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FAR AWAY GALAXIES AS A POSSIBLE SUBSTITUTE FOR DARK MATTER

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Abstract. We give a short report on a recent work which shows how some effects usually attributed to the gravitational action of local dark matter may be explained in a conservative way as due to the gravitational action of far away galaxies, dealt with according to the standard prescriptions of general relativity. We also mention some new perspectives.

1. INTRODUCTION

The conception that a large amount of dark matter be spread in the Universe, with a density even five times that of the visible one, was enforced by the realization that the gravitational action of local visible matter is not sufficient to account for some fundamental qualitative features of cosmology, such as the flattening of the rotation curves at the edges of galaxies, and the excess of velocity of galaxies in clusters. The gravitational action exceeding that attributable to local visible mass was thus attributed to local unobserved mass (see for example Peebles 1993).

The conception that the additional gravitational action may instead be attributed to far away galaxies, was advanced and discussed in the paper Carati, Cacciatori & Galgani 2008a. Such an apparently new idea found its inspiration in some results recently obtained in classical electrodynamics of point particles (see Carati & Galgani 2003; Marino, Carati & Galgani 2007), where it was pointed out that the retarded electromagnetic action of far away charges produces at any place a very relevant local effect, inasmuch as it accounts for the radiation reaction of an accelerated charge, as first proposed by Wheeler and Feynman (see Wheeler & Feynman 1945; Carati, Cacciatori & Galgani 2008b).

In this paper, which was presented at the Conference held on the occasion of the seventieth anniversary of John D. Hadjidemetriou, we give a short review of the available results, and also mention some new perspectives.

2. THE MODEL

According to the main principle of general relativity, the whole information about the gravitational action on a test particle at a point of spacetime is assumed to be contained in the form of the metric tensor $g_{\mu\nu}$, inasmuch as it determines the geodesics governing the particle motion. In turn, the metric tensor is assumed to be a solution of Einstein's equations having as a source the energy momentum tensor due to the whole matter present in the Universe.

The first point in which our approach differs from the most common ones is that, in describing the contribution of the far away matter to the energy momentum tensor, the matter is thought of as a finite system of N point particles (each particle corresponding typically to one galaxy), and not as a continuum. Then, following literally the treatment given by Einstein in his Princeton lectures in connection with Mach's principle (see Einstein 1922), we perform a perturbation treatment. The metric tensor $g_{\mu\nu}$ is written as a perturbation of the Minkowskian one $\eta_{\mu\nu}$, i.e., as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and the perturbation $h_{\mu\nu}$ is estimated in the linear (or weak field) approximation. This leads for $h_{\mu\nu}$ to an equation which is essentially the wave equation, and this in turn is solved *"by the method. familiar in electrodynamics, of retarded potentials"* through a formula (formula (101), page 87 of Einstein 1922), which involves the positions $\mathbf{q}_j$ and the velocities $\dot{\mathbf{q}}_j$ of the source particles. The formula reads

$$h_{\mu\nu} = \frac{-2G}{c^4} \sum_{j=1}^{N} \frac{M_j}{\gamma_j} \left. \frac{2\dot{q}_\mu^{(j)}\dot{q}_\nu^{(j)} - c^2\eta_{\mu\nu}}{|\mathbf{x} - \mathbf{q}_j|} \right|_{t=t_{\mathrm{ret}}} \tag{1}$$

(with $\mathbf{q}^{(j)} \equiv \mathbf{q}_j$), the dot denoting derivative with respect to proper time, where M_j and γ_j are the mass and the Lorentz factor of the j–th source particle.

The problem is then how to deal with the velocities and the positions of the sources. Concerning velocities, Einstein points out (page 88) that *"The previous developments are valid however rapidly the masses which generate the field may move relatively to our chosen system of quasi–Galilean coordinates"*. Having however in mind the application to astronomy, he added: *" But in astronomy we have to do with masses whose velocities, relatively to the coordinate system employed, are always small compared to the velocity of light, ... We therefore get an approximation which is sufficient for nearly all practical purposes if in (101) we replace the retarded potential by the ordinary (non–retarded) potential ..."*. This is the way in which the restriction to the Newtonian, fast decaying, potential was introduced, and consequently only the near matter, and not the far away one, appeared to play a role in connection with Mach's principle (see page 100 of Einstein 1922). Notice that Hubble's law had not yet been discovered at that time.

