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Intensity correlations from linear interactions

Abstract: We address the generation of intensity correlations arising from the interference between two optical states interacting through a beam splitter. We consider both phase insensitive and sensitive states and write the intensity correlations as functions of the mean number of photons and field quadratures of the input states. In particular we consider Fock number states and squeezed states as a paradigm of non-classical states and mixture of coherent states, which can be experimentally accessible. We show that, under certain conditions on the input states, intensity correlations may vanish or turn into anti-correlations.

Keywords: Intensity correlations; photon number statistics

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1 Introduction

A beam splitter (BS) is a linear, passive, optical device that finds wide applications in quantum optics experiments. The BS can be used to generate entangled states in continuous variable regime (see, e.g., [1]) as well as to implement advanced detection techniques such as homodyne and heterodyne detection. The interference at a BS of a probe input state with a local oscillator generates a bipartite state which, in general, displays some kind of correlations, which may be measured by various detectors. In turn, in the last years photon number resolving detectors [2, 3] have been successfully employed in quantum optics experiments both for state characterization [4, 5] and enhancement of optimal quantum receivers [6, 7].

Intensity correlations are strictly related to the photon number statistics of the bipartite states and have fundamental interest for state characterization [8] but also for practical and technical applications [9–12]. In this paper we investigate the intensity correlations of a couple of optical states interfering at a BS focusing on different classes of input states: thermal, coherent, Fock and squeezed states. In particular, we investigate the behavior of the intensity correlations in the case in which one of the two inputs is excited in a coherent state (local oscillator). This scenario corresponds to a homodyne-like detection scheme with low intensity local oscillator [13], however here we are not interested in the difference photocurrent and its variance, but on the actual intensity correlations between the detected states. More in details, we derive analytical results which link the intensity correlation function with the mean values and variances of the number of photons and field quadratures of the input states. Our results can be useful not only from the theoretical point of view, but can be of some interest also for the experimentalist that is interested in a feasible way to characterize a bipartite state through a BS starting from a basic description of the input states.

The paper is structured as follows. In Section 2 we review the dynamics of two optical beams through a BS and write the general expression of the intensity correlations as a function of the number and quadrature operators of the input fields. In Section 3 we focus on states that displays a symmetry in the phase space leading, in particular, to vanishing quadrature first moments. In Section 4 we consider the class of phase insensitive states, which are diagonal in the Fock number basis, such as thermal, phase-averaged coherent states [14, 15] and Fock states. Phase sensitive states, as the so-called bracket states [16] and squeezed vacuum states, are addressed in Section 5. We close the paper drawing some concluding remarks in Section 6.

2 Intensity correlations for a factorized input state

Let us consider a generic factorized state $\rho_{\text{in}} = \rho_a \otimes \rho_b$ of the input radiation modes a and b of a BS. The BS acts as a linear mixer of the two input modes through the unitary

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operator $U_{BS} = \exp\{\theta(a^\dagger b - a b^\dagger)\}$, with the assumption $\theta \in \mathbb{R}$, and the output state is given by $\rho_{out} = U_{BS}\,\rho_{in}\,U^\dagger_{BS}$. A simple way to compute average values of proper observables at the output ports of the BS is provided by the Heisenberg picture, according to which the radiation modes a and b transforms through U_{BS} as

$$\begin{pmatrix} c \\ d \end{pmatrix} = U^\dagger_{BS} \begin{pmatrix} a \\ b \end{pmatrix} U_{BS} = \begin{pmatrix} \sqrt{\tau}\,a + \sqrt{1-\tau}\,b \\ \sqrt{\tau}\,b - \sqrt{1-\tau}\,a \end{pmatrix}, \qquad (1)$$

where $\tau \equiv \cos^2\theta$ is the transmittance of the BS.

