Brief Announcement: Optimal Uniform Circle Formation by Asynchronous Luminous Robots

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Abstract

We study the Uniform Circle Formation (UCF) problem for a swarm of n autonomous mobile robots operating in *Look-Compute-Move* (LCM) cycles on the Euclidean plane. We assume our robots are *luminous*, i.e. equipped with a persistent light that can assume a color chosen from a fixed palette, and *opaque*, i.e. not able to see beyond a collinear robot. Robots are said to *collide* if they share positions or their paths intersect within concurrent LCM cycles. To solve UCF, a swarm of n robots must autonomously arrange themselves so that each robot occupies a vertex of the same regular n-gon not fixed in advance. In terms of efficiency, the goal is to design an algorithm that optimizes (or provides a tradeoff between) two fundamental performance metrics: *(i)* the execution time and *(ii)* the size of the color palette.

In this paper, we develop a deterministic algorithm solving UCF avoiding collisions in $O(1)$-time with $O(1)$ colors under the asynchronous scheduler, which is asymptotically optimal with respect to both time and number of colors used, the first such result. Furthermore, the algorithm proposed here minimizes for the first time what we call the *computational SEC*, i.e. the smallest circular area where robots operate throughout the whole algorithm.

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1 Introduction

The *Look-Compute-Move* (LCM) model [13, 14] is a theoretical model used to study swarms of mobile robots and design distributed algorithms for solving collaborative problems for such systems. Robots are idle by default, but they can be activated by a scheduler. When a robot is activated, it performs an LCM cycle: it first obtains a snapshot of its surroundings (*Look*), then computes the new destination based on the snapshot (*Compute*), and finally moves straight to the computed destination (*Move*). After that, the robot becomes idle again. The scheduler can be fully synchronous ($\mathcal{FSYNC}$), semi-synchronous ($\mathcal{SSYNC}$), or asynchronous ($\mathcal{ASYNC}$). Most of the literature considers very simple and limited robots: they are assumed to be punctiform agents that can operate in the Euclidean plane, *autonomous* (no external control), *anonymous* (no internal identifiers), *indistinguishable* (no external identifiers), *homogeneous* (execute the same algorithm), and *disoriented* robots (each robot has its local coordinate system without any assumption of global orientation).

Table 1 Existing UCF deterministic solutions for $n \geq 1$ luminous-opaque robots on the plane, avoiding collisions. $x \in [1, O(\log\log n)]$.

Algorithm	Time (in epochs)	Number of Colors	Computational SEC	Scheduler
[8]	$O(1)$	$O(1)$	Not minimized	$\mathcal{FSYNC}$
[11]	$O(1)$	$O(1)$	Not minimized	$\mathcal{SSYNC}$
[11]	$O(n)$	$O(1)$	Not minimized	$\mathcal{ASYNC}$
[9]	$O(\log n)$	$O(1)$	Not minimized	
Generic [20]	$O(x)$	$O\left(n^{1/2^x}\right)$	Not minimized	
OptTime [20]	$O(1)$	$O(\sqrt{n})$	Not minimized	
OptColor [20]	$O(\log\log n)$	$O(1)$	Not minimized	
OptTime&Color (this paper)	$O(1)$	$O(1)$	Minimized	

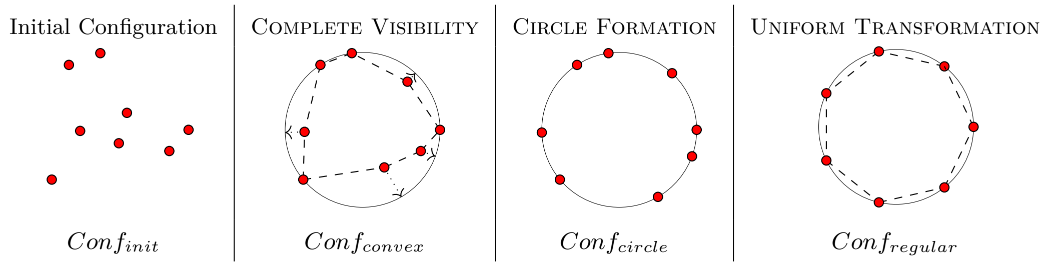
In this work, we consider *opaque* robots [1, 7, 8, 9, 20, 22, 23] thus they experience *obstructed visibility* in case of collinearities (if robots a, b, c are collinear, then a and c cannot see each other). To cope with this restrictive condition, we assume each robot is equipped with a light whose color can be updated at the beginning of its *Move* step choosing it from a fixed palette and persists until its next update. Since such a light is visible to both the robot itself and the other robots, the *luminous model* [3, 9, 16, 20, 21, 22, 23] grants robots both a persistent internal state (memory) and a direct communication means with other robots. Except for lights, robots have no other persistent memory or communication means. We say that two robots *collide* if either *(i)* they share the same position at a given time or *(ii)* their paths towards their destinations intersect within concurrent LCM cycles. We assume that our robots do not tolerate collisions and that robot movements are *rigid*, i.e., in each *Move*, the robot stops only after reaching its computed destination.

