

Constraints on cosmological parameters and CMB first acoustic peak in conformal Killing gravity

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In the frame of conformal Killing gravity cosmology, we performed a Bayesian analysis on two different datasets of Baryon Acoustic oscillations (DESI and SDSS DR16), two datasets of SNeIa (Pantheon+ and Union3), and using the Cosmic Microwave Background (CMB) Planck likelihood. The results for H_0 and Ω_M in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) background are consistent with the Λ CDM scenario. We obtain a non-negligible negative value for the novel density of dark sector, Ω_D , and its relevance in the evolution of the cosmological observables, thus finding quantitatively what its contribution is on real data to match the standard scenario. The results confirm the dynamical character of dark energy. We also calculate the deceleration parameter q_0 and the present time dark energy equation of state parameter w_0 : the latter belongs to the quintessence regime. The evaluation of the first acoustic peak of CMB places it near to the best value provided by the Planck collaboration. In this scenario, we can conclude that late time and early time data can be successfully matched under the same standard.

Keywords: Conformal Killing gravity, Friedmann equations, cosmological parameters, dynamical dark energy, first acoustic peak of CMB.

I. INTRODUCTION

In the last decades General Relativity (GR) and deep space surveys have radically changed our understanding and view of the Universe. Since its beginning, GR has predicted and has been confirmed in crucial experiments: from early ones, such as the precession anomaly of Mercury, the bending of light, gravitational redshift, to recent ones, as gravitational lensing [1], the black hole shadow, gravitational waves from compact mergers [2]. Despite the successes, GR encounters challenges that precision measures make compelling: the flat rotation curves of stars in galaxies and the stability of galactic clusters (the dark matter puzzle), the present accelerated phase of the Universe (the dark energy puzzle) [3, 4]. The standard cosmological model faces two mainstreams: either introduce new forms of matter and energy, or modify the gravitational field equations [5, 6].

In the first instance, the cosmological constant Λ has been the primary candidate for dark energy [7], but it meets difficulties, like the *fine-tuning problem*¹ and the *coincidence problem*² [8, 9]. For this reason, other dark

energy models have been proposed, such as quintessence [10, 11], phantom energy [12] oscillating quintom [13], ghost [14], Chaplygin gas [15].

The alternatives are the theories of modified gravity, where the shortcomings of GR in fitting astrophysical and cosmological structures require more “geometric” degrees of freedom that modify the gravitational interaction at large scale and avoid the huge, yet undetected, amount of exotic matter [16]. Some popular models are: scalar-tensor theories [17], Einstein-aether theory [18], $f(R)$ gravity [19–22], Gauss - Bonnet gravity [23, 24], mimetic gravity [25]. For reviews see [5, 26]. Dark matter and dark energy can be both investigated in $f(R)$ framework as discussed in [27, 28].

A breakthrough in the comprehension of dark energy are the results of the Dark Energy Spectroscopic Instrument (DESI) [29–31]. In particular, Ref. [30] provides the transverse comoving distances and the Hubble rates of over 6 million extragalactic objects in the redshift range $0.1 < z < 4.2$. The dark energy dynamics is tested in the flat w CDM model assuming an equation of state (EoS) for dark energy $p_D = w\mu_D$ with constant w , and in the w_0w_a CDM model, where the EoS is linearly scale-dependent: $w = w_0 + w_a(1 - a)$. The DESI data release 2 presents measures of baryon acoustic oscillation (BAO) in more than 14 million galaxies and quasars [29].

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¹ The fundamental constants match the stability range for stars and planets and for life to form.

² The dark matter and energy densities are of the same order of

magnitude.

A large literature eventually flourished, with main focus on the persistent deviations of DESI data from standard Λ CDM cosmology (see for example [32–43]). Several papers test the evolution of dark energy parametric models for $w(a)$, like the one by Chevallier-Polarski-Linder [44, 45] or by Jassal-Bagla-Padmanabhan [46], with experimental data.

In [39], the implications of BAO measurements by DESI are investigated for models with energy-momentum flow among dark matter and dark energy. In [33], a non-parametric, model independent reconstruction of the dark energy density evolution, using DESI, is performed. In [47], a generalisation of exponential $f(R)$ gravity is considered and compared with CDM model by using the latest data from DESI. The analysis suggests that the model provides much better fits than CDM model. A similar result is achieved in [48] where data seem to challenge the standard Λ CDM paradigm.

In this context, Harada [49, 50] introduced the Conformal Killing gravity (CKG) in 2023 to face the issue of dark energy. It is defined by the following field equations, that are third order in the derivatives of the metric tensor³:

$$H_{jkl} = T_{jkl} \quad (1)$$

$$H_{jkl} = \nabla_j R_{kl} + \nabla_k R_{lj} + \nabla_l R_{jk} - \frac{1}{3}(g_{kl}\nabla_j R + g_{lj}\nabla_k R + g_{jk}\nabla_l R) \quad (2)$$

$$T_{jkl} = \nabla_j T_{kl} + \nabla_k T_{lj} + \nabla_l T_{jk} - \frac{1}{6}(g_{kl}\nabla_j T + g_{jl}\nabla_k T + g_{jk}\nabla_l T) \quad (3)$$

R_{jk} is the Ricci tensor with trace R , T_{kl} is the stress-energy tensor with trace T .

The Bianchi identity $\nabla_j R^j_k = \frac{1}{2}\nabla_k R$ implies $\nabla_j T^j_k = 0$. Solutions of the Einstein equations are solutions of the new theory. The cosmological constant Λ arises as an integration constant.

Soon after, Mantica and Molinari [51] found a parametrization showing that the Harada equations are equivalent to the Einstein equations modified by a supplemental conformal Killing tensor that is divergence-free:

$$R_{kl} - \frac{1}{2}Rg_{kl} = T_{kl} + K_{kl} \quad (4)$$

$$\begin{aligned} &\nabla_j K_{kl} + \nabla_k K_{jl} + \nabla_l K_{jk} \\ &= \frac{1}{6}(g_{kl}\nabla_j K + g_{jl}\nabla_k K + g_{jk}\nabla_l K) \end{aligned} \quad (5)$$

The name of the theory is after this feature. The reformulation makes the extension of GR explicit through the conformal Killing term, that satisfies $\nabla^k K_{kl} = 0$. It appears in the equation as a source term, and suggests itself as a candidate for the dark sector.

Geometric properties of conformal Killing tensors are discussed in refs.[52].

Most of the studies in CKG are in static spacetimes. Let us recap some important ones: Barnes [53] found the most general spherically symmetric solution both in vacuum and with a Maxwell source [54]. Junior, Lobo and Rodriguez investigated regular black hole solutions [55, 56], as well as black bounce coupled to non-linear electrodynamics and scalar fields [57]. Dynamics of thin-shell wormholes made of two CKG black holes was pursued in [58].

In Ref. [59], Mantica and Molinari gave a parametrization of CKG in static spherically symmetric background, with a conformal Killing tensor of the form of energy-momentum for an anisotropic fluid. The fact that the field equations turn to second order considerably simplifies the search of solutions with respect to the original formulation. The same authors derived a Tolman-Oppenheimer-Volkoff equation for CKG in static spherically symmetric spacetimes, and a solution for a perfect fluid sphere with uniform matter density [60].

