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Gamma decay of giant resonances by using Skyrme functionals

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Abstract. A microscopic model, entirely based on Skyrme energy density functionals, has been developed to study the γ-decay of giant resonances. In particular, it treats the ground-state decay within the fully self-consistent Random Phase Approximation (RPA) and the decay to low-lying states at the lowest order beyond RPA. This model is applied to ^{208}Pb and the results are compared with experimental data and with previous theoretical results obtained, in the past decades, using phenomenological models.

1. Introduction

Nuclear giant multipole resonances had been widely investigated in the past decades both theoretically and experimentally (see, e.g., [1, 2]). These states have finite lifetime and consequently they carry a width of 3-5 MeV. They lie in general above the threshold for emitting a nucleon, so they can decay directly by emission of particles, mainly neutrons in medium-heavy nuclei (being proton emission hindered by the large Coulomb barrier). Then, they can emit photons by coupling to the electromagnetic field. Eventually, the resonances can also relax by coupling to progressively more complicated states lying at the same energy (of 2p-2h, 3p-3h, ..., np-nh character), and this is the major contribution to the width, called $\Gamma^{\downarrow}$. The particle decay and the γ-emission processes are characterized by a width $\Gamma^{\uparrow}$ (escape width) and Γ_{γ}, respectively. While the escape width is not negligible, although small with respect to the total width ($\Gamma^{\uparrow}/\Gamma \approx 10^{-1}$, in medium-heavy nuclei), the γ decay has an even lower probability ($\Gamma_{\gamma}/\Gamma \approx 10^{-4}$). In this paper we will focus on this latter process because, although small, it can convey some useful information and it has been the object of recent experimental studies. We will not consider the actual strength function of the giant resonance and the compound gamma decay [3].

A completely microscopic model, based on Skyrme functionals, has been developed to face this problem. In particular, the ground state decay is treated within the fully self-consistent Random Phase Approximation (RPA); for the decay to low-lying states, on the other hand, we include the lowest order correction beyond RPA.

The model has been applied to ^{208}Pb, in particular we focus on the γ-decay of the isoscalar giant quadrupole resonance (ISGQR) to the ground-state and the low-lying collective octupole vibration. In section 2 we briefly summarize the basic features of our formalism and in section 3 we give our results for the two processes at hand.

2. Formalism

In this section we briefly sketch our theoretical framework, referring the reader to a forthcoming paper [4]. The transition amplitude for the emission of a photon of given multipolarity from the nucleus, is proportional to the matrix element of the electric multipole operator $Q_{\lambda\mu}$. The long wavelength approximation is appropriate in our case, then this latter operator takes the form

$$Q_{\lambda\mu} \equiv \frac{1}{2}\sum_{i=1}^{A} e_i^{eff} i^\lambda r_i^\lambda Y_{\lambda\mu}\left(\hat{\boldsymbol{r}}_i\right) \tag{1}$$

where the effective charges due to the recoil of the center of mass of the nucleus are defined e.g. in Ref. [5].

The gamma decay width, summed over the magnetic substates of the photon and of the final nuclear state, is then given by

$$\Gamma_\gamma(E\lambda; i \to f) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E}{\hbar c}\right)^{2\lambda+1} B(E\lambda; i \to f), \tag{2}$$

where E is the energy of the transition and the reduced transition probability B associated with the above operator $Q_{\lambda\mu}$ is

$$B(E\lambda; i \to f) = \frac{1}{2J_i+1}|\langle J_f\|Q_\lambda\|J_i\rangle|^2. \tag{3}$$

2.1. The γ-decay to the ground state

We consider in this subsection the decay of an excited RPA state to the ground-state. We limit ourselves to spherical systems, so that the RPA states have quantum numbers JM (only natural parity, or non spin-flip, are considered); in addition, they are labelled by an index n. At the RPA level, in the case of the decay of the state $|nJ\rangle$ to the ground-state, we obtain for the reduced matrix of Eq. (3),

$$\langle 0\,\|Q_J\|\,nJ\rangle = \sum_{ph}\left(X_{ph}^{nJ} + Y_{ph}^{nJ}\right) e_{ph}^{eff}\langle j_p\|i^J r^J Y_J\|j_h\rangle, \tag{4}$$

where X (Y) are the forward (backward) RPA amplitudes (see, e.g., [6]). It is possible to give a diagrammatic representation of the ground state decay (see Fig. 1).