Contributions. We consider the Uniform Circle Formation (UCF) problem [4, 5, 6, 15, 17, 18, 19, 24]: starting from an arbitrary configuration where n robots lie on distinct points on a plane, robots must autonomously arrange themselves to form a regular n-gon, independently of its position, orientation, and scale. We propose a deterministic algorithm solving UCF in the luminous-opaque model under $\mathcal{ASYNC}$, avoiding collisions. Our algorithm runs in $O(1)$ time using a $O(1)$-size palette, and it minimizes a spatial metric that we call *computational SEC*, i.e. the smallest circle containing all the points the robots touch during the execution of the algorithm. Note that forcing the swarm to act within the circular area delimited by their initial configuration may represent a realistic requirement in critical scenarios (e.g. lack of space or no guarantee about the safety of the space around robots). Previous works [8, 9, 10, 11, 20] have investigated UCF under the same model: their results are summarized in Table 1 in comparison with our contribution.

Challenge and techniques. The main challenge of this work was to make robots exploit parallelism (thus achieving a $O(1)$ runtime) even in conditions of asynchrony and obstructed visibility, always keeping the size of the color palette constant and avoiding collisions among robots. For this purpose, the key techniques adopted along our algorithm include the arrangement of robots along an (inner) circle in a mirror-symmetric pattern, the Beacon Directed Curve Positioning procedure [22], and a novel *rank encoding* technique (existing techniques in [2, 9] do not fit our assumptions and requirements).

2 Algorithm Overview

Table 2 Sub-problems composing Uniform Circle Formation.



Let $Conf_{init}$ be an arbitrary initial configuration of n robots on distinct points on $\mathbb{R}^2$, all with the same color `off`. Given a configuration $Conf$, we indicate with $SEC(Conf)$ the *smallest circle enclosing* all the robots in $Conf$. Our algorithm ensures that any robot acts within the circular area delimited by $SEC(Conf_{init})$, thus minimizing the computational SEC. We now provide an overview of the different phases and procedures composing our algorithm which transforms $Conf_{init}$ into a regular configuration $Conf_{regular}$ (see Figure 1). Such procedures work in $O(1)$ time and use a $O(1)$-size palette of colors.

We factorize UCF into three sub-problems (see Table 2): *(i)* Complete Visibility, *(ii)* Circle Formation, and lastly *(iii)* Uniform Transformation. Starting from $Conf_{init}$, we *(i)* exploit the Complete Visibility solution in [22] to arrange robots on the vertices of a convex polygon, forming the configuration $Conf_{convex}$. After that, *(ii)* a simple procedure safely transforms $Conf_{convex}$ into $Conf_{circle}$ where all the robots lie on $SEC(Conf_{convex})$. From now on, let us call this circle as Cir: no robot will move out from Cir. Step *(iii)* aims to equally distribute the robots on the perimeter of Cir, thus solving UCF.

Our Uniform Transformation solution entails a different algorithmic approach according to the geometric properties of $Conf_{circle}$. Specifically, we classify $Conf_{circle}$ into three categories: $Conf_{regular}$ (where robots already form a regular polygon, so they do nothing), $Conf_{biangular}$ (biangular configuration presented in [5], where there exist two different angles α, β such that each robot forms a central angle α with one of its two adjacent robots and β with the other one), and $Conf_{periodic}$. $Conf_{biangular}$ (see Figure 2a) can be converted into a regular configuration through a similar approach to the strategy introduced in [5]: our approach guarantees robots to minimize the computational SEC. The most challenging case is the periodic configuration $Conf_{periodic}$, for which we developed a sequence of multi-step procedures to form the target regular polygon, as depicted in Figure 1.

