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Introduction

In this section I will provide a brief description of my Ph.D. Thesis. Economic growth is an important field in economic studies. Nowadays, economic growth is in the middle of the public discussions since it is a necessary target in order economies to overcome the negative consequences of the economic crisis. However, economic growth is not just a method or something that can happen fast. It is a long run procedure and due to this it is very important societies to understand what kind of determinants and policy parameters can stimulate in the long run economic growth.

There are of course theories about exogenous economic growth where important variables for explaining economic growth is the growth rate of physical capital, openness to trade and population growth. The only drawback of this branch of economic growth is that there is not a deep investigation on the determinants of economic growth. The branch of economic growth which tries to explain in more detail the mechanisms through which economic growth can become true is the so called theories of endogenous economic growth. These theories are more interested to explain how technical progress take place and affect economic growth. Important determinant of both endogenous technical progress and endogenous economic growth in general is the human capital accumulation. The endogenous human capital accumulation and the role of it in the growth rate of the economies, since human capital is considered as an important input for production of goods or as a prerequisite for technical progress, has been used by many authors like Uzawa (1965), Lucas (1988), Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2011) and many others not mentioned here.

However, the empirical evidence do not provide a clear result on the relationship between human capital and economic growth, depending on different measures of human capital, different time period and different data sets. Two important papers in this direction are those of Benhabib and Spiegel (1994) and Islam (1995). Under this ambiguity in the first chapter of this dissertation I am trying both at a theoretical aggregate level and by using econometric techniques to investigate possible distortions of other factors on human capital accumulation.

In the first chapter of my thesis I am trying to propose that population growth has a nonlinear role in the formation of human capital and not only negative and linear as it has been proposed in the current empirical literature. Seminal papers are those of Rozenzweig and Schultz (1987), Stafford (1987), Brander and Dowrick (1994), Kelley and Schmidt (2003) and Goux and Maurin (2004).

In the second chapter of this dissertation I am trying to investigate theoretically and empirically at an aggregate model the impact of corruption in some important components of human capital. I am interested in the issue of corruption because corruption has been proposed in the literature as an important negative factor in the misallocation of talent. Papers which highlight this impact of corruption are the papers of Murphy, Sleifer and Vishny (1991), Murphy and Vishny (1993) and Acemoglu and Verdier (1998). However, these papers deal with occupational choice and not how the effect of corruption through different channels (public expenditure on education and physical capital investment) can affect the human capital accumulation at an aggregate level.

In both the first and in the second chapter of this dissertation I am also applying nonparametric econometric methods such as the Local Linear nonparametric method proposed by Racine and Li (2004), Hsiao *et al.* (2007) test for checking for misspecification of the linear model and also the Smooth Coefficient Semiparametric Method proposed by Stone (1977), Li *et al.* (2002) and many others.

Finally, in the last chapter of this dissertation I am dealing with endogenous technical progress by using an endogenous R&D model with both endogenous labor supply (leisure choice) and endogenous human capital accumulation. The first important assumption of this chapter is that leisure can have a direct negative effect by reducing the available time for both working and schooling but also a positive one, depending to which sector is used. The second important assumption is that the stock of human capital in the R&D sector can increase the quality of formal education and therefore the human capital accumulation. The main idea in this chapter comes from the historical fact that leisure time can be used also in productive way. Davis (2009) finds in an empirical survey that leisure can be beneficial for inventions. Below, there is a more detailed general description of the three chapters and afterwards the whole papers are provided.

In the first paper of my thesis I have worked on a joint paper with A. Bucci and T. Stengos about the non-monotonic role of fertility in the human capital accumulation. In the existing theoretical and empirical literature fertility is assumed to play a negative impact on economic growth and also in the accumulation of human capital. This is the well-known *quantity-effect* according to the literature of endogenous fertility. However, children are more prone to be educated at low of birth rate levels, whereas at high birth rate levels, children become part of the labor force in order to contribute to their family income. Furthermore, in countries with low growth rates and shorter life expectancy, parents choose to have more children to insure themselves for their retirement. In other words, having more children is perceived by parents as a future source of income flows. Under these arguments fertility should be used in a way that can capture both positive and negative effects in the accumulation of human capital depending on the size of birth rate. We build a two-sector growth model in which we employ birth rate in the main equation for human capital. Then we test empirically for the existence of non-linear effect of birth rate in the human capital accumulation at an aggregate level by using fully non-parametric and semi-parametric (GAM) methods. Our empirical results support the idea of non-monotonical effect of birth rate on human capital accumulation and therefore on economic growth, which provides important policy implications such as to provide subsidies for encouraging births or extra taxation for reducing them.

The second paper of my Thesis tries to explore the effect of corruption on human capital accumulation through the distortion that is observed due to corruption's impact on the allocation of public expenditures. We build a two-sector growth model with both physical and human capital in which the distribution of human capital to different sectors is the main determinant of economic growth. At the same time we assume that important factor for the distribution of human capital is the allocation of public expenditures, which is also distorted by the level of corruption. This distortion, we assume, is different for different types of public expenditures since some components of the public expenditure are more affected by corruption relatively more than other parts. In a general environment, the theoretical results show that if corruption is stronger by reducing by more the net amount of public expenditure for final output, then corruption is mainly bad for economic growth because is affected by more the investment in physical capital relative to investment in human capital. However, when corruption is stronger in the public expenditures that are used in the education sector the effects on economic growth are not negative. This is because in a regime of corruption exists extra incentive from agents to invest in education for having the opportunity to become bureaucrats. Then we test empirically, by using the smooth coefficient semi-parametric methods, the effect of corruption into human capital accumulation by looking how corruption affects the public expenditure on education and the physical capital investment. The empirical results support the main assumptions of the model. In this paper I am the sole author.

In the last paper of my Thesis in which I am the sole author, I propose that leisure time may have also positive externalities in the productivity of labor which is employed in an R&D sector contrary to endogenous labor supply literature which considers that leisure just reduces the available working time for working and schooling. The examples of Archimedes and Newton support the idea of developing theories of nature during their leisure time. The main assumption of the model is that agents with higher human capital are more mature to exploit more efficiently their leisure time together with the nature of their work. By using an endogenous R&D growth model with leisure externalities in the R&D sector together with R&D externalities into human capital accumulation, I recast different effects of leisure on the economy. The effects of leisure on the growth rate of an economy depend on the complexity of performing research and on how much is connected the R&D activity with the education sector. This paper has also policy implications in the debate for the differences in the labor productivity between the EU and the U.S. economy.

CHAPTER 1

NON-MONOTONICITY OF FERTILITY IN HUMAN CAPITAL ACCUMULATION AND ECONOMIC GROWTH*

Abstract

This paper investigates the relationship between per-capita human capital investment and the birth rate. Since the consequences of higher fertility (birth rate) on per-capita human capital accumulation (the so-called dilution effect) are not the same (in sign and magnitude) across different groups of countries with different birth rates, we analyze the growth impact of a non-linear dilution-effect. The main predictions of the model (concerning the relationship between population and economic growth rates) are then compared with those of a standard model in which the exogenous birth rate affects linearly and negatively (as postulated by most of the existing theoretical literature) human capital investment at the individual level. By using non-parametric techniques, we find evidence of strong nonlinearities in the total effect of fertility on human capital accumulation. This supports the idea that fertility plays a non-monotonic role in the accumulation of human capital and hence in the growth rate of an economy. The non-monotonic effect of fertility on human capital appears to be valid for OECD, as well as non-OECD countries according to our empirical results.

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1. INTRODUCTION

The analysis of the impact of demographic change (population growth) on the growth rate of real per capita income represents an old, but still unsettled topic of research. Malthus (1798) was among the first to recognize that a higher population growth rate would ultimately (in the very long-run) have led to economic stagnation. According to his view, in a world in which economic resources are in fixed supply and technological progress is very slow or totally absent, the food-production activity would, sooner or later, have been overwhelmed by the pressures of a rapidly growing population. In this scenario, the available diet of each single individual in the population would have fallen below a given *subsistence level*, so leading to a fall of the productivity growth rate as well.[1] Unlike Malthus, proponents of the optimistic view[2] emphasize, instead, the positive effect that a larger population can exert on the rate of technological progress (an endogenous variable) and, thus, on economic growth**:** *"…More people mean more Isaac Newtons and therefore more ideas. More ideas, because of nonrivalry, mean more per capita income. Therefore, population growth, combined with the increasing returns to scale associated with ideas delivers sustained long-run growth"* (Jones, 2003, p. 505).[3] Besides the pessimistic and optimistic ones, there also exists another belief about the long-run effects of population growth on economic growth: the neutralist one. The advocates of this view claim that population growth has in general only little significant impact on economic growth and that such impact can be either positive, or negative, or else wholly inexistent (Srinivasan, 1988; Bloom *et al.*, 2003, p. 17).

The pessimistic prediction of Malthus has fortunately never become reality. Nonetheless, it is now well known (Bloom *et al.*, 2003, Fig.1.1, p. 13; United Nations, 2004) that, unlike the industrialized world, the less (and especially the least) developed regions of the planet are rapidly and increasingly gaining shares of the world population. Since these regions are those that actually exhibit the highest fertility and the

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1 More recent examples of the view that population growth is detrimental to economic development include the neoclassical growth theory with exogenous technological progress (Solow, 1956), some empirical applications of this theory (notably, Mankiw *et al*., 1992), and Barro and Becker (1988, 1989). The last two authors consider an environment in which fertility is endogenous. Under the assumption that children are *normal goods* and parents are altruistic (they gain utility from having kids), they conclude that a faster population growth, by implying a *dilution* of the capital endowment of each individual in the population, harms long-run growth in real per capita incomes.

2 See Kuznets (1960, 1967), Simon (1981), Boserup (1981), Kremer (1993), Jones (2001) and Tamura (2002, 2006), just to mention a few examples.

3 In models with endogenous technological change population can affect economic growth in two distinct ways. In some papers (Romer, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991) real per capita income growth depends positively on *population size*. This result (known as *strong scale effect*) is rejected on empirical grounds (Jones, 1995). In other papers, notably the *semi-endogenous* growth models (Kortum, 1997; Segerstrom, 1998), it is *population growth* (as opposed to population size) to sustain economic growth in the very long-run. Another branch of endogenous growth theory has recently analyzed the impact of population growth on economic growth in environments in which there is also, along with technological progress, human capital accumulation. Such papers (Dalgaard and Kreiner, 2001; Strulik, 2005 and Bucci, 2008) find that population growth is not necessary for long-run economic growth and that the effect of population growth on economic growth can be either non-positive (Dalgaard and Kreiner, 2001), or ambiguous (Strulik, 2005 and Bucci, 2008). In all these papers, however, population (size and/or growth) is an exogenous variable.

lowest literacy and economic growth rates, the following question becomes of paramount importance: what is the effect that a further increase in the fertility rate (thus, in the population growth rate) may have on human capital accumulation and, through this channel, on sustained long-run economic growth?

The main objective of the present paper is to tackle this issue, from a theoretical as well as an empirical perspective. In order to achieve this objective we briefly present the main results of a simple benchmark model in which individuals can invest solely in human capital, the only input in the aggregate production function. We also assume that final output (*i.e.*, aggregate GDP) can be only consumed, that the economy is closed (there is no international trade in goods and services and no international migration of people) and, finally, that there exists no governmental activity. Thus, in such an environment there cannot be any investment in physical capital. In this model the birth rate, and as a consequence population growth, is exogenous and affects linearly and negatively per-capita human capital investment (linear dilution effect). This is a rather standard assumption in the growth literature with human capital accumulation. The main prediction of the model is that the impact of population growth on economic growth is monotonic and negative.[4]

Then we extend the benchmark model both by allowing for an endogenous birth rate, and for a nonlinear effect of this rate on per-capita human capital investment (nonlinear dilution effect). We see that, through these changes, the impact of population growth on economic growth becomes itself nonlinear, implying that the growth-effect of higher fertility may well differ (in sign and in magnitude) across different groups of countries at different levels of birth rate. Kelley (1988, p. 1686) was among the first to admit that the relationship between population and economic growth rates might have been somehow nonlinear.[5]

Finally, in the last section of the paper we test empirically the hypothesis of exogeneity of the birth rate and the hypothesis of linearity of its relationship with per-capita human capital investment. In order to do so we use semi-parametric and fully non-parametric methods. Our empirical investigation finds strong evidence against the exogeneity of the birth rate, as well as against linearity (the two building–blocks of the simple benchmark model).[6]

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4 At most, a rise of the growth rate of population has no effect on economic growth. This occurs when agents are perfectly altruistic towards future generations, a very special case.

5 *"[…] In some countries population growth may on balance contribute to economic development; in many others, it will deter development; and in still others, the net impact will be negligible"* (Kelley, 1988, p. 1686).

6 The economic consequences of demographic change are at the hearth of many empirical studies. While in some of them the focus is on the economy as a whole (for instance, Brander and Dowrick, 1994; Kelley, 1988 and Kelley and Schmidt, 2003), in others country-based, household-level surveys are presented in order to shed light on the link between family size/fertility and human capital investment/children's performance at school (see, among others, Rosenzweig and Schultz, 1987 for Malaysia; Goux and Maurin, 2004 for France and Stafford, 1987 for the US). In comparison to Brander and Dowrick (1994), who analyze the impact of the birth rate on physical capital investment, we study the possible nonlinear effect of the birth rate on human

Kalaitzidakis *et al.* (2001), using non-parametric techniques, were among the first to search empirically for a nonlinear effect of human capital accumulation on economic growth. In their paper, the authors split human capital by gender (male *vs.* female human capital) and by category (primary *vs.* tertiary education). They maintain that the nonlinearities come especially from the distinction of human capital by gender (as a result of the discrimination between males and females in the labor market), and suggest that overall human capital investment has a negative impact on economic growth (due to the fact that, particularly in countries with low levels of human capital, acquiring skills is generally considered as a rent-seeking economic activity). In our paper, we follow a similar strategy as in Kalaitzidakis *et al.* (2001), since we try to understand the presence of possible nonlinearities of demographic factors on human capital accumulation and, hence, economic growth. Starting from the idea that a change in the birth rate affects the decision of how much to invest in per-capita human capital, and realizing that a higher birth rate leads to depletion of available resources and so investing in education is more costly, we claim that it is the relation between birth rate and per-capita human capital accumulation to be nonlinear. The presence of this nonlinearity, in turn, implies nonlinearity in the relationship between the birth rate and economic growth as well, and suggests that the opportunity cost of having children is different across countries which have different birth rates.

The article is organized as follows. In section 2 we present briefly the main assumptions and predictions of a simple (benchmark) model in which all the demographic components of population growth are taken as exogenous and summarized in a single parameter, n. This benchmark model represents a useful theoretical starting point and offers itself for comparison with a richer model (section 3) in which the birth rate is made endogenous and affects in a nonlinear manner per-capita human capital accumulation. This model is compatible with the existence of a nonlinear relationship between population and economic growth rates. In section 4, we empirically estimate the effect of the birth rate on human capital accumulation. According to this analysis, the impact of the birth rate on human capital accumulation at the individual level is found to be nonlinear, a result that is contrary to the linear relationship assumed in the simple benchmark model. Furthermore, there is strong evidence that the birth rate is endogenous. The last section summarizes and concludes.

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capital accumulation. Contrary to Kelley (1988) and Kelley and Schmidt (2003) we use data covering a larger time-span (1960/2000) and non-parametric methods. Finally, unlike the household-level, country-based surveys (that in general predict a negative and linear relationship between the two variables being analyzed), our main empirical motivation in the present paper is to search for possible nonlinearities in the relation between birth rate and per-capita skill investment.

2. Analytical Framework: A simple (bemchmark) Model with Exogenous Fertility and Linear Dilution in Per-Capita Human Capital investment

2.1 The Environment

Consider an economy in which households purchase consumption goods and choose how much to invest in human capital. Each individual in the population offers inelastically one unit of labor-services per unit of time. Hence, population (N) coincides with the available number of workers and grows at an exogenous and constant rate $\gamma = n - d$. Population growth (γ) depends on three fundamental variables: fertility (the birth rate, n), mortality (the death rate, d), and migration. To start with, in this model we abstract from any fertility decision (whereby rational agents choose the number of their descendants by weighing costs and benefits of rearing children), we neglect migration (the economy is closed to international trade in goods and services and to international mobility of people) and consider the mortality rate as exogenous. Therefore, we take the growth rate of population as given. Moreover, since in this paper we focus on the birth rate as the fundamental variable affecting agents' investment in education (in the empirical section of this work we estimate the effect that a change in the birth rate has on per-capita human capital investment[7]), we simplify further the analysis by setting $d = 0$.[8] Hence, in the remainder of the paper the population growth rate equals the birth rate $\dot{N}_t / N_t \equiv \gamma = n$.[9]

Following Barro and Sala-i-Martin (2004, Chap.5, p. 240) the total stock of human capital existing in the economy at time *t* ($H_t \equiv N_t h_t$) changes not only because population size can change, but also because the average quality of each worker (or per capita human capital, $h_t = H_t / N_t$) may increase over time.

Consumption goods (or final output) are produced competitively by using solely human capital (H_Y) as an input. The aggregate technology for the production of final output is:

$$Y_t = AH_{Yt}, \qquad H_{Yt} = u_t H_t \tag{1}$$

with Y_t denoting the GDP of the economy at time *t*, $A > 0$ representing a positive productivity parameter (total factor productivity), and u_t being the share of the total stock of human capital devoted to the production of goods at *t*.

[7] The birth rate is the main source of the human capital *dilution-effect* according to some empirical surveys in which it is clearly shown that an increase in the family size leads to a reduction in children's participation to education (Radyakin, 2007; Booth, 2005).

[8] For a comprehensive analysis of the effects of declining mortality (rising longevity) on investment in education and economic growth see, among others, Ehrlich and Lui (1991), Zhang and Zhang (2005), Kalemli-Ozcan (2002).

[9] In the subsequent model we endogenize the birth rate while continuing to leave migrations out of the analysis and to normalize the death rate to zero. Therefore, in the next model population growth is endogenous, as well.

There is no physical capital investment and final output (the *numeraire* in this economy) can be only consumed. The aggregate production function (Eq. 1) is linear in the amount of human capital devoted to goods manufacturing. However, unlike *"AK"-type* growth models, individuals choose endogenously at each time *t* how to allocate the existing stock of (human) capital between production of consumption goods (u_t) and production of new (human) capital (1-u_t). Under our assumptions, the economy-wide budget-constraint reads as:

$$Y_t = AH_{Yt} = Au_tH_t = Au_tN_th_t = C_t, \tag{2}$$

where C is aggregate consumption.

Concerning human capital accumulation, we follow Uzawa (1965) and Lucas (1988) and assume that the law of motion of human capital at the economy-wide level is:

$$\dot{H}_t = \sigma(1-u_t)H_t, \qquad \sigma > 0, \tag{3}$$

where σ is a technological parameter representing the productivity of human capital in the production of new human capital.[10] For the sake of simplicity we assume that human capital is not subject to any form of material obsolescence. Given $\dot{H}_t$ and the definition of per capita human capital h_t, the law of motion of human capital in per-capita terms is:

$$\dot{h}_t = \frac{\dot{H}_t}{N_t} - nh_t = \left[\sigma(1-u_t) - n\right]h_t.$$

In the last equation, with $n > 0$, the term $-nh_t$ represents the cost (in terms of per-capita human capital investment) of upgrading the degree of education of the newborns (who are uneducated) to the average level of education of the existing population (linear and negative dilution in human capital accumulation at the individual level).

With a *Constant Intertemporal Elasticity of Substitution* (CIES) instantaneous utility function, the objective of the family-head is to maximize under constraint the household's inter-temporal utility deriving from per-capita consumption:

$$\underset{\{c_t,u_t,h_t\}_{t=0}^{+\infty}}{Max} U \equiv \int_0^{+\infty} \left(\frac{c_t^{1-\theta}}{1-\theta}\right) N_t^{\nu} e^{-\rho t} dt, \qquad \rho > 0; \qquad \nu \in [0;1]; \qquad \theta > 0 \tag{4}$$

$$s.t.: \quad \dot{h}_t = \left[\sigma(1-u_t) - n\right]h_t, \qquad \sigma > 0; \qquad n \geq 0; \qquad u_t \in [0;1], \quad \forall t \tag{5}$$

along with the transversality condition: $\lim_{t\to\infty} \lambda_{ht} h_t = 0$ (6)

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10 *"…Uzawa (1965) worked out a model very similar to this one. The striking feature of his solution, and the feature that recommends his formulation to us, is that it exhibits sustained per-capita income growth from endogenous human capital accumulation alone: no external 'engine of growth' is required"* (Lucas, 1988, p. 19).

and the initial condition:[11] $h(0)>0$. (7)

The household decides on the amount of per-capita consumption and on the share of human capital to be devoted to production activity (respectively $c_t \equiv C_t / N_t$ and u_t). Eq. (4) is the household's inter-temporal utility function and Eq. (5) represents the per-capita human capital accumulation function. We denoted by $1/\theta$ the constant inter-temporal elasticity of substitution in consumption. Following many existing examples in the literature (for instance, Razin and Sadka, 1995, Chap. 13, footnote 1, among others) we make a formal distinction between two types of altruism: *inter-temporal* (the pure rate of time preference, ρ) and *intra-temporal altruism*, ν. The limiting case of $\nu=1$ defines the situation of *perfect altruism* (the family-head maximizes the discounted value of total utility, *i.e.* per capita utility multiplied by the aggregate family size), whereas the opposite limiting case of $\nu=0$ defines the minimal degree of altruism (the family-head maximizes solely the discounted value of per capita utility). Clearly, $\nu \in (0;1)$ describes an intermediate degree of intra-temporal altruism. Since u is a fraction, it must belong to the closed set $[0;1]$. Finally, in Eq. (5) the exogenous birth rate (n) is set at a value being positive or, at most, equal to zero.

By solving[12] the benchmark model, it is possible to show that:

$$u=\frac{\sigma(\theta-1)+\rho-(\theta+\nu-1)n}{\sigma\theta}$$

$$g=\frac{(\sigma-\rho)-(1-\nu)n}{\theta}$$

Hence, exogenous population growth (n) may have either a negative or no effect on real per-capita income growth (g). Intuitively, the impact of population growth on per-capita income growth can be explained as follows. Along the balanced growth path (BGP), economic growth is driven by human capital accumulation:

$$\frac{\dot{h}_t}{h_t} \equiv g=\sigma(1-u)-n$$

So, an increase in n has two opposing effects:

[11] We normalize population size (and, therefore, the number of workers) at time zero, $N(0)$, to one. Under this assumption the objective function can also be written as: $U \equiv \int_0^{+\infty}\left(\frac{c_t^{1-\theta}}{1-\theta}\right)e^{-(\rho-\nu n)t}dt$. In this case $\rho>\nu n$ ensures that U is bounded away from infinity if c remains constant over time.

[12] The solution of the model is very simple, and available upon request.

- On the one hand, it deters human capital investment and economic growth through the direct, linear and negative *'dilution'* effect (the term $-n$ in the equation above).
- On the other hand, for $\theta+v>1$, something that can be justified empirically, it fosters human capital investment and economic growth through the indirect and positive *'accumulation'* effect – the term $(1-u)$ in the equation above. Alternatively, if $\theta+v=1$, then there is no positive effect of population growth on human capital accumulation.[13]

The above argument suggests that in the benchmark model the linear and negative dilution effect always prevails over (or, at most, is equal to) the positive accumulation effect, e.g. $\partial g / \partial n \leq 0$, $\forall n \geq 0$.

3. THE MODEL WITH ENDOGENOUS FERTILITY AND NONLINEAR DILUTION IN PER-CAPITA HUMAN CAPITAL INVESTMENT

In this section we extend the previous model along two directions: *i)* We endogenize the birth rate (n) and, more importantly, *ii)* We generalize our analysis by considering a nonlinear *dilution-effect* of the birth rate on the human capital investment equation expressed in per-capita terms. The main implication of this assumption is that at low birth rate levels children are more likely to be educated, whereas at high birth rate levels children enter more quickly into the labor force in order to contribute to their family income. Furthermore, in countries with low growth rates and shorter life expectancy, parents prefer to invest in the number of children as a form of security for their retirement. As far as we know, this is the first attempt in the literature at analyzing and characterizing a similar model.[14] To be more concrete, we postulate that the dilution-effect of the birth rate on human capital accumulation is represented by a generic nonlinear function of n, $f(n_t)$, in the law of motion of per-capita human capital:

$$\dot{h}_t = \left[\sigma(1-u_t) + f(n_t)\right] h_t \tag{8}$$

We assume that $f'(n_t) < 0$ and $f''(n_t) < 0$, which guarantees that we have a maximum solution for the agent's problem. The structure of the economy remains the same as in the benchmark model. In particular,

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13 *"…The above literature review is by no means complete; it is included here to point out that, although the assumption of* $\gamma=1$ *is knife-edge and disputable, the alternative assumption of* $\gamma<1$ *is also quite troublesome. Empirical investigations bring somewhat convincing evidence, that* $\gamma>1$ " (Growiec, 2006, p.18). The author also states: "…*We note that for CRRA utility functions, u can be everywhere positive only if* $\gamma<1$ " (Growiec, 2006, p.7). In Growiec (2006)'s framework $1/\gamma$ denotes the intertemporal elasticity of substitution in consumption. In order to ensure the positivity of the instantaneous utility function $u(c_t)=c_t^{1-\theta}/(1-\theta)$, we (like Growiec, 2006) assume in the rest of the analysis that $0<\theta<1$. Moreover, since we also know that (according to empirical studies) θ is sufficiently large, in principle assuming $\theta+v\geq 1$ does not appear so unrealistic.

14 The original Lucas' (1988, Eq. 13, p. 19) formulation does not include any *dilution-effect*. Lucas' assumption (newborns enter the work-force endowed with a skill-level proportional to the level already attained by older members of the family, so population growth *per se* does not reduce the current skill level of the representative worker) is based on the *social nature* of human capital accumulation, which has no counterpart in the accumulation of physical capital and of any other form of tangible assets. On the other hand, the assumption of a linear and negative *dilution-effect* of population growth on per-capita human capital investment can be found in Strulik (2005, p. 135, Eq. 24) and Bucci (2008, p. 1134, Eq. 12'), just to mention a few examples.

the aggregate production function and the economy's budget constraint (Eqs. 1 and 2, respectively) are unchanged, and we continue to make the following main assumptions: agents devote their own time-endowment in part (u_t) to production activities and in part ($1-u_t$) to produce new human capital; the economy is closed; there is no migration of people across countries; we still set the death rate equal to zero ($d=0$). Therefore, the rate of population growth coincides again with the birth rate, now being an endogenous variable.

