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*to my father...*

## TABLE OF CONTENTS

<b>INTRODUCTION</b> .....	3
<b>CHAPTER 1</b>	
<b>HUMAN CAPITAL ACCUMULATION AND MIGRATION: A REVIEW</b> .....	7
1. Introduction .....	8
2. Migration and Its Implications on Human Capital Accumulation.....	9
3. Human Capital Accumulation in Home Country Analyses.....	11
3.1 The Main Channels.....	12
3.1.1 Educational Incentives under Probabilistic Migration Models.....	12
3.1.2 Endogenous Fertility and Income Differences .....	16
4. Host Country Analyses.....	20
5. How do results affect economic growth? .....	23
6. Conclusion.....	25
References .....	27
<b>CHAPTER 2</b>	
<b>HUMAN CAPITAL ACCUMULATION UNDER SKILL-BIASED TECHNOLOGICAL CHANGE WITH MIGRATION</b> .....	32
1. Introduction .....	33
2. The Literature Review .....	34
3. The Model .....	37
3.1 Production and Technological Structure .....	37
3.2 Skill-Biased Technological Change .....	41
3.3 Migration Decision under SBTC.....	42
3.4 Human Capital Accumulation under SBTC .....	42
3.4.1 Cost of Education .....	43
3.4.2 Optimal Education Decision.....	44
4. Equilibrium with Human Capital Accumulation under SBTC with Migration.....	45
4.1 Less Developed (Migrant Sending) Country Case .....	46
4.2 Advanced (Migrant Receiving) Country Case .....	50
5. Conclusion.....	51
References .....	53

## CHAPTER 3

DILUTION EFFECTS, POPULATION GROWTH AND ECONOMIC GROWTH UNDER HCA AND ENDOGENOUS TECHNOLOGICAL CHANGE .....	55
1. Introduction .....	56
2. The Model .....	58
2.1 Production.....	58
2.2 Research and Development (R&D).....	61
2.3 Households .....	64
3. General Equilibrium and BGP Analysis.....	66
4. Population Growth and Economic Growth.....	70
5. Empirical Analyses.....	72
5.1 An Exercise: Testing the Theory for Advanced Countries.....	72
5.2 Panel Fixed Effect Analysis with Penn World Table Data (2013).....	73
5.2.1 Methodology and Data .....	73
5.2.2 Empirical Results.....	75
5.3 Panel Structural VAR Analyses with Barro-Lee (2013) Data.....	75
5.3.1 Methodology and Data .....	75
5.3.2 Empirical Results.....	78
6. Conclusion.....	79
References .....	81
APPENDIX A: .....	86
APPENDIX B:.....	92
APPENDIX C:.....	93

## **INTRODUCTION**

This section provides an extended summary of my Ph.D. thesis, which is composed of three independent chapters. First chapter is considered as a review paper. Chapter 2 and 3 are the original research papers. The common denominator of the three papers is population and its impact on economic growth under the endogenous models with human capital accumulation. The next section will describe the incentives, the structure of the models, including the main assumptions, and the main results of each chapter.

First chapter is a review-article that is largely associated with two literatures: International migration and economic growth. It is well known that human capital takes an important part in endogenous growth models. It makes a direct contribution to production with physical capital and is vital to sustain technological progress and innovation, which are crucial for the economic growth. Additionally, over many years, international migration has been one of the growing concerns of population change, which has various economic implications that can be either at macro or micro level. Given these facts, this chapter confines itself to consider solely the effects of migration on human capital accumulation. The motivation of this review is to identify how and to what extent migration is capable of impacting human capital formation in both migrant sending (home) and receiving (host) countries and thus, economic growth.

The main purpose of this paper is to show that the linkage between migration and human capital accumulation has been evolving due to the changes in economic roles of migrants. Due to the methods of complexity in international migration research, this study required a multi-dimensional perspective in order to evaluate the results in interactions between human capital accumulation and economic growth. The literature revealed that it is not easy to say that migration brings a net gain or leads to a net cost to the countries. Thus, we can conclude that the human capital-based growth effects of migration ranges according to the type of country analyzed (sending vs. receiving country), the duration of migration (temporary or permanent) and the heterogeneity in skill types of migrants (low or high-skilled).

The main finding of this review is that that migration can affect the human capital accumulation via its impacts on three important factors: (i) Skill formation, (ii) Fertility and (iii) Wages (income levels). This review addresses precisely those points, that the changes in those factors generally drive many of the differences in results (either positive or negative or neutral) across studies. In home country analyses we evidenced that the increase in the return to education, investment in human capital may stimulate the migration decisions of individuals that have both



direct and indirect effects on skill composition, trade-off between quality and quantity of children, and the current (future) income levels. In addition to these findings, we also highlight the spill-over effects of migrants on natives' and on companies' investments in education and technology. Differences between natives' and migrants' preferences for having children have also critical roles in explaining the changes in human capital accumulation in host countries.

The present review also shows that the modelling frameworks, and the assumptions which are subject to above-mentioned categorizations, leads to ambiguity in theoretical and empirical results across studies. For example, uncertainty and different migration probabilities are the key assumptions determining the strength of the impacts of migration on human capital accumulation. Most, recent relevant literature and case studies consider an economy with homogenous agents and exogenously given migration probabilities, however; a model with differentiated individuals and endogenous probability of migration may lead to more realistic results. It is also emphasized in the review that the assumptions are needed to be qualified by empirical observations. For instance, the results may also differ according to what extent migrants and natives are perfect (or imperfect substitutes) within skill groups. Chapter 1 reveals that further empirical studies would be important to verify the currently available theoretical results. Additionally the impacts of low-skilled migration and return migration on human capital accumulation may be paid much attention in endogenous growth models.

Chapter 2 is an original theoretical paper entitled "Human Capital Accumulation under Skill-Biased Technological Change with Migration". As the previous chapter points out, human capital-based growth effect of migration is steadily becoming one of the most interesting questions in the literature. This chapter intends to provide a complementary dimension to the directed technological change literature by incorporating human capital accumulation with skilled migration. The underlying reason for this chapter is the absence of the ongoing analyses on how skill-biased technological change (SBTC, hereafter) can potentially have an impact on human capital accumulation. The approach used in the model allows us to see the impact of technological change on human capital accumulation in both migrant receiving and sending countries jointly. Therefore, to the best of our knowledge, this research is the one of the very first attempts in trying to fill the aforementioned gap in economic growth and migration literatures.

The present chapter has been built upon the following two arguments: First, technological development demands relatively more high skilled labour; thus, the return to education is higher under the skill-biased technological change. Second, as we know from the literature that, such an increase in the labour demand, which is mostly driven by SBTC, leads to an increase in skill-

specific migration (brain drain) from technologically less developed countries to technologically advanced countries. Therefore, chapter 2 mainly rests on the following assumptions: First, the probability of getting high (expected) wages is the only motivation for labour migration. Second, investment in education not only increases the skill level but also increases the mobility of individuals and third, only skilled labours are able to migrate due to the skill-specific pull factor of technological change.

Consequently, the most important novelty of this chapter consists in the fact that in an Acemoglu (2002) type growth model we explicitly integrate the model of the human capital accumulation by taking into account the possibility of skilled-biased migration. The results support the positive impact of SBTC on human capital accumulation in less developed countries due to the existence of the possibility of migration. Both skilled and less skilled parents increase their investment in education for their children in spite of that SBTC has not occurred in their countries. This result shows that an SBTC in an advanced economy stimulates the investment in education in less developed country, as long as the probability of getting high wages via migration is sufficiently high. At the same time, the present chapter has figured out that the net effect of SBTC on human capital accumulation is ambiguous (either positive or negative or neutral) for advanced countries under skilled migration. This result depends on the two opposite effects: First, a higher current wage ratio due to SBTC leads to a higher relative education cost for less skilled parents. This implies a negative effect on human capital accumulation. Second, the probability of earning skilled wages in the future, keeps the parents from investing in higher levels of education. The approach we follow in this study does not bring any negative externalities of skilled migration on the decisions of investment in education of local parents at advanced countries under the implications of SBTC.

Chapter 3 is a joint paper with *Professor Alberto Bucci*. We analyze the impact of population growth as an exogenous variable on economic growth under endogenous technological change with human capital accumulation. This chapter is composed of two parts. In the first part of the paper, we set up a theoretical model with expanding variety of products and endogenous human capital formation. In the second part, the theoretical results are tried to test through a system of simultaneous equations by employing a panel fixed effect and structural panel VAR analyses for a set of countries.

The novelty of this study is the inclusion of a *dilution effect* of population growth which is not presented in the original Uzawa-Lucas model of human capital accumulation. Unlike Lucas (1988), we illustrate that population growth has a direct and negative effect which decreases human capital investment of economic agents, and thus economic growth. Additionally the paper explains

the population growth may have an opposite effect on economic growth which is called *indirect ideas effect* of population growth.

The main objectives of this paper are first to provide an alternative but complementary theoretical framework explaining why an increase in the population growth rate may yield an ambiguous (positive, negative or neutral) impact on the growth rate of per-capita income in the very long-run, and second, given this ambiguity, to assess exactly to what extent the *dilution effect* of population growth may affect countries' economic growth differently. The theoretical results have demonstrated (i) *dilution effect* has a central role in explaining the ambiguous sign of the impact of population growth on economic growth along the BGP equilibrium. (ii) A threshold level for the dilution effect exists. (iii) Correlation between population and economic growth rates may be either, positive or negative or else neutral as long as the dilution effect is greater than one. (iv) There exists an unambiguously positive correlation between population growth and economic growth as long as the *dilution effect* is sufficiently low (below one). Empirical results also have supported the later finding that a dilution effect of population growth on human capital accumulation exists and is below the threshold level in advanced economies.

## CHAPTER 1

### HUMAN CAPITAL ACCUMULATION AND MIGRATION: A REVIEW

***Abstract:** This paper is a survey article that mainly reviews the literatures of international migration and economic growth. Over the last fifteen years there has been a growing concern about the effects of migration on human capital formation. The main motivation of the present article is to identify how and to what extent migration affects human capital accumulation, and thus, economic growth. The paper reveals that the research on migration requires a multi-dimensional perspective in explaining the human capital related economic questions. The paper shows that that the impact of migration on human capital accumulation can be observed via three channels: The impacts of migration on (i) skill formation, (ii) fertility decisions and (iii) wage (income) levels. This review addresses that the changes in those factors generally drive many of the differences in results (either positive or negative or neutral) across studies. Lastly, this review figures out that endogenous growth models do not take the impacts of different skill and durations of migration into account sufficiently, and the current theoretical results need to be verified by further empirical researches.*

## 1. Introduction

The literature on migration is one of the most extensive in economics. Even though there are many reasons such as ageing population, demographic decline, global competition and other economic and political factors making this topic so interesting, the most popular topic have been questioned in the literature is whether immigrants lead to an adverse effect on labor market outcomes or not. Contrary to the popular belief, recent studies show that immigrants have positive effects on productivity, income and wage inequality. Currently we can say that international labor movements have several economic implications lead to ambiguity in results. In this survey we confine ourselves to figure out the impacts of migration on human capital accumulation. Therefore, this paper is largely associated with two literatures: international migration, and economic growth.

Many factors (technological change, income distribution, trade-offs between work and fertility, public policies and education) have become subjects to research for their effects on human capital accumulation. As we know that human capital is one of the most vital factors for production because of its significant roles in economic activity. First, it makes a direct contribution to production with physical capital. Second, stock of human capital is essential to sustain technological progress and innovation, which are substantial for economic growth. Therefore, human capital accumulation is directly linked to output growth, and these two reasons sufficiently explain why the studies on human capital accumulation take an important place in the endogenous growth literature. Given this, the present survey attempts to find an instructive answer to the following question: How and to what extent does migration affect human capital accumulation and thus, economic growth? This study also aims to identify the knowledge gaps in this field to make some suggestions for future researches.

The paper reveals that mainly three channels drive many of the differences in results across studies focusing on the impacts of migration on human capital accumulation. These channels can be categorized as follows: (i) skill formation, (ii) fertility decisions and (iii) wage (income) differentiation. This survey also concludes that in growth literature, the effects of return migration on human capital accumulation is not sufficiently discussed<sup>1</sup>, and expanding the empirical analyses by considering the diversity in various types of migration may also be an important contribution to the literature.

The remaining part of the paper is organized as follows. The next section presents the main implications of migration on human capital accumulation briefly. Sections 3 and 4 detail the basic

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<sup>1</sup> Dos Santos and Postel-Vinay (2003) point out the positive role of return migration to origin countries via the additional skills acquired abroad.

structures of the studies with regard to home and host country analyses respectively. Section 5 explains the differences in findings with a summary of the results. Finally, section 6 concludes the review.

## **2. Migration and Its Implications on Human Capital Accumulation**

The linkage between migration and human capital accumulation has two major dimensions (implications in the country of destination and in the origin country) which can also be categorized by means of the duration (temporary vs. permanent) and the heterogeneity in skill types (low vs. high-skilled) of migration. With regard to human capital accumulation issue there are several questions to be answered: For instance, to what extent is a brain drain good for home countries? What are the human capital externalities of migration on non-migrants, and which impacts of migration on natives have already been observed? What is the net human capital stock after migration flows in receiving countries? Does high-skilled immigration absolutely raise the accumulation of skills in those countries in the long run? or do low-skilled migrants affect human capital formation negatively?

As Mountford (1997), Stark *et al.* (1997, 1998), Vidal (1998) and Stark and Wang (2002) argued, the migration prospects may bring an incentive to accumulate skills, and make a contribution to human capital formation in less developed countries. Or as recent studies Maria and Stryszowski (2009), Magris and Russo (2009), Maria and Lazarova (2011), Azarnert (2011) showed, migration and selective immigration policies may distort the agent's incentives to accumulate the most appropriate skills<sup>2</sup> for their country of origin. These are some of the findings at the forefront of policy making debates which require a multi-dimensional perspective in explaining the human capital related economic discussions.

Empirical evidences suggest that the impacts of migration on labor market outcomes is one of the controversial topics in the literature at present<sup>3</sup>, and it is clear that the effects of migration on human capital investment of natives depends on how the expectations have been defined by the theoretical models. For instance, if there is a strong belief that the impact of migration on native wages is negative, this would result, *ceteris-paribus*, a negative effect on optimal education decisions of natives in receiving countries. However, Docquier *et al.* (2010) find that immigration has zero to small positive long-run effect on the average wages of natives in the rich OECD

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<sup>2</sup> Skills are generally classified as lower and higher-skills. In addition to this classification, higher-skills can also be categorized in two groups according to their specializations: "technical and general skills".

<sup>3</sup> Some references on this literature can be found, see Friedberg and Hunt (1995), Ottaviano and Peri (2006), Peri (2006). On the contrary, see Borjas (2005).

countries and reduces the wage inequality among the workers. It is likely to find a number of studies showing positive impacts of migration on optimal education decision of natives. This literature argues that high-skilled immigration exerts a positive effect on the return of investment in higher education among natives. Like Regets (2001), Bellettini and Ceroni (2003) show that opening up the borders to the immigrants brings an upward shift in the steady-state fraction of skilled native workers as long as the host economy succeeds in attracting high skilled foreign agents. As Chander and Thangavelu (2004) emphasize that due to the policy interactions between education and migration, and complementarities in technological development some governments may prefer to stimulate high-skilled migrants to move into their countries as an alternative policy tool. Conversely, in an early study (Van Dalen, 1993) receiving countries are advised to train their own labor force instead of introducing a selective immigration policy due to widening of the divergence in dynamic efficiency in capital accumulation. Contrary to Regets (2001), Borjas (2007, 2009), Azarnert (2010a, 2010b) also emphasize immigration induced adverse effects on educational incentives for native population.

The relation between technological progress and brain drain is the other controversial issue in economics. We know that brain drain can be simply defined as the emigration of high-skilled brains from one country to another, and here, skilled-biased technological change is a pull factor on these flows. However, technological progress may not always favor the skilled workers, and this point is often overlooked in the analyses. Funk and Vogel (2004) deal with this question, and show that technological progress can also increase the efficiency of unskilled workers. In other words, many developed countries at the same time attract the unskilled foreign workers. Therefore; taking solely skilled migration into consideration on the analyses of human capital accumulation may not always reflect what is likely to happen in the receiving countries at the present time. Very few theoretical studies such as Cipriani (2006), and Azarnert (2011) draw attention to this point.

It is interesting to note that Sato *et al.* (2008) explain how a skill biased technological change plays a role in explaining the key stylized facts of economic development. Increases in the fraction of educated people, and declines in the fertility, and rises followed by declines in income inequality is the simple explanation of this cycle. Like the relationship between fertility and technological change, there is also a large literature on the relationship between fertility and migration addressing the potential impacts of migration on native population.<sup>4</sup> The studies focusing on the implications of low-skilled immigration on host countries argue that this type of migration

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<sup>4</sup> Particularly relevant for Europe, where fertility rates are low, and population is ageing.

may increase the labor supply of high-skilled native women and may reduce the trade-off between work and fertility among high skilled workers.<sup>5</sup>

Fertility decisions are also crucial in explaining the human capital accumulation. Parents are free to choose the number of children they want to raise. The ultimate aim for them is to provide a necessary education, which will provide a sufficient economic condition to their children during the life time. Investing in education of the same proportion with a high fertility rate is understandably more costly than investing with a low fertility rate. In general, much of the relevant studies raise the following question: How does migration affect the fertility decisions of parents, and thus, the human capital accumulation in source and receiving countries via fertility? Chen (2006, 2009) show that change in the migration probability leads to a trade-off between the quality and the quantity of children. As a result of this trade-off, not only fertility and education decisions but also economic growths of source countries are affected. Marchiori *et al.* (2008) and Zak *et al.* (2002) also analyze the impact of migration on the dynamics of the distribution of human capital via fertility. As recent studies Azarnert (2010b, 2011) and Mountford and Rapoport (2011) are the influential ones for the theoretical analysis.

After having looked at the literature, we now conclude this section by underlining that the results are responsive to the model assumptions and the channels mentioned in the studies. The following two sections; section three and four discuss the studies with regard to home and host country analyses, respectively.

### **3. Human Capital Accumulation in Home Country Analyses**

Home country -or origin country- analyses focus on the economic effects of migration on migrant sending countries. It is important to highlight Regets (2001)'s statement here that "receiving" and "sending" are not meant be equal to "developed" and "less developed". Today retaining human capital has become a challenging issue for many developed countries as a result of global competition. Many less developed countries are also successful to attract foreign talents in areas where they are able to offer opportunities. Indeed, many countries may be both net receivers and net senders in different skill areas. However in the migration literature migrant sending countries are often defined as small economies and the wage efficiency unit of labor in these countries is always lower than that in the world economies. Due to the smallness of their economies, they are reasonably open to push and pull factors, which lead people to leave their countries.

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<sup>5</sup> For the literature, see Furtado and Hock (2010).



Talking about the effects of emigration on human capital accumulation at any research or policy debate, the attention tends to focus on two major topics: “brain drain and brain gain”. The brain drain issue is not a new phenomenon in the literature, and studies date back to late 60s such as Grubel and Scott (1966), Johnson (1967). The primary emphasis on the importance of skilled human capital for economic growth has led Miyagiwa (1991), Burda and Wyplosz (1992), Haque and Kim (1995) and Galor and Tsiddon (1997) to presume that a brain drain has detrimental effects on economic development of low income countries. However, with the late 90s the migration literature including Mountford (1997), Stark *et al.* (1997, 1998), Vidal (1998) and Stark and Wang (2002) has evolved to beneficial brain drain economics for home countries.<sup>6</sup>

**Table 1. Brain Drain vs. Brain Gain: Historical Evolution of Brain Drain Studies**

Classification of Brain Drain Literature	Relevant References
Classical Brain Drain Studies [late 60s – late 90s]	Grubel and Scott (1966), Johnson (1967), Miyagiwa (1991), Burda and Wyplosz (1992), Haque and Kim (1995), Galor and Tsiddon (1997)
Beneficial Brain Drain Studies [late 90s – 2000s]	Mountford (1997), Stark <i>et al.</i> (1997, 1998), Vidal (1998), Stark and Wang (2002), Chen (2006, 2009), Marchiori <i>et al.</i> (2008) Sorger <i>et al.</i> (2013), Stark and Dorn (2013)
Empirical Studies	Beine <i>et al.</i> (2001, 2008), Maria and Lazarova (2011)

As a consequence of the absence of reliable data on international migration by educational attainment, the discussions on brain drain have long remained purely theoretical in the literature. Over the last decade, a few empirical analyses such as Maria and Lazarova (2011), Beine *et al.* (2001, 2008) have attempted to test these theoretical predictions of new brain drain economics.

### 3.1 The Main Channels

#### 3.1.1 Educational Incentives under Probabilistic Migration Models

Using overlapping generations (OLG) models Mountford (1997), Stark *et al.* (1997, 1998), Vidal (1998) has theorized the mechanism of how emigration affects the human capital formation in migrant sending countries. The main assumptions are as the following: Education decisions are endogenous, human capital is acquired instantly in home country, and wages are independent from migration. The later implies that the effects of migration on labor market outcomes are neglected in explaining the human capital formation mechanism in those countries. This approach depends on

<sup>6</sup> For an extensive overview about emigration, brain drain and development issue; see Docquier and Rapoport (2012), Clemens (2011), and Gibson and McKenzie (2011).

workers' possibilities of enjoying higher wages from rich countries through migration. In short, higher returns to skill in a foreign country increase the migration prospects and incentives to invest in education at home. If we reconsider the overlapping generation frameworks of Stark *et al.* (1997, 1998) the saving option has not been placed in their settings. Savings may lead to weaken the decisions of making additional investment in education. Latest study, Stark and Dorn (2013) therefore examine whether saving crowds out the effect of the prospect of migration on human capital formation. The authors show that even agents have the option to save the possibility of migration raises the human capital acquired by workers in developing countries.