On the other hand, in our approach which has a cosmological character, with the source particles corresponding to single galaxies, the motion of galaxies has to be taken into account. To this end we introduced the simplest model we could conceive, namely, the one in which all peculiar velocities are altogether neglected, and the single galaxies are assumed to follow Hubble's law. So we assume that a galaxy, having a position vector $\mathbf{q}_j$, correspondingly has a velocity

$$\dot{\mathbf{q}}_j = H_0 \mathbf{q}_j \ , \quad j = 1, \ldots, N \ , \tag{2}$$

where H_0 is the Hubble constant which, in our extremely simplified model, we take fixed at its present value.

For what concerns the position vectors $\mathbf{q}_j$, following the approach introduced by Chandrasekhar and von Neumann in the context of stellar dynamics (see the review Chandrasekhar 1943), and taken also by Davis and Peebles in the context of galactic dynamics (see Davis & Peebles 1977), we assume them to be random variables. We now describe the results which are obtained by introducing further assumptions on the probability distribution of the positions of the galaxies.

3. FIRST RESULT: THE EFFECTIVE DENSITY

The most natural assumption one is led to introduce concerning the probability distribution of the positions of galaxies, is that of the one-particle probability distribution be isotropic, just in analogy with the corresponding isotropy assumption for the matter density in the standard continuum model for the source matter. From this assumption one easily deduces the form of the mean metric corresponding to the metric $g_{\mu\nu}$ defined by (1), and this mean metric turns out to be just a flat Friedmann–Robertson–Walker one. This then leads to the first result. This is obtained by requiring a consistency condition, namely, that the expansion rate corresponding to such a mean metric should coincide with the expansion rate that had been introduced into the model through Hubble's law (2). Now, the formula giving the expansion rate corresponding to the mean metric, happens to contain certain sums involving the positions of the source galaxies, in which the dominant contribution just comes from the extremely far away ones. On the other hand, the estimate of such sums naturally leads to introducing a certain effective density $\rho_{\rm eff}$, and the compatibility condition then leads to the result that such an effective density has to be about five times the observed one ρ_0:

$$\rho_{\rm eff} \simeq 5\rho_0 \ . \tag{3}$$

In other words, through their contribution to the metric tensor (and thus to the expansion rate), the extremely far away galaxies turn out to produce at each point a local effect which can be described by saying that *everything goes as if there existed an effective density which is about five times the locally observed one*. This is the first instance in which, through their gravitational action, the far away galaxies were found to play a role usually attributed to local dark matter.

4. SECOND RESULT: cH_0 AS A TYPICAL VALUE OF THE ACCELERATION

Then our attention was addressed to the force per unit mass acting on a test particle (in the approximation of velocities small compared to that of light). From the formula (1) for the metric tensor, combined with Hubble's law (2), one finds out that the force contains a term decreasing as $1/r^2$ (near–field), which is proportional to the velocity of the source, and also a term decreasing as $1/r$ (far–field), which is proportional to the acceleration of the source. Thus, as Hubble's law with a time–independent H_0 implies that also the acceleration of the source is proportional to distance, the force due to the far field actually doesn't depend on distance at all. This is the main reason why the far away matter may give the dominant contribution to the gravitational field of force (leaving aside the Newtonian contribution from the local observable mass).

The force per unit mass corresponding to such a dominant term (which we denote by $\mathbf{f}$) turns out to have the form

$$\mathbf{f} = \frac{4GH_0^2\,M}{c^2}\;\mathbf{u} \tag{4}$$

where we have introduced the vector $\mathbf{u}$, depending on the number N of source galaxies, defined by

$$\mathbf{u}(N) = \sum_{j=1}^{N} \frac{\mathbf{q}_j}{|\mathbf{q}_j|}\bigg|_{t=t_{\mathrm{ret}}} \quad . \tag{5}$$

So, everything depends on the sum of the unit vectors pointing to each galaxy, at the position where we see it, a formula in which the dependence on distance did completely disappear. In formula (5), the masses of the sources were all put equal to a common value M, and the Lorentz factors γ_j were approximated by 1, because in practice this theoretical formula was actually estimated through an extrapolation from partial sums restricted to non extremely large distances (see later). Our attention was actually addressed to one component of the force $\mathbf{f}$ per unit mass, say its projection along a given direction, which we denote by f.

In order to estimate f, the force per unit mass acting on a test particle along a given direction, further assumptions on the distribution of matter have to be introduced. Obviously, the force vanishes if matter were approximated by an isotropic continuum, and the same is easily seen to occur for the mean value of the force in our discrete stochastic model, just due to the isotropy assumption for the one–particle probability distribution of position. An estimate of the "typical" value of the modulus $|f|$ of the force is then provided by the standard deviation σ_f. One has then to provide an estimate for the variance σ_f^2.

