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Stratification Effects on Momentum Transfer in the Roughness Sublayer over Tall Forested Canopies

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A new perspective for the Roughness Sublayer: the Cospectral Budget Model

INTRODUCTION

The scale-wise impact of thermal stratification on turbulent momentum fluxes is explored using a Co-spectral Budget (CSB) model applied to the Amazon Tall Tower Observatory (ATTO, Manaus, Brazil).

NOVELTY

The CSB model addresses stratification scale-by-scale using the momentum budget instead of the TKE budget.

KEY FINDING

The momentum flux co-spectrum $F_{wu}(k_x)$ is impacted by the energy spectrum of the vertical velocity $E_{ww}(k_x)$ and the much less studied co-spectrum of the longitudinal heat flux $F_{u\theta_v}(k_x)$.

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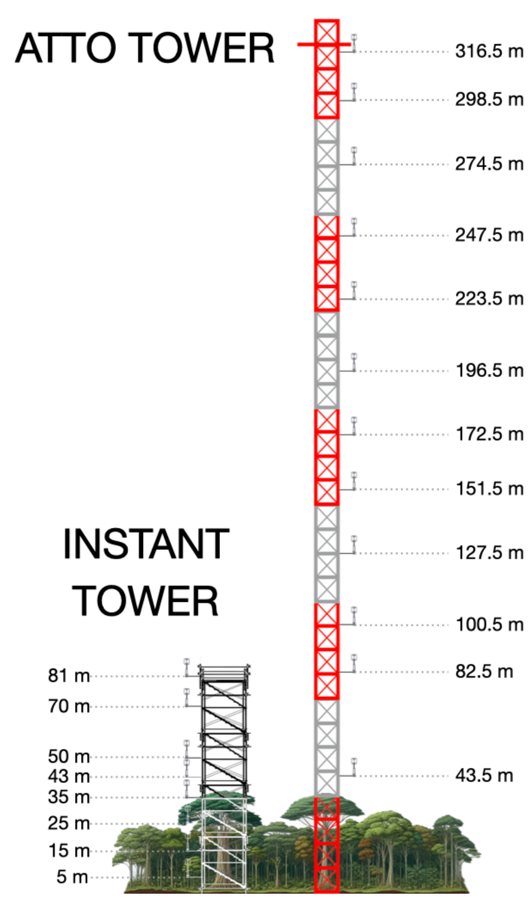