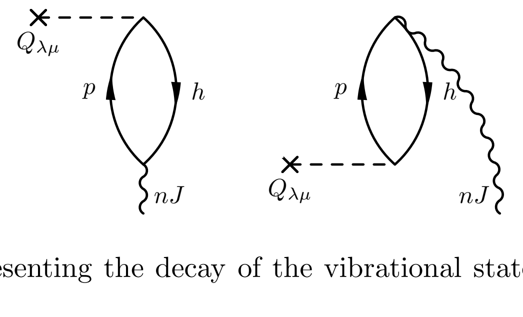

Figure 1. Diagrams representing the decay of the vibrational state $|nJ\rangle$ state to the ground state.

2.2. The γ-decay to low-lying states

By construction RPA is an appropriate theory to describe transition amplitudes between states that differ only by one vibrational (phonon) state. For this reason, the description has to be improved to describe the case of a decay between two vibrational states, extending to a treatment beyond RPA. A consistent framework which is available is the one provided by the Nuclear Field Theory (NFT) [7, 8], since this framework takes into account the interweaving, by means of particle-vibration vertices, between phonons and single-particle degrees of freedom, considered as the relevant independent building blocks of the low-lying spectrum of finite nuclei. The particle-vibration coupling idea [9] has been already applied to the study of γ-decay in Ref. [10]. However, in the present work a consistent use of a microscopic effective interaction of Skyrme type is undertaken for the first time. We consider all the lowest-order contributions to the γ-decay of a phonon state to a different phonon, i.e., all the diagrams involving single-particle states and phonon states, that can lead from the initial to the final state by the action of the external electromagnetic field.

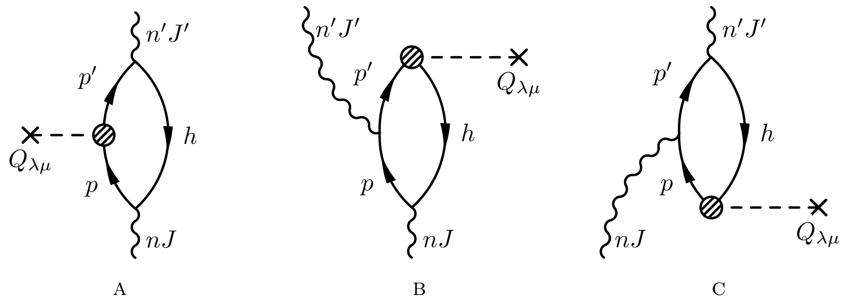

Figure 2. Processes associated with the decay of the $|nJ\rangle$ state to the $|n'J'\rangle$ state. A similar group of diagram are obtained by interchanging the role of particles and holes.

The perturbative diagrams associated with the λ-pole decay of the initial RPA state $|nJ\rangle$ (at energy E_J) to the final state $|n'J'\rangle$ (at energy $E_{J'}$) can be grouped into three families. A representative of each family is shown in Fig. 2 and corresponding analytical expressions read

$$\langle n'J' \| Q_\lambda \| nJ\rangle_{\text{A}} = \sum_{pp'h} (-)^{J+\lambda+J'+1} \begin{Bmatrix} J & \lambda & J' \\ j_{p'} & j_h & j_p \end{Bmatrix} \frac{\langle p\|V\|h, nJ\rangle\langle h, n'J'\|V\|p'\rangle Q^{\lambda pol}_{p'p}}{(E_J - \epsilon_{ph} + i\eta)\left(E_{J'} - \epsilon_{p'h}\right)}, \tag{5.A}$$

$$\langle n'J' \| Q_\lambda \| nJ\rangle_{\text{B}} = \sum_{pp'h} (-) \begin{Bmatrix} J & \lambda & J' \\ j_{p'} & j_p & j_h \end{Bmatrix} \frac{\langle p\|V\|h, nJ\rangle\langle p', n'J'\|V\|p\rangle Q^{\lambda pol}_{hp'}}{(E_J - \epsilon_{ph} + i\eta)\left(E_J - E_{J'} - \epsilon_{p'h} + i\eta'\right)}, \tag{5.B}$$

$$\langle n'J' \| Q_\lambda \| nJ\rangle_{\text{C}} = \sum_{pp'h} \begin{Bmatrix} J & \lambda & J' \\ j_h & j_{p'} & j_p \end{Bmatrix} \frac{\langle p'\|V\|p, nJ\rangle\langle h, n'J'\|V\|p'\rangle Q^{\lambda pol}_{ph}}{\left(E_{J'} - \epsilon_{p'h}\right)(E_J + \epsilon_{ph} - E_{J'} + i\eta)}. \tag{5.C}$$

