Quantum quench for inhomogeneous states in the nonlocal Luttinger model

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In the Luttinger model with nonlocal interaction we investigate, by exact analytical methods, the time evolution of an inhomogeneous state with a localized fermion added to the noninteracting ground state. In the absence of interaction the averaged density has two peaks moving in opposite directions with constant velocities. If the state is evolved with the interacting Hamiltonian, two main effects appear. The first is that the peaks have velocities which are not constant but vary between a minimal and maximal value. The second is that a dynamical "Landau quasiparticle weight" appears in the oscillating part of the averaged density, asymptotically vanishing with time, as a consequence of the fact that fermions are not excitations of the interacting Hamiltonian.

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I. INTRODUCTION

Recent experiments on cold atoms [1] have motivated increasing interest in the dynamical properties of many-body quantum systems which are *closed* and isolated from any reservoir or environment [2]. Nonequilibrium properties can be investigated by *quantum quenches*, in which the system is prepared in a state and its subsequent time evolution driven by a many-body Hamiltonian is observed. As the resulting dynamical behavior is the cumulative effect of the interactions between an infinite or very large number of particles, the computation of local observables averaged over time-evolved states poses typically great analytical difficulties; therefore, apart for some analysis in two dimensions [3,4], the problem is mainly studied in one dimension [5–29]. A major difference with respect to the *equilibrium* case relies on the fact that in such a case a form of *universality* holds, ensuring that a number of properties are essentially *insensitive* to the model details; for instance, a large class of one-dimensional systems, named *Luttinger liquids* [30], have similar equilibrium properties irrespective of the exact form of the Hamiltonian, and this fact can even be proven rigorously under certain hypotheses using constructive Renormalization Group methods [31]. Universality and independence from the details also explain why even crude approximations are able to capture the essential physics of such systems. At nonequilibrium the behavior depends instead on model details; for instance, integrability in spin chains dramatically affects the nonequilibrium behavior [13,32,33], while it does not alter the $T = 0$ equilibrium properties [34]. This extreme sensitivity to details or approximations requires a certain number of analytically exact results at nonequilibrium to provide a benchmark for experiments or approximate computations.

One of the interacting fermionic systems where nonequilibrium properties can be investigated is the Luttinger model, which provides a great deal of information in the equilibrium case. In this model the quadratic dispersion relation of nonrelativistic fermions is replaced with a linear dispersion relation, with the idea that the properties are mainly determined by the states close to the Fermi points, where the energy is essentially linear; a Dirac sea is introduced, filling all the states with negative energy. It is important to stress that there exist two versions of this model, the *local* Luttinger model (LLM) and the *nonlocal* Luttinger model (NLLM); in the former a local δ-like interaction is present, while in the latter the interaction is short ranged but nonlocal. The finite range of the interaction plays the role of an ultraviolet cutoff. At equilibrium two such models are often confused as they have similar behavior due to the above-mentioned insensitivity to model details; there is, however, no reason to expect that this is also true at nonequilibrium. It should be also stressed that the LLM is plagued by *ultraviolet divergences* typical of a Quantum Field Theory, and an *ad hoc* regularization is necessary to get physical predictions; the short time or distance behavior depends on the chosen regularization.

The quantum quench of *homogeneous* states in the LLM was derived in [9,10], and that in the NLLM was derived in [19,20]; the effects of other ultraviolet cutoffs on the dynamics, like a lattice, after a quench have also been discussed [21,22,29]. Regarding the quantum quench of *inhomogeneous* states, in [15] the dynamical evolution in the LLM of a *domain-wall* state was considered as an approximate description for the analogous problem in the spin-XXZ spin chain. It was found in [15] that the evolution in the free or interacting case is the same up to a finite renormalization of the parameters; in particular, the front evolves with a *constant* velocity. Nonconstant velocities appear, from numerical simulations, in more realistic models like the XXZ chain [13,23,29].

In this paper we consider the evolution of *inhomogeneous* states in the NLLM, using exact analytical methods in the infinite-volume limit. In particular, we consider the state obtained by adding a particle to the noninteracting ground state or the vacuum. In the absence of interaction the particle moves in a ballistic motion with a *constant* velocity, showing a "light-cone" dynamics [13,35,36]. In the presence of interaction, the dynamics is still ballistic (in agreement with the fact that the conductivity computed via the Kubo formula is diverging [13,32–34]), but the evolution is not simply the free one with a renormalized velocity; on the contrary, the evolution is driven by velocities which are *nonconstant* and energy dependent. Moreover, the interaction produces a dynamical "Landau quasiparticle weight" in the oscillating part, asymptotically

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vanishing with time; no vanishing weight is present in the nonoscillating part. Note also that the expressions we get do not require any ultraviolet regularization and also correctly capture the short-time dynamics.

The plan of this paper is as follows. We introduce the NLLM in Sec. II, and we derive by this method the ground-state two-point function and the average over a homogeneous quenched state. Section III contains our main result, namely, the time evolution of an inhomogeneous state. In the appendices the analytical derivation of our results is exposed.