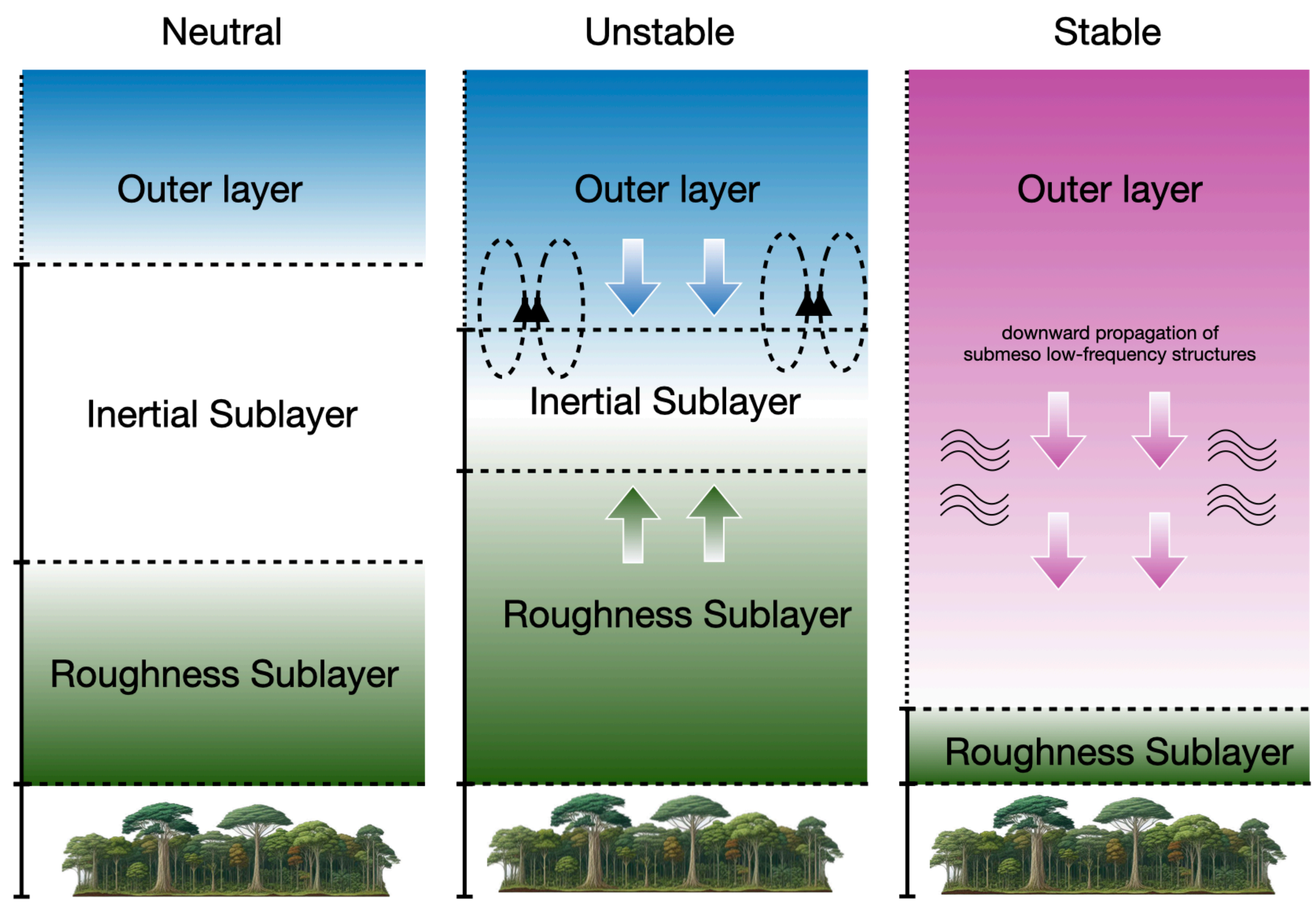

The turbulent stress budget

Buoyancy contribution on momentum transfer - the turbulent stress budget.

$$\frac{\partial \overline{w'u'}}{\partial t} = 0 = -\sigma_w^2 \Gamma(z) + R_{u,w} + \beta_o \overline{u'\theta_v'} - \left[\frac{\partial \overline{w'w'u'}}{\partial z} + 2\epsilon_{uw} \right]$$

$$\frac{\partial \overline{e}}{\partial t} = 0 = -\overline{u'w'}\Gamma(z) + \beta_o \overline{w'\theta_v'} - \epsilon - \left[\frac{\partial \overline{w'e}}{\partial z} + \frac{\partial \overline{w'p'}}{\partial z} \right]$$

The pressure-velocity interaction terms act differently in the budgets:

- In the TKE budget, they redistribute energy.
- In the momentum stress budget, they de-correlate u' and w'.

$R_{u,w}$ **is way more efficient than** ϵ_{uw}.

The CSB - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber k_x read as:

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\bar{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\bar{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

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Local rate of change | Molecular destruction | Transfer | Generation by mean flow | Buoyancy Contribution | Pressure decorrelation

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Molecular destruction

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Pressure decorrelation

Neglected

Derived from Data

Parameterized

The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),

II) stationary planar homogeneous flow (only vertical gradients considered)

III) standard closure for $R_{u,w}(k)$ using the Rotta scheme

IV) the flux transfer terms across scales is ignored

<u>Simplified CSB:</u>

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Simplified CSB:

Momentum Flux

$$-F_{wu}(k) = A^{-1}\tau(k)\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