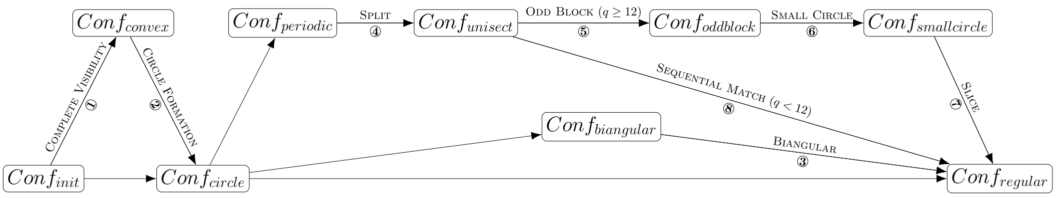
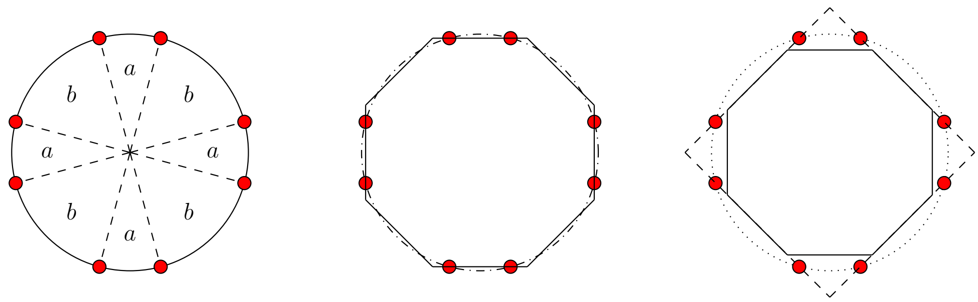


Figure 1 Transition diagram among configurations while solving UCF. The arrows without numbering denote a transition with only color change (no robot moves). The parameter q is the number of robots in each uniform sector of $Conf_{unisect}$.

3 Uniform Transformation

Biangular case

We propose a new approach to transform $Conf_{biangular}$ into $Conf_{regular}$, taking inspiration from [5]. Let P be the n-gon formed by the robots in $Conf_{biangular}$. In [5], robots spot the target regular n-gon P' which encloses P, such that robots lie on alternative edges of P', and then slide on the edges of P' until they stop on its vertices (Figure 2b). This simple approach however does not guarantee to minimize the computational SEC. Thus, we make robots spot a n-gon P'' *inscribed* in P, so that robots can slide on the larger edges of P until they reach the vertices of P'', without moving outside $SEC(Conf_{biangular})$ (Figure 2c).



(a) $Conf_{biangular}$. **(b)** Regular n-gon P' [5]. **(c)** Regular n-gon P''.

Figure 2 Arrangement of $Conf_{biangular}$ in a regular n-gon.

Periodic case

We propose a sequence of procedures to transform $Conf_{periodic}$ into $Conf_{regular}$. In $Conf_{periodic}$, all the n robots non-uniformly lie on the same circle Cir, in a periodic pattern[1], without forming a biangular configuration.

Procedure Split. $Conf_{periodic}$ is partitioned into $k \geq 2$ circular sectors $\Upsilon_0, \ldots, \Upsilon_{k-1}$ such that *(i)* they have the same arc length and *(ii)* they are size-balanced (i.e. containing the same number of robots), and *(iii)* they are chiral (i.e. the robots are arranged in an asymmetric pattern along the arc of each Υ_i). The number of sectors k depends on the degree of periodicity of $Conf_{periodic}$. We call such sectors as *uniform sectors*. Within each Υ_i, some robots will be elected as *leaders* to fix its boundaries B_i, B_{i+1}, whereas two other robots (`left`- and `right`-colored) will be elected and made to move to fix the chirality of Υ_i. Let q be the number of robots inside each Υ_i (except for its boundaries). From now on, each group of q robots works independently and in parallel within each Υ_i. The next procedures aim to uniformly arrange the q robots of each Υ_i along the arc of Υ_i, in order to cover the vertices of the target regular n-gon (also called *uniform positions*).