In [61] Altas and Tekin claimed that field equations based on a tensor with rank greater than 2 are problematic for the interpretation of integration constants, such as the parameter m in Schwarzschild’s metric, in terms of conserved quantities of the theory. Starting from Harada’s equations they showed that $\nabla^k \nabla^j H_{jkl} = \nabla^k \Phi_{kl}$, and argued that the current $J_k = \Phi_{kl} \xi^l$ inferred via a Killing vector is zero for Einstein spaces and for the Schwarzschild metric. In [62] the authors verified that J_k is actually nonzero for Harada’s vacuum solution and, in static spherically symmetry as well as in cosmological background, CKG is (because of parametrization) not third order [59, 60]. This permits to introduce conserved currents in CKG as standard contractions of the divergence-free stress-energy and of the conformal Killing tensors with the available Killing vectors.

It is worth mentioning the works on wave and Kundt metrics in CKG [63, 64], and the discussion of the cosmological constant [65].

Only a few papers have been devoted so far to cosmology in CKG. Vacuum cosmological solutions were discussed by Clemént and Nouicer [66] together with wormhole and black hole ones. In Ref. [51] a conformal Killing parametrization was obtained in a Robertson-Walker (RW) background: a perfect fluid K_{jl} describing the dark sector. The Friedmann equations with the Killing source gave the same forecast for the dark fluid as Harada in [50]. In particular, a Λ term arises, and the field equations are second order.

The first attempt to obtain quantitative forecasts of cosmological parameters in CKG was developed in [67]. The analytic expression of the Hubble function versus redshift $H(z)$ was fitted with cosmological data based on cosmic chronometers (CC) or CC and baryon acoustic oscillations (CC+BAO). The dark energy density and pressure were found to vary with the scale, with large dependence on the sign of Ω_D .

Similar dichotomic results were obtained for the function $\sigma_8(z)$: it was shown that the negative value of Ω_D im-

³ We use Latin indices for space-time components, and Greek indices for space components.

plies a peak of the function $g(z)$, sharper than the peak in Λ CDM, both in the range $0 < z < 1$, while $\Omega_D > 0$ gives a local plateau in $z = 0$. The equations in the linear

	Λ CDM	CKG	Λ CDM	CKG
H_0	68.16	71.42	69.83	64.17
q_0	-0.516	-0.854	-0.549	-0.130
Ω_M	0.323	0.311	0.301	0.306
Ω_Λ	0.677	0.368	0.699	1.103
Ω_D	-	+0.321	-	-0.410
Age	13.59	13.47	13.92	14.10

TABLE I. The central values of the cosmological parameters for Λ CDM and CKG evaluated in Ref.[67] by fit with CC (left) and CC+BAO (right) data. Note the opposite signs of Ω_D and the relative stability of Ω_M .

approximation for the evolution of the density contrast in a matter dominated Universe were also solved: the dark and the Λ terms gave no significant deviation from the Λ CDM results. The role of dark energy remained quite elusive.

In the present paper, we clarify this ambiguity. We use the BAO datasets from DESI DR2 and the Sloan Digital Sky Survey (SDSS) DR16 combined with the Supernovae type Ia (SNeIa) data of Pantheon+ and Union3 to fix the cosmological parameters of CKG in a spatially flat RW background. We complement these results by also including the Cosmic Microwave Background (CMB) Planck [68] likelihood, and drop the assumption of flatness for one specific computation.

The paper is organized as follows. In Section II we shortly recap the Friedmann equations as they appear in CKG. In Section III we present the datasets utilised in this work. Section IV is devoted to the methodology and the main cosmological results for the CKG parameters. In Section V some considerations are drawn for the growth of perturbations in CKG cosmology and of σ_8 . Discussions and conclusions are drawn in Section VI.

II. THE FRIEDMANN EQUATIONS IN CONFORMAL KILLING GRAVITY

The cosmological principle fixes the space-time metric as Robertson-Walker (RW)

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (6)$$

where $a(t)$ is the scale factor and $k = 0, \pm 1$ is the parameter for the space curvature. An alternative covariant characterization is through the existence of a unit timelike vector field, $u_k u^k = -1$, such that (see [69]):

$$\nabla_j u_k = H(g_{jk} + u_j u_k), \quad (7)$$

$$\nabla_j H = -\dot{H} u_j \quad (8)$$

with zero Weyl tensor.

In the coordinates (6), $H = \dot{a}/a$ is the Hubble parameter, $u^0 = -1$ and $u^\mu = 0$. The dot means $u^k \nabla_k$, i.e. a derivative in the cosmic time t (for a scalar field $f = u^k \nabla_k f = \partial_t f$).

Eq.(8) is equivalent to $R_{ij} u^j = \xi u_i$, with eigenvalue proportional to the cosmic acceleration

$$\xi = 3(H^2 + \dot{H}) = 3 \frac{\ddot{a}}{a} \quad (9)$$

In a RW spacetime the Ricci tensor and the scalar curvature are

$$R_{kl} = \frac{R - 4\xi}{3} u_k u_l + \frac{R - \xi}{3} g_{kl} \quad (10)$$

$$R = \frac{R^*}{a^2} + 6H^2 + 2\xi \quad (11)$$

$R^* = 6k$ is the curvature of the spacelike hypersurface. In a RW spacetime, the divergence-free conformal Killing tensor is found to be

$$K_{kl} = g_{kl} \left[\frac{5}{6} C a^2 - \Lambda \right] + u_k u_l \frac{C a^2}{3} \quad (12)$$

being C and Λ two integration constants [51, 67].

Remark 1 Eq.(12) is inherited from the conformal Killing symmetry of a RW spacetime. It is a matter of fact that the spacetime is endowed by a unique timelike conformal Killing vector $\xi_j = a(t) u_j$, i.e.

$$\nabla_i \xi_j + \nabla_j \xi_i = 2\dot{a} g_{ij}$$

The associated tensor $K_{ij} = \frac{1}{3} C \xi_i \xi_j + \beta(t) g_{ij}$ with arbitrary constant C and function $\beta(t)$ is, by construction, a conformal Killing tensor. The divergence-free condition imposes $\dot{\beta} = (5/3) \dot{a} \dot{a}$. Integration gives another arbitrary constant: $\beta(t) = \frac{5}{6} C a^2(t) - \Lambda$. Eq.(12) is recovered.

The perfect-fluid tensor (12) is purely geometric and supposedly describes the *dark perfect fluid*: $K_{kl} = (p_D + \mu_D) u_k u_l + p_D g_{kl}$, with dark energy density and dark pressure

$$\mu_D = -\frac{1}{2} C a^2 + \Lambda \quad (13)$$

$$p_D = +\frac{5}{6} C a^2 - \Lambda \quad (14)$$

Remarkably, in CKG, they depend on the scale (i.e. cosmic time). The EoS parameter is:

$$w_D(a) = \frac{p_D}{\mu_D} = -1 + \frac{2C a^2}{6\Lambda - 3C a^2} \quad (15)$$

While the parameterizations of $w(a)$ by Chevallier-Polarski-Linder [44, 45] or Jassal-Bagla-Padmanabhan [46] or others in literature, are dictated by convenience to mimic observed data, the expression (15) is a direct consequence of CKG.