In these equations ϵ_{ph} is equal to the difference of the Hartree-Fock (HF) single-particle energies $\epsilon_p - \epsilon_h$, and V is the residual particle-hole interaction. In all the energy denominators we include finite imaginary parts η to avoid accidental divergences and to take into account the coupling to more complicated configurations, not included in the model space. The other contributions are obtained interchanging the role of particles and holes, applying the usual rules of diagrammatic expansion.

In all the above equations, the matrix elements of the operator Q_λ include the contribution from the nuclear polarization (consequently they carry the label *pol*).

$$
\begin{aligned}
Q_{ij}^{\lambda pol} =& \langle i \| Q_\lambda \| j \rangle + \\
&+ \sum_{n'} \frac{1}{\sqrt{2\lambda+1}} \left[\frac{\langle 0 \| Q_\lambda \| n'\lambda \rangle \langle i, n'\lambda \| V \| j \rangle}{(E_J - E_{J'}) - E_{n'} + i\eta} - \frac{\langle i \| V \| j, n'\lambda \rangle \langle n'\lambda \| Q_\lambda \| 0 \rangle}{(E_J - E_{J'}) + E_{n'} + i\eta} \right],
\end{aligned}
\tag{6}
$$

where $|n'\lambda\rangle$ are the RPA states having multipolarity λ (and lying at energy $E_{n'}$), while the bare operator Q_λ has been defined in Eq. (1). The polarization contribution, that is, the second and third term in the latter equation, has the effect of screening partially the external field. In a diagrammatic way, the bare and the polarization contributions to Eq. (6) are drawn in Fig. 3.

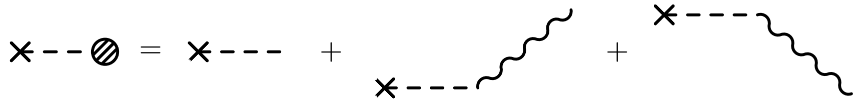

Figure 3. Polarization contribution to the operator $Q_{\lambda\mu}$

As mentioned above, in the present implementation of the formalism we use consistently different zero-range Skyrme interactions. The single-particle energies ϵ_i, and the corresponding wavefunctions, come from the solution of the HF equations. The energies (and X and Y amplitudes) of the vibrational states are obtained through fully self-consistent RPA [11]. These quantities enter the reduced matrix elements associated with the PVC vertices. The basic one, that couples the single-particle state i to the particle-vibration pair j plus nJ, is

$$
\langle i \| V \| j, nJ \rangle = \sqrt{2J+1} \sum_{ph} X_{ph}^{nJ} V_J(ihjp) + (-)^{j_h - j_p + J} Y_{ph}^{nJ} V_J(ipjh). \tag{7}
$$

V_J is the particle-hole coupled matrix element

$$
V_J(ihjp) = \sum_{\{m\}} (-)^{j_j - m_j + j_h - m_h} \langle j_i m_i j_j - m_j | JM \rangle \langle j_p m_p j_h - m_h | JM \rangle v_{ihjp}, \tag{8}
$$

while v_{ihjp} is a shorthand notation for $\langle j_i m_i j_h m_h | V | j_j m_j j_p m_p \rangle$. For the detailed derivation of the the reduced matrix element of Eq. (7) we refer to the Appendix of Ref. [12].

3. Results

In this section, the results obtained from our numerical calculations in ^{208}Pb are discussed. In particular, we focus on the γ-decay width Γ_γ associated with the decay of the isoscalar giant quadrupole resonance (ISGQR) either to the ground-state or to the 3^- low-lying state. We have employed four different Skyrme forces: SLy5 [13], SGII [14], SkP [15] and LNS [16].

