Geomechanical surveys and geostatistical analyses in Valchiavenna (Italian Central Alps)

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Abstract

The present study deals with the forecast of geomechanical features in rock masses, out from survey points, and how mechanical properties can be regarded as regionalized variables. It considers an area of about 200 km^2, located in the Italian Central Alps, along the San Giacomo Valley (SO), where different civil and mining works are present.

The regional geological setting is related to the Pennidic Nappe arrangement, characterized by the emplacement of sub-horizontal gneissic bodies, separated by a metasedimentary cover unit.

Almost one hundred geomechanical field surveys were carried out in order to characterize the rock masses, in accordance with the I.S.R.M. suggested methods. This procedure allowed to identify the number of joint sets and their average orientations, supplying a quantitative description of the discontinuities in terms of spacing, persistence, roughness, aperture, filling, wall strength, weathering and moisture conditions. From collected data, the rock mass quality indexes were evaluated in each surveyed site.

Geostatistical methods were applied to study the spatial distribution of main rock mass characteristics, such as the horizontal intercept and the Volumetric Joint Count, being the direct survey data local. Where no data were available the rock mass features were estimated; the results obtained by kriging and conditional simulation techniques are here presented and compared.

1 Introduction

The forecast of geomechanical properties can be an important goal in civil and mining engineering planning, especially when a wide area is involved. The most common measurement techniques of rock mass properties give punctual values, but know the distribution of these properties in the entire study area can be very important and useful in different fields of geosciences and geo-engineering. In particular, the spatial distribution of rock fractures must be known in solving hydro-geological problems of fracture-affected flow channels, in resource exploration activities for vein-type mineral deposits and fluids in fractured reservoirs (National Research Council, 1996; Adler and Thovert, 1999), but also in slope stability evaluation.

Fractures with different origins and various scales are generally developed in rock masses, but the quantitative description of their feature values is limited by the number and distribution of outcrops; the estimate of fracture properties in a whole area can be made using geostatistical techniques (Isaaks and Srivastava, 1989; Villaescusa and Brown, 1990). Indeed the study of fracture attributes, in relation with distance between survey points, can reveal spatial correlation structures. Geostatistics can incorporate these structures, which mean spatial dependence of regionalized variable at different location in space. Some authors have applied geostatistical approach to problems of rock mass fracture-distribution modelling (Long and Billaux, 1987; Young, 1987; Chilès, 1988; Gringarten, 1996) or to rock mass specific properties. For instance La Pointe (1980) used geostatistics to indicate the degree of inhomogeneity in frequencies and orientation of two distinct joint sets. Barla et al. (1987) applied geostatistical analysis to rock mass characterization, using Rock Mass Rating index. Young (1987) applied indicator kriging to evaluate the local probability distribution of rock joint orientations. Hoerger et al. (1987) furnished local estimates of rock mass conditions obtained through geostatistics. Yu and Mostyn (1993) reviewed concepts and models used to model the spatial correlation of joint geometric parameters. They concluded that rock mass parameters can be estimated using geostatistical interpolation methods.

This work is a contribute in assessing how the properties of rock masses can be regarded as regionalized variables; focusing particularly on the fracture density, which is the parameter that best summarizes rock mass characteristics.

2 The research area: geological and structural setting

Figure 1: the location of study area, red circle represents the Chiavenna Valley

The study area is located in the Italian Central Alps; in particular it is lengthen along the Chiavenna Valley (Provence of Sondrio), which is a glacial valley, situated between Lake Como and the Splügen Pass (Figure 1). The Chiavenna Valley consists of two main valleys (San Giacomo and Bregaglia valleys), which connect Italy to Switzerland. In this paper we focus on the San Giacomo Valley, whose extension is about 200 km^2.

The Central Northern Alps are a fold and trust system. The major trust sheets were created during the Alpine compressional phase and were imbricated from South to North, forming, in the region of interest, the Pennidic Nappe arrangement. The regional geological setting is characterized by the emplacement of sub-horizontal gneissic bodies resulting from the Mesoalpine isoclinalic folding of crystalline basements (the "Tambò" and "Suretta" Units) emplaced throw east and separated by a metasedimentary cover unit, called "Spluga Syncline" or also the Tambò cover Unit. The tectonic contact between the two main nappes gently dips to E–NE.

The investigated rock masses belong to the Upper Pennine Nappe, in particular to the Tambò Unit, overlapped by its meta-sedimentary cover and to the Suretta Unit. The Tambò basement is mainly constituted by polycyclic and poly-metamorphic rocks: two micas paragneiss, micaschist and metagranite with subordinated anphibolitic levels. Its metasedimentary cover (the Spluga Syncline) is formed by highly laminated micaschist, phillades and mylonitic rocks. Levels of hard metavolcanic rocks are included in the cover and subordinately in the basement. The lithological features of Suretta basement are almost the same of Tambò Unit.

Alpine pressure-dominated metamorphism did not reach conditions higher than blue-schist facies, and the eclogite facies present in the Upper Pennine Units (Tambò and Suretta) are ascribed to the Pre-alpine metamorphic events.

Four main Alpine deformation phases were recognised in the upper eastern Pennine Units (Huber and Marquer, 1988) related to: the closure of the Valais Pennine basin, the north-westward thrust structure formation during the Eocene subduction; the Oligo–Miocene collision accompanied with a syn-collisional E–W extension. The second deformation phase induced the most penetrative ductile structures and is responsible of the main regional schistosity which is parallel to the contact between the Suretta and Tambò nappes. Major ductile detachment zones cross-cut the nappe tectonic contact. Subsequent deformation structures are related to the late and Post-alpine deformation and are due to the vertical extrusion of crustal block at north of the Insubric lineament and to the brittle–ductile E–W extension parallel to the Forcola line. The two late deformation phases overprinted and steepened the previous structures, and produced an extensive fracturing pattern, dominated by two sets orientated NW-SE and NE-SW, mainly expressed by normal faults which cross-cut all previous structures.

The San Giacomo Valley, furrowed by the Liro Stream, follows an almost N–S striking tectonic lineament, which is accompanied by minor parallel sub-vertical structural elements responsible for a series of geomorphologic terraces on both sides of the valley. Deep seated flank deformations, structurally controlled, are present especially on the upper portion of the valley, while rockfalls sometimes occur chiefly on the left hydrographical side of San Giacomo Valley, characterized by high rock walls.

3 Local rock mass properties

In San Giacomo Valley, geomechanical surveys were carried out in 97 different sites, mainly located on the left side of the Liro Stream; 78 surveys involve the Tambò basement, 7 the Spluga Syncline, and 12 the Suretta basement (Figure 2). Measurement points are very scattered, because they are strongly affect by position and accessibility of outcrops.