Mountford (1997) shows that when the productivity of home country economy is an increasing function of the proportion of educated people in the previous time, and the probability of successful emigration is not a certainty ( $0 < p < 1$ ), general emigration increases the human capital accumulation, and is unambiguously good for home countries. Additionally, another point is also emphasized by Mountford: As a result of multiple steady-state levels of human capital accumulation, as long as emigration is permitted for agents who have a certain level of ability, brain drain migration increases the amount of educated agents higher than the general emigration does. This finding shows that a source economy can benefit from a brain drain under certain conditions (such as high wage differences among countries and low proportion of educated people in the previous period with a small prospect of emigration) which present a positive level of emigration; otherwise, a brain drain will have negative effects. Vidal (1998) also merely rests on the model of Mountford (1997). The author points out that the higher the probability of emigration the higher the level of human capital formation. But due to the presence of multiple steady-states, the opening the economy to labor emigration does not always imply a convergence to the highest possible level of human capital accumulation. In order to guarantee the convergence to upper level of human capital, the probability of emigration must be high enough. In the words of Stark and Wang: "*What migration can entail is that the gains from migration to the home country accrue neither from migrant's remittances nor from migrants' return home with amplified skills acquired abroad*" (2002, p. 30). The main idea of Stark and Wang (2002) is that higher prospective returns to human capital in a developed country stimulate more human capital formation in a developing country via migration, and therefore, migration can also be considered as a substitute for the provision of public subsidies as a means of bringing about the formation of a socially preferred level of human capital. Complementing Stark and Wang (2002), Sorger *et al.* (2013) show that human capital gain confers a long-run growth and a welfare gain in the home country beyond some threshold level of human capital.

Another fact is that the duration of stays of emigrant workers in a foreign country is either permanent or temporary. However, in the new brain drain literature it is a common assumption that emigration is given permanent. Stark *et al.* (1997) explain the human capital formation in home countries by highlighting this point and reproduce the framework by introducing the return migration as a new step which follows the phases of acquiring human capital in home country and emigration.<sup>7</sup> Unlike Mountford (1997), the authors consider a successful emigration case where the probabilistic migration rate is given, and unity throughout the model. Additionally, Stark *et al.* (1997) assume that the human capital acquired in home country cannot be equally efficient for all educated emigrant workers in host economies, and may not be fully distinguished by the employers due to the asymmetric information ( $m$ ). As a consequence of this asymmetry, relatively low-skilled workers may enjoy a high wages in pre-discovery period but a lower wage following discovery when the asymmetry is abated ( $0 < m \leq 1$ ). If this new condition (the probability of discovering true productivity levels of migrants) makes staying abroad costly rather than living in home country, the emigrant workers will find it convenient to return their home countries. Therefore it makes a contribution to the human capital accumulation in home countries through a return migration of relatively low skilled educated workers. Stark *et al.* (1997) figure out that the higher probability of perfect information ( $m$ ) in the second period, higher the permanent migration of high skill workers, but the opposite holds for low skilled workers.

People are not identical in abilities. Unless there is a unique education system in the home country, the cost of acquiring human capital for talent and less talent individuals differ. We can say that the cost of education is decreasing in personal ability, and the correlation between the probability of emigration and the average level of human capital is positive. However, in general the probability of emigration is given exogenous in theoretical models. Individuals with a low average level of human capital should require a lower probability than those with a high average level of human capital. Vidal (1998) amends the basic model to endogenise the probability of emigration:

$$p(h_i) = \begin{cases} p^* & \text{if } h_i < h^\# \\ \bar{p} > p^* & \text{if } h_i \geq h^\# \end{cases}$$

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<sup>7</sup> Dustman *et al.* (2011) highlight that efficiency considerations, which has been largely overlooked in the literature, determine who leaves and who returns. Emigration and return migration decisions mostly depend on the extent of transferability of working and schooling experiences acquired in one country to another one. Therefore, wage differentials are not only motive behind the emigration decisions. A person may move to a country to acquire high-skills, and then may come back if the return to human capital is high in the origin country.

In Vidal's demonstration the probability of emigration is a function of human capital level that the presence of a threshold externality ( $h^\#$ ) affects the convergence patterns. It is interesting to note that even though emigration is a basic human right which is established by the United Nations Universal Declaration of Human Rights, is often restricted by the governments, and is regulated by the national laws of destination countries. Magris and Russo (2009) also define an endogenous probability by introducing the weight of immigration policies ( $\psi$ ) on the entries of emigrants to destination countries.

$$p(\pi(h), \psi) = \psi\pi(h) + (1-\psi) \quad \psi \in [0,1]$$

The crucial thing under Magris and Russo (2009)'s framework is that selective immigration policies either foster or harm the equilibrium level of human capital under the case of restricted entry. In their analysis, they obtain some conditions that make returns to education uncertain, and they argue that selective policies may harm incentives to invest in human capital. For instance the analysis by Bertoli *et al.* (2014) reveals that there is an optimal degree of selectivity in immigration policies when migrants are positively selected on unobservable abilities, and that further increases in selectivity have detrimental effects on migrants' quality.

To the best of our knowledge, Beine *et al.* (2001) is the first attempt providing an empirical validation of the conditions of a positive brain drain in home countries. They test the direct effect of emigration on education and its indirect effect on growth by using cross-section data including 37 developing countries. The results show that the possibility of brain gain in theoretical analyses cannot be rejected on the base of available data.<sup>8</sup> Following this study, with a new data set for 127 developing countries, the authors find an evidence of a positive effect of skilled migration prospects on gross human capital levels at pre-migration period in home countries. The empirical evidence suggest that countries combining relatively low levels of human capital and low rates for skilled emigration are more likely to experience a beneficial brain drain, however this hypothesis is supported only for a small group of countries in Beine *et al.* (2008). Beine *et al.* (2001, 2008) have theoretically figured out two fundamental results: First, there is a positive non-linear relation between migration opportunities and education. Second, the growth rate is positively linked to share of educated people, and is negatively affected by migration of these educated agents.

The question of why the brain gain has been observed in a small number of countries draws Maria and Stryszowski (2009)'s attention. They focus on the existence of winners and losers among

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<sup>8</sup> Beine *et al.* (2008) reemphasize that in the absence of reliable data on international migration by educational level, the discussions on the impacts of brain drain for home countries have long remained purely theoretical in the literature. For empirical estimations, the following sources can be used: Beine *et al.* (2007), Docquier and Marfouk (2006).

sending countries. If we look at the brain gain literature, the common point of the previous studies is the assumption of an equal benefit from all human capital in fostering the productivity growth in home countries. However, Maria and Stryszowski (2009) underline that the improvement in productivity relies on the composition of human capital in each country, and migration may distort the agents' incentives to accumulate the most appropriate skills, and therefore changes the composition of human capital in origin countries.<sup>9</sup> The authors argue that in early stages of development, specialization in technical skills may be helpful, but at later stages (such as innovative phases of development), a broader range of skills such as financial, managerial, legal and political have important contribution on productivity. Skill allocation is crucial for developing world and is influenced by the prospects of migration.<sup>10</sup> The gap between technological frontier and actual level of productivity is an important determinant for the gain in those countries under migration. As a consequence of the change in the composition of human capital, educational incentives may have detrimental effects on the growth of origin countries. In other words, the more gap between technological frontier and actual level the stronger the negative effects of migration on growth. For instance, Maria and Lazarova (2011) explicitly reveal that the level effect of migration on the stock of human capital and the composition effect on skills that agents choose to acquire (general or technical) depend on technological development of sending countries. Maria and Lazarova concludes this evidence: “...*Differences in wages and the degree of marketability of migrants' skills depend on the level of technological development, thus the effect of migration needs to be discussed taking explicitly into account the technological gap of each sending country*” (2011, p. 952). This finding is in line with the theoretical setting of Maria and Stryszowski (2009).

### **3.1.2 Endogenous Fertility and Income Differences**

As it is observed, in probabilistic migration models have tended to assume a constant population growth -in every time period a new generation grows at a constant rate- and trade-offs between human capital accumulation (quality) and fertility (quantity) is neglected. However, in fact there is a quality quantity trade-off faced by parents. Investing in education of the same proportion with a high fertility rate is more costly than investing with a low fertility rate. It can be said that as long as the cost of investment in education increases, fertility decisions tend to decrease because of

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<sup>9</sup> For a detailed analysis on the importance of allocation of human capital for economic growth please see Murphy, Shleifer and Wishty (1991). The authors argue that talents in an economy are allocated between rent seeking sectors (unproductive) and entrepreneurial (productive) sectors; this implies that both sectors have different effects in explaining the growth performance of countries.

<sup>10</sup> For a theoretical explanation of allocation of talents under the possibility of emigration, please see Mariani (2007).

the increasing schooling expenditures of the parents.<sup>11</sup> In contrast to previous literature, Chen (2006, 2009) and Marchiori *et al.* (2008) allow the fertility decisions in order to see how a change in the migration probability affects fertility and human capital accumulation. To the best of our knowledge, Chen (2006) is the first study which attempts to examine the growth differences of developing countries under OLG and dynamic models of migration with different type of education regimes and endogenous fertility. It is important to note here that, Chen in his both studies follows several approaches, such as discrete time OLG, stochastic partial equilibrium models and numeric simulations. By using OLG models, Chen (2006) shows that with endogenous fertility decisions, an increase in the probability of migration reduces the population growth; however, contrary to the expectation, fertility and school expenditures are independent from the migration probability. In the later study, Chen (2009) develops his model's constraints by including the average costs in involved in preparing each migrating child to overcome the migration barriers (such as language skill preparations, paper works and so on) and figures out that both educational investment and fertility are dependent upon parental human capital, the extent of migration costs and the probability of migration. The result is as the following: An increase in the migration costs and an increase in the probability of migration increase the educational investment while fertility reduces.

Chen (2006)'s model and numerical simulation figure out three important results: First, in an economy with homogenous agents (with the same probability of migration for all agents), an increase in  $p$  leads to a trade-off between quality and quantity. As a result of this trade-off, fertility declines, school expenditures increase<sup>12</sup>, and thus, human capital accumulation is positively affected under both education regimes. Second, a brain gain can happen if there exists at least one of the following three conditions: (i) When the wage ratio is quite large between home and host countries or (ii) when parents support children to migrate or (iii) when  $p$  is sufficiently strong. Otherwise, an increase in  $p$  causes a brain drain in which the converse conditions hold. Relaxation of restrictions on high-skilled emigration contributes the economic growth unless the labor market will be dominated by low-skilled workers in the long run. On the other hand allowing more low-skilled workers to emigrate can have positive results on growth if probability of low-skilled migration is sufficiently high in that economy. Chen (2009) also reemphasizes the same results.

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<sup>11</sup> Chen (2006) demonstrates that education regimes also affect the fertility rates in home countries. When education is under public regimes, the cost of having children is lower; therefore, fertility is higher under a public education regime than under a private education regime.

<sup>12</sup> Chen explains this mechanism as the following: Having fewer children while induces parents to make more investment for each child's education under private schooling, under public schooling it provides more time for adults to work. This means a high amount of tax revenue for governments at the same time. Therefore, school expenditure per student will increase due to an increase in tax revenues as well.

The study of Marchiori *et al.* (2008) differs from Chen (2006, 2009) by incorporating migrants' remittances into the analyses. They assume that only high-skilled individuals can emigrate with a certain probability and remit. The remittances are considered as a kind of welfare improving element. Both skilled emigration and the prospect of remittances have effects on the household's fertility and education choices in the home countries. Their main result is based on the following condition: Since the marginal gain of educating one child is higher than the marginal cost of education parents will raise more children. This implies that migrants' remittances play a significant role in explaining the human capital accumulation in home countries. Receiving more remittances outweighs the increased education expenditures and contributes the human capital accumulation. Marchiori *et al.* (2008) figures out two distinct effects of migration on the number of children. On the one hand the more probability the more remittances which leads to an increase in the number of high educated children, on the other hand the more probability the more educational cost that reduces the number of children. Therefore the effect on the number of children is ambiguous. General differences in approaches in the literature are summarized in *Table 2, p. 18*.

It is important to note that Azarnert (2011) distinguishes itself from the literature by focusing only emigration of individuals from the middle class of the wealth distribution in source countries. As the previous studies point out that the motivation for individuals to generate incentives for investment in human capital is the wage differences between foreign and source countries but, contrary to the standard approach Azarnert shows that, the possibility of migration to a higher wage foreign country may lower the relative attractiveness of the skilled employment in the home countries. It means that low-skilled agents of home country may prefer to work in relatively low-skilled sectors in foreign countries and they may give up their educational investments for high-skilled employments in home countries.<sup>13</sup>

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<sup>13</sup> Azarnert (2011) takes the guest-worker migration as a reference point for this study, and assumes that unskilled wages in foreign country are also higher than the unskilled wages in the home country.

**Table 2. Migration induced Human Capital Accumulation effects in Home Country Analyses**

References	Methods of Analyses <sup>1</sup>	Duration of Migration <sup>2</sup>	Population Growth <sup>3</sup>	Individual Ability <sup>4</sup>	Probability of Migration <sup>5</sup>	Fertility <sup>6</sup>	HCA Effects via <sup>7</sup>	Economic Growth Effect via HCA <sup>8</sup>	Other Variables
Positive Effects on Human Capital Accumulation (HCA)	Mountford (1997)	T	P	C	HT	X	-	H	P
	Stark <i>et al.</i> (1997)	T	P,R	C	HT	X	-	H	-
	Stark <i>et al.</i> (1998)	T	P	C	HM	X	-	H	-
	Vidal (1998)	T	P	C	HM	E	-	H	-
	Beine <i>et al.</i> (2001)	T, E	P	C	HT	X	-	H	P, N
	Stark and Wang (2002)	T	P	C	HM, HT	X	-	H	-
	Dos Santos and Postel-Vinay (2003)	T, N	P, R	C	HM	X	-	H	P
	Chen (2006)	T, N	P	F	HM, HT	X	E	F	P, N
	Beine <i>et al.</i> (2008)	E, N	P	-	HM	-	-	H	-
	Marchiori <i>et al.</i> (2008)	T, N	P	F	HT	X	E	F, WI	-
	Chen (2009)	T, N	P	F	HM, HT	X	E	H, F	P, N
	Dustman <i>et al.</i> (2011)	T	R	C	HT	-	-	H, WI	-
Mountford and Rapoport (2011)	T	P	F	HM	X	E	H, F	P	
Sorger <i>et al.</i> (2013)	T	P	C	HM, HT	X	-	H	P	
Stark and Dorn (2013)	T	P	C	HM	X	-	H	-	
									Imperfect information
									The role of public subsidies on HCA.
									Positive effect of knowledge diffusion.
									Different education systems are considered.
									The role of remittances on HCA and Fertility.
									Equilibrium with migration costs.
									Migration decisions depend on efficiency considerations.
									With a possibility of saving option
Detrimental Effects on HCA	Maria and Stryszowski (2009)	T	P	C	HT	X	-	H	N
	Magris and Russo (2009)	T, N	P, R	C	HM, HT	E, X	-	H	-
	Maria and Lazarova (2011)	T, E	P	-	HM	-	-	H	N
	Azarnert (2011)*	T	R	F	HM	-	E	H	N
									The role of the composition of human capital and technological gap.
									Selective policies may cause a brain loss.
									Growth effect depends on the level of technological development (gap) of the sending country.
									*Low-skilled emigration

<sup>1</sup> Methods of Analyses : T = Theoretical; E = Empirical; N = Numerical Simulations.

<sup>2</sup> Duration of Migration : P = Permanent; R = Return Migration

<sup>3</sup> Population Growth : C = Constant growth; F = Fertility induced Population Growth;

<sup>4</sup> Individual Ability : HM = Homogeneous; HT= Heterogeneous Agents

<sup>5</sup> Probability of Migration : E = Endogenous; X= Exogenous

<sup>6</sup> Fertility : E= Endogenous; X= Exogenous

<sup>7</sup> HCA Effects via : H= Higher Expected Returns; F= Fertility Effect; WI= Wage/ Income Effect

<sup>8</sup> Growth Effect : P= Positive Effect; N= Negative Effect



#### 4. Host Country Analyses

The host or destination country analyses focus on the economic implications of migration in receiving countries. As previous section reveals that home country analyses assume sufficiently small migration flows that do not affect the wages both in receiving and in home countries. However, in contrast to these analyses, migration literature has mainly discussed the uncertainties with regard to the labor market outcomes of natives in destination countries under labor migration. For example, Dustmann and Preston (2006) and Borjas (2007, 2009) generally emphasize the reductions in economic opportunities of natives.

The main issue is therefore to understand to what extent the skills are interchangeable and transferable among natives and immigrants. In other words, substitutability within occupational skill levels, and complementarities between natives and immigrants have become a long standing concern of economists. Regarding this discussion Regets (2001) underlines the asymmetries in arguments for the sake of simplicities. Regets explains his statement in the context of static supply and demand equilibrium with the following example: *“If high-skilled immigrants are substitute for natives in the domestic labor market, it would lead a reduction in the human capital investment return for natives (wages) due to the increase in the supply of higher-skills in the domestic labor market. Then, return of human capital would reduce the incentives for natives in human capital. If we take the lower-skilled immigrants as substitutes for lower-skilled natives and as complements the higher-skilled natives, this would increase the inequality between higher-skilled and lower-skilled workers. However, if we consider both propositions at the same time, the following argument would be also possible: While lower-skilled migrants would increase the incentives for natives to invest in human capital, the high-skilled immigrants reduce the inequality”* (2001, p. 15). The literature on host country analyses shows that human capital level of natives is generally affected by two main groups of factors which can be called as internal and external factors. Internal factors which are composed of political and economic conditions (such as physical and human capital stocks, the distribution of human capital and fertility choices, technological environments) are important in defining the education decisions of natives. It is clear that immigration belongs to external factors that also have effects on these internal factors. As Chiswick (1989) states that the implications of immigration depends on the immigrants’ size and quality.

In general, much of the relevant studies ask the following question: How and in which forms do immigrants affect the incentives for investment of natives? Chander and Thangavelu (2004) emphasize the importance of attracting high skilled migrants for educational attainment of natives as natural consequences of interaction between education and immigration policies, and

complementarities between technology and education. Authors demonstrate that higher level of investment in education increases returns from the new technology relative to the old, and entrepreneurs may indeed adopt the new technology when workers invest more in education. Conversely, low investment in education may indeed lead entrepreneurs to decide not to switch to the new technology. From a welfare point of view, governments can coordinate the expectations of workers and entrepreneurs' decisions on adoption of the new technologies. If governments commit themselves to attract high-skilled immigrant workers then natives will also increase their investment in education and the economy will move from low technology equilibrium to welfare improving technology.<sup>14</sup>

Bellettini and Ceroni (2003) show that instead of increasing the barriers, reducing the restrictions to foreign workers may be beneficial to generate positive spillover effects on natives' incentives to invest in education. Their OLG dynamic model reveals that opening up the borders to immigrants brings an upward shift in the steady-state fraction of skilled natives when the host economy succeeds in attracting the skilled foreign agents sufficiently. Below a threshold level of the immigration quota, the steady-state fraction of skilled natives is lower than in the closed economy. In order to characterize the migration incentives of skilled and unskilled agents Bellettini and Ceroni define some threshold values of unit wages for domestic labor market. Under certain assumptions<sup>15</sup> they show that a minimum wage, which is compatible with immigration, stimulates only skilled immigrants to enter into domestic labor markets. This leads a direct brain gain which is determined by the level of restrictions in host countries, and by the average human capital level of immigrants. The reason of an increase in educational incentives for natives in Bellettini and Ceroni (2003) relies on the reduction in individual costs of acquiring education through an increase in average level of human capital by skilled migration. While skilled immigrants increase the average level of human capital in host countries, at the meantime they reduce the threshold levels of ability that makes the schooling decisions preferable by natives.<sup>16</sup> However, some studies argue that immigration policies may not be as effective as expected. In a former study, (Van Dalen, 1993) receiving countries are advised to train their own labor force instead of introducing selective immigration policies due to widening of the divergence in dynamic efficiency in capital accumulation.

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<sup>14</sup> Like Maria and Stryzowski (2009), Chander and Thangavelu (2004) also use two forms of human capital such as "general" and "technology-specific".

<sup>15</sup> Immigration is temporary and costly. In particular, subjective costs are independent of skills of foreign agents. Total costs of migration are higher for the unskilled individuals, and a perfect substitutability of skilled and unskilled labor exists in production.

<sup>16</sup> Bellettini and Ceroni (2003) also define the threshold levels of abilities for skilled and unskilled parents' children in order to clarify the education decisions of parents.

The study of Zak *et al.* (2002) is one of the seminal works which analyzes the impact of immigration on the dynamics of the distribution of human capital in receiving countries when fertility and migration decisions are endogenous. In an open-economy endogenous growth model with heterogeneity in skills, they show that fertility and immigration can cause an economy to be caught in a poverty trap, but under specified conditions may also lead to a balanced growth. In Zak *et al.* (2002), the depreciation of human capital, the cost of emigration and the preferences for children have significant effects in explaining the impacts of immigration on human capital accumulation of natives. The authors argue that when immigrants and natives are identical<sup>17</sup>, economies with strong political capacity and sufficient initial stocks of physical and human capital are attracted to the balanced growth path (BGP), and a constant level of immigration has no effect on the dynamics of balanced growth but affects the rate of convergence to the BGP. On the other hand, their model analytically shows that as long as the immigrants' human capital less than the native workers, it leads a negative impact on growth because of the higher fertility rate of immigrants that reduces the rate of accumulation of human capital. Zak *et al.* (2002) reveals that if human capital depreciates upon emigration and the preference for children is higher than the destination country, immigrants save less and have more children than natives, therefore; this leads an inequality reducing growth via fertility.