In typical quantum optical setups, like homodyne or heterodyne detections, the accessible observable is the number of photons $N_c = c^\dagger c$ and $N_d = d^\dagger d$ at the output ports of the BS [17]. We are interested in measuring intensity correlations between the two output radiation modes c and d and analyzing its dependence on different input states of optical radiation. A suitable definition for an intensity correlation function is given by

$$\Gamma = \frac{\langle (N_c - \langle N_c\rangle)(N_d - \langle N_d\rangle)\rangle}{\sqrt{\mathrm{Var}(N_c)\,\mathrm{Var}(N_d)}} = \frac{\langle N_c N_d\rangle - \langle N_c\rangle\langle N_d\rangle}{\sqrt{\mathrm{Var}(N_c)\,\mathrm{Var}(N_d)}}, \qquad (2)$$

where the averages $\langle\cdots\rangle$ are performed, in the Heisenberg picture, over the input state ρ_{in} and with $\mathrm{Var}(N) = \langle N^2\rangle - \langle N\rangle^2$ being the variance associated with the mean number of photons. The definition in Eq. (2) quantifies the amount of intensity correlation ($\Gamma > 0$) or anti-correlation ($\Gamma < 0$) between the output light beams whenever the two statistics are not independent $\langle N_c N_d\rangle \neq \langle N_c\rangle\langle N_d\rangle$.

In order to obtain a very general expression of Γ, we compute the number operators of the output modes N_c and N_d in the Heisenberg representation:

$$N_c = \tau N_a + (1-\tau)N_b + \sqrt{\tau(1-\tau)}(x_a x_b + p_a p_b) \qquad (3a)$$
$$N_d = \tau N_b + (1-\tau)N_a - \sqrt{\tau(1-\tau)}(x_a x_b + p_a p_b). \qquad (3b)$$

We introduced the quadrature operators of the input modes $k = a, b$ as

$$x_k(\phi_k) = \frac{k\,e^{-i\phi_k} + k^\dagger e^{i\phi_k}}{\sqrt{2}} \qquad (4)$$

which correspond, respectively, to the position x_k and to momentum p_k, for $\phi_k = 0$ and for $\phi_k = \pi/2$.

Now we have all the ingredients necessary to the computation of the intensity correlation function (2), but its expression for a generic input state is somehow complicated and not insightful. In the following we will consider large classes of states that give rise to intensity correlations obtaining simple forms of Eq. (2).

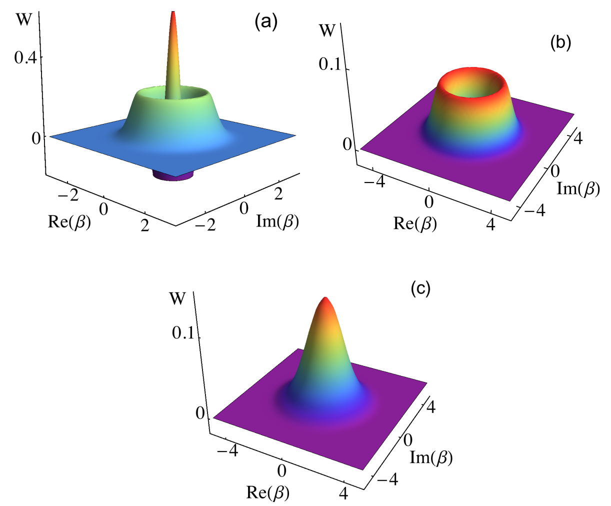

Figure 1: (Color online) Phase-space representation (Wigner functions) of a Fock state $|n\rangle$ (with $n = 2$) (a), a PHAV state with $\langle N\rangle = 2$ (b) and a thermal state with $\langle N\rangle = 2$ (c).

3 Phase-space symmetric states

In this Section we focus our analysis on the class of *phase-space symmetric states* which we define as the symmetric mixture of a state ρ_0 and the π-shifted state $\rho_\pi = \Pi\rho_0\Pi$:

$$\rho_{in} = \frac{\rho_0 + \rho_\pi}{2} \qquad (5)$$

where $\Pi = \exp(i\pi N_k)$ is the parity operator. The effect of the operator Π on a bosonic mode $k = a, b$ is to reverse its parity:

$$\Pi\,k\,\Pi = \sum_n \sqrt{n}\,e^{-i\pi k^\dagger k}|n\rangle\langle n+1|e^{i\pi k^\dagger k} = -k \qquad (6)$$