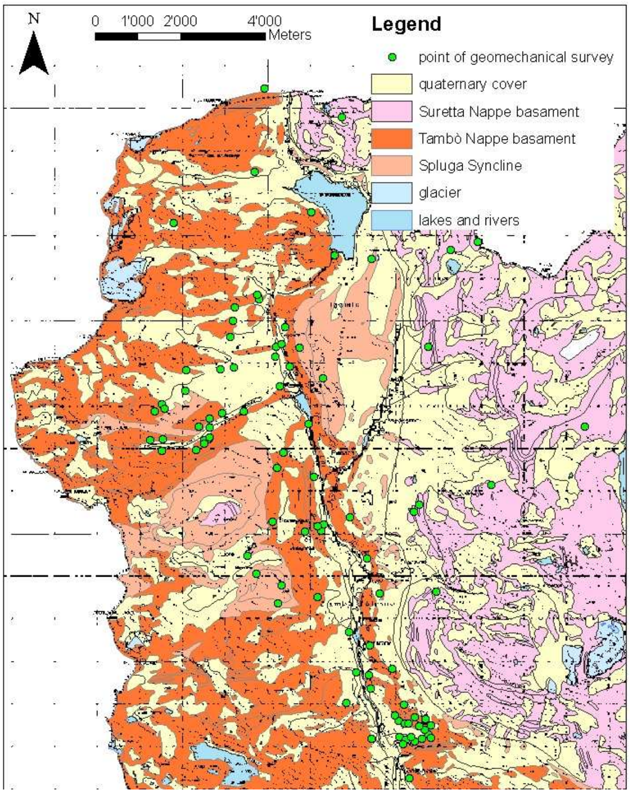


Figure 2: geological sketch map of study area with superimposed the location of surveys

Detailed geomechanical field surveys, performed according to the ISRM suggested methods (ISRM, 1978), allowed to characterize each investigated rock mass, its intact rock and discontinuities, in terms of: number of main joint sets, their representative orientation, vertical and horizontal intercept, average set spacing, persistence, aperture, degree of weathering, moisture conditions, roughness coefficient, wall strength, presence and nature of infill. From collected data, rock mass quality indexes, such as the Rock Mass Rating (Bieniawski, 1989) and the Geological Strength Index (Hoek and Brown, 1997), were evaluated in each surveyed site.

Some general considerations can be outlined to describe analogies and differences in the investigated rock masses. The examined rock masses, belonging to the Tambò and Suretta basement Units, show a similar behaviour. Joint orientations and properties are similar, and the little variability in lithological characters does not control significantly the discrepancy in rock mass quality. The rock masses of meta-sedimentary cover show a general greater state of deformation. The high lithological variability is obviously responsible for a wider variation in rock mass quality, but it is worth to note that the groundwater conditions appear to be an important controlling factor.

The Rock Mass Rating (RMR) values ranges from 45 to 77, half of them belong to the “fair quality” class (with RMR included between 41 and 60), while the other half belong to the “good quality” class (RMR ranges from 61 to 80), mostly of these values are below 70, with the highest value (equal to 77) found in correspondence of a quartzite outcrop.

The intercept, the average joint spacing and consequently the Volumetric Joint Count (Palmstrom, 1982) are the factors mainly responsible for the regional variation of rock mass quality. The distance from the local tectonic lineaments seems to play a significant role in joint intensity of fracturing (Apuani et al., 2009), but this statement needs more insights, it is necessary to investigate the regional trend of the rock mass features that are till now punctual. For this reason the geostatistical analyses start to study intercept and Volumetric Joint Count, which is derived from average spacing.

4 Geostatistical analyses

Geostatistical approach, which investigates the spatial behaviour of regionalized variables, has just been used several times in rock mass characterisation.

Geostatistical analyses were performed in order to reconstruct rock mass mechanical properties, considering many different features, particularly the fracture density, which is the parameter that more influences the mechanical and hydro-geological rock mass behaviour. The fracture density is studied using two different parameters: the horizontal intercept, which is the mean distance between all fractures in a rock mass, independently from their orientation, measured along an horizontal scan line, and the Volumetric Joint Count (Jv) derived from the average spacing of each discontinuity set. The Jv is a measure of the number of joints within a unit volume of rock mass, defined by the following formula:

$$Jv = \Sigma\ (1/Si)$$

where S is the joint spacing in metres for the each joint set i. Since Jv is based on joint measurements of spacings or frequencies, it can easily be calculated.

The geostatistical analyses, performed using as regionalized variables the horizontal intercept and Jv, were developed by the following phases: exploratory spatial data analyses, variography, prediction and finally validation.

4.1 Exploratory spatial data analyses

First of all, for each defined regionalized variable, a study of main statistical parameters was carried out; the descriptive statistical parameters of horizontal intercept and Jv measurements can be summarized as follow. The Jv was calculated in every sampling location, but only in 61 sites was possible to measure the horizontal intercepts. Sampling values range from 5.2 to 41.2centimeters for the horizontal intercept and from 6.7 to 66.6fractures/m^3 for the Jv. The resulting mean value is 19.16cm for intercept, with a standard deviation of 10.02, and 25.27fractures/m^3 for Jv, with a standard deviation of 13.27. The frequency distributions are clearly uni-modals (Figure 3), with positive asymmetry, skewness of 0.70 for the horizontal intercept and of 0.65 for the Jv, and kurtosis of -0.49 for intercept and -0.44 for Jv.