II. THE NONLOCAL LUTTINGER MODEL

The nonlocal Luttinger model (NLLM) Hamiltonian is

$$H = \int_{-L/2}^{L/2} dx i(:\psi^+_{x,1}\partial_x\psi^-_{x,1}: - :\psi^+_{x,2}\partial_x\psi^-_{x,2}:) + \lambda\int_{-\frac{L}{2}}^{\frac{L}{2}} dxdy v(x-y):\psi^+_{x,1}\psi^-_{x,1} :: \psi^+_{y,2}\psi^-_{y,2}:, \quad (1)$$

where $\psi^\pm_{x,\omega} = \frac{1}{\sqrt{L}}\sum_k a_{k,\omega}e^{\pm ikx-0^+|k|}$, $\omega = 1,2$, $k = \frac{2\pi n}{L}$, with $n \in N$, are fermionic creation or annihilation operators and :: denotes Wick ordering.

In the NLLM the potentials $v(x-y)$ must verify conditions which are more easily expressed in terms of the Fourier transform; that is, one requires $\hat{v}(p)$ to be even, such that $\hat{v}(p)$ and $\partial\hat{v}(p)$ are continuous and decay faster than p^{-1} for large p. The potential $v(x-y)$ is therefore nonlocal and short ranged; it can be, for instance, quartic (i.e., $[1+(x-y)^4]^{-1}$), Gaussian, or Schwartz class. The main difference from the LLM is in the choice of the potential. In the LLM $v(x-y) = \delta(x-y)$; that is, the Fourier transform does not decay for large momenta, and this produces *ultraviolet divergences*, resulting in the expectations being expressed in terms of ill-defined integrals. This is, at first sight, surprising as ultraviolet divergences are usually not present in many-body problems. They are forbidden by the presence of the lattice or nonlinear dispersion relations, and this has the effect that there is essentially no difference between local and short-range interactions. This is, however, not true in the Luttinger model due to the linear relativistic dispersion relation, and as a consequence, the physical properties in the two models, at least at small times, differ in several aspects [37]. We are choosing units so that $v_F = 1$, where v_F is the Fermi velocity.

The Hamiltonian can be rewritten as

$$H = H_0 + V = \sum_{k>0} k[(a^+_{k,1}a^-_{k,1} + a^-_{-k,1}a^+_{-k,1}) + (a^+_{-k,2}a^-_{-k,2} + a^-_{k,2}a^+_{k,2})] + \frac{2\lambda}{L}\sum_{p>0}\hat{v}(p)[\rho_1(p)\rho_2(-p) + \rho_1(-p)\rho_2(p)] + \frac{\lambda}{L}\hat{v}(0)N_1N_2, \quad (2)$$

where, if $p > 0$,

$$\rho_\omega(p) = \sum_k a^+_{k+p,\omega}a^-_{k,\omega},$$
$$N_\omega = \sum_{k>0}(a^+_{k,\omega}a^-_{k,\omega} - a^-_{-k,\omega}a^+_{-k,\omega}).$$

The regularization implicit in the above expressions is that $\rho_\omega(p)$ must be thought of as $\lim_{\Lambda\to\infty}\sum_k\chi_\Lambda(k)\chi_\Lambda(k+p)a^+_{k+p,\omega}a^-_{k,\omega}$, where $\chi_\Lambda(k)$ is 1 for $|k| \leqslant \Lambda$ and 0 otherwise. The Hamiltonian H and $\rho_\omega(p)$ can be regarded as operators acting on the Hilbert space $\mathcal{H}$ constructed as follows: let $\mathcal{H}_0$ be the abstract linear span of the vectors obtained by applying finitely many creation or annihilation operators to

$$|0\rangle = \prod_{k\leqslant 0} a^+_{k,1}a^+_{-k,2}|\text{vac}\rangle. \quad (3)$$

We get an abstract linear space to which we introduce the scalar product between two vectors by considering them to be Fock space vectors; then $\mathcal{H}$ is the completion of $\mathcal{H}_0$ in the scalar product just introduced.

We define

$$\hat{\psi}^\pm_{\mathbf{x}} = e^{iH_0t}\psi^\pm_{\omega,x}e^{-iH_0t} = \frac{1}{\sqrt{L}}\sum_k a^\pm_{\omega,k}e^{\pm i(kx-\varepsilon_\omega kt)-0^+|k|}, \quad (4)$$

where $\varepsilon_1 = +, \varepsilon_2 = -$, so that

$$\langle 0|\psi^\varepsilon_{\omega,\mathbf{x}}\psi^{-\varepsilon}_{\omega,\mathbf{y}}|0\rangle = \frac{(2\pi)^{-1}}{i\varepsilon_\omega(x-y) - i(t-s) + 0^+}. \quad (5)$$

The basic property of the Luttinger model is the validity of the following anomalous commutation relations, first proved in [38] ($p \geqslant 0$):

$$[\rho_1(-p),\rho_1(p')] = [\rho_2(p),\rho_2(-p')] = \frac{pL}{2\pi}\delta_{p,p'}. \quad (6)$$