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Neutral Stratification
Mortarini et al. (2023)

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NEW TERM:
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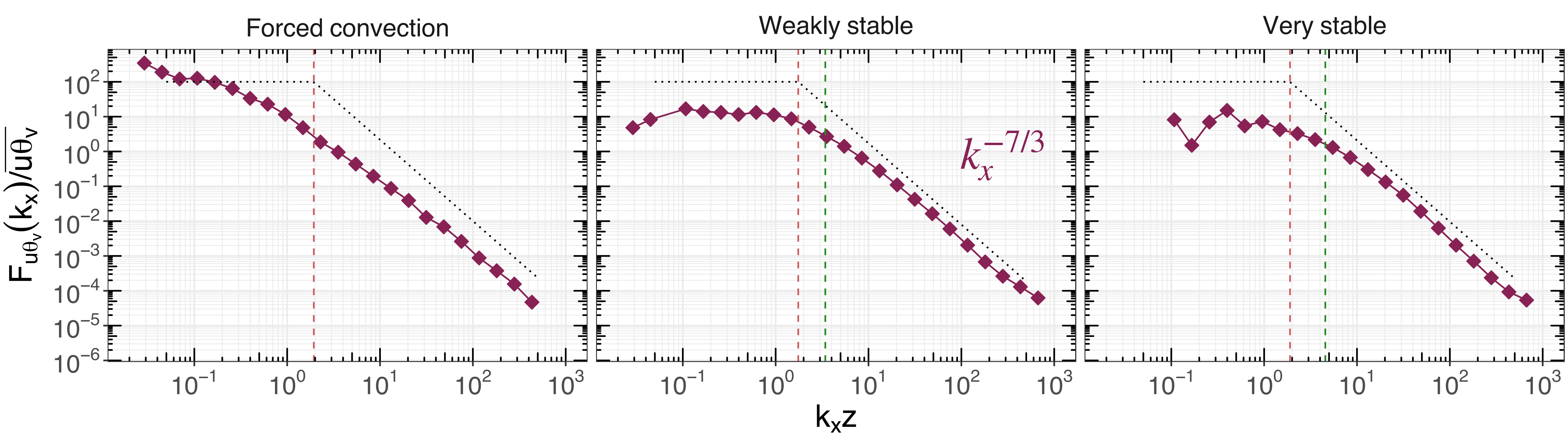


The CSB - corollaries (I - II)

$$\overline{w'u'} = \frac{\tau}{A}\left[-\sigma_w^2\frac{dU}{dz} + \frac{g}{\overline{\theta}_v}\overline{u'\theta_v'}\right] \qquad -F_{wu}(k) = \alpha A^{-1}\epsilon^{-1/3}k^{-2/3}\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