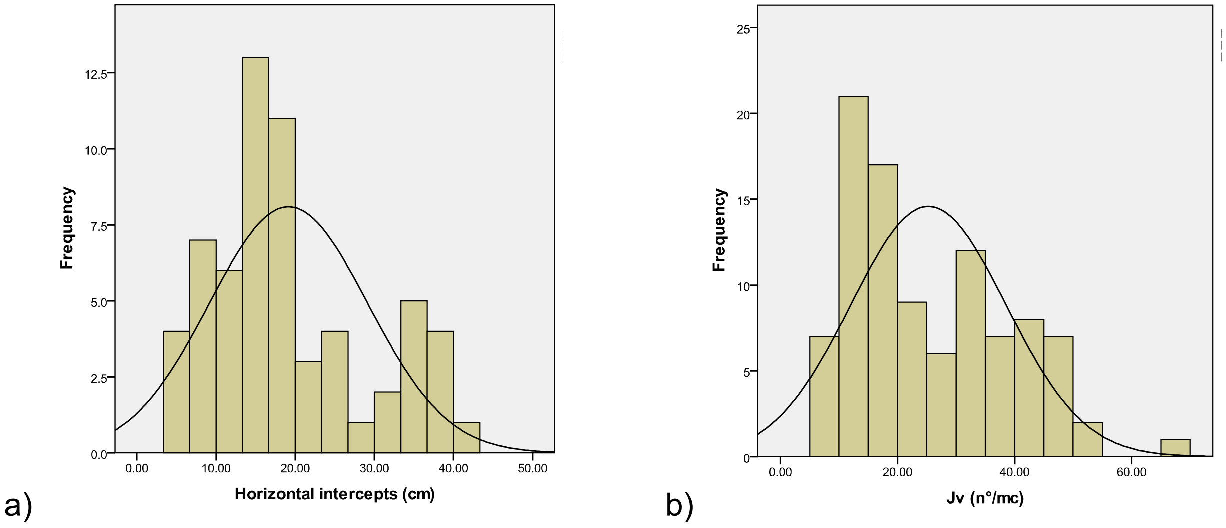
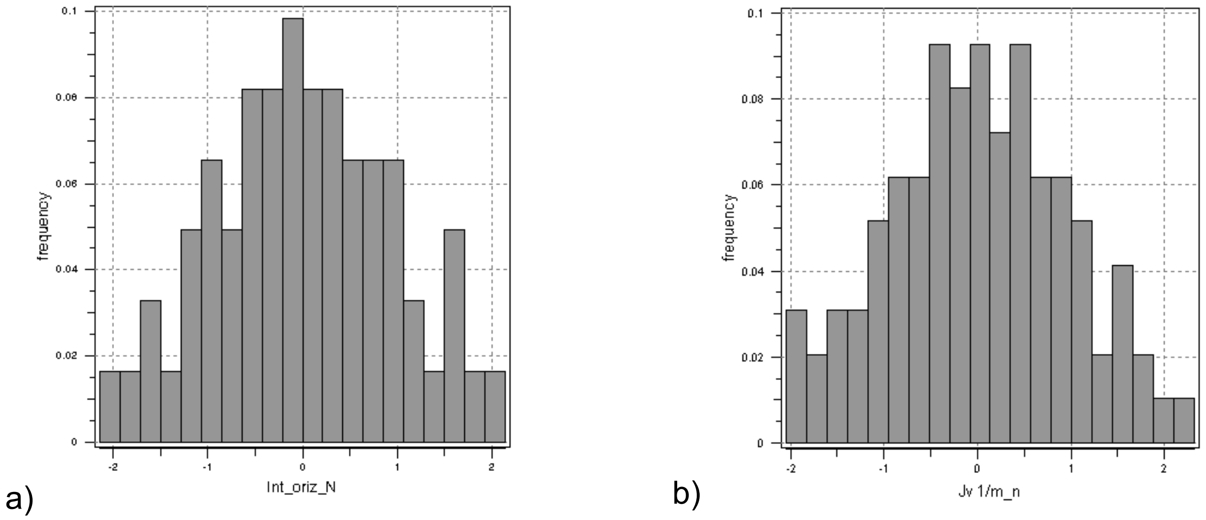


Figure 3: frequency distribution histograms of: raw horizontal intercept data (a) and raw Jv data (b); continuous lines represent the best-fitted normal distribution functions

Since many geostatistical techniques are more reliable if the variable of interest have a Gaussian distribution, it is necessary to verify if the variable has a normal distribution and if it is not the transformation of data in to a Gaussian one is essential. It is rare in modern geostatistics to consider untransformed data. The use of Gaussian techniques requires a prior Gaussian transformation of the data and the reconstruction of semivariogram model on these transformed data; this has some important advantages: the difference between extreme values is dampened and the theoretical sill is known to be 1 (Gringarten and Deutsch, 2001). Also, systematic trends should be removed from the variable prior to transformation and semivariogram calculation.

Both intercept and Jv values approximate a log-normal distribution, so the values were transformed using their logarithm. The normality of transformed data was verified using various graphical and statistical tests, such as Shapiro-Wilk test (Shapiro and Wilk, 1965) and Kolmogorov-Smirnov test with Lilliefors correction (Lilliefors, 1967).

Transformed data were then used in geostatistical analyses, their frequency distributions are shown in Figure 4. The absence of trends allowed to confirm the stationarity property of the considered variables over the studied domain.



a) b)

Figure 4: frequency distribution histograms of: transformed horizontal intercept data (a) and transformed Jv data (b)

4.2 Variography

The construction of semivariogram, a mathematical model that captures the spatial correlation between data, is a very important step in any geostatistical analysis. The semivariogram is a measure of variability; it increases as samples become more dissimilar. The variogram is defined as

$$2\gamma(\mathbf{h}) = \mathrm{Var}\,[Y(\mathbf{u}) - Y(\mathbf{u} + \mathbf{h})] = E\,\{[Y(\mathbf{u}) - Y(\mathbf{u} + \mathbf{h})]^2\}$$

where Y is a stationary random function with known mean m and variance σ^2, which are independent of location, so $m(\mathbf{u}) = m$ and $\sigma^2(\mathbf{u}) = \sigma^2$ for all locations $\mathbf{u}$ in the study area, therefore the variogram function depends only on the distance $\mathbf{h}$ and so the intrinsic hypothesis occurs.

The variogram is the expected squared difference between two data values separated by a distance vector. The semivariogram $\gamma(\mathbf{h})$ is one half of the variogram $2\gamma(\mathbf{h})$, to avoid excessive jargon we simply refer to it with the term variogram.

If a variable is correlated, initially the variogram increases and than becomes stable beyond a distance **h** called the “range”. Beyond this distance, the mean square deviation between two quantities $Y(\mathbf{u})$ and $Y(\mathbf{u} + \mathbf{h})$ no longer depends on the distance **h** between them and the two quantities are no longer correlated. When the range is different in some directions of space, the examined regionalized variable exhibits a geometric anisotropic structure. The range corresponds to a variance value called “sill”, which corresponds to zero correlation.

Variography is here applied to recognize the fracture density spatial distribution of the examined rock masses. The tool applied to assess the spatial structure of intercept and Jv is the variogram, which was constructed using the transformed data. The correlation structures of variables were investigated at different scales, taking into account the possible occurrence of anisotropies.

First of all, both for the horizontal intercept and for the Jv, independently, an omni directional variogram was constructed in order to individuate if a correlation of the variable in the research area exists. The presence of any preferential correlation direction was firstly sought graphically using a 2D variogram map (Figure 5), which is a plot of experimental variogram values in a coordinate system (h_x ; h_y) with the centre of the map corresponding to the variogram at lag 0.0 (Goovaerts, 1997). A more detailed research of major correlation direction was conducted through the construction of several directional variograms.