As we know that, agents want to maximize their utilities in their life times and they do make their migration decisions by taking the expected benefits of migration into consideration under certain constraints such as budget, education and migration costs. Indeed in Zak *et al.* (2002), the moving cost has an important role in explaining and structuring the the model however, it is given exogenously. Like Zak *et al.* (2002), Cipriani (2006) also analyzes the effects of immigration on growth where the source of growth is human capital accumulation under endogenous fertility. Cipriani (2006)'s work distinguishes itself from the rest of the analyses by taking the various type of migration (skilled-unskilled, temporary-permanent) into account. Contrary to Zak *et al.* (2002), Cipriani assumes that migration cost is a proportion of the level of human capital and inversely related to immigrants' ability. This endogeneity implies that the more skilled agents can adjust to the new environment with less cost.

Mountford and Rapoport (2011) show the linkage between equilibrium level of ability and human capital accumulation in a dynamic two country model of the world economy where agents in both countries make optimal fertility and human capital decisions. Like Maria and Stryszowski

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<sup>17</sup> Zak *et al.* (2002) assume here a benchmark model which considers a case when there is no immigration cost. Natives and immigrants have the same level of human capital and human capital does not depreciate when moving from a country to another.

(2009) and Maria and Lazarova (2011), Mountford and Rapoport (2011) also argue that an increase in growth of frontier technology in a receiving economy with no migration increases the equilibrium level of human capital via decreases in the threshold level of ability for natives to become an educated person, and decreases the fertility rates. By opening the economy to the skilled immigrants, their result shows two distinct effects, namely static and dynamic effect. The first defines an increase in fertility and a reduction in the proportion of native agents becoming skilled because of an increase in the threshold level of ability and a decrease in the equilibrium of skilled workers. The second effect shows that even a permanent immigration of a small amount of skilled foreign workers leads an increase in the equilibrium growth rate of economy because of an increase in the equilibrium level of human capital, and therefore; a positive effect on human capital accumulation and a negative effect on fertility occur in receiving country. The net effect of skilled immigration on human capital accumulation depends on which effect outweighs the other in the long run.

On the other side, Azarnert (2010a) also mentions immigration induced adverse effects on educational incentives for native population. He shows that the arrival of a sizable mass of potential foreign workers is likely to increase the perceived probability of a possible occupational mismatch for skilled native individuals. This reduces the returns to human capital investment for natives. Again in another study Azarnert (2010b) shows that income redistribution from low-fertility natives to high-fertility immigrants is one of the reasons for opposition to immigration in host countries. Azarnert's growth model with endogenous fertility shows that while tax-financed income transfers lower the fertility rate of skilled natives increases the fertility of unskilled immigrants with the result of a decline in human capital accumulation among natives. Addition to Azarnert (2010a, 2010b), Borjas (2007, 2009) emphasizes different point of the discussion. He indicates that skilled foreign students in advanced education also adversely affect the educational attainments of natives in those areas, and crowd out them from several high-skilled fields in the US.

In short, the review shows that different approaches lead to differences in results across studies. The following section explains the differences in findings with a summary of the implications of migration on economic growth via human capital accumulation.

## **5. How do results affect economic growth?**

Until now we have categorized the main channels identifying the interactions between migration and human capital accumulation in home and host country analyses. In this section, we will sum up the main factors - such as assumptions, parameters, variables or analytical frameworks

- that have critical roles in explaining the differences in results. Listed below, accompanied by a synopsis of supporting explanations, are several points that determine the results in the abovementioned studies.

*1. An optimized emigration policy can play an important role in increasing the growth rate of the home country.* It is crucial to reemphasize here that the human capital level of a country is one of the determinants of the productivity level of an economy; thus, the impact of migration on economic growth can merely be investigated via its impact on human capital accumulation. In this review, we have figured out that many theoretical models use OLG framework in explaining the optimality conditions of individuals, and it is more likely to see multiple steady-state levels of human capital accumulation. The possibility of multiple steady-state equilibrium is one of the interesting features of OLG approaches that only high-skilled emigration guarantees the high education levels under certain conditions.

*2. Uncertainty and different migration probabilities are the key assumptions that determine the impacts of migration on human capital accumulation.* In the absence of restriction on labor mobility ( $p = 1$ ), both general and high-skilled migrations affect human capital formation negatively. Without taking the return migration into account, the intuition behind the uncertainty is quite obvious. Otherwise, every individual who invest in education leaves their country due to the certain employment opportunities in host countries. As a result of this, an economy with differentiated abilities arises the necessity of using endogenous probability of migration in order to pursue more realistic results in economic models.

*3. Trade-off between quality and quantity of children matters under changes in migration probabilities since the economic growth depends on the human capital.* The studies show that fertility in general decreases with education, and an increase in investment in human capital also leads an increase in the probability of emigration. Thus, the fertility differentials between natives and immigrants matters because of the effect on the average level of human capital at both sending and receiving countries.

*4. The distance from technological frontier plays a major role in determining the effects of migration on growth via distribution of skill types.* Recent studies such as Maria and Stryszowski (2009) and Maria and Lazarova (2011) point out that the probability of migration has two main effects on human capital accumulation: “level effect” and “composition effect”. While the first one positively affects the shared of skilled workers in the home country, the second one affects the proportion of technical skills either positive or negative according to the development stage of a

country. If an economy is far from the technological frontier, the sign of the composition effects would be negative, and the level effect would be high. However migration induced changes may distort the agents' incentives to accumulate the most appropriate skills for the country of origin. This implies that although there is an increase in human capital accumulation in home country, the composition of the workforce induced by the probability of migration may lead to detrimental effects on growth.

## **6. Conclusion**

The main objective of this paper is to show “how international migration affects human capital accumulation in both the sending and receiving countries”. This paper is mainly based on two strands of literature. First, it is related to the literature on international migration including the papers that analyzing the impacts of migration on economic growth and second, it is related to the papers that analyze the human capital accumulation under endogenous growth models. This review shows that the relationship between migration and human capital accumulation requires a multi-dimensional perspective including many aspects and types of migration. The present paper reveals that migration affects the human capital accumulation via its effects on three important factors: (i) Skill formation, (ii) fertility decisions and (iii) wage (income) levels. This review has showed that changes in these factors drive many of the differences in results across studies. Moreover, this paper point outs that the ambiguity in results mainly depends on the modeling structure and the assumptions.

This review also identifies the knowledge gaps in this field by assessing the existing studies under the light of evidential facts. As one can observe that under certain conditions, the brain drain migration theoretically can lead to positive effects for home countries, and can dominate its negative effects. Even though the absence of reliable data on high-skilled migration, a limited number of empirical studies supports the theoretical results of the beneficial brain drain analyses. This review reveals that further empirical research (e.g. panel data analyses over longer time periods) is strongly needed to verify current theoretical results.

This paper demonstrates that the interaction between education and immigration policies may also be considered as a new research topic in policy analyses on the sustainability of economic growth. For instance, the overlapping generation models have showed that optimal migration policies can succeed in increasing the steady-state level of human capital in both the sending and receiving countries. At the policy level, this result suggests that migration of high skilled agents can

also be considered as a policy instrument which may substitute the income related public subsidies in home and host countries.

Lastly, this review concludes that the impacts of permanent migration on human capital accumulation and growth have been widely studied in the literature. However, further attention to studies on temporary migration is needed. From a methodological point of view, the impact of the return migration on human capital accumulation, and thus growth would be an interesting extension for future empirical studies. Furthermore, like permanent migration, taking only skilled migration into considerations may not always reflect what is likely to happen in receiving countries at the present time. Analyses allowing and combining all types of migration (skilled-unskilled, permanent-temporary) may provide a robust estimation on the impacts of increased heterogeneity in populations.

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## CHAPTER 2

### HUMAN CAPITAL ACCUMULATION UNDER SKILL-BIASED TECHNOLOGICAL CHANGE WITH MIGRATION

***Abstract:** The impact of migration on human capital accumulation has become one of the most important aspects of economic growth. By introducing the human capital accumulation and migration to an Acemoglu-type growth model this paper provides a complementary dimension to the directed technological change literature. The motivation behind this paper is the lack of economic analyses on how skill-biased technological change (SBTC) affects the human capital accumulation in both sending and receiving countries; therefore the paper attempts to answer a new question: How does a SBTC affect human capital accumulation in developed and less developed countries undergoing skill-biased migration? The main results show that, while a SBTC in a technologically advanced country has positive effects on human capital accumulation in technologically less developed countries via migration, it may adversely affect the incentives of local (natives) low skilled individuals to invest in education at advanced countries. Therefore, the net effect of SBTC is ambiguous (either positive or negative or neutral) in developed countries under migration.*

## 1. Introduction

This paper builds upon the skill-biased technological change (hereafter SBTC) literature developed by Acemoglu (2002) to provide a “complementary dimension” between the studies of human capital accumulation and migration by using a theoretical framework. The motivation behind this work is the lack of economic analyses on how SBTC affects human capital accumulation in both migrant sending and receiving countries in an Acemoglu-type growth model.

Factor endowments -as a force shaping technological change- play an essential role in the economic development. The scarcity of high qualified persons, inappropriate domestic policies and mismatches between technologies and resources increase productivity differences between the rich and poor countries.<sup>1</sup> Lack of well functioning markets, low employment rates, increasing costs of technological adoption and insufficient investment in R&D are some factors that push the human capital of less developed countries to search for jobs in advanced economies. As Acemoglu (1998, 2002) shows, technological change favors high skilled labor, and increases the labor demand. Such an increase in demand for skilled labor which is mostly driven by SBTC leads to an increase in skill-specific migration (brain drain) from less developed countries to technologically advanced countries as well.

Acemoglu (1998) emphasizes that profit incentives determine the amount of research and development directed towards different factors and sectors. The main determinants for these incentives are relative prices and market sizes. Similarly for human capital, wage incentives determine the level of investment in education for acquiring appropriate skills before entering the labor market. Beine *et al.* (2001) have addressed this issue and show that increasing the possibility to work abroad increases the stock of human capital in the source countries. Therefore, we can infer that, the underlying reason for the migration of skilled labors is the probability of getting high wages accompanied by a well functioning technological environment.<sup>2</sup>

This paper aims to bring a new perspective to the economic literature by asking the following question: How does an SBTC affect human capital formation in less developed and developed countries under skill-biased migration? One important novelty of the present paper is that this study introduces migration to the Acemoglu (2002) model of skill-biased technological change to see how this affects the predictions on relative wages and human capital accumulation in both sending and receiving countries. Consequently, this paper makes a contribution to the literature by

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<sup>1</sup> For a detailed analysis on the reasons for productivity differences across countries, see Gancia and Zilibotti (2009).

<sup>2</sup> For an empirical study, see Murakami (2009).

extending the limited discussions on human capital accumulation in endogenous growth theory including migration dimension.

The remaining part of the paper is organized as follows. The next section reviews the existing literature and presents the main frameworks used in the paper. Section 3 details the benchmark model by highlighting the skill-biased migration decision and optimal education decision of parents. Section 4 presents the main results of the impact of technological change on human capital accumulation under certain conditions in both migrant sending and receiving countries. Finally, section 5 concludes the paper.

## **2. The Literature Review**

International migration not only changes the population size but also affects the human capital stock of countries. Besides resource allocation, migration can be considered as a movement of knowledge, abilities, talents and productivity. In the economic literature, human capital accumulation plays an important role as one of the pivotal engines of growth. The concepts of human capital and migration are, therefore, interconnected issues making it essential to understand the economic consequences of migration on human capital accumulation in both host and home countries.

Public discussions do not solely focus on illegal migration based economic issues, but also express doubts about economic contribution of legal migration. For example, the question of whether skilled migrants affect the returns to human capital investments of natives adversely or not has been started to address in the economic literature in recent years. Azarnert (2010) reveals the underlying reason of this question by showing two opposite growth effects of skilled immigration that can be either growth enhancing or growth depleting. If skilled immigration has an adverse effect on educational incentives for natives, this effect reduces the positive contribution of the foreign brains to the receiving country's human capital stock, and thus, hampers economic growth of that country. Azarnert emphasizes this negative relation by underlining the fact that the size of skilled immigration flows is not small. The numbers for international migration increased from 74 million to 188 million between 1960 and 2010, which refers a slightly increase, from 2.7% to 2.8% with respect to the share of world population. However, the figures change when only high skilled migration is taken into consideration. As a result of globalization, technological changes, and selective immigration policies the type of world migration has been evolving to skilled migration. The number of highly skilled immigrants living in the OECD countries increased by 70% during the

1990s while low skilled immigrants increased by 30%.<sup>3</sup> Another factor is mentioned that there exists an uncertainty with regard to the labor market opportunities for the native population (occupational mismatch for skilled natives) as a result of the arrival of the foreign competitors.

Earlier studies such as Regets (2001), Bellettini and Ceroni (2002), Borjas (2007, 2009) also provide different results about the impacts of migration on educational incentives in receiving countries at micro and macro level. According to Regets (2001) a positive correlation exists between increased enrolments in graduate programs by foreign and by native students in the US. Bellettini and Ceroni (2002) argue that the opening up of borders to immigrants brings an upward shift in the steady-state fraction of skilled native workers when the host economy succeeds in attracting highly skilled foreign agents above a certain threshold. Otherwise the steady-state fraction of skilled natives would be unchanged. Unlike Regets (2001), Bellettini and Ceroni (2002), Borjas (2007, 2009) state that an exogenous increase in the supply of skilled labor due to the skilled migration may adversely affect the educational incentives of natives. In line with Azarnert (2010)'s occupational mismatch hypothesis, Borjas finds that skilled migrants crowd out the native labor in several high skilled fields in the US job market.

There are several studies that also examine the impacts of skilled migration on economic growth via the implications on human capital accumulation. Addition to receiving country analyses, Mountford (1997), Stark *et al.* (1997, 1998), Vidal (1998) and Beine *et al.* (2001, 2008) are influential studies showing that migration prospects can raise the expected return on human capital in home countries. The effects of skilled migration on economic growth are identified in two groups such as static effects and dynamic effects by Mountford and Rapoport (2011). While static effects imply an increase in fertility rates, and a reduction in the incentives to become a skilled worker; dynamic effects imply positive effects on the proportion of agents who choose to become skilled workers. The authors argue that if dynamic effects are sufficiently high than static effects, human capital accumulation and thus, growth in receiving country increase in the long run. For the case of sending country, the emigration of skilled workers might reduce the human capital stock; however, on the other hand, the possibility of emigration to a developed country may increase the incentive to accumulate human capital, and may decrease the fertility rate in the origin country. Therefore, whether brain drain decreases growth or not is an ambiguous statement for sending countries.

After having formed an opinion about the relationship between skilled migration and human capital accumulation, we can now formulate the theoretical proposition upon which we base the main argument in this paper. Acemoglu (2002) shows that biased technological change shifts out

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<sup>3</sup> For details see Gibson and McKenzie (2011) and Docquier and Rapoport (2012).



the relative demand curve for a factor (here skilled workers) so its relative marginal product (relative wages) increases at given factor proportions (given the relative quantity of factors). The key point is that technological change demands high skilled labor; and therefore, the return to education would be higher under skill-biased technology. Docquier (2007) emphasizes that the educational structure of international migration is getting more skill-biased as a result of drastic rises in educational attainment in developing countries accompanied by an increase in demographic sizes. As Beine *et al.* (2008) state, the choice of migration has been affected by skill-biased technology. It is important to mention here that in a recent study, Fadinger and Mayr (2012) – which will also play an important role in defining the migration decisions in our model – both empirically and theoretically show that depending on the elasticity of substitution between skilled and unskilled workers, an SBTC decreases the brain drain. In other words, what Fadinger and Mayr explicitly say that incentive to emigration (outflow) decreases under an SBTC. This finding is also supported by earlier Beine *et al.* (2008) and Mountford and Rapoport (2011). Both studies argue that higher technological growth in advanced economies increases the incentives for foreign agents to migrate to advanced economies. Individuals in less developed countries may invest in education to get high-skill jobs and high wages through an increase in their probabilities of migration. So this makes a contribution to the human capital accumulation in home countries. Additionally, the growth rate of technology in the advanced economy increases due to the raise in the accumulated stock of skilled labor, and this in turn increases the incentives for skilled agents to migrate to the advanced economy, and also increases the incentives for human capital in the advanced economy itself.

What Fadinger and Mayr (2012) demonstrate us is that SBTC leads a reduction in migration incentives of natives. From a different perspective it means that SBTC increases the migration incentives of foreigners. Throughout this paper we assume that SBTC is a pull factor for skilled foreigners and implicitly we accept that incentives for migration (inflow) increase to a country where an SBTC exists. However, we still need to point out how SBTC affects the human capital stock in both home and host countries. If we return to Acemoglu (2002), one can notice that human capital is exogenously given, and the international labor mobility is not taken into account in the model. On the other hand, we have a perfect knowledge of how the relative wage ratio and technological development increase the incentives for skilled individuals to move towards the advanced economies. This immigration might have either positive or negative effects on human capital formation in sending and receiving countries. Nevertheless, a theoretical illustration problem related to the current model appears. We need a demonstration for human capital accumulation in the Acemoglu model.

Most of the existing literature does not explicitly account for this issue, but Eggebrecht (2009) and Gancia and Zilibotti (2009). Only these two studies attempt to extend the Acemoglu model by identifying a mechanism which augments the model of directed technical change with educational investments. Eggebrecht (2009) develops a closed economy framework, and focuses on wage differences between skilled and unskilled workers. The author argues that expected wages are one of the crucial determinants of the current and future education decisions of individuals, and therefore, the impact of SBTC on human capital accumulation can be demonstrated via changes in wage ratios. The model of Gancia and Zilibotti (2009) differs from Eggebrecht (2009) in explaining the mechanism. Their argument is that human capital is a complementary factor in production; therefore, the extent to which it is involved in production depends on the degree of competition in markets. Firms, which are stimulated by rising monopoly power, seek high profits, and put pressure on wages; thus, the return to human capital.

To summarize, both empirical and theoretical findings raise the following questions: (i) In an Acemoglu-type of growth model including migration, what kind of changes can be examined in human capital accumulation in both sending and receiving countries? (ii) How can be these changes modeled in an open economy allowing skilled labor migration, and (iii) To what extent the optimal education decisions of parents are affected under SBTC with migration? The following section lays out the basic model and introduces the conditions of the economies to answer the aforementioned questions.

### **3. The Model**

This section presents the benchmark model, which is based on a simplified version of the Acemoglu-type growth model (1998, 2002) and in turn related to Fadinger and Mayr (2012) and also Gancia and Zilibotti (2009). The main assumption in this model is that human capital may also be accumulated through migration prospects. The argument underlying this assumption is based on the studies mentioned in section 2 and the report of the International Organization for Migration (IOM 2003) as well. This report accurately shows that *“prospects of working abroad have increased the expected return to additional years of education and led many people to invest in more schooling, especially in occupations in high demand overseas.”*

#### **3.1 Production and Technological Structure**

In Acemoglu model (1998, 2002), an economy is given with two different factors of production,  $L$  and  $H$ , corresponding to unskilled and skilled workers respectively. Output is categorized as

final and sectoral final outputs. The final output sector is perfectly competitive, and aggregate production function is given by a CES production function:

$$Y = \left[ Y_L^{\frac{\varepsilon-1}{\varepsilon}} + Y_H^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (1)$$

where  $Y_L$  and  $Y_H$  are sectoral final outputs which demonstrate  $L$  and  $H$  intensive productions, and  $\varepsilon$  is the elasticity of substitution between the factors and  $\varepsilon \in (0, \infty)$ . The product markets clearing condition gives the aggregate demand and the relative demand for sectoral aggregates:

$$P_H = \left[ \frac{Y}{Y_H} \right]^{\frac{1}{\varepsilon}} \quad (2)$$

$$P_L = \left[ \frac{Y}{Y_L} \right]^{\frac{1}{\varepsilon}} \quad (3)$$

$$\frac{P_H}{P_L} = \left[ \frac{Y_L}{Y_H} \right]^{\frac{1}{\varepsilon}} \quad (4)$$

The price of the final output is assumed as a numeraire, which implies

$$P = P_H^{1-\varepsilon} + P_L^{1-\varepsilon} = 1 \quad (5)$$

Sectoral final outputs are produced under perfect competition and given by the following production functions:

$$Y_L = \left[ \int_0^{A_L} y_L(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \quad (6)$$

$$Y_H = \left[ \int_0^{A_H} y_H(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \quad (7)$$

where  $L$  and  $H$  intensive outputs are produced by using a range of sector specific differentiated inputs (or “machines”)  $y_L(i)$  and  $y_H(i)$  with elasticity of substitution  $\sigma > 1$ . The range of intermediate inputs  $A_L$  and  $A_H$  (used with unskilled and high skilled labors) allow technical change to be biased. We assume that technology monopolist supplies these inputs by using skilled and unskilled labor during the production process.  $A_L$  and  $A_H$  are given exogenously now, but with the

discussion of skill bias of technology,  $A_L$  and  $A_H$  will be determined endogenously in the following subsection.