Eqs. (10) and (12) give the perfect fluid energy-momentum tensor of ordinary matter with energy density μ and pressure p :

$$\begin{aligned} T_{kl} &= R_{kl} - \frac{1}{2}Rg_{kl} - K_{kl} \\ &= -\frac{1}{6}(R + 2\xi + 5Ca^2 - 6\Lambda)g_{kl} \\ &\quad + \frac{1}{3}(R - 4\xi - Ca^2)u_k u_l \\ &\equiv (p + \mu)u_k u_l + pg_{kl} \end{aligned} \quad (16)$$

After specifying R with (11) and ξ with (9), the Friedmann equations for CKG with ordinary matter are:

$$\mu = \frac{R^*}{2a^2} + 3H^2 + \frac{1}{2}Ca^2 - \Lambda \quad (17)$$

$$p = -\frac{R^*}{6a^2} - 3H^2 - 2\dot{H} - \frac{5}{6}Ca^2 + \Lambda \quad (18)$$

With $C = 0$ they reduce to the standard GR equations with cosmological constant Λ .

In the standard analysis, ordinary matter is composed of pressure-less dust with energy density μ_M , and radiation with $p_R = \frac{1}{3}\mu_R$. Conservation in the expanding Universe gives: $\mu_M/a^3 = \mu_{M0}/a_0^3$ and $\mu_R/a^4 = \mu_{R0}/a_0^4$. The Friedmann Eq. (17) of CKG with energy density $\mu = \mu_M + \mu_R$, the dark energy (with Λ term) and the curvature term is [67]

$$\mu_{M0} \left(\frac{a}{a_0}\right)^{-3} + \mu_{R0} \left(\frac{a}{a_0}\right)^{-4} = \frac{R^*}{2a^2} + 3H^2 + \frac{1}{2}Ca^2 - \Lambda \quad (19)$$

Divide by $3H_0^2$ (the value at scale a_0) and obtain:

$$\begin{aligned} \left(\frac{H}{H_0}\right)^2 &= \Omega_R \left(\frac{a}{a_0}\right)^{-4} + \Omega_M \left(\frac{a}{a_0}\right)^{-3} \\ &\quad + \Omega_k \left(\frac{a}{a_0}\right)^{-2} + \Omega_\Lambda + \Omega_D \left(\frac{a}{a_0}\right)^2 \end{aligned} \quad (20)$$

$$\Omega_M = \frac{\mu_{M0}}{3H_0^2}, \quad \Omega_R = \frac{\mu_{R0}}{3H_0^2}, \quad \Omega_k = -\frac{R^*}{6H_0^2 a_0^2},$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \quad \Omega_D = -\frac{Ca_0^2}{6H_0^2}$$

$$\Omega_M + \Omega_R + \Omega_k + \Omega_\Lambda + \Omega_D = 1$$

While Ω_M and Ω_R are true energy densities and are positive, Ω_Λ , Ω_k and Ω_D have geometric origin and their sign is not automatically positive.

Eq.(20), with $H = \dot{a}/a$, gives the time evolution of the scale function $a(t)$. In terms of the redshift parameter $1 + z = a_0/a$, it becomes:

$$\begin{aligned} \left(\frac{H(z)}{H_0}\right)^2 &= \Omega_R(1+z)^4 + \Omega_M(1+z)^3 \\ &\quad + \Omega_k(1+z)^2 + \Omega_\Lambda + \frac{\Omega_D}{(1+z)^2} \end{aligned} \quad (21)$$

A. Spatially flat FLRW

We consider a spatially flat FLRW space-time ($\Omega_k = 0$), and neglect the vanishing contribution of radiation in the late Universe. The relevant terms in the equation for $H(z)$ are

$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_M(1+z)^3 + \Omega_\Lambda + \frac{\Omega_D}{(1+z)^2} \quad (22)$$

Besides H one may consider the cosmographic parameters “deceleration” q , “jerk” j and “snap” s (see [70]) through the expansion

$$\begin{aligned} \frac{a(t)}{a_0} &= 1 + H_0(t - t_0) - \frac{H_0^2}{2}q_0(t - t_0)^2 \\ &\quad + \frac{H_0^3}{3!}j_0(t - t_0)^3 + \frac{H_0^4}{4!}s_0(t - t_0)^4 + \dots \end{aligned} \quad (23)$$

The deceleration is $q = -a\ddot{a}/\dot{a}^2 = -1 - (\dot{H}/H^2)$. With $\dot{H} = (dH/dz)\dot{z}$ and $\dot{z} = -H(z+1)$, it is

$$q(z) = \frac{\Omega_M(1+z)^5 - 2\Omega_\Lambda(1+z)^2 - 4\Omega_D}{2\Omega_M(1+z)^5 + 2\Omega_\Lambda(1+z)^2 + 2\Omega_D} \quad (24)$$

Remark 2 The contraction $R_{ij}u^i u^j = -\xi = -3\ddot{a}/a = 3H^2q$ is related to the “strong energy condition”, which is the physical requirement

$$(T_{ij} - \frac{1}{2}Tg_{ij})u^i u^j \geq 0$$

For a perfect fluid it is $3p + \mu \geq 0$.

If $R_{ij} = T_{ij} - \frac{1}{2}Tg_{ij}$, the condition $3H^2q \geq 0$ contradicts the present time acceleration. This is overturned in standard cosmology by the introduction of the cosmological constant, with negative pressure.

In CKG cosmology the source is $T_{ij} + K_{ij}$. Considering radiation, dust, the dark fluid, and $R^* = 0$, the parameter q is eq.(24). In Table IV we show that q_0 is negative for all datasets.

The “weak energy condition” $T_{ij}u^i u^j \geq 0$ means that the energy density T^{00} is positive in the comoving frame. Neglecting radiation it is $\mu_M + \mu_D \geq 0$ i.e.

$$\Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_D(1+z)^{-2} \geq 0$$

By eq.(22) this is the requirement $H(z)^2 \geq 0$. The appearance of a zero $H(z) = 0$ is discussed in the next remark.

The “lookback time” of a photon emitted at time t_e at scale a_e , and received at time t_0 at scale a_0 , is:

$$t_L = \int_{t_e}^{t_0} dt = \int_{a_e}^{a_0} \frac{da}{\dot{a}} = \int_0^{z_e} \frac{dz}{H(z)(1+z)} \quad (25)$$

The age of the Universe is the limit (whenever it exists) $z \rightarrow +\infty$ of the previous expression.