Because the birth rate is endogenous, we follow Palivos and Yip (1993) in considering the birth rate (net population growth), along with per-capita consumption, as an argument of the individual instantaneous utility function. However, we depart from Palivos and Yip (1993) in that we analyze the predictions of a model with endogenous allocation of human capital between production and education sectors. More specifically, the instantaneous utility function of each individual in the economy is:

$$u\left(c_t;n_t\right)\equiv\frac{\left(c_t^{\beta}n_t^{1-\beta}\right)^{1-\theta}}{1-\theta},\qquad \beta\in\left(0;1\right],\qquad \theta>0\qquad \text{and}\qquad \theta\neq 1 \tag{9}$$

where $c_t \equiv C_t / N_t$ is per-capita consumption, n_t is the net population growth rate (the birth rate), β and $\left(1-\beta\right)$ determine, respectively, the weights by which c and n enter an individual's utility function. As it is well-known, the utility function of Eq. (9) exhibits a constant elasticity of marginal utility for both c and n.[15] The hypothesis $\beta\in\left(0;1\right]$ suggests that per-capita consumption is a fundamental argument of each individual's instantaneous utility. As in Palivos and Yip (1993), θ must be different from one for an equilibrium value of n to exist (see next Eq. 14). For positive values of c_t and n_t, and with $\beta\in\left(0,1\right)$, the assumption $0<\theta<1$[16] guarantees that the second order conditions for a maximum are satisfied and that $u\left(\cdot\right)$ remains always strictly positive. The evolution of the population size (*N*) over time is governed by:

$$\dot{N}_t = n_t N_t,\qquad\qquad n_t\geq 0. \tag{10}$$

The inter-temporal problem faced by the representative family-head is now:

$$\underset{\{c_t,u_t,n_t,h_t,N_t\}_{t=0}^{+\infty}}{Max}\; U\equiv\int_0^{+\infty}\frac{\left(c_t^{\beta}n_t^{1-\beta}\right)^{1-\theta}}{1-\theta}e^{-\rho t}N_t^{\nu}dt,\qquad \rho>\nu n\geq 0;\qquad \nu\in\left[0;1\right];\qquad \beta\in\left(0;1\right];\qquad \theta\in\left(0;1\right) \tag{11}$$

$$s.t.:\qquad \dot{N}_t = n_t N_t,\qquad\qquad n_t\geq 0\quad \forall t \tag{10}$$

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15 The elasticity of marginal utility of consumption equals $\left[1-\beta\left(1-\theta\right)\right]$, whereas n has an elasticity of marginal utility equal to $\left[\beta+\theta\left(1-\beta\right)\right]$. When $\beta=1$ the utility function becomes $u\left(c_t;n_t\right)\equiv\frac{c_t^{1-\theta}}{1-\theta}$ and $1/\theta>0$ is the usual elasticity of substitution in consumption.

16 See Growiec (2006), pp.10,16 and 17 and Jones (2001), p. 4, Eq. (1).

$$\dot{h}_t = \left[\sigma\left(1-u_t\right)+f\left(n_t\right)\right]h_t, \qquad \sigma>0; \quad u_t\in\left[0;1\right],\ \forall t \tag{8}$$

along with the two transversality conditions: $\lim_{t\to\infty}\lambda_{ht}h_t=0$; $\lim_{t\to\infty}\lambda_{Nt}N_t=0$.

and the initial condition: $h\left(0\right)>0$.

3.1 BGP Analysis

We employ the following definition of BGP equilibrium.

Definition: BGP Equilibrium

A BGP equilibrium is an equilibrium path along which: (i) All variables depending on time grow at constant (possibly positive) exponential rates; (ii) The allocation of human capital between production of consumption goods and production of new human capital is constant ($u_t=u$ *,* $\forall t$ *).*

Proposition 1

Along the BGP equilibrium, we have:

$$u=\frac{\beta\left(\theta-1\right)\left[\sigma+f\left(n\right)\right]+\rho-vn}{\sigma\left[\beta\left(\theta-1\right)+1\right]} \tag{12}$$

$$g_c=g_y=g_h\equiv g=\frac{\left(\sigma-\rho\right)+vn+f\left(n\right)}{\left[\beta\left(\theta-1\right)+1\right]} \tag{13}$$

$$f^{'}\left(n\right)=\frac{v}{\beta\left(\theta-1\right)}-\left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta\left(\theta-1\right)+\beta\left(\theta-1\right)f\left(n\right)+\rho-vn}{\beta\left(\theta-1\right)+1}\right] \tag{14}$$

Proof: See *Appendix A.* ■

Equations (12) and (13) provide respectively the fraction of human capital (u) devoted by each individual in the population to non-educational activities (namely, production of consumption goods), and the common growth rate (g) attained by the economy in per-capita terms over the very long-run (BGP equilibrium), whereas (14) gives, by knowing the exact form of the nonlinear function $f\left(n\right)$, the BGP equilibrium values of the endogenous birth rate, $n\geq 0$. Note that in this model (as in the benchmark case) economic growth is driven by human capital accumulation (that is, $\dot{h}_t/h_t=g$).

It is apparent from Eq. (14) that the current model may display multiplicity of equilibria. Indeed, it is immediate to see that if $f^{'}\left(n\right)$ is a quadratic function of n, in general Eq. (14) is satisfied (for given β,

ν, θ, σ and ρ) for different values of n.[17] However, it is not the objective of this paper to build a theory of the role of multiplicity in economic growth, since we are mainly focused here in highlighting the possible non-linear effects of the birth rate on real per-capita income growth through per-capita human capital investment. In this respect, it is worth observing that now economic growth is a nonlinear function of n (Eq. 13). This result is entirely explained by the nonlinearity of the effect of population growth (the net birth rate) on per-capita human capital investment.

Proposition 2 ensures that the endogenous variables of the model undertake economically meaningful values along the BGP.[18] The condition found in the following proposition is very important for conducting comparative statics.

PROPOSITION 2
Assume $\sigma > \rho - \nu n > 0$. The following inequality:

$$-(\sigma - \rho + \nu n) < f(n) < \frac{\rho - \nu n - \sigma\beta(1-\theta)}{\beta(1-\theta)} \tag{15}$$

ensures that along the BGP equilibrium $g > 0$ and $u \in (0;1)$ hold simultaneously.

Proof: Immediate from (12) and (13). ■

In this economy the key policy-parameter is σ which captures the productivity of the education sector. A high value of σ ensures that the growth rate of the economy is positive. However, the total effect of σ as well as the other parameters β, ρ, ν and θ on the growth rate of the economy can be broken down into a direct and an indirect effect due to the endogenous nature of the birth rate.

Proposition 3 shows the direct and indirect effects of the parameters on the balanced growth rate of the economy.

PROPOSITION 3
In this economy we have the following results:

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[17] In Appendix A (Table A.1) we show that, under specific combinations of the parameter values, multiple (real and positive) roots for n coming from Eq. (14) do exist.

[18] The birth rate (n) is an endogenous variable in this economy. In particular, Eq. (14) provides the endogenous value(s) of n as a function of β, ρ, ν, σ and θ. Since at the moment we do not do any assumption about the specific functional form undertaken by $f(n)$, we cannot compute in closed form the possible solutions of $n(\beta, \rho, \nu, \sigma, \theta)$. Hence, inequality (15) provides ultimately restrictions on the relations between the parameters of the model.

DIRECT EFFECT	INDIRECT EFFECT	TOTAL EFFECT
$\frac{\partial g}{\partial \sigma} > 0$	$\frac{\left[v+f'(n)\right]\frac{\partial n}{\partial \sigma}}{\left[\beta(\theta-1)+1\right]}$>0; $\frac{dn}{d\sigma}<0$	$\frac{dg}{d\sigma} > 0$
$\frac{\partial g}{\partial v} > 0$	$\frac{\left[v+f'(n)\right]\frac{\partial n}{\partial v}}{\left[\beta(\theta-1)+1\right]}$<0; $\frac{dn}{dv}>0$	$\frac{dg}{dv} > 0$
$\frac{\partial g}{\partial \rho} < 0$	$\frac{\left[v+f'(n)\right]\frac{\partial n}{\partial \rho}}{\left[\beta(\theta-1)+1\right]}$<0; $\frac{dn}{d\rho}>0$	$\frac{dg}{d\rho} < 0$
$\frac{\partial g}{\partial \beta} > 0$	$\frac{\left[v+f'(n)\right]\frac{\partial n}{\partial \beta}}{\left[\beta(\theta-1)+1\right]}$>0; $\frac{dn}{d\beta}<0$	$\frac{dg}{d\beta} > 0$
$\frac{\partial g}{\partial \theta} < 0$	$\frac{\left[v+f'(n)\right]\frac{dn}{d\theta}}{\left[\beta(\theta-1)+1\right]}$>0; $\frac{dn}{d\theta}<0$	$\frac{dg}{d\theta}<0$, for small altruism $\frac{dg}{d\theta}>0$, for high altruism

Proof: See Appendix A. ■

Proposition 3 highlights a number of interesting issues. First of all, the productivity parameter in the education sector (σ) has a positive effect on the growth rate of the economy, and the higher it is the lower is the birth rate. This implies that in more developed countries, where the education sector is probably more productive, parents would prefer to have fewer children as the opportunity cost of having children is quite high. When the altruistic parameter (v) increases, for a given level of population, parents will care more both of their own utility and that of their descendants, and hence the higher this parameter the higher the birth rate in equilibrium. In that case, the direct effect of (v) dominates the negative dilution-effect on economic growth.

The other two results are quite intuitive. It is well known that an increase in the time-preference parameter ρ reduces growth because people are more impatient and they do not save enough. Normally, the expected sign of $\frac{\partial n}{\partial \rho}$ would be negative because people with high(ρ) are not taking into account future generations. However, in our model it is positive, something that suggests that the motivation of having children goes beyond the simple consideration of available resources. The higher the weight individuals assign to consumption per capita (β) relatively to the number of births, the higher will be the positive impact of (β) on economic growth, and the lower the birth rate in equilibrium. Finally θ, which describes the relative risk aversion of individuals, has a negative effect on growth for small values of altruism (high β and low v), and a positive effect for high values (low β and high v). Children are considered to be a risky asset and the higher the value of θ, the lower the birth rate in equilibrium**.**

4. From Theory to Evidence: Empirical Methodology and Data Description

By focusing on the effect of the birth rate n on human capital accumulation,[19] our main interest in this section is to estimate Eq. (8):

$$\dot{h}_t / h_t = \sigma(1-u_t) + f(n_t).$$

Since there are no available data for u (which is a function of n according to Eq. 12), simply replacing in the equation above the equilibrium value of u (Eq. 12), yields:

$$\dot{h}_t / h_t = \underbrace{\sigma(1-u_t)}_{\text{accumulation}} + \underbrace{f(n_t)}_{\text{dilution}} = \underbrace{\sigma - \frac{\beta(\theta-1)\left[\sigma + f(n)\right] + \rho - vn}{\left[\beta(\theta-1)+1\right]}}_{\text{accumulation}} + \underbrace{f(n_t)}_{\text{dilution}} \equiv \phi(n_t)$$

Hence, the equation we ultimately estimate is of the form:

$$\frac{\dot{h}_t}{h_t} = \phi(n_t).$$

This equation captures at the same time the combined effect of the *accumulation* and *dilution* effects. If the total effect of a change in n on $\dot{h}_t / h_t$ is positive, then the dilution effect is smaller than the accumulation effect, whereas if it is negative then the dilution effect of having a higher fertility rate dominates the accumulation effect. We use an estimation method more flexible than OLS in order not only to check whether the relationship written above is mis-specified, but also to search for nonlinearities in the link between the birth rate and human capital investment at the individual level. In addition, we also examine the possible endogeneity of n. However, before going into the details of the econometric techniques and the specification tests used, we start by describing briefly the variables employed and the sources of our data.

Our sample consists of a panel of ninety-nine countries (both OECD and non-OECD) for the period 1960–2000. The observations are averages over a 5 years-interval.[20] Hence, we have 9 time periods. For

[19] Even though in the theory presented above we set $d=0$, in our empirical analysis we include the death rate as a control variable. More specifically, we use life expectancy as a proxy for the death rate. Alternatively, and for the sake of robustness, we also employed the crude death rate. Contrary to life expectancy, the crude death rate displays negative sign and is statistically significant. In both cases, however, we do find a nonlinear effect of the birth rate on per-capita human capital accumulation. The results with the death rate as a control variable are available upon request.

human capital accumulation (which is our dependent variable) we use enrollment rates for population aged between 15 and 65 years.[21] The data about this variable come from the Barro–Lee (2000) dataset. Barro and Lee (2000) fill the missing observations for some specific years and countries by using information taken from United Nations on school enrollment ratios and the structure of population by age-groups. Furthermore, they use adjusted gross enrollment ratios in order to take into account a significant fraction of students who either start earlier school attendance, or change their own level of education with delay. It is well-known that the Barro–Lee dataset for human capital has an important limitation in the fact that it does not explicitly take into account differences in schooling-quality across countries. For this reason we use group dummy variables for controlling for any potential difference between groups of countries. Despite its limitations, the Barro–Lee (2000) dataset has been extensively used in recent years and as such allows us to make direct comparisons with other empirical studies that explore the role of human capital in economic growth.[22]

According to de la Fuente and Domenech (2000), the stock measure of human capital (total mean years of schooling data) suffer from serious measurement error problems, something that would be exacerbated if we were to obtain growth rates from differencing the stock series. Hence, instead of measuring human capital accumulation in growth rates as it is the case for per-capita income, we prefer to use enrollment rates. Human capital accumulation for country *i* at time *t* is denoted by $HUM_{i,t}$. The data for the crude birth rate (*cbr*) come from the UN dataset (2008). The crude birth rate is measured as the number of births over 1000 people.[23] This is the variable that we estimate by using both OLS and semi/non-parametric methods.[24] All the other variables enter linearly in our econometric specification.

We use time dummies (d_t) in order to capture any time specific effects and group dummies ($dOECD_i$) to capture any difference between OECD and non-OECD countries. Following Kalaitzidakis *et al.* (2001),

[20] The use of averages over periods of 5-years, as opposed to the use of annual data, has been proposed in the literature as a strategy for reducing possible business cycle effects.

[21] When working with fertility it is better to use human capital data for population which is above 15 years old. We have tried another dataset for human capital from the Barro-Lee (2000) data base for population above 25 years and the nonlinearities are also present there. The results for population over 25 years old are available upon request**.**

[22] We use total human capital, which is the sum of primary, secondary and higher education. This is done for three major reasons. The first is mainly theoretical in nature and relies on the fact that in the model of the previous section there is no distinction of human capital by category of education. Secondly, non-OECD countries exhibit very small participation rates at higher levels of education. Finally, because in every country participation at primary and secondary level of education is a necessary requirement in order for people to proceed into higher levels of schooling, we believe it is preferable to include also primary and secondary education in our measure of human capital.

[23] The crude birth rate is expressed in percentage terms for comparability with the enrollment rates.

[24] We have also used the *adjusted birth rate* proposed by Brander and Dowrick (1994):*"…Adjusted births are simply crude or gross births net of infant mortality"* (Brander and Dowrick, 1994, p. 5). The results do not change and are available upon request.

we also employ regional dummies (dRegion_i) – such as dummies for Latin America (*la*) and Sub-Saharan Africa (*af*) countries – to control for possible heterogeneity in our sample.

Finally, we use a vector $X_{i,t}$ = (lifexp, hum60, infmort) of control variables that are used at a later stage in order to check for the robustness of our results. These variables are life expectancy (*lifexp*), human capital at 1960 (*hum60*),[25] and infant mortality (*infmort*), respectively. The data concerning the two demographic variables (life expectancy and infant mortality), like those on the birth rate, come from the UN (2008) dataset. Life expectancy (*lifexp*) is defined as *"the average number of years of life expected by a hypothetical cohort of individuals who would be subject during all their lives to the mortality rates of a given period"* (United Nations, 2008). This variable is expected to have a positive impact on human capital accumulation (intuitively, when expecting a higher chance of surviving, rational individuals invest more in education to enhance earnings for consumption over a longer life. Moreover, the longer life expectancy, the longer the time-horizon within which the costly investment in human capital can be re-paid). Infant mortality (*infmort*) is the probability of dying between birth and age one. It is expressed as the number of deaths per 1000 births and it is expected to have a negative impact on human capital accumulation, since a rise of infant mortality would imply that the *quantity-effect* (having more children) dominates the *quality-effect* (having lower fertility, but more education per child). The average years of schooling at 1960 (*hum60*) is a stock measure of human capital and is used in order to capture the difference in initial conditions across countries. Contrary to initial GDP per capita (used in empirical exercises on the neoclassical growth theory to predict convergence), in our endogenous growth model in the long-run (the BGP equilibrium) there exists no convergence across countries starting from different levels of development. Hence this variable reveals the preference of a country towards human capital accumulation and as such it is expected to have a positive impact on our dependent variable.

The equation we estimate by OLS is the following:

$$HUM_{i,t} = b_0 + b_1\left(dOECD_i\right) + b_2\left(dRegion_i\right) + b_3 d_t + b_4 n_{i,t} + \varepsilon_{i,t} ,$$

with $\varepsilon_{i,t}$ being an *iid* error term. The equation we estimate in the semi-parametric framework is:

$$HUM_{i,t} = b_0 + b_1\left(dOECD_i\right) + b_2\left(dRegion_i\right) + b_3 d_t + \phi\left(n_{i,t}\right) + \varepsilon_{i,t} .$$

In the above equation, $\phi(n_{i,t})$ is used to capture possible nonlinearities in the relationship between birth rate and human capital accumulation (all the other variables enter linearly in our specification). We also

[25] Human capital at 1960 (*hum60*) is the stock of human capital in the year 1960. It is measured as the average years of schooling in that year. We use this variable, and not the enrollment rate at 1960, in order to reduce any possible bias due to the correlation with the dependent variable. However, we have also used enrollment rates at 1960, but results are not affected at all by this change.

augment the model to include additional available explanatory variables to guard against omitted variable bias and we further test within the linear framework for possible endogeneity of n, using a Durbin-Wu-Hausman test. Having established the presence of a nonlinear relationship, we will try to best approximate it by different polynomials both in the model with and without the control variables, selecting the one with the highest explanatory power. Subsequently, we use the specification with the extra control variables for conducting sensitivity analysis. The following equation is the one used for checking the robustness of the linear specification estimates, and a similar equation is used when we include the optimal polynomial of birth rate:

$$HUM_{i,t} = b_0 + b_1\left(dOECD_i\right) + b_2\left(dRegion_i\right) + b_3 d_t + b_4 n_{i,t} + b_5 X_{i,t} + \varepsilon_{i,t}$$

where $X_{i,t}$ = (lifexp, hum60, infmort) is the vector of control variables. Finally, the equation we estimate by considering all the variables non-parametrically is:

$$HUM_{i,t} = f(b_0, dOECD_i, dRegion_i, d_t, n_{i,t}) + \varepsilon_{i,t}.$$

The fully non-parametric method that is used in this paper was proposed first by Racine and Li (2004) and is appropriate for mixed data (discrete and continuous variables). We also use the Hsiao *et al.* (2007) test to check whether the linear model is well specified against semi/non parametric alternatives. Finally, we use a significance test[26] for the nonparametric regression model in order to ensure that the birth rate is significant in the nonparametric framework.

4.1 RESULTS

A first glance at Table B.1 (all Tables are in *Appendix B*) reveals that the birth rate (*cbr*) is statistically significant and appears in the regression with a negative sign.[27] This is consistent with the benchmark theoretical model (see Eq. 5). The dummies both for Latin America and Sub-Saharan Africa countries are statistically significant and positive in all the specifications, except when human capital is measured at the secondary-higher level. In this case the region dummies have negative sign. This is an indication that in these groups of countries the *dilution-effect* is stronger at the secondary-higher level of education.[28] We proceed to estimate a semi-parametric and a fully nonparametric version of the above benchmark model.

26 For this test see Racine (1997).

27 As we are going to mention in a moment, the birth rate (*cbr*) appears with negative sign in the linear specification and there exist nonlinearities (a polynomial of the 3rd power). Furthermore, we have the same kind of nonlinearities also when we use total fertility (instead of the crude birth rate), which confirms our hypothesis of a nonlinear effect of newborns on per capita human capital accumulation in per-capita terms. We do not present the results with total fertility here, but they are available upon request.

28 The results for primary and tertiary level of education are not presented but are available upon request.

Clearly, the semi-parametric and the fully non-parametric specifications[29] increase the explanatory power of the model (both have a very similar explanatory power). Furthermore, the Hsiao *et al.* (2007)[30] test strongly rejects the hypothesis of a linear specification against the semi-parametric and non-parametric alternatives, see the p-values of P(Specific) in Tables B.1 and B.4. We proceed in Table B.2 to use polynomial terms for the birth rate and also a dummy-variable for OECD countries which interacts with the birth rate in order to control for any heterogeneity between OECD and non-OECD group of countries. Clearly, using polynomial terms increases the explanatory power of the model. We only present here results concerning the inclusion of a third degree polynomial, since higher-order polynomials do not appear to be statistically significant. In column B of Table B.2 the polynomial terms are statistically significant, both individually and jointly (see the *F - Joint test*). In column C of the same Table we add an extra variable which captures the interaction between the OECD-dummy and the birth rate. This term is statistically significant and positive. Since the negative coefficient on the birth rate is larger than the positive coefficient on the interaction term, we infer that the *dilution-effect* (*i.e.*, the negative impact of the birth rate on human capital investment at the individual level) in OECD countries is smaller than in non-OECD countries. Finally, in column D we consider the interaction between the OECD-dummy and the higher-power terms of the birth rate. By looking at the correspondent t-statistics and the F-Joint test, all the interaction terms are statistically insignificant both individually and jointly, which means that the birth rate behaves similarly across the different groups of countries.[31] We further proceed to test for possible endogeneity of the birth rate in the extended linear model with the extra control variables at our disposal (lifexp, hum60, infmort). The results are presented in Table B.3. In columns C and D we have similar results as in Table B.2 (without the control variables). Again the interaction terms of the polynomial with the OECD dummy, is not statistically significant both individually and jointly and also the third power polynomial remains statistically significant jointly and the signs are the same as in Table B.2. Finally, the effect of the birth rate is still nonlinear. This result appears to be robust and is the same for OECD as well as for non-OECD countries. Since the specification with the control variables displays by far the best fit we use it as the basis to conduct the test of endogeneity. We find strong evidence against the null hypothesis of no endogeneity, a result that contradicts the main premise of the benchmark model that was based on an exogenous birth rate.[32] Note that as instruments we use the lagged values of the birth rate**.** We

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29 The optimal bandwidths, for these methods are found by cross validation and are available upon request. The choice of kernel is the Gaussian kernel and the method of estimation relies on a constant kernel approach.

30 The Hsiao test results are showed in the row P(Specific.). In column A, there is the comparison between OLS and fully-non parametric and in column B the comparison is between OLS and semi parametric.

31 We have also done separate regressions for OECD and non-OECD countries and still the same third power polynomial of the birth rate appears to be the correct specification. These results are available upon request.

32 The Prob>chi2 value of Durbin-Wu-Hausman test for endogeneity is 0.0476 which implies that the null hypothesis for the absence of correlation between birth rate "*n*" and the error term is rejected.

also estimated the semi-parametric model using the lagged birth rate variable in place of the current birth rate in the semi-parametric specification; see the semi-parametric graphs (Figures 1 and 2). Figure 1 is the semi-parametric graph for the contemporaneous value of birth rate and Figure 2 for the lagged birth rate**.** Interestingly, both graphs are quite similar, suggesting that the nature of the nonlinear relationship is preserved for different measures of the birth rate, its current and lagged forms.[33] A closer look at either figure reveals that a low birth rate has a small positive contribution if any at all but after a given point the dilution effect appears to become strong. This is in line both with theories that support birth control in overpopulated countries and also with those that claim that rich countries need to do the opposite.

4.2 Sensitivity Analysis

In order to check for the robustness of our results we follow two different ways. Firstly, we use for the initial sample (792 observations) the extra control variables mentioned earlier (*i.e.*, life expectancy, infant mortality and human capital in 1965) to test the significance of the polynomial terms (compare Tables B.2 and B.3). Secondly, we exclude the *OPEC* countries as income earned from oil production potentially distorts the fertility–human capital nexus and we also exclude the *Ex-socialist* countries (Eastern Europe and Cuba), since for these countries the existence of a centrally-planned regime might have influenced more effectively the dynamics of the birth rate in the past. Results appear in Table B.4.

Table B.3 allows us to draw some additional conclusions. First of all, from column B, we observe that the polynomial for the birth rate preserves the same order and its terms are jointly statistically significant. Furthermore, in columns C and D we have similar results as in Table B.2 (without the control variables). Finally, the effect of the birth rate still appears nonlinear. This result seems to be robust and the same for OECD as well as for non-OECD countries.

From Table B.4 we see that some of the time-dummies are statistically significant. All the other results are qualitatively the same as in Table B.1. In particular, we observe that the birth rate is statistically significant and still has a negative sign. Yet, as before the Hsiao *et al.* (2007) test strongly rejects the linear specification of the benchmark model. However, the current nonlinear effect appears stronger (see Figure 3). This is in line with our conjecture that the excluded countries have been able to control more effectively their own birth rates, hence mitigating the presence of nonlinear effects.

We can summarize the findings of this section as follows. Superficially, at first there seems to be a negative relationship between the birth rate and human capital by using a linear specification corresponding to the benchmark model developed in section 2. The benchmark specification is strongly rejected when subjected to rigorous testing and re-estimation of the human capital–birth rate nexus using

[33] We have also tried additional lagged values of the birth rate and the results remained qualitatively the same.

semi-parametric and nonparametric methods reveals the presence of a strong nonlinear relationship. However, even in the presence of nonlinearities the dilution effect dominates any positive effect that comes from the accumulation effect for most of the observations in our sample. The linear benchmark specification also suffers from endogeneity bias, as the birth rate cannot be taken as exogenous. Adding extra control variables does not change the overall nonlinear pattern of the relationship. The latter result invalidates the main premise of the benchmark model that the relationship between birth rate and per-capita human capital investment would be expected to be linear.

5. Summary and Conclusions

This paper has reassessed the long-run correlation between demographic change and economic growth from an empirical as well as a theoretical point of view. In order to accomplish this task, we focused on the fertility rate as our demographic variable of interest (the birth rate is, indeed, one of the most important demographic variables, as it affects directly population growth), and on human capital accumulation as the fundamental driver of sustained long-run economic growth. From the empirical point of view, our motivation was to search for possible nonlinearities in the (negative) relation between birth rate and human capital investment at the individual level. From the theoretical point of view, instead, our main objective has been to compare the results (concerning the long-run relationship between population and economic growth rates) of two completely different models: the first is a simple (although quite standard) model in which the birth rate is exogenous and affects linearly and negatively per-capita human capital accumulation (linear and negative *dilution-effect*); the second, instead, is a model in which the birth rate is endogenous and the *dilution-effect* is nonlinear. Since countries experience different birth rates and have different economic performances as well, our major conjecture in building the second model was that fertility can have a nonlinear impact on economic growth through its nonlinear effect on per-capita human capital investment. This result accords well with the idea that an increase in population growth may have a differential impact on economic growth across countries (Kelley, 1988).

The benchmark model is strongly rejected by the data, according to which there exist strong nonlinearities in the relationship between the fertility rate and per-capita human capital investment. Because of these nonlinearities, we can say, for any different value of n (and, hence, for any different country in the sample), which effect (whether the positive accumulation effect, or the negative dilution effect) dominates the other one.

As a whole, our analysis suggests that in more developed economies higher birth rates can have a positive growth-effect and that the main policy initiatives here should concentrate on easing the opportunity costs of having children, since the *accumulation-effect* is stronger than the *dilution-effect*. Of course, for most of the other countries, and especially for the least developed ones, birth rates should be reduced substantially. Our proposal for the group of countries with high birth rates and low life expectancy is that they need simultaneously to reduce the birth rate and to improve the health conditions of their people in order to be able to achieve higher growth rates**.**

This paper can be extended along different directions, three of which are listed here. The first would be to consider a human capital production technology different from the Lucas' (1988), and more similar to the Mincer (1974)'s specification (Bils and Klenow, 2000, Eq. 3, p. 1162). Secondly, in this article we treated human capital as the sole reproducible factor, entering the aggregate production function as the only input. Though, it is well known that economic agents (individuals and firms) do accumulate and produce by way of a variety of other factor-inputs (think of physical and technological capital, just to mention some notable examples). Building and testing a theory of endogenous fertility in which there is more than one reproducible factor-input and where there exist nonlinear *dilution-effects* of the birth rate in the law of motion of one or all of these factor-inputs still remains at the top of future theoretical research agenda and, thus, represents a further possible extension of the present paper. Finally, there is scope to develop a model based on micro foundations in which nonlinearities will be derived from individual behavior, something that we leave for future research.