Since sectoral output markets are competitive, profit maximization functions of firms can be written as

$$\max_{L, \{y_L(i)\}} P_L Y_L - w_L L - \int_0^{A_L} p_L(i) y_L(i) d_i \quad (8)$$

$$\max_{H, \{y_H(i)\}} P_H Y_H - w_H H - \int_0^{A_H} p_H(i) y_H(i) d_i \quad (9)$$

where  $p_L$  and  $p_H$  are given and denote the rental prices of machines, and  $w_L$  and  $w_H$  are the wages of unskilled and skilled workers. From the first order condition we obtain the following inverse demand functions:

$$p_L(i) = y_L(i)^{-\frac{1}{\sigma}} Y_L^{\frac{1}{\sigma}} P_L \quad (10)$$

$$p_H(i) = y_H(i)^{-\frac{1}{\sigma}} Y_H^{\frac{1}{\sigma}} P_H \quad (11)$$

Gancia and Zilibotti (2009) and Fadinger and Mayr (2012) give the state of technology for the intermediate inputs as  $l(i)$  and  $Zh(i)$ .  $Z$  is a parameter which ensures the equilibrium skill premium is always positive ( $Z > 1$ ). Each intermediate input  $y_L(i)$  and  $y_H(i)$  is subject to the resource constraints  $\int_0^{A_L} y_L(i) d_i \leq L$  and  $\int_0^{A_H} y_H(i) d_i \leq H$ . Here we will follow the same demonstration<sup>4</sup> of Fadinger and Mayr (2012) to obtain an expression of wage premium. Later, we will see that the wage premium will play a crucial role to express the human capital accumulation under SBTC. In this respect, the production function for each input can be written as

$$y_L(i) = l(i) \quad (12)$$

$$y_H(i) = Zh(i) \quad (13)$$

Substituting (12) and (13) into the demand functions (10) and (11), we can define the revenue of technology monopolists as the following functions:

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<sup>4</sup> Fadinger and Mayr define the labor market clearing conditions by the following equities:  $\int_0^{A_L} l(i) di = L_E$  and  $\int_0^{A_H} h(i) di = H_E$  where  $H_E (L_E) \leq H (L)$  denotes the aggregate employment level of skilled (unskilled) workers in the labor market. Therefore,  $x_H = \frac{H_E}{H}$ ,  $x_L = \frac{L_E}{L}$  shows the employment rates measuring the labor market tightness which also express the probability of finding a job for a skilled (unskilled) individual in that labor market.

$$p_L(i)y_L(i) = l(i)^{\frac{\sigma-1}{\sigma}} Y_L^{\frac{1}{\sigma}} P_L \quad (14)$$

$$p_H(i)y_H(i) = Zh(i)^{\frac{\sigma-1}{\sigma}} Y_H^{\frac{1}{\sigma}} P_H \quad (15)$$

The monopolists tend to choose an optimal employment level that maximizes their profits. Fadinger and Mayr assume that the firms and workers get fractions  $\sigma/(2\sigma-1)$  and  $(\sigma-1)/(2\sigma-1)$  of the revenue respectively. Then the firm chooses an optimal level of employment, and this allows the firm to set an optimal price equal to

$$p_L(i) = p_L(i) = \left(1 - \frac{1}{\sigma}\right)^{-1} w_L \quad (16)$$

$$p_H(i) = p_H(i) = \left(1 - \frac{1}{\sigma}\right)^{-1} \frac{w_H}{Z} \quad (17)$$

Given the pricing and labor market conditions, optimal profits of the firms can also be shown as

$$\pi_L = \frac{\sigma}{2\sigma-1} p_L l(i) \quad \text{and} \quad \pi_H = \frac{\sigma}{2\sigma-1} p_H Zh(i) \quad (18)$$

From the labor market clearing conditions we get  $l(i) = L_E/A_L$  and  $h(i) = H_E/A_H$ . Using the symmetry, we can express the final output production functions as

$$Y_L = A_L L_E \quad \text{and} \quad Y_H = A_H Z H_E \quad (19)$$

Substituting these sectoral production functions in (4) we can rewrite the relative demand function as

$$\left[ \frac{P_H}{P_L} \right] = \left[ \frac{A_L L_E}{A_H Z H_E} \right]^{\frac{1}{\varepsilon}} \quad (20)$$

For given levels of technology -  $A_L, A_H$  - relative wages or skill premium  $\omega$  (21) and relative profitability (22) can be obtained from (16), (17), (18), and (20) by noting the fact that the revenue of the intermediate sectors equals to expenditure on sectoral intermediates,  $p_L L_E = P_L Y_L, p_H Z H_E = P_H Y_H$ .

$$\frac{w_H}{w_L} \equiv \omega = \frac{P_H Z A_H}{P_L A_L} = \left[ \frac{Z A_H}{A_L} \right]^{\frac{\varepsilon-1}{\varepsilon}} \left[ \frac{L_E}{H_E} \right]^{\frac{1}{\varepsilon}} \quad (21)$$

$$\frac{\pi_H}{\pi_L} = \frac{P_H Z H_E}{P_L L_E} = \left[ \frac{A_L}{A_H} \right]^{\frac{1}{\varepsilon}} \left[ \frac{Z H_E}{L_E} \right]^{\frac{\varepsilon-1}{\varepsilon}} \quad (22)$$

### 3.2 Skill-Biased Technological Change

As Acemoglu (1998, 2002) mentions, firms make innovations for new technologies to get higher profits, and technological progress is directed towards abundant factors. If we look at the relative profitability function (22) we see that profits depend on two components: prices and market size.

In endogenous technological change, we assume that innovation can take the form of new varieties of intermediate inputs and requires a fixed cost ( $\mu$ ) in each sector  $L$  and  $H$ . Intermediate producers make zero profit due to free entry; therefore, what is relevant for them is the net present discounted value ( $V_L, V_H$ ) of all profits in the future.  $V_L$  and  $V_H$  can be expressed as

$$rV_L - \dot{V}_L = \pi_L \quad \text{and} \quad rV_H - \dot{V}_H = \pi_H \quad (23)$$

The term  $r$  is the interest rate which is assumed constant in the future, and in the case of steady state the  $\dot{V}$  terms are equal to 0. Then we can write,

$$V_L = \pi_L / r \quad \text{and} \quad V_H = \pi_H / r \quad (24)$$

Note that the net present value of the firms cannot exceed the innovation costs ( $V_L = V_H = \mu$ ) and this requires the following condition,  $\pi_H / \pi_L = 1$ .

$$\frac{\pi_H}{\pi_L} = \frac{P_H Z H_E}{P_L L_E} = \left[ \frac{A_L}{A_H} \right]^{\frac{1}{\varepsilon}} \left[ \frac{Z H_E}{L_E} \right]^{\frac{\varepsilon-1}{\varepsilon}} = 1 \quad (25)$$

From (25) we can obtain the skill bias of technology:

$$\left[ \frac{A_H}{A_L} \right] = \left[ \frac{Z H_E}{L_E} \right]^{\varepsilon-1} \quad (26)$$

And lastly, substituting (26) into (21) we get an expression for the skill premium ( $\omega^*$ ) which depends on relative skilled employment and technology  $Z$ .

$$\frac{w_H}{w_L} \equiv \omega^* = Z^{\varepsilon-1} \left[ \frac{H_E}{L_E} \right]^{\varepsilon-2} \quad (27)$$

According to (26), Acemoglu argues that if the elasticity of substitution between factors is greater than unity, the market size dominates the price effect; therefore, the firms choose to produce

technologies that are biased towards the abundant factor. One can also infer from (27) that the skill premium is increasing in relative skilled workers as long as elasticity of substitution between the factors is greater than 2. This shows the relative demand for skilled labor has to be sufficiently elastic for the skill premium to increase in relative skills.<sup>5</sup>

### 3.3 Migration Decision under SBTC

As it is demonstrated in section 2, we can find strong theoretical and empirical evidences for the impacts of SBTC on migration incentives. The probability of getting high-skilled intensive jobs and high wages stimulate the choice of migration. Therefore, we allow migration decisions (as an exogenous variable) in the model with the skill specific emigration rates  $s_i$  as illustrated by Fadinger and Mayr (2012). They assume that workers decide about emigration to maximize their utility: “If expected utility for an individual  $i$  with skill type  $H, L$  associated with migration ( $M$ ) is greater than the utility associated with staying ( $S$ ) in the country of origin, the individual  $i$  chooses to emigrate or otherwise stays at home”.<sup>6</sup>

$$s_i = \text{prob}(U_i^M(i) > U_i^S) \quad i \in H, L \quad 0 \leq s_i \quad (28)$$

### 3.4 Human Capital Accumulation under SBTC

Previous sections show that skill biased technological progress leads to an increase in demand for skilled labor and thus, an increase in the wages of skilled labor. If a greater wage differential occurs among different labor markets, this also leads an incentive for immigration of skilled workers following the rise in wage premium at the host country. But first, we need model how to see the impact of SBTC on human capital accumulation. Eggebrecht (2009) explains the link between technological change and human capital accumulation through the changes in the level of wages and wage inequality. Therefore this section will follow identically Eggebrecht’s argument. As a rational expectation, the wage differential between skilled and unskilled workers affects the present and future education decisions of the agents because of the fact that the return from the accumulation of skill is greater than the return from not accumulating skill.

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<sup>5</sup> Acemoglu (2002) points out that whether technological change is skill biased or not depends on the elasticity of substitution between the factors. If this elasticity is greater than 1, the factors are gross substitutes, and an increase in  $A_H$  relative to  $A_L$  increases the marginal productivity of skilled workers. On the contrary, if the elasticity is less than 1, the factors are gross complements, and therefore an increase in  $A_H$  relative to  $A_L$  reduces the marginal productivity of skilled workers.

<sup>6</sup> Note that migration itself has some skill-specific individual costs like traveling costs, getting working permissions, moving homes, the unification of families, etc. For a variety of reasons these costs can differ according to the skill levels of the migrants. For the sake of simplicity, in this study we assume that skill-specific subjective costs are exogenously given and the same for all skilled individuals.

After this brief clarification on the relation between human capital accumulation and wage premium, the effects of SBTC on human capital accumulation can be expressed as the following steps.

### 3.4.1 Cost of Education

To obtain an expression of cost of education with respect to skill levels of parents this section demonstrates the approach of Eggebrecht (2009). Parents are assumed to always behave rationally while they are making education decisions for their children. Therefore there is no uncertainty, and parents have a perfect foresight over future wage levels for skilled and unskilled jobs. Being skilled or unskilled not only depends on parents' decisions but is also subject to parents' budget constraint, which is given by

$$w_i = c_i + \lambda_i n e_i \quad \lambda_i \in [0,1] \quad (29)$$

where the variables  $w_i$ ,  $c_i$  and  $e_i$  denote parent's  $i$  wage income, consumption and education cost per child respectively.  $n$  denotes the number of children (fertility rate) and has only a multiplier effect on children's average wage income. This multiplier does not change the results qualitatively. For simplicity reasons, fertility decision of parents is given exogenously. Optimal education decision  $\lambda_i$  shows the proportion of children from parent  $i$  who receive education and endogenously determined by the parents. Skilled adults have a key role in education, and they can transfer the skill and knowledge to their children like teachers. Eggebrecht assumes that since skilled wages are greater or equal to unskilled wages, the skilled parents never invest less in education compared to unskilled ones; therefore, education cost is supposed to be equal a proportion of the wage of skilled parents like ( $\delta$ ) and can be shown as the following equation:

$$e_i = \delta w_H \quad \delta \in (0,1) \quad (30)$$

Regarding (30), there are two crucial points. First, the education decision depends positively on parental wage income. The second is whether this cost is affordable for parents or not. Using (30), we can get relative education cost for skilled and unskilled parents according to the ratio ( $e_i/w_i$ ).



For skilled parents, relative education cost is independent on wage premium and is determined by the parameter  $\delta$ . For unskilled parents, relative education cost is given by  $\frac{\delta w_H}{w_L}$ .<sup>7</sup>

### 3.4.2 Optimal Education Decision

Parents' optimal education decision is simply a constrained maximization problem. It is assumed that each parent  $i \in H, L$  shares the same utility function and maximizes it subject to a budget constraint.

$$\max_{\lambda_i} U_i^i = \ln [c_t^i] + \beta \ln [w_t^L + \lambda_i (w_{t+1}^H - w_{t+1}^L)] \quad \text{s.t.} \quad w_i = c_i + \lambda_i n e_i \quad (31)$$

As can be seen from (31) parents have a utility function for parent  $i$  which includes two components. The first is their own consumption and the second is the quality of their children. The parameter  $\beta$  denotes the altruism of the parents, and  $\beta \in [0, 1]$ .  $w_{t+1}^H$  and  $w_{t+1}^L$  represent the wage incomes of skilled and unskilled children of the parents in the future period. The solution of this maximization problem with respect to optimal education decision is obtained as

$$\lambda_i \begin{cases} 0 & \text{if } \frac{1}{1+\beta} [\bullet] < 0, \\ 1 & \text{if } \frac{1}{1+\beta} [\bullet] > 1, \\ \frac{1}{1+\beta} \left[ \beta \frac{w_t^i}{n e_i} - \frac{w_{t+1}^L}{w_{t+1}^H - w_{t+1}^L} \right] & \text{else.} \end{cases} \quad (32)$$

Eggebrecht (2009) mentions that parents have three different decision type for the education of their children. First, they may not invest in the education of any children ( $\lambda_i = 0$ ); second, they may invest in the education all their children ( $\lambda_i = 1$ ). Finally, they may invest in education of some of

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<sup>7</sup> If  $\delta = 1$  then  $e_i$  would be equal to  $w_H$  so that such an education cost together with the expenditure for consumption, which would be over budget constraint, could not be plausible for both skilled and unskilled parents. Under skill-biased technical change, the demand curve for skilled labor is upward sloping, and this increases the high skilled wage faster than unskilled wage ( $w_H > w_L$ ). This implies that, under ( $\delta w_H > w_L$ ), the relative education cost for unskilled parents would be greater than 1, and this would be not affordable. In addition,  $\delta = 0$  is also not plausible for skilled and unskilled parents. For a while, suppose that the wage premium  $\frac{w_H}{w_L}$  is less than 1. Thus, ( $w_H < w_L$ ). In this case,

skilled agents would tend to apply for unskilled positions which do not correspond to their skill levels. As long as the labor demand corresponds the labor supply there would be no more intention to accumulate skills under a closed economy framework. However, in reality the prospects of getting higher wages from abroad through international labor mobility can provide the necessary condition ( $w_H < w_L < w_H^F$ ), which leads parents to invest more in education of their children.

their children ( $0 < \lambda_i < 1$ ). It should be noted that even though the fertility rate  $n$  is given exogenously, higher fertility rates imply that investing in education of the same proportion  $\lambda_i$  is getting more costly for parents compared to the lower fertility rate. Therefore, Eggebrecht argues that the future proportion of skilled adults in the population is determined by the current education decisions of skilled ( $\lambda_H$ ) and unskilled adults ( $\lambda_L$ ) and is derived as the following:

$$\lambda_H = \frac{1}{1+\beta} \left[ \frac{\beta}{n} \frac{1}{\delta} - \frac{1}{\frac{w_{t+1}^H}{w_{t+1}^L} - 1} \right] \quad \text{and} \quad \lambda_L = \frac{1}{1+\beta} \left[ \frac{\beta}{n} \frac{w_t^L}{\delta w_t^H} - \frac{1}{\frac{w_{t+1}^H}{w_{t+1}^L} - 1} \right] \quad (33)$$

These equations make it obvious that - while optimal education decisions of - skilled parents are affected by future wage inequality, the education decisions of low skilled parents depend on both current wage ratio and the future wage inequality relative to unskilled labor<sup>8</sup> .

#### 4. Equilibrium with Human Capital Accumulation under SBTC with Migration

The brain drain is one of the growing concerns of development issues because of its detrimental effects on public finance, growth and productivity at origin countries. We accept that this concept is much more apparent in less developed countries rather than developed countries (Gibson and McKenzie 2011). Skilled people in technologically less developed countries hardly find the proper jobs to show their knowledge and to perform at their maximum capabilities and capacities. Additionally, we know from the theory and eq. (27) that the countries in which the elasticity between the factors remains below a certain threshold, the demand curve for high skilled labor is downward sloping. Thus, an increase in the relative number of skilled workers leads to a decrease in their wages. As a result of this, the employment opportunities for skilled workers with high wages lessen in that labor market; therefore, educated individuals show an intention to move to the countries where skill biased technology has actually occurred.

This paper differs from Fadinger and Mayr (2012) with regard to the direction of the migration flow according to the place where the SBTC occurs. Here, SBTC only occurs in technologically advanced economies (host countries). Therefore, we do not assume a brain drain issue in these countries. However, developing and less developed countries struggle against brain drain due to the lack of sufficient technological development favoring high-skilled labor. The effort of these countries consists entirely of the adaptation (or imitation) of these technologies. We argue

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<sup>8</sup>  $\left( \frac{w_{t+1}^L}{w_{t+1}^H - w_{t+1}^L} \right)$

that the direction of brain drain moves towards technologically abundant countries from less developed countries.

After having defined the behaviors of firms and of individuals' decisions on investment in education in previous sections, we can now write the equilibrium for the effects of SBTC on human capital accumulation under migration for sending and receiving countries.

#### 4.1 Less Developed (Migrant Sending) Country Case

In this case we examine how an SBTC in an advanced economy affects the human capital formation of a less developed country under skill-biased migration. The main argument of this case is that the SBTC does not occur in the migrant sending country but in the advanced (receiving) country. Therefore, the skill premium function is given as obtained earlier in equation (21) in the less developed country. We assume that skilled wages ( $w_t^H$ ) and unskilled wages ( $w_t^L$ ) are initially the same in both countries. There are two periods,  $t$  and  $t+1$ . In period  $t$ , we suppose that the relative number of skilled individuals increases in the less developed country while an SBTC occurs in the advanced economy. As it can be seen from equation (21), an increase in the relative supply of skilled labor in time  $t$  results a decrease in their wages relative to the unskilled in time  $t+1$ . But in the meanwhile according to equation (27), the relative wages of skilled labor are increasing in the advanced economy due to presence of SBTC. This would lead to a difference in wages of skilled labor among the countries at time  $t+1$ . It should be noted that relative skill premium functions (21) and (27) are also an expression of relative marginal productivity of workers. According to equation (21), relative marginal productivity of skilled workers decreases in the relative abundance of high skilled employment in the less developed country. Therefore, skilled workers in less developed country would have an incentive to emigrate to the advanced economy to maximize their utility.

In addition to this intention, the prospects of getting higher wages through being a skilled worker in an advanced economy would encourage the parents living in less developed countries to invest in education of their children. For the sake of simplicity issue, unskilled wages are taken constant in all time periods. Therefore at time  $t+1$ , the wages of unskilled labor will remain the same as the wage level ( $w_t^L$ ) at time  $t$ .<sup>9</sup>

In order to see the indirect effects of SBTC on optimal education decisions in a less developed country under skill-biased migration, an expression of expected wages as a function of a skill-

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<sup>9</sup> Unskilled wages are assumed as the minimum wage that low-skilled workers can sell their labor. This wage is determined by the negotiations between governments and labor unions. In the short run these wage level is always constant.

specific immigration rate must be defined because of the presence of the terms  $(w_{t+1}^H)$  and  $(w_{t+1}^L)$  in (33). Based on the modified standard model of Fadinger and Mayr (2012), a skilled (unskilled) individual searches for a work with the probability of  $x_H = H_E/H$  ( $x_L = L_E/L$ ). Thus, when an SBTC occurs, the expected wages under skill biased migration for a skilled (unskilled) worker in less developed country  $w_{t+1}^{H,D}$  ( $w_{t+1}^{L,D}$ ) can be derived as a function of wages of the advanced country.

$$w_{t+1}^{H,D} = w_t^{H,A} x_H = w_H \frac{H_E}{(1+s_H)H} \quad 0 \leq s_H \quad (34)$$

$$w_{t+1}^{L,D} = w_t^{L,A} x_L = w_L \frac{L_E}{(1+s_L)L} \quad 0 \leq s_L \quad (35)$$

Note that SBTC is one of the pull factors of high skilled immigration. It is crucial to re-emphasize here that in a technologically advanced country, a brain drain issue does not take place because of the occurrence of the SBTC. Unlike Fadinger and Mayr (2012), the sign of the term  $s_H$  is given positive in Eq. (34). If there is no migration possibility from less developed country to advanced economy,  $s_H$  would be equal to 0. Additionally, as a natural result of SBTC, in the short run only skilled labor is able to move. Due to the high migration costs (such as language barriers, visa costs, government fees and etc.) low-skilled workers cannot migrate in the short run. Hence,  $s_L$  are given 0 in the model, and the cost of migration for skilled labor is assumed very low. To simplify the analysis this cost is also taken as zero.

After having defined the expected wages, we can obtain the relative expected wage as the following:

$$\frac{w_{t+1}^H}{w_{t+1}^L} = \frac{w_H x_H}{w_L x_L} = \left[ \frac{w_H}{w_L} \right] \left[ \frac{H_E}{(1+s_H)H} \right] \left[ \frac{L}{L_E} \right] \quad (36)$$

Substituting the relative skill premium function (27) into (36), we can rearrange the expression of relative expected wages as a function of the skill-specific emigration rate under the impact of endogenous technology.

$$\frac{w_{t+1}^H}{w_{t+1}^L} = \left[ \frac{ZH_E}{L_E} \right]^{\varepsilon-1} \left[ \frac{L}{(1+s_H)H} \right] \quad (37)$$

The first expression on the right hand side in (37) is the skill-bias of technology that we already defined for  $(A_H/A_L)$  in (26), and the latter gives the relative endowments of unskilled workers under skill-biased migration. Note that, there is an adverse effect of skilled migration on relative expected wages according to equation (37), but an increase in  $A_H$  relative to  $A_L$  with a sufficient elasticity of substitution increases the marginal product of skilled workers in future periods and smooths over the adverse effect of migration on skilled wages; thus, relative expected wages increase.