Moreover, one can verify that, if $p > 0$,

$$\rho_2(p)|0\rangle = 0, \quad \rho_1(-p)|0\rangle = 0. \quad (7)$$

Other important commutation relations are as follows:

$$[H_0,\rho_1(\pm p)] = \pm\rho_1(\pm p), \quad [H_0,\rho_2(\pm p)] = \mp\rho_2(\pm p), \quad [\rho_\omega,\psi^\pm_{\omega,x}] = e^{ipx}\psi^\pm_{\omega,x}. \quad (8)$$

It is convenient (see [38]) to introduce an operator $T = \frac{2\pi}{L}\sum_{p>0}[\rho_1(p)\rho_1(-p) + \rho_2(-p)\rho_2(p)]$ and write $H = (H_0 - T) + (V + T) = H_1 + H_2$. Note that H_1 commutes with ρ_ω and H_2 can be written in diagonal form with the following transformation:

$$e^{iS}H_2e^{-iS} = \widetilde{H}_2 = \frac{2\pi}{L}\sum_p \text{sech}2\phi_p \times [\rho_1(p)\rho_1(-p) + \rho_2(-p)\rho_2(p)] + E_0, \quad (9)$$

so that

$$e^{iS}e^{iHt}e^{-iS} = e^{i(H_0+D)t}, \quad (10)$$

where

$$S = \frac{2\pi}{L}\sum_{p\neq 0}\phi_p p^{-1}\rho_1(p)\rho_2(-p), \quad \tanh\phi_p = -\frac{\lambda\hat{v}(p)}{2\pi}.$$

Defining $D = T + \widetilde{H}_2$, we can write

$$D = \frac{2\pi}{L}\sum_p \sigma_p[\rho_1(p)\rho_1(-p) + \rho_2(-p)\rho_2(p)] + E_0, \quad (11)$$

where $\sigma_p = \mathrm{sech}2\phi(p) - 1$ and $[H_0, D] = 0$.

The *ground state* of H is

$$|\mathrm{GS}\rangle = e^{iS}|0\rangle, \quad (12)$$

while $|0\rangle$ is the ground state of H_0.

The relation between the creation or annihilation fermionic operators and the quasiparticle operators is

$$\psi_x = e^{ip_F x}\psi_{x,1} + e^{-ip_F x}\psi_{x,2}, \quad (13)$$

and we set $e^{ip_F x}\psi_{x,1} = \widetilde{\psi}_{x,1}$ and $e^{-ip_F x}\psi_{x,-1} = \widetilde{\psi}_{x,2}$, where p_F is the *Fermi momentum*. In momentum space this simply means that the momentum k is measured from the Fermi points; that is, $c_{k,\omega} = \widetilde{c}_{k+\varepsilon_\omega p_F,\omega}, \varepsilon_\omega = \pm$. Finally, we recall that the XXZ spin-chain model can be mapped in an interacting fermionic system; when the interaction in the third direction of the spin is missing (XX chain), the mapping is over a noninteracting fermionic system with Fermi momentum $\cos p_F = h$. Therefore, $|0\rangle$ corresponds to the ground state of the XX chain with magnetization m such that $p_F = \pi(\frac{1}{2} - m)$.

In the NLLM the average of the two-point function over the ground state (see [38]), in the $L \to \infty$ limit (see Appendix B), is

$$\langle \mathrm{GS}|\psi^+_{\omega,x}\psi^-_{\omega,0}|\mathrm{GS}\rangle = \frac{1}{2\pi}\frac{1}{i\varepsilon_\omega x + 0^+}\exp\int_0^\infty dp\frac{1}{p}\{2\sinh^2\phi_p(\cos px - 1)\}. \quad (14)$$

Asymptotically, for large distances

$$\langle \mathrm{GS}|\psi^+_{\omega,x}\psi^-_{\omega,0}|\mathrm{GS}\rangle \sim O(|x|^{-1-\eta}), \quad \eta = \sinh^2\phi_0, \quad (15)$$

implying that the average of the occupation number over the interacting ground state is $n_{k'+\varepsilon_\omega p_F} \sim a + O(k'^\eta)$.

We now consider a quantum quench in which the interaction is switched on at $t = 0$. An intersting object is the non interacting ground state evolved the interacting Hamiltonian [9],

$$\langle O_t|\psi^+_{\omega,x}\psi^-_{\omega,y}|O_t\rangle = \langle 0|e^{-iHt}\psi^+_{\omega,x}\psi^-_{\omega,y}e^{iHt}|0\rangle. \quad (16)$$

One finds (see Appendix D), in the limit $L \to \infty$,

$$\langle O_t|\psi^+_{\omega,x}\psi^-_{\omega,0}|O_t\rangle = \frac{1}{2\pi}\frac{1}{i\varepsilon_\omega x + 0^+}\exp\int_0^\infty dp\frac{\gamma(p)}{p} \times\{(\cos px - 1)[1 - \cos 2p(\sigma_p + 1)t]\}, \quad (17)$$

where $\gamma(p) = 4\sinh^2\phi_p\cosh^2\phi_p$. Keeping x fixed (see Appendix A),

$$\lim_{t\to\infty}\langle O_t|\psi^+_{\omega,x}\psi^-_{\omega,0}|O_t\rangle = \frac{1}{2\pi}\frac{1}{i\varepsilon_\omega x + 0^+}\exp\int_0^\infty dp\frac{1}{p}\{\gamma(p)(\cos\ px - 1)\}. \quad (18)$$