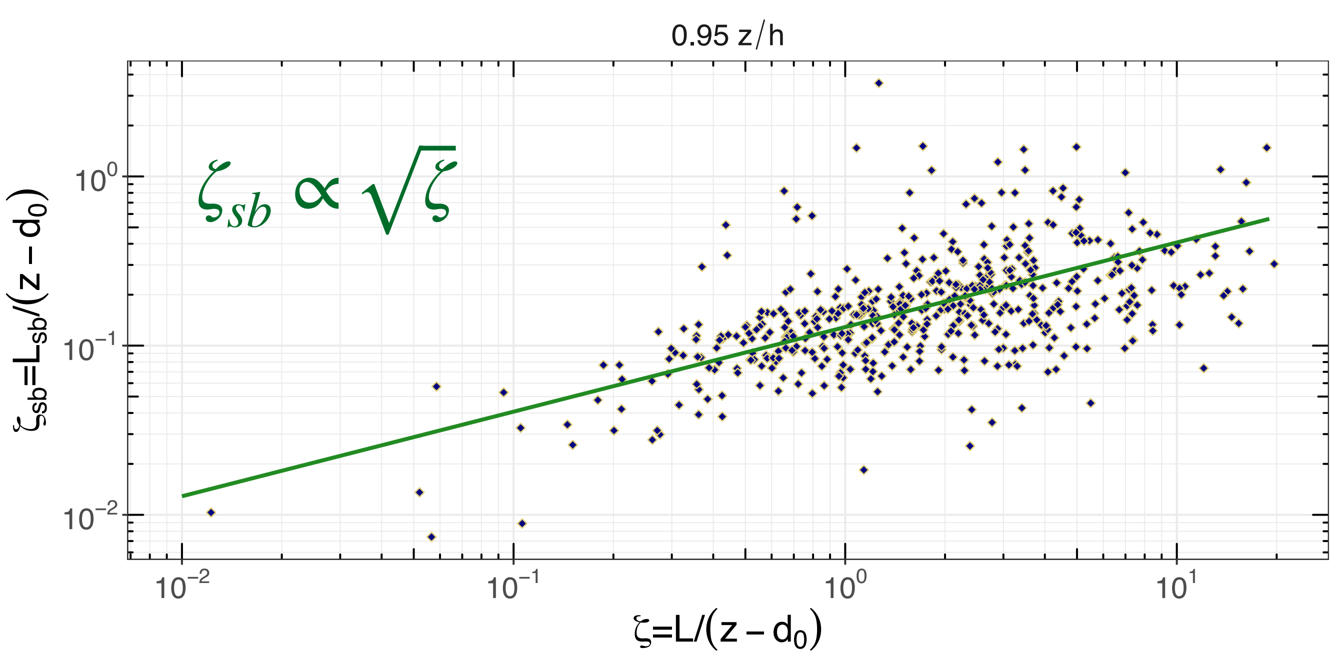
I) Gradient-diffusion break down:

$$\overline{w'u'} = -\nu_T\frac{dU}{dz} + \left[\frac{\tau}{A}\beta_o\overline{u'\theta_v'}\right] \qquad \nu_T = A^{-1}(\tau\sigma_w^2) = A^{-1}\int \tau(k)E_{ww}(k)dk$$

This cannot be attributed to the flux transport terms.
Instead, it is entirely driven by thermal stratification.

II) Emergence of a length scale that reflects the contribution of mechanical production and buoyancy production or destruction terms to the turbulent stress budget

$$L_{sb} = \frac{\sigma_w^2 u_*}{\kappa\beta_o\overline{u'\theta_v'}}\phi_m(\zeta)$$