For given levels of technology, relative wages in the less developed country at time  $t$  has been given by (21). Using the inverse function of (21) and expected wages as a function of skill-bias of technology with migration (37), we can rewrite the optimal education decisions (33) of skilled and low-skilled parents in the less developed country as the following equations (38) and (39) respectively.

$$\lambda_H = \frac{1}{1+\beta} \left[ \frac{\beta}{n} \frac{1}{\delta} - \frac{1}{\left[ \frac{ZH_E}{L_E} \right]^{\varepsilon-1} \left[ \frac{L}{(1+s_H)H} \right]^{-1}} \right] \quad (38)$$

$$\lambda_L = \frac{1}{1+\beta} \left[ \frac{\beta}{n\delta} \left[ \frac{ZA_H}{A_L} \right]^{\frac{1-\varepsilon}{\varepsilon}} \left[ \frac{L_E}{H_E} \right]^{-\frac{1}{\varepsilon}} - \frac{1}{\left[ \frac{ZH_E}{L_E} \right]^{\varepsilon-1} \left[ \frac{L}{(1+s_H)H} \right]^{-1}} \right] \quad (39)$$

Skill-bias of technological change depends on the elasticity of substitution between the factors as mentioned in section 3. If the elasticity of substitution is sufficiently high, ( $\varepsilon > 2$ ) with an increase in relative employment workers ( $H_E/L_E$ ), relative technology ( $A_H/A_L$ ) is biased towards the employment factor, which is relatively more abundant. Breaking the Eq. (39) into two parts, we can observe that the initial part (before minus) shows the impact of current wages on education decisions, the following part (after minus) shows the effect of expected wages determined according to advanced county. Making a clarification here is very important: when the migration possibility is getting higher for the skilled individuals, they do not want to work in less qualified jobs and they tend to migrate to technologically developed countries. This means that the factors become gross

complements and therefore, the elasticity of substitution between the factors is always less than 1. This clearly explains why some countries have a brain drain issue.

To see the impacts of SBTC on human capital accumulation in the less developed country, we have to define  $\lambda_H$  and  $\lambda_L$  in terms of relative technologies ( $A_H/A_L$ ) according to Eq. (26), with an elasticity of substitution  $\varepsilon < 1$  for the less developed country for the current wage part of optimal education decision functions in the case for migrant sending country.

$$\lambda_H = \frac{1}{1+\beta} \left[ \frac{\beta}{n\delta} \frac{(1+s_H)H}{\left[\frac{A_H}{A_L}\right] L - (1+s_H)H} \right] \quad (40)$$

$$\lambda_L = \frac{1}{1+\beta} \left[ \frac{\beta}{n\delta} \left[\frac{A_H}{A_L}\right]^{1-\varepsilon} \left[\frac{H_E}{L_E}\right]^{\frac{1}{\varepsilon}} Z^{\frac{1}{\varepsilon}-1} - \frac{(1+s_H)H}{\left[\frac{A_H}{A_L}\right] L - (1+s_H)H} \right] \quad (41)$$

And, lastly, these two equations (40) and (41) enable us to see the effects of technological change. Taking derivatives of education decisions  $\lambda_H$  and  $\lambda_L$  with respect to  $(A_H/A_L)$ , we can make an intuition on the direction of the education decisions of parents.

$$\frac{\partial \lambda_H}{\partial \frac{A_H}{A_L}} = \underbrace{\left(\frac{1}{1+\beta}\right)}_+ \underbrace{\left(\frac{(1+s_H)H}{L}\right)}_+ \underbrace{\left(\frac{1}{(A_H/A_L)^2}\right)}_+ > 0$$

$$\frac{\partial \lambda_L}{\partial \frac{A_H}{A_L}} = \underbrace{\left(\frac{1}{1+\beta}\right)}_+ \underbrace{\left(\frac{\beta}{n\delta}\right)}_+ \underbrace{\left(\frac{1-\varepsilon}{\varepsilon}\right)}_+ \underbrace{\left(\frac{A_H}{A_L}\right)^{\frac{1-2\varepsilon}{\varepsilon}}}_+ \underbrace{\left(\frac{H_E}{L_E}\right)^{\frac{1}{\varepsilon}}}_+ \underbrace{\left(Z^{\frac{1}{\varepsilon}-1}\right)}_+ + \underbrace{\left(\frac{1}{1+\beta}\right)}_+ \underbrace{\left(\frac{(1+s_H)H}{L}\right)}_+ \underbrace{\left(\frac{1}{(A_H/A_L)^2}\right)}_+ > 0$$

As can be seen from the derivatives, both skilled and less-skilled parents increase their investment in education for their children when  $\varepsilon$  is sufficiently low in their countries. This result shows that an SBTC in an advanced economy can stimulate the investment in education and has positive effects on human capital formation in less developed countries, as long as the probability of getting high wages through migration is high.

## 4.2 Advanced (Migrant Receiving) Country Case

As the previous case showed, by substituting the inverse relative wages (27) and expected wages (37) under prospects of skilled migration, we obtain the expressions for the optimal education decision (33) of high skilled and unskilled parents in the receiving country, as in the following equations (42) and (43) respectively.

$$\lambda_H = \frac{1}{1+\beta} \left[ \frac{\beta}{n} \frac{1}{\delta} - \frac{1}{\left[ \frac{ZH_E}{L_E} \right]^{\varepsilon-1} \left[ \frac{L}{(1+s_H)H} \right]^{-1}} \right] \quad (42)$$

$$\lambda_L = \frac{1}{1+\beta} \left[ \frac{\beta}{n\delta} \left[ \frac{1}{Z^{\varepsilon-1}} \right] \left[ \frac{L_E}{H_E} \right]^{\varepsilon-2} - \frac{1}{\left[ \frac{ZH_E}{L_E} \right]^{\varepsilon-1} \left[ \frac{L}{(1+s_H)H} \right]^{-1}} \right] \quad (43)$$

In this case we will follow the same analysis employed for the less developed country. Taking the derivative of optimal education decisions of skilled and less skilled parents with respect to technological change shows how human capital accumulation is affected under skilled migration. It is a natural consequence of Eggebrecht's model that optimal education decision formulation of skilled parents is identical for both developed and less developed country cases. Therefore according to Eq. (42) we observe that skilled parents always have the same characteristics, and they increase their investment in education for their children under SBTC.

For less skilled parents' education decisions, if we rewrite the Eq. (43) in terms of skill-bias technology, we obtain the following:

$$\lambda_L = \frac{1}{1+\beta} \left[ \frac{\beta}{n\delta} \left[ \frac{A_H}{A_L} \right]^{-1} \left[ \frac{H_E}{L_E} \right] - \frac{1}{\left[ \frac{A_H}{A_L} \right] \left[ \frac{L}{(1+s_H)H} \right]^{-1}} \right] \quad (44)$$

And lastly, taking derivative of  $\lambda_L$  with respect to  $(A_H/A_L)$ , we have the following result:

$$\frac{\partial \lambda_L}{\partial \frac{A_H}{A_L}} = \underbrace{\left( \frac{1}{1+\beta} \right) \left( \frac{\beta}{n\delta} \right) \left( -\frac{A_H}{A_L} \right)^{-2} \left( \frac{H_E}{L_E} \right)}_{-} + \underbrace{\left( \frac{1}{1+\beta} \right) \left( \frac{(1+s_H)H}{L} \right) \left( \frac{1}{(A_H/A_L)^2} \right)}_{+} = 0 >$$

From equation (27) we observe that SBTC affects the skilled wages positively thus, skill premium increases. This brings an additional cost to less skilled parents to afford the same amount of investment in education as skilled parents do. In other words, a higher current wage ratio leads to a higher relative education cost for less skilled parents. This implies a negative effect on human capital accumulation. An interesting feature of the current result is that, adding migration component would not have a negative effect on the impact of SBTC on educations decisions of less-skilled parents. These findings are in line with the results of Eggebrecht (2009).

As can be seen in equation (44), the exponent of relative technology ( $A_H/A_L$ ) in the first bracket is minus one, and this reduces the human capital accumulation. However the later effect, the probability of earning skilled wages in the future, keeps the parents from investing more in education according to their budget constraint. Therefore, the net effect of SBTC on human capital accumulation for less skilled parents can be either positive, negative or zero.

## 5. Conclusion

This paper introduces a new dimension to the effects of SBTC on human capital accumulation from the perspectives of an Acemoglu-type growth model with migration. Inclusion of migration decision enables us to see the effects of SBTC on human capital accumulation in migrant sending countries as well. The model depends on two main assumptions: First, SBTC occurs only in technologically advanced countries that are also characterized as migrant receiving countries. Second, SBTC stimulates the incentives of individuals to migrate technologically advanced economies.

The paper draws several conclusions: (i) Optimal education decisions of high skilled parents in both host and home countries are positively affected by an SBTC. (ii) However, results differ for low skilled parents. While an SBTC affects the optimal education decisions of less-skilled parents positively in migrant sending countries, the net effect in receiving countries is ambiguous. (iii) Our findings also reveal that the net impact of SBTC on human capital accumulation in receiving countries depends on the modelling structure of the sign relationship between expected wages and migration impact. We can also show a negative effect of skilled migration on human capital accumulation. The intuition behind this result is straightforward: If there is a strong expectation in the labor market of receiving country that the impact of migration on wages would be negative (as it



is also modeled in this paper), skilled-migration also leads to a negative effect on optimal education decisions of local parents. However, with an increase in relative technologies due to SBTC, the marginal product of skilled workers increases, and expected wages would continue to increase under skilled migration until the skilled-wage levels come to the equilibrium among countries in the long run *(iii)* Consequently, since the probability of getting high wages through migration is high, and migration is permissible among the countries, SBTC positively affects the human capital accumulation in less developed countries. However, the net effect of SBTC on human capital accumulation is uncertain in advanced countries under skilled migration.

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## CHAPTER 3

### DILUTION EFFECTS, POPULATION GROWTH AND ECONOMIC GROWTH UNDER HUMAN CAPITAL ACCUMULATION AND ENDOGENOUS TECHNOLOGICAL CHANGE\*

***Abstract:** The paper analyzes the impact of population growth on economic growth under endogenous technological change and human capital investment. The novelty of this chapter is the inclusion of a “dilution effect” of population growth on per capita human capital accumulation, which is not present in the original Uzawa-Lucas model. The present paper has showed that an increase in the population growth rate yields an ambiguous impact on the growth rate of per-capita income due to the relative contribution of two distinct effects of population growth: The direct dilution effect and the indirect ideas effect. This study revealed that the dilution effect has a central role in explaining the ambiguous impact of population growth on economic growth. When the dilution effect is sufficiently low, an unambiguously positive correlation between population growth and economic growth is obtained. When it is sufficiently high the correlation may be either positive or negative or neutral. Another result is that more population growth generates an indirect ideas effect on the rate of innovation and economic growth. These evidences are empirically checked for the advanced countries.*

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## 1. Introduction

In recent years, the United Nations has consistently called attention to the importance of addressing the economic issues which mainly rely upon the reality in the world population. Population division of the UN captures that even under the assumption of decreasing fertility rates it is a fact that *the world population reached to 7.2 billion in mid-2013 [...] and it is projected to reach 9.6 billion in 2050 by increasing more than 2.4 billion more than in 2013* (UN, 2013a, pp.1-5). Additionally, in another report of the UN it is emphasized that new population patterns and trends strongly demand differentiated policies and programmes both at national and international levels. According to this report “*In the past two decades, many governments in less developed regions have realized the importance of reducing high rates of population growth, while a growing number of governments in more developed regions have expressed concerns about low rates of population growth...*” (UN, 2013b, p. 1). The latter fact shows that the correlation between population and economic prosperity may vary with the level of economic progress.

The relationship between population and economic growth has always been comprehensively taken into account by the economists and the policy makers. From the economic point of view, - starting from the Malthusian theory - studies on the impact of population growth on economic growth can be categorized as follows: Pessimist, optimist and neutralist views. According to the pessimistic views, population growth has detrimental effects on economic growth. Simply they claim that economic resources (such as food supply) are fixed in the long run, and technological progress is also limited against increasing population.<sup>1</sup> Unlike pessimists, optimists argue that population growth affects economic growth positively due to the endogenous technological progress and scale effects of larger populations.<sup>2</sup> Last group, neutralists claim that the impact of population growth on economic growth is so little (either positive or negative or non-existent) that can be negligible.<sup>3</sup>

Until now, the literature revealed that “[...] *population growth is not all good or all bad for economic growth*” as Kelly and Schmidt (1995, p. 554) argue in their paper. Instead of asking “what is the net impact of population growth on economic growth?”, asking the question of “why does population growth affect countries’ economic growth differently?” would be much more significant in order to get an accurate answer about the sign of the relationship between population growth and

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<sup>1</sup> For the main proponents of this group, see Malthus (1798), Coale and Hoover (1958), Ehrlich (1968). In the standard growth theory where savings rate and technological change are exogenous, population growth lowers income because of the (physical) capital dilution. For some empirical studies, see Mankiw *et al.* (1992), Ahituv (2001), Li and Zhang (2007) and Herzer *et al.* (2012).

<sup>2</sup> For some models (with endogenous technological change) in which population size (and/or growth) affects economic growth positively; see Kuznets (1967), Boserup (1981), Simon (1981), Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Kremer (1993), Jones (1995).

<sup>3</sup> For an example; see Srinivasan (1988).

economic growth. Prettner (2013), Romero (2013) and Mierau and Turnovsky (2014) argue that the role of demography is another factor to evaluate the relation between population and economic growth. However; the nature of the demographic changes (mortality, fertility and ageing) is not at the focus of this paper.<sup>4</sup> Both theoretical and empirical studies in the (endogenous growth) literature have showed that “...*Whether population growth or population size foster or hamper economic growth strongly depends on the modelling framework...*” (Prettner and Prskawetz, 2010, p. 607).

In the light of this wide literature, this paper attempts to analyze the impact of population growth on economic growth under endogenous technological change and human capital investment by agents. The model we study in this paper is based on an endogenous growth model with expanding variety of products<sup>5</sup> where return to specialization is always positive. We consider a closed economy in which final output, intermediate and research sectors are vertically integrated, and there are three types of homogenous agents which are perfectly mobile and fully employed. In this economy governmental activity does not exist, population grows exogenously, and there is no external shock such as migration. Individuals are assumed to spend their time to work and invest in human capital.

The important novelty of this paper comes from a critic which is not presented in the original Uzawa (1965) and Lucas (1988) model. Lucas (1988) argues that newborns do not reduce the current skill level of individuals hence; population growth does not exist in the formulation of human capital accumulation. Unlike Lucas, Bucci (2008, p. 2029), Strulik (2005 p. 137) and Dalgaard and Kreiner (2001, p. 190) illustrate that population growth decreases the average human capital level of an economy, and therefore, has a dilution effect on the accumulation of per-capita human capital. Additionally, we know that there are some empirical studies also concluding that the population growth has a direct and negative dilution effect on human capital investment.<sup>6</sup> Our explanation mainly rests on the inclusion of an explicit dilution effect of population growth on human capital accumulation. Then, we extend our benchmark formulation by introducing a parameter which measures the strength of this negative effect of population growth on per-capita human capital investment.

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<sup>4</sup> To address the question, Kelly (1988) provides an extensive review. For some recent substantial analyses, see also Prettner and Canning (2014) and Prettner *et al.* (2013). Lastly, Strulik *et al.* (2013)'s “*child quantity-quality trade off*”, and Prettner (2014)'s *schooling intensity* approaches are important examples for showing the adverse effect of population growth on economic growth under the R&D-based growth models with human capital accumulation.

<sup>5</sup> For a detailed explanation of the model structure see Barro and Sala-i Martin (2004, Chp.6, p. 285).

<sup>6</sup> See Boikos *et al.* (2013, pp. 52-56). See also Coale and Hoover (1958) for the types of dilution effect of population growth.

The objective of the present paper is therefore twofold. First it answers the latter question by providing an alternative but complementary theoretical framework that explains why an increase in the population growth rate - regardless of the source of demographic change<sup>7</sup> - may yield an ambiguous (positive, negative or neutral) impact on the growth rate of per-capita income in the very long run. Second, it aims to evaluate that to what extent the dilution effect of population growth explains the different rates of economic growth across countries. The results have demonstrated that the strength of this dilution effect has a central role in accounting for the ambiguous impact of population growth on economic growth along the BGP equilibrium. Another result of the paper is that population growth has an indirect ideas effect on real per-capita income.

The remaining part of the paper is organized as follows. In Section 2 we lay out the benchmark model whose predictions are analyzed along the BGP in Section 3. Section 4 we demonstrate the relationship between population growth and per capita income growth under the BGP equilibrium. Section 5 is the second part of the paper which aims to test the theoretical results empirically. Finally, section 6 concludes the paper, and provides a ground for possible future extensions.

## 2. The Model

### 2.1 Production

Consider an environment in which three sectors of activity are vertically integrated. The research sector is characterized by free entry. Here, firms combine human capital and (eventually) the existing number of ideas to engage in innovative activity that results in the invention of new blueprints for firms operating in the intermediate sector. The intermediate sector is composed of monopolistically competitive firms. There is a distinct firm producing each single variety of intermediates/durables and holding a perpetual monopoly power over its sale. In the competitive final output sector, atomistic firms produce a homogeneous consumption/ final good/output by employing human capital and all the available varieties of intermediate inputs. The representative firm producing final output has the following technology:<sup>8</sup>

$$Y_t = n_t^{\bar{\alpha}} H_{Yt}^{1-Z} \int_0^{n_t} (x_{it})^Z di, \quad \bar{\alpha} \geq 0, \quad 0 < Z < 1 \quad (1)$$

<sup>7</sup> Notice that, Boucekkine *et al.* (2002) also follow a similar method of approach to investigate the effects of population growth on economic growth. The authors find a population growth rate which maximizes the growth rate.

<sup>8</sup> We follow Ethier (1982) and Romer (1987, 1990).

In Eq. (1)  $Y$  denotes the total production of the homogeneous final good (the *numeraire* in the model),  $x_i$  and  $H_Y$  are, respectively, the quantity of the  $i$ -th intermediate and human capital input employed in the sector. The number of ideas existing at a certain point in time ( $n_t$ ) coincides with the number of intermediate-input varieties and represents the actual stock of *non-rival* knowledge capital available in the economy. Here, we assume that having a larger number of intermediate-input varieties do not lead any detrimental effect on aggregate productivity in the production process. As a whole, the aggregate production function (1) displays constant returns to scale to the two private and rival factor-inputs ( $H_Y$  and  $x_i$ ), with  $1 - Z$  and  $Z$  corresponding to their shares in GDP.<sup>9</sup> When  $Z \in (0;1)$ , final output production takes place by using simultaneously human capital and intermediates.

The inverse demand function for the  $i$ -th intermediate reads as:

$$p_{it} = Zn^{\bar{\alpha}} H_{Yt}^{1-Z} (x_{it})^{Z-1} \quad (2)$$

Eq. (2) represents that  $i$ -th intermediate producer receives its own marginal product at time  $t$ , since the industry is competitive. In the absence of any strategic interaction across firms in the intermediate sector<sup>10</sup>, the demand for the  $i$ -th durable has price elasticity (in absolute value) equal to  $1/(1-Z) > 1$ , which coincides with the elasticity of substitution between any two generic varieties of capital goods in the final output production.

In the intermediate sector, firms engage in monopolistic competition. Each of them produces one (and only one) horizontally differentiated durable and must purchase a patented design before producing its own output. Thus, the price of the patent represents a fixed entry cost. Following Grossman and Helpman (1991, Ch. 3), we assume that local monopolists have access to the same one-to-one technology:

$$x_{it} = h_{it}, \quad \forall i \in [0; n_t], \quad n_t \in [0; \infty) \quad (3)$$

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<sup>9</sup> Since final output is produced competitively under constant returns to scale to rival inputs, at equilibrium  $H_Y$  and  $x_i$  are rewarded according to their own marginal products. Hence,  $(1-Z)$  is the share of  $Y$  going to human capital and  $Z$  is that accruing to intermediate inputs.

<sup>10</sup> That amounts to assuming that the number of intermediate firms ( $n$ ) is so large that each of them produces only a very negligible share of the total supply of intermediates.



where  $h_i$  is the amount of skilled labor (human capital) required in the production of the  $i$ -th durable, whose output is  $x_i$ . For given  $n$ , Eq. (3) implies that the total amount of human capital used in the intermediate sector at time  $t$  ( $H_t$ ) is:

$$\int_0^{n_t} (x_{it}) di = \int_0^{n_t} (h_{it}) di \equiv H_t \quad (4)$$

By continuing to assume that there exists no strategic interaction across intermediate firms, and making use of Eq. (2), maximization of the generic  $i$ -th firm's instantaneous flow of profits leads to the usual *constant markup* rule:

$$p_{it} = \frac{1}{Z} w_{it} = \frac{1}{Z} w_t = p_t, \quad \forall i \in [0; n_t], \quad n_t \in [0; \infty) \quad (5)$$

Eq. (5) says that the price is the same for all intermediate goods  $i$  and is equal to a constant markup ( $1/Z > 1$ ) over the marginal cost of production ( $w_t$ ). In a moment it will be explained that in this economy the whole available stock of human capital ( $H$ ) is employed and spread across production of consumption goods ( $H_Y$ ), durables ( $H_I$ ), and new ideas ( $H_n$ ). Since it is assumed to be perfectly mobile across sectors, at equilibrium human capital will be rewarded according to the same wage rate  $w_{Yt} = w_{It} = w_{nt} \equiv w_t$ , with  $w_t$  denoting the wage paid to any generic unit of human capital employed in the intermediate sector. Under the hypothesis of symmetry – i.e.,  $p$  and  $x$  equal across  $i$ 's – Eq. (4) leads to:

$$x_{it} = H_t / n_t = x_t, \quad \forall i \in [0; n_t] \quad (4')$$

$$\pi_{it} = [Z(1-Z)H_{Yt}^{1-Z}H_t^Z]n_t^{\bar{\alpha}-Z} = \pi_t, \quad \forall i \in [0; n_t] \quad (6)$$

Thus, each intermediate firm will decide at time  $t$  to produce the same quantity of output ( $x$ ), to sell it at the same price ( $p$ ), accruing the same instantaneous profit ( $\pi$ ). The symmetry across durables is a direct consequence of the fact that each intermediate firm uses the same production technology (3) and faces the same demand function (see 2 and 5). Notice that,  $Z \in (0; 1)$  and the product within the square brackets is therefore, greater than zero.  $\pi_t$  would have been equal to zero if  $Z$  had been equal to one (instantaneous profit are zero in a perfectly-competitive market). Under symmetry, Eq. (1) can be recast as:

$$Y_t = (H_{Y_t}^{1-Z} H_{I_t}^Z) n_t^R, \quad R \equiv \bar{\alpha} + 1 - Z > 0 \quad (1')$$

where  $R$  measures the degree of returns to specialization, that is “[...]The degree to which society benefits from ‘specializing’ production between a larger number of intermediates” (Benassy, 1998, p. 63). In the present paper, it is immediate to verify that  $R$  is always positive. The hypothesis  $R > 0$  implies that the impact on aggregate productivity ( $Y$ ) of having a larger number of intermediate-input varieties is always positive ( $n > 0$ ) for any  $H_I > 0$  and  $H_Y > 0$  (see Eq. 1’). According to Eq. (1’), the aggregate production function exhibits constant returns to  $H_Y$  and  $H_I$  together, but either increasing ( $R > 1$ ), or decreasing ( $0 < R < 1$ ), or else constant ( $R = 1$ ) returns to an expansion of variety, while holding the quantity employed of each other input fixed. With respect to other settings, this article introduces important novelties. Unlike Devereux *et al.* (1996a, 1996b, 2000) where, if all intermediates are hired in the same quantity  $x$  the returns to specialization are either unambiguously increasing<sup>11</sup> or at most constant,<sup>12</sup> we allow for the possibility that the returns to specialization might also be decreasing. Unlike Bucci (2013), we explicitly rule out the possibility that the returns to specialization  $R$  are negative.<sup>13</sup>

## 2.2 Research and Development (R&D)

There is a large number of small competitive firms undertaking R&D activity. These firms produce ideas indexed by zero through an upper bound  $n \geq 0$ . Ideas take the form of new varieties of intermediate inputs that are used in the production of final output. They are partially excludable, but nonrival. With access to the same stock of knowledge,  $n$ , a representative research–firm uses only human capital to develop new ideas:

$$\dot{n}_t = \psi_t H_{nt}, \quad n(0) > 0 \quad (7)$$

In Eq. (7) is  $H_n$  the number of people attempting to discover new ideas, and  $\psi$  is the rate at which a single researcher can generate a new idea. Since the representative R&D-firm is small with respect to the whole sector, it takes  $\psi$  as given. Hence, Eq. (7) suggests that R&D-activity is conducted

<sup>11</sup> In Devereux *et al.* (1996a, p. 236, Eq. 1; 2000, p. 549, Eq. 1), under symmetry ( $x_i = x, \forall i$ ) the aggregate production function reads as:  $Y = xN^{1/\rho}, \rho \in (0;1)$ . Therefore, the degree of returns to specialization equals  $1/\rho$ , a number clearly larger than one. This is the “increasing returns to specialization case” in Devereux *et al.* (1996b, p. 633, Eq. 4b, with  $\lambda = 0$ ).