The two-point function over $|0_t\rangle$ reaches, for $t \to \infty$, a limit that is similar but different with respect to the average over the ground state (14); thermalization does not occur, and memory of the initial state persists. The difference between the limit of the quench and the ground-state average is that the prefactor in the integrand (related to the critical exponent) is in one case $\gamma(p) = 4\sinh^2\phi_p\cosh^2\varphi_p$ and in the other $2\sinh^2\phi_p$.

The value of $\langle O_t|\psi^+_{\omega,x}\psi^-_{\omega,y}|O_t\rangle$ in the LLM can be obtained from (18) by replacing $\gamma(p),\sigma_p$ with $\gamma(0),\sigma_0$. After doing that, the integral in the exponent of (14) becomes ultraviolet divergent, and it requires a regularization; we find (see [9])

$$\frac{1}{2\pi}\frac{1}{i\varepsilon_\omega x}\frac{1}{|x|^{\gamma(0)}}\left[\frac{x^2 - v^2t^2}{v^2t^2}\right]^{\frac{\gamma(0)}{2}}, \quad (19)$$

where $v = 1 + \sigma_0$. Comparing (17) with (19), we see that the expressions in the LLM and the NLLM are rather different at short times; in the LLM there is a divergence at $t = 0$ due to the *ad hoc* regularization which is, of course, absent in the NLLM. The expressions qualitatively agree if the limit $t \to \infty$ is performed first but only if we consider the large-distance behavior; in contrast, for small distances the behavior is radically different. In the NLLM one sees that the interaction has *no effect* at small distances [the integral in (18) is equal to zero as $x = 0$]; this is what one expects in a solid-state model, as there are no high-energy processes altering the short-distance (or high-momentum) behavior. In contrast, from (19) we see that the interaction has a strong effect even for small x, as a singularity $O(x^{-1-\gamma})$ is present, which is a consequence of the absence of an intrinsic cutoff in such a model.

III. QUANTUM QUENCH FOR THE SINGLE-PARTICLE STATE

Let us consider now an *inhomogeneous* state obtained by adding a particle to the noninteracting ground state with Fermi momentum p_F; the case in which the particle is added to the vacuum is obtained by setting $p_F = 0$. The state is the evolution of $\psi^+_x|0\rangle$, which using (13) can be written as

$$|I_{\lambda,t}\rangle = e^{iHt}(e^{ip_F x}\psi^+_{1,x} + e^{-ip_F x}\psi^+_{2,x})|0\rangle, \quad (20)$$

and we consider the average of the number operator $n(z)$,

$$\langle I_{\lambda,t}|n(z)|I_{\lambda,t}\rangle, \quad (21)$$

where $n(z)$ is the regularized version of the particle number $\psi^+_z\psi^-_z$, namely,

$$n(z) = \sum_{\rho=\pm}(\widetilde{\psi}^+_{1,z+\rho\varepsilon}\widetilde{\psi}^-_{2,z} + \widetilde{\psi}^+_{2,z+\rho\varepsilon}\widetilde{\psi}^-_{1,z} + \widetilde{\psi}^+_{2,z+\rho\varepsilon}\widetilde{\psi}^-_{2,z} + \widetilde{\psi}^+_{1,z+\rho\varepsilon}\widetilde{\psi}^-_{1,z}). \quad (22)$$

One needs to introduce a point splitting (the sum over $\rho = \pm$) that plays the same role as Wick ordering, and at the end the limit $\varepsilon \to 0$ is taken. Note that using the correspondence with the XXZ spin modes, the state $|I_{\lambda,t}\rangle$ corresponds to adding an excitation to the ground state of the XX chain with total magnetization $m = 1/2 - p_F/\pi$. It turns out that $\langle I_{\lambda,t}|n(z)|I_{\lambda,t}\rangle$ is the sum of several terms,