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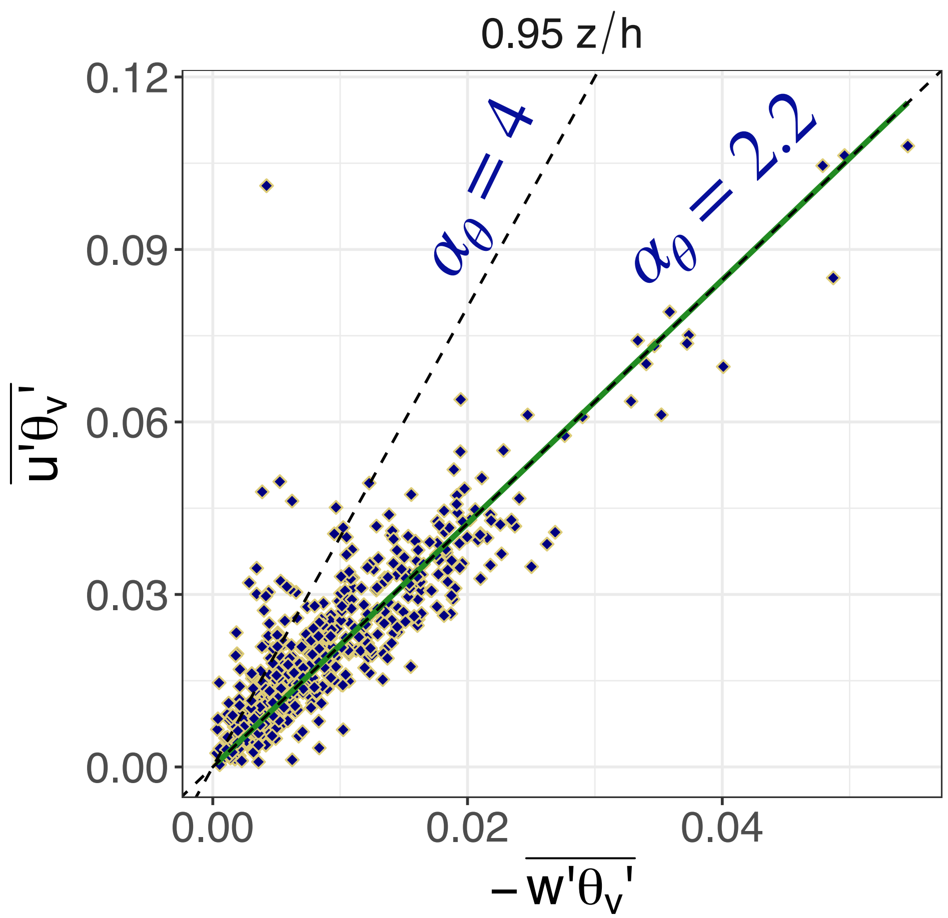
$$\overline{w'u'} = \frac{\tau}{A}\left[-\sigma_w^2\frac{dU}{dz} + \frac{g}{\bar{\theta}_v}\overline{u'\theta_v'}\right] \qquad -F_{wu}(k) = \alpha A^{-1}\epsilon^{-1/3}k^{-2/3}\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\bar{\theta}_v}F_{u\theta}(k)\right)$$

III) Maximum sustainable heat flux in stable stratification:

Assuming a negative momentum flux and $\overline{u'\theta'} = -\,\alpha_\theta\overline{w'\theta'}$ results in

$$-\sigma_w^2\frac{dU}{dz} + \frac{g}{\bar{\theta}_v}\overline{u'\theta_v'} \leq 0 \longrightarrow \overline{u'\theta'} \leq \frac{\bar{\theta}_v}{g}\,\sigma_w^2\frac{dU}{dz} \longrightarrow \overline{w'\theta'} \geq -\frac{1}{\alpha_\theta}\frac{\bar{\theta}_v}{g}\,\sigma_w^2\frac{dU}{dz}$$



The concept of maximum sustainable heat flux was introduced by Van de Wiel et al. (2007, 2012a, b). Here, this flux is:

$$\overline{w'\theta'}_{max} = -\frac{1}{\alpha_\theta}\frac{\bar{\theta}_v}{g}\,\sigma_w^2\frac{dU}{dz}$$