and the same holds for the corresponding Hermitian $\Pi\,k^\dagger\,\Pi = -k^\dagger$. The corresponding Wigner function is symmetric with respect to the origin of the phase space $\{\mathrm{Re}(\beta_k), \mathrm{Im}(\beta_k)\}$ as

$$W_{\rho_{in}}(\beta_k) = \frac{W_{\rho_0}(\beta_k) + W_{\rho_0}(-\beta_k)}{2}, \qquad (7)$$

from which the name given to this class of states. Due to Eq. (6) the average value of an observable depending on an odd number of annihilation and creation operators k and $k^\dagger$ over the state ρ_π is $\langle f_{odd}(k,k^\dagger)\rangle_{\rho_\pi} = -\langle f_{odd}(k,k^\dagger)\rangle_{\rho_0}$, whereas for an even number of k and $k^\dagger$ operators we have $\langle f_{even}(k,k^\dagger)\rangle_{\rho_\pi} = \langle f_{even}(k,k^\dagger)\rangle_{\rho_0}$. Therefore the average values of such observables over the phase-space symmetric state (5) are

$$\langle f_{odd}(k,k^\dagger)\rangle_\rho = 0 \qquad (8a)$$
$$\langle f_{even}(k,k^\dagger)\rangle_\rho = \langle f_{even}(k,k^\dagger)\rangle_{\rho_0}. \qquad (8b)$$

Since the average values of observables like $\langle x_k(\phi_k)\rangle$, $\langle x_k(\phi_k)N_k\rangle$ and $\langle N_k x_k(\phi_k)\rangle$ (with $k=a,b$) are null, we can simplify the expression of Eq. (2). As a consequence, it is sufficient to have just one of the two input states ρ_a or ρ_b in the form (5) to obtain a simplified expression of the quantities appearing in Eq. (2):

$$\frac{\langle N_c N_d\rangle - \langle N_c\rangle\langle N_d\rangle}{\tau(1-\tau)} = \mathrm{Var}(N_a) + \mathrm{Var}(N_b) - \langle X_{a,b}\rangle \quad (9a)$$

$$\frac{\mathrm{Var}(N_c)}{\tau(1-\tau)} = \frac{\tau}{1-\tau}\mathrm{Var}(N_a) + \frac{1-\tau}{\tau}\mathrm{Var}(N_b) + \langle X_{a,b}\rangle \quad (9b)$$

$$\frac{\mathrm{Var}(N_d)}{\tau(1-\tau)} = \frac{\tau}{1-\tau}\mathrm{Var}(N_b) + \frac{1-\tau}{\tau}\mathrm{Var}(N_a) + \langle X_{a,b}\rangle \quad (9c)$$

where

$$\langle X_{a,b}\rangle \equiv 2\langle N_a\rangle\langle N_b\rangle + \langle N_a\rangle + \langle N_b\rangle + \langle a^2\rangle\langle b^{\dagger 2}\rangle + \langle a^{\dagger 2}\rangle\langle b^2\rangle. \quad (10)$$

In the following we consider of phase-symmetric states, that are the phase-insensitive and phase-sensitive states.

4 Phase-insensitive states

Diagonal states in the Fock basis, namely $\rho_{\mathrm{PI}} = \sum_n p_n |n\rangle\langle n|$, are phase-insensitive as their representation in phase-space, through the Wigner function, has only a radial dependence (see Fig. 1):

$$W_{\rho_{\mathrm{PI}}}(\beta) = \frac{2}{\pi} e^{-2|\beta|^2} \sum_{n=0}^{\infty} (-1)^n p_n L_n(4|\beta|^2) \quad (11)$$

with $L_n(x)$ Laguerre polynomials. Fock states $|n_b\rangle$ with $p_n = \delta_{n,n_b}$, thermal states with $p_n = \frac{1}{1+\langle N^{(th)}\rangle}\left(\frac{\langle N^{(th)}\rangle}{1+\langle N^{(th)}\rangle}\right)^n$ and phase-averaged (PHAV) states [14] with $p_n = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}$ belong to this family of states and their corresponding Wigner functions are shown in Fig. 1.