Procedure Sequential Match. This procedure is executed if $q < 12$, i.e., the number of robots is relatively small compared to the number of robots involved along the other procedures of the algorithm. In this case, we adopt a sequential schema to make robots reach their uniform positions along the arc of Υ_i. Specifically, following the orientation of Υ_i, robots reach their target vertex in turn.

[1] We consider an asymmetric configuration as a special case of $Conf_{periodic}$.

Procedure Odd Block. This procedure (and the following ones) is executed if $q \geq 12$. Within each sector Υ_i, two robots are elected as *guards* to fix the boundaries and chirality of a structure called *odd block*. An odd block for Υ_i is a circular sector completely contained in Υ_i, having the same origin and radius as Υ_i. Moreover, the arc of the odd block contains an odd number of uniform positions. Let $\mathcal{L}$ be the chord joining the left guard (`blockL`-colored) with the right guard (`blockR`-colored) of the odd block. One robot is elected as the *median* robot and reaches the midpoint of the arc cut by $\mathcal{L}$. The other robots on the sector arc of Υ_i are now moved to the chord $\mathcal{L}$ by implementing the Beacon Directed Curve Positioning strategy (BDCP) [22], setting their color as `chord`. See Figure 3.

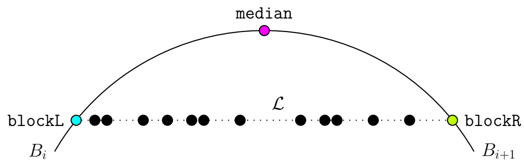


Figure 3 Odd block built inside Υ_i, delimited by the guards `blockL` and `blockR`. All the robots of the sector (except for the `median` one) have migrated to the block chord $\mathcal{L}$.

Procedure Small Circle. Let $\mathcal{C}$ be the circle within the odd block such that it passes through the median robot and such that $\mathcal{L}$ becomes its tangent. This procedure aims to place all the `chord` robots on $\mathcal{L}$ on the two halves of $\mathcal{C}$, $\mathcal{C}_w$ and $\mathcal{C}_e$, in perfect mirror-symmetry. Firstly, all the `chord` robots reach $\mathcal{C}$ traveling along the trajectories connecting their initial position on $\mathcal{L}$ with the median robot. After that, all the robots on $\mathcal{C}_w$ migrate towards $\mathcal{C}_e$ by implementing BDCP. Eventually, the robots on $\mathcal{C}_e$ split into two equal groups, and one of the groups comes back to $\mathcal{C}_w$ forming a mirror symmetric configuration on $\mathcal{C}$. See Figure 4.

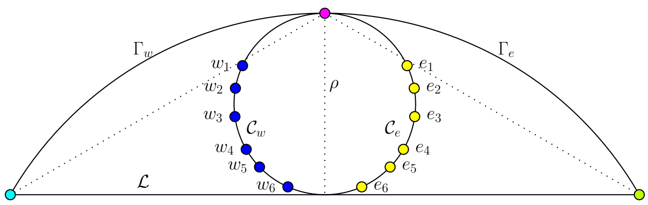


Figure 4 The small circle $\mathcal{C}$ built inside the odd block. All the robots (except for the block guards) are equally distributed on $\mathcal{C}_w$ and $\mathcal{C}_e$.

Procedure Slice. Let ρ be the diameter of $\mathcal{C}$ passing through the median robot. Let Γ_w and Γ_e be the left and right halves of the odd-block arc, cut by ρ. This procedure aims to uniformly arrange robots from $\mathcal{C}$ on the arc of the odd block. We now use a strategy to provide a rank to the robots on $\mathcal{C}_e$ ($\mathcal{C}_w$, resp.) so that a robot with rank j moves to the j-th uniform position on Γ_e (Γ_w, resp.). In particular, the robots on $\mathcal{C}_w$ move to new positions on $\mathcal{C}_w$ to encode their rank using the angular distance with a fixed robot. Thus, the robots from $\mathcal{C}_e$ can obtain their ranks using the $\mathcal{C}_w$ group as a reference. Then, the robots of each group (first the right one, then the left one) migrate on ρ on their projections, then they recompute their rank and reach their target vertices on Γ_e and Γ_w. See Figure 5.