Figure 5: 2D variogram maps of: horizontal intercept (a) and Jv transformed data (b)

For each variable three experimental variograms were constructed at different scales, varying the lag distance from 250meters to 1000meters, therefore the maximum distance under study increases. The lag tolerance was assumed to be equal to half of the lag distance.

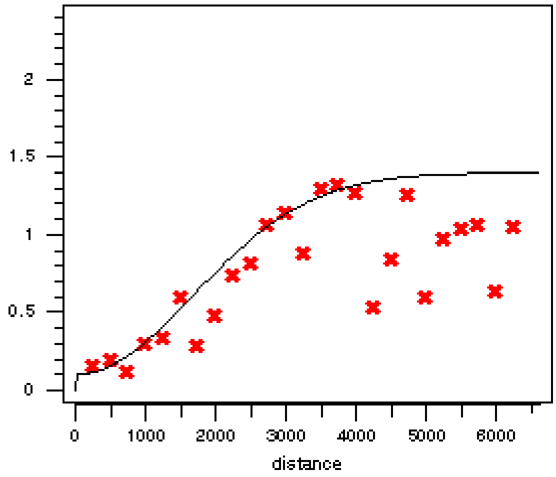
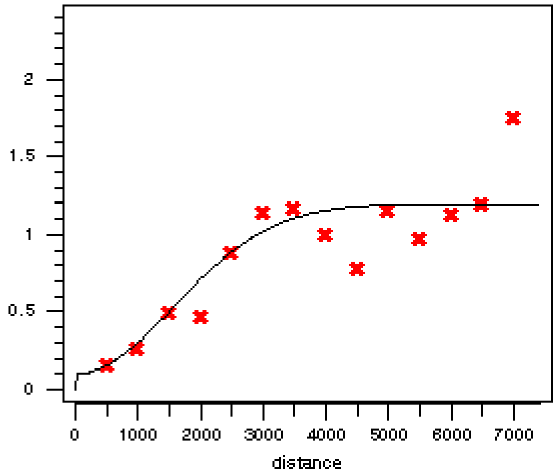
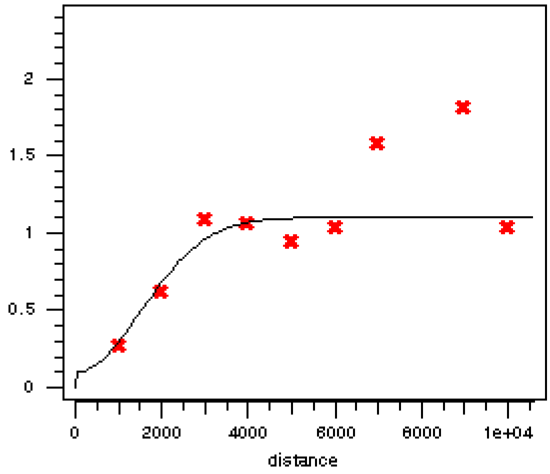
A good regionalized variable should show an invariance of scale; in other words the variograms should not show important changes varying the scale, the structure and the maximum correlation direction should remain approximately the same, although the small heterogeneities, which are neglected in the variograms with large lag, could be better highlight in the variograms created using small lag.

The variogram analysis, carried out, separately, for each variable, allowed us to assess:

- the behaviour of variograms near to origin: all variograms not tend towards zero when **h** is zero. This discontinuity of the variogram at the origin, which corresponds to short scale variability, is called “nugget effect” and can be due to local heterogeneity of the geology structures, with correlation ranges shorter than the sampling resolution, or to measurements errors; it is worth to notice that the nugget effect is bigger for Jv than horizontal intercept and it could be related to the fact that while intercept derived from direct measurements, the Jv is calculated from the mean of many measurements carried out on many different sets;
- the structure of variograms: the variance values increases with the lag, this indicates that the variability of horizontal intercept and Jv increases as the distance **h** among sampling points grows; the experimental variogram behaviour allows to identify the variogram model which best fits data; the horizontal intercept and Jv values disposition go near to a nested model composed by a nugget effect model and by a Gaussian one for the intercept, while by a spherical one for the Jv;
- the principal axes of anisotropy: the maximum correlation direction occurs where the range is major, while the minimum correlation direction was assumed perpendicular to maximum correlation direction; the maximum correlation has direction WSW-ENE for the horizontal intercept and approximately perpendicular for Jv, this is a good results because, although these two parameters are independent, they describe the fracturation degree in different ways: increasing intercept, fracturation degree decreases, while rising Jv, fracturation degree increases;
- the sill: if the maximum sill value should be equal to the variance, and thus to 1 in transformed variables, is a debated topic considered by several authors (Journel and Huijbregts, 1978; Barnes, 1991; Goovaerts, 1997; Grigarten and Deutsch, 2001). We constructed both a model using maximum sill equal to sample variance and a model using a sill value bigger than sample variance; since validation shows that, in our case, the sill major than 1 provides the best results, in following phases we considered only the model with sill bigger than sample variance. The experimental variograms show

that sill decreases when lag distance increase; Jv has an higher sill, and so a major variability, than horizontal intercept;

- the range: the maximum correlation distance of Jv is bigger than the horizontal intercept range, while the range along minimum correlation distance is smaller for Jv than intercepts; the Jv is so characterized by a major anisotropy ratio.

Horizontal intercept	
Lag = 250meters Gaussian model Nugget effect = 0.1 Sill = 1.3 Maximum correlation direction: 67.5° – 247.5° Maximum range: 4125m Minimum range: 1875m	
Lag = 500meters Gaussian model Nugget effect = 0.1 Sill = 1.1 Maximum correlation direction: 67.5° – 247.5° Maximum range: 3850m Minimum range: 2450m	
Lag = 1000meters Gaussian model Nugget effect = 0.1 Sill = 1 Maximum correlation direction: 67.5° – 247.5° Maximum range: 3700m Minimum range: 2200m	