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APPENDIX A:

1) EQS. (12) – (14)

Since $c_t \equiv \frac{C_t}{N_t} = \frac{Y_t}{N_t} \equiv y_t = Au_t h_t$, the Hamiltonian function (J_t) is:

$$J_t = \frac{\left[\left(Au_t h_t\right)^{\beta} n_t^{1-\beta}\right]^{1-\theta}}{1-\theta} e^{-\rho t} N_t^{\nu} + \lambda_{Nt} n_t N_t + \lambda_{ht}\left[\sigma\left(1-u_t\right)+f\left(n_t\right)\right] h_t .$$

The necessary FOCs read as:

(A1) $\frac{\partial J_t}{\partial n_t} = 0 \quad \Leftrightarrow \quad \left(1-\beta\right)\left(Au_t h_t\right)^{\beta(1-\theta)} n_t^{(1-\beta)(1-\theta)-1} e^{-\rho t} N_t^{\nu} + \lambda_{Nt} N_t + \lambda_{ht} h_t f'\left(n_t\right) = 0$

(A2) $\frac{\partial J_t}{\partial u_t} = 0 \quad \Leftrightarrow \quad \beta\left(Ah_t\right)^{\beta(1-\theta)} u_t^{\beta(1-\theta)-1} n_t^{(1-\beta)(1-\theta)} e^{-\rho t} N_t^{\nu} - \sigma\lambda_{ht} h_t = 0$

(A3) $\frac{\partial J_t}{\partial N_t} = -\dot{\lambda}_{Nt} \quad \Leftrightarrow \quad \nu\frac{\left(Au_t h_t\right)^{\beta(1-\theta)} n_t^{(1-\beta)(1-\theta)}}{1-\theta} e^{-\rho t} N_t^{\nu-1} + \lambda_{Nt} n_t = -\dot{\lambda}_{Nt}$

(A4) $\frac{\partial J_t}{\partial h_t} = -\dot{\lambda}_{ht} \quad \Leftrightarrow \quad \beta\left(Au_t\right)^{\beta(1-\theta)} n_t^{(1-\beta)(1-\theta)} h_t^{\beta(1-\theta)-1} e^{-\rho t} N_t^{\nu} + \lambda_{ht}\left[\sigma\left(1-u_t\right)+f\left(n_t\right)\right] = -\dot{\lambda}_{ht}$.

Along the BGP equilibrium all variables depending on time grow at constant exponential rates. Therefore, *n* and *u* are constant in the long-run (see Eqs. 10 and 8 in the text). With this in mind, after taking logs of both sides of (A2) and differentiating with respect to time, we obtain:

(A5) $-\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = \left[\beta\left(\theta-1\right)+1\right]\left[\sigma\left(1-u\right)+f\left(n\right)\right]+\rho-\nu n$, $\qquad \frac{\dot{h}_t}{h_t} \equiv g_h = \left[\sigma\left(1-u\right)+f\left(n\right)\right]$.

We solve (A2) with respect to λ_h:

(A2') $\lambda_{ht} = \sigma^{-1}\beta A^{\beta(1-\theta)} h_t^{\beta(1-\theta)-1} n^{(1-\beta)(1-\theta)} u^{\beta(1-\theta)-1} e^{-\rho t} N_t^{\nu}$.

Using (A2') into (A4) and dividing both sides of the resulting equation by λ_{ht}, yields:

(A4') $-\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = \sigma + f\left(n\right)$.

We now divide both sides of (A3) by λ_{Nt}:

(A3') $-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = \frac{\nu\left(Auh_t\right)^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)} e^{-\rho t} N_t^{\nu-1}}{\left(1-\theta\right)\lambda_{Nt}} + n$,

and both sides of (A1) by λ_{ht}:

$$\frac{\left(1-\beta\right)}{\lambda_{ht}}\left(Auh_t\right)^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)-1} e^{-\rho t} N_t^{\nu} + \frac{\lambda_{Nt}}{\lambda_{ht}} N_t + h_t f'\left(n\right) = 0 .$$

If we substitute (A2') for the first term of the last equation we obtain:

(A1') $\frac{\lambda_{Nt} N_t}{\lambda_{ht} h_t} = -\left[f'\left(n\right) + \frac{\left(1-\beta\right)\sigma u}{\beta n}\right]$.

In the BGP equilibrium the right hand side of (A1') is a constant. Therefore, we can take logs of both sides of (A1'), and derive with respect to time:

$$\frac{\dot{\lambda}_N}{\lambda_N} + \frac{\dot{N}}{N} - \frac{\dot{\lambda}_h}{\lambda_h} - \frac{\dot{h}}{h} = 0 .$$

By using (A4') and Eqs. (8) and (10) in the main text, we can recast the last expression as:

(A1'') $-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = n + \sigma u$.

By equating (A4') with (A5) we obtain the BGP equilibrium value of u:

(A6) $u = \frac{\beta(\theta-1)\left[\sigma + f(n)\right] + \rho - vn}{\sigma\left[\beta(\theta-1)+1\right]}$

Eq. (A2') can be recast as:

$$\frac{\sigma u \lambda_{ht} h_t}{\beta} = A^{\beta(1-\theta)} h_t^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)} u^{\beta(1-\theta)} e^{-\rho t} N_t^{v}.$$

This expression can be replaced into (A3'), yielding:

(A3'') $-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = \frac{\lambda_{ht} h_t}{\lambda_{Nt} N_t}\left[\frac{v\sigma u}{\beta(1-\theta)}\right] + n$.

Using (A1') into (A3'') gives:

$$-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = \frac{-nv\sigma u + n(1-\theta)\left[\beta n f'(n) + (1-\beta)\sigma u\right]}{(1-\theta)\left[\beta n f'(n) + (1-\beta)\sigma u\right]}.$$

After equating the last expression to (A1''), we get:

(A7) $f'(n) = \frac{v}{\beta(\theta-1)} - \frac{(1-\beta)\sigma u}{\beta n}$,

Substituting u from (A6) into (A7):

(A7') $f'(n) = \frac{v}{\beta(\theta-1)} - \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1) + \beta(\theta-1) f(n) + \rho - vn}{\beta(\theta-1)+1}\right].$

From the fact that:

$$c_t \equiv \frac{C_t}{N_t} = \frac{Y_t}{N_t} \equiv y_t = Auh_t,$$

we conclude:

$$g_c = g_y = g_h \equiv g = \sigma(1-u) + f(n).$$

After replacing in the last equation u from (A6) we finally obtain:

(A8) $g_c = g_y = g_h \equiv g = \frac{(\sigma-\rho) + vn + f(n)}{\left[\beta(\theta-1)+1\right]}$.

We now check that the two transversality conditions:

$$\lim_{t\to+\infty} \lambda_{ht} h_t = 0 \text{ and } \lim_{t\to+\infty} \lambda_{Nt} N_t = 0$$

do hold. Using Eq. (A4') and the fact that $g_h \equiv g = \sigma(1-u) + f(n)$, the first condition can be recast as:

$$\lim_{t\to+\infty} \lambda_{ht} h_t = \lambda_h(0) h(0) \lim_{t\to+\infty} e^{\left(\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} + g\right)t} = \lambda_h(0) h(0) \lim_{t\to+\infty} e^{-(\sigma u)t} = 0.$$

Hence, for each $\lambda_h(0) > 0$, $h(0) > 0$ and $\sigma > 0$, the transversality condition is clearly satisfied for each $u > 0$.

The second transversality condition, after employing Eqs. (A1'') and (10) in the text, can be re-written as:

$$\lim_{t\to+\infty} \lambda_{Nt} N_t = \lambda_N(0) \lim_{t\to+\infty} e^{\left(\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} + \frac{\dot{N}_t}{N_t}\right)t} = \lambda_N(0) \lim_{t\to+\infty} e^{-(\sigma u)t} = 0, \qquad N(0) \equiv 1.$$

Again, for each $\lambda_N(0) > 0$ and $\sigma > 0$, this transversality condition is also satisfied for each $u > 0$. In the main text we guarantee that $u > 0$ does hold.

Going back to Eq. (A7') we see that this equation gives, by knowing the exact form of the nonlinear function $f(n)$, the BGP equilibrium values of the endogenous birth rate, $n \geq 0$. If $f'(n)$ were a square function of n, Eq. (A7') would be satisfied, for given β, ν, θ, σ and ρ, for three different values of n.

Since empirically the relationship between fertility and human capital accumulation is mainly negative (the *dilution-effect* dominates the *accumulation-effect*), we assume that the empirical results we find provide a proxy for the magnitude of the dilution-effect. By looking at these empirical results (Table B2, column B), we observe that the dilution-effect can be approached by the following polynomial function: $f(n) = 26.35n - 1143n^2 + 9224n^3$. Then, Table A.1 in this Appendix provides the solutions (the real and positive roots) of Eq. (A7'), for specific combinations of the values of the parameters. For some of them we have, indeed, empirical estimates (or baseline specifications) coming from previous works. For others, we consider a set of possible parameterizations. More precisely, we use the following parameter-values:

- $\theta = 0.8$;

When $\beta = 1$, the utility function used in the text becomes $u(c;n) = \frac{\left(c^{\beta} n^{1-\beta}\right)^{1-\theta}}{1-\theta} = \frac{c^{1-\theta}}{1-\theta} = u(c)$ and $1/\theta > 0$ is the elasticity of substitution in consumption. So, θ represents the inverse of the intertemporal elasticity of substitution in consumption when $\beta = 1$. In the endogenous fertility literature many authors suggest θ to take values lower than one in order for the flow instantaneous utility function to be everywhere positive. A value of $\theta = 0.8$ is used by Growiec (2006, p.16, Table 1) who, unlike the present paper, considers a CRRA utility function which is separable in its two arguments: per capita consumption and the birth rate, respectively. When $u(c;n) = \frac{\left(c^{\beta} n^{1-\beta}\right)^{1-\theta}}{1-\theta}$, the elasticity of substitution in consumption becomes: $\frac{1}{\left[1-\beta(1-\theta)\right]}$. Therefore, the value of this elasticity depends both on θ and β.

- $\rho = 0.08$;

This parameter comes also from the numerical example discusses in Growiec (2006, p.16, Table 1).

- $\sigma = 0.12$.

This parameter-value come from Mulligan and Sala-i-Martin (1993, p.761).

- $\nu \in [0;1]$

As far as we know, Altonji *et al.* (1997) represents one of the very few attempts at obtaining a direct estimate of agents' degree of altruism. Their paper tests for a specific form of altruism, namely that of parents who make money transfers to their children. According to the theory of *inter-vivos* transfers, we would face perfect altruism if an increase, say, by one-dollar in the income of parents making transfers to a child, coupled with a simultaneous one-dollar decrease in that child's income, resulted in the parents' increasing their transfer to the child by exactly one dollar. To test for this hypothesis, the authors use the 1968-89 Panel Study of Income Dynamics (PSID) data-set, which contains a supplementary survey on family transfers. The PSID collects separate panel data on parents and most of their adult children. Consequently, the authors can control for the principal theoretical determinants of money transfers (the current and permanent incomes of the parents, the child, and the child's siblings). The effective sample consists in the end of 3402 parent-child pairs, including 687 pairs with positive transfers. The findings of this study say that redistributing one dollar from a recipient child to donor parents leads to only about a 13-cent increase in the parents' transfer to the child, far less than the one-dollar increase we would observe in the presence of perfect altruism. Using panel data on bequests, rather than *inter-vivos* transfers from parents to children, Laitner and Ohlsson (2001) obtain a similar result. On the basis of such literature, we use four different values of ν, namely $\nu = 0$; $\nu = 0.13$; $\nu = 0.5$; $\nu = 1$.

- $\beta(0;1]$

Like ν, we use three different values for β, as well: $\beta = 0.25$; $\beta = 0.5$; $\beta = 0.75$; $\beta = 1$ (this parameter cannot be equal to zero). Results are as follows:

	$\theta = 0.8$ $\rho = 0.08$ $\sigma = 0.12$
$\nu = 0$ $\beta = 0.25$	$n_1 = 0.0214$ $n_2 = 0.0653$
$\nu = 0$ $\beta = 0.5$	$n_1 = 0.0163$ $n_2 = 0.0673$
$\nu = 0$ $\beta = 0.75$	$n_1 = 0.0145$ $n_2 = 0.0682$
$\nu = 0$ $\beta = 1$	$n_1 = 0.0138$ $n_2 = 0.0688$
$\nu = 0.13$ $\beta = 0.25$	$n_1 = 0.0230$ $n_2 = 0.0634$
$\nu = 0.13$ $\beta = 0.5$	$n_1 = 0.0170$ $n_2 = 0.0665$
$\nu = 0.13$ $\beta = 0.75$	$n_1 = 0.0150$ $n_2 = 0.0676$
$\nu = 0.13$ $\beta = 1$	$n_1 = 0.0143$ $n_2 = 0.0683$
$\nu = 0.5$ $\beta = 0.25$	$n_1 = 0.0286$ $n_2 = 0.0572$
$\nu = 0.5$ $\beta = 0.5$	$n_1 = 0.0193$ $n_2 = 0.0640$
$\nu = 0.5$ $\beta = 0.75$	$n_1 = 0.0166$ $n_2 = 0.0660$
$\nu = 0.5$ $\beta = 1$	$n_1 = 0.0155$ $n_2 = 0.0671$
$\nu = 1$ $\beta = 0.25$	$n_1 = 0.0408$ $n_2 = 0.0439$
$\nu = 1$ $\beta = 0.5$	$n_1 = 0.0230$ $n_2 = 0.0600$
$\nu = 1$ $\beta = 0.75$	$n_1 = 0.0190$ $n_2 = 0.0635$
$\nu = 1$ $\beta = 1$	$n_1 = 0.0174$ $n_2 = 0.0652$

Table A1: *The roots ($n \geq 0$) of the equation* $f^{'}(n) = \frac{\nu}{\beta(\theta-1)} - \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-\nu n}{\beta(\theta-1)+1}\right]$ *for possible combinations of* ν *and* β*, and for given* $\theta = 0.8$, $\rho = 0.08$ *and* $\sigma = 0.12$.

For each single combination of the parameters reported in Table A1, it is possible to show that the restrictions $\rho - vn > 0$ and $\sigma > \rho - vn$ (see Proposition 2 in the text) do hold. Moreover, the elasticity of substitution in consumption, $\frac{1}{\left[1-\beta\left(1-\theta\right)\right]}$, is equal to: 1.05 (when $\beta = 0.25$); 1.11(when $\beta = 0.5$); 1.18 (when $\beta = 0.75$), and 1.25(when $\beta = 1$). ■

2) Second Order Conditions for Maximized Hamiltonian

In our model we may have many interior solutions for *n*, due to the specific form taken by the nonlinear function $f\left(n\right)$. Hence, we cannot solve explicitly for *n* and as such it is quite hard to solve explicitly for the Arrow concavity condition. For this reason we use Corollary 3.1 from Caputo (1995) ch. 3, p. 55 **,** which requires the intertemporal utility function to be concave in u,n,h and N and the two dynamic constraints for population and human capital to be also concave on u,n,h and N:

$$U_{NN} = \frac{v\left(v-1\right)}{1-\theta}\left(Auh\right)^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)} N_t^{v-2} \leq 0, \qquad \text{for } \theta < 1,$$

$$U_{hh} = \left[\beta\left(1-\theta\right)-1\right]\beta\left(Au\right)^{\beta(1-\theta)} h^{\beta(1-\theta)-2} n^{(1-\beta)(1-\theta)} N_t^{v} \leq 0,$$

$$U_{nn} = \left[\left(1-\beta\right)\left(1-\theta\right)-1\right]\left(1-\beta\right)\left(Auh\right)^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)-2} N_t^{v} \leq 0,$$

$$U_{uu} = \beta\left[\beta\left(1-\theta\right)-1\right]\left(Ah\right)^{\beta(1-\theta)} u^{\beta(1-\theta)-2} n^{(1-\beta)(1-\theta)} N_t^{v} \leq 0.$$

The first constraint $g_1 = n_t N_t$ is linear in all the variables $\left(u,n,h,N\right)$ and therefore concave, whereas the second constraint $g_2 = \left[\sigma\left(1-u_t\right)+f\left(n_t\right)\right]h_t$ is linear in $\left(u,h,N\right)$ and for *n* takes the following value: $g_{2nn} = f^{''}\left(n_t\right)h_t \leq 0$ for $n \in \left[0, 0.0413\right]$, which contains the average value of birth rate in our sample. For very high values of *n* it is easy to show that the growth rate of the economy is negative and the human capital accumulation will decrease, so $g_{2nn} = f^{''}\left(n_t\right)h_t \to 0$. That implies that the second constrain is linear and therefore concave in *n*. ■

3) Conditions for Proposition 3

By differentiating Eq. (13) with respect to σ, v, ρ, and θ we get the total effects as follows:

$$\frac{dg}{d\sigma} = \frac{1+\left[v+f'\left(n\right)\right]\frac{\partial n}{\partial \sigma}}{\left[\beta\left(\theta-1\right)+1\right]}; \quad \frac{dg}{dv} = \frac{n+\left[v+f'\left(n\right)\right]\frac{\partial n}{\partial v}}{\left[\beta\left(\theta-1\right)+1\right]}; \quad \frac{dg}{d\rho} = \frac{-1+\left[v+f'\left(n\right)\right]\frac{\partial n}{\partial \rho}}{\left[\beta\left(\theta-1\right)+1\right]};$$

$$\frac{dg}{d\beta} = \frac{-\left(\theta-1\right)\left[\left(\sigma-\rho\right)+vn+f\left(n\right)\right]}{\left[\beta\left(\theta-1\right)+1\right]^2} + \frac{\left[v+f'\left(n\right)\right]\frac{\partial n}{\partial \beta}}{\left[\beta\left(\theta-1\right)+1\right]}; \quad \frac{dg}{d\theta} = -\frac{\left[\sigma-\rho+vn+f\left(n\right)\right]\beta}{\left(\beta\left(\theta-1\right)+1\right)^2} + \frac{\left[v+f'\left(n\right)\right]\frac{dn}{d\theta}}{\left[\beta\left(\theta-1\right)+1\right]}$$

The direct effects are:

$$\frac{\partial g}{\partial \sigma}=\frac{1}{\left[\beta(\theta-1)+1\right]}>0; \qquad \frac{dg}{dv}=\frac{n}{\left[\beta(\theta-1)+1\right]}>0; \qquad \frac{dg}{d\rho}=\frac{-1}{\left[\beta(\theta-1)+1\right]}<0;$$

$$\frac{dg}{d\beta}=\frac{-(\theta-1)\left[(\sigma-\rho)+vn+f(n)\right]}{\left[\beta(\theta-1)+1\right]^2}>0; \qquad \frac{dg}{d\theta}=-\frac{\left[\sigma-\rho+vn+f(n)\right]\beta}{\left(\beta(\theta-1)+1\right)^2}<0$$

The indirect effects are:

For σ: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\sigma}}{\left[\beta(\theta-1)+1\right]}$;

For v: $\dfrac{\left[v+f'(n)\right]\frac{dn}{dv}}{\left[\beta(\theta-1)+1\right]}$;

For ρ: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\rho}}{\left[\beta(\theta-1)+1\right]}$;

For θ: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\theta}}{\left[\beta(\theta-1)+1\right]}$;

For β: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\beta}}{\left[\beta(\theta-1)+1\right]}$.

In order to find the signs for the indirect and total effects we consider the result from Proposition 2 (*i.e.*, $\rho-vn>0$) and $f'(n)<0$, $f''(n)<0$, which is true for the range $n\in(0.0138;\ 0,0413)$ containing the average value of the birth rate ($n=0.03123$).

From Eq. (14) in the text we have:

$$f'(n)=\frac{v}{\beta(\theta-1)}-\left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right]$$

or alternatively:

$$\Psi(n)=f'(n)-\frac{v}{\beta(\theta-1)}+\left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right]$$

By using the *implicit function theorem*:

$$\frac{dn}{d\sigma}=-\left(\frac{d\Psi}{d\sigma}\Big/\frac{d\Psi}{dn}\right) \tag{3.1}$$

$$\frac{d\Psi}{d\sigma}=\frac{(1-\beta)\beta(\theta-1)}{\beta n\left(\beta(\theta-1)+1\right)}=\frac{(1-\beta)(\theta-1)}{n\left(\beta(\theta-1)+1\right)}<0 \tag{3.2}$$

$$\frac{d\Psi}{dn}=f''(n)-\left(\frac{1-\beta}{\beta n^2}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right]+\left(\frac{1-\beta}{\beta n}\right)\left[\frac{\beta(\theta-1)f'(n)-v}{\beta(\theta-1)+1}\right] \tag{3.3}$$

We now prove that $\frac{d\Psi}{dn}<0$.

Proof:

Use $f''(n)<0$ and $\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right]>0$ (see Proposition 2).

Hence:

$$-\left(\frac{1-\beta}{\beta n^2}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right]<0 \qquad \text{(A)}$$

The term $\left(\frac{1-\beta}{\beta n}\right)\left[\frac{\beta(\theta-1)f'(n)-v}{\beta(\theta-1)+1}\right]>0$ (B)

Then, we have to compare term (A) and term (B):

From Proposition 2:

$\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn>0 \Rightarrow \sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho>vn\geq 0$

(A) > 0 $\Rightarrow$ $\rho-\sigma\beta(1-\theta)-\beta(1-\theta)f(n)>0$

(B) > 0 $\Rightarrow$ $-nf'(n)\beta(1-\theta)>0$, for $\theta<1$ and $f'(n)<0$.

Then (A) > (B) iff: $\left|\rho-\sigma\beta(1-\theta)-\beta(1-\theta)f(n)\right|>\left|nf'(n)\beta(1-\theta)\right|$.

A (sufficient) condition for the last inequality to be checked is:

$\left|f(n)\right|/n\geq\left|f'(n)\right|$,

which implies that the average dilution effect is higher or equal than the marginal dilution effect. So:

$$\frac{d\Psi}{dn}<0.$$

Since $\frac{\left[v+f'(n)\right]}{\left[\beta(\theta-1)+1\right]}<0$, the indirect effect for σ is positive and the total effect is positive, as well.

In the same way, we can prove: the following:

i)

$$\frac{d\Psi}{dv}=-\frac{1}{\beta(\theta-1)}-\frac{(1-\beta)n}{\beta n(\beta(\theta-1)+1)}\Rightarrow\frac{1}{\beta(1-\theta)}-\frac{(1-\beta)}{\beta(\beta(\theta-1)+1)}=\frac{1-\beta(1-\theta)-(1-\beta)(1-\theta)}{\beta(1-\theta)(\beta(\theta-1)+1)}\Rightarrow$$

$$\frac{d\Psi}{dv}=\frac{\theta}{\beta(1-\theta)(\beta(\theta-1)+1)}>0\Rightarrow\frac{dn}{dv}>0$$

ii) $\frac{d\Psi}{d\rho}=\frac{1-\beta}{\beta n(\beta(\theta-1)+1)}>0\Rightarrow\frac{dn}{d\rho}>0$.

For θ we have:

$$\frac{d\Psi}{d\theta}=-\frac{v}{\beta(\theta-1)^2}-\left(\frac{1-\beta}{\beta n}\right)\frac{\left[(\sigma+f(n))\beta-(\rho-vn)\beta\right]}{(\beta(\theta-1)+1)^2}$$ <0 since the numerator in the second term is:

$(\sigma+f(n))\beta-(\rho-vn)\beta=\beta\left[\sigma+f(n)-\rho+vn\right]>0$ by using the result from Proposition 2. So, $\frac{d\Psi}{d\theta}<0$ and $\frac{dn}{d\theta}<0$. Hence the indirect effect is positive, while the direct effect is negative.

In order to check for the total effect we compare two extreme cases:

i) $v=0;\ \beta=0.75$, which denotes a situation with very low altruism towards children, and
ii) $v=1;\ \beta=0.25$, which denotes the case where there is more altruism towards children.

In the former case the total effect of θ is negative, whereas in the latter it is positive.

Finally, for β we also consider these two extreme cases:
i) $v=0;\ \beta=0.75$
ii) $v=1;\ \beta=0.25$.

$$\frac{d\Psi}{d\beta}=\frac{v}{\beta^2(\theta-1)}+\frac{\left\{\left[\sigma+f(n)\right](\theta-1)-2\left[\sigma+f(n)\right](\theta-1)\beta-\rho+vn\right\}\left[\beta n\left(\beta(\theta-1)+1\right)\right]}{\left[\beta n\left(\beta(\theta-1)+1\right)\right]^2}-$$
$$\frac{(1-\beta)\left\{\left[\sigma+f(n)\right](\theta-1)\beta+\rho-vn\right\}\left[2\beta n(\theta-1)+n\right]}{\left[\beta n\left(\beta(\theta-1)+1\right)\right]^2}$$

After considerable amount of algebra and by using different parameter values one can show the following. For case i) $v=0;\ \beta=0.75$**,** $\frac{dn}{d\beta}<0$**,** the indirect effect is positive, the direct effect is negative but the total effect is positive.

For case ii) $v=1;\beta=0.25$, one can show that $\frac{dn}{d\beta}<0$, both direct and indirect effects are positive and the total is positive as well. ■

APPENDIX B

OECD COUNTRIES: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Republic of Korea, Spain, Sweden, Switzerland, Turkey, U.K., U.S.

NON-OECD COUNTRIES: Afghanistan, Algeria, Argentina, Bahrain, Bangladesh, Barbados, Bolivia, Botswana, Brazil, Bulgaria, Cameroon, Central African Republic, Colombia, Costa Rica, Cuba, Cyprus, Dominican Republic, Ecuador, El Salvador, Fiji, Ghana, Guatemala, Guyana, Haiti, Honduras, Hong Kong, India, Indonesia, Iran (Islamic Republic of), Iraq, Israel, Jamaica, Jordan, Kenya, Kuwait, Lesotho, Liberia, Malawi, Malaysia, Mali, Mauritius, Mozambique, Myanmar, Nepal, Nicaragua, Niger, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Romania, Senegal, Sierra Leone, Singapore, South Africa, Sri Lanka, Sudan, Swaziland, Syrian Arab Republic, Thailand, Togo, Trinidad and Tobago, Tunisia, Uganda, Uruguay, Venezuela, Zambia, Zimbabwe.

OPEC COUNTRIES: Algeria, Ecuador, Indonesia, Iran (Islamic Republic of), Iraq, Kuwait, Venezuela.

EASTERN EUROPEAN/EX-SOCIALIST COUNTRIES: Bulgaria, Czech Republic, Cuba, Hungary, Poland, Romania.

LATIN AMERICA COUNTRIES: Argentina, Barbados, Bolivia, Brazil, Chile, Colombia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Trinidad and Tobago, Uruguay, Venezuela.

SUB-SAHARAN AFRICA COUNTRIES: Botswana, Cameroon, Central African Republic, Ghana, Kenya, Lesotho, Liberia, Malawi, Mali, Mauritius, Mozambique, Niger, Senegal, Sierra Leone, South Africa, Sudan, Swaziland, Togo, Uganda, Zambia, Zimbabwe.