$$\begin{aligned}&\langle 0|\widetilde{\psi}^-_{1,x}e^{iHt}\widetilde{\psi}^+_{1,z+\rho\varepsilon}\widetilde{\psi}^-_{2,z}e^{-iHt}\widetilde{\psi}^+_{2,x}\,|0\rangle\\&+\langle 0|\,\widetilde{\psi}^-_{2,x}e^{iHt}\widetilde{\psi}^+_{2,z+\rho\varepsilon}\widetilde{\psi}^-_{1,z}e^{-iHt}\widetilde{\psi}^+_{1,x}\,|0\rangle\\&+\langle 0|\,\widetilde{\psi}^-_{1,x}e^{iHt}\widetilde{\psi}^+_{2,z+\rho\varepsilon}\widetilde{\psi}^-_{2,z}e^{-iHt}\widetilde{\psi}^+_{1,x}\,|0\rangle\\&+\langle 0|\,\widetilde{\psi}^-_{2,x}e^{iHt}\widetilde{\psi}^+_{1,z+\rho\varepsilon}\widetilde{\psi}^-_{1,z}e^{-iHt}\widetilde{\psi}^+_{2,x}\,|0\rangle\\&+\langle 0|\,\widetilde{\psi}^-_{1,x}e^{iHt}\widetilde{\psi}^+_{1,z+\rho\varepsilon}\widetilde{\psi}^-_{1,z}e^{-iHt}\widetilde{\psi}^+_{1,x}\,|0\rangle\\&+\langle 0|\,\widetilde{\psi}^-_{2,x}e^{iHt}\widetilde{\psi}^+_{2,z+\rho\varepsilon}\widetilde{\psi}^-_{2,z}e^{-iHt}\widetilde{\psi}^+_{2,x}\,|0\rangle. \qquad (23)\end{aligned}$$

In the noninteracting case $\lambda = 0$, the first term can be written as

$$\langle 0|\widetilde{\psi}^-_{1,x,t}\widetilde{\psi}^+_{1,z+\rho\varepsilon}\,|0\rangle\langle 0|\widetilde{\psi}^-_{2,z}\widetilde{\psi}^+_{2,x,t}\,|0\rangle, \qquad (24)$$

so that in the limit $\varepsilon \to 0$ this term is equal to $e^{2ip_F(x-y)}(4\pi^2)^{-1}[(x-z)^2 - t^2]^{-1}$; a similar result is found for the second term. The third and fourth terms are vanishing as $\sum_\rho \frac{1}{\rho\varepsilon} = 0$; similarly, the last two terms give $(4\pi^2)^{-1}[(x - z) \pm t]^{-2}$. Therefore, in the absence of interaction, one gets

$$\begin{aligned}&\lim_{L\to\infty}\langle I_{0,t}|n(z)|I_{0,t}\rangle\\&= \frac{1}{2\pi^2}\frac{\cos 2p_F(x-y)}{(x-z)^2 - t^2}\\&\quad + \frac{1}{4\pi^2}\left[\frac{1}{[(x-z)-t]^2} + \frac{1}{[(x-z)+t]^2}\right]. \qquad (25)\end{aligned}$$

The average of the density is the sum of two terms, an oscillating part and a nonoscillating part (when the particle is added to the vacuum, there are no oscillations, $p_F = 0$). At $t = 0$ the density is peaked at $z = x$, where the average is singular. As the time increases, the particle peaks move in the left and right directions with constant velocity $v_F = 1$ (ballistic motion); that is, the average of the density is singular at $z = x \pm t$ and a light-cone dynamics is found.

The interaction addresses in a quite nontrivial way the above dynamics. We get in the $L \to \infty$ limit (see Appendix C)

$$\begin{aligned}&\lim_{L\to\infty}\langle I_{\lambda,t}|n(z)|I_{\lambda,t}\rangle\\&= \frac{1}{4\pi^2}\left[\frac{1}{[(x-z)-t]^2} + \frac{1}{[(x-z)+t]^2}\right]\\&\quad + \frac{1}{4\pi^2}\frac{e^{Z(t)}}{(x-z)^2 - t^2}[e^{2ip_F(x-z)}e^{Q_a(x,z,t)}\\&\quad + e^{-2ip_F(x-z)}e^{Q_b(x,z,t)}], \qquad (26)\end{aligned}$$

where

$$Z(t) = \int_0^\infty \frac{dp}{p}\gamma(p)[\cos 2p(\sigma_p + 1)t - 1] \qquad (27)$$

and $\gamma(p) = \frac{e^{4\phi_p}-1}{2}$; moreover,

$$\begin{aligned}Q_a = \int_0^\infty dp\frac{e^{-p0^+}}{p}&[(e^{ip(x-z)+ip(\sigma_p+1)t} - e^{ip(x-z)+ipt})\\&+ (e^{ip(x-z)-ip(\sigma_p+1)t} - e^{ip(x-z)-ipt})]\end{aligned}$$

and

$$\begin{aligned}Q_b = \int_0^\infty dp\frac{e^{-p0^+}}{p}&[(e^{-ip(x-z)+ip(\sigma_p+1)t} - e^{-ip(x-z)+ipt})\\&+ (e^{-ip(x-z)-ip(\sigma_p+1)t} - e^{-ip(x-z)-ipt})].\end{aligned}$$

By looking at (26) we see first that the interaction does not modify the nonoscillating part. Regarding the oscillating part, it produces two main effects. First of all, the velocity of the peaks of is no longer constant but *varies* between a maximal and a minimal value. This is an effect which is absent in the LLM; indeed, if we replace σ_p with σ_0, we have

$$\frac{1}{(x-z)^2 - t^2}e^{Q_a} = \frac{1}{(x-z)^2 - (1+\sigma_0)^2 t^2}, \qquad (28)$$

so that one gets the same expression as in the free case with a renormalized velocity (a similar expression is valid for Q_b). The presence of nonconstant velocity is in agreement with the result of numerical simulations in the XXZ chain [13,23,29].