It is immediately seen that the variance σ_f^2 vanishes in the limit of a large number N of galaxies, if their positions are assumed to be independent random variables. This is a simple consequence of the law of large numbers, just because in such a case the sum defining σ_f^2 increases as N, whereas a nonvanishing result can be obtained only if it increases as N^2, as is shown by the formula given below. In turn, the sum can increase as N^2 if the positions of the galaxies are assumed to be correlated. Nature comes to our help in this case, because it is actually well known that the positions of the galaxies do present some correlations (see Mandelbrot 1977), the only open observational problem being that of establishing up to which scale does this occur (see e.g. Peebles 1993; Sylos Labini, Montuori & Pietronero 1998; Ruffini, Song & Taraglio 1998; Joyce, Anderson, Montuori, Pietronero & Sylos Labini 2000). Without entering discussions of such a type, we just limit ourselves to investigate which consequences follow from a correlation assumption for the positions.

Being unable at the moment to perform analytical estimates, we limited ourselves to numerical ones, by building up samples of N particles (with N ranging from 1000 to 512,000) presenting a spatial correlation. Concretely, we took a distribution having a fractal dimension, and actually chose the fractal dimension to be 2, just because in such a case the constructions of the samples are particularly manageable (see

Mandelbrot 1977). The result turns out to be $\sigma_u^2 \simeq 0.2\, N^2$, which leads to

$$\sigma_f \simeq \sqrt{0.2}\ \frac{4GH_0^2}{c^2} MN \ . \tag{6}$$

Notice that this formula involves the product MN, i.e., the total mass of the sample of galaxies considered, just because the variances increase as N^2. With the independence hypothesis, the variances would instead increase as N, and this would produce in σ_f a factor $M\sqrt{N}$, which would not do the job.

Indeed, we took formula (6), which was obtained for small values of N (limited by the available computation power), and extrapolated it up to the present horizon $R_0 = c/H_0$, i.e., we inserted in formula (6) the actual value of N, so that the quantity MN could be identified with the total visible mass of the Universe. Inserting also

$$MN = \frac{4}{3}\pi\ \rho_{\rm eff}\ R_0^3\ ,$$

in terms of the previously determined effective density $\rho_{\rm eff}$, one gets $\sigma_f \simeq 0.2\ cH_0$. So, for the typical value of the modulus $|f|$ of a component of the force per unit mass, one gets

$$|f| \simeq 0.2\ cH_0\ . \tag{7}$$

This constitutes the second result. Namely, *on the assumption that the positions of the galaxies do present a correlation (and, specifically, according to a fractal distribution with fractal dimension 2), the component of the force per unit mass along a given direction has a typical value whose modulus is of the order of* cH_0.

This seems not to be trivial at all, because it is well known that dark matter usually appears to be a necessary ingredient, in providing a gravitational contribution additional to the Newtonian one due to local visible mass, just when the corresponding acceleration is of the order of cH_0 (see Milgrom 1983). Now, the fact that local dark matter has something to do with cosmology, as witnessed by such a connection with Hubble's constant, might appear as a curious coincidence, whereas it appears somehow as "explained" in the present approach.

5. THIRD RESULT: VELOCITY DISPERSION IN CLUSTERS

So our main result is that, in our model with spatially correlated galaxies, the far away galaxies produce at any point a gravitational acceleration, which we may call "cosmic acceleration", of the order cH_0. The only application made in the paper Carati, Cacciatori & Galgani 2008a concerns the velocity dispersion in clusters of galaxies. Let us recall that the problem with clusters of galaxies essentially amounts to the fact that the single galaxies of a cluster do present velocities (with respect to the center of mass) which exceed the escape velocity determined by the visible matter, so that more mass is needed to keep them there. We showed that our model explains the observations, provided a further assumption of a quite general character be introduced, namely, that the forces acting on two different galaxies of a cluster be uncorrelated.

That such a decorrelation property may well be plausible in our model, will be discussed in a moment. Here, let us previously explain why such a property is necessary in order to fit the observations. The first point in this connection is that the gravitational action of the far away galaxies should have on the internal galaxies of a cluster an effect analogous to that of the pressure exerted by the external walls on the particles of a gas in a box. Now, this can occur only if the field of cosmic acceleration does form, in the region of the cluster, patterns which are of central–like type. We are thinking of the field of cosmic acceleration as a random one, which in some regions might present a pattern convergent towards a center, because only in such a case could the external acceleration play the role of a pressure, and thus could a cluster exist. On the other hand, central–like fields can exist only if one has spatial decorrelation, because otherwise the acceleration at a generic point inside the cluster would be essentially parallel to the acceleration at the center of mass.