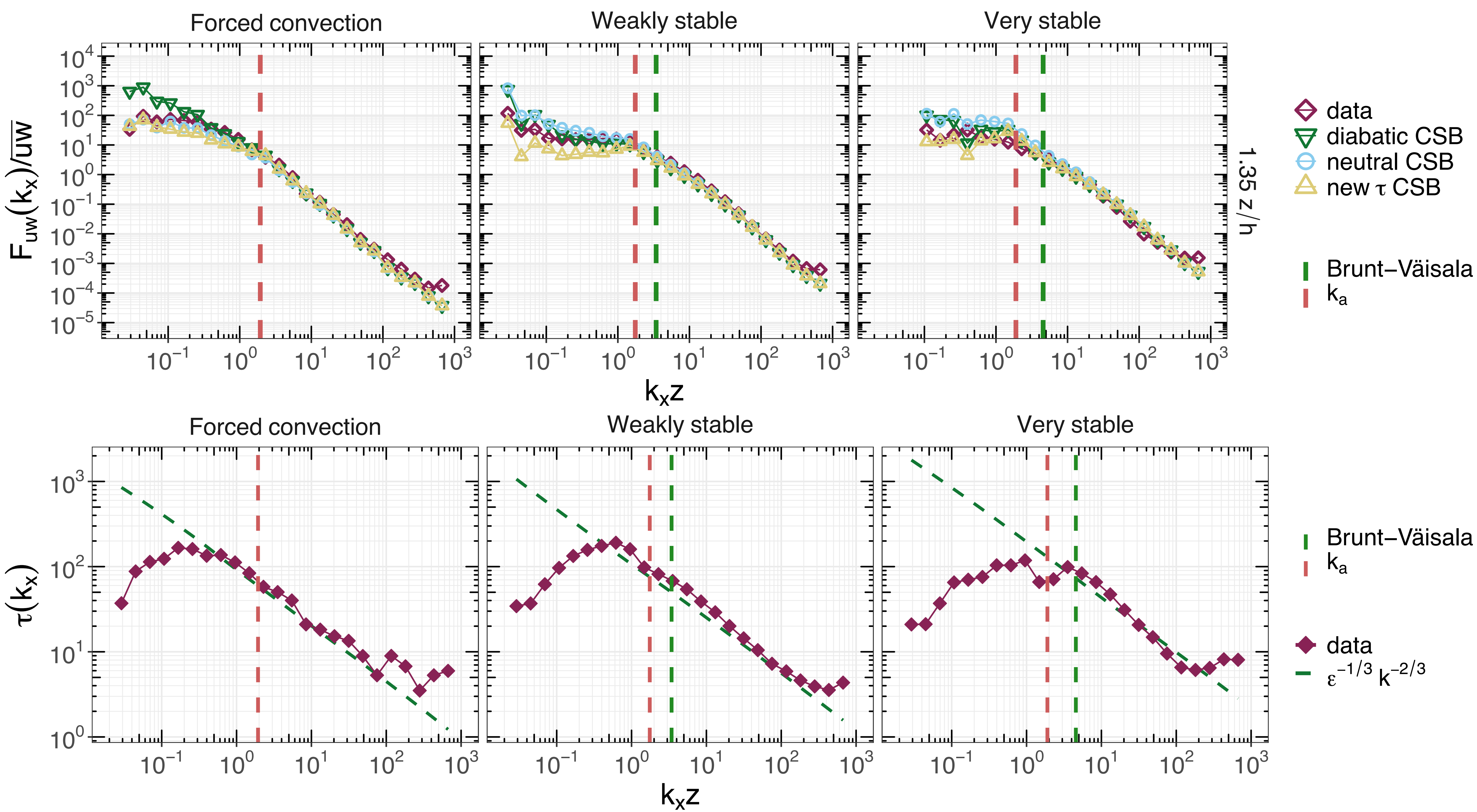
And represents the maximum heat flux the flow can provide for a given shear.

The de-correlation time-scale

$$\tau(k_x) = -\frac{AF_{wu}(k_x)}{\Gamma(z)E_{ww}(k_x) - \frac{g}{\bar{\theta}_v}F_{u\theta_v}(k_x)}$$

$$\tau(k_x) = \epsilon^{-1/3}k_x^{-2/3} \quad \forall k_x$$

$$\begin{cases} \tau(k_x) = \epsilon^{-1/3}k_a^{-2/3} & k_x \leq k_a \\ \tau(k_x) = \epsilon^{-1/3}k_x^{-2/3} & k_x > k_a \end{cases}$$