In all cases, we start by solving the HF equations in a radial mesh that extends up to 20 fm with a radial step of 0.1 fm. Once the HF solution is found, the RPA equations are solved in the usual matrix formulation. Vibrations (or phonons) with multipolarity L ranging from 1 to 3, and with natural parity, are calculated. The RPA model space consists of all the occupied states, and all the unoccupied states lying below a cutoff energy E_C equal to 50 MeV. The states at positive energy are obtained by setting the system in a box of 20 fm, that is, the continuum is discretized. These states have increasing values of the radial quantum number n, and are calculated for those values of l and j that are allowed by selection rules. With this choice of the model space the energy-weighted sum rules (EWSRs) satisfy the double commutator values at the level of about 99%; moreover, the energy and the fraction of EWSR of the states which are relevant for the following discussion are well converged.

Table 1. Energy of the ISGQR and γ-decay width associated with its transition to the ground state. The first four rows correspond to the present RPA calculations performed with different Skyrme parameter sets. In this case we show both the bare Γ_γ from Eq. (2) as well as the renormalized value which is discussed in the main text. The next two rows report the results of previous theoretical calculations [10, 17]. In the last row the experimental value from [18] is displayed.

	E_{GQR} [MeV]	Γ_γ [eV]	Γ_γ^{ren} [eV]
SLy5	12.28	231.54	127.58
SGII	11.72	163.22	113.57
SkP	10.28	119.18	159.72
LNS	12.10	176.57	104.74
Ref. [17]	10.60	112	
Ref. [10]	11.20	142	
Ref. [18]	10.60	130±40	

3.1. Ground-state decay

We group in table 1 the results obtained for the decay of the ISGQR to the ground state. In general, our calculations reproduce the experiment quite well, without any parameter adjustment. They tend to overestimate the decay width, and this is true in particular for SLy5; however, even in this worst case, our result are still compatible with the experimental value.

More importantly, this discrepancy is entirely due to the fact that the properties of the giant resonance (energy and fraction of EWSR) do not fit accurately the experimental findings. In particular, the critical quantity turns out to be the resonance energy, since in Eq. (2) the energy of the transition is raised to the fifth power: consequently, an increase of the energy by 1 MeV produces an increase of the gamma decay width by about 50% (at 10 MeV). To substantiate this point, in the last column of table 1 we report the values obtained for the decay width after having rescaled the ISGQR energy to the experimental value (shown in table 2).

Since for all interactions the experimental value of the ground-state decay width can be obtained simply by scaling to the experimental value the energy and the fraction of EWSR, we conclude that this kind of measurement is not able to discriminate between models more than other integral observables (e.g. energy and strength).

For completeness, in table 1 the previous theoretical values found in the literature [10, 17] are listed as well. In Ref. [17], the theory of finite Fermi systems [19] is implemented with a separable interaction in order to obtain the decay width. In Ref. [10], in which the nuclear field theory with a separable interaction at the particle-vibration vertex is used, the result is obtained from the values of energy and EWSR in table 1.

3.2. ISGQR decay to the low-lying 3^- state

In table 3, the results obtained for the decay of the ISGQR to the low-lying octupole state are shown, together with theoretical values from Refs. [10, 17]. These latter are obtained using different models: in [17], the theory of finite Fermi systems (cf. Ref. [19]) with a separable interaction is used, while in [10], the nuclear field theory with a separable interaction at the particle-vibration coupling vertex is implemented.

It can be noticed that only two interactions, namely SLy5 and SkP, can reasonably reproduce the experimental value for the decay width. Nevertheless, all these forces are able to produce a

Table 2. Detailed properties of the ISGQR and of the low-lying 3^- state in ^{208}Pb.

	ISGQR		3^-	
	E [MeV]	EWSR [%]	E [MeV]	B(E3↑) [10^5e^{2}fm^6]
Experiment	10.9 ± 0.3	100	2.6145 ± 0.0003	6.11 ± 0.09
SLy5	12.28	69.27	3.62	6.54
SGII	11.72	72.31	3.14	6.58
SkP	10.28	81.79	3.29	5.11
LNS	12.10	66.98	3.19	5.67

Table 3. γ-decay width associated with the transition from the ISGQR to the low-lying 3^- state. The first four rows correspond to the present calculation. The energy of the transition is also reported in the second column. The next two rows report the results of previous theoretical calculations [10, 17] are also listed. In the last row, the experimental value from [18] is provided.