TABLE B.1: *SEMI /NON-PARAMETRIC REGRESSION RESULTS*

	OLS	SP	NP
Variables	(A)	(B)	(C)
constant	1.1521*** (0.0176)	0.6132*** (0.0164)	-
doecd	0.0528*** (0.0115)	0.0883*** (0.0168)	-
la	0.1709*** (0.0136)	0.1547*** (0.0135)	-
af	0.0153 (0.0197)	0.0355*** (0.0162)	-
d1965	0.0044 (0.0221)	0.0078 (0.0216)	-
d1970	0.0250 (0.0211)	0.0281 (0.0214)	-
d1975	0.0226 (0.0194)	0.0219 (0.0209)	-
d1980	0.0280 (0.0185)	0.0252 (0.0204)	-
d1985	0.0274 (0.0176)	0.0268 (0.0268)	-
d1990	0.0317 (0.0175)	0.0318 (0.0318)	-
d1995	0.0181 (0.0172)	0.0168 (0.0168)	-
cbr	-16.3436*** (0.5328)	-	-
N	792	792	792
R2/R2adj.	72.37/71.98	74.2/73.7	77.66/-
F-test	185.8***		
Heterosked.$^{(1)}$	158.96***		
$\mathrm{P}\left(\mathrm{Specific.}\right)^{(2)}$	2.22e-16***	2.22e-16***	
$\mathrm{NP}\left(Test\right)^{(3)}$	0.012531*		

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). Robust standard errors are in parentheses. (2): The P(specific.) shows the p-values if the null of the parametric linear (linear-OLS) is correctly specified in comparison to a fully *non-parametric* (NP) model, using the Hsiao *et al.* (2007) test for continuous and discrete data models after 399 Bootstrap replications. The P(specific.) at column B, checks if the parametric model (linear-OLS) is well specified when compared to the *semi-parametric* (SP) model. (3): The NP(Test) is a significance test for the explanatory variable (*cbr*) in the locally linear nonparametric specification. It is like the *t-test* in the parametric regression framework and is based on Racine (1997). We drop the time dummy for 2000.

TABLE B.2: *REGRESSION RESULTS BY USING POLYNOMIAL TERMS FOR THE BIRTH RATE* (cbr)

	OLS	OLS	OLS	OLS
Variables	(A)	(B)	(C)	(D)
constant	1.1521*** (0.0176)	0.6958*** (0.0699)	1.1926*** (0.0205)	0.6108*** (0.1305)
doecd	0.0528*** (0.0115)	0.0883*** (0.0115)	-0.0630 (0.02428)	0.1007 (0.1563)
la	0.1709*** (0.0136)	0.1551*** (0.0140)	0.1626*** (0.0140)	0.1546*** (0.0143)
af	0.0153 (0.0197)	0.0301 (0.0195)	0.0227*** (0.0198)	0.0312 (0.0195)
d1965	0.0044 (0.0221)	-0.0023 (0.0219)	0.0022 (0.0219)	-0.0018 (0.0222)
d1970	0.0250 (0.0211)	0.0180 (0.0209)	0.0236 (0.0210)	0.0194 (0.0211)
d1975	0.0226 (0.0194)	0.0139 (0.0193)	0.0222 (0.0193)	0.0147 (0.0195)
d1980	0.0280 (0.0185)	0.0199 (0.0182)	0.0280 (0.1844)	0.0237 (0.0172)
d1985	0.0274 (0.0176)	0.0234 (0.0171)	0.0290 (0.0175)	0.0237 (0.0172)
d1990	0.0317 (0.0175)	0.0300 (0.0170)	0.0335 (0.0173)	0.0307 (0.0171)
d1995	0.0181 (0.0172)	0.0165 (0.0166)	0.0195 (0.0169)	0.0163 (0.0166)
cbr	-16.3436*** (0.5328)	26.35** (7.670)	-17.4017*** (0.6085)	34.04* (13.3694)
$(cbr)^2$		-1143*** (265.99)		-1355** (419.65)
$(cbr)^3$		9224** (2868.025)		11050** (4147.205)
$oecd(cbr)$			5.1988*** (1.040)	4.665 (18.14)
$oecd(cbr)^2$				-403.77 (674.02)
$oecd(cbr)^3$				-6856 (7950)
N	792	792	792	792
R2/R2adj.	72.37/71.98	73.87/73.43	72.84/72.42	73.91/73.37
F-test	185.8***	169.2***	174.1***	137.2***
F-Joint	158.96***	22.269***	13.371***	0.4312
Heterosked.[(1)]	133.24***	183.37***	167.11***	185.55***

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). Robust standard errors are in parentheses. We drop the time dummy for 2000.

TABLE B.3: *REGRESSION RESULTS USING POLYNOMIAL TERMS FOR THE BIRTH RATE (CBR) AND CONTROL VARIABLES*

	OLS	OLS	OLS	OLS
Variables	(A)	(B)	(C)	(D)
constant	0.33319** (0.1454)	0.0449 (0.1504)	0.3824*** (0.1413)	0.0161 (0.1754)
doecd	0.0038 (0.0097)	0.0205 (0.0094)	-0.0816*** (0.0205)	0.1449 (0.1339)
la	0.1260*** (0.0093)	0.1158*** (0.0094)	0.1204*** (0.0094)	0.0113*** (0.1138)
af	0.0610*** (0.0119)	0.0704*** (0.0114)	0.0643*** (0.0118)	0.0709*** (0.0115)
d1965	-0.0180 (0.0164)	-0.0298* (0.0165)	-0.0233 (0.0163)	-0.0274 (0.0162)
d1970	-0.0132 (0.0162)	-0.0231* (0.0161)	-0.0172 (0.0160)	-0.0204 (0.0162)
d1975	-0.0154 (0.0155)	-0.0246 (0.0154)	-0.0179 (0.0153)	-0.0218 (0.0155)
d1980	-0.0098 (0.0146)	-0.0170 (0.0141)	-0.0113 (0.0144)	-0.0149 (0.0144)
d1985	-0.0110 (0.0139)	-0.0147 (0.0136)	-0.0108 (0.0137)	-0.0131 (0.0136)
d1990	-0.0029 (0.0133)	-0.0048 (0.0132)	-0.0020 (0.0131)	-0.0035 (0.0131)
d1995	-0.0021 (0.0135)	-0.0492*** (0.0133)	-0.0012 (0.0133)	-0.0030 (0.0132)
hum65	0.0466*** (0.0030)	19.17*** (0.0033)	0.0478*** (0.0030)	0.0496*** (0.0032)
infmort	-0.002*** (0.003)	-0.0016*** (0.003)	-0.002*** (0.0003)	-0.0017*** (0.0003)
lifexp	0.0048*** (0.0018)	0.0054** (0.0017)	0.0045*** (0.0017)	0.0051*** (0.0017)
cbr	-0.5468 (0.6897)	19.17*** (5.2137)	-1.4363*** (0.7230)	24.78* (9.9806)
$(cbr)^2$		-487.7*** (189.44)		-665* (306.6897)
$(cbr)^3$		3384 (1895)		5067 (2985.376)
$oecd(cbr)$			3.7724*** (0.8013)	-14.82 (15.7202)
$oecd(cbr)^2$				488.5 (585.4747)
$oecd(cbr)^3$				-4455 (6899.486)
N	792	792	792	792
R2/R2adj.	88.34/88.13	88.8/88.57	88.58/88.36	88.84/88.57
F-test	420.5***	384.2***	401.2***	322.6***
F-Joint			16.153***	0.9286
F-Joint 3rd Polyn.	-	16.049***	-	7.8147***
Heterosked.(1)	33.30***	27.01***	29.20***	22.99***

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). The standard errors are in parentheses and t-statistics are available upon request. We drop the time dummy for 2000,. Instead of life expectancy we have used also crude death rate. This variable has negative sign contrast to the sign of life expectancy, is statistically significant and the results for the variable of birth rate (*cbr*) remain the same.

FIGURE 1: *SEMI-PARAMETRIC PLOT FOR THE CONTEMPORANEOUS BIRTH RATE IN THE WHOLE SAMPLE*

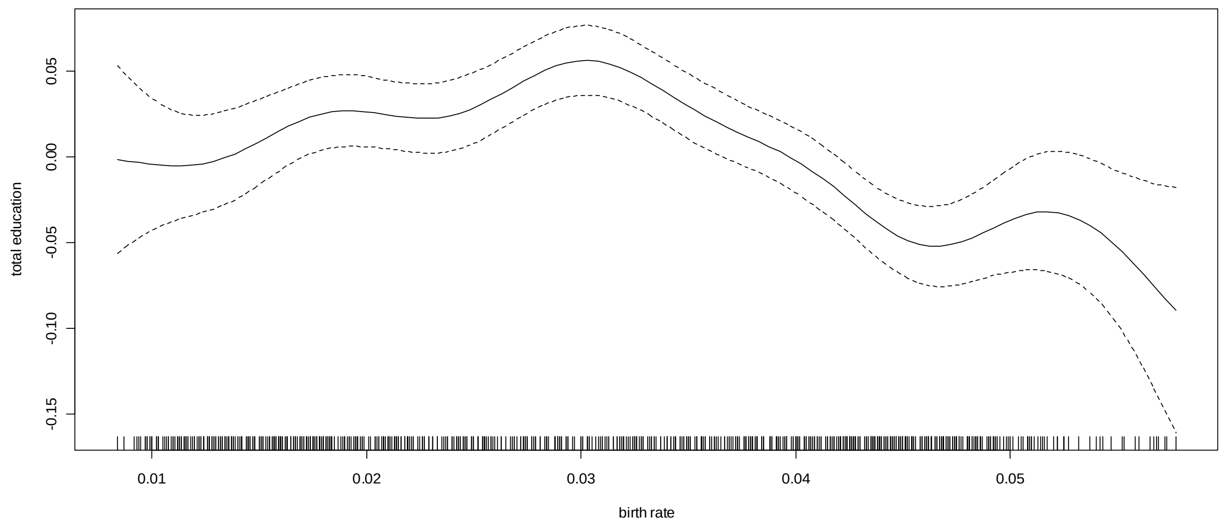


FIGURE 2: *SEMI-PARAMETRIC PLOT FOR THE LAGGED BIRTH RATE IN THE WHOLE SAMPLE*

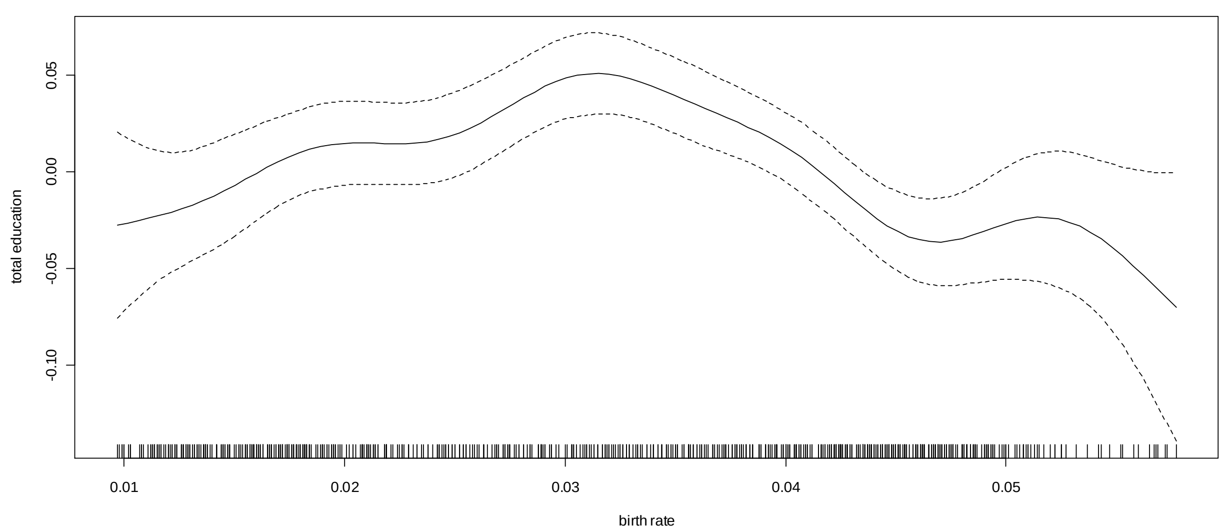
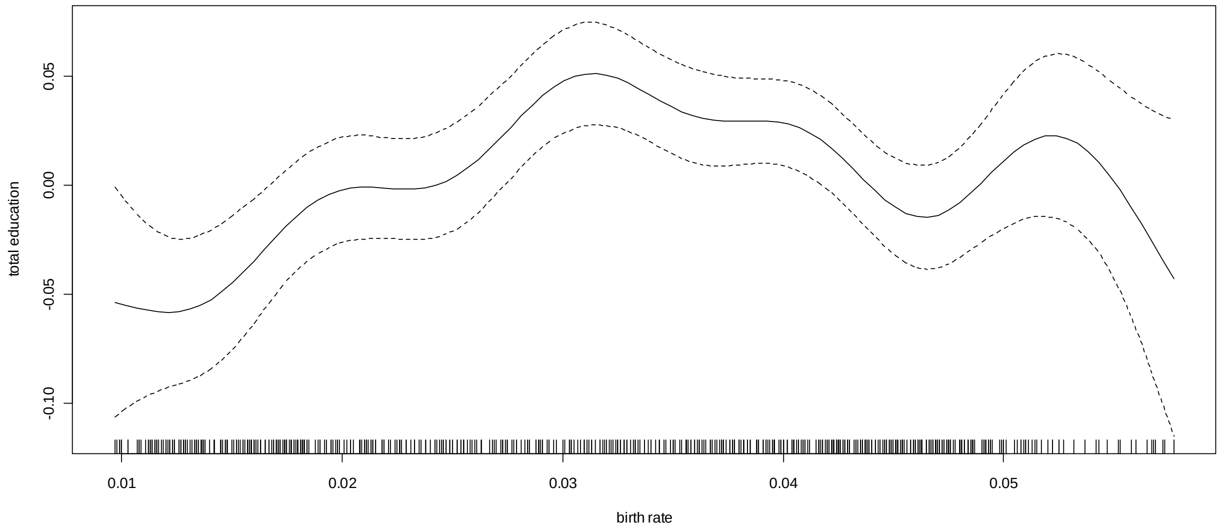


TABLE B.4: *SEMI/NON-PARAMETRIC REGRESSION RESULTS IN THE NEW SAMPLE*

	OLS	SP	NP
Variables	(A)	(B)	(C)
Constant	1.1559*** (0.0211)	0.6101*** (0.0188)	-
doecd	0.0503*** (0.0142)	0.0938*** (0.0199)	-
la	0.1644*** (0.0160)	0.1500*** (0.0152)	-
af	0.0101 (0.0222)	0.0316 (0.0181)	-
d1965	0.0015 (0.0248)	0.0035 (0.0240)	-
d1970	0.0214 (0.0236)	0.021 (0.0238)	-
d1975	0.0175 (0.0218)	0.0147 (0.0233)	-
d1980	0.0183 (0.0208)	0.0154 (0.0227)	-
d1985	0.0187 (0.0198)	0.0194 (0.0223)	-
d1990	0.0271 (0.0198)	0.0287 (0.0222)	-
d1995	0.0136 (0.0194)	0.0145 (0.0220)	-
cbr	-16.2055*** (0.6248)	-	-
N	688	688	688
R2/R2adj.	70.85/70.37	72.8/72.2	74.71/-
F-test	149.4***		
Heterosked.[(1)]	130.38***		
$\mathrm{P}\left(\mathrm{Specific.}\right)^{(2)}$	2.22e-16***	2.22e-16***	
$\mathrm{NP}\left(Test\right)^{(3)}$	0.0025063**		

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). Robust standard errors are in parentheses. (2): The P(specific.) shows the p-values if the null of the parametric linear (linear-OLS) is correctly specified in comparison to a fully *non-parametric* (NP) model, using the Hsiao *et al.* (2007) test for continuous and discrete data models after 399 Bootstrap replications. The P(specific.) at column B, checks if the parametric model (linear-OLS) is well specified when compared to the *semi-parametric* (SP) model. (3): The NP(Test) is a significance test for the explanatory variable (*cbr*) in the locally linear nonparametric specification. It is like the *t-test* in the parametric regression framework and is based on Racine (1997). We drop the time dummy for 2000.

FIGURE 3: *SEMI-PARAMETRIC GRAPH IN THE RESTRICTED SAMPLE*



CHAPTER 2

CORRUPTION, PUBLIC EXPENDITURE, AND HUMAN CAPITAL ACCUMULATION*

Abstract

The main purpose of this paper is to investigate the effect of corruption on human capital accumulation through two channels. The first channel is through the effect of corruption on the public expenditure on education and the second channel is through the effect of corruption on the growth rate of physical capital. Public expenditure on education is an important determinant for human capital accumulation, while physical capital can have different impact on human capital depending on possible complementarities or substitutability between the two capitals. Initially, we construct an endogenous two-sector growth model where human capital is the engine of growth and by considering corruption as an exogenous variable we try to explore the impact of corruption on the allocation of public resources and as such on the distribution of human capital across different sectors. The results from the theoretical model suggest that there exist conditions where public expenditure on education can have both positive and negative effects on human capital accumulation due to corruption. Then we test empirically the equation of human capital accumulation from the model by using a smooth coefficient semi-parametric model to capture possible non-linearities. The results support the existence of non-linearities between human capital and corruption.

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1. INTRODUCTION

The topic of corruption has received recently a lot of attention in the economic literature, with some recent surveys on the topic being the papers by Svensson (2005), Aidt (2009) and Campos et al. (2010). Corruption is closely related with rent-seeking behavior and the misallocation of talent. According to Murphy, Sleifer and Vishny (1991), Murphy and Vishny (1993) and Acemoglu and Verdier (1998) in the presence of corruption investment in the more innovative sectors of the economy is less profitable due to higher transaction costs and this reduces the incentives for investment in R&D. Furthermore, due to corruption there is misallocation of public resources and in general public expenditures are less efficient, something that may also lead to a reduction of investment in physical capital. All the previous arguments formulate the general framework which examines the negative effect of corruption on economic growth. In this context important empirical contributions on this area are among others Mauro (1995), Mironov (2005) and Mo (2001). However, there are also arguments that support the idea that corruption can have a positive effect on in economic growth and can be seen as a mechanism which increases efficiency in countries where institutions are not functioning well by reducing barriers due to high red tape, see Aidt et al. (2008)[34].

The theoretical literature of the corruption and economic growth nexus is very rich. Some papers in that context are that of Sarte (2000), who analyses the effect of bureaucratic corruption on growth, Angeletos and Kollintzas (2000), who investigate a broader definition of corruption which includes rent-seeking in an endogenous growth model with R&D and find that corruption slows down innovation and growth, without taking into account any additional effects that corruption may have in the composition of public expenditures. Mauro (2004) tries to explain the persistence of corruption in some countries by arguing that when corruption is widespread then there is no incentive for individuals to fight against it. In that case, in the presence of wide spread corruption a corrupted individual is more difficult to be caught and so he or she can allocate more time on rent-seeking activities than on more productive activities. An endogenous growth and corruption paper worth mentioning is that of Barreto (2000), who uses a neoclassical growth model to find that effect of corruption is mainly to redistribute income and that it may also have positive effects in the presence of much bureaucratic red tape as found by Aidt et al. (2008). Blackburn et al. (2006) show that corruption negatively affects growth since some public resources are being spent in mechanisms for corruption reduction and are not used directly for investment in physical capital. Finally, Ebben and Vaal (2011) show that in an environment of low quality of institutions, corruption can be useful for economic growth as it overcomes red tape.

[34] Some older paper papers that were making similar arguments are Leff (1964) and Huntington (1968).

The purpose in this paper is to investigate the effect of corruption on human capital accumulation. It is well known from the empirical literature that human capital has an unclear role in economic growth,[35] even if (endogenous) growth theories[36] consider human capital as the main engine of growth. In this context very little attention has been given to the effect of corruption on human capital accumulation. One of the very few attempts is Rogers (2008) who used standard OLS regression methods but his results are not robust to the change of sample or to alternatively measures of corruption.

One important theoretical paper which takes into account the effect of corruption on human capital accumulation is that of Ehrlich and Lui (1999). They investigate the negative effect of bureaucratic corruption on human capital accumulation by assuming that corruption affects negatively human capital because more time is invested in political capital to improve the bureaucratic power of individuals than on productive education sector. Pecorino (1992) investigates the impact of rent seeking on human capital accumulation, while Ghosh and Gregoriou (2010) find that corruption affects exogenously the productivity of different types of public spending but not the allocation of human capital among sectors as we do in our paper. Empirical papers of Devarajan et al. (1996) and Ghosh and Gregoriou (2008) justify empirically that public spending on sectors different from education has more positive impact on growth. That may be due to the fact that there may be already too much public investment on education and its marginal effect in that case would be small. Alternatively, it may be because of the presence of corruption in the education sector. However, contrary to our model, they do not use human capital accumulation and do not check for the impact of corruption in the allocation of government spending. Blackburn et al. (2011) consider an OLG model of economic growth with endogenous corruption which arises due to the ability of bureaucrats to steal public funds and to reduce the quality of public goods provision. However, their model does not incorporate human capital accumulation as we do in our model.

The contribution of our paper is that we make two hypotheses for the formation of human capital: i) the allocation of public expenditure among different sectors affects the fraction of human capital that is distributed between final output sector and education sector and ii) we assume that the growth rate of physical capital[37] can affect positively or negatively human capital accumulation. Though these channels corruption can create distortions which can affect indirectly human capital accumulation. Furthermore, we check empirically our specification of human capital and we uncover a non-linear effect of corruption on the two determinants of human capital: public expenditure on education and growth rate of physical capital

[35] Benhabib and Spiegel (1994), Islam (1995) and Pritchett (1996) provide controversial empirical results for the role of human capital on economic growth.

[36] Uzawa (1965), Lucas (1988) and Romer (1990) provide a very good exposition of theoretical models of endogenous growth.

[37] For justification of physical capital entering in the accumulation of human capital see Albelo (1999).

Empirically, we try to investigate potentially the presence of non-linear effects of corruption on the two components of human capital. To carry out our empirical analysis we use semi-parametric methods in order to explore potentially non-linearities. Our results, suggest that public expenditures have a positive but declining effect on human capital accumulation. This may be due to the fact that either corruption cannot distort at the same level the effect of public expenditures on education as it does in other sectors or that because of corruption individuals have an incentive to accumulate more human capital. Furthermore, corruption is detrimental to human capital mainly through the deterioration of physical capital investment.

In section 2 we provide a theoretical model with exogenous corruption. In section 3 we provide the analytical framework needed for the empirical part and in section 4 we provide a brief exposition of the econometric methods that we use and the empirical results. Finally, in section 5 we conclude.

2. THEORETICAL MODEL WITH EXOGENOUS CORRUPTION

2.1 SET UP OF THE MODEL

In our economy total public expenditures are given as: $G = G_H + G_Y$ where $\left(G_H\right)$ is the public expenditure on education and $\left(G_Y\right)$ is the public expenditure on other activities, whereas $s_H = \frac{G_H}{G}$ and $s_Y = \frac{G_Y}{G}$ are the shares of public expenditures on total public expenditures which are used in education and final output respectively. A necessary constraint is: $s_H + s_Y = 1$. Corrupted bureaucrats are able to steal a fraction $\left(1-\zeta\right)s_H$ from the education sector and $\left(1-\delta\right)s_Y$ from the final output sector with $\delta, \zeta \in \left(0,1\right)$. So, the actual fraction which appears in the education sector is ζs_H and that in the final output sector is δs_Y. At a more general level of the analysis we assume that the corruption level is different between sectors or in other words that bureaucrats can steal more easily public resources from one sector than from another.[38] In our economy households take into account the actions of corrupted bureaucrats when they choose how to allocate their human capital but corruption in our model is exogenous and we do not analyze the incentives of bureaucrats to be corrupted and therefore how corruption appeared in the first place. Furthermore, we assume no population growth since we are not interested to examine demographic issues and also that the population level is normalized to one: ($\frac{\dot{L}}{L} = n = 0$ and $L = 1$) and as

[38] For the different magnitude of corruption between sectors see Croix and Delavallade (2006). However, we prefer not make any assumption regarding which the relative sizes of ζ or δ .

such the aggregate and per capita variables coincide. Individuals in order to allocate their human capital between sectors take into account the net fraction of resources which is provided in each sector after corruption. The fraction of human capital which enters into the education sector is:

$$u_1 = f\left(\zeta s_H\right), \tag{1}$$

The fraction of human capital which enters into the production of final output sector is:

$$u_2 = g\left(\delta s_Y\right) = g\left(\delta\left(1-s_H\right)\right), \tag{2}$$

One important assumption that guarantees a simultaneous determination of s_H and γ in the balanced growth path equilibrium (BGPE) as it is shown in proposition 4 is that the functions $f\left(\zeta s_H\right)$ and $g\left(\delta\left(1-s_H\right)\right)$ are monotonic and therefore invertible functions, which in addition agrees with the constraint of the allocation of human capital across sectors: $f\left(\zeta s_H\right)+g\left(\delta s_Y\right)=1$. An extra assumption which verifies that the Hamiltonian corresponds to a maximization problem is that the function $f\left(\zeta s_H\right)$ is concave $\left[f''(\bullet)<0\right]$. This assumption implies that we have smaller increase in the fraction of human capital which enters into education sector due to an increase in the net from corruption public resources which are oriented for education. An intuition is that individuals can postpone working by entering into education but this cannot be a situation which lasts for long period, since they need to find a job and to live into a more stable working environment. From the constraint $f\left(\zeta s_H\right)+g\left(\delta s_Y\right)=1$, if the function $f\left(\zeta s_H\right)$ is concave then the function $g\left(\delta s_Y\right)$ is convex. It can be shown that an extra necessary condition in order the solution of the Hamiltonian function to correspond to a maximum is that: $\left(1-\alpha\right)\left[g'\left(\delta\left(1-s_H\right)\right)\right]^2 > g\left(\delta\left(1-s_H\right)\right)\bullet g''\left(\delta\left(1-s_H\right)\right)$. In order $f\left(\zeta s_H\right)$ to be both concave and monotonic increasing we need to assume that $f\left(\zeta s_H\right)$ is bounded from above. This condition in mathematical form is like: for $\zeta s_H \rightarrow 1 \Rightarrow f\left(1\right) \rightarrow f_{\max} < 1$ and similarly, $\delta s_Y \rightarrow 1 \Rightarrow g\left(1\right) \rightarrow g_{\max} < 1$. In order always to have an interior solution for the allocation of human capital across sectors we need $f\left(0\right)>0$ and $g\left(0\right)>0$. Another important implication of this assumption is that even if corruption is zero or at a maximum level or even if an economy has public expenditures or not to allocate among sectors always individuals will decide to distribute their human capital among sectors. The social planner indirectly affects the distribution of human capital among sectors through the functions $f\left(\zeta s_H\right)$ and $g\left(\delta s_Y\right)$ by deciding on s_H and taking as given the level of corruption. The restrictions for defining well the problem are the followings:

$$u_1 + u_2 = 1 \Rightarrow u_2 = 1 - u_1 = 1 - f\left(\zeta s_H\right), \tag{3}$$

$$f\left(\zeta s_H\right)+g\left(\delta s_Y\right)=1, \tag{4}$$

$$f\left(\zeta s_H\right), g\left(\delta s_Y\right)\in\left(0,1\right) , \tag{5}$$

Together with $f'>0$, $g'>0$ and f, g are bounded and invertible functions. (6)

So in this economy the human capital accumulation is described from the following equation:

$$\dot{H}=\sigma H_H+\varphi\gamma_K H \Rightarrow \dot{H}=\sigma\left(f\left(\zeta s_H\right)\right)H+\varphi\gamma_K H \tag{7}$$

We have that $\left(H_H\right)$ is the human capital that is used in the education sector and its fraction into that sector is equal to: $\frac{H_H}{H}=f\left(\zeta s_H\right)$. The parameter $\sigma>0$ represents the productivity parameter of human capital and $\varphi\in\mathbb{R}$ represents a scale parameter that describes possible complementarities or substitutabilities between physical and human capital growth. Actually, as we shall show in section 2.2, this parameter plays a crucial role in the results we are going to derive in the long run equilibrium and should be negative and less than one in absolute value which means that the growth rate of physical capital plays the role of an endogenous depreciation rate. The fraction of human capital which is employed into the final output sector is a function as well of the net resources after corruption and equals to: $\frac{H_Y}{H}=g\left(\delta s_Y\right)=g\left(\delta\left(1-s_H\right)\right)$.