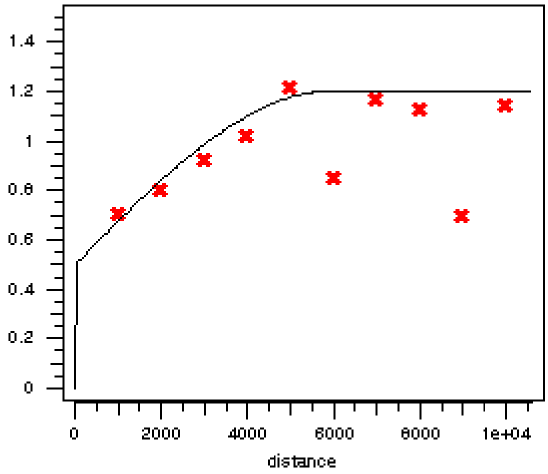
Jv	
Lag = 1000meters Spherical model Nugget effect = 0.5 Sill = 1.2 Maximum correlation direction: 135° – 315° Maximum range: 5900m Minimum range: 1800m	

Table 1: variogram of horizontal intercept transformed data, for different lags, and of Jv transformed data for 1000m lag distance; on the left are reported the parameters of the variogram model which best fits the experimental variogram

Experimental and theorical variograms along the maximum correlation direction, obtained using different lag sizes, are shown in Table 1, together whit a summary of the parameters used to create variogram models. To avoid excessive length of the article, for Jv is reported only the variogram with lag distance equal to 1000metres.

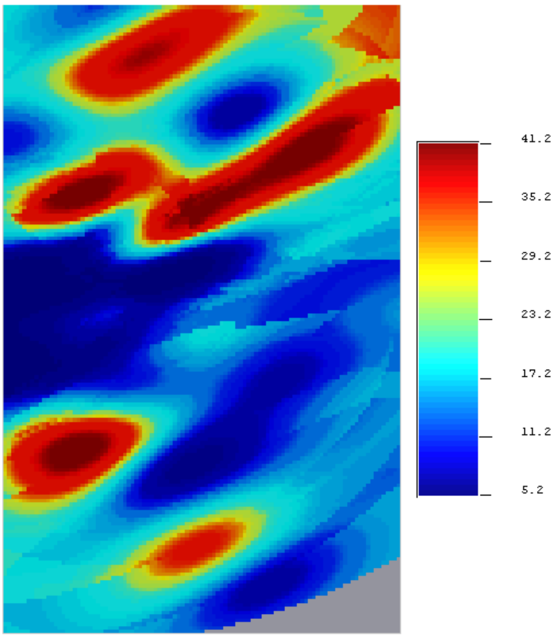
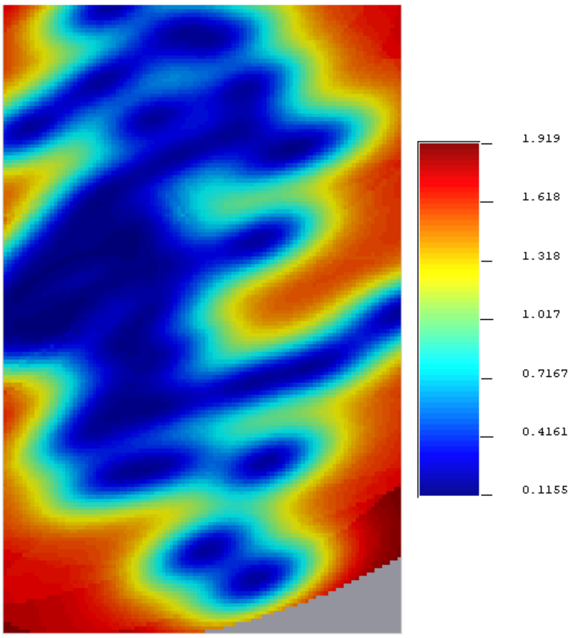
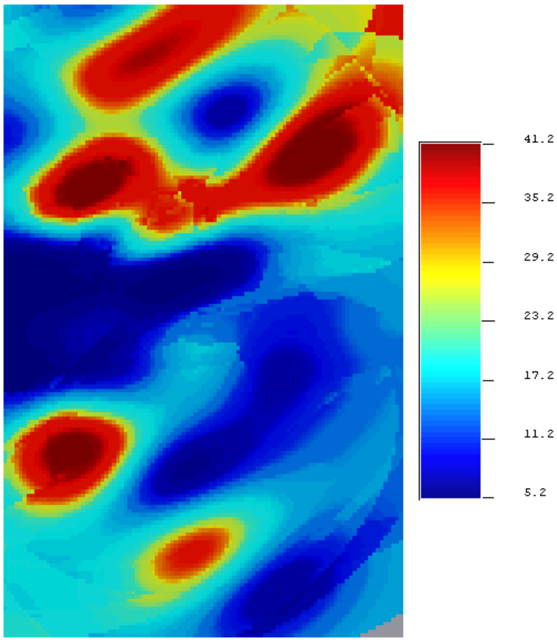
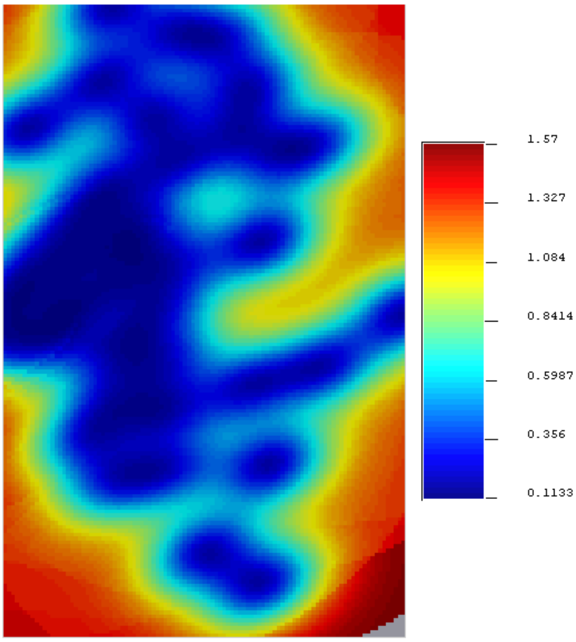
4.3 Prediction

The variogram models described above were employed for the subsequent spatial interpolation of horizontal intercept and Jv values, among survey points. Using the parameters of variogram models initially ordinary kriging method was performed for horizontal intercept and Jv predictions. Among different kriging methods, the ordinary was chosen, being the technique that provides the Best Linear Unbiased Estimator of unknown fields (Journel and Huijbregts, 1978; Kitanidis, 1997), furthermore ordinary kriging is a local estimator that provides the interpolation and extrapolation of the originally sparsely sampled data in whole the domain, assuming that the values are reasonably characterized by the Intrinsic Statistical Model.

Since the variables under study show a strong spatial anisotropy, the measurements inside a research elliptic region, with axes parallel to maximum and minimum correlation direction individuated by the variograms, were considered to perform the estimation process, doubling the axes length and including in the calculation of every point a minimum of 3 and a maximum of 20 samples, so to take in account irregularity of data distribution and nugget effect.