The dark energy density (13) and the dark pressure (14) versus redshift are

$$\begin{aligned}\mu_D &= 3H_0^2 \left[\Omega_\Lambda + \frac{\Omega_D}{(1+z)^2} \right] \\ p_D &= -3H_0^2 \left[\Omega_\Lambda + \frac{5\Omega_D}{3(1+z)^2} \right]\end{aligned}\quad (26)$$

with EoS parameter (15)

$$w_D(z) = -1 - \frac{2}{3} \frac{\Omega_D}{\Omega_D + \Omega_\Lambda (1+z)^2} \quad (27)$$

The first order expansion in the scale parameter determines the empirical coefficients of the Chevallier-Polarski-Linder model [44, 45]:

$$\begin{aligned}w_D(a) &= w_{D0} + w_{Da}(1-a) \\ &= -\frac{1}{3} \frac{5\Omega_D + 3\Omega_\Lambda}{\Omega_\Lambda + \Omega_D} + \frac{4}{3} \frac{\Omega_D \Omega_\Lambda}{(\Omega_D + \Omega_\Lambda)^2} (1-a).\end{aligned}\quad (28)$$

See Table IV for the numerical values.

Remark 3 *The equations show a limit value $z = -1$, at which p_D and μ_D diverge with ratio $w_D(-1) = -\frac{5}{3}$.*

However, with $\Omega_D/\Omega_\Lambda < 0$ there is another barrier: the ratio $(H/H_0)^2$ in Eq.(21) becomes zero for a critical value z_c that solves the quintic equation

$$\frac{\Omega_M}{\Omega_\Lambda} (1+z_c)^5 + (1+z_c)^2 + \frac{\Omega_D}{\Omega_\Lambda} = 0. \quad (29)$$

The critical value z_c is evaluated and discussed in Sect.IV B.

III. OBSERVATIONAL DATA AND STATISTICAL ANALYSIS

In this section, we detail the datasets used in our analysis as well as the analytical inference of fundamental cosmological quantities using the CKG model.

A. Supernovae type Ia

SN Ia are a specific type of supernova explosion occurring in binary systems after the accretion of mass of a white dwarf from its companion star. Given their intrinsic mechanisms, it is possible to infer their intrinsic luminosity (and thus their absolute magnitude M) from the shape of the detected light curve. For this reason, they are considered standard candles and are of fundamental importance in observational cosmology [4].

The difference $m - M$ among the observed and the absolute magnitudes is related to the luminosity distance D_L

(in Mpc) by the relation $m - M = 5 \log_{10} D_L + 25$. The luminosity distance is defined as

$$D_L(z) = \frac{(1+z)}{H_0 \sqrt{|\Omega_k|}} S_k \left(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z')/H_0} \right) \quad (30)$$

$$S_k(x) = \begin{cases} \sin x & k = 1 \\ x & k = 0 \\ \sinh x & k = -1 \end{cases} \quad (31)$$

B. The Pantheon+ dataset

Refs.[71, 72] present one of the most recent compilations of spectroscopically confirmed type Ia Supernovae. It contains 1701 light curves of 1550 SNe Ia in the redshift range $0 < z < 2.3$. It is worth noticing that 77 of them are in galaxies that host Cepheids in the range $0.00122 < z < 0.01682$ (see [42]). It comprises 18 different samples, loosely defined as data sets produced by single SNe surveys in definite periods of time. This sample presents distance moduli $\mu = M - m$ anchored to the Classical Cepheids of SH0ES collaboration [73], as well as magnitudes already corrected for the Tripp [74] formula.

C. The Union3 dataset

This compilation is a comprehensive dataset of 2087 Type Ia Supernovae from 24 samples in the range $0.01 < z < 2.26$, reported in [75]. Up to the time of this analysis, the complete dataset has not yet been released. Only the 22 bins representative of the entire catalog have been used in our analysis.

D. BAO & DESI

In the early Universe before decoupling, the density fluctuations of baryonic matter produced acoustic waves traveling in the primordial plasma. These are the baryonic acoustic oscillations (BAO). The maximum distance covered from the Big Bang to the time of decoupling (drag epoch) is the *comoving sound horizon* at the drag epoch r_d :

$$r_d = \int_{z_d}^{+\infty} \frac{c_s(z') dz'}{H(z')}. \quad (32)$$

Here, $c_s(z)$ is the speed of sound in the photon-baryon fluid, and $z_d \approx 1080$ (the last scattering surface). Before the last scattering it is

$$c_s(z) = \frac{c}{\sqrt{3}} \frac{1}{\sqrt{1 + \frac{3\mu_b}{4\mu_R(1+z)}}}$$

where μ_b is the baryonic energy density. The results of the Planck collaboration [68] give $r_d = (147.21 \pm$

0.48) Mpc. The maximum distance r_d travelled by these acoustic waves left an imprint which manifests in correlations on the large scale structure of the Universe. It can be measured to infer the cosmological parameters. In this case, the BAO can be considered distance rulers. For instance, the BAO measurements from DESI DR2 [29] are described by the following quantities:

- the transverse comoving distance D_M/r_d , with

$$D_M(z) = c \int_0^z \frac{dz'}{H(z')}$$

- the Hubble horizon D_H/r_d

$$D_H(z) = \frac{c}{H(z)}$$

- the angle averaged distance D_V/r_d

$$D_V(z) = (z D_M(z)^2 D_H(z))^{1/3}$$

These measures target tracers of bright galaxy samples, luminous red galaxies, quasars, and the Lyman- α forest.

E. SDSS DR16

The 16th Data release from the SDSS contains the extended Baryon Oscillation Spectroscopic Survey (eBOSS) data [76], which includes all data from eBOSS and its predecessor, the Baryonic Oscillation Spectroscopic Survey. The BAO scale is measured both in the autocorrelation of Lyman- α absorption and in its cross-correlation with 341,468 quasars with redshift $z_q > 1.77$.

F. CMB

The CMB data used in our analysis come from the Planck Collaboration [68]. Planck was a full-sky mission that in 4 years mapped the CMB temperature and polarization power spectra with a precision never achieved before, looking at the anisotropies in these spectra. The latest 2018 data release not only provides data but also built-in likelihoods to be used for cosmological calculations. They already address possible systematics like foreground effects and use a plethora of nuisance parameters for the marginalization of the proper, cosmological ones.

G. Sound horizon and CMB first acoustic peak in CKG cosmology

The value of comoving sound horizon r_d in Eq.(32) strongly depends upon the early time expansion rate. For $z > 1080$, in a spatially flat spacetime, the terms Ω_Λ

and $\frac{\Omega_D}{(1+z)^2}$ in Eq. (21) are negligible, while radiation and matter dominate:

$$\left(\frac{H(z)}{H_0}\right)^2 \approx \Omega_R(1+z)^4 + \Omega_M(1+z)^3 \quad (33)$$

The radiation energy density $\Omega_R = \Omega_\gamma + \Omega_\nu$ is the same of the Λ CDM model.

The CMB term Ω_γ is evaluated with the present CMB temperature $T_0 = 2.7255 K$ so that $\Omega_\gamma = 5.45 \times 10^{-5}$.

Ω_ν is the neutrino term

$$\Omega_\nu = \frac{7}{8} N_\nu^{eff} (4/11)^{4/3} \Omega_\gamma = 0.2271 N_\nu^{eff} \Omega_\gamma. \quad (34)$$

where the effective number of relativistic species is fixed to $N_\nu^{eff} = 3.046$ in the standard cosmological model (see [77, 79, 80]). Then $\Omega_\nu = 0.6917 \Omega_\gamma$ giving

$$\Omega_R = 9.22 \cdot 10^{-5}$$

With this estimate, for $z > 1080$, it is $\Omega_R(1+z)^4 > 1.25 \cdot 10^8$ and, with $\Omega_M \approx 0.3$, it is $\Omega_M(1+z)^3 > 3.77 \cdot 10^8$. Then the dark sector is completely negligible and the comoving sound horizon can be evaluated with the formula 2 in [29].