The second reason why a spatial decorrelation property, and thus some nonsmoothness property, is required, is that otherwise the field at a generic point inside the cluster could be well approximated by the corresponding linear term in a Taylor expansion about the center of mass, and this would lead, for the velocity dispersion, to a dependence on the linear dimension L of the cluster which doesn't fit the observations. This is seen as follows. For a cluster composed of n galaxies, the "typical" velocity of a galaxy is estimated by the corresponding dispersion σ_v, with $\sigma_v^2 = (1/n)\sum_i v_i^2$. On the other hand, in the virial theorem one considers the virial of the tidal forces per unit mass $\mathcal{V} = \sum_i (\mathbf{f}_i - \mathbf{f}^*) \cdot \mathbf{x}_i$ where $\mathbf{f}_i$ is the force per unit mass acting on the i–th galaxy, and $\mathbf{f}^*$ is the force at the center of mass. According to the virial theorem, for a confined system one has $\overline{\sigma_v^2} = -\overline{\mathcal{V}}/n$, where overline denotes time average. So, with a smooth field, expanded at first order about the center of mass, one would find $f_i - f^*$ of the order of L, the linear dimension of the cluster, and correspondingly one would have $\overline{\mathcal{V}}/n \simeq H_0^2 L^2$, whereas the observations seem to give σ_v^2 proportional to L (see e.g. Kazanas & Manhein 1991). On the other hand, with the decorrelation assumption one gets that the variance of $f - f^*$ is independent of L, being just just given by $\sqrt{2}$ the variance of f. So, inserting the previously estimated value of f, one immediately gets

$$\overline{\sigma_v^2} \simeq 0.07\, cH_0 L \ . \tag{8}$$

Thus with the decorrelation assumption one gets a velocity variance proportional to L, as seems to be required by the observations. Moreover, in the paradigmatic case of the Coma cluster from (8) one gets a value $8 \cdot 10^5$ km^2/sec^2, which quantitatively fits rather well the value $5 \cdot 10^5$ km^2/sec^2 reported by Zwicky already in the year 1933.

So, how could such a spatial decorrelation property for the cosmic acceleration be made plausible in our model? One rather qualitative argument is the following one. It was already pointed out that the dominant contribution to the acceleration at a given point comes from the extremely far away galaxies, or, more properly, from the galaxies near the corresponding horizon. Now, horizons corresponding to different galaxies do not coincide, and on the other hand the distributions of matter about two different horizons, being non causally connected, should be considered as independent. This seems to imply decorrelation.

6. FURTHER PERSPECTIVES

So we have illustrated, in a rather colloquial way, the results obtained in the paper Carati, Cacciatori & Galgani 2008a, where it was first proposed that the far away galaxies may be a substitute for the local dark matter, in providing the observed acceleration which is not accounted for by the Newtonian attraction due to local visible mass. This was obtained through a completely conservative approach, in which the gravitational action of the galaxies is estimated according to the standard prescriptions of general relativity in the weak field approximation, with the source galaxies described as point particles, instead than as a continuum. The positions of the galaxies are considered as random variables, and their velocities are assumed to follow Hubble's law.

Further assumptions had then to be introduced concerning the positions as random variables: isotropy of the one–particle distribution, and a correlation for the two–particle distribution. Such assumptions already allow one to get two results, namely: i) in connection with the expansion rate, the gravitational action of the far away galaxies has the same effect as if there existed a distributed matter having a density five times that of the observed one, and ii) at any point there exists a force per unit mass acting on a test particle, a kind of cosmic acceleration, a typical value of which can be estimated to be $0.2cH_0$. Finally, this fact appears to explain the observed velocity dispersion in clusters of galaxies, if the further assumption is introduced that the cosmic acceleration due to the far away galaxies be spatially uncorrelated (at least at the scale of galaxy distances).

We are now working at two more problems. The first one is of a technical character, and concerns the estimate of the variance of the force per unit mass due to N point particles. In the mentioned paper the estimate was obtained through numerical methods, and was restricted to the particular case of a fractal dimension of dimension 2. In the meantime, the numerical estimates were extended to other values of the fractal dimensions. The preliminary indications are that the results be essentially independent of the dimension. Furthermore, a rather simple method was devised in order to perform the estimates analytically; in the case of fractal dimension 2, the estimate $\sigma_u^2 \simeq 0.2\,N^2$ was in particular confirmed.

The main problem that remains open is to establish whether the gravitational influence of the far away galaxies may prove able to explain also the second main phenomenon that enforced the introduction of dark matter, namely, the flattening of the rotation curves in galaxies. We are now working in this direction, and are very confident that a positive answer may be provided rather soon, and in quite a simple and elegant way.

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