With just one of the input modes a or b prepared in a phase-insensitive state, Eq. (10) reduces to $\langle X_{a,b}\rangle \equiv 2\langle N_a\rangle\langle N_b\rangle + \langle N_a\rangle + \langle N_b\rangle$, given the diagonal form of these kind of states. We thus obtain simplified expressions of Eqs. (9) in terms of the Mandel-$\mathcal{Q}$ parameter, defined as $\mathcal{Q}_k = \mathrm{Var}(N_k)/\langle N_k\rangle - 1$ (with $k=a,b$). For example Eq. (9a) becomes:

$$\frac{\langle N_c N_d\rangle - \langle N_c\rangle\langle N_d\rangle}{\tau(1-\tau)} = \langle N_a\rangle\mathcal{Q}_a + \langle N_b\rangle\mathcal{Q}_b - 2\langle N_a\rangle\langle N_b\rangle \quad (12)$$

and similar expressions can be obtained for the other quantities. In this way the intensity correlations of the output modes are related to the statistics of the input states, via simple observable quantities which can be measured by standard photon detectors commonly employed in quantum optical experiments. Interesting features arise considering just one input mode (say, mode b) in a phase-insensitive state.

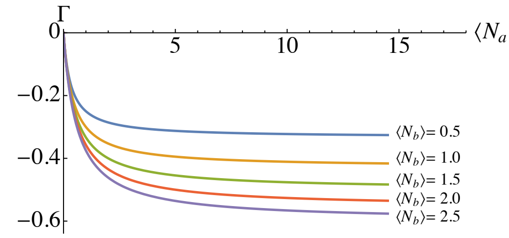

Figure 2: (Color online) Plot of the intensity correlation function Γ for a PHAV state on mode b and a coherent state (or a PHAV state with the same statistics) on mode a, as a function of the average photon number N_a, for different fixed energy N_b of the coherent state.

As a first example, we address the case of the vacuum state $|0\rangle_b$ interacting at the BS with a generic state ρ_a. The intensity correlation function has a very simple expression and uniquely depends on the Mandel-$\mathcal{Q}$ parameter associated to the input state:

$$\Gamma = \frac{\sqrt{\tau(1-\tau)}\mathcal{Q}_a}{\sqrt{\left[1+\tau\mathcal{Q}_a\right]\left[1+(1-\tau)\mathcal{Q}_a\right]}}. \quad (13)$$

A non-zero correlation arises whenever the state ρ_a displays super- or sub-Poissonian statistics, bringing to, respectively, correlation ($\mathcal{Q}_a > 0$, e.g. for a thermal state) or anti-correlation ($\mathcal{Q}_a < 0$, e.g. for a Fock state).

As a second significative example we consider the case in which a PHAV state (on the mode b) is mixed with a coherent state or another PHAV state (on the mode a), given that the two possess the same features from the point of view of the statistics (same mean photon number and variance). In this case the two outgoing light beams are anti-correlated, independently on the transmittance τ of the BS, as Eq. (12) becomes negative $-2\langle N_a\rangle\langle N_b\rangle < 0$, provided that the corresponding Mandel-$\mathcal{Q}$ parameters are $\mathcal{Q}_a = \mathcal{Q}_b = 0$. In Fig. 2 we show that the amount of anti-correlation between two PHAV states increases with the average photon number of the two input modes N_a and N_b.

Another interesting case is provided by a thermal state mixed at the BS with a local oscillator in a coherent state. In particular, the Mandel-$\mathcal{Q}$ parameter of a thermal state is $\mathcal{Q}_b = \langle N_b^{(\mathrm{th})}\rangle$ and, therefore Eq. (12) becomes $\langle N_b^{(\mathrm{th})}\rangle\big(\langle N_b^{(\mathrm{th})}\rangle - 2\langle N_a\rangle\big)$. As one may expect, the strongest correlation is achieved when a thermal state is mixed at the BS with the vacuum. The presence of a local oscillator of energy $\langle N_a\rangle$ drives down the correlations, up to a condition of zero correlation achieved for $2\langle N_a\rangle = \langle N_b^{(\mathrm{th})}\rangle$ (see Fig. 3). If the energy of the coherent state is increased the outgoing light