The first acoustic peak is found at the angular scale (see [68] Sec. 3.1)

$$\theta_s = \frac{r_d}{D_M(z_d)} \quad (35)$$

The value of $D_M(z_d)$ depends strongly upon the late time expansion rate, where radiation is negligible and the Hubble parameter evolves as Eq.(22). Now Ω_D and Ω_Λ are the relevant parameters with Ω_M :

$$D_M(z_d) = \frac{c}{H_0} \int_0^{z_d} \frac{dz}{\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda + \frac{\Omega_D}{(1+z)^2}}} \quad (36)$$

The Planck collaboration results [68] give $100 \theta_s = 1.04097 \pm 0.00046$. It is observed at the corresponding multipole

$$\ell \simeq \frac{\pi}{\theta_s} \quad (37)$$

Measurements of the first acoustic peak can be found in [81, 82]

IV. COSMOLOGICAL RESULTS

In this section, we detail the Bayesian methodology followed in our analysis and the main cosmological results.

A. Methodology

We used a Bayesian approach to derive the posterior probabilities for the cosmological quantities studied in

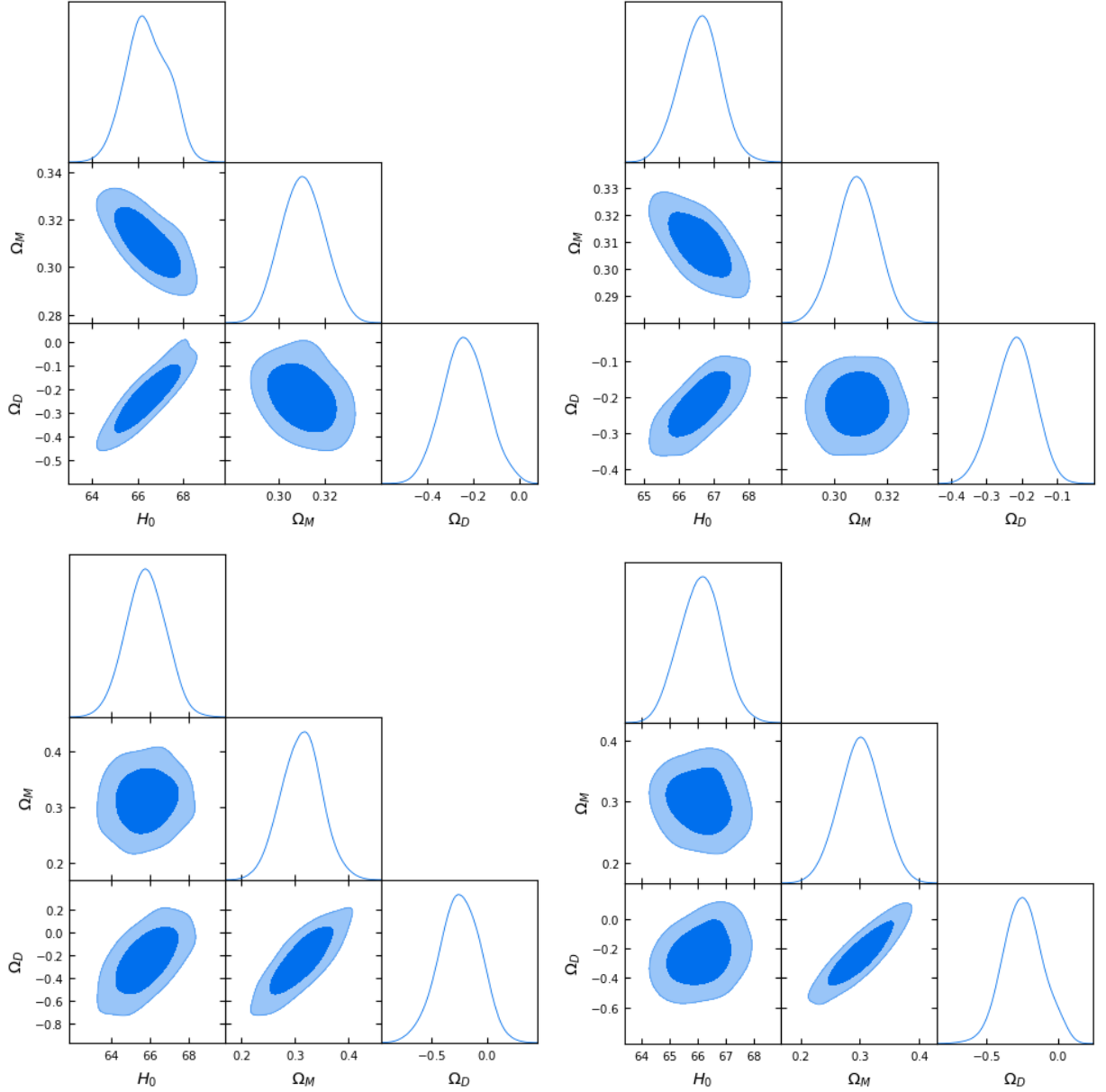


FIG. 1. Results of the cosmological analysis: DESI+Union3 (top left panel); DESI+Pantheon+ (top right panel); DR16+Union3 (bottom left panel); DR16+Pantheon+ (bottom right panel). The contours represent the 68% and 95% probability levels, respectively.

our analysis: H_0 , Ω_M , and Ω_D . We fixed $\Omega_\Lambda = 1 - \Omega_M - \Omega_D$ and assumed spatial flatness ($\Omega_k = 0$).

Regarding the SNeIa, we used a marginalized chi-squared function to break the degeneracy between H_0 and the absolute magnitude M . We followed the method outlined in [83, 84]. This is defined as

$$\chi_{\text{SN}}^2 = \Delta^T \mathbf{C}^{-1} \Delta - \frac{[\mathbf{1}^T \mathbf{C}^{-1} \Delta]^2}{\mathbf{1}^T \mathbf{C}^{-1} \mathbf{1}}, \quad (38)$$

where \mathbf{C}^{-1} is the inverse of the covariance matrix for the SNeIa datasets, and

$$\Delta = m_B^{\text{corr}} - \mu^{\text{th}} \quad (39)$$

Here, m_B^{corr} is the standardized observed magnitude, and μ^{th} is the theoretical distance modulus computed from the cosmological model:

$$\mu^{\text{th}} = 5 \log_{10} \left(\frac{D_L}{\text{Mpc}} \right) + 25.$$

The difference Δ contains an unknown additive offset $\mathcal{M} = M_B - 5 \log_{10} H_0 + 25$ which is analytically marginalized in the likelihood.

Regarding the BAO, we fixed $r_d = 147.5$ Mpc, being consistent with the Planck results, and used the D_M and D_H values of the DR16 dataset. In analogy with Eq.(38)

for SNeIa, we introduce a χ_{BAO}^2 .