The grid used is defined by regular square cells, west-east and south-north oriented; each cell has dimensions of 100 meters for 100 meters.

Results of kriging, showed in Table 2, are expressed with the map of expected values and related variance. To avoid excessive length of the article, intercept results with lag of 500meters and Jv results with 250 and 500meters lags are not reported.

Horizontal intercept	
Expected values	Variance
Lag = 250meters	
	
Lag = 1000meters	
	

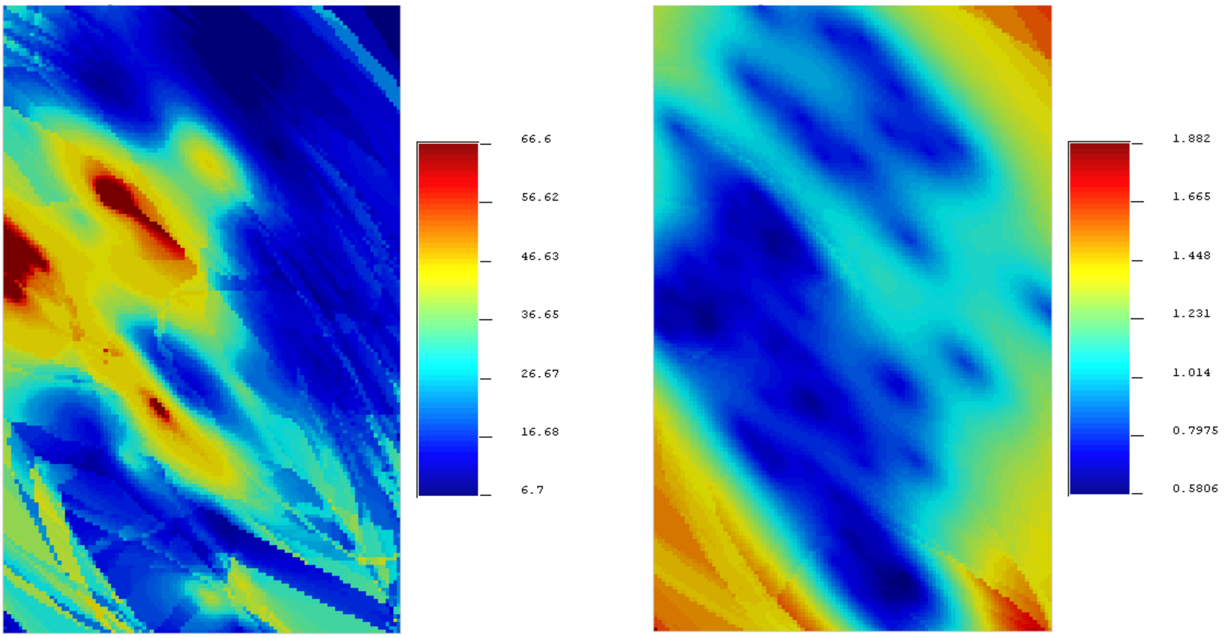
Jv (Lag = 1000meters)


Table 2: on the left side there are the expected values of horizontal intercept (expressed in centimetres) and Jv (expressed in number of fractures/m^3) estimated by ordinary kriging, with their associated variances on the right side. About intercept maps, the lag distance increases from the top to the bottom of the table

The plausibility of the interpolation models was investigated using a cross-validation procedure, which consists of sequentially estimation at each of *n* known locations using remaining *n*-1 sampled locations in the domain. This analysis, which compares estimates and actual known sampled values, shows that the estimation method adopted tends to overestimate low values and underestimate high ones, producing a marked smoothing effect; that leads to neglect the extreme values of sample distribution and therefore does not preserve the variability of the parameters under investigation.

Because we are not particularly interested in finding the best estimate of actual fracture density in a given location, but rather, we could be interested in the spatial variability of these parameters, a geostatistical simulation technique was also applied; this method does not provide the best linear unbiased estimate but does create realizations with the same variability as that observed in the field (Long & Billaux, 1987).

Among the various methods of simulation, after the searching and reading of articles concerning the simulation of the rock mass fracturing index (Chilès, 1988; Billaux et al., 1989; Gringarten, 1996; Escuder Viruete et al., 2003; Koike and Ichikawa, 2006; Stavropoulou et al., 2007; Ellefmo, 2009), we used the sequential Gaussian simulation, a conditional technique, that is forced to take the measured values of the variable in the sampling points. Geostatistical simulations (or stochastic representations) can be seen as possible realizations of a spatially correlated random field, they all honour the spatial moments (mean, variogram) of the field.

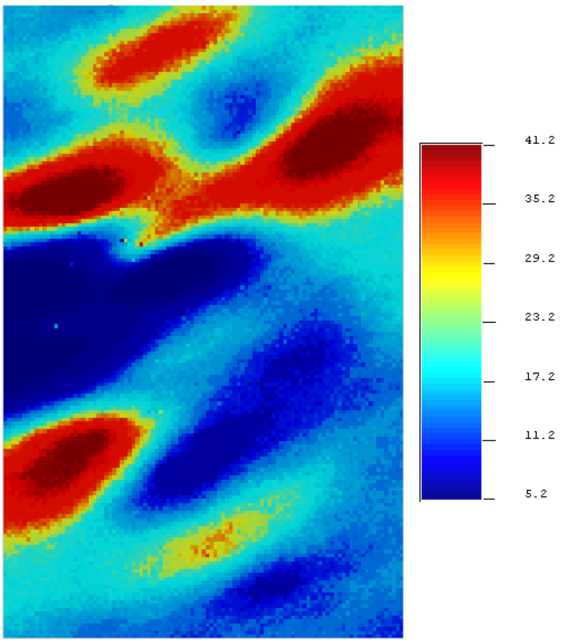
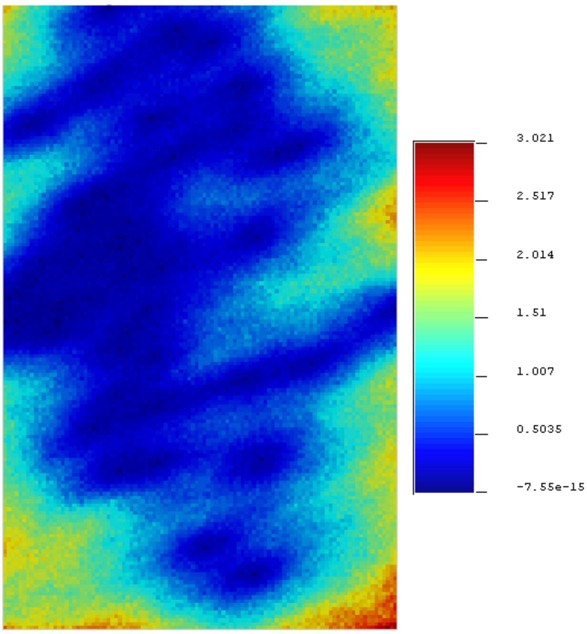
With the parameters of spatial continuity models previously defined through variogram analysis, Gaussian conditional simulation was used to model intercept and Jv distributions, separately, using the same grid and research ellipse of those used for ordinary kriging.