We consider an economy in which the social planner chooses consumption goods, how much to invest in physical capital and how to allocate public expenditures between sectors by taking into account that this choice affects indirectly the allocation of human capital between sectors.

The production function of the economy at the aggregate level is the following:

$$Y=AH_Y^{\alpha}K^{1-\alpha} \text{ with A=1 } \rightarrow Y=H_Y^{\alpha}K^{1-\alpha} \tag{8}$$

Where (A) is the total factor productivity and we normalize it equal to 1, since we have exogenous technological progress. The physical capital accumulation is described by:

$$\dot{K}=Y-G-C, \tag{9}$$

By following Barro (1990) we consider a balanced budget constraint for government and implement the following condition: $G=G_H+G_Y=\tau Y \Rightarrow$

$$\dot{K}=\left(1-\tau\right)H_Y^{\alpha}K^{1-\alpha}-C=\left(1-\tau\right)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha}H^{\alpha}K^{1-\alpha}-C, \tag{10}$$

Distortionary taxes[39] are considered exogenous and τ is assumed to be constant over time. In this model,

[39] In this model we are not interested in optimal taxation and we do not make any assumption on possible effects of corruption on taxes. For the connection between corruption and tax structure, see among others Fisman and Svensson (2002), Gordon and Li (2005), and Litina and Palivos (2011). Furthermore we avoid from the analysis to use total public expenditures because we are not interested in the optimal public size. For this literature see Johnson et al. (1999) and Tanzi and Davoodi (1997).

we consider that both G_Y and G_H have only an indirect effect on the production function and on the equation of human capital accumulation through $s_Y = \frac{G_Y}{G}$ and $s_H = \frac{G_H}{G}$ respectively. This is a simplification assumption which is very useful in order to estimate later the equation of human capital accumulation. Furthermore, for simplicity we assume a zero depreciation rate for both human and physical capital.[40]

The instantaneous utility function of the representative agent is: $U(C) = \frac{C^{1-\theta} - 1}{1-\theta}$, with $\theta > 0$ to be the inverse of the inter-temporal elasticity of substitution. So the social planner has to maximize the following problem:

$$\underset{\{C_t, s_H, H_t, K_t\}_{t=0}^{+\infty}}{Max} U \equiv \int_0^{+\infty} \left(\frac{C_t^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} dt, \qquad \rho > 0; \qquad \theta > 0 \tag{11}$$

s.t.: $$\dot{H} = \sigma\left(f\left(\zeta s_H\right)\right) H + \varphi \gamma_K H, \qquad \sigma > 0; \qquad f \in [0;1], \qquad \forall t \tag{12}$$

$$\dot{K} = (1-\tau) H_Y^{\alpha} K^{1-\alpha} - C = (1-\tau)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha} H^{\alpha} K^{1-\alpha} - C, \qquad \forall t \tag{13}$$

along with the transversality conditions: $\lim_{t\to\infty} \lambda_t H_t = 0$ and $\lim_{t\to\infty} \mu_t K_t = 0$ (14)

and the initial conditions: $\mathrm{H}(0) > 0$ and $\mathrm{K}(0) > 0$ given. (15)

For notational simplicity we avoid the time subscripts and we define the following current value Hamiltonian function that the social planner has to maximize:

$$\underset{\{C_t, s_H, H_t, K_t\}}{Max} J = \frac{C^{1-\theta} - 1}{1-\theta} + \lambda\left[\left[\sigma\left(f\left(\zeta s_H\right)\right) + \varphi \gamma_K\right]\right] H + \mu\left[(1-\tau)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha} H^{\alpha} K^{1-\alpha} - C\right] \tag{16}$$

DEFINITION: *BALANCED GROWTH PATH* (BGP) *EQUILIBRIUM*

A Balanced Growth Path (BGP) equilibrium in this economy is a framework where:

i) All time-dependent variables grow at a constant possibly positive exponential rate; ii) The ratio of the two endogenous state variables, K_t / H_t, remains invariant[41] over time.

[40] One can say that the term $\varphi\gamma_K$ in the equation of human capital can play the role of a depreciation rate; however the concept of depreciation rate is broader than the term $\varphi\gamma_K$ in the equation of human capital accumulation. This term shows possible negative effects of corruption on human capital through the accumulation of physical capital and it's not just an obsolesence effect. Furthermore, the addition of a depreciation rate in the equation of physical capital accumulation will not add anything extra in the analysis.

[41] In a decentralized equilibrium framework it can be proved that the returns of the two assets $(K \text{ and } H)$ are equal and so in equilibrium the two assets should grow in the same way.

The reason of having the same growth rate for the human capital and the physical capital is because both factors are necessary for the aggregate production function. If for example physical capital grows faster than human capital then in the long run $(t \to \infty)$ human capital will become infinitely small relative to physical capital (especially in the case where physical capital acts as an endogenous depreciation rate for human capital). This assumption will lead in the long run in the coexistence of the two forms of capital in the aggregate production function.

2.2 BGP ANALYSIS

PROPOSITION 1

Along the BGP equilibrium, we have:

$$\gamma = \frac{\sigma\left[f(\bullet) \right]}{1-\varphi}\ ,\ \varphi \neq 1 \text{ and } \varphi < 1,\ \text{for } \gamma > 0 \tag{17}$$

Proof: It comes by considering $\gamma_H = \gamma_K = \gamma$ in the equation of human capital accumulation. ■

The next proposition shows the growth rate of the economy described only from the parameters of the model and from values of variables which are in BGP. It is derived by using all the appropriate FOCS and the result of equation (17).

PROPOSITION 2

Along the BGP equilibrium, we have:

$$\gamma = \frac{\sigma\left(f\theta - 1 + \varphi \right)}{(1-\varphi)\varphi} + \frac{\rho}{\varphi}, \tag{18}$$

Proof: This result comes by using the FOCS conditions of the maximization problem and also the equation (17). So it is the actual formula for the growth rate of the economy in BGP. For the derivations of this expression see *Appendix A*. ■

The next proposition shows under what conditions the growth rate from equation (18) is positive and both $\frac{d\gamma}{d\rho}<0$ and $\frac{d\gamma}{d\theta}<0$ and also that TVC holds.

PROPOSITION 3

Along the BGP equilibrium, we have:

For $\gamma>0$; $\frac{d\gamma}{d\rho}<0$ and $\frac{d\gamma}{d\theta}<0$, we need $\varphi<0$, $1\text{-}\varphi > f\bullet\theta$ and $\rho<\frac{\sigma(1-\varphi-f\theta)}{(1-\varphi)}$. The TVC always holds.

Proof: It comes immediate from Eq. (18). ■

The parameter restriction of $\varphi<0$ [42], implies that the human capital and the physical capital are substitutes. This is important condition for the structure of this model, otherwise if they are complements then in case that an economy is very impatient $($high values of ρ and θ $)$, then the growth rate of the economy will be negative, $\downarrow\gamma_K \Rightarrow\downarrow\gamma_H \Rightarrow \gamma_Y<0$.

The next proposition shows what extra restriction should exist on φ in order $\frac{d\gamma}{d\rho}<0$ and $\frac{d\gamma}{d\theta}<0$.

PROPOSITION 4

Along the BGP equilibrium, we need $|\varphi|<1$ in order $\frac{d\gamma}{d\rho}<0$ and $\frac{d\gamma}{d\theta}<0$.

Proof: The proof is mainly intuitive. When $\rho\uparrow$ and/or $\theta\uparrow\Rightarrow\downarrow$ *savings* $\Rightarrow\downarrow\gamma_K\Rightarrow\uparrow\gamma_H$ (because physical and human capital are substitutes). In order $\frac{d\gamma}{d\rho}<0$ we need the increase of human capital accumulation to be less than the reduction of physical capital accumulation due to the reduction in savings. We know from equation (12) that $\frac{d\gamma_H}{d\gamma_K}=\varphi<0$ and in order the total effect of ρ to be negative on γ we need: $\left|\frac{d\gamma_H}{d\gamma_K}\right|<\left|\frac{d\gamma}{d\rho}\right|\Rightarrow|\varphi|<\frac{1}{|\varphi|}\Rightarrow|\varphi|<1$. Similary, for θ, in order $\frac{d\gamma}{d\theta}<0$ we need again the increase of human capital accumulation to be less than the reduction of physical capital accumulation due to the reduction in savings. From equation (12) as usual we have $\frac{d\gamma_H}{d\gamma_K}=\varphi<0$ and in order the total effect of θ to be negative on γ we need:

42 Bartel and Sicherman (1995) find empirically that human and physical capitals are substitutes, because it is a common practice to exist training on the job in the more advanced economies and therefore physical capital is a substitute of official education. Also, in our empirical part we find the same negative relationship between the growth rate of human and physical capital.

$\left|\frac{d\gamma_H}{d\gamma_K}\right| < \left|\frac{d\gamma}{d\theta}\right| \Rightarrow |\varphi| < \frac{\sigma f(\bullet)}{|\varphi|(1-\varphi)} \Rightarrow |\varphi|^2 < \frac{\sigma f(\bullet)}{(1-\varphi)} < 1$, since $f(\bullet) \in (0,1)$ and $(1-\varphi) > 1$ for $\varphi < 0$, it is expected that the second restriction for satisfying $\frac{d\gamma}{d\theta} < 0$ quarantees that $|\varphi| < 1$ still holds.[43] ■

The intuition behind these conditions and this proposition is that the decrease of savings in a more impatient economy creates a higher decrease on physical capital accumulation than the possible increase on human capital accumulation due to the existence of substitutabilities of the two factors. The next proposition shows that there exists simultaneously an endogenous growth rate (γ) and an endogenous share of public expenditure on education (s_H).

PROPOSITION 5

Along the BGP equilibrium, we have a simultaneous determined value for (γ^*) and for (s_H^*) given by $\gamma^* = \frac{\sigma\left(f\left(\zeta s_H^*\right)\theta - 1 + \varphi\right)}{(1-\varphi)\varphi} + \frac{\rho}{\varphi}$, and $\left(1-s_H^*\right) = s_Y^* = \frac{\zeta - f^{-1}\left[\frac{(1-\varphi)[\varphi\gamma + \sigma - \rho]}{\sigma\theta}\right]}{\zeta}$ (19), respectively, when we have $\{\sigma > \rho \text{ and } \rho - \varphi > \sigma\}$, which is always true iff $\varphi < 0$, and $\left\{0 < \frac{\rho - \sigma}{\varphi} < \gamma\right\}$.

Proof: The proof is in the *Appendix A*. The important assumption for this proof is that the function $f(\bullet)$ is invertible. ■

The next proposition shows the effect of the parameters of the model $(\theta; \zeta; \rho; \sigma \text{ and } \varphi)$ on the endogenous share of public expenditure on education.

PROPOSITION 6

By using the results from the previous propositions along the BGP equilibrium, we have:

i) $\frac{ds_H}{d\zeta} < 0$ and $\frac{ds_Y}{d\zeta} > 0$, ii) $\frac{ds_H}{d\varphi} > 0$ and $\frac{ds_Y}{d\varphi} < 0$ if $\gamma > \frac{\sigma - \rho}{1 - 2\varphi} > 0$, otherwise $\frac{ds_H}{d\varphi} < 0$ and $\frac{ds_Y}{d\varphi} > 0$ if $\gamma < \frac{\sigma - \rho}{1 - 2\varphi}$, iii) $\frac{ds_H}{d\rho} < 0$ and $\frac{ds_Y}{d\rho} > 0$, iv) $\frac{ds_H}{d\theta} < 0$ and $\frac{ds_Y}{d\theta} > 0$ and finally $\frac{ds_H}{d\sigma} > 0$ and $\frac{ds_Y}{d\sigma} < 0$.

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[43] Mulligan and Sala-i-Martin (1993) suggest that σ should have a value less than 1.

Proof: The proof comes from an immediate differentiation of equation (19). Some derivations are provided in the *Appendix A*. ■

When due to corruption more public resources are stolen from the education sector then the public expenditure in education should increase in order to maintain enough fraction of human capital into education sector. If an economy has growth rate over $\frac{\sigma-\rho}{1-2\varphi}$ then if increases φ, which means that $\varphi \to 0$ since $\varphi<0$, then the growth rate of physical capital is less detrimental for human capital, and in that case more public resources should be oriented into the education sector since human capital is the engine of growth for this group of countries. On the contrary, in countries with lower growth than $\frac{\sigma-\rho}{1-2\varphi}$, less public resources shoud go into the education sector relatively to the final output sector. The intuition behind this result is that in less developed countries the stock of physical capital is quite low and therefore there is no negative impact of physical capital on human capital. Economies which are more impatient$\left(\text{higher values of } \theta \text{ and } \rho\right)$, have less savings and lower physical capital acccumulation and in that cases it is more important to increase the public resources which are attributed into the final output sector relative to the public resources for education. Finally, when the productivity of education sector is high $\left(\text{higher values of } \sigma\right)$, then the higher is the public expenditure on education the higher are the potential positive externality effects of human capital into the economy.

The next proposition shows the effect of corruption in the two sectors in the growth rate of the economy.

PROPOSITION 7

$\frac{d\gamma}{d\delta} = -\frac{\sigma g'\theta s_Y}{(1-\varphi)\varphi} > 0,$ for $s_Y \in (0,1); g'>0,$ $|\varphi|<1$ and $\varphi<0,$ which means

$\frac{d\gamma}{d(1-\delta)} < 0$, and

$\frac{d\gamma}{d\zeta} = \frac{\sigma f'\theta s_H}{(1-\varphi)\varphi} < 0,$ for $s_H \in (0,1); f'>0,$ $|\varphi|<1$ and $\varphi<0,$ which means $\frac{d\gamma}{d(1-\zeta)} > 0$.

Proof: This result comes by differentiating equation (18) with respect to δ and ζ respectively. ■

We know from equation (13), that $\gamma_K = (1-\tau)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha} H^{\alpha}K^{-\alpha} - C/K$.

Then $\frac{d\gamma_K}{d\delta}=(1-\tau)\alpha\delta(1-s_H)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha-1} g' H^{\alpha} K^{-\alpha}>0$. In case that corruption increases in that sector $\left[\uparrow(1-\delta)\Rightarrow\downarrow\delta\right]$ we have a reduction on $\left(\downarrow\gamma_K\right)$ and since the physical capital and human capital are substitutes then $\frac{d\gamma_H}{d\delta}=\frac{\partial\gamma_H}{\partial\gamma_K}\frac{\partial\gamma_K}{\partial\delta}=\varphi\frac{\partial\gamma_K}{\partial\delta}<0$, because $\varphi<0$ and $\frac{\partial\gamma_K}{\partial\delta}>0$. But since

$$|\varphi|<1 \Rightarrow\left|\frac{d\gamma_H\uparrow}{d\delta\downarrow}\right|<\frac{d\gamma_K\downarrow}{d\delta\downarrow}\Rightarrow\frac{d\gamma}{d(1-\delta)}<0,$$

because the increase of γ_H is smaller than the decrease of γ_K, the total effect of corruption that exists in the final output sector on economic growth is negative. Similarly, if we substitute in equation 13 the constraint:

$$g\left(\delta(1-s_H)\right)=1-f\left(\zeta s_H\right),$$

then $\gamma_K=(1-\tau)\left[1-f\left(\zeta s_H\right)\right]^{\alpha} H^{\alpha}K^{-\alpha}-C/K$ and

$\frac{d\gamma_K}{d\zeta}=-(1-\tau)\alpha\zeta s_H\left[1-f\left(\zeta s_H\right)\right]^{\alpha-1} f' H^{\alpha}K^{-\alpha}<0$. In that case if corruption increases in the education sector $\left[\uparrow(1-\zeta)\Rightarrow\downarrow\zeta\right]$ it leads into more public resources in the final output sector relative to education sector and therefore we have an increase on $\left(\uparrow\gamma_K\right)$. However, since the physical and human capital are substitutes, $\frac{d\gamma_H}{d\zeta}=\frac{\partial\gamma_H}{\partial\gamma_K}\frac{\partial\gamma_K}{\partial\zeta}=\varphi\frac{\partial\gamma_K}{\partial\zeta}>0$ because $\varphi<0$ and $\frac{\partial\gamma_K}{\partial\zeta}<0$. Together with the important parameter restriction for φ proven in proposition 4, $\left[|\varphi|<1\right]\Rightarrow\left|\frac{d\gamma_H\downarrow}{d\zeta\downarrow}\right|<\frac{d\gamma_K\uparrow}{d\zeta\downarrow}\Rightarrow\frac{d\gamma}{d(1-\zeta)}>0$,

because the decrease of γ_H is smaller than the increase of γ_K, the total effect of corruption that exists in the education sector on economic growth is positive.

The above analysis indicates that the lower is the corruption in the final output sector the higher is economic growth. On the contrary, the higher is the corruption in the education sector, the less detrimental are the effects on economic growth. This result comes from the assumptions of the model that the two inputs (human and physical capital) are rivals, and that the role of physical capital as a depreciation rate for human capital is quite small. However, we believe that these results can be more close to reality since it is expected that when corruption is high there are more incentives for individuals to acquire education in order to become bureaucrats[44] and to have the possibility to steal public resources. It is also easier to steal

44 According to Woodrow (1887) and other official definitions: bureaucracy is a group of specifically non-elected officials within a government or other institution that implements the rules, laws, ideas, and functions of their institution. In other words,

public resources oriented for the final output for example by reducing the quality of the constructed infrastructure, which is also proposed as an idea by De la Croix and Delavallade (2009). An extension of the current model by incorporating R&D sector can capture also the case where human capital and physical capital are complements and therefore $\varphi > 0$ as well.

3. ANALYTICAL FRAMEWORK

In this section of the paper, we explain how important are some of the assumptions of the theoretical model in order the theoretical model to be consistent with the empirical part. First of all, as in Albelo (1999) the equation for human capital accumulation is:

$\dot{h} = \sigma(1-u)h + \varphi \dot{k}$, where $\varphi \dot{k}$ is known as the "learning by using" effect. Contrary, to Albelo (1999) we make the assumption that one of the key determinants of the allocation of human capital between sectors is the net of corruption share of public expenditure which is devoted to different sectors. We prefer to use the growth rate of physical capital $\left(\frac{\dot{k}}{k}\right)$, because it is more convenient empirically. Furthermore, in this paper the term $\varphi\gamma_K$ plays a role as an endogenous depreciation rate for human capital accumulation. The other component of our specification for human capital accumulation is public expenditure on education as a share of total public expenditures (s_{it}^h).

In the theoretical model, we assume that the public expenditure on education as a share of total public expenditures $\left(s_{it}^h = \frac{G_h}{G}\right)$ affects the decisions of individuals on how to allocate their human capital among sectors and not the public expenditure on education as a share of *gdp* $\left(s_{it}^h = \frac{G_h}{Y}\right)$. We follow this assumption for the following reasons: i) by following the reasoning of Delavallade (2006), since corruption increases the size of public sector $\left(\frac{G}{Y}\right)$[45], and $\left[\frac{G_h}{G} = s_{it}^h\right]$ is expected to be reduced due to

a government administrative unit that carries out the decisions of the legislature or democratically-elected representation of a state. Because of this definition, a minimum level of education is required for someone to become bureaucrat.

45 For the positive impact of corruption on total public expenditures as a share of *gdp* see among others: Tanzi and Davoodi (1997), and Tanzi (1998).

corruption, then $\left(\frac{G_h}{Y}=\frac{G}{Y}\times\frac{G_h}{G}\right)$ has potentially ambiguous sign, and ii) if we use the variable $\left(s_{it}^h=\frac{G_h}{Y}\right)$ instead of $\left(s_{it}^h=\frac{G_h}{G}\right)$ it is possible to exist endogeneity bias since Y and γ_{it}^h are correlated.

In the introduction we mentioned that according to the empirical literature both s_{it}^h and γ_{it}^k are negative functions of corruption. Here we introduce a flexible way to allow for a link between corruption and s_{it}^h and γ_{it}^k respectively by allowing corruption to be the main determinant of their regression coefficient. In that context we use a flexible semi-parametric econometric model that allows for an unknown smooth coefficient function of corruption for both s_{it}^h and γ_{it}^k to capture a potentially different effect of corruption among sectors. The equation we are going to estimate for human capital accumulation is:

$$\gamma_{it}^h = f\left(corrup_{it}\right)s_{it}^h + g\left(corrup_{it}\right)\gamma_{it}^k.$$

Where $f\left(corrup_{it}\right)$ is the coefficient of s_{it}^h estimated non-parametrically as a function of corruption, and $g\left(corrup_{it}\right)$ is the coefficient of γ_{it}^k estimated non-parametrically as a function of corruption.

The main point of this specification is that it captures simultaneously the two effects of corruption into the economy: i) distortion in the allocation of public expenditure (our interest is for public expenditure on education) and ii) on the physical capital investment due to barriers implemented to firms by bureaucrats and which is consistent with the existent empirical evidence.

4. DATA, ESTIMATION METHOD AND EMPIRICAL RESULTS

4.1 DATA

The main equation of interest that we want to estimate is the following:

$$\gamma_{it}^h = a_0 + \sum_{i=1}^{N-1} a_i D_i + \sum_{j=1}^{Z-1} a_j D_j + \sum_{t=1}^{T-1} a_t D_t + \sum_{s=1}^{2} b_s X_{sit} + \theta_1\left(corrup_{it}\right)\gamma_{it}^k + \theta_2\left(corrup_{it}\right)s_{it}^h + u_{it}, \qquad (1)$$

where (D_i) is a group dummy separating the sample into OECD and non-OECD countries, and (D_j) is a region dummy in order to capture specific characteristics of sub-Saharan, Latin America and Eastern-European countries which were in transition during the period of our sample. The data have been averaged over 5 years for the following periods: 1995-1999, 2000-2004 and 2005-2010. We use time-specific dummy (D_t) in order to avoid business cycle effects. The vector of (X_{sit}) consists of two control variables that are used later in order to examine the robustness of our results. More specifically, the two variables are infant mortality (*infmort*) and political stability (*polstab*). The former is an important variable which affects human capital accumulation in the fertility and growth literature and as Gupta et al. (2000) and Rajkumar et al. (2008) argue it is also affected by corruption due to the low public investment in health services. The latter variable captures the general political framework where corruption[46] can thrive and have an important effect on human capital. The data for infant mortality (*infmort*) come from the United Nations dataset (2010) and this variable is defined as the probability of dying between birth and the age of one. Moreover, it is expressed as deaths per 1000 births. The data for political stability (*polstab*) come from the work of Kaufmann et al. (2010). The higher values of this index correspond to less political instability.

As was mentioned earlier the growth rate of physical capital (γ_{it}^{k}) or alternatively physical investment is worsened by corruption and similarly is the public expenditure on education as a share of total public expenditures (s_{it}^{h}).[47] Our main goal is to check the two effects of corruption through the two variables (γ_{it}^{k}) and (s_{it}^{h}) on human capital accumulation simultaneously by allowing these effects to be non-linear and variable over time and for different group of countries. The two unknown functions $\theta_1(.)$ and $\theta_2(.)$ depend on the level of corruption and are estimated by a smooth coefficient semi-parametric model (see Fan (1992) and Fan and Zhang (1999)). The difference of this method with OLS in which corruption affects directly and linearly human capital accumulation is that now we assume that both of the coefficients of (γ_{it}^{k}) and (s_{it}^{h}) vary directly with the level of corruption. In that way, since countries have different levels of corruption it is expected that the effect of (γ_{it}^{k}) and (s_{it}^{h}) not to be constant across countries and time.

Before presenting briefly the mechanics of the smooth coefficient semi-parametric model we want to stress some other data-related points. First of all, the data for public expenditure on education as a share of total public expenditures come from the United Nations dataset (2010). This variable includes government spending on educational institutions (both public and private), educational administration as well as subsidies for private entities. It is expressed as a percentage. The variable of the growth rate of physical

46 Mo (2001) finds that in countries with high corruption there exists higher political instability. The index of political stability has been used as a control variable of corruption by C. Bjørnskov (2003).

47 For the negative impact of corruption on physical capital investment see the paper of Mauro (1995) and for the impact of corruption on public expenditure on education as a share of total public expenditures see Delavallade (2006).

capital (γ_{it}^{k})[48] is constructed by using data from Heston et al. (2011). Finally the data for corruption are obtained from the database of Kaufmann et al. (2010). This variable is defined in such a way as to capture the perceptions of the extent to which public power is exercised for private gain. The original scores range from -2.5 to 2.5, with higher values corresponding to better outcome[49]. For human capital accumulation (γ_{it}^{h}) (which is our dependent variable) we use enrollment rates for population aged between 25 and 65 years. The data for this variable come from the Barro–Lee (2010) dataset. The Barro–Lee (2010) dataset has been extensively used in recent years and as such allows us to make direct comparisons with other empirical studies that explore the role of human capital in economic growth[50].

According to de la Fuente and Domenech (2000), the stock measure of human capital (total mean years of schooling data) suffer from serious measurement error problems, something that would be exacerbated if we were to obtain growth rates from differencing the stock series. Hence, instead of measuring human capital accumulation in growth rates as it is the case for per-capita income, we prefer to use enrollment rates instead.

Finally, due to the possible existence of endogeneity bias which stems from the fact that corruption may relate to human capital in a two-way causality pattern, we also use an instrumental variable approach. Aidt et al. (2008) used the index of voice and accountability (*voaac)* as an appropriate instrument of corruption, variables that are also obtained from the Kaufmann et al. (2010) data set. Voice and accountability captures perceptions of the extent to which a country's citizens are able to participate in selecting their government, as well as freedom of expression, freedom of association and a free press. The higher the value of this index, the higher is the quality of institutions. We first proceed to perform a test of

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48 For the construction of $\gamma_{it}^{k} = \frac{k_{t+1} - k_t}{k_t}$, we have used the following formula: $k_{i,t} = k_{i,t-1}\left(1-\delta\right) + I_{i,t}$. We have set $\delta = 0.06$ which is standard in the literature. The initial value for physical capital has been constructed by $k_{i,t_0} = \frac{Y_{i,t_0}}{\alpha}$, where α takes the value 2 but we have tried different values around 2 and the results are the same and available upon request. $I_{i,t}$ is the total investment at 2005 constant prices, and $Y_{i,t}$ is the real GDP (Laspeyres), at 2005 constant prices. We have also used data for population in thousands from the same data base. Then we average the constructed series of γ_{it}^{k} in the following intervals: 1995-1999, 2000-2004 and 2005-2010.

49 In order higher values of the index to represent higher corruption we have transformed the data according to the following formula: corruption=2.5-corruption [Index].