The chi-squared functions for SNeIa and BAO are summed and used for the Bayesian computation. For this purpose, we used COBAYA [85], a code for Bayesian analysis tailored for cosmological computations, which contains both the datasets used in our analysis as well as the Markov Chain Monte Carlo (MCMC) sampler for the cosmological posterior contours.

For the analysis which includes the CMB, we used two of the official Planck 2018 likelihoods in COBAYA: the low Temperature-temperature (TT) for low coefficients of the power spectra (i.e. the ones associated with the largest angular scales), and the Plik TTTEEE-lite for the rest. We used the lite likelihood because it automatically marginalizes over the nuisance parameters, allowing us to derive only the cosmological ones for our tests. For the Theory part of the code, i.e. the one properly computing the spectra starting from the likelihood and the cosmological parameters, we used CLASS [86], a Boltzmann solver able to derive the anisotropies. For the CMB computations, we fixed all the other fundamental cosmological quantities to marginalize only Ω_M , H_0 , and Ω_D as in the computations using SNeIa and BAO. The CMB was used in conjunction with the two other probes to test the reliability of the results, as well as a case where the flatness condition is not set a priori, as we will show.

B. Results

We combined the two datasets of BAO and SNeIa in four different ways and derived the posterior for the cosmological parameters. The main results are shown in Fig. 1, while the mean and one-dimensional standard deviations are shown in Table II.

The results obtained using also the CMB are shown in Fig. 2 and in Table III. For these, the SNeIa and BAO sets used are the Pantheon+ and DESI, respectively.

Datasets	H_0	Ω_M	Ω_D
DESI+Union3	66.44 ± 0.93	0.310 ± 0.009	-0.235 ± 0.095
DESI+Pantheon+	66.61 ± 0.58	0.309 ± 0.008	-0.221 ± 0.057
DR16+Union3	65.77 ± 1.04	0.311 ± 0.037	-0.25 ± 0.19
DR16+Pantheon+	66.11 ± 0.74	0.301 ± 0.036	-0.243 ± 0.14

TABLE II. One-dimensional mean and standard deviation for the cosmological parameters derived for the CKG cosmology using 4 different combinations of the SNeIa and BAO datasets.

The results obtained for all the cosmological parameters are each other consistent within 1σ in all the considered combinations of datasets.

The values for Ω_M and H_0 that were obtained for the CKG cosmological model, are consistent with the results achieved by the Planck collaboration [68]: this is an important consistency test confirming the reliability of the results.

By comparing the contours in Fig. 1, it is worth noticing that their shapes depend on the combination of datasets. Ω_D and Ω_M correlate strongly for the sets DR16+Union3 and DR16+Pantheon+, but they do not show the same correlation for results involving DESI. Although this can be due to the fact that more data are needed in both cases to achieve stable shapes of contours, this is still an interesting difference worth mentioning.

The results for Ω_D are consistent with what found in [67] with the CC+BAO set, presumably because BAO datasets have also been used in the present work. Specifically, we note that Ω_D is always negative, at least for more than 2σ levels according to our results. Given that we fixed $\Omega_\Lambda = 1 - \Omega_M - \Omega_D$, the ratio Ω_D/Ω_Λ is negative for all our results, thus the discussion in Sec. II applies.

Case	H_0	Ω_M	Ω_D	Ω_Λ
Flat	66.53 ± 0.57	0.321 ± 0.007	-0.101 ± 0.038	.
Not-Flat	66.74 ± 0.53	0.321 ± 0.009	-0.131 ± 0.050	0.808 ± 0.049

TABLE III. One-dimensional mean and standard deviation for the cosmological parameters derived for the CKG cosmology using CMB and the SNeIa and BAO datasets Pantheon+ and DESI.

We now describe the results obtained using also the CMB in conjunction with SNeIa and BAO. As previously mentioned, the computations have been performed using the CLASS theory code. From its point of view, adding the CKG contribution with Ω_D is equivalent to add a second cosmological fluid with $w = -5/3$, that acts together with the standard one of the Λ CDM model, for which we recall $w = -1$. A $w = -5/3$ corresponds to a phantom dark energy ($w < -1$), which in our case complements the standard Equation of State evolution. It is interesting to note that, for high values of the redshift, the phantom contribution becomes negligible with respect to the usual term, while its relative contribution grows with time according to its dependence upon the scale factor.

On the left panel of Fig. 2, we show the results obtained by varying H_0 , Ω_M , and Ω_D as in the previous computations. We note how the values for Ω_M and H_0 are consistent with those obtained using only SNeIa and BAO. We also note that Ω_D is centered around -0.1 , thus still being negative but closer, albeit significantly different, to 0. We also note how adding the CMB allows us to deduce more clearly the various correlations between parameters. In particular, we note the clear anti-correlation between Ω_M and Ω_D , and the correlation between H_0 and Ω_D .

In the right panel of Fig. 2, we drop the flatness assumption we used up to now and derive Ω_Λ alongside the other cosmological parameters. This has been done by imposing $\Omega_K = 1 - \Omega_M - \Omega_D - \Omega_\Lambda$. We note how the results are remarkably consistent with those obtained by imposing flatness, which is important because it means that, if we derive Ω_K from our computations, we recover the flatness even if we do not impose it. Indeed, by com-

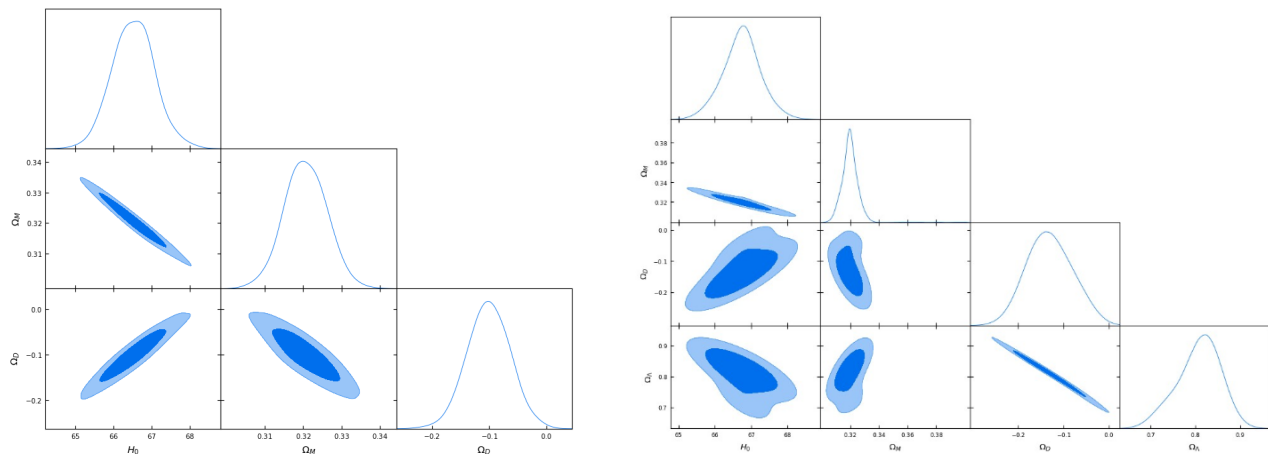


FIG. 2. Results of the cosmological analysis combining DESI+PantheonPlus+CMB: keeping Ω_Λ fixed (left panel), and letting it vary (right panel). The contours represent the 68% and 95% probability levels, respectively.

puting Ω_K from the mean values of Ω_M , Ω_D , and Ω_Λ shown in Tab. III, we get $\Omega_K = 0.002$, remarkably consistent with a flat universe in the error range. This is an important reliability check for the observational results of CKG cosmology.