The optimal number of simulations was chosen comparing the results of 10, 100 and 1000 simulations, through a validation process. In the present study the optimal number of simulations is 100, because it provides better results than those obtained using only 10 realizations and only little worse than those obtained from 1000 simulations which, however, require a gigantic times to run with only a small improvement of results.

The various realizations might initially seem to be quite different, nevertheless the variability and distribution of estimated values are very similar to those of original data, and the smoothing effect, which was observed using the kriging method, does not occur.

Each simulation, even if maintains the variability and distribution of samples, provides a different map, hence to get a final map, is necessary to calculate, in each location of grid, a single estimated value of least squared error-type: the conditional expectation.

Final results of sequential Gaussian simulations, showed in Table 3, are so expressed both in term of the map of expected values and related variance.

<table>
<tr><td colspan="2">Horizontal intercept</td></tr>
<tr><td>Expected values</td><td>Variance</td></tr>
<tr><td colspan="2">Lag = 250meters</td></tr>
<tr><td> </td><td> </td></tr>
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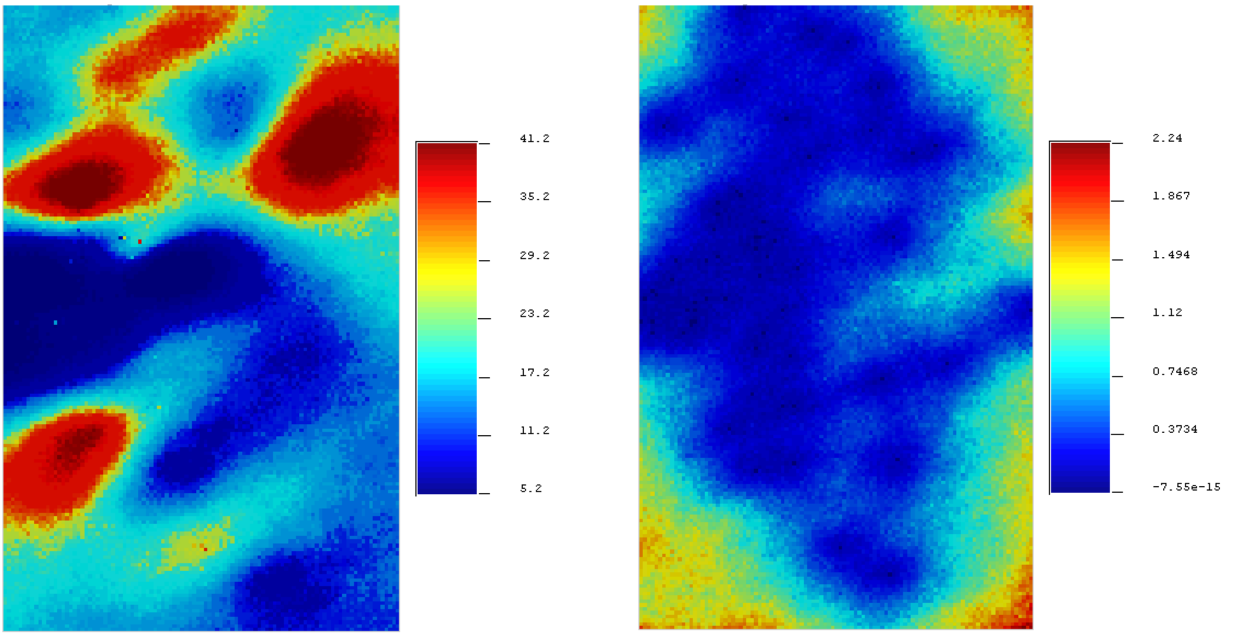
Lag = 1000meters



Jv (Lag = 1000meters)

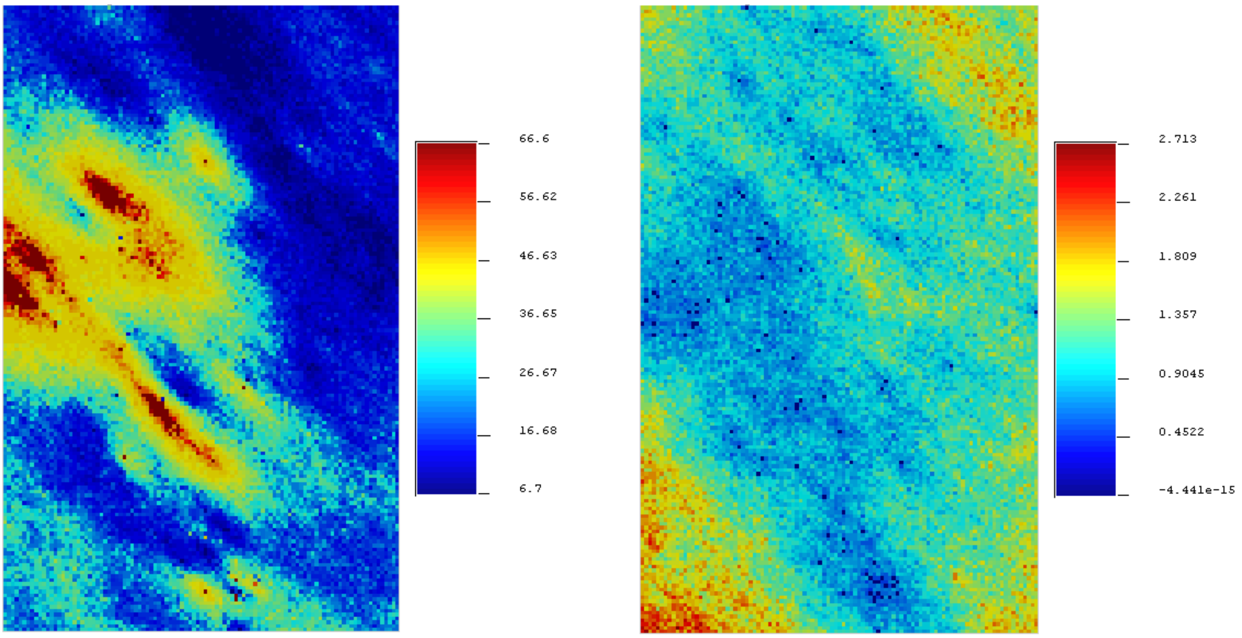


Table 3: on the left side there are the expected values of horizontal intercept (expressed in centimetres) and Jv (expressed in number of fractures/m^3) estimated by sequential Gaussian simulation, with their associated variances on the right side

The two methods (ordinary kriging and sequential Gaussian simulation) provide quite similar outcomes for the central values of variable frequency distribution, while remarkable differences occur for the extreme values of data, indeed these are neglected in kriging results, while are maintained in those coming from simulation.