50 We use total human capital, which is the sum of primary, secondary and higher education. This is done for two major reasons. Firstly, non-OECD countries exhibit very small participation rates at higher levels of education. Finally, because in every country participation at primary and secondary level of education is a necessary requirement in order for people to proceed into higher levels of schooling, we believe it is preferable to include also primary and secondary education in our measure of human capital. We have estimated our results for different categories of human capital and the non-linearities still appear and are even stronger in the tertiary level of education. These results are not presented here but are available upon request.

endogeneity,[51] and we do not find any evidence for its presence. However, we still proceed to follow a two stage least squares approach using the instrument mentioned above in order to compare the results with the case when we ignore the possibility of endogeneity. The first stage in the two stage approach that uses the following equation

$$corrup_{it} = f\left(D_i, D_j, D_t, \gamma_{it}^{k}, s_{it}^{h}, voacc_{it}\right) \qquad (2).$$

This is an OLS regression which includes all the exogenous variables and dummies plus the instrument which is the variable of voice and accountability (*voacc*). Then this regression provides us with fitted values for corruption ($\hat{corrup}$) which is the new index that we can use now for corruption in the second stage of the analysis. We will then compare the results from the two approaches, using *corrupt* and $\hat{corrup}$ to see how the results differ, if they differ at all. Given that our test failed to detect the presence of endogeneity we expect that the two sets of results will not differ by much

Now we provide a quick exposition of the semi-parametric method of smooth coefficients. Equation (1) in a more compact form becomes:

$$\gamma_{it}^{h} = \Psi_{it}\beta + \theta_1\left(corrup_{it}\right)\gamma_{it}^{k} + \theta_2\left(corrup_{it}\right)s_{it}^{h} + u_{it} \qquad (1'),$$

where $\Psi_{it} = \left(D_i, D_j, D_t, X_{sit}\right)$, X_{sit} is the vector of the two control variables (*polstab ,infmort*) which is used when we perform the robustness check, and the error term satisfies $E\left(u_{it} \middle| \Psi_{it}, corrup_{it}, \gamma_{it}^{k}, s_{it}^{h}\right) = 0$. We define $z_{i,t} = \left(\gamma_{it}^{k}, s_{it}^{h}\right)$. The most important is to estimate $\theta_1(.)$ and $\theta_2(.)$. It is a generalized method of varying coefficient models and it is based on local polynomial regression.[52] For details of the method has been used extensively in the literature, see Li et al (2002), Mamuneas, Kalaitzidakis and Stengos (2006) among others.

The data are given as $\left\{Y_{i,}W_i\right\}$, given *i=1,…,n*, with $n = N \times T$, for notational simplicity[53]. Furthermore, we can define $V_i = \left(corrup_i\right)$, then $W_i = \left(V_i, \Psi_i\right)$. The regression function is given by:

$$E\left(Y \middle| \Psi = \psi, V = v, Z = z\right) = \psi\beta + \partial\left(v\right)z \qquad (3).$$

We use a standard multivariate kernel density estimator with Gausian kernel and the rule of thumb suggested by Silverman (1986) as the choice of the bandwidth. The non-parametric element of equation

[51] The Prob>chi2 value of Durbin-Wu-Hausmann test for endogeneity is 0.3724 which means that the initial hypothesis for the absence of correlation between "*corruption"* and the error term is not rejected, which means that there is no endogeneity bias of corruption. We use the Durbin-Wu-Hausmann test for endogeneity see Davidson and MacKinnon (1993).

[52] For taking into account the equivalence between the smooth coefficient method and the local polynomial method see among others, Stone (1977), Fan (1992), and Gozalo and Linton (2000).

[53] Our sample consists of 101 countries. We have 244 observations in total, and the panel is not balanced.

(1') will be examined graphically. Finally, we have performed the Hsiao *et al.* (2007) test in order to check if the linear model is well specified or alternatively to use a more flexible non-parametric model such as the smooth coefficient method described above.

4.2 EMPIRICAL RESULTS

In column A of Table 1B, there are the OLS results when we use the corruption index directly without the use of any control variable. From this table we observe that all the main variables of interest enter negatively and corruption moreover is statistically significant[54] using robust standard errors. In column B of Table 1B, we use the new index of corruption from the first stage in order to compare these results with those of using the corruption index directly. The results are qualitative similar with those in column 1. Finally, in column C of Table 1B, there are the results with the inclusion of the two control variables (political stability and infant mortality). The results are quantitatively[55] similar both in OLS and in the semi-parametric framework. Moreover, the results of Hsiao et al. (2007) test suggest that the linear specification is rejected.

In order to see the smooth coefficient semi-parametric results, we proceed to the graphical analysis. According to Figure 1, in low levels of corruption where the more developed countries belong, a marginal increase of corruption leads into a positive effect of public expenditure as a share of total public expenditure (s_{it}^{h}) on human capital accumulation. In general the impact of public expenditure on education is positive but decreasing for high levels of corruption. A possible explanation is that even in more corrupted environment the increase of public expenditure on education can be a signal to acquire more knowledge since this can be opening the way to enter into the bureaucrat[56] sector and to gain from rent reaping activities or people may simply want to improve their chances for better jobs.

In the case where we check for the impact of physical capital growth (γ_{it}^{k}) on human capital accumulation, Figure 2 still suggests the presence of a non-linear relationship. In low levels of corruption, where countries are more developed, an increase of corruption has negative but small impact on the

54 In the paper of Lin (1998), someone can find a theoretical justification how an increase in public expenditure on education can lead into a reduction in the incentive of individual to accumulate human capital.

55 Even if the variable of political stability is not statistically significant it is important to mention that by including this variable as a control variable the non-linearities are prevalent but in countries with high corruption and high political instability the negative effect of public expenditure on education is more severe. The semi-parametric results for the control variables are not presented here but are available upon request.

56 The various tasks of bureaucrats require minimum level of skills and education in order to be implemented according to Woodrow (1887).

coefficient of γ_{it}^{k} on human capital accumulation. However, for higher levels of corruption the impact of γ_{it}^{k} on human capital investment is negative and more severe. In that case, corruption is possible to have much more negative spillovers in the economy as a whole. Figure 3 and Figure 4 present the smooth coefficient semi-parametric results when we use the instrumented index of corruption. From a direct comparison of Fig. 3 and 4 with Fig. 1 and 2 respectively, it is easily observed that the results do not change when we have instrumented for corruption.

5. SUMMARY AND CONCLUSIONS

In this paper we try to propose an endogenous growth model where corruption, which is considered as an exogenous variable, reduces the net amount of public resources which go to different sectors of the economy. This creates a distortion in the allocation of human capital across sectors since one of the main assumptions of our model is that individuals decide on how to allocate their human capital between sectors by considering the net (from corruption) amount of public resources. In that way, corruption affects human capital accumulation directly by reducing the share of public expenditure on education but also indirectly by deterring investment on physical capital. So even if, the fraction of human capital entering into education sector increases due to the fact that the net from corruption public resources oriented for education are more than those oriented for final output, corruption has negative effect on human capital through the deterioration of physical investment.

In the empirical section of the paper we use a semi-parametric smooth coefficient model in order to capture possible non-linearities of corruption on the important elements of human capital. The empirical results suggest the existence of non-linearities between corruption and human capital accumulation which means that the magnitude of the effect of corruption varies across different groups of countries. More specifically, corruption until an upper bound level has a declining but positive effect on the coefficient of the public expenditure on education. Moreover, it has mainly negative but non linear effect on the coefficient of private capital. This comes into line with our intuition that when corruption increases there is incentive for individuals to enter into education sector. It can be also the case that corruption has less negative effects on public expenditure on education relative to other kind of public expenditures. Another possible explanation is that corruption has damaged the final output sector to such an extent that individuals prefer to postpone their job career and to remain into education sector.

To conclude, corruption has negative and non-linear effects on human capital accumulation. From a policy perspective a higher index for educational achievement does not indicate always higher growth if corruption is a dominance practice in an economy, and it would be of paramount importance the existence of policy practices which reduce corruption firstly in order public expenditure to be enhancing factor for economic growth. Further, extensions of this paper for research are the followings: first of all, at the current stage we try to see how the theoretical results will change if we have an extra sector (sector of bureaucrats) which can have both positive effects on production function by performing useful activities for the private sector and negative effects as in the current version of the model by appropriating public resources. Secondly, an additional extension would be to introduce R&D activity and to check both theoretically and empirically the effect of corruption on R&D. In that case one could take into account the possible positive effects of corruption on R&D activity through the increase on market power for innovating firms and the negative effects through the misallocation of talented people into the bureaucrat sector. Another, important point that has been raised from the current paper is to check for conditions where human capital and hence higher public investment on education can reduce corruption. We think that for analyzing the above issue it would be useful to pursue a more micro founded model with generations' conflict.

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Appendix A: Eqs. (17) – (19)

The current value Hamiltonian is:

$$J = \frac{C^{1-\theta}-1}{1-\theta} + \lambda\left[\sigma f\left(\zeta s_H\right) + \varphi\gamma_K\right]H + \mu\left[\left(1-\tau\right)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha} H^{\alpha}K^{1-\alpha} - C\right]$$

In the Hamiltonian function written above a_1 and C are the control variables and H and K are the state variables. The necessary first order conditions read as:

(A1) $\quad \frac{\partial J}{\partial C} = 0 \;\Leftrightarrow\; C^{-\theta} = \mu \Rightarrow \frac{\dot{C}}{C} = -\frac{1}{\theta}\frac{\dot{\mu}}{\mu}$

(A2) $\quad \frac{\partial J}{\partial s_H} = 0 \;\Leftrightarrow\; \lambda\sigma f'\zeta \mathrm{H} = \mu\alpha\left(1-\tau\right)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha-1} g'\delta H^{\alpha}K^{1-\alpha}$

(A3) $\quad \dot{\mu} = \mu\rho - \frac{\partial J}{\partial K} = \mu\rho - \mu\left(1-\tau\right)\left(1-\alpha\right)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha} H^{\alpha}K^{-\alpha}$

(A4) $\dot{\lambda} = \lambda\rho - \frac{\partial J}{\partial H} = \lambda\rho - \lambda\left[\sigma f\left(\bullet\right) + \varphi\gamma_K\right] - \mu\alpha\left(1-\tau\right)\left[g\left(\bullet\right)\right]^{\alpha} H^{\alpha-1}K^{1-\alpha}$.

Divide (A3) with μ:

(A3') $\frac{\dot{\mu}}{\mu} = \rho - \left(1-\tau\right)\left(1-\alpha\right)\left[g\left(\delta(1-s_H)\right)\right]^{\alpha} H^{\alpha}K^{-\alpha}$. With $g\left(\bullet\right)$ to be constant in BGP, in order $\frac{\dot{\mu}}{\mu}$ to be constant as well in BGP, we need: $\frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \gamma$, (A3'').

Divide (A4) with λ:

(A4') $\frac{\dot{\lambda}}{\lambda} = \rho - \left[\sigma f\left(\bullet\right) + \varphi\gamma_K\right] - \frac{\mu}{\lambda}\alpha\left(1-\tau\right)\left[g\left(\bullet\right)\right]^{\alpha} H^{\alpha-1}K^{1-\alpha}$.

Log-linearize and differentiate (A2) wrt time:

$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{H}}{H} = \frac{\dot{\mu}}{\mu} + \alpha\frac{\dot{H}}{H} + \left(1-\alpha\right)\frac{\dot{K}}{K}$ and together with (A3''):

(A2') $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}$.

We solve (A2) wrt. $\frac{\mu}{\lambda}$:

(A2'') $\frac{\mu}{\lambda} = \frac{\sigma f'\left(\bullet\right)\zeta}{\alpha\left(1-\tau\right)\left[g\left(\bullet\right)\right]^{\alpha-1} g'\delta H^{\alpha-1}K^{1-\alpha}}$.

By replacing (A2'') into (A4') we get after some algebra:

(A4'') $\frac{\dot{\lambda}}{\lambda} = \rho - \left[\sigma f\left(\bullet\right) + \varphi\gamma_K\right] - \sigma g\left(\bullet\right)$. Which is constant is BGP.

Then we equate (A4'') with (A3') and solve wrt $\left(\frac{g\left(\bullet\right)H}{K}\right)^{\alpha}$:

(A5) $\left(\frac{g\left(\bullet\right)H}{K}\right)^{\alpha} = \frac{\sigma\left(f+g\right) + \varphi\gamma_K}{\left(1-\alpha\right)\left(1-\tau\right)} = \frac{\sigma + \varphi\gamma_K}{\left(1-\alpha\right)\left(1-\tau\right)}$, since $f + g = 1$.

From the constraint of human capital and (A3'') we get:

(A6) $\gamma = \frac{\sigma f\left(\bullet\right)}{1-\varphi}$, with $\varphi \neq 1$ and φ<1, for γ>0.

From the definition of BGP and from (A3'') together with (A6) $\frac{\dot{K}}{K} = \gamma$ is constant iff:

(A7) $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \gamma$.

By inserting (A3') into (A1), equating the resulting expression with (A6) and then solve wrt $\left(\frac{g\left(\bullet\right)H}{K}\right)^{\alpha}$:

(A8) $\left(\frac{g(\bullet)H}{K}\right)^{\alpha}=\frac{\sigma f(\bullet)\theta+\rho(1-\varphi)}{(1-\varphi)(1-\tau)(1-\alpha)}$.

Then by equating expression (A5) with (A8) we get:

(A9) $\gamma=\frac{\sigma(f\theta-1+\varphi)}{(1-\varphi)\varphi}+\frac{\rho}{\varphi}$.

In order $\frac{d\gamma}{d\rho}<0$ and $\frac{d\gamma}{d\theta}<0$ we need $\varphi<0$, we need 1-$\varphi>\theta f$, for $\frac{d\gamma}{d\sigma}>0$ and for $\gamma>0\Rightarrow\rho<\sigma\frac{(1-\varphi-f\theta)}{(1-\varphi)}$.

We now check the two transversality conditions: $\lim_{t\to\infty}\lambda_t H_t=0$ and $\lim_{t\to\infty}\mu_t K_t=0$. Because of (A2') and (A7) if the transversality condition holds for the one state variable it holds for the other as well.

$$\lim_{t\to\infty}e^{-\rho t}\lambda_t H_t=\lambda(0)H(0)\lim_{t\to\infty}e^{\left(-\rho+\frac{\dot{\lambda}}{\lambda}+\frac{\dot{H}}{H}\right)t}=\lambda(0)H(0)\lim_{t\to\infty}e^{\left(-\rho+\frac{\dot{\lambda}}{\lambda}+\gamma\right)t}.$$

By replacing into the previous expression (A4''), $\sigma f+\varphi\gamma=\gamma$ from (A6) and the constraint $f+g=1$, we have:

$$\lim_{t\to\infty}e^{-\rho t}\lambda_t H_t=\lambda(0)H(0)\lim_{t\to\infty}e^{\left(-\rho+\frac{\dot{\lambda}}{\lambda}+\frac{\dot{H}}{H}\right)t}=\lambda(0)H(0)\lim_{t\to\infty}e^{-(\sigma(1-f(\bullet)))t}=0,\text{ since } f(\bullet)\in(0,1).$$

From (A9) we have: $\gamma=\frac{\sigma(f\theta-1+\varphi)}{(1-\varphi)\varphi}+\frac{\rho}{\varphi}$. Since $f(\bullet)$ is an invertible function by assumption, then from (A9) we get:

(A10) $\frac{(1-\varphi)[\varphi\gamma+\sigma-\rho]}{\sigma\theta}=f(\zeta s_H)\Rightarrow f^{-1}\left[\frac{(1-\varphi)[\varphi\gamma+\sigma-\rho]}{\sigma\theta}\right]=\zeta s_H\Rightarrow s_H=\frac{f^{-1}(\bullet)}{\zeta}$,

and (A11) $1-s_H=s_Y=\frac{\zeta-f^{-1}(\bullet)}{\zeta}$.

In order the expression inside the brackets in equation (A10) to be a positive number we need:

$\varphi\gamma+\sigma-\rho>0\Rightarrow<0<\frac{\rho-\sigma}{\varphi}<\gamma$, since $\varphi<0$ and $\sigma-\rho>0$ as it is shown below.

Since $1-s_H=s_Y\in(0,1)\Rightarrow\zeta>f^{-1}$; $f^{-1}>0$, with $f'>0$ and since $f(\bullet)$ is invertible $\left(f^{-1}\right)'>0$ as well.

From equation (A9) by using the result from (A6) we get the following result:

(A9') $\gamma=\frac{\sigma(f\theta-1+\varphi)}{(1-\varphi)\varphi}+\frac{\rho}{\varphi}=\frac{\sigma f\theta-\sigma(1-\varphi)}{(1-\varphi)\varphi}+\frac{\rho}{\varphi}=\frac{\gamma\theta}{\varphi}-\frac{\sigma}{\varphi}+\frac{\rho}{\varphi}=\frac{\gamma\theta}{\varphi}+\frac{\rho-\sigma}{\varphi}$

We define:

$\Psi(\gamma)=\frac{\gamma\theta}{\varphi}+\frac{\rho-\sigma}{\varphi}$, so (A9') becomes $\Omega(\gamma)=\gamma-\Psi(\gamma)$. In order to exist an endogenous solution of γ, we need or $\{\Psi'(\gamma)>0 \text{ if } \Psi(0)<1\}\, or\, \{\Psi'(\gamma)<0 \text{ if } \Psi(0)>1\}$.

Since from (A8) $[(1-s_H)=s_Y]$ is constant on BGP, then $f(\bullet)$ and $g(\bullet)$ are constant on BGP as well.

$\Psi'(\gamma) = \dfrac{\theta}{\varphi} < 0$ for φ<0 . Then we need $\Psi(0) > 1$.

$\Psi(0) = \dfrac{\rho - \varphi}{\varphi} > 1 \Rightarrow \{\sigma > \rho$ and $\rho - \varphi > \sigma\}$, these two inequalities hold simulataneously iff $-\varphi > 0$ which is true since $\varphi < 0$.

If the above conditions hold then an endogenous growth rate of the economy γ^* exists in BGP and if we replace γ^* inside (A10) it exists an endogenous $\left(1 - s_H^*\right) = s_Y^* = \dfrac{\zeta - f^{-1}\left[\dfrac{(1-\varphi)[\varphi\gamma + \sigma - \rho]}{\sigma\theta}\right]}{\zeta}$ as well. So, γ^* and $\left(1 - s_H^*\right) = s_Y^*$, are determined simultaneously.

The comparative statics for equation (A10) are straightforward for the parameters $(\theta;\zeta;\rho;\sigma$ and $\varphi)$:

ii) $\dfrac{ds_H}{d\zeta} < 0$ and $\dfrac{ds_Y}{d\zeta} > 0$, ii) $\dfrac{ds_H}{d\varphi} = \dfrac{1}{\zeta}\left(f^{-1}(\bullet)\right)' \bullet \left[\dfrac{(1-2\varphi)\gamma - (\sigma - \rho)}{\sigma\theta}\right]$.

$\dfrac{ds_H}{d\varphi} > 0$ and $\dfrac{ds_Y}{d\varphi} < 0$ if $\gamma > \dfrac{\sigma - \rho}{1 - 2\varphi}$ otherwise $\dfrac{ds_H}{d\varphi} < 0$ and $\dfrac{ds_Y}{d\varphi} > 0$ if $\gamma < \dfrac{\sigma - \rho}{1 - 2\varphi}$ since $\varphi < 0$ and $\sigma > \rho$,

iii) $\dfrac{ds_H}{d\rho} = -\dfrac{(1-\varphi)}{\sigma\theta\zeta}\left(f^{-1}(\bullet)\right)' < 0$ and $\dfrac{ds_Y}{d\rho} > 0$,

iv) $\dfrac{ds_H}{d\theta} = -\dfrac{(1-\varphi)[\varphi\gamma + \sigma - \rho]}{\sigma\theta^2\zeta}\left(f^{-1}(\bullet)\right)' < 0$ and $\dfrac{ds_Y}{d\theta} > 0$ and finally

$\dfrac{ds_H}{d\sigma} = \dfrac{\left[(1-\varphi)\rho - (1-\varphi)\varphi\gamma\right]}{\sigma^2\theta\zeta}\left(f^{-1}(\bullet)\right)' > 0$ for φ<0 and $\dfrac{ds_Y}{d\sigma} < 0$.

The comparative statics for the equation (A9) are straightforward:

$\dfrac{d\gamma}{d\delta} = -\dfrac{\sigma g'\theta s_Y}{(1-\varphi)\varphi} > 0,$ for $s_Y \in (0,1); g' > 0$ and $\varphi < 0 \Rightarrow \dfrac{d\gamma}{d(1-\delta)} < 0$ and

$\dfrac{d\gamma}{d\zeta} = \dfrac{\sigma f'\theta s_H}{(1-\varphi)\varphi} < 0,$ for $s_H \in (0,1); f' > 0$ and $\varphi < 0 \Rightarrow \dfrac{d\gamma}{d(1-\zeta)} > 0$.

APPENDIX B

OECD COUNTRIES: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Finland, France, Greece, Hungary, Iceland, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Republic of Korea, Spain, Sweden, Switzerland, U.K., U.S.

NON-OECD COUNTRIES: Algeria, Argentina, Bahrain, Bangladesh, Barbados, Benin, Bolivia, Botswana, Brazil, Bulgaria, Burundi, Cameroon, Central African Republic, Colombia, Costa Rica, Cote d' Ivoire, Cuba, Cyprus, Dominican Republic, Ecuador, Egypt, El Salvador, Fiji, Ghana, Guatemala, Guyana, Hong Kong, India, Indonesia, Iran (Islamic Republic of), Israel, Jamaica, Kenya, Kuwait, Lao Republic, Latvia, Lesotho, Liberia, Malawi, Malaysia, Mali, Mauritius, Moldova, Morocco, Mozambique, Namibia, Nepal, Nicaragua, Niger, Pakistan, Panama, Paraguay, Peru, Philippines, Romania, Russian Federation, Rwanda, Saudi Arabia, Senegal, Sierra Leone, Singapore, South Africa, Swaziland, Tajikistan, Thailand, Togo, Trinidad and Tobago, Tunisia, United Ar. Emirates, Uganda, Uruguay, Yemen, Zambia.

LATIN AMERICA COUNTRIES: Argentina, Barbados, Bolivia, Brazil, Chile, Colombia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Trinidad and Tobago, Uruguay.

Sub-Saharan Africa Countries: Benin, Botswana, Burundi, Cameroon, Central African Republic, Ghana, Cote d' Ivoire, Kenya, Lesotho, Liberia, Malawi, Mali, Mauritius, Mozambique, Namibia, Niger, Senegal, Sierra Leone, South Africa, Swaziland, Togo, Uganda, Zambia.

Transition Countries: Bulgaria, Croatia, Cuba, Czech Republic, Estonia, Hungary, Latvia, Poland, Romania, Russian Federation, Slovak Republic.

Table 1B: OLS Empirical Results

	OLS	OLS	OLS
Variables	(A)	(B)	(C)
constant	1.067*** (0.052)	1.149*** (0.070)	1.065*** (0.042)
oecd	0.036 (0.021)	0.001 (0.032)	0.024 (0.013)
la	0.148*** (0.024)	0.143*** (0.024)	0.106*** (0.020)
af	-0.136*** (0.040)	-0.136*** (0.043)	0.153*** (0.042)
tran	0.169*** (0.022)	0.123*** (0.022)	0.053*** (0.021)
d00	-0.082** (0.022)	-0.084*** (0.022)	-0.019 (0.020)
d05	-0.036 (0.021)	-0.037 (0.022)	-0.013 (0.018)
pubexp	-0.068 (0.259)	-0.171 (0.284)	-0.305 (0.193)
gk	-0.777* (0.398)	-0.911*** (0.405)	-1.060*** (0.411)
corruption	-0.096*** (0.012)	-0.116*** (0.020)	-0.012 (0.013)
polstab	-	-	-0.012 (0.015)
infmort	-	-	-0.006*** (0.000)
R2/R2adj.	58.15/56.54	54.25/52.49	73.08/71.81
Observations	244	244	244
F-test	36.13***	30.38***	57.27***
Heteroskedasticity	105.13***	94.33***	136.91***
P(Specific.)	2.22e-16***	2.22e-16***	2.22e-16***

Notes: ***,** and * denote the 1%,5% and 10% significance level. Heteroskedasticity is present by conducting Breusch-Pagan test. In the parentheses are the robust standard errors. Pubexpg is the public expenditure of education as a share of total public expenditures, and gk is the growth rate of physical capital. P(Specific) shows the p-values if the null of the parametric linear OLS is correctly specified in comparison to a fully non-parametric model, using the Hsiao *et al.* (2007) test for continuous and discrete data models after 399 Boostrap replications.