<sup>12</sup> See Devereux *et al.* (1996b, p. 633, Eq. 4b, with  $\lambda = 1 - 1/\rho$ ).

<sup>13</sup> A negative  $R$  means that an increase in  $n$  would lead to some sort of ‘inefficiency’ in the economy since, following a rise of the number of intermediate-good varieties, aggregate GDP ( $Y$ ) would *ceteris paribus* decline in this case.

under constant returns to scale to the human capital input ( $H_n$ ). We postulate that the arrival rate  $\psi$  has the following specification:

$$\psi_t = \frac{1}{\chi} \frac{H_n^{\mu-1}}{H_t^\Phi} n_t^\eta, \quad \chi > 0, \quad \mu > 0, \quad \Phi \geq 0, \quad \eta < 1 \quad (7')$$

Using together (7) and (7'), the R&D-technology (the production-function of new ideas) reads as:

$$\dot{n}_t = \frac{1}{\chi} \frac{H_n^\mu}{H_t^\Phi} n_t^\eta, \quad n(0) > 0, \quad \chi > 0, \quad \mu > 0, \quad \Phi \geq 0, \quad \mu \neq \Phi, \quad \eta < 1 \quad (8)$$

In the equations above,  $\chi$  is a strictly positive technological parameter and  $H$  is the aggregate amount of human capital available in the economy. The rate at which a researcher can generate a new idea ( $\psi$ ) is related to three different effects. The parameter  $\eta$  measures the traditional *intertemporal spillover-effect* arising from the existing stock of knowledge,  $\eta < 0$  reflects the case where the rate at which a new innovation arrives declines with the number of ideas already discovered (“*fishing-out effect*”); if  $0 < \eta < 1$ , previous discoveries raise the productivity of current research effort (“*standing-on-shoulders effect*”);  $\eta = 0$  represents the situation in which the arrival rate of new ideas is independent of the available stock of knowledge.<sup>14</sup> The case  $\eta = 1$  is ruled out from the analysis in order to avoid possible scale effects, whereby an increase in the level of available human capital may affect the rate at which new ideas are produced over time. The parameter  $\mu$  captures the effect on the arrival rate of a new innovation of the actual size of the R&D process (measured by the number of units of skilled labor-input actually devoted to it). A value  $\mu = 0$  would imply that  $H_n$  is not an input to R&D-activity (Eq. 8). We rule out this unrealistic case by assuming that research human capital is indispensable to the discovery of new designs and that its contribution to the production of new ideas is always positive (*i.e.*,  $\mu > 0$ ). If  $\mu = 1$ , doubling the number of researchers  $H_n$  would not affect the arrival rate of a new idea in Eq. (7'), so leading to exactly double the production of new innovations per unit of time (Eqs. 7 and 8); if  $\mu \in (0;1)$  due to the existence of congestion/duplication externalities (“*stepping-on-toes effect*”), increasing the number of researchers leads to a reduction of the rate at which each of them can discover a new idea (Eq. 7') and to a simultaneous increase (but less than proportional) in the total number of new innovations produced in the unit of time (Eq. 8).<sup>15</sup> In accordance with Jones (2005,

<sup>14</sup> For a detailed discussion of the “*fishing out*” and “*standing on shoulders*” effects, see Jones (1995, 2005).

<sup>15</sup> Likewise, if  $\mu > 1$ , increasing the number of researchers would imply an increase (more than proportional) in the total number of new innovations produced in the unit of time (Eq. 8).

p. 1074, Eq. 16) we keep our analysis as much general as possible and impose no upper-bound to  $\mu$ . According to Eq. (8), inventing the latest idea requires a skilled-labor input equal to  $H_n = (\chi H^\Phi / n^\eta)^{1/\mu}$ , which can change over time either because of the growth of  $n$  (*intertemporal knowledge-spillover effect*), or because of the growth of  $H$ . If  $\Phi$  is positive, an increase in population size would *ceteris paribus* lead to a rise of  $H$  and, ultimately, to a decrease of research human capital productivity (an increase in  $H_n$ ). The hypothesis that the productivity of human capital employed in research may fall due to an increase of population size can be justified by the fact that it becomes increasingly difficult to introduce successfully new varieties of (intermediate) goods in a more crowded market (*R&D-difficulty* grows also with the size of population, as suggested by Dinopoulos and Segerstrom, 1999, p. 459). In Eq. (8) a positive  $\Phi$  measures the strength of this effect: all the rest being equal, the larger  $\Phi$  and the bigger the decline in the R&D human capital productivity following an increase of population size. On the other side, a negative  $\Phi$  shows that the productivity of human capital employed in research sector increases because of the fact that growing human capital stock leads to an increase in the ease of exchanging of ideas and expanding the possibilities for creating interactions between researchers. Notice that the Jones' (2005) formulation of the R&D process does not take these important features of the inventive activity into account.<sup>16</sup>

The R&D sector is competitive and there is free-entry. A representative R&D firm has instantaneous profits equal to:

$$\text{R\&D firm profits} = \underbrace{\left( \frac{1}{\chi} \frac{H_n^\mu}{H_t^\Phi} n^\eta \right)}_n V_{nt} - w_{nt} H_{nt} \quad (9)$$

where:

$$V_{nt} = \int_t^\infty \pi_{i\tau} e^{-\int_t^\tau r(s) ds} d\tau, \quad \tau > t \quad (10)$$

In the last two equations,  $V_n$  denotes the value of the generic *i-th* intermediate firm (the one that has got the exclusive right of producing the *i-th* variety of capital goods by employing the *i-th* blueprint),  $\pi_{i\tau}$  is the flow of profits accruing to the same *i-th* intermediate firm at date  $\tau$ ,

<sup>16</sup> When  $\Phi = 0$ , Eq. (8) becomes:  $\dot{n}_t = \frac{1}{\chi} H_n^\mu n^\eta$ ,  $\chi > 0$ ,  $\mu > 0$ , and  $\eta < 1$ . This specification coincides with Jones (2005, Eq. 16, p. 1074).

$\exp\left[-\int_t^\tau r(s)ds\right]$  is a present value factor which converts a unit of profit at time  $\tau$  into an equivalent unit of profit at time  $t$ ,  $r$  denotes the instantaneous interest rate (the real rate of return on households' asset holdings, to be introduced in a moment), and  $w_n$  is the wage rate going to one unit of research human capital. Eq. (9) says that profits of a representative R&D firm are equal to the difference between total R&D revenues (R&D output,  $n$ , times the price of ideas,  $V_n$ ) minus total R&D costs related to *rival* inputs (human capital employed in research,  $H_n$ , times the wage accruing to one unit of this input,  $w_n$ ). Eq. (10), instead, reveals that the price of the generic  $i$ -th idea is equal to the present discounted value of the returns resulting from the production of the  $i$ -th variety of capital-goods by profit-making intermediate firm  $i$ .

Using Eq. (9), the zero-profit condition in the R&D sector implies:

$$w_{nt} = \frac{1}{\chi} \frac{H_{nt}^{\mu-1}}{H_t^\Phi} n^\eta V_{nt} = \psi_t V_{nt} \quad (9')$$

### 2.3 Households

The economy is closed and consists of many structurally-identical households. Therefore, we focus on the choices of a single infinitely-lived family with perfect foresight whose size coincides with the size of the whole population ( $L$ ) and that owns all the firms operating in the economy. Each member of the household can purposefully invest in human capital. Consequently, the aggregate stock of this factor-input ( $H_t = h_t L_t$ ) can rise either because population grows at a constant and exogenously given rate  $g_L > 0$ , or because per capita human capital,  $h_t$ , endogenously increases over time. The household uses the income it does not consume to accumulate new assets that take the form of ownership claims on firms. Thus:

$$\dot{A}_t = (r_t A_t + w_t H_{Et}) - C_t, \quad A(0) > 0 \quad (11)$$

where  $A$  and  $C$  denote, respectively, household's asset holdings and consumption and  $H_E \equiv uH = H_Y + H_I + H_n$  is the fraction of the available human capital employed in production activities (namely, production of consumption goods and intermediate inputs, and discovery of new ideas).<sup>17</sup> Eq. (11) suggests that household's investment in assets (the left hand side) equals

<sup>17</sup> As already mentioned, at equilibrium all human capital employed in production activities ( $H_E$ ) is rewarded according to the same wage,  $w$ .

household's savings (the right hand side). Household's savings, in turn, are equal to the difference between household's total income -the sum of interest income,  $rA$ , and human capital income,  $wH_E$  - and household's consumption ( $C$ ). Given Eq. (11), the law of motion of assets in per-capita terms ( $a_t \equiv A_t/L_t$ ) reads as:

$$\dot{a}_t = (r_t - g_L)a_t + (u_t h_t)w_t - c_t, \quad a(0) > 0 \quad (11')$$

where  $h_t \equiv H_t/L_t$  denote consumption and human capital per capita, respectively. The term  $-g_L$  in (11') captures the *dilution* occurring in per-capita asset holdings accumulation due to population growth, and reflects the 'cost' of bringing the amount of per-capita assets of the newcomers up to the average level of the existing population. This formulation implies that, *ceteris paribus*, population growth tends to slow down the investment in assets of the average individual in the population.

At each time  $t \geq 0$ , the household uses the remaining fraction  $(1 - u_t)$  of  $H_t$  in educational assignments. Human capital per capita accumulates as:

$$\dot{h}_t = [\sigma(1 - u_t) - \xi g_L]h_t, \quad \sigma > 0, \quad \xi \geq 0, \quad h(0) > 0 \quad (12)$$

where  $\sigma$  and  $\xi$  are parameters. The first measures the productivity of education, whereas the second reveals the strength, if any, of the negative effect of population growth on per-capita human capital investment. When  $\xi = 1$ , Eq. (12) shows the existence of a linear, one-to-one, dilution effect of population growth on per capita human capital accumulation (similar to that of Eq. 11'). A possible explanation of such effect would be that since newborns enter the world uneducated they naturally reduce, *ceteris paribus* and at a given point in time, the existing stock of human capital per capita. Indeed, this effect is not presented in the original Lucas' (1988, Eq. 13, p. 19) formulation. Lucas' assumption (newborns enter the work-force endowed with a skill-level proportional to the level already attained by older members of the family, so population growth *per se* does not reduce the current skill level of the representative worker) is based on the *social nature* of human capital accumulation, which has no counterpart in the accumulation of physical capital and of any other form of tangible assets. When  $\xi = 0$ , Eq. (12) is able to recover this idea (Lucas, 1988, p.19). A value of  $\xi \in (0;1)$  represents an intermediate case between the previous two.

With a *Constant Intertemporal Elasticity of Substitution (CIES)* instantaneous felicity function, the problem faced by a representative infinitely-lived family seeking to maximize the utility (attained from consumption) of its members is:

$$\text{Max}_{\{c_t, u_t, a_t, h_t\}_{t=0}^{\infty}} U \equiv \int_0^{\infty} \left( \frac{c_t^{1-\theta} - 1}{1-\theta} \right) e^{-(\rho-g_L)t} dt, \quad \rho > g_L \geq 0, \quad \theta > 0 \quad (13)$$

$$\text{s.t.}: \quad \dot{a}_t = (r_t - g_L)a_t + (u_t h_t)w_t - c_t, \quad u_t \in [0;1], \quad \forall t \geq 0; \quad \dot{L}_t / L_t \equiv g_L > 0$$

$$\dot{h}_t = [\sigma(1-u_t) - \xi g_L] h_t, \quad \sigma > 0; \quad \xi \geq 0$$

$$a(0) > 0, \quad h(0) > 0 \text{ given.}$$

In Eq. (13) population at time 0,  $L(0)$ , has been normalized to one. The household chooses the optimal path of per-capita consumption ( $c$ ) and the share of human capital to be devoted to production activities ( $u$ ). The other symbols have the following meaning:  $U$  and  $\left( \frac{c_t^{1-\theta} - 1}{1-\theta} \right)$  are the household's intertemporal utility function and the instantaneous felicity function of each member of the dynasty. We indicate by  $\rho > 0$  the pure rate of time-preference and by  $1/\theta > 0$  the constant intertemporal elasticity of substitution in consumption. The hypothesis  $\rho > g_L$  ensures that  $U$  is bounded away from infinity if  $c$  remains constant over time.

### 3. General Equilibrium and BGP Analysis

Since human capital is fully employed and there exists perfect mobility of this factor-input across sectors, the following equalities must hold at equilibrium:

$$H_E \equiv u_t H_t = H_{Y_t} + H_{It} + H_{nt} \quad (14)$$

$$w_{It} = w_{nt} \quad (15)$$

$$w_{It} = w_{Y_t} \quad (16)$$

Eq. (14) says that aggregate labor demand (the right hand side) should equal the fraction of the available human capital stock employed in production and R&D activities (the left hand side). Eqs. (15) and (16) together state that, for the previous equality to be checked, wages do adjust in such a way that the salary earned by one unit of skilled labor in the intermediate sector should be equal to

the salary earned by the same unit of skilled labor if employed in research or in the production of final goods. Moreover, since household's asset holdings must equalize the aggregate value of firms, the following equation should also be met in equilibrium:

$$A_t = n_t V_{nt} \quad (17)$$

Where is given by Eq. (10) and satisfies the usual *no-arbitrage condition*:

$$\dot{V}_{nt} = r_t V_{nt} - \pi_t$$

In the model, the *i-th* idea allows the *i-th* intermediate firm to produce the *i-th* variety of durables. This explains why in Eq. (17) total assets ( $A$ ) equal the number of profit-making intermediate firms ( $n$ ) times the market value ( $V_n$ ) of each of them (equal, in turn, to the price of the corresponding idea). On the other hand, the *no-arbitrage condition* suggests that the return on the value of the *i-th* intermediate firm ( $r_t V_{nt}$ ) must be equal to the sum of the instantaneous monopoly profit accruing to the *i-th* intermediate input producer ( $\pi_t$ ) and the capital gain/loss matured on  $V_{nt}$  during the time interval  $dt$ ,  $\dot{V}_{nt}$ . We are now able to move to a formal definition and characterization of the model's BGP equilibrium.

**DEFINITION: BGP EQUILIBRIUM**

*A BGP Equilibrium in this economy is a long-run equilibrium path along which:*

- (i) *All variables depending on time grow at constant (possibly positive) exponential rates;*
- (ii) *The sectoral shares of human capital employment ( $s_j = H_j/H$ ,  $j = Y, I, n$ ) are constant.*

From this definition, Proposition 1 follows:

**PROPOSITION 1**

*Along the BGP, the fraction of the aggregate stock of human capital employed in production activities is constant (that is,  $u_t = u$ ,  $\forall t \geq 0$ ). ■*

*Proof:* Immediate from Eq. (12), and the fact that the growth rate of all time-dependant variables is constant along the BGP equilibrium. The following results do hold along the BGP (mathematical derivation can be found in the *Appendices, Appendix A*):



$$\frac{\dot{H}_{Yt}}{H_{Yt}} = \frac{\dot{H}_{It}}{H_{It}} = \frac{\dot{H}_{mt}}{H_{mt}} = \frac{\dot{H}_t}{H_t} \equiv \gamma_H = \frac{[(\sigma - \rho) - (\xi - 1 - \theta)g_L]}{\Upsilon R(\theta - 1) + \theta} \quad (18)$$

$$\frac{\dot{n}_t}{n_t} \equiv \gamma_n = \frac{\Upsilon [(\sigma - \rho) - (\xi - 1 - \theta)g_L]}{\Upsilon R(\theta - 1) + \theta} = \Upsilon \gamma_H \quad (19)$$

$$r = \frac{\sigma\theta + \Upsilon R(\sigma\theta - \rho) - \{\theta[\xi(1 + \Upsilon R) - (1 + 2\Upsilon R)]\}g_L}{\Upsilon R(\theta - 1) + \theta} \quad (20)$$

$$\gamma_a \equiv \frac{\dot{a}_t}{a_t} = \gamma_c \equiv \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r - \rho) \quad (21)$$

$$\gamma_y \equiv \frac{\dot{y}_t}{y_t} = \gamma_a = \gamma_c = \frac{(1 + \Upsilon R)(\sigma - \rho) - [\Upsilon R(\xi - 2) + (\xi - 1)]g_L}{\Upsilon R(\theta - 1) + \theta} \quad (22)$$

$$u = 1 - \frac{(\sigma - \rho) - [\Upsilon R(1 - \xi)(\theta - 1) + \xi(1 - \theta) - 1]g_L}{\sigma[\Upsilon R(\theta - 1) + \theta]} \quad (23)$$

$$s_n = \frac{Z(1 - Z)\gamma_n}{[1 - Z + Z^2][r + (1 - R)\gamma_n - \gamma_H] + Z(1 - Z)\gamma_n} u \quad (24)$$

$$s_I = \left[ \frac{Z^2}{1 - Z + Z^2} \right] (u - s_n) \quad (25)$$

$$s_Y = \left[ \frac{1 - Z}{1 - Z + Z^2} \right] (u - s_n) \quad (26)$$

$$\frac{H_t^{\mu - \Phi}}{n^{1 - \eta}} = \frac{\chi}{s_n^\mu} \gamma_n \quad (27)$$

$$R \equiv \bar{\alpha} + 1 - Z \quad \Upsilon \equiv \frac{\mu - \Phi}{1 - \eta}$$

Eq. (18) gives the BGP-equilibrium growth rate of the economy's human capital stock ( $H$ ), and of the human capital employment in final output, intermediate and research sectors. Eq. (19) gives the BGP-equilibrium growth rate of the economy's stock of knowledge ( $n$ ). Eq. (20) provides the equilibrium real rate of return on asset holdings ( $r$ ). According to Eqs. (21) and (22) per capita

consumption ( $c$ ), per capita asset holdings ( $a$ ) and per capita real income ( $y \equiv Y/L$ ) all grow at the same constant rate. Eq. (23) gives the allocation of the available stock of human capital between production and educational activities along the BGP. The equilibrium shares of the existing human capital stock devoted to production of ideas ( $s_n$ ), production of intermediates ( $s_l$ ) and production of consumption goods ( $s_y$ ) are reported in Eqs. (24), (25) and (26), respectively. Finally, Eq. (27) expresses the ratio of (some function of) the two state-variables in terms of the growth rate of the number of ideas ( $\gamma_n$ ), and the share of the available human capital stock devoted to R&D-activity ( $s_n$ ). It is evident from this equation that the restriction  $\mu \neq \Phi$  prevents, *ceteris paribus*,  $\gamma_n$  to be independent of  $H_t$ .

*Assumption A* introduces constraints on the (relationship among) the feasible values of the model's parameters.

**ASSUMPTION A.** *Assume*

(i)  $\Upsilon > 0$

(ii)  $\sigma > 0$

(iii)  $\theta > \text{Max} \left\{ 0; \frac{\Upsilon R}{1 + \Upsilon R}; \frac{\Upsilon R \rho}{\sigma(1 + \Upsilon R) - [\xi(1 + \Upsilon R) - (1 + 2\Upsilon R)]g_L}; \right\}$

(iv)  $(\sigma - \rho) > \text{Max} \left\{ (\xi - 1 - \theta)g_L; \frac{[\Upsilon R(\xi - 2) + (\xi - 1)]g_L}{1 + \Upsilon R}; [\Upsilon R(1 - \xi)(\theta - 1) + \xi(1 - \theta) - 1]g_L \right\}$

The assumption  $\Upsilon > 0$  comes directly from the assumptions  $\mu > \Phi$  and  $\eta < 1$ . This also coincides with Jones (2005, p. 1074, Chap. 16, Eq.16).