The interaction also has another nontrivial effect; it introduces a dynamical Landau quasiparticle weight in the oscillating part that asymptotically vanishes with time. Indeed, for large t

$$\exp Z(t) = O(t^{-\gamma(0)}), \qquad (29)$$

while $Z(0) = 1$. This vanishing weight can be physically interpreted as a consequence of the fact that fermions are not excitations of the interacting Hamiltonian. Finally, note that the quasiparticle weight is 1 at $t = 0$ and decreases at large t.

IV. CONCLUSIONS

We have computed by exact analytical methods the time evolution of an inhomogeneous state with a localized fermion added to the noninteracting ground state in the nonlocal Luttinger model. The interaction does not produce a simple renormalization of the parameters of the noninteracting evolution; on the contrary, it generates nonconstant velocities, and a dynamical Landau quasiparticle weight appears in the oscillating part of the averaged density that asymptotically vanishes with time. We believe that similar phenomena would also be present in the evolution of more complex initial states such as a domain-wall profile, and we plan to extend our methods to such a case.

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APPENDIX A

In order to prove (18) we set $z = p(1+\sigma)$, and we note that $\partial_p z(p) = H_p$ is bounded and different from zero; moreover, z is an increasing function of p such that p/z tends to a constant for $p \to 0$ and $p \to \infty$. Integrating by parts and using $\frac{x \sin px}{p} \sim x^2$, (18) follows.

In order to evaluate the large-distance behavior of $Z(t)$ we use $\gamma(p) = \frac{v(p)}{2\pi}$, and we write $\int_0^\infty \frac{dp}{p}\gamma(p)[\cos 2\omega(p)pt - 1]$ as $\int_0^1 + \int_1^\infty$, where the second integral is bounded by a constant. In the first term we can write $\gamma(p) = \gamma(0)e^{-\kappa p} + r(p)$, with $r(p) = o(p)$, and the integral containing $r(p)$ is again bounded by a constant. Note that

$$\gamma(0)\int_0^1 dp\frac{e^{-\kappa p}}{p}[\cos 2\omega(0)pt - 1] \sim \frac{\gamma(0)}{2}\log\frac{\kappa^2}{\kappa^2 + 4\omega(0)^2t^2}. \tag{A1}$$

Moreover, we can write $\omega(p) = 1 + \sigma_p = \omega(0) + f(p)$, with $f(p) = O(p)$ and

$$\begin{aligned}\int_0^1 dp\frac{e^{-\kappa p}}{p}(\cos(2\omega(p)pt) - \cos(2\omega(0)pt)) &= \int_0^1 dp\frac{e^{-\kappa p}}{p}(\cos(2\omega(0)pt)(\cos f(p)pt - 1)\\ &\quad + \int_0^1 dp\frac{e^{-\kappa p}}{p}\sin(2\omega(0)pt)\sin(f(p)pt).\end{aligned} \tag{A2}$$

Integrating by parts we find that both integrals are bounded by a constant.

APPENDIX B

In order to derive (14) we write $\langle GS|\psi^+_{1,x}\psi^-_{1,y}|GS\rangle$ as

$$\langle 0|e^{iS}\psi^+_{1,x}e^{-iS}e^{iS}\psi^-_{1,y}e^{-iS}|0\rangle \tag{B1}$$

and

$$e^{i\varepsilon S}\psi^-_{1,x}e^{-i\varepsilon S} = W_{1,x}R_{1,x}\psi^-_{1,x}, \tag{B2}$$

with $c(\phi) = \cosh\varepsilon\phi - 1$, $s(\phi) = \sinh\varepsilon\phi$,

$$W^\varepsilon_{1,x} = \exp\left\{-\frac{2\pi}{L}\sum_{p>0}\frac{e^{-0^+p}}{p}[\rho_1(-p)e^{ipx} - \rho_1(p)e^{-ipx}]c(\phi)\right\},$$

$$R^\varepsilon_{1,x} = \exp\left\{-\frac{2\pi}{L}\sum_{p>0}\frac{e^{-0^+p}}{p}[\rho_2(-p)e^{ipx} - \rho_2(p)e^{-ipx}]s(\phi)\right\},$$

so that (B1) becomes

$$\langle 0|\psi^+_{1,x}W^{-1}_{1,x}R^{-1}_{1,x}R_{1,y}W_{1,y}\psi^-_{1,y}|0\rangle. \tag{B3}$$

By using the commutation relations (6) and $e^Ae^B = e^Be^Ae^{[A,B]}$ to carry $\rho_1(p)$ [$\rho_2(p)$] to the left (right) and $\rho_1(-p)$ [$\rho_2(-p)$] to the right (left) and using (7), we get (14).