FIGURE 1: THE MARGINAL EFFECT OF PUBLIC EXPENDITURE ON EDUCATION ON HUMAN CAPITAL ACCUMULATION

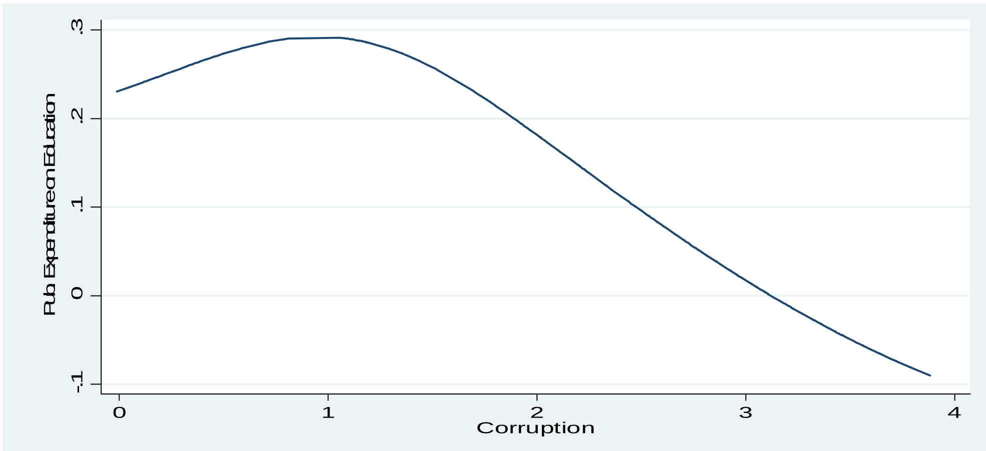


FIGURE 2: THE MARGINAL EFFECT OF THE GROWTH RATE OF PHYSICAL CAPITAL ON HUMAN CAPITAL ACCUMULATION

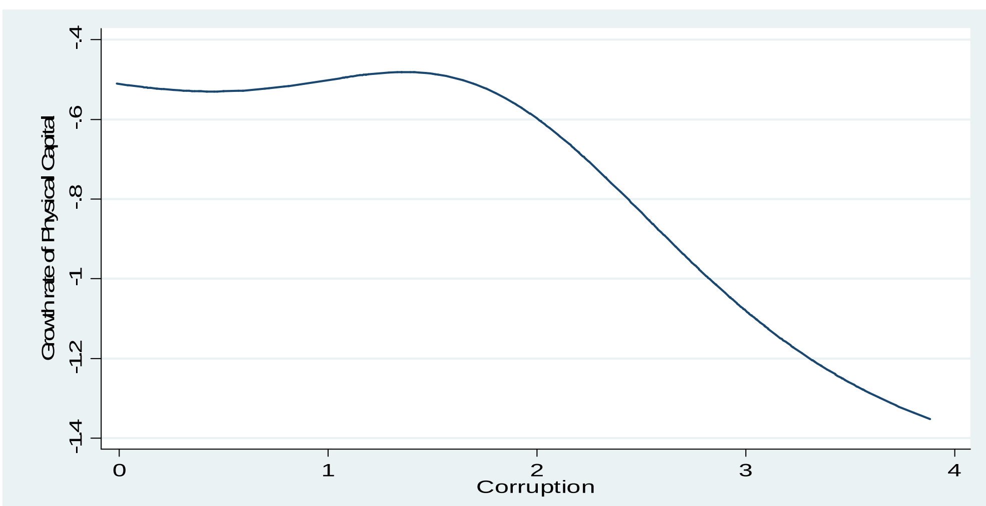


FIGURE 3: THE MARGINAL EFFECT OF PUBLIC EXPENDITURE ON EDUCATION ON HUMAN CAPITAL ACCUMULATION WITH INSTRUMENTED CORRUPTION

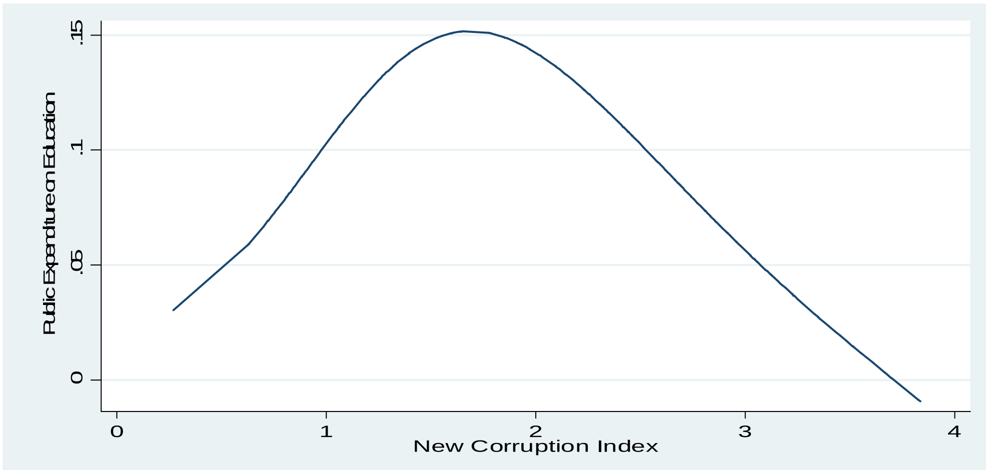
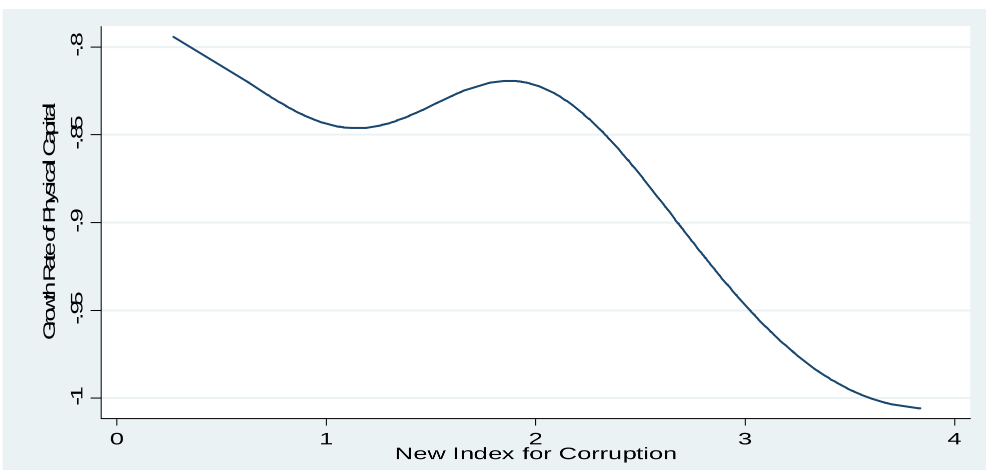


FIGURE 4: THE MARGINAL EFFECT OF THE GROWTH RATE OF PHYSICAL CAPITAL ON HUMAN CAPITAL ACCUMULATION WITH INSTRUMENTED CORRUPTION



CHAPTER 3

LEISURE EXTERNALITIES AND HUMAN CAPITAL ACCUMULATION IN AN ENDOGENOUS R&D GROWTH MODEL*

Abstract

This paper considers leisure externalities in an endogenous growth model with R&D investment and elastic labor supply. We consider that leisure can be used in a more efficient way for the kind of human capital which is employed in the R&D sector, and it can play a positive role in the production of new ideas. An extra assumption we implement in this paper is that the higher is the stock of human capital in the research sector the higher is the externality in the current accumulation of human capital. The theoretical results support the idea that leisure can be beneficial for economic growth if there is a positive externality of leisure in the R&D sector above a threshold in order to cancel out the negative role of leisure in the available time for working and schooling. However, the most important factor for economic growth is the accumulation of human capital through the official education sector. Finally, this paper can provide alternative arguments in explaining the differences in leisure choices and labor productivity between European and US workers.

* In this paper I am the only author, but I would like to thank Alberto Bucci, Xavier Raurich and Thanasis Stengos for their useful comments.

1. INTRODUCTION

Leisure is an important condition which affects the labor supply and it plays a major role in the theory of business cycles mainly from a quantitative perspective, as it can explain substantial amount of output fluctuations (e.g. Hansen and Wright, 1992). Furthermore, the structure of taxation can affect the decisions of allocation of time between productive and less-productive activities. The available also amount of time for working and education can affect economic growth in the endogenous growth theories.

There is an important attempt of Azariadis et al. (2009), who try to investigate in a two-sector growth model the effect of leisure externalities which appear inside the utility of the representative agent into the growth rate of the economy. The same idea with leisure externalities inside the utility function has been employed both by Gómez (2006) and Pintea (2006). However, according to the Canadian organization about Human Resources and Skills Development Indicators, leisure is defined as "*the time we set aside to socialize, exercise our minds and bodies, and pursue our own interests. It is time free from work,… . Having time for leisure and the ability to exercise some choice on how we spend our time are important components of mental and physical well-being.*" According to this definition it is clear that leisure not only reduces the available time from working and schooling but also it can improve the abilities of individuals. Furthermore, according to Lucas (1988), human capital accumulation is not only part of formal education but also is affected by social interactions, which are related with the availability of leisure time. Under this perspective, it is not wise enough to use leisure only as an element of utility, with only negative impact through the reduction of available time for productive activities. To the same direction is the paper of Sánchez-Losada (2000) which considers that leisure time of parents is important for transmission of knowledge to children.

According to Biddle and Hamermesh (1990), in a both theoretical and empirical paper it is advised to take into account of sleep as an important part of leisure time. In addition, Gershuny (2005) in an empirical study suggests that most of the time in West societies is spent into working. Because of the above arguments since the available time for leisure is specific, it is important to assume that the effect of leisure in the labor productivity depends on the way people can exploit their leisure time. Regarding this point, in this paper we try to put in an endogenous growth model with elastic labor supply, leisure externalities.

In order to capture possible different effects of leisure time on the productivity of human capital, we build a model with human capital in which human capital is employed in different sectors. This is one of the main reasons why we incorporate in our analysis a model with endogenous R&D activity. Important papers in this area are those of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), but without human capital accumulation and with the empirically rejected result of scale effects.

Models which try to avoid the strong scale effects are those of Young (1998), Peretto (1998), Dinopoulos and Thompson (1998) and Howitt (1999). R&D growth models which incorporate human capital accumulation include papers such as Dalgaard and Kreiner (2001), Strulik (2005) and Bucci (2011). However, all of the above papers consider fixed labor supply without having the choice of endogenous leisure. The main structure of the model is related to that of Bucci (2011)[57] because not only it contains a combination of both endogenous human capital and endogenous innovation but also it is able to capture the non-monotonic behavior between product market competition and economic growth and therefore it is more consistent with the theoretical and empirical literature.[58] An important paper with endogenous labor supply and R&D investment is the paper of Zeng and Zhang (2007), but this paper not only does not contain human capital but also abstain from the use of non-monotonic effect of leisure through leisure externalities.

Furthermore, contrary to Azariadis et al. (2009), we do not use leisure externalities inside the utility function but inside the technology of producing new ideas. Important historical examples are Archimedes and Isaac Newton, who both had important ideas during their leisure time. Important paper which suggests that leisure can have a positive externality into the process of invention is that of Davis *et al.* (2009). They find, by using data for a survey of 3,000 German inventors, that even if most of the invention process happens during working time, during leisure time it appears a substantial amount of research as well. They also distinguish invention into two different categories: conceptual-based and science-based. The former type is enhanced at leisure time where the latter one is promoted mainly during working time. In our model, we do not make any distinction of innovation into different types, since we are interested at aggregate level, but by incorporating the positive leisure externality in the production of R&D sector, we are able to capture the two different types of innovation at aggregate level. Other arguments which can explain the possible positive effect of leisure time into innovation process are the following two: firstly, individuals who work in the R&D sector need leisure because their job is heavy mainly mentally, and secondly they are mature enough to choose specific kinds of leisure which are productivity enhancing.

The main structure of the model is related to that of Bucci (2011) but apart from the leisure externalities in the R&D sector and the elastic labor supply, we make an extra assumption in the accumulation of human capital. Even though, we employ as well human capital accumulation like Uzawa (1965) and Lucas (1988), we add an externality in the accumulation of human capital which comes from

57 Contrary to Bucci (2011) we are not interested to stress into the combined effect of population growth and product market competition into economic growth but to use this model because of its nice properties to analyze the effect of leisure externalities into the economy.

58 Papers which conclude that product market competition and economic growth face up a non-monotonic relation are the followings among others: Aghion, Harris and Vickers (1997), Aghion *et al*. (2001), Aghion *et al.* (2005) and Aghion *et al.* (2009).

the stock of human capital which is employed in the R&D sector. We justify this assumption with two arguments. First of all, human capital accumulation at micro-level is happening to younger agents, and according to the empirical paper of Guryan *et al.* (2008), more educated parents spend more time with their children[59] and secondly at aggregate level the higher is the stock of human capital that works in the R&D sector the more ideas will be produced, which in turn will increase the available stock of knowledge which can pass though education sector. To put it differently, when teachers have gained appropriate training in private sector, then the quality of education and therefore the stock of knowledge will increase.

The main theoretical implications of our model is that leisure cannot have only negative effects by reducing the available time for working and schooling but also in some respect can be beneficial for sector specific human capital, and this can improve economic growth. More precisely, leisure can have positive effect on economic growth when the productivity parameter of leisure is over a threshold value to offset the negative effects from reducing the available time for working and schooling. Another important finding is that leisure has positive effects on economic growth when the difficulty parameter of doing research is relative small to the positive leisure externality in the R&D sector. An extra implication that arrives from the model is that economies with higher connection[60] between the education sector and the R&D sector can face higher economic growth. Furthermore, even in the presence of positive leisure externalities, the main engine of growth is the accumulation of human capital through the official education sector, because from the assumption of the model human capital appears as an important factor in all the sectors (final output, intermediate goods and that of R&D). Finally, the steady-state value of leisure depends negatively on the leisure externality because leisure has positive effect only on the human capital employed in the R&D sector. This paper also can propose alternative argument in the different levels of leisure and labor productivity between Europeans and US workers.

In section two we present the main assumptions of the model and in section 3 we conclude.

2. THE MODEL ECONOMY

In this economy the total amount of time that individuals have available for working, education and leisure has been normalized to one. The available time for working and spending on education is $(1-l)$, where l is leisure. The fraction of human capital which is employed in the education sector after deciding

59 The paper of Guryan *et al.* (2008) provides empirical results for 14 OECD countries, since the data limitation for leisure.

60 The higher connection between human capital accumulation and R&D sector is captured in this paper by introducing the positive externality of the stock of human capital employed in R&D sector into the equation for the human capital accumulation.

on leisure is: $(1-u)(1-l)H = H_E$. The rest amount of human capital $u(1-l)H = H_N + H_I + H_Y$, is distributed between the R&D, intermediate and final output sector. We define the following fractions:

$\zeta_1 u(1-l) = H_N / H; \zeta_2 u(1-l) = H_Y / H;$ and $\zeta_3 u(1-l) = H_I / H$, with
$\zeta_1 u(1-l)H = H_N;\ \zeta_2 u(1-l)H = H_Y;$ and $\zeta_3 u(1-l)H = H_I$,

by adding all together and using $(1-u)(1-l)H = H_E$ we have:

$$(\zeta_1 + \zeta_2 + \zeta_3)u(1-l)H + H_E = (\zeta_1 + \zeta_2 + \zeta_3)u(1-l)H + (1-u)(1-l)H = (1-l)H,$$

which is true for $(\zeta_1 + \zeta_2 + \zeta_3) = 1$. The shares of available working time that goes into the different productive sectors of the economy are: ζ_1, ζ_2 and ζ_3. These fractions are constant in BGP, and they are determined by the mobility of human capital across sectors.

The equation of human capital accumulation follows Uzawa (1965) and Lucas (1988) with an extra assumption that the stock of human capital is accumulated in a higher way if it is higher the fraction of human capital employed by firms into the R&D sector. This plays a role of a positive externality of working into the R&D sector to the whole economy for instance by increasing the quality of teachers in the education sector. The representative agent cannot choose H_N directly but only firms decide on it, however by choosing the fraction of available time $u(1-l)$ which is spent on working activities indirectly can affect the available amount of human capital that can be employed in the R&D sector and this can lead into positive externalities into the accumulation of human capital.

The main idea behind this, is that it is not only important to invest in education but also to use the knowledge acquired into the sector which produces the ideas in the economy. Furthermore, this assumption implies that the higher is the training of teachers in the real economy and mainly in the R&D sector, the higher is the quality of education and the accumulation of human capital will be increased. After the analysis above the equation for the human capital accumulation is the following:

$$\dot{H}_t = BH_E + \phi H_N = \left[B(1-u_t)(1-l)H\right] + \phi\zeta_1 u_t (1-l)H = \left[B(1-l) + (\phi\zeta_1 - B)u_t (1-l)\right]H$$

With $B > 0$, to be the technological parameter representing the productivity of human capital in the production of new human capital, and $\phi > 0$ to represent the positive effects that the employment of human capital in R&D sector can have in the total stock of human capital.

2.1 PRODUCTION

Consumption (or final) goods represent the *numeraire* in this economy (their price is normalized to one) and are produced competitively by combining human capital (H_Y) and specialized inputs (indexed by i and employed in quantity x_i). Furthermore, we try to capture the *trade-off* between positive (*specialization* gains) and negative (*complexity* costs). The aggregate technology for the production of final output is:

$$Y_t = H_{Yt}^{1-\alpha} \left[\frac{1}{N_t^{\varepsilon}} \int_0^{N_t} \left(x_{it} \right)^{1/m} di \right]^{\alpha m}, \qquad \alpha \in (0,1), \qquad \text{m>1.} \tag{1}$$

In Eq. (1) Y_t denotes aggregate output (GDP) at time *t*, N_t is the number of intermediate-input varieties discovered until time *t*, while α and $(1-\alpha)$ represent the shares of output going respectively to intermediate inputs, x_i, and human capital, H_Y. The term ε measures the net effect of the contrast between *specialization* gains and *complexity* costs arising from horizontal differentiation. An increase in the number of (*N*) of available varieties of intermediate goods may have either a positive impact on *Y* ($\varepsilon < 1$, the *specialization* gains are larger than the *complexity* costs), or no impact on *Y* ($\varepsilon = 1$, the *specialization* gains exactly offset the *complexity* costs), or else a negative impact on *Y* ($\varepsilon > 1$, the *specialization* gains are smaller than the *complexity* costs The intuition behind this formulation is that the higher the number of intermediate varieties the more complicated is to use all these different types into the production due to organizational complexity, but the more types of intermediate goods can potentially increase the productivity of human capital that is employed in the production of final output. The term *m* is a technological parameter that determines the elasticity of substitution between any pair of varieties of differentiated intermediates, and this elasticity equals $m/(m-1)$. A decrease in *m*, by increasing the substitutability between durables, leads to a tougher competition across capital-goods producers and to lower prices. Thus, *m* can be used as a (inverse) measure of the degree of competition in the intermediate product market. More precisely, in a moment we show that *m* is the optimal mark-up on the production marginal cost in the durables sector. As a whole, the aggregate production function (1) displays constant returns to scale to private and rival inputs (H_Y and x_i) and allows disentangling the measure of product market concentration (*m*) from the factor-shares in GDP $(\alpha$ and 1-$\alpha)$. The inverse demand function for the *i*-th intermediate is:

$$p_{it}=\alpha H_{Yt}^{\;1-\alpha}\frac{1}{N_t^{\varepsilon}}\left[\frac{1}{N_t^{\varepsilon}}\int_0^{N_t}\left(x_{it}\right)^{1/m}di\right]^{\alpha m-1}\left(x_{it}\right)^{(1-m)/m},\qquad \forall i\in\left[0,N_t\right],\qquad N_t\in[0,\infty) \tag{2}$$

The firms in the intermediate sector face monopolistic competition. Each of them produces one and only one horizontally differentiated intermediate good and must purchase a patented design before producing its own specialized durable. Thus the price of the patent represents a fixed entry cost. The generic firm *i* employs a one-to-one technology in human capital (Grossman and Helpman, 1991, Chap. 3):

$$x_{it}=h_{it},\qquad \forall i\in\left[0,N_t\right],\qquad N_t\in[0,\infty) \tag{3}$$

Because technology and demand are the same for all intermediates we can focus on a symmetric equilibrium in which $x_{it}=x_t$, $\forall i\in\left[0,N_t\right]$ (3) implies that the total amount of human capital used in the intermediate sector at time t $\left(H_{It}\right)$ is:

$$\int_0^{N_t}x_{it}di=\int_0^{N_t}h_{it}di\equiv H_{It} \tag{3'}$$

By assuming that there exists no strategic interaction across firms, maximization of the generic *i-th* firm's profit leads to the following mark-up rule:

$$p_{it}=mw_{It}=mw_t=p_t,\qquad \forall i\in\left[0,N_t\right]. \tag{4}$$

This expression says that the price is the same for all intermediate goods and, as already said, equal to a constant mark-up (*m*) on the marginal cost (w_t). In this model human capital is employed and produces respectively, consumption goods$\left(H_Y\right)$, durables$\left(H_I\right)$, and ideas$\left(H_N\right)$. Since it is assumed perfectly mobility for human capital across sectors the wage is the same in the different sectors, $w_{Yt}=w_{It}=w_{Nt}\equiv w_t$. The hypothesis of symmetry (*i.e.,* x_t and p_t equal across *i*) leads to:

$$x_{it}=\frac{H_{It}}{N_t}=x_t,\qquad \forall i\in\left[0,N_t\right]. \tag{3''}$$

Given x_t, the instantaneous profit accruing to the generic intermediate firm *i* can be recast as:

$$\pi_{it}=\alpha\left(\frac{m-1}{m}\right)H_{Yt}^{\;1-\alpha}H_{It}^{\;\alpha}N_t^{\alpha\left[m(1-\varepsilon)-1\right]-1}=\pi_t,\qquad \forall i\in\left[0,N_t\right]. \tag{5}$$

Under symmetry, Eq. (1) can be recast as:

$$Y_t=H_{Yt}^{\;1-\alpha}H_{It}^{\;\alpha}N_t^{R},\qquad R\equiv\alpha\left[m\left(1-\varepsilon\right)-1\right], \tag{1'}$$

2.2 R&D ACTIVITY

This sector produces *ideas* (N) for new varieties of intermediate inputs and is populated by a large number of atomistic competitive firms. The representative R&D firm employs a research technology similar to (Bucci, 2011) with the extra assumption that leisure has positive effect in the accumulation of ideas for the people who work in that sector. We make this assumption not only because the work in the R&D sector is more intensive and therefore leisure can relax workers but also because people who work at this sector are able to exploit in a more efficient way their leisure time (this can improve their skills) and this can result in a higher production of new ideas. The accumulation of ideas follows the technology below:

$$\dot{N}_t = \left[\frac{H_N + \eta l H}{\chi H}\right] N, \qquad N(0) > 0, \qquad \eta > 0, \qquad l \in (0,1) \qquad \text{and} \qquad \chi > 0, \tag{6}$$

where H_N is the amount of human capital employed in R&D activity, (lH) is the positive effect of the amount of human capital that is in leisure activities, (η) is a positive productivity parameter[61] that captures the externality of leisure in the production of new ideas and the term χH is a measure of the difficulty of performing R&D and has been introduced by Dinopoulos and Segerstrom (1999). Since the human capital into the economy grows with the higher available time $(1-l)$ that exists into the economy, the higher the stock of human capital the more difficult is to perform research (complexity effect). By replacing $\left[H = \frac{H_N}{\zeta_1 u(1-l)}\right]$ into equation (6) we get:

$$\dot{N}_t = \left[\frac{\zeta_1 u(1-l) + \eta l}{\chi}\right] N, \tag{6'}$$

Because this sector is competitive there is free entry into R&D activity. This implies:

$$w_{Nt} = \left[\frac{\zeta_1 u(1-l) + \eta l}{\zeta_1 u(1-l)\chi}\right] . \frac{N V_N}{\chi H}, \tag{7}$$

where:

$$V_{Nt} = \int_t^\infty \exp\left[-\int_t^\tau r(s)\, ds\right] \pi_\tau d\tau, \qquad \tau > t. \tag{8}$$

[61] Leisure cannot be considered as a direct input in the production function of ideas since it is not input like labor and physical capital, but as a possible positive externality term, which enhances the productivity of human capital in that sector.

In Eqs. (7) and (8) V_{Nt} is the market value of the generic *i-th* blueprint, π is the instantaneous profit of the i-th intermediate firm, *r* denotes the real rate of return on individuals' asset holdings and w_N going to one unit of human capital which is employed in the R&D sector.

2.3 CONSUMERS

The economy is closed and there is no migration across countries. The number of households populating this economy is constant and normalized to unity. We assume no population growth. The representative agent uses the income which is not consumed to accumulate more assets (taking the form of ownership claims on firms). Thus:

$$\dot{A}_t = r_t A_t + \left((1-l)u_t H_t\right) w_t - C_t, \qquad A(0) > 0, \tag{9}$$

where A denotes total assets, r is the real rate of return on household's asset holdings and u is the employed fraction of the available household's stock of human capital, (H) and $(1-l)$ is the available amount of time after the household has decided for the amount of leisure (l). According to this equation, agent's investment in assets (the left hand side) equals agent's savings (the right hand side). In turn, savings are equal to the difference between total income (the sum of interest income, rA, and human capital income, $u(1-l)Hw$) and aggregate consumption, *C*. Since human capital obtains at equilibrium the same reward across sectors, we denote the wage rate going to one unit of human capital simply by *w*, without any sector-specific subscript.

As it has been mentioned before, the law of motion of human capital at the economy-wide level is:

$$\dot{H}_t = BH_E + \phi H_N = \left[B(1-u_t)(1-l)H\right] + \phi\zeta_1 u_t (1-l)H = \left[B(1-l) + (\phi\zeta_1 - B)u_t(1-l)\right]H, \; B > 0,$$

$$\phi > 0, \qquad H(0) > 0 \tag{10}$$

By following Benhabib and Perli (1994) we employ the following utility:

$$U \equiv \int_0^{+\infty} u(C_t, l_t) e^{-\rho t} dt \tag{11}$$

with $u(C_t, l_t,) = lnC_t + \Psi \dfrac{(l_t)^{1-\sigma} - 1}{1-\sigma}$ and σ>0 and Ψ>0 (12)

The parameter Ψ shows the relative preference of leisure versus consumption and it is positive because we assume that leisure is a normal good. Furthermore, σ is the inverse of intertermporal elasticity of substitution of leisure.

So the representative agent has to solve following problem:

$$\underset{\{C_t, u_t, l_t, A_t, H_t\}_{t=0}^{+\infty}}{\text{Max}} U \equiv \int_0^{+\infty} \left[ln C_t + \Psi \frac{(l_t)^{1-\sigma} - 1}{1-\sigma} \right] e^{-\rho t} dt, \quad \rho > 0;$$

$$s.t.: \quad \dot{A}_t = r_t A_t + ((1-l_t) u_t H_t) w_t - C_t, \quad u_t \in [0,1] \quad \forall t; \tag{9}$$

$$\dot{H}_t = [B(1-l_t) + (\phi\zeta_1 - B) u_t (1-l_t)] H_t, \tag{10}$$

The current value Hamiltonian is the following:

$$J = \ln C_t + \Psi \frac{(l_t)^{1-\sigma} - 1}{1-\sigma} + \lambda_t [r_t A_t + ((1-l_t) u_t H_t) w_t - C_t] + \mu_t [[B(1-l_t) H_t] + (\phi\zeta_1 - B) u_t (1-l_t) H_t]$$

Where $\lambda > 0$ and $\lambda(0) > 0$ is the costate variable for the equation of assets and $\mu > 0$ and $\mu(0) > 0$ is the costate variable for the equation of human capital.

along with the transversality conditions:

$$\lim_{t \to +\infty} e^{-\rho t} \lambda_t a_t = 0; \qquad \lim_{t \to +\infty} e^{-\rho t} \mu_t h_t = 0$$

and the initial conditions:

$$a(0) > 0, \qquad h(0) > 0.$$

Finally, as u is a fraction, it must belong to the closed set $[0;1]$. For an interior solution to exist u should be strictly between zero and one.

2.4 GENERAL EQUILIBRIUM AND BGP ANALYSIS

All markets clear. Since human capital is perfectly mobile across sectors and fully employed between production and educational activities, in equilibrium the following equalities must hold:

$$u_t (1-l_t) H_t = H_{Nt} + H_{It} + H_{Yt} \tag{13}$$

$$w_{Yt} = w_{It} \tag{14}$$

$$w_{It} = w_{Nt} \tag{15}$$

Moreover, total household's asset holdings (A) must equal the aggregate value of firms (NV_N):

$$A_t = N_t V_{Nt}, \tag{16}$$

where V_{Nt} is given by Eq. (8) and satisfies the usual *no-arbitrage equation*:

$$\dot{V}_{Nt} = r_t V_{Nt} - \pi_{it}. \tag{16'}$$

In the model, the *i-th* idea allows the *i-th* intermediate firm to produce the *i-th* variety of intermediates. This explains why in Eq. (16) total assets (A) equal the number of profit-making intermediate firms (N)

times the market value (V_N) of each of them (equal, in turn, to the market value of the corresponding idea). The no-arbitrage equation suggests that the return on the value of the *i-th* intermediate firm $(r_t V_{Nt})$ at equilibrium must equal the sum of the instantaneous monopoly profit accruing to the *i-th* input producer (π_{it}) and the capital gains/losses matured on V_N during the time interval dt ($\dot{V}_{Nt}$).

We can now move to a formal definition and characterization of the model's BGP equilibrium. In what follows we denote by g_Z the growth rate of the generic time-dependent variable Z_t.