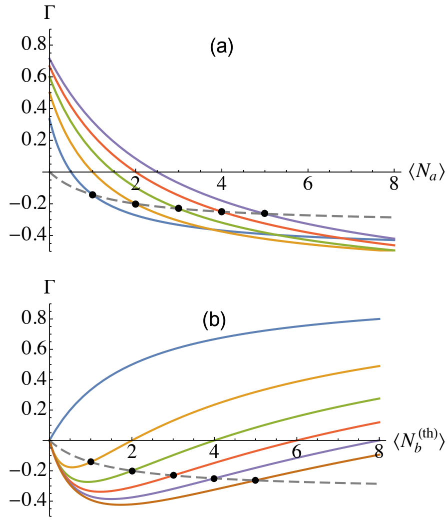

Figure 3: (Color online) Plot of the intensity correlation function Γ for a thermal state on mode b and a coherent state on mode a, as a function of the coherent state (a) and thermal state (b) average photon numbers. The dashed black curve represent anti-correlation for equally increasing energy of the two input states $\langle N_a \rangle = \langle N_b^{(\mathrm{th})} \rangle$.

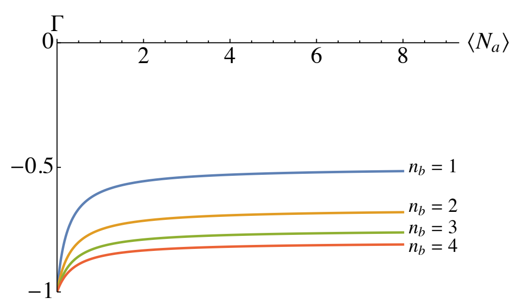

Figure 4: (Color online) Plot of the intensity correlation function Γ for a Fock state $|n_b\rangle$ and a coherent state (or a PHAV state with the same statistics) on mode a, as a function of the average photon number $\langle N_a \rangle$, for different number of photons n_b of the Fock state.

beams become anti-correlated. The condition for complete anti-correlation is fulfilled when $\langle N_b^{(\mathrm{th})} \rangle < 2\langle N_a \rangle$. In Fig. 3 is highlighted (dashed-dotted black curve) the case in which the average photon number of the thermal and the coherent states are equal $\langle N_a \rangle = \langle N_b^{(\mathrm{th})} \rangle$.

As last example of phase-insensitive state, we consider a Fock number state $|n_b\rangle$ on mode b with $\mathcal{Q}_b = -1$, mixed at the BS with a local oscillator in a coherent state $|\alpha\rangle$ on mode a with $\mathcal{Q}_a = 0$. Now the two outgoing light beams show anti-correlation, as Eq. (12) becomes $-n_b(1 + \langle N_a \rangle) < 0$, as shown in the plot of Fig. 4. In the limit $\langle N_a \rangle \gg 1$ the correlation function becomes $\Gamma = -n_b/(1 + n_b)$ for a balanced BS ($\tau = 1/2$).

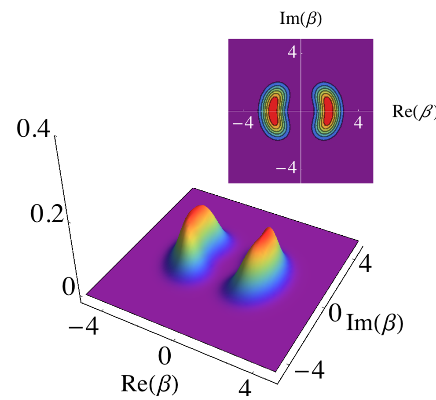

Figure 5: (Color online) Phase-space representation (Wigner function) of a bracket state with $\langle N \rangle = |\alpha|^2 = 2$ and $\gamma = \pi/2$.