In solving Eq.(29) for z_c with the results obtained with the DESI+Union3 datasets, we find $z_c = -0.510$. If we apply the same conditions to the dark energy and pressure, eq.(26), we find

$$\begin{aligned} \mu_D > 0 & \quad \text{if } z > -1 + \sqrt{|\Omega_D/\Omega_\Lambda|} \approx -0.496, \\ p_D > 0 & \quad \text{if } z > -1 + \sqrt{\frac{5}{3}|\Omega_D/\Omega_\Lambda|} \approx -0.349. \end{aligned}$$

Table IV reports the calculated values of the deceleration parameter q_0 (24) with $z = 0$, the parameters w_{D0} and w_{Da} in the linear approximation (28) for the EoS parameter of the dark energy, the first acoustic peak θ_s of CMB in (35), and the ages of the Universe evaluated with the central values of Table II. For θ_s we used the value of comoving sound horizon $r_d = 147.50$ Mpc as fixed by DESI DR2 in [29], and the value $z_d = 1080$ in computing the integral (36).

The results show that the deceleration parameter q_0 is greater than the one of Λ CDM model, and the present value of the dark energy EoS parameter w_0 belongs to the quintessence regime. The values w_{Da} for the linear approximation may be compared for example with the corresponding ones of the CPL parametrization in [29] (Sect VII A) or in [38]. The calculated ages of the Universe are fully compatible with Planck results.

The values of the first acoustic peak in Table IV are very close to the best value provided by [68]. In our opinion, this feature is very important because our analysis, despite considering for this comparison only the datasets gathered at low and intermediate redshifts (SNeIa and BAO), is in agreement with CMB data, in particular with the Planck data release. In other words,

we need not shape the dark sector, and CKG cosmology seems fully predictive in matching early and late Universe dynamics.

Datasets	q_0	w_{D0}	w_{Da}	$100\theta_s$	Age (Gy)
DESI+Union3	-0.300	-0.773	-0.609	1.0454	13.78
DESI+Pantheon+	-0.315	-0.787	-0.563	1.0460	13.77
DR16+Union3	-0.283	-0.758	-0.657	1.0368	13.89
DR16+Pantheon+	-0.305	-0.768	-0.625	1.0284	13.95

TABLE IV. The deceleration parameter (24) at $z = 0$, the parameters w_{D0} and w_{Da} in (28), the angle θ_s in (35), the age of the Universe, evaluated at the central values of Table II.

Datasets	q_0	w_{D0}	w_{Da}	Age (Gy)
Flat	-0.417	-0.901	-0.228	13.79
non-flat	-0.385	-0.871	-0.308	13.70

TABLE V. The deceleration parameter (24) at $z = 0$, the parameters w_{D0} and w_{Da} in (28), the age of the Universe, evaluated at the central values of Table III.

V. GROWTH OF PERTURBATIONS IN CKG COSMOLOGY

In this section we investigate the equations governing the growth of perturbations, in the framework of spherical collapse illustrated by Abramo *et al.* [87]. The procedure has been used in several theories of extended gravity [88–92].

In a spatially flat FLRW Universe with a pressure-less dust, the Λ and the dark terms, a local perturbation of the matter density changes the background value μ_M to $\mu_M(1 + \delta_M)$. The parameter δ_M is the “density contrast” [93]. Its evolution was derived in [67], in the linear ap-

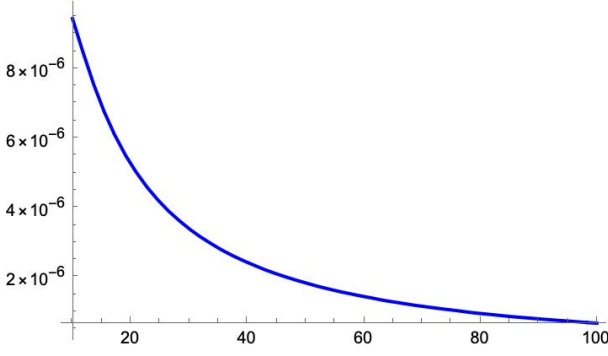


FIG. 3. The deviation of the density contrast for CKG from the values in GR, $\delta_M(z) - \delta_{M,GR}(z)$. The first one is the numerical solution of (40), the GR function is $c_2/(1+z)$. Both are evaluated for $z > 10$, with initial conditions $\delta_M(400) = 0.0001$ and $\delta'_m(400) = -\delta_M(400)/401$.

proximation:

$$0 = \frac{d^2 \delta_M}{dz^2} (1+z)^2 + \frac{1}{2} \frac{d\delta_M}{dz} (1+z) \frac{\Omega_M(1+z)^5 - 2\Omega_\Lambda(1+z)^2 - 4\Omega_D}{\Omega_M(1+z)^5 + \Omega_\Lambda(1+z)^2 + \Omega_D} - \frac{\delta_M}{2} \frac{3\Omega_M(1+z)^5 + 8\Omega_D}{\Omega_M(1+z)^5 + \Omega_\Lambda(1+z)^2 + \Omega_D} \quad (40)$$

The following analytic solutions were studied:

- 1) $\Omega_D = \Omega_\Lambda = 0$ (GR evolution):

$$\delta_{M,GR}(z) = c_1(1+z)^{3/2} + c_2(1+z)^{-1} \quad (41)$$

The constants c_1 and c_2 are determined by initial conditions at a reference redshift $z_i \gg 1$ (matter dominated era). The requirement that the fluctuation is small and the derivative is negative and small (initial growth of structures) rules out c_1 as unphysical and poses the ‘‘adiabatic condition’’

$$\delta'_M(z_i) = -\frac{\delta_M(z_i)}{1+z_i} \quad (42)$$

that is used as a criterion to fix the constants in more general conditions.

- 2) $\Omega_\Lambda \neq 0, \Omega_D = 0$ (Λ CDM, Peebles [93], Martel [94]). With $\alpha = \Omega_M/\Omega_\Lambda$, matter dominance means $\Omega_M(1+z)^3 \gg \Omega_\Lambda$ i.e. $\alpha(1+z)^3 \gg 1$. The admissible solution is

$$\delta_M(z) = c_3 \frac{1}{1+z} {}_2F_1\left(1, \frac{1}{3}; \frac{11}{6}; -\frac{1}{\alpha(1+z)^3}\right) \quad (43)$$

In the dominant matter regime the hypergeometric function is around unity i.e. the Λ CDM density contrast becomes GR for large z . For large z_i it fulfills the adiabatic condition (42).