*If Assumption A is satisfied, then:*

**PROPOSITION 2**

- $\gamma_H$  and  $\gamma_n$  are positive;
- $\gamma_y$  is positive;
- $r$  is positive;
- $0 < u < 1$ ;

- $r > \gamma_H - (1-R)\gamma_n$ . *Ceteris paribus*, this condition allows  $V_{nt}$  to be positive at any time  $t \geq 0$  along the BGP;
- The two transversality conditions:  $\lim_{t \rightarrow +\infty} \lambda_{at} a_t = 0$  and  $\lim_{t \rightarrow +\infty} \lambda_{ht} h_t = 0$  are simultaneously checked along the BGP.

*Proof:* When (i) and (iii) in Assumption A are met, then the denominator of Eqs. (18), (19), (20) and (22) is positive, i.e.  $[\Upsilon R(\theta-1) + \theta] > 0$ . Given this, and the fact that in the model  $g_L > 0$  and  $\sigma > 0$ , we conclude that: (i)-(ii)-(iii) ensure  $r > \gamma_H - (1-R)\gamma_n$ ,  $r > 0$ ,  $u > 0$ , and the respect of the two transversality conditions; (i)-(iii)-(iv) ensure  $\gamma_y > 0$ ,  $\gamma_H > 0$  and  $\gamma_n > 0$ . Finally, (i)-(ii)-(iii)-(iv) ensure  $u < 1$ . ■

#### 4. Population Growth and Economic Growth

The following theorem analyzes the interaction between population and economic growth rates in this economy.

##### **THEOREM**

Assume that parameter-restrictions (i) and (iii) of Assumption A are checked for  $\xi \geq 0$  and  $\Upsilon > 0$ .

Then;

- When the dilution effect of population growth on human capital investment is greater than one ( $\xi > 1$ ), the correlation between population and economic growth rates is ambiguous,

$$\text{i.e. } \frac{\partial \gamma_y}{\partial g_L} > 0, \quad \frac{\partial \gamma_y}{\partial g_L} < 0, \quad \frac{\partial \gamma_y}{\partial g_L} = 0.$$

- When  $0 \leq \xi \leq 1$ , there exists an unambiguously positive correlation between population growth and economic growth, i.e.  $\frac{\partial \gamma_y}{\partial g_L} > 0$ .

Results are summarized in **Table I** (mathematical derivation can be found in Appendix B).

When $\Upsilon R > 0$ ; $\xi \geq 0$	$\partial \gamma_y / \partial g_L$
$\xi = 0$	$\frac{\partial \gamma_y}{\partial g_L} > 0$
$\xi \leq 1$	$\frac{\partial \gamma_y}{\partial g_L} > 0$

$1 < \xi < \frac{1+2YR}{1+YR}$	$\frac{\partial \gamma_y}{\partial g_L} > 0$
$\xi = \frac{1+2YR}{1+YR}$	$\frac{\partial \gamma_y}{\partial g_L} = 0$
$\xi > \frac{1+2YR}{1+YR}$	$\frac{\partial \gamma_y}{\partial g_L} < 0$

The intuition behind the results of Theorem is as follows. By using again the BGP-equilibrium relation:

$$\gamma_y = \gamma_H + R\gamma_n - g_L \quad (28)$$

One can observe that

$$\begin{aligned} \frac{\partial \gamma_y}{\partial g_L} &= \left( \frac{\partial \gamma_H}{\partial g_L} + R \frac{\partial \gamma_n}{\partial g_L} \right) - 1 \\ &= \left( \frac{1}{Y} \frac{\partial \gamma_n}{\partial g_L} + R \frac{\partial \gamma_n}{\partial g_L} \right) - 1 \\ \frac{\partial \gamma_y}{\partial g_L} &= \underbrace{\left( \frac{1+YR}{Y} \right)}_{\substack{\text{under Assumption A} \\ > 0}} \frac{\partial \gamma_n}{\partial g_L} - \underbrace{1}_{\substack{\text{ideas} \\ \text{effect}}} \quad \text{dilution effect} \end{aligned} \quad (29)$$

According to Eq. (29), the impact of population growth on real per-capita income depends on the relative contribution of two distinct effects:

- The direct dilution effect: This effect is always negative since when newborns enter the world they reduce the existing per-capita stock of any reproducible factor–input. So, in order to equip every single member of the growing population with a given (per capita) amount of such input, some resources need to be explicitly devoted to this aim, which slows productivity growth down.
- The indirect ideas effect: This effect describes the impact that at a certain point in time an exogenous change of population size (due to a change of  $g_L$ ) may have on the economy’s growth rate of ideas ( $\gamma_n$ ), and hence on  $N$ : “...More people means more Isaac Newtons and

therefore more ideas...” (Jones, 2003, p. 505). Unlike the previous one, this effect is always positive as long as  $1 + \theta - \xi > 0$ .<sup>18</sup>

According to *Table 1*, when  $\xi \leq 1$  the impact of population growth on economic growth is always positive. However; when  $\xi > 1$ , a threshold level of  $\xi$  has emerged. In particular, when  $\xi$  is below the threshold, the ideas effect of population growth is positive and greater than the dilution effect. As a result of this, the impact of population growth on economic growth continues to be positive under a certain threshold level of dilution effect. When  $\xi$  equals to the threshold, the dilution effect neutralizes the ideas effect which is still positive, and thus; the impact of population growth on per-capita income growth is neutral. And lastly when  $\xi$  is above the threshold, the dilution effect of population growth is quite strong that results a negative impact on economic growth. Note that If  $\xi$  is sufficiently high ( $1 + \theta - \xi < 0$ ); the ideas effect of population growth can also turn to negative.

## 5. Empirical Analyses

### 5.1 An Exercise: Testing the Theory for Advanced Countries

This empirical part is a modest attempt to analyze the sign relationship between population growth and economic growth under the light of our theoretical model. As it is demonstrated in *Table 1*, the dilution effect of population growth on human capital investment has a key role in explaining the non-linear effects of population growth on economic growth. Eq. (29) also reveals that under BGP-equilibrium relation, unambiguous impacts of population growth on economic growth can be explained through the net impact of two distinct effects of population growth; the indirect ideas effect and the direct dilution effect.

The objective of this part is also twofold. First, it aims to demonstrate the impact of population growth on economic growth. Second is to point out what extent the empirical results can confirm the *Table 1* results. In order to check our objectives, we benefitted from two different approaches and two different data sets: In the first approach we intend to analyze the impact of population growth on economic growth in advanced countries. We employed a panel fixed effect

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<sup>18</sup> The ideas effect is given by  $\frac{\partial \gamma_n}{\partial g_L}$ . From Eq.(19),  $\frac{\partial \gamma_n}{\partial g_L} = \frac{\Upsilon(1+\theta-\xi)}{\Upsilon R(\theta-1)+\theta}$ , the ideas effect is positive as long as  $\xi < 1 + \theta$  and *Assumption A* are met. According to Eq.29, for a while suppose that the indirect ideas effect is zero and hence  $\partial \gamma_y / \partial g_L < 0$ . Therefore, we can accurately say that when  $\xi$  is below or equal to the threshold level,  $\xi \leq \frac{1+2\Upsilon R}{1+\Upsilon R}$ , the ideas effect is strictly positive under the requirement of (i) and (iii) of *Assumption A*, in particularly when  $\theta > \frac{\Upsilon R}{1+\Upsilon R}$ .

analysis for twenty-two countries by using *Penn World Table Data* set. Then, in order to compare the results of first approach with *Table 1*, a panel structural VAR model based on Blanchard and Quah (1989) was employed by using an updated data set of *Barro and Lee (2013)*.

## 5.2 Panel Fixed Effect Analysis with Penn World Table Data (2013)

### 5.2.1 Methodology and Data

In order to measure the effects of population growth in advanced countries we mainly focus on equations (12) and (22) in which the human capital accumulation and per-capita income growth rate are given respectively. The crucial point is that population growth  $g_L$  takes place in both equations, and is the only explanatory variable in Eq. (22) along the BGP analysis.

$$\begin{aligned} \frac{\dot{h}}{h} &= F_1((1-u), g_L) \Rightarrow \text{From Eq.(12)} \\ \frac{\dot{y}}{y} &= F_2(g_L) \Rightarrow \text{From Eq.(22)} \end{aligned}$$

According to our theoretical part, the population growth not only has a direct dilution effect on human capital accumulation but also has an indirect effect coming from its impact on the allocation of the available stock of human capital between production ( $u$ ) and educational activity ( $1-u$ ). If we reconsider the Eq. (23) the indirect effect of population growth can be shown as the following form:  $(1-u) = F(g_L)$ . Since there are no available data for  $(1-u)$  what we estimate for Eq. (12) would give us the direct effect of population growth on human capital accumulation. In order to distinguish the allocation effect  $(1-u)$  and the direct dilution effect of population growth ( $-g_L$ ) on human capital accumulation we need a good proxy for  $(1-u)$  which may be obtained from micro founded data from surveys.

Barro and Lee (2001), Gong *et al.* (2004), Cohen and Soto (2007), Kubik (2010) and Hartwig (2014) reveal that defining a proxy for human capital is the most challenging and controversial part in empirical studies because of the lack of a clear description on how to measure human capital variables, and because of the questions on the quality of the data on human capital itself. To reduce the questions on the quality and the reliability issue and to explain the sign relationship between population growth and economic growth we use *Penn World Table Data* (Feenstra *et al.* 2013) which also provides an index of human capital per-person based on years of schooling (Barro and Lee, 2013) and returns to education (Psacharopoulos, 1994).

We ultimately estimate the following equations by panel least squares method:

$$\begin{aligned}
 gy_{i,t} &= c_0 + c_1(gL_{i,t}) + \varepsilon_{i,t} \\
 gy_{i,t} &= c_0 + c_1(gL_{i,t}) + c_2(gh_{i,t}) + \varepsilon_{i,t} \\
 gy_{i,t} &= c_0 + c_1(gL_{i,t}) + c_2(gh_{i,t}) + c_3(capf_{i,t}) + \varepsilon_{i,t} \\
 gh_{i,t} &= c_0 + c_1(gL_{i,t}) + \varepsilon_{i,t}
 \end{aligned}$$

In above mentioned equations, variables  $gy_{i,t}$ ,  $gL_{i,t}$ ,  $gh_{i,t}$  and  $capf_{i,t}$  are per-capita income growth, population growth, per-capita human capital growth and share of gross capital formation for country  $i$  at time  $t$  respectively. According to Eq. (22), population growth is the only independent variable explaining the per-capita income growth rate. However for the regressions of economic growth, it is even more important to include some control variables such as share of gross capital formation (or in other words; physical capital investment) and per-capita human capital growth. In particular the physical capital is one the most significant variable in explaining the growth differences among countries. Therefore estimating the regressions without adding control variables would probably lead misspecifications in results. The data about the all variables in the analysis are taken from the Penn World Table Dataset and our sample consists of a panel of twenty-two advanced countries including *Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New-Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States* for the period of 1951 – 2010. The observations are taken annually.

**Table 2.** Panel Fixed Effect Estimations with Penn Table Data (2013)

Variables	gy (1)	gy (2)	gy (3)	gh (4)
<i>constant</i>	0.024581*** (0.0000)	0.024557*** (0.0000)	0.006073 (0.3290)	0.005837*** (0.0000)
<i>gL</i>	0.642992** (0.0117)	0.642666** (0.0119)	0.442827* (0.0908)	0.079216** (0.0194)
<i>gh</i>		0.004117 (0.9843)	0.023852 (0.9089)	
<i>capf</i>			0.077039*** (0.0012)	

Note: The values in parentheses give the p-values.

\*\*\* Variables are significant at 1% level

\*\* Variables are significant at 5% level

\* Variable is significant at 10% level

## 5.2.2 Empirical Results

At first glance, *Table 2* reveals that population growth is statistically significant and appears in all regressions with positive sign. The regression result at column (4) demonstrates that the impact of population growth on human capital accumulation is statistically significant and positive. The reason lying behind this result may come from the construction method of the index of human capital per-capita. Feenstra *et al.* (2013) might have not taken a human capital formulation such as what we based on the modified version of Uzawa-Lucas into account. However, from this result it can be inferred that in advanced countries, an exogenous change of population size leads an increase in stock of human capital, and as a result of an increase in human capital stock, we can infer that population growth leads more ideas in advanced economies. In theoretical part, we found that as long as the indirect ideas effect of population growth is greater than the direct dilution effect of population growth, the economic growth is affected positively by population growth. As it is seen from *Table 2* columns (1), (2) and (3), the impact of population growth on economic growth is statistically significant and positive in advanced countries. From this perspective, this result is therefore consistent with our benchmark theoretical model (see Eqs. (28) and (29)).

## 5.3 Panel Structural VAR Analyses with Barro-Lee (2013) Data

### 5.3.1 Methodology and Data

In previous section we showed that an exogenous increase in population growth leads a positive impact on economic growth in advanced countries. In the light of this finding, according to our theoretical results (*Table 1*) we can infer that the parameter showing the strength of the dilution effect of population growth ( $\xi$ ) must correspond a value between zero and the threshold level,  $0 \leq \xi < \frac{1+2\Upsilon R}{1+\Upsilon R}$  where the indirect ideas effect is greater than the direct dilution effect of population growth.

In order to check the theoretical and empirical results we obtained, a system of simultaneous equations composed of above-mentioned equations (12), (22) including Eq. (23) comes as a necessity.

$$\frac{\dot{h}}{h} = F_3((1-u), g_L) \Rightarrow \text{From Eq.(12)}$$

$$\frac{\dot{y}}{y} = F_1(g_L) \Rightarrow \text{From Eq.(22)}$$

$$(1-u) = F_2(g_L) \Rightarrow \text{From Eq.(23)}$$



This section uses a panel structural vector autoregression (panel SVAR) model based on a long run identification restrictions provided by Blanchard and Quah (1989) and in turn related with Lee *et al.* (2012) to estimate relationships among four variables; population growth, human capital accumulation, average years of schooling (allocation effect) and economic growth. Before going into the details of the econometric technique, a brief description of data we used in this analysis is summarized below.

The challenging part of the estimation is that since there are no good proxy for  $(1-u)$ , here we used the average years of schooling data of Barro and Lee (2013) to replace the allocation effect variable. The reason why we used Barro-Lee data set can be justified as the followings: First, This data is composed of the distribution of educational attainment of the adult population over age 15 with all levels of education – non-educated, primary, secondary and tertiary – for 146 countries. Second, their earlier estimates (Barro and Lee, 1993, 1996 and 2001) have been widely used in many studies related to growth models with human capital investment. Third, their estimations on educational attainment provide a reasonable and detailed proxy for human capital in a broad group of countries that also ease to make comparisons among different country groups over the long run. And forth, they provide average years of schooling data which is considered useful in constructing the index of human capital per-capita (Feenstra *et al.* , 2013) and to measure the stock of human capital in growth regressions (De la Fuente and Doménech, 2006).

To estimate the linkage among the equations (12), (22) and (23) we need data for four variables: Population growth rate  $gL_{i,t}$ , the time allocated to build up human capital accumulation  $(1-u)$ , the growth rates of per-capita income  $gy_{i,t}$ , and of per-capita human capital  $gh_{i,t}$ . As we stated above for  $(1-u)$  we used average years of schooling. For the growth rate of per-capita human capital or in other words, the human capital accumulation; we compute the growth rates of a fraction  $(h = H/L)$  which is obtained by dividing the total number of individuals who completed enrolments to the numbers of individuals who enrolled at least even one day in total population over age 15. This is done for two major reasons. First in our theoretical model we do not make any distinction of human capital according to the level of educational attainment. Therefore in this part we assume that an individual who completed at least primary education or more can be accepted as a human capital. This means that the stock of human capital  $H$  would equal to the sum of the individuals who completed either primary or secondary or tertiary education. Second, in our theoretical model we assume that the allocation of available stock of human capital between production and educational activities are restricted by the condition  $0 < u < 1$  of *Proposition 2*. This

proposition implies that while constructing the per-capita human capital proxy we cannot consider a population including the individuals who did not enrol to any specific level of education. Therefore this non-educated population rate of Barro-Lee (2013) can correspond to the case where  $u = 1$  in our theoretical setting.<sup>19</sup> Therefore; the reason of considering the total population  $L$  as the numbers of total enrolled people in the population over age 15 is also consistent with what set up theoretically in the previous part. For example, if an increase in population growth affects schooling (time spent for education or in other words allocation effect,  $1-u$ ) negatively, and due to this negative effect if completion rates have been falling, we may have an idea to consider a direct dilution effect of population growth on human capital accumulation as we assumed in theoretical model. For the other two variables; the growth rate of per-capita income and the growth rate of population (exogenous variable), we benefitted from the *World Bank* data set for the years between 1965 and 2010. All annual observations have been transformed to averages over periods of 5 years in order to avoid the possible business cycle effects.

As it is stated before, we use a panel structural VAR model with four mentioned variables to analyze the relationship between the variables in a system of simultaneous equations. The model, which is theoretically identified from the reduced-form of VAR model (see *Appendix C*) takes the following form:

$$X_{i,t} = \Gamma(L)X_{i,t} + u_{i,t}$$

where  $X_{i,t}$  is a  $(4 \times 1)$  vector including  $gy, gL, sch, gh$ ,  $L$  is a lag operator,  $\Gamma(L)$  is a  $(4 \times 4)$  coefficient matrix and  $u_{i,t}$  is the vector for random shocks for country  $i$  at time  $t$ . We consider structural restrictions for the theoretical identification of a typical VAR model. First we estimate a panel VAR through pooled fixed effects. Then, the theoretical restrictions are imposed according to the model based on Blanchard and Quah (1989). This method allows us to add an exogenous variable- such as population growth- into the estimation. The reason why we used Panel SVAR analysis can be justified because of this relaxation. However this estimation deals with 7 restrictions where the model is an over-identified with one more restriction than a just-identification.

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<sup>19</sup> Gong et al. (2004 p. 406) emphasize that human capital comprises a person's stock of knowledge and abilities which is not only acquired by formal education (schooling) but also acquired by informal education which may take job trainings, physical and mental fitness, social services into account as well. However the authors also underline that a human capital formulation which is based on the Uzawa-Lucas model merely rests on the result of the time dedicated to the formal education. Therefore, we do not consider the other forms of human capital in this analysis.

The coefficient matrix  $\Gamma(L)$  for the vector  $X_{i,t} = [gy, gL, sch, gh]'$  and long run constraints are given as the following:

$$\begin{pmatrix} \Delta X_{i,t}^{gy} \\ \Delta X_{i,t}^{gL} \\ \Delta X_{i,t}^{sch} \\ \Delta X_{i,t}^{gh} \end{pmatrix} = \underbrace{\begin{pmatrix} NA & C(1) & 0 & C(2) \\ 0 & NA & 0 & 0 \\ 0 & C(3) & NA & 0 \\ 0 & C(4) & C(5) & NA \end{pmatrix}}_{\Gamma(L)} \begin{pmatrix} u_{i,t}^{gy} \\ u_{i,t}^{gL} \\ u_{i,t}^{sch} \\ u_{i,t}^{gh} \end{pmatrix}$$

According to this form, economic growth is affected by population growth and per-capita human capital growth (as a control variable) but not influenced by schooling variable (first restriction denoted by 0 in the first row of coefficient matrix). Population growth is not influenced by any other variable because it is exogenously given in the theoretical model. (second, third and fourth restrictions are applied). Schooling or allocation effect  $(1-u)$  is influenced by population growth (fifth and sixth restrictions are applied for economic growth and per-capita human capital growth respectively). And lastly, population growth and schooling affect the change in growth of per-capita human capital in the long run (seventh restriction is applied for economic growth).

**Table 3. Panel SVAR Estimations with Barro-Lee Data (2013)**

1965 – 2010 N: 168	Coefficients	Std. Error	z-Statistic	Prob.
$C(1)$	1.236241	0.073457	16.82946	0.0000
$C(2)$	0.103713	0.028556	3.631950	0.0003
$C(3)$	-6.380836	0.388033	-16.44404	0.0000
$C(4)$	-0.025457	0.005164	-4.929374	0.0000
$C(5)$	0.036318	0.004420	8.216095	0.0000
Chi-Square:	293.7415	Prob. 0.0000	Long-run test for over identification	

### 5.3.2 Empirical Results

The results from the data of 21 advanced countries (excluding Germany)<sup>20</sup> for the period of 1965 – 2010 have revealed that there exists a dilution effect of population growth on human capital accumulation which is negative and significant (coefficient  $C(4)$ : -0.025,  $\xi = 0.025$ ). This estimation is consistent with our *Table 1* results for  $0 \leq \xi \leq 1$  in which population growth affects per-capita income growth rate positively (coefficient  $C(1)$ : 1.24). We also found that the impact of

<sup>20</sup> We exclude Germany due to the data availability.

population growth on time spent for education  $(1-u)$  statistically significant and negative. (coefficient C(3): -6.38). *Table 2* also reveals that the sigma parameter of Eq. (12) in the theoretical part is statistically significant and positive  $\sigma > 0$  (coefficient C(5) 0.036). It implies that the marginal product of education in the process of human capital accumulation is positive in advanced countries. This finding is consistent with our *parametric restriction (ii) of Assumption A*.<sup>21</sup>

## 6. Conclusion

Given the last population facts of the UN and all the results of the literature, it seems that population growth will hold the questions about its effects on economic prosperity. This paper therefore attempts to understand the sign of the relationship between population growth and per-capita income growth. While doing this, the paper provides an alternative but complementary theoretical framework explaining the impacts of population growth on economic growth under endogenous technological change and human capital investment by the economic agents along the BGP equilibrium.