5 Phase-sensitive states

The first phase-sensitive state we consider is the classical bracket state, defined as [16]

$$\rho = \int_{-\gamma/2}^{\gamma/2} \frac{d\psi}{\gamma} \frac{||\alpha|e^{i\psi}\rangle\langle|\alpha|e^{i\psi}| + |-|\alpha|e^{i\psi}\rangle\langle -|\alpha|e^{i\psi}|}{2} \tag{14}$$

which is a phase-space symmetric state (Fig. 5) with $\rho_0 = \int_{-\gamma/2}^{\gamma/2} \frac{d\psi}{\gamma} ||\alpha|e^{i\psi}\rangle\langle|\alpha|e^{i\psi}|$. In the limiting cases of $\gamma = \pi$ and $\gamma \to 0$ it corresponds, respectively, to a PHAV state and to a mixture of coherent states $|\pm|\alpha|\rangle\langle\pm|\alpha||$.
According to the rules (8) derived for a generic phase-space symmetric state, the only average values different from zero are the following

$$\langle N \rangle_{\rho_0} = \mathrm{Var}(N)_{\rho_0} = |\alpha|^2 \tag{15a}$$

$$\langle x^2(\phi) \rangle_{\rho_0} = \frac{1}{2} + |\alpha|^2 \left[1 + \mathrm{sinc}(\gamma)\cos(2\phi)\right]. \tag{15b}$$

The correlation function (2) for two bracket states at the inputs of a balanced BS with $\tau = 1/2$, is:

$$\Gamma = -\frac{\langle N_a \rangle \langle N_b \rangle [1 + \mathrm{sinc}(\gamma_a)\mathrm{sinc}(\gamma_b)]}{\langle N_a \rangle + \langle N_b \rangle + \langle N_a \rangle \langle N_b \rangle [1 + \mathrm{sinc}(\gamma_a)\mathrm{sinc}(\gamma_b)]} \tag{16}$$

which displays anti-correlation for every value of the parameters characterizing the two bracket states, as shown in Fig. 6. We note that anti-correlation is more pronounced when both phase noise in the bracket state is reduced ($\gamma \to 0$) and its average intensity $\langle N \rangle$ increases.

As a second representative example of phase-sensitive states on mode a, we consider a quantum state, namely the squeezed vacuum state:

$$|r\rangle = \frac{1}{\sqrt{\mu}} \sum_{n=0}^{\infty} \left(\frac{\nu}{2\mu}\right)^n \frac{\sqrt{(2n)!}}{n!} |2n\rangle \tag{17}$$

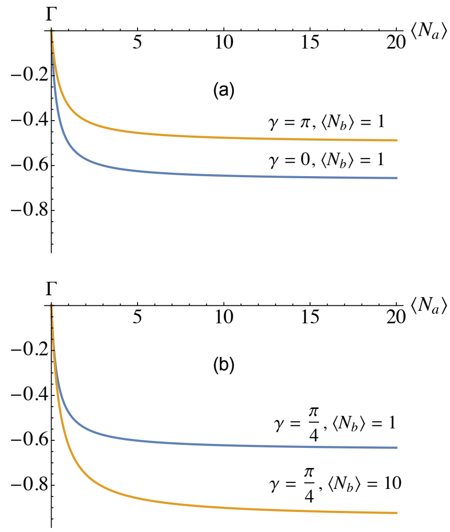

Figure 6: (Color online) (a) Input states: two bracket states with $\langle N_b \rangle = 1$, at the limiting cases of a mixture of symmetric coherent states ($\gamma = 0$) and PHAV states ($\gamma = \pi$). (b) Input states: two bracket states with $\langle N_b \rangle = 1, 10$ at a fixed range of phases of integration $\gamma = \pi/4$.

where $\mu = \cosh r$ and $\nu = \sinh r$ (with $r \in \mathbb{R}$). The squeezed state is characterized by its average intensity and variance as follows

$$\langle N_a \rangle = \sinh^2 r \tag{18a}$$

$$\mathrm{Var}(N_a) = 2\langle N_a \rangle \big(\langle N_a \rangle + 1\big). \tag{18b}$$

In order to investigate the intensity correlations after the BS, we consider two relevant cases for the input state on mode b: a Fock state $|n_b\rangle$ and a coherent state $|\beta\rangle = |\sqrt{\langle N_b \rangle}\, e^{i\phi}\rangle$. In the first case, since we have a phase-insensitive state on mode b, Eq. (10) reduces to

$$\langle X_{a,b} \rangle = 2\langle N_a \rangle \langle N_b \rangle + \langle N_a \rangle + \langle N_b \rangle,$$

whereas in the presence of a coherent state we have

$$\langle X_{a,b} \rangle = 2\langle N_a \rangle \langle N_b \rangle + \langle N_a \rangle + \langle N_b \rangle + 2\sqrt{\langle N_a \rangle \big(\langle N_a \rangle + 1\big) \langle N_b \rangle}\cos(2\phi).$$