Interaction	$E_{trans}\,[MeV]$	$\Gamma_\gamma\,[eV]$
SLy5	8.66	3.39
SGII	8.58	29.18
SkP	6.99	8.34
LNS	8.90	39.87
Ref. [17]	7.99	4.00
Ref. [10]	8.59	3.50
Ref. [18]	7.99	5.00±5.00

Table 4. Quenching factors that combine to produce the decay width Γ_γ from the typical particle-hole dipole transition at the energy of interest of ≈ 8.5 MeV: they are discussed in the main text. The same quantities from [10] are displayed.

	SLy5	SGII	SkP	LNS	Ref. [10]
p − h transition [eV]	$1.2{\cdot}10^3$	$3{\cdot}10^3$	$1.5{\cdot}10^3$	$3.0{\cdot}10^3$	$1.5{\cdot}10^3$
Recoupling coefficient	3	3	3	3	3
$\pi-\nu$ cancellation	5	4	3–4	4	4
p − h cancellation	3–4	2–3	2–3	3–4	2–3
Polarization	6	3	7–8	4	15
Γ_γ[eV]	3.39	29.18	8.34	39.87	3.50

total $\Gamma_\gamma(\mathrm{GQR} \rightarrow 3^-)$ which is only a few percent of $\Gamma_\gamma(\mathrm{GQR} \rightarrow gs)$, as the experiment indicates.

The factors in the amplitude, obtained summing all the contributions reported in section 2.2, combine to produce a rather small width in the end: they are displayed in table 4, for the four forces used, together with similar factors from [10]. A typical particle-hole dipole transition would produce at the energy of interest of ≈ 8.5 MeV the γ-decay width reported in the first row. However, several effects intervene to quench this strong coupling. The Wigner $6-j$ symbol times the other geometrical and statistical factors in Eq. (5) reduces the width by a factor 3 (second row). The third row displays the quenching deriving from the fact that proton and neutron amplitudes tend to cancel, being the initial and final state predominantly isoscalar and the electromagnetic operator (1) isovector. As it is well known [20], another source of interference is the cancellation between particles and holes: the associated quenching factor is displayed in the fourth row. Eventually, the polarization contribution (6) reduces the width by the factors list in the fifth entry, to get a width of the order of electronvolts (or tens of electronvolts), as reported in the last row. In particular, the most important effect is the polarization of the nuclear medium.

4. Conclusions

In our work we deal with the de-excitation of nuclear excited RPA states via electromagnetic emission, by using a fully microscopic approach: in fact, the γ-decay has been studied in the past decades using only phenomenological models. In particular, in our scheme, within the Skyrme framework, the single particle states are obtained within HF, the vibrations are calculated using fully self-consistent RPA and the whole Skyrme force is employed at the particle-vibration vertices. We treat the ground-state decay within the fully self-consistent RPA and the decay to low-lying collective vibrations at the lowest contributing order of perturbation theory beyond RPA.

We have applied our model to the γ-decay of the isoscalar giant quadrupole resonance in ^{208}Pb into the ground-state and the first low-lying octupole vibration. Concerning the ground-state decay, we found that our outcomes are consistent with previous theoretical calculations, based on phenomenological models, and with the experimental data. In particular, all the Skyrme parametrizations give a decay width of the order of hundreds of electronvolts, though, at the same time, they tend to overestimate it: these discrepancies are due to the fact that the energy of the resonance does not totally agree with the experimental data. For this reason, we conclude that the γ-decay to the ground-state is not so able to discriminate between different models, at least not more than other inclusive observable (as energy and strength).

On the other hand, the γ-decay to low-lying collective states is more sensitive to the interaction used. As a matter of fact, only two interactions (namely SLy5 and SkP) manage to achieve a decay width of few electronvolts, consistently with the experimental finding. Nevertheless, the other interactions give a width that is some percent of the one corresponding to the ground-state decay. In particular, the description of the dipole spectrum is a crucial point, because small differences in the strength of the dipole states introduced as intermediate states change significantly the polarization of the nuclear medium, as it is discussed in [4].

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