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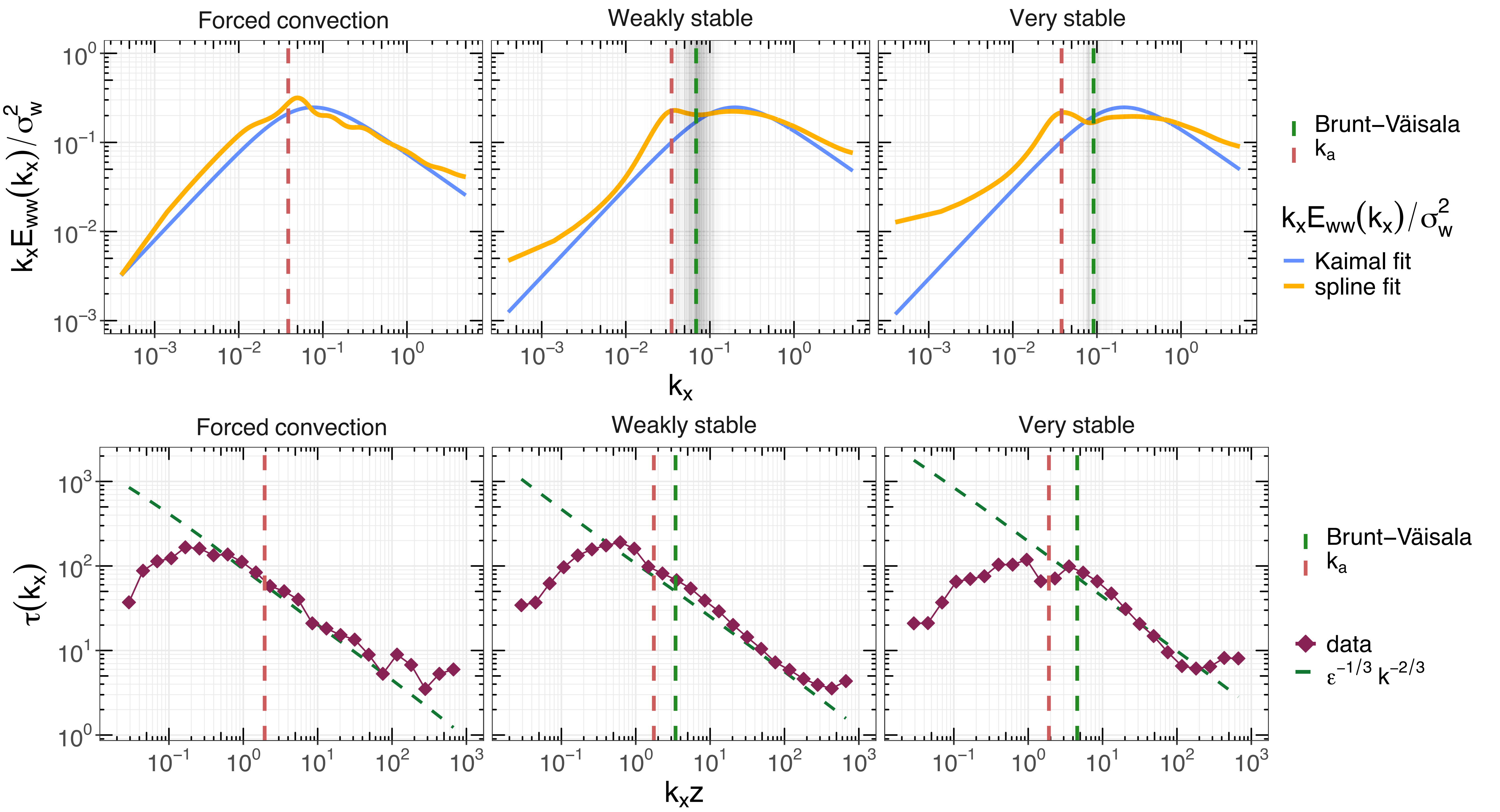

Conclusions

1. Newly proposed CSB model shows that thermal stratification on the turbulent momentum flux is **direct** and **indirect**:

Direct: through the longitudinal heat flux

Indirect: through the effects of thermal stratification on the vertical velocity energy spectrum, the mean velocity gradient, and the turbulent kinetic energy dissipation rate through a de-correlation time.

2. In presence of buoyancy $\overline{w'u'}$ may no longer be explained by the mean velocity gradient and gradient-diffusion breaks down.

3. The analysis shows that stability characterization is not only through the Obukhov length. The latter was developed from considerations of the turbulent kinetic energy budget, not the turbulent stress budget.

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Thank you!

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