Isogeometric methods for acoustic waves with absorbing boundary conditions

Elena Zampieri

Department of Mathematics, University of Milan, Italy, `elena.zampieri@unimi.it`

In recent years there has been an increasing attention to high order simulation of acoustic and elastic wave propagation [1, 2]. In this presentation we consider the Galerkin and Collocation Isogeometric approximation of the acoustic wave equation with absorbing boundary conditions in cartesian and curvilinear 2D regions, while the time discretization is based on explicit or implicit Newmark schemes.
Since both the IGA Galerkin and Collocation mass matrices are not diagonal, the main difference between explicit and implicit IGA Newmark schemes is related to the stability bounds for the time step Δt, rather than to the solution of the linear systems arising at each temporal instant.
In this respect we briefly illustrate some stability estimates both for the semidiscrete and fully discrete schemes that are only partially based on proven results, due to the lack of theoretical estimates regarding eigenvalues and conditioning of the mass and stiffness IGA matrices [3, 4].
Furthermore, we present a detailed numerical study on the properties of the IGA methods as concerns stability thresholds, convergence errors, accuracy [3, 4], and spectral properties of the IGA matrices [6] varying the polynomial degree p, mesh size h, regularity k, and time step Δt.
Finally, we focus on two meaningful examples in the framework of wave propagation simulations: a test problem with an oscillatory exact solution having increasing wave number, and the propagation of one or two interfering Ricker wavelets [5].
Numerical results show that the IGA Collocation method retains the convergence and stability properties of IGA Galerkin. Moreover, IGA Collocation is in general less accurate when we adopt the same choices of discretization parameters. On the other hand, regarding the computational cost and the amount of memory required to achieve a given accuracy, we observe that the IGA Collocation method often outperforms the IGA Galerkin method, especially in the case of maximal regularity $k = p - 1$ with increasing NURBS degree p.

References

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