. Furthermore, the more the number of pendulums the more the normal modes tend to be equally spaced in frequency. In fact, in the limit case of a continuous system, as the vibrating string, the frequency of each mode is an integer multiple of the frequency of the first normal mode.

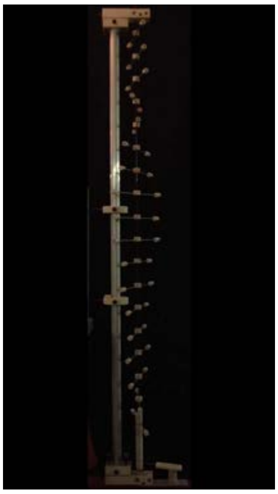


Figure 16. The Shive machine

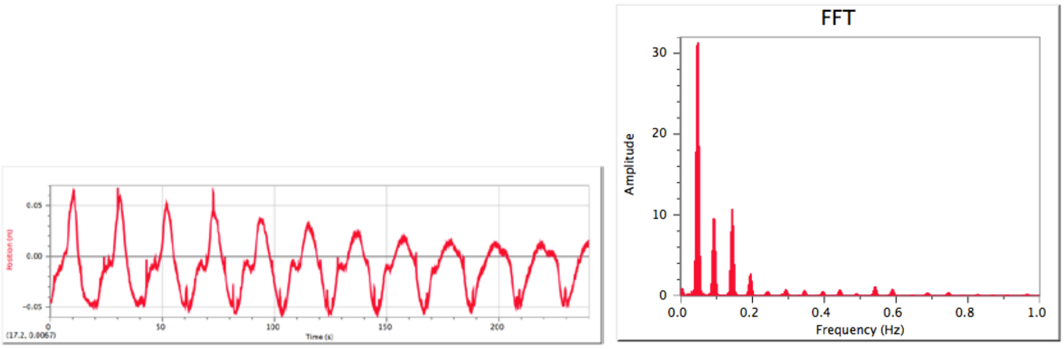


Figure 17. The Shive machine. From left to right: the waveform of one of the pendulums and its FFT.

Results

In the initial brainstorming, students were asked to group the oscillating systems they observed by the naked eye. Most of them decided to put together oscillators with similar trajectories. For instance, the vertical mass-spring, the ball bouncing on the floor and the bouncing disk were grouped together “*because all move along a straight line*”; the simple pendulum, the rod tilting on a flat pivot and the ball running along a semi-circular rail, were grouped together “*because they describe an arc*”. In the final test, on the contrary, over 60% of the students grouped oscillators taking into account the forces acting on the system, being them restoring forces or not.

Another interesting fact emerged from the initial brainstorming is that about 80% of the students thought that the oscillation frequency of a vertical mass-spring oscillator does depend on the initial displacement. In particular, they thought that the greater the initial amplitude, the greater the frequency. Some students said: "*when the amplitude is bigger, the frequency is higher because the movement of the mass is faster*". A few students thought that the frequency of oscillation decreases with the initial displacement because: "*the velocity is the same but the space is longer, so the oscillation takes more time*". Only less than 20% of the students decided that the frequency is constant, regardless the initial displacement. They stated this fact on the base of direct observation by naked eye: "*looking at the oscillation I can't see difference*".

Analogous results and similar comments were obtained with the simple pendulum, despite the fact that almost all the students already knew the pendulum isochronism law.

The situation greatly changed after the didactical intervention as at the end of the path nearly 90% of the students were able to recognize that the frequency of a harmonic oscillator does not depend on amplitude.

Regarding normal modes, while many students were able to imagine "some special motion configurations" of a system of two coupled pendulums before the topic was introduced and the experiments performed, only a couple of them were also able to predict the motion of the third normal mode of a system of three coupled pendulums. None could predict higher modes in more complex systems (five pendulums). Anyway, after introducing the graphic technique (see Figure 13), the number of students able to predict the motion of all normal modes increased significantly. In the final test and in the interviews was proposed a question on a system of five coupled oscillators. All the students were able to describe the motion configuration of the first normal mode by words and/or by sketches. Over 80% described correctly the second normal mode, over 60% the third and the fourth and nearly 50% the fifth one. Most of the wrong answers on higher normal modes were due to inaccuracy in drawing the sketches.

Conclusions

This experimental approach here described, allows to overcome most of the mathematical difficulties that one encounters in treating coupled oscillations with secondary school students. Moreover, the use of data logging software can also help students to get over some difficulties in representing and interpreting graphics. In addition, in our experience, we have noticed that the use of techniques, such as video-tracking generates great enthusiasm in students. In fact it needs only a smartphone as "probe", and we all know that smartphones represent a technology very friendly to young students. This is further proved by the many works students performed at home, by themselves, even without having been asked.

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