DEFINITION: Balanced Growth Path (BGP) Equilibrium

A BGP equilibrium in this economy is a situation in which: (i) *All variables depending on time grow at constant (possibly positive) exponential rates;* (ii) *The existing number of varieties* (N_t) *and the available stock of human capital* (H_t) *grow at the same rate,* $g_B = g_H \equiv g$ *(so that the aggregate human to technological capital ratio,* H_t / N_t*, remains invariant over time);* (iii) *The sectoral shares of human capital* ($s_D \equiv H_{Dt} / H_t$*, D = Y, I, N) are constant.*

From this definition Proposition 1 follows immediately:

PROPOSITION 1

The results below give the BGP values for the variables of interest. The mathematical derivation is in the Appendix.

$$\gamma_H = \gamma_N \equiv \gamma = \left[\frac{(B-\phi\zeta_1)(B-\rho)\eta + B\zeta_1\rho}{(B-\phi\zeta_1)(B\chi+\eta)} \right] \tag{17}$$

$$\gamma_C = \gamma_A = \gamma_Y = \left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\left[\frac{(B-\phi\zeta_1)(B-\rho)\eta + B\zeta_1\rho}{(B-\phi\zeta_1)(B\chi+\eta)} \right] \tag{18}$$

$$r = \rho + \left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\left[\frac{(B-\phi\zeta_1)(B-\rho)\eta + B\zeta_1\rho}{(B-\phi\zeta_1)(B\chi+\eta)} \right] \tag{19}$$

$$H_{It} = \frac{\alpha}{\left[m(1-\alpha)+\alpha\right]}\left[\frac{(1-\zeta_1)\rho\chi\left[B\zeta_1\rho + (B-\phi\zeta_1)(B-\rho)\eta\right]}{\left\{\zeta_1\rho(B\chi+\eta)+\eta\left[(B-\rho)(B-\phi\zeta_1)\chi-\zeta_1\rho\right]\right\}(B-\phi\zeta_1)} \right] H \tag{20}$$

$$H_{Yt} = \frac{m(1-\alpha)}{\left[m(1-\alpha)+\alpha\right]}\left[\frac{(1-\zeta_1)\rho\chi\left[B\zeta_1\rho + (B-\phi\zeta_1)(B-\rho)\eta\right]}{\left\{\zeta_1\rho(B\chi+\eta)+\eta\left[(B-\rho)(B-\phi\zeta_1)\chi-\zeta_1\rho\right]\right\}(B-\phi\zeta_1)} \right] H \tag{21}$$

$$H_{Nt}=\left[\frac{\zeta_1\rho\chi\left[B\zeta_1\rho+\left(B-\phi\zeta_1\right)\left(B-\rho\right)\eta\right]}{\left\{\zeta_1\rho\left(B\chi+\eta\right)+\eta\left[\left(B-\rho\right)\left(B-\phi\zeta_1\right)\chi-\zeta_1\rho\right]\right\}\left(B-\phi\zeta_1\right)}\right]H \tag{22}$$

$$u=\left[\frac{\left(B\chi+\eta\right)\rho}{\left(B-\phi\zeta_1\right)\left(\eta+\rho\chi\right)+\zeta_1\rho}\right] \tag{23}$$

$$l=\left[\frac{\left(B-\rho\right)\left(B-\phi\zeta_1\right)\chi-\zeta_1\rho}{\left(B-\phi\zeta_1\right)\left(B\chi+\eta\right)}\right]. \tag{24}$$

By solving the system of equations (20)-(22) and by using the values of (u) and (l) from equations (23) and (24) one can obtain values for ζ_1,ζ_2 and ζ_3.

Proposition 2 ensures under what parameter restrictions the main endogenous variables of the model undertake economically meaningful values in the BGP equilibrium.

PROPOSITION 2

Assume that $\left(1+\alpha\left[m\left(1-\varepsilon\right)-1\right]\right)>0$.

For $B-\phi\zeta_1>\rho$, we can verify:

$\gamma>0$; $\mathrm{r}>0$; $\gamma_C=\gamma_A=\gamma_Y>0$; $u\in\left(0,1\right)$; $l\in\left(0,1\right)$; $\left(H_{It}/H\right)\in\left(0,1\right)$ and $\left(H_{Yt}/H\right)\in\left(0,1\right)$. An extra condition which is necessary to verify $\left(H_{Nt}/H\right)\in\left(0,1\right)$ is: $\phi\eta>\rho$.

Proof: It comes immediately by solving the results in Proposition 1. The first condition considers that the productivity parameter of the education sector is higher than the positive externality of the R&D sector into the accumulation of human capital. The intuition behind is that if this condition doesn't hold then it is optimal nobody to invest in schooling and to try to be employed in the R&D sector. But, this can shrink the growth rate of the economy because human capital from schooling is a prerequisite both for working in R&D sector but also in the other sectors of the economy. The second condition implies that the parameter of leisure externality in the R&D sector together with the parameter of externality that appears into the accumulation of human capital due to the human capital employed in the R&D sector should exceed a critical value of the time preference parameter (ρ). This means that the externalities can give an economically value for the human capital employed in the R&D sector if the economy is not very impatient, which implies that the saving decisions are crucial for the R&D sector.

The next proposition provides the effect of product market competition parameter in the growth rate of the economy.

PROPOSITION 3

Table 1:

ε	$\varepsilon < 0$	$\varepsilon = 0$	$\varepsilon = 1$	$\varepsilon > 1$
$\frac{\partial \gamma_Y}{\partial m}$	+	+	0	-

In this paper is not considered population growth and also the utility function is additive separable with intertemporal elasticity of substitution for consumption equal to one, something which does not enable us to fully compare the results of Table 1 with the Table 1 of Bucci (2011). In contrast to Bucci (2011), in this paper we capture only the '*trade off*' effect. According to this effect, product market competition has different effects on economic growth depending on the value of the parameter (ε) which shows if the *specialization gains* are larger or not to the *complexity costs*. However for the main values of the parameter(ε), we have the same results. The first point that we have to mention is that the effect of product market competition is stronger in all the cases, the higher it is the magnitude of the leisure externality (η) and the higher is the externality (ϕ) in the accumulation of human capital due to the presence of human capital employed in the R&D sector. For instance, by looking into Table 1, we can see that when $(\varepsilon > 1)$ which represents the case where the *specialization gains* are smaller to the *complexity costs*, the decrease in the product market competition leads into a reduction in the growth rate of the economy because the increase in the number of intermediate varieties due to the less competition together with high complexity costs lead into a reduction in the productivity of human capital in the final output and therefore into the total amount of production.

The next proposition summarizes the main comparative statics of the model for the main parameters of interest.

PROPOSITION 4

It can be proven: $\frac{\partial \gamma_Y}{\partial B} > 0$, $\frac{\partial \gamma_Y}{\partial \phi} > 0$, $\frac{\partial \gamma_Y}{\partial \eta} > 0$ always and $\frac{\partial \gamma_Y}{\partial \rho} < 0$ iff $B > \frac{\phi \zeta_1}{\eta - \zeta_1}$, which can verified for low values of ϕ and ζ_1 and/or for high values of η.

Proof: It comes immediately by differentiate equation (18) with respect to the parameters. The results are standard in the growth theory. First of all, one of the main enhancing growth parameters is the productivity of the official educational system(B). Secondly, if the human capital in the R&D sector improves the accumulation of human through education then the economy can face a higher economic growth. Thirdly, if the possible positive productivity parameter of leisure on the accumulation of new ideas is increasing in magnitude, the higher will be economic growth. This is verified by the next two propositions where in that case leisure has positive effects on economic growth. Finally, as usual the higher the impatience in the economy $\left(\text{high value of } \rho\right)$, the lower is economic growth with a requirement condition that the fraction of human capital which works in the R&D sector to be low relatively to the leisure externality in the same sector. A possible explanation can be the following: if the agents do not save enough which is important for R&D investment and in addition the R&D sector is quite small, then any accumulation of human capital through education will have small effect in the real economy.

In Proposition 5, it is provided the effect of the parameters in the steady-state value of leisure.

PROPOSITION 5

It can be proven: $\frac{\partial l}{\partial B} > 0$, $\frac{\partial l}{\partial \phi} < 0$, $\frac{\partial l}{\partial \eta} < 0$, $\frac{\partial l}{\partial \rho} < 0$, and $\frac{\partial l}{\partial \chi} > 0$.

Proof: It comes immediately by differentiating equation (24) with respect to the parameters. To sum up these results, we observe that the higher is the productivity in the education system the easier is to obtain leisure time. Also, the higher is the complexity parameter for research $\left(\chi\right)$ the higher is the leisure time that individuals will require in order to continue the tired program in their work. On the contrary, the higher is the externality in the human capital accumulation $\left(\phi\right)$ due to the human capital which works in the R&D sector; the lower will be the leisure. This is because with extra working time and also with education the growth rate of the economy will be increased substantially. Furthermore, if extra leisure can increase the production of new ideas through the parameter $\left(\eta\right)$, contrary to what is expected, the leisure will be reduced in the steady-state. This can be the case because it appears, according to our assumptions, leisure to have a positive effect only in the R&D's sector human capital. This point can be an alternative perspective why workers in US have less leisure time than European workers, in the sense that leisure can have less positive effect in the research sector in US economy relative to European economies and in addition leisure can reduce by more the productivity of less specialized workers in US relative to European workers.[62] Finally, the more impatient is the economy the lower should be the leisure time in order the agents to be able through their wages to consume.

Finally, Proposition 6 provides parameter restrictions under which the endogenous variable of leisure can have positive effects on the growth rate of the economy.

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62 The seminal paper of Alesina et al. (2005) provides a thorough exposition of arguments in the debate about the differences in labor productivity between US and European economies.

PROPOSITION 6

It can be proven the followings: i) for $\eta > \eta^*$, $\frac{\partial \gamma_Y}{\partial l} > 0$ otherwise if $\eta < \eta^*$, $\frac{\partial \gamma_Y}{\partial l} < 0$

with $\eta^* = \dfrac{\rho\chi\left[\sqrt{1+\dfrac{4\zeta_1 B}{(B-\zeta_1)\rho\chi}}\right]}{2}$,

ii) for $\eta > \zeta_1$ the higher is (B) it exists $\frac{\partial \gamma_Y}{\partial l} > 0$, iii) the higher is (ϕ) it exists $\frac{\partial \gamma_Y}{\partial l} < 0$ and iv) the higher is (χ) relative to (ϕ) it exists $\frac{\partial \gamma_Y}{\partial l} < 0$.

Proof: The derivations of these results are provided in the end of the Appendix. The first result shows that leisure can have positive effect on economic growth if the productivity parameter of leisure[63] is over a threshold value. This comes from the fact that leisure has also direct negative effects by reducing the available time for working and schooling. By looking at the formula of the threshold value of leisure, we infer that the higher is the productivity parameter of the education sector (B), the higher is the opportunity cost of having extra leisure because the foregone human capital and amount of production are going to increase. Secondly, the increase of the productivity parameter of the education sector (B) can lead into a positive effect of leisure on the growth rate of the economy if the productivity parameter of leisure in the R&D sector (η) is quite high. On the contrary, the higher is the externality in the human capital accumulation (ϕ) due to the human capital which works in the R&D sector; the lower will be the growth rate of the economy due to a further increase of leisure. A possible intuition for this result is that the tighter is the connection between the official educational sector and the R&D sector of the economy; the lower should be the available leisure.[64] Finally, an increase of leisure can have a positive effect on economic growth if the complexity to perform research on R&D sector (χ) is smaller than the positive productivity externality of leisure in the same sector (η) and vice versa. This is because in case that the complexity of doing research is smaller relative to the positive impact of leisure in the R&D production, then leisure is productivity enhancing for the growth rate of the economy.

63 In the text we call also the productivity parameter of leisure into the R&D sector as leisure externality, since leisure cannot be considered as a direct input in the production of new ideas.

64 Of similar intuition is the result in Proposition 5.

3. SUMMARY AND CONCLUSIONS

Leisure plays important role in economic theory. In economic growth it is used leisure mainly to have a negative effect because it reduces the available time for working and education. Few theoretical models incorporate leisure into R&D. Furthermore, leisure it has been used as a parameter in the utility function. In the current paper we consider that leisure can play a positive role in the production of new ideas. We use Uzawa (1965) and Lucas (1988) type of human capital, with an extra assumption that the higher is the stock of human capital that works in the R&D sector the more ideas will be produced, which in turn will increase the available stock of knowledge which can pass though education sector. This assumption tries to make a connection between education and R&D sector.

The main findings of our model are: first of all, leisure can have also a positive impact on the economy. More precisely, leisure can have positive effect on economic growth in the following cases: when the productivity parameter of leisure is over a threshold value to overcome the negative effects from reducing the available time for working and schooling and when the complexity cost of doing research is relative small to the positive leisure externality in the R&D sector. Secondly if the education sector is more connected with the R&D sector the higher is economic growth and more intensive is the work characterized in terms of less leisure. Furthermore, in this model the existence of a positive productivity parameter of leisure does not lead into an increase of leisure time in the steady-state. The assumption that leisure has a positive effect only in the R&D's sector human capital can justify the result above. However, this point raises important policy implications in the debate for the difference in labor productivities and leisure choices between Europeans and US workers. Finally, as usual the productivity in the official sector is the enhancing growth parameter.

For future research the current paper can be extended into some further dimensions. First of all, in this model can be added different forms of taxation like in the paper of de Hek (2006), and to check the effect of taxes on the choice of different types of leisure. Secondly, we can assume different types of leisure which are sector specific and we can incorporate them in a model with labor mobility and economic growth, since labor mobility is the main source of diffusion of technology between sectors. In addition, we can extend this model by assuming that the type of leisure is a function of the duration of unemployment in a model with frictions in labor market. Finally, an extra important point which raises from this paper for future research is to investigate empirically how sector specific workers choose different leisure activities and together with possible differences in the complexity of doing research into R&D sector between Europe and US, to try to explain differences in leisure choices and labor productivity.

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APPENDIX : EQS. (17)-(24)

In this appendix we derive results (17)-(24) in the main text. The time subscript is not used for convenience. Letting λ and μ denote the shadow prices of asset holdings and human capital, respectively, the Hamiltonian function (J) reads as:

$$J = \ln C + \Psi \frac{(l)^{1-\sigma} - 1}{1-\sigma} + \lambda\left[rA + \left((1-l)uH\right)w - C\right] + \mu\left[\left[B(1-l)H\right] + (\phi\zeta_1 - B)u(1-l)H\right]$$

The necessary first order conditions are:

(A1) $\quad \dfrac{1}{C} = \lambda ,$

(A2) $\quad \Psi l^{-\sigma} - \lambda uHw - \mu\left[B + (\phi\zeta_1 - B)u\right]H = 0 ,$

(A3) $\quad \lambda(1-l)Hw + \mu(1-l)(\phi\zeta_1 - B)H = 0 \Rightarrow \lambda w = \mu(B - \phi\zeta_1) ,$

(A4) $\quad \dot{\lambda} = \rho\lambda - \dfrac{\partial J}{\partial A} \Rightarrow \dfrac{\dot{\lambda}}{\lambda} = \rho - r ,$

(A5) $\quad \dot{\mu} = \rho\mu - \dfrac{\partial J}{\partial H} = \rho\mu - \lambda\left[(1-l)uw\right] - \mu\left[B + (\phi\zeta_1 - B)u\right](1-l) ,$

From (A1) we get:

(A1') $\quad \dfrac{\dot{C}}{C} = -\dfrac{\dot{\lambda}}{\lambda}$

Combining (A1') together with (A4):

(A1'') $\quad \dfrac{\dot{C}}{C} = r - \rho$

By considering ζ_1 being constant in BGP, and by log-differentiating (A3) with respect to time we get:

(A3') $\quad \dfrac{\dot{\lambda}}{\lambda} + \dfrac{\dot{w}}{w} = \dfrac{\dot{\mu}}{\mu} .$

Dividing (A5) by μ yields:

(A5') $\quad \frac{\dot{\mu}}{\mu} = \rho - B(1-l)$.

By replacing λw from (A3') into (A2) we get:

(A2') $\quad \Psi l^{-\sigma} = \mu B H$.

Since l is constant in BGP, we log-differentiate with respect to time (A2'):

(A6) $\quad \frac{\dot{\mu}}{\mu} = -\frac{\dot{H}}{H}$.

By combining (A3'), (A4) and (A5') we get the following value for *r:*

(A7) $\quad r = \frac{\dot{w}}{w} + B(1-l)$.

Eq. (5) from the text becomes:

(A8) $\quad \pi_i = \alpha\left(\frac{m-1}{m}\right)\left(\frac{H_Y}{N}\right)^{1-\alpha}\left(\frac{H_I}{N}\right)^{\alpha} N_t^{\alpha[m(1-\varepsilon)-1]} = \pi$.

Since *r* is constant in BGP Eq. (8) from the text becomes:

(A9) $\quad V_N = \pi_i = \alpha\left(\frac{m-1}{m}\right)\left(\frac{H_Y}{N}\right)^{1-\alpha}\left(\frac{H_I}{N}\right)^{\alpha} \frac{N_t^{\alpha[m(1-\varepsilon)-1]}}{\left[r-\alpha\left[m(1-\varepsilon)-1\right]\right]\gamma}$, with $\gamma = \frac{\dot{N}}{N} = \frac{\dot{H}}{H}$.

By inserting (A9) into Eq. (7) from the text we get:

(A10) $\quad w_N = \alpha\left(\frac{m-1}{m}\right)\left(\frac{H_Y}{N}\right)^{1-\alpha}\left(\frac{H_I}{N}\right)^{\alpha} \frac{N_t^{\alpha[m(1-\varepsilon)-1]}}{\left[r-\alpha\left[m(1-\varepsilon)-1\right]\gamma\right]} \bullet \frac{N}{\chi H}\left[\frac{\zeta_1 u(1-l)+\eta l}{\zeta_1 u(1-l)\chi}\right]$.

By assuming $\frac{\partial\left[\int_0^N (x_i)^{1/m}\, di\right]^{\alpha m-1}}{\partial x_i} = 0$ from Eq. (2) in the text we get:

(2') $\quad p = \left(\frac{H_Y}{N}\right)^{1-\alpha}\left(\frac{H_I}{N}\right)^{\alpha-1} N_t^{\alpha[m(1-\varepsilon)-1]}$, and by inserting this one inside Eq. (4) from the text we get:

(A11) $\quad w_I = \frac{\alpha}{m}\left(\frac{H_Y}{N}\right)^{1-\alpha}\left(\frac{H_I}{N}\right)^{\alpha-1} N_t^{\alpha[m(1-\varepsilon)-1]}$.

From the fact that $\frac{\partial Y}{\partial H_Y} = w_Y \Rightarrow$

(A12) $\quad w_Y = (1-\alpha)\left(\frac{H_Y}{N}\right)^{-\alpha}\left(\frac{H_I}{N}\right)^{\alpha} N_t^{\alpha[m(1-\varepsilon)-1]}$.

From (15) in the text $w_I = w_N$,we get:

(A13) $\quad H_I = \left[r-\alpha\left[m(1-\varepsilon)-1\right]\gamma\right]\left(\frac{\chi}{m-1}\right)\left[\frac{\zeta_1 u(1-l)\chi}{\zeta_1 u(1-l)+\eta l}\right] H$.

From (14) in the text $w_I = w_Y$,we get:

(A14) $\quad H_Y = \frac{m(1-\alpha)}{\alpha} H_I$.

If we replace (A14) into (A11) and (A12) and also (A14) together with (A13) into (A10), we can prove that:

(A15) $$\frac{\dot{w}_Y}{w_Y} = \frac{\dot{w}_I}{w_I} = \frac{\dot{w}_N}{w_N} = \frac{\dot{w}}{w} = \alpha\left[m(1-\varepsilon)-1\right]\gamma .$$

From Eq. (16) from the text together with (A9) we get:

(A16) $$\gamma_A = \left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\gamma .$$

From Eq. (9) from the text together with (A4) we get:

(A17) $$\frac{\dot{\lambda}}{\lambda} = \rho - \gamma_A + \frac{H(1-l)uw}{A} - \frac{C}{A}$$, with u, l constant in BGP.

From the combination of (A5') with (A6) and by using Eq. (10) from the text we get:

(A18) $$u(1-l) = \frac{\rho}{(B-\phi\zeta_1)}$$, from this condition we need $B > \phi\zeta_1$, for having $u(1-l) > 0$ and $\rho < B - \phi\zeta_1$ for $u(1-l) \in (0,1)$.

From (A18) we can see that u and $(1-l)$ are constant in BGP.

By combining (A3'), (A15), (A17), replacing γ_A from (A16) and using also (A6) after some algebra we get:

(A19) $$\frac{C}{A} = \rho + \frac{H(1-l)uw}{A} .$$

From (A16): $A(t) = A(0)e^{\left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\gamma t}$, $H(t) = H(0)e^{\gamma t}$, and from (A13): $w(t) = w(0)e^{\left(\alpha\left[m(1-\varepsilon)-1\right]\right)\gamma t}$, by combining all these and replacing inside (A19) we get:

(A20) $$\frac{C}{A} = \rho + \frac{H(0)(1-l)uw(0)}{A(0)}$$, which is constant in BGP since u and l are constant, and we get:

(A21) $$\frac{\dot{C}}{C} = \frac{\dot{A}}{A} .$$

Using (A21) and equating (A16) with (A1'') we get a value for *r*:

(A22) $$r = \rho + \left[1+\alpha\left[m(1-\varepsilon)-1\right]\right]\gamma .$$

Then by replacing in (A7) *r* from (A22) and $\frac{\dot{w}}{w}$ from (A15) we can get a solution for γ:

(A23) $$\gamma = B(1-l) - \rho .$$

By replacing (A23) into (A22) we compute *r*:

(A22') $$r = \rho + \left[1+\alpha\left[m(1-\varepsilon)-1\right]\right]\left[B(1-l)-\rho\right],$$ with $r > 0$ iff $B(1-l) > \rho$. This condition guarantees that $\gamma > 0$ and V_N .

From (A21), (A1'') and by using *r* from (A22') we get:

(A21') $$\gamma_A = \gamma_C = r - \rho = \left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\left[B(1-l)-\rho\right].$$

By equating Eq. (10) with (A23):

(A24) $$u(1-l) = \frac{\rho}{(B-\phi\zeta_1)}$$, which is the same as (A18).

From the definition of $\gamma = \frac{\dot{N}}{N}$ and Eq. (6) from the text we get:

(A25) $$\gamma = \frac{\zeta_1 u(1-l) + \eta l}{\chi}.$$

By replacing $u(1-l)$ from (A24) into (A25), and then equating with (A23) we get a value for l:

(A26) $$l = \left[\frac{(B-\rho)(B-\phi\zeta_1)\chi - \zeta_1\rho}{(B-\phi\zeta_1)(B\chi+\eta)} \right].$$

If we replace (A26) into (A24) and solve with respect to u :

(A27) $$u = \left[\frac{(B\chi+\eta)\rho}{(B-\phi\zeta_1)(\eta+\rho\chi)+\zeta_1\rho} \right].$$

From Eqs. (1'), (3''), and (6) from the text together with the BGP definition and by using (A21'):

(A21'') $$\gamma_Y = \gamma_A = \gamma_C = \left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\left[B(1-l)-\rho\right].$$

By replacing (A27) into (A23), (A21') and (A22') we get $\gamma; \gamma_Y = \gamma_A = \gamma_C; r$:

(A28) $$\gamma_H = \gamma_N \equiv \gamma = \left[\frac{(B-\phi\zeta_1)(B-\rho)\eta + B\zeta_1\rho}{(B-\phi\zeta_1)(B\chi+\eta)} \right].$$

(A29) $$\gamma_C = \gamma_A = \gamma_Y = \left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\left[\frac{(B-\phi\zeta_1)(B-\rho)\eta + B\zeta_1\rho}{(B-\phi\zeta_1)(B\chi+\eta)} \right].$$

(A30) $$r = \rho + \left(1+\alpha\left[m(1-\varepsilon)-1\right]\right)\left[\frac{(B-\phi\zeta_1)(B-\rho)\eta + B\zeta_1\rho}{(B-\phi\zeta_1)(B\chi+\eta)} \right].$$

(A31) $$H_N \left[\frac{\zeta_1 u(1-l) + \eta l}{\zeta_1 u(1-l)} \right] = \chi\gamma H.$$

By replacing (A14) into Eq. (13) from the text we get:

(A32) $$H_I = \frac{\alpha}{\left[m(1-\alpha)+\alpha\right]}\left[u(1-l)H - H_N\right].$$

By inserting (A14) into (A32) we get:

(A33) $$H_Y = \left[\frac{m(1-\alpha)}{m(1-\alpha)+\alpha} \right]\left[u(1-l)H - H_N\right].$$

By replacing (A24), (A26) and (A28) into (A31), we get a value for H_N:

(A34) $$H_{Nt} = \left[\frac{\zeta_1\rho\chi\left[B\zeta_1\rho + (B-\phi\zeta_1)(B-\rho)\eta\right]}{\left\{\zeta_1\rho(B\chi+\eta) + \eta\left[(B-\rho)(B-\phi\zeta_1)\chi - \zeta_1\rho\right]\right\}(B-\phi\zeta_1)} \right] H.$$

By replacing (A24) and (A34) into both (A32) and (A33), we get respectively a value for H_I and H_Y:

(A35) $$H_{It} = \frac{\alpha}{\left[m(1-\alpha)+\alpha\right]}\left[\frac{(1-\zeta_1)\rho\chi\left[B\zeta_1\rho + (B-\phi\zeta_1)(B-\rho)\eta\right]}{\left\{\zeta_1\rho(B\chi+\eta) + \eta\left[(B-\rho)(B-\phi\zeta_1)\chi - \zeta_1\rho\right]\right\}(B-\phi\zeta_1)} \right] H, \text{ and}$$

(A36) $$H_{Yt} = \frac{m(1-\alpha)}{\left[m(1-\alpha)+\alpha\right]}\left[\frac{(1-\zeta_1)\rho\chi\left[B\zeta_1\rho + (B-\phi\zeta_1)(B-\rho)\eta\right]}{\left\{\zeta_1\rho(B\chi+\eta) + \eta\left[(B-\rho)(B-\phi\zeta_1)\chi - \zeta_1\rho\right]\right\}(B-\phi\zeta_1)} \right] H.$$

In the end of this appendix we verify that the two transversality conditions are checked. By using (A6) and (A1') these two conditions can be recast as:

(A37) $\lim_{t\to+\infty} \lambda_t A_t = \lambda(0) A(0) \lim_{t\to+\infty} e^{-\rho t} e^{(\gamma_A-\gamma_A)t} = 0$, for ρ>0 and

$\lim_{t\to+\infty} \mu_t H_t = \mu(0) H(0) \lim_{t\to+\infty} e^{-\rho t} = 0$, for ρ>0,

where $A(0) > 0$, $H(0) > 0$.

By differentiating with respect to leisure (l) from (A25) and replace (u) from (A27) we get:

(A38) $$\frac{\partial \gamma_Y}{\partial l} \equiv \Omega = \left[\eta^2 (B - \phi\zeta_1) + \eta\rho\chi (B - \phi\zeta_1) - B\zeta_1\rho\chi\right]\left(1 + \alpha\left[m(1-\varepsilon) - 1\right]\right).$$

The above equation is a polynomial of second power for the term(η). By solving the discriminant together with the assumption of$(B - \phi\zeta_1) > 0$, $\left(1 + \alpha\left[m(1-\varepsilon) - 1\right]\right) > 0$ and for being interested only in positive values of(η) by assumption we get:

(A39) for $\eta > \eta^*$, $\frac{\partial \gamma_Y}{\partial l} > 0$ with $\eta^* = \dfrac{\rho\chi\left[\sqrt{1 + \dfrac{4\zeta_1 B}{(B - \zeta_1)\rho\chi}}\right]}{2}$.

By differentiating (A38) with respect to (B), we need $\eta > \zeta_1$ as a sufficient condition in order the increase of (B) to lead into $\frac{\partial \gamma_Y}{\partial l} > 0$. By differentiating (A38) with respect to (ϕ), we get:

(A40) $\frac{\partial \Omega}{\partial \phi} = -\eta^2\zeta_1 - \eta\rho\chi\zeta_1 < 0$, which means that that an increase of (ϕ) leads to $\frac{\partial \gamma_Y}{\partial l} < 0$.

Finally, by differentiating (A38) with respect to (ϕ) we get the following condition:

(A41) $\frac{\partial \Omega}{\partial \chi} = \rho\left[B(\eta - \chi\zeta_1) - \eta\phi\zeta_1\right] < 0$, which indicates that the higher is (χ) relative to (η), the increase of (χ) leads into $\frac{\partial \gamma_Y}{\partial l} < 0$. ■