APPENDIX C

We consider one term in (23) (the others are studied in a similar way),

$$\langle 0|\psi^-_{1,x}e^{iHt}\psi^+_{1,z,}\psi^-_{2,z}e^{-iHt}\psi^+_{2,x}\,|0\rangle, \tag{C1}$$

which can be rewritten as

$$\begin{aligned}&\langle 0|\psi^-_{1,x}e^{-iS}e^{iS}e^{iHt}e^{-iS}e^{iS}\psi^+_{1,z}e^{-iS}e^{iS}e^{-iHt}e^{-iS}\\ &\quad\times e^{iS}e^{iHt}e^{-iS}e^{iS}\psi^-_{2,z}e^{-iS}e^{iS}e^{-iHt}e^{-iS}e^{iS}\psi^+_{2,x}|0\rangle.\end{aligned} \tag{C2}$$

We use the relation

$$e^{i(H_0+D)t}e^{iS}\psi^+_{1,x}e^{-iS}e^{-i(H_0+D)t} = \bar\psi^+_{1,x,t}W^{-1}_{1,x,t}R^{-1}_{1,x,t},$$

where $e^{i(H_0+D)t}\psi^+_{1,x}e^{-i(H_0+D)t} = \bar\psi^+_{1,\mathbf{x}}$ and, setting $c(\phi) = \cosh\phi - 1$, $s(\phi) = \sinh\phi$,

$$W_{1,x,t} = \exp\left\{-\frac{2\pi}{L}\sum_{p>0}\frac{1}{p}[\rho_1(-p,t)e^{ipx} - \rho_1(p,t)e^{-ipx}]c(\phi)\right\},$$

$$R_{1,x,t} = \exp\left\{-\frac{2\pi}{L}\sum_{p>0}\frac{1}{p}[\rho_2(-p,t)e^{ipx} - \rho_2(p,t)e^{-ipx}]s(\phi)\right\},$$

where $\rho_1(\pm p,t) = e^{\pm ip(\sigma_p+1)t}\rho_1(\pm p)$, $\rho_2(\pm p,t) = e^{\mp ip(\sigma_p+1)t}\rho_2(\pm p)$; moreover,

$$\bar\psi^\varepsilon_{1,\mathbf{x}} = z_b\hat\psi^\varepsilon_{1,\mathbf{x}}B_{1,+,\mathbf{x}}B_{1,-\mathbf{x}} = z_aB_{1,+,\mathbf{x}}B_{1,-,\mathbf{x}}\hat\psi^\varepsilon_{1,\mathbf{x}}, \tag{C3}$$

where

$$B^\varepsilon_{1,+,\mathbf{x}} = \exp\varepsilon\frac{2\pi}{L}\sum_{p>0}e^{-0^+p}\times[\rho_1(p)(e^{-ipx+ip(\sigma_p+1)t} - e^{-ipx+ipt})],$$

$$B^\varepsilon_{1,-,\mathbf{x}} = \exp-\varepsilon\frac{2\pi}{L}\sum_{p>0}e^{-0^+p}\times[\rho_1(-p)(e^{ipx-ip(\sigma_p+1)t} - e^{ipx-ipt})],$$

and $\hat\psi^+_{\mathbf{x},\omega} = e^{iH_0t}\psi^+_{x,\omega}e^{-iH_0t}$, $z_a = \exp\frac{2\pi}{L}\sum_p\frac{1}{p}(e^{ip\sigma_pt} - 1)$, and $z_b = \exp\frac{2\pi}{L}\sum_p\frac{1}{p}(e^{-ip\sigma_pt} - 1)$.

We write

$$e^{-iS}\bar\psi^+_{1,x,t}W^{-1}_{1,x,t}R^{-1}_{1,x,t}e^{iS} = e^{-iS}\bar\psi^+_{1,x,t}e^{iS}\bar W^{-1}_{y,t}\bar R^{-1}_{y,t}, \tag{C4}$$

where $\bar W_{y,t},\bar R_{y,t}$ are equal to $W_{y,t},R_{y,t}$ in (C3) with $\rho(p)$ replaced by

$$\begin{aligned}e^{-iS}\rho_1(\pm p)e^{iS} &= \rho_1(\pm p)\cosh\phi(p) - \rho_2(\pm p)\sinh\phi,\\ e^{-iS}\rho_2(p)e^{iS} &= \rho_2(\pm p)\cosh\phi(p) - \rho_1(\pm p)\sinh\phi.\end{aligned} \tag{C5}$$

Note that $\bar W_{1,y,0}\bar R_{1,y,0} = W^{-1}_{1,y}R_{1,y,0}$, so that $(e^{-iS}\bar\psi^+_{1,x}e^{iS})\bar W_{1,y,0}\bar R_{1,y,0} = \psi^+_{1,x}$.