4.4 Validation

To compare results obtained using these two geostatistical techniques, a validation process was performed, using an independent data set. Almost 10 new geomechanical surveys were carried out in the research area to form the training point data set.

The validation process was performed comparing measures of new sampling points with estimated values in their locations. The difference between actual and estimated values allowed computing, for each applied technique, the mean error and his related root-mean-square, average standard error, mean standardized error and root-mean-square standardized error. The minimum mean error was obtained performing kriging with small lag distance, while minimum standard deviation of errors coming from sequential Gaussian simulation technique based on medium lag distance. The diagram which relates measured and estimated values is presented in Figure 6; the line closer to bisector is the regression line obtained from ordinary kriging with small lag. Nevertheless is important observe that training point dataset does not contain extreme values which should have lower correspondence with kriging method.

Generally the validation reveals a quite good accordance between estimated and measured data, especially for horizontal intercepts.

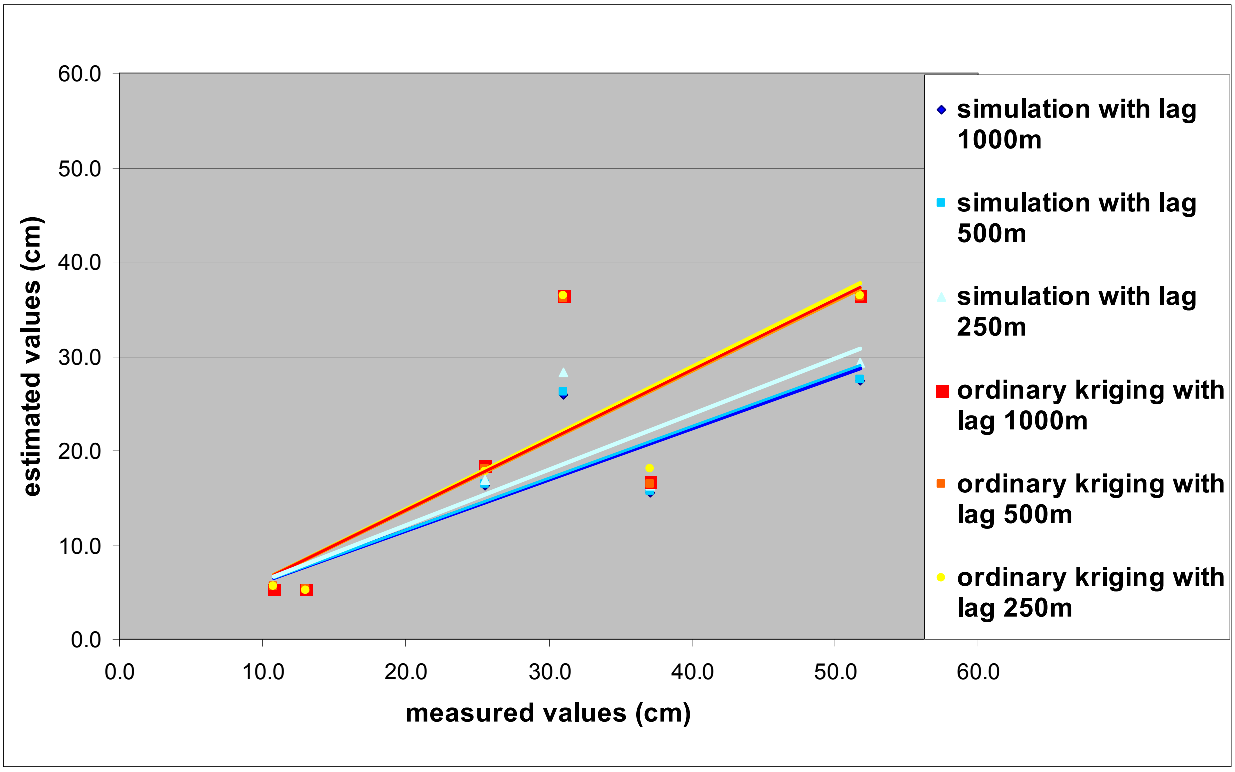


Figure 6: validation of horizontal intercept, this graph relates the measured with estimated values of an independent dataset

5 Conclusion

A rock mass fracture density characterization in an Italian Alpine valley has been here presented.

Geomechanical work was carried out by surveying rock discontinuities in 97 different sites and by classifying, according to RMR system, the examined rock masses, which exhibit both good qualities and similar geometrical and mechanical parameters in each surveyed sites.

A geostatistical application was carried out to examine the spatial variability of fracture density, described using two different and independent parameters: the horizontal intercept and the Volumetric Joint Count (Jv), derived from spacing measurements. The structure of the distribution of each parameter was investigated by means of a variogram analysis. Some correlations in the space were determined at different scales, although the general correlation structure was constant at all scales. The maximum correlation direction is toward N-E for intercept and exactly perpendicular for Jv, this property is respected at each scale.

The modelling of experimental variograms allowed to estimate the variables out from survey points, using two different techniques: ordinary kriging and sequential Gaussian simulation. A validation process, carried out on an independent dataset, reveals a quite good accordance between estimated and measured data, especially for horizontal intercepts. In particular, in the case under study, both ordinary kriging and sequential Gaussian simulation supplied the best result using short lag distance, which permits to consider also small heterogeneities. The simulation technique seems to be more influenced by differences in lags than the kriging.

Geostatistical methods allowed forecasting the distribution of fracturation density out from points of survey, but a geological reason for the disposition of areas with different fracturation degree needs further investigations. The fracturation density maps, obtained from the application of geostatistical methods, should be superimposed with the structural map of major tectonic lineament of the area. A first trial with the available preliminary structural map was attempted and no univocal correspondence between high fracturation degree and proximity with local fault systems was revealed. More geo-structural measurements are therefore necessary to better understand the geological significance of dispositions of high fracturation degree areas.

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