The theoretical part of the paper has showed that an increase in the population growth (regardless of the source of demographic change such as fertility, mortality or ageing) rate yields an ambiguous (positive, negative or else neutral) impact on the growth rate of per-capita income. This ambiguity comes from the relative contribution of two distinct effects of population growth: *The direct dilution effect* and *the indirect ideas effect*. The direct dilution effect mainly rests on the modification of Lucas (1988) formulation of human capital accumulation. The present paper has showed that (i) The dilution effect has a central role in explaining the ambiguous impact of population growth on economic growth. (ii) There exists a threshold level of the dilution effect that the correlation between population and economic growth rates may be either positive or negative or neutral according to this threshold. (iii) When the dilution effect is sufficiently low,  $0 \leq \xi \leq 1$ , an unambiguously positive correlation between population growth and economic growth is obtained.

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<sup>21</sup> It is important to here that Gong *et al.* (2004 p. 403) argue that estimating the original Uzawa-Lucas models yields a negative  $\sigma < 0$  value which does not make sense since it would imply a negative marginal product of education in the process generating human capital. According to them, the original Uzawa-Lucas model will not be compatible with the time series. Unlike Gong *et al.* (2004), we are capable of finding a positive marginal product of education for the advanced countries by employing a panel structural VAR model. However in developing countries, like Gong *et al.* (2004), we also find a negative sigma value. In the benchmark theoretical model we do not assume a negative impact of education. Negativity in  $\sigma$  violates *the parameter-restriction (ii) of Assumption A* and as a result of this, an estimation for developing countries not cannot be done.

Another result of the paper is that more population growth generates an indirect ideas effect (ambiguous) on the rate of innovation and economic growth.

The second part of the paper is a modest attempt to test our theoretical findings empirically by benefitting from two different approaches: Panel fixed effect analysis and panel structural VAR analysis. First approach reveals that the impact of population growth on economic growth has been positive in twenty-two advanced countries between the years of 1951-2010. This result implies that the dilution effect of population growth on human capital accumulation should be sufficiently low in those countries. In order to measure the strength of the dilution effect, a system of simultaneous equations is set and a panel SVAR analysis (with long-run restrictions) is employed. This analysis supports that a dilution effect of population growth on human capital accumulation exists and is below then 1 and thus, threshold level. This result partially confirms the *Table 1* results of the first part. We empirically demonstrate that *when  $0 \leq \xi \leq 1$ , there exists an unambiguously positive correlation between population growth and economic growth  $\partial\gamma_y/\partial g_L > 0$  in advanced countries.* This finding also confirms why indirect ideas effect of population growth is greater than the direct dilution effect of population growth in advanced economies.

In order to confirm the theoretical results for  $\xi > 1$  we need further empirical investigations for different country groups instead of making tests based on regional classifications. In both two empirical analyses, advanced countries do not solely depend on a regional block, and we are able to get partial results, however in Barro-Lee (2013) data set, developing countries are categorized in regional blocks. By using different classifications - such as income levels - one may find statistically significant results for the impact of dilution effect of population growth on economic growth in developing countries.

Lastly, we believe that these findings shed new lights on the determinants of the ambiguous impacts of population growth on economic growth, and will help to introduce more realistic models to the literature of modern growth theory with human capital accumulation. We underline that further empirical research (e.g. a panel data or standard VAR analyses with good proxies for human capital and allocation effect) would be a good extension to verify the all theoretical results we obtained.

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## APPENDICES

### APPENDIX A: EQS. (18) – (27)

The *Hamiltonian function* ( $J_t$ ) related to the intertemporal problem (13) in the body-text is:

$$J_t = \left( \frac{c_t^{1-\theta} - 1}{1-\theta} \right) e^{-(\rho-g_L)t} + \lambda_{at} \left[ (r_t - g_L) a_t + (u_t h_t) w_t - c_t \right] + \lambda_{ht} \left[ \sigma(1-u_t) - \xi g_L \right] h_t,$$

Where  $\lambda_{at}$  and  $\lambda_{ht}$  are the *co-state variables* associated, respectively, to the *state variables*  $a_t$  and  $h_t$ .

The necessary *FOCs* are:

$$(A1) \quad \frac{\partial J_t}{\partial c_t} = 0 \quad \Leftrightarrow \quad \frac{e^{-(\rho-g_L)t}}{c_t^\theta} = \lambda_{at}$$

$$(A2) \quad \frac{\partial J_t}{\partial u_t} = 0 \quad \Leftrightarrow \quad \lambda_{at} = \frac{\sigma}{w_t} \lambda_{ht}$$

$$(A3) \quad \frac{\partial J_t}{\partial a_t} = -\dot{\lambda}_{at} \quad \Leftrightarrow \quad \lambda_{at} (r_t - g_L) = -\dot{\lambda}_{at}$$

$$(A4) \quad \frac{\partial J_t}{\partial h_t} = -\dot{\lambda}_{ht} \quad \Leftrightarrow \quad \lambda_{at} u_t w_t + \lambda_{ht} \left[ \sigma(1-u_t) - \xi g_L \right] = -\dot{\lambda}_{ht}$$

along with the two transversality conditions:

$$\lim_{t \rightarrow +\infty} \lambda_{at} a_t = 0, \quad \lim_{t \rightarrow +\infty} \lambda_{ht} h_t = 0,$$

and the initial conditions:

$$a(0) > 0, \quad h(0) > 0.$$

Combining (A2) and (A4) yields:

$$(A5) \quad \frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = -(\sigma - \xi g_L)$$

Eqs. (A3) and (A2) imply, respectively:

$$(A6) \quad \frac{\dot{\lambda}_{at}}{\lambda_{at}} = -(r_t - g_L)$$

$$(A7) \quad \frac{\dot{\lambda}_{at}}{\lambda_{at}} = \frac{\dot{\lambda}_{ht}}{\lambda_{ht}} - \frac{\dot{w}_t}{w_t}$$

Combination of (A5), (A6) and (A7) leads to:

$$(A8) \quad r_t = (1 - \xi) g_L + \sigma + \frac{\dot{w}_t}{w_t}$$

Since human capital is perfectly mobile across sectors, at equilibrium it will be rewarded according to the same wage:  $w_{Yt} = w_{It} = w_{nt} \equiv w_t$ . Moreover, along the BGP this common wage would grow at

a constant exponential rate, implying that  $\frac{\dot{w}_{Yt}}{w_{Yt}} = \frac{\dot{w}_{It}}{w_{It}} = \frac{\dot{w}_{nt}}{w_{nt}} \equiv \frac{\dot{w}_t}{w_t}$  is constant. Accordingly, in the

BGP equilibrium the real rate of return on asset holdings,  $r$  will be constant (Eq. A8). With  $r$  constant, and making use of Eqs. (6) and (10) in the main text, we find that along the BGP:

$$(A9) \quad V_{nt} = Z(1-Z) \left( \frac{H_{Yt}}{n_t} \right)^{1-Z} \left( \frac{H_{It}}{n_t} \right)^Z \frac{n_t^R}{[r + (1-R)\gamma_n - \gamma_H]}, \quad R \equiv \bar{\alpha} + 1 - Z, \quad \frac{\dot{n}_t}{n_t} \equiv \gamma_n, \quad \frac{\dot{H}_t}{H_t} \equiv \gamma_H$$

For any  $0 < Z < 1$ ,  $H_Y > 0$ ,  $H_I > 0$ ,  $n > 0$ , and  $R > 0$ ,  $V_{nt}$  is positive provided that:

$$(A9') \quad r > \gamma_H - (1-R)\gamma_n$$

Given  $V_{nt}$ , from Eq. (9') in the main text:

$$(A10) \quad w_{nt} = \frac{Z}{\chi} (1-Z) s_n^{\mu-1} H_t^{\mu-1-\Phi} n_t^\eta \left( \frac{H_{Yt}}{n_t} \right)^{1-Z} \left( \frac{H_{It}}{n_t} \right)^Z \frac{n_t^R}{[r + (1-R)\gamma_n - \gamma_H]}$$

where  $s_n \equiv H_{nt}/H_t$  is constant along the BGP. We can now use Eqs. (5), (2) and (4') in the main text, obtaining:

$$(A11) \quad w_{It} = Z^2 \left( \frac{H_{Yt}}{n_t} \right)^{1-Z} \left( \frac{H_{It}}{n_t} \right)^{Z-1} n_t^R$$

From Eq. (15) in the main text, by equalizing (A11) and (A10) in this appendix one gets:

$$(A12) \quad s_I \equiv \frac{H_{It}}{H_t} = \frac{Z\chi}{(1-Z)} \frac{[r + (1-R)\gamma_n - \gamma_H]}{s_n^{\mu-1}} \frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}}$$

Combining Eqs. (1) and (4') in the text:

$$(A13) \quad w_{Yt} \equiv \frac{\partial Y_t}{\partial H_{Yt}} = (1-Z) \left( \frac{H_{Yt}}{n_t} \right)^{-Z} \left( \frac{H_{It}}{n_t} \right)^Z n_t^R$$

From (16) in the main text and (A12) above, equalization of Eqs. (A11) and (A13) in this appendix delivers:

$$(A14) \quad s_Y \equiv \frac{H_{Yt}}{H_t} = \left( \frac{1-Z}{Z^2} \right) s_I = \frac{\chi}{Z} \frac{[r + (1-R)\gamma_n - \gamma_H]}{s_n^{\mu-1}} \frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}}$$

Along the BGP all variables depending on time grow at constant rates and the sector shares of human capital employment are also constant. Therefore, from Eq. (8) in the main text:

$$(A15) \quad \frac{\dot{n}_t}{n_t} \equiv \gamma_n = \left( \frac{\mu - \Phi}{1 - \eta} \right) \gamma_H, \quad \gamma_H \equiv \frac{\dot{H}_t}{H_t}$$

If  $\mu - \Phi = 1 - \eta$  we have a very special case of the model in which human and technological capital grow at the same rate  $\gamma_n = \gamma_H \equiv \gamma$  along the BGP. We rule out this possibility and analyze the most general possible case:  $\mu \neq \Phi \neq \Phi + 1 - \eta$ .

Using Eqs. (A10), (A11), (A13) and (A15) we see that along the BGP wages grow at a common and constant rate:

$$(A15') \quad \frac{\dot{w}_{nt}}{w_{nt}} = \frac{\dot{w}_{It}}{w_{It}} = \frac{\dot{w}_{Yt}}{w_{Yt}} \equiv \frac{\dot{w}_t}{w_t} = R\gamma_n$$

Combining Eqs. (A1) and (A6), the usual Euler equation follows:

$$(A16) \quad \frac{\dot{c}_t}{c_t} \equiv \frac{1}{\theta}(r - \rho), \quad c \equiv \frac{C}{L}$$

From (17) in the text and (A9) in this appendix we conclude that along the BGP:

$$(A17) \quad \frac{\dot{a}_t}{a_t} \equiv \gamma_a = \gamma_H + R\gamma_n - g_L, \quad a_t \equiv A_t/L_t, \quad g_L \equiv \dot{L}_t/L_t$$

Merging (11') in the main text and (A6) in this appendix yields:

$$(A18) \quad \frac{\dot{\lambda}_{at}}{\lambda_{at}} = -\gamma_a + u_t \frac{h_t w_t}{a_t} - \frac{c_t}{a_t}, \text{ where } u_t = u, \forall t \geq 0 \text{ along the BGP.}$$

Instead, from the combination of (12) in the text and (A5) in this appendix we get:

$$(A19) \quad \frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = -\gamma_h - \sigma u, \quad h_t \equiv H_t/L_t, \quad \gamma_h \equiv \dot{h}_t/h_t$$

Eqs. (A7), (A15'), (A17), (A18) and (A19) together lead to:

$$(A20) \quad \frac{c_t}{a_t} = u \left[ \frac{h_t w_t}{a_t} + \sigma \right], \text{ where } \gamma_h = \gamma_H - g_L \text{ has been used.}$$

Using (12) in the main text, Eqs. (A15'), (A17) and (A20) and the fact that  $\gamma_H = \gamma_h + g_L = \sigma(1-u) + (1-\xi)g_L$  one obtains:

$$(A20') \quad \frac{c_t}{a_t} = u \left[ \frac{h(0)w(0)}{a(0)} + \sigma \right],$$

where  $h(0)$ ,  $w(0)$  and  $a(0)$  are the initial values (*i.e.*, at  $t=0$ ) of  $h_t$ ,  $w_t$  and  $a_t$ , respectively. With  $u$  constant, the just-mentioned initial values given, and  $\sigma > 0$  the last equation implies:

$$(A21) \quad \gamma_c = \gamma_a$$

This means that along the BGP  $a_t$  and  $c_t$  grow at the same rate. Using (A21) and equating (A16) and (A17) it is possible to get:

$$(A22) \quad r = \rho + \theta(\gamma_H + R\gamma_n - g_L)$$

Next, by equalizing (A22) to (A8), and using (A15'):

$$(A23) \quad \gamma_n \equiv \frac{\dot{n}_t}{n_t} = \frac{[(\sigma - \rho) - (\xi - 1 - \theta)g_L - \theta\gamma_H]}{R(\theta - 1)}$$

Equating (A23) to (A15), and solving for  $\gamma_H$ , we finally obtain:

$$(A23') \quad \gamma_H \equiv \frac{\dot{H}_t}{H_t} = \frac{[(\sigma - \rho) - (\xi - 1 - \theta)g_L]}{\Upsilon R(\theta - 1) + \theta}, \quad \Upsilon \equiv \left[ \frac{\mu - \Phi}{1 - \eta} \right]$$

Given  $\gamma_H$ , it is possible to re-cast  $\gamma_n$  as:

$$(A23'') \quad \gamma_n \equiv \frac{\dot{n}_t}{n_t} = \frac{\Upsilon [(\sigma - \rho) - (\xi - 1 - \theta)g_L]}{\Upsilon R(\theta - 1) + \theta} = \Upsilon \gamma_H$$

Eqs. (A23') and (A23'') confirm that  $\gamma_H = \gamma_n = \frac{[(\sigma - \rho) - (\xi - 1 - \theta)g_L]}{R(\theta - 1) + \theta}$  in the special case

$\Upsilon \equiv \left[ \frac{\mu - \Phi}{1 - \eta} \right] = 1$ . The BGP equilibrium- value of  $r$  is obtained by combining (A22), (A23') and (A23''):

$$(A22') \quad r = \frac{\sigma\theta + \Upsilon R(\sigma\theta - \rho) - \{\theta[\xi(1 + \Upsilon R) - (1 + 2\Upsilon R)]\}g_L}{\Upsilon R(\theta - 1) + \theta}$$

Eqs. (A21) and (A16) together imply:

$$(A21') \quad \gamma_a \equiv \frac{\dot{a}_t}{a_t} = \gamma_c \equiv \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r - \rho), \quad \text{where } r \text{ is given by Eq. (A22').}$$

After using Eq. (12) in the main text, the definition of  $h \equiv H/L$  and the fact that  $\dot{L}/L \equiv g_L$ , we conclude:

$$(A24) \quad \gamma_H \equiv \frac{\dot{H}}{H} = \sigma(1 - u) + (1 - \xi)g_L$$

Equalization of (A23') and (A24) allows obtaining the BGP equilibrium value of  $u$ :

$$(A25) \quad u = 1 - \frac{(\sigma - \rho) - [\Upsilon R(1 - \xi)(\theta - 1) + \xi(1 - \theta) - 1]g_L}{\sigma[\Upsilon R(\theta - 1) + \theta]}$$

From (1) in the text, (A22'), (A23') and (A23'') in this appendix, the hypothesis of symmetry (Eq. 4' in the text), the definitions of  $y(\equiv Y/L)$ ,  $R \equiv \bar{\alpha} + 1 - Z$  and  $\dot{L}/L \equiv g_L$ , we obtain the growth rate of real per-capita output along the BGP:

$$(A26) \quad \gamma_y \equiv \frac{\dot{y}_t}{y_t} = \frac{(1 + \Upsilon R)(\sigma - \rho) - [\Upsilon R(\xi - 2) + (\xi - 1)]g_L}{\Upsilon R(\theta - 1) + \theta} = \gamma_a = \gamma_c = \frac{1}{\theta}(r - \rho)$$

We now compute the BGP-equilibrium values of  $s_n$ ,  $s_I$  and  $s_Y$ . Eq. (14) in the main text suggests:

$$u = s_Y + s_I + s_n$$

From (A14) in this appendix we use  $s_Y = \left(\frac{1-Z}{Z^2}\right)s_I$  into the expression above and obtain:

$$(A27) \quad s_I = \left[ \frac{Z^2}{1-Z+Z^2} \right] (u - s_n), \quad \text{where } u \text{ is given by (A25).}$$

Hence:

$$(A28) \quad s_Y = \left[ \frac{1-Z}{1-Z+Z^2} \right] (u - s_n)$$

According to (A14), however, it is also true that:

$$s_Y \equiv \frac{H_{Yt}}{H_t} = \frac{\chi}{Z} \frac{[r + (1-R)\gamma_n - \gamma_H]}{s_n^{\mu-1}} \frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}}$$

Equating this expression to (A28) yields:

$$(A29) \quad \frac{H_t^{\mu-\Phi}}{n_t^{1-\eta}} = \frac{\chi [1-Z+Z^2]}{Z(1-Z)} \frac{[r + (1-R)\gamma_n - \gamma_H]}{s_n^{\mu-1} (u - s_n)}$$

From Eq. (8) in the body-text:

$$(A30) \quad \frac{H_t^{\mu-\Phi}}{n_t^{1-\eta}} = \frac{\chi}{s_n^\mu} \gamma_n$$

Equalization of (A29) and (A30) leads to:

$$(A31) \quad s_n = \frac{Z(1-Z)\gamma_n}{[1-Z+Z^2][r + (1-R)\gamma_n - \gamma_H] + Z(1-Z)\gamma_n} u$$



Given Eqs. (A22'), (A23'), (A23''), (A25) and (A31), it is possible to compute the BGP ratio  $\frac{H_t^{\mu-\Phi}}{n^{1-\eta}}$  (by using either Eq. 29 or Eq. A30), along with  $s_t$  and  $s_y$  (Eqs. A27 and A28).

Finally, by employing Eqs. (A6), (A7), (A15'), (A17) and the definition of  $h \equiv H/L$ , it can be showed that along the BGP the two transversality conditions  $\lim_{t \rightarrow +\infty} \lambda_{at} a_t = 0$ ,  $\lim_{t \rightarrow +\infty} \lambda_{ht} h_t = 0$  are simultaneously checked when:  $r > \gamma_H + R\gamma_n$

In turn, when the two transversality conditions are met, then the requirement (Eq. A9'):  $r > \gamma_H - (1-R)\gamma_n$  is also met, for any positive. ■

## APPENDIX B: TABLE 1

$$\frac{\partial \gamma_y}{\partial g_L} = \frac{-[\Upsilon R(\xi - 2) + (\xi - 1)]}{\Upsilon R(\theta - 1) + \theta}$$

When (i) and (iii) in Assumption A in the main text are met,  $[\Upsilon R(\theta - 1) + \theta] > 0$  is always satisfied.

With  $\Upsilon > 0$ ,  $R > 0$ , and  $\xi \geq 0$  we conclude:

- $\frac{\partial \gamma_y}{\partial g_L} > 0 \Rightarrow -\Upsilon R(\xi - 2) - (\xi - 1) > 0 \Rightarrow \begin{aligned} & -\Upsilon R(\xi - 2) > (\xi - 1) \\ & \Upsilon R\xi + \xi < 1 + 2\Upsilon R \\ & \xi(1 + \Upsilon R) < 1 + 2\Upsilon R \\ & \xi < \frac{1 + 2\Upsilon R}{1 + \Upsilon R} \end{aligned}$
- $\frac{\partial \gamma_y}{\partial g_L} < 0 \Rightarrow -\Upsilon R(\xi - 2) - (\xi - 1) < 0 \Rightarrow \xi > \frac{1 + 2\Upsilon R}{1 + \Upsilon R}$
- $\frac{\partial \gamma_y}{\partial g_L} = 0 \Rightarrow -\Upsilon R(\xi - 2) - (\xi - 1) = 0 \Rightarrow \xi = \frac{1 + 2\Upsilon R}{1 + \Upsilon R}$
- if  $\xi = 1 \Rightarrow \frac{\partial \gamma_y}{\partial g_L} = \frac{-[\Upsilon R(-1)]}{\Upsilon R(\theta - 1) + \theta} = \frac{\Upsilon R}{\Upsilon R(\theta - 1) + \theta} \Rightarrow \frac{\partial \gamma_y}{\partial g_L} > 0$

- if  $\xi = 0 \Rightarrow \frac{\partial \gamma_y}{\partial g_L} = \frac{-[YR(-2) + (-1)]}{YR(\theta - 1) + \theta} = \frac{2YR + 1}{YR(\theta - 1) + \theta} \Rightarrow \frac{\partial \gamma_y}{\partial g_L} > 0$  ■

### APPENDIX C: REDUCED FORM OF PANEL VAR MODEL

An unrestricted VAR model where  $X_t$  is the vector of variables in question and  $u_t$  is the vector of random shocks

$$X_t = A^{-1}\Gamma(L)X_t + A^{-1}Bu_t$$

After having arranged the unrestricted VAR model, we obtain:

$$X_t = [I - A^{-1}\Gamma(L)]^{-1}A^{-1}Bu_t$$

Equation above reflects how random shocks affect the long run levels of the variables. Given the matrix  $C = [I - A^{-1}\Gamma(L)]^{-1}A^{-1}B$ , the aggregate effect of the random shock  $u_t$  is given by the matrix  $C$ . Long run impact is obtained by the cumulative effect of the random shock  $u_t$  on  $X_t$ . The matrix  $C$  for the vector  $X_t = [gy, gL, sch, gh]'$  is therefore given in the main text as the as the following form:

$$C = \begin{pmatrix} NA & NA & 0 & NA \\ 0 & NA & 0 & 0 \\ 0 & NA & NA & 0 \\ 0 & NA & NA & NA \end{pmatrix}$$

■