- 3) CKG with $\Omega_\Lambda \neq 0, \Omega_M + \Omega_\Lambda + \Omega_D = 1$.

Equation (40) is solved numerically with NDSolve of Mathematica13. The solution does not show significative difference with the expanding mode of GR in the matter-dominated phase (see Fig.3).

A robust measurable quantity in red-shift surveys, that is related to the density contrast is the product $f(z)\sigma_8(z)$ [95] (for a simple account of the theory see [96]). $\sigma_8(z)$ is the root mean square mass fluctuation at the scale $R_8 = 8h^{-1}\text{Mpc}$:

$$\sigma_8^2(z) = 4\pi \int_0^\infty k^2 dk \widetilde{W}^2(k) P_{\delta_M}(k, z)$$

In the integral, $P_{\delta_M}(k, z)$ is the mass power spectrum, i.e. the Fourier transform of the correlator $\langle \delta_M(\mathbf{x})\delta_M(\mathbf{x}') \rangle$ at redshift z . $\widetilde{W}(k)$ is the Fourier transform of a window function $W(r)$ that filters the spatial scale; a choice is $W(r) = 3/(4\pi R_8^3)$ if $r < R_8$ and $W(r) = 0$ if $r > R_8$.

The tension in $\sigma_8 = \sigma_8(0)$ is the discrepancy between the values measured in the late Universe, that are smaller than the values found in CMB (early Universe). Planck data [68] give $\sigma_8 = 0.8120 \pm 0.0073$. A complete survey of σ_8 tension is contained in [97], Sec. 3.1, together with a wide list of measures of σ_8 (Table 2 and fig. 21).

f is the growth rate of linear perturbations:

$$f = \frac{d \log \delta_M}{d \log a} = -\frac{1+z}{\delta_M(z)} \frac{d\delta_M}{dz},$$

In the linear perturbation regime, $\delta_M(z)$ and $\sigma_8(z)$ are proportional (see Eq.2.21 of [98]). Then, we consider the function

$$g(z) = \frac{f(z)\sigma_8(z)}{\sigma_8(0)} = -\frac{1+z}{\delta_M(0)} \frac{d\delta_M(z)}{dz} \quad (44)$$

Experimental measures of $f(z)\sigma_8(z) \propto g(z)$ show a weak maximum, as in Fig.1 of [99].

The function is plotted for Λ CDM and CKG in Fig.4. Both curves show a maximum. The maximum occurs in a redshift range that is borderline of the dominant matter era, wherein the linear evolution equations for $\delta_M(z)$ were obtained. Since the factors $\delta_M(0)$ for Λ CDM and CKG are very close (respectively 0.0316 and 0.0295 with the data of Fig.3), the two curves can be compared in the same plot Fig.4.

The function $g(z)$ and datasets for $f(z)\sigma_8(z)$ allow in principle a determination of σ_8 in CKG.

According to the linear theory, Eq.(44), the ratio $f(z)\sigma_8(z)/g(z)$ is constant and equal to σ_8 . We join two datasets: Benisty (Table 1 with 47 points in [99]) and Kazantzidis & Perilovopoulos (Table II with 63 points in [100]), and evaluate $g(z)$ at the same points with (44). Different points yield different ratios; the best fitting values σ_8 (omitting error bar analysis of input data) are: $\sigma_8 = 0.775$ (Λ CDM) and $\sigma_8 = 0.843$. With these data, the plots for $g(z)$ are rescaled to obtain plots for $f(z)\sigma_8(z)$, that are shown with experimental data in Fig.5.

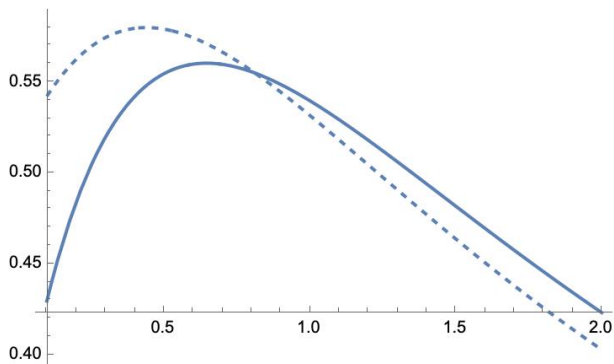


FIG. 4. The function $g(z)$, Eq.(44), for Λ CDM (dashed) and CKT (full).

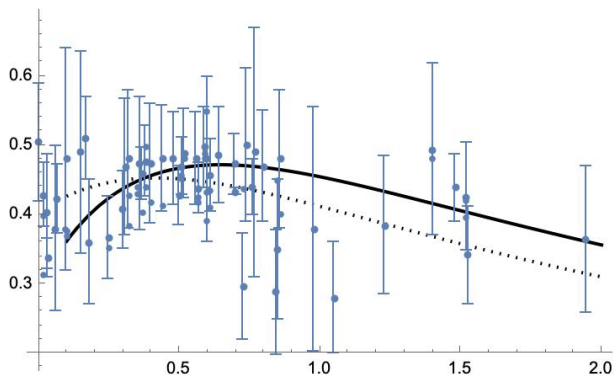


FIG. 5. The function $f(z)\sigma_8(z)$ for Λ CDM (dotted) and CKG (full), with data points. The error bars are from Table 1 by Benisty, ref.[99].

VI. DISCUSSION AND CONCLUSIONS

In this paper we present posterior results for the cosmology derived from CKG with a MCMC methodology and different combinations of cosmological datasets, namely BAO, SNeIa, and the CMB.

We first considered a flat FLRW spacetime, with parameters H_0 , Ω_R , Ω_M , and the novel dark parameter Ω_D coming from the CKG cosmology. The results are consistent with the standard scenario for H_0 and Ω_M .

We obtain negative values for Ω_D in all the analysed cases. This is consistent with the previous study on

CKG cosmology [67], where a negative value for Ω_D was found using CC+BAO and a qualitative analysis of σ_8 was done. We also complemented our analysis with one case where we drop the a priori flatness assumption, but we recover it from the results, passing another important test for the CKG cosmology.

A consequence of $\Omega_D < 0$ is the existence of a negative critical value for the redshift z_c that nullifies the ratio H/H_0 and thus the second Friedmann equation.

The evaluation of the present time deceleration parameter and the dark energy EoS parameter confirm a quintessence regime.

The lookback time is used to evaluate the age of the Universe, and confirms Planck results.

Finally, the first acoustic peak of the CMB is estimated, with remarkable proximity to the best value of the Planck collaboration.

The CKG evolution of the density contrast $\delta_M(z)$ in the linear regime shows no sensible deviation in the matter dominated-regime from the Λ CDM or GR solutions. The CKG curve for $f(z)\sigma_8(z)$ shows a maximum that would not properly occur with $\Omega_D > 0$, in qualitative agreement with datasets.

The present results call for a thorough investigation of the peak of σ_8 and its deviation from Λ CDM.

In conclusion, it appears that CKG might consistently explain the cosmic expansion up to the last scattering surface. In a forthcoming paper, we will further detail the model by considering more datasets.

DATA AVAILABILITY STATEMENT

All data used in this work are openly available in Refs.[29–31, 68, 71, 72, 75, 76].

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