It remains to evaluate $e^{-iS}\bar\psi_{1,x,t}e^{iS}$; we use (C3) so that it can be written as

$$z_a\bar B^+_{1,+,x,t}\bar B^+_{1,-,x,t}(e^{-iS}\hat\psi^+_{1,\mathbf{x}}e^{iS}), \tag{C6}$$

where $\bar{B}^{\varepsilon}_{1,+,x,t}, \bar{B}^{\varepsilon}_{1,-,x,t}$ are equal to $B^{\varepsilon}_{1,+,x,t}, B^{\varepsilon}_{1,-,x,t}$ with $\rho(p)$ replaced by (C5). Moreover,

$$(e^{-iS}\hat{\psi}^{+}_{1,\mathbf{x}}e^{iS}) = \hat{\psi}^{+}_{1,\mathbf{x}}W_{1,0,x,t}R^{-1}_{1,0,x,t}, \tag{C7}$$

and $W_{1,0,x,t}$, $R_{1,0,x,t}$ are equal to $W_{1,x,t}$, $R_{1,x,t}$, with $\sigma_p = 0$. In conclusion, (C2) can be rewritten as

$$\begin{aligned}&\langle 0|\psi_{1,x}\big(\bar{B}^{+}_{1,+,z,t}\bar{B}^{+}_{1,-,z,t}\hat{\psi}^{+}_{z,t,1}W_{1,0,z,t}R^{-1}_{1,0,z,t}\big)\\&\times \bar{W}^{-1}_{1,z,t}\bar{R}^{-1}_{1,z,t}\bar{W}_{2,z,t}\bar{R}_{2,z,t}\\&\times\big(W^{-1}_{2,0,z,t}R_{2,0,z,t}\hat{\psi}^{-}_{z,t,2}\bar{B}^{-}_{2,+,z,t}\bar{B}^{-}_{2,-,z,t}\psi^{+}_{2,x}\big)|0\rangle.\end{aligned}$$

By using

$$\begin{aligned}&e^{\frac{2\pi}{L}\sum_p \frac{1}{p}F\rho_\omega(\pm p)}\psi^{-}_{\omega,x,t}\, e^{-\frac{2\pi}{L}\sum_p \frac{1}{p}F\rho_\omega(\pm p)}\\&\quad= \psi^{-}_{\omega,x,t}e^{-\frac{2\pi}{L}\sum_p \frac{1}{p}Fe^{\pm(ipx-ipt)}},\\&e^{\frac{2\pi}{L}\sum_p \frac{1}{p}F\rho_\omega(\pm p)}\psi^{\dagger}_{\omega,x,t}\, e^{-\frac{2\pi}{L}\sum_p \frac{1}{p}F\rho_\omega(\pm p)}\\&\quad= \psi^{\dagger}_{\omega,x,t}e^{\frac{2\pi}{L}\sum_p \frac{1}{p}Fe^{\pm(ipx-ipt)}},\end{aligned} \tag{C8}$$

where F is an arbitrary regular function, and the Backer-Hausdorff formula to carry $\rho_1(p)$ $[\rho_2(p)]$ to the left (right) and $\rho_1(-p)$ $[\rho_2(-p)]$, we finally get (26).

APPENDIX D

We can write (16) as

$$\begin{aligned}&\langle 0|e^{-iS}\{e^{iS}e^{iHt}e^{-iS}[e^{iS}\psi^{+}_{1,x}e^{-iS}]e^{iS}e^{iHt}e^{-iS}\}e^{iS}\\&\quad\times\{e^{-iS}\{e^{iS}e^{iHt}e^{-iS}[e^{iS}\psi^{-}_{1,y}e^{-iS}]e^{iS}e^{-iHt}e^{-iS}\}e^{iS}|0\rangle,\end{aligned}$$

which is equal to

$$\begin{aligned}&\langle 0|e^{-iS}e^{i(H_0+D)t}[e^{iS}\psi^{+}_{1,x}e^{-iS}]e^{-i(H_0+D)t}e^{iS}\\&\quad\times e^{-iS}e^{i(H_0+D)t}[e^{iS}\psi^{-}_{1,y}e^{-iS}]e^{-i(H_0+D)s}e^{iS}|0\rangle\end{aligned}$$

and by (C3)

$$\langle 0|e^{-iS}\bar{\psi}_{1,x,t}W^{-1}_{1,x,t}R^{-1}_{1,x,t}e^{iS}e^{-iS}W_{1,y,t}R_{1,y,t}\bar{\psi}_{1,x,t}e^{iS}|0\rangle,$$

from which we finally obtain

$$\begin{aligned}&\langle 0|\{e^{-iS}\bar{\psi}_{1,x,t}e^{iS}\bar{W}^{-1}_{1,x,t}\bar{R}^{-1}_{1,x,t}\}\\&\quad\times\{\bar{W}_{y,t}\bar{R}_{1,y,t}(e^{-iS}\bar{\psi}_{1,x,t}e^{iS})|0\rangle.\end{aligned}$$

As in the previous case, we now use the commutation relations (6) and the relation $e^Ae^B = e^Be^Ae^{[A,B]}$ to carry $\rho_1(p)$ $[\rho_2(p)]$ to the left (right) and $\rho_1(-p)$ $[\rho_2(-p)]$ to the right (left), and using (7), we get (14). The final expression coincides with the one found in [20] with a different method, namely, using a bosonization identity expressing the fermionic field in terms of bosons and Majorana operators.

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