

Spectral properties and preconditioning of Galerkin and Collocation Isogeometric approximations of acoustic wave problems

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In recent years, Isogeometric Analysis (IGA) discretizations of PDEs have focused also on wave problems [1]. In our previous works we have studied stability and convergence properties of IGA Galerkin [2] and Collocation [3] methods for the acoustic wave equation with absorbing boundary conditions and Newmark's time-advancing scheme. Since both the IGA collocation and Galerkin mass matrices are not diagonal, the solution of the linear systems at each time step is a key issue regardless of whether the Newmark scheme is explicit or implicit. Unfortunately, there is still a lack of theoretical results for IGA matrices' properties in the literature, and most of the known estimates are only conjectures, even for the Poisson problem with Dirichlet boundary conditions.

In this presentation, we illustrate a numerical comparison of collocation and Galerkin IGA methods with respect to the condition numbers of their mass and iteration matrices, varying the polynomial degree p , mesh size h and regularity k [4]. We show that the same trends hold for the condition numbers of the IGA collocation and Galerkin mass and stiffness matrices with respect to the mesh size h , and these results are comparable to those available for the Poisson problem. If we consider instead the behavior of the condition numbers for increasing values of p , we observe that they are better for the IGA collocation case than for IGA Galerkin, both for minimal and maximal regularity k . Since the linear systems to be solved at each time step are ill conditioned, we propose a two-level Overlapping Schwarz (OS) preconditioner [5], accelerated with GMRES in the IGA collocation case and with the conjugate gradient method in the IGA Galerkin case. We investigate numerically the robustness of the OS preconditioner with respect to discretization parameters h , p , k , overlap parameter γ and heterogeneous acoustic propagation velocity c_0 [6].

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