In Fig. 7 and Fig. 8 we plot the behavior of the intensity correlation function Γ as a function of the involved parameters. We note that, also in these cases, the outgoing beams may display correlation or anti-correlation. The corresponding thresholds for vanishing correlations can be easily computed. In the case in which a squeezed state is mixed with a Fock state, the intensity correlations vanish when $\langle N_b \rangle = \langle N_a \rangle$. When the squeezed state is mixed with the coherent state the threshold is:

$$\langle N_b \rangle = \frac{\langle N_a \rangle \big(1 + 2\langle N_a \rangle\big)}{2\left[\langle N_a \rangle + \sqrt{\langle N_a \rangle (1 + \langle N_a \rangle)}\cos(2\phi)\right]}.$$

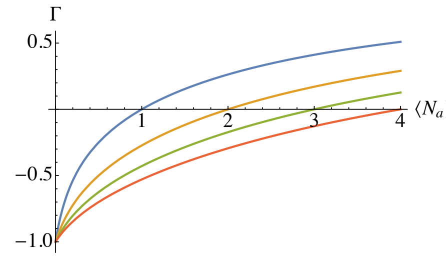

Figure 7: (Color online) Plot of the intensity correlation function Γ for a squeezed state $|r\rangle$ on mode a and a Fock state $|n_b\rangle$, as a function of the average photon number $\langle N_a \rangle$, for $\tau = 1/2$. From top to bottom $n_b = 1, 2, 3, 4$.

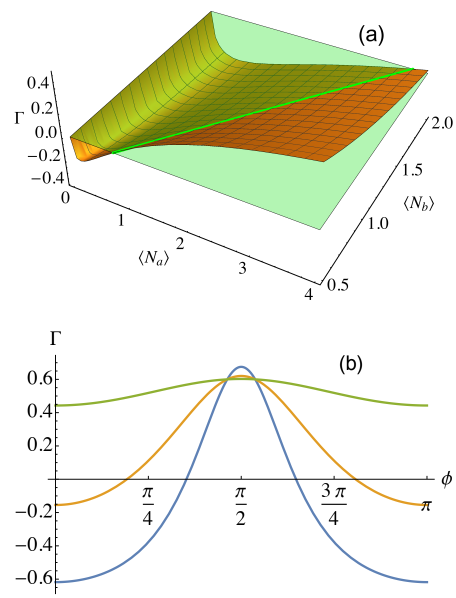

Figure 8: (Color online) (a) Plot of the intensity correlation function Γ for a squeezed state $|r\rangle$ on mode a and a coherent state $|\beta\rangle$ on mode b, as a function of the corresponding average photon numbers $\langle N_a \rangle$ and $\langle N_b \rangle$, with fixed $\phi = 0$ and $\tau = 1/2$. The horizontal green plane corresponds to $\Gamma = 0$. (b) Plot of the intensity correlation function Γ as a function of the coherent state phase ϕ, for $\langle N_a \rangle = 1$ and, from bottom to top, $\langle N_b \rangle = 0.1, 1, 10$.

We note that in the last case the threshold exists only when the coherent state phase satisfies the condition $\cos(2\phi) > -\sqrt{\langle N_a \rangle / \big(1 + \langle N_a \rangle\big)}$.

6 Concluding remarks

In this paper we investigated the generation of intensity correlations by mixing two optical states at a BS. More in details, we found analytical expressions for intensity correlation functions in the case of one or both the inputs prepared in phase-space symmetric states. We have shown that there are conditions leading to correlation or anti-correlation at the outputs and, where possible, we obtained the analytical expression of the threshold corresponding to vanishing correlations as a function of the input energies. Our work can be interesting for further investigations of intensity correlations also related to the birth of quantum correlations, both from the theoretical and experimental points of view.

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