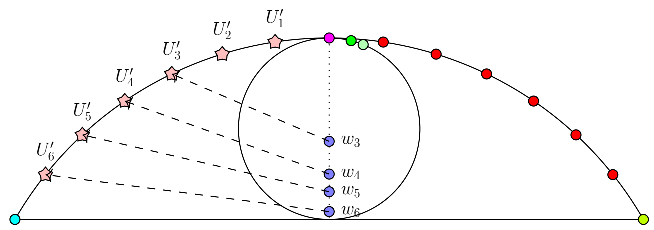


Figure 5 Each robot on ρ uses the two robots on $\mathcal{C}_e$ (here *green*) to recompute its rank j and its target uniform position U'_j.

After each uniform sector Υ_i completes the algorithm, the n robots are equally distributed on Cir, thus solving UCF. Theorem 1 summarizes our result.

▶ **Theorem 1** (Uniform Circle Formation). *Given any $Conf_{init}$ of n `off`-colored robots on distinct points on a plane, the robots reposition to $Conf_{regular}$ solving UCF in $O(1)$ epochs using $O(1)$ colors under $\mathcal{ASYNC}$, avoiding collisions, always performing within $SEC(Conf_{init})$.*

As a corollary, our UCF algorithm asymptotically optimizes both the computational time (number of epochs) and the size of the palette (number of colors), and minimizes the computational SEC.

References

1 Kaustav Bose, Manash Kumar Kundu, Ranendu Adhikary, and Buddhadeb Sau. Arbitrary pattern formation by asynchronous opaque robots with lights. *Theor. Comput. Sci.*, 849:138–158, 2021. doi:10.1016/J.TCS.2020.10.015.

2 Quentin Bramas, Anissa Lamani, and Sébastien Tixeuil. Stand up indulgent gathering. *Theoretical Computer Science*, 939:63–77, 2023. doi:10.1016/j.tcs.2022.10.015.

3 Shantanu Das, Paola Flocchini, Giuseppe Prencipe, Nicola Santoro, and Masafumi Yamashita. Autonomous mobile robots with lights. *Theor. Comput. Sci.*, 609:171–184, 2016. doi:10.1016/J.TCS.2015.09.018.

4 Yoann Dieudonne, Ouiddad Labbani-Igbida, and Franck Petit. Circle formation of weak mobile robots. *ACM Transactions on Autonomous and Adaptive Systems*, 3, August 2006. doi:10.1145/1452001.1452006.

5 Yoann Dieudonné and Franck Petit. Swing words to make circle formation quiescent. In *SIROCCO*, volume 4474, pages 166–179, 2007. doi:10.1007/978-3-540-72951-8_14.

6 Yoann Dieudonné and Franck Petit. Squaring the circle with weak mobile robots. In *ISAAC*, pages 354–365, 2008. doi:10.1007/978-3-540-92182-0_33.

7 Caterina Feletti, Lucia Mambretti, Carlo Mereghetti, and Beatrice Palano. Computational power of opaque robots. In *3rd Symposium on Algorithmic Foundations of Dynamic Networks, SAND 2024*, volume 292, pages 13:1–13:19, 2024. doi:10.4230/LIPICS.SAND.2024.13.

8 Caterina Feletti, Carlo Mereghetti, and Beatrice Palano. Uniform circle formation for swarms of opaque robots with lights. In *20th International Symposium on Stabilization, Safety, and Security of Distributed Systems, SSS*, volume 11201 of *Lecture Notes in Computer Science*, pages 317–332. Springer, 2018. doi:10.1007/978-3-030-03232-6_21.

9 Caterina Feletti, Carlo Mereghetti, and Beatrice Palano. O(log n)-time uniform circle formation for asynchronous opaque luminous robots. In *27th International Conference on Principles of Distributed Systems, OPODIS*, volume 286 of *LIPIcs*, pages 5:1–5:21, 2023. doi:10.4230/LIPICS.OPODIS.2023.5.

10 Caterina Feletti, Carlo Mereghetti, and Beatrice Palano. Uniform Circle Formation for Fully, Semi-, and Asynchronous Opaque Robots with Lights. *Applied Sciences*, 13(13):7991, 2023. doi:10.3390/app13137991.

11 Caterina Feletti, Carlo Mereghetti, Beatrice Palano, and Priscilla Raucci. Uniform circle formation for fully semi-, and asynchronous opaque robots with lights. In *23rd Italian Conference on Theoretical Computer Science, ICTCS 2022*, volume 3284, pages 207–221, 2022. URL: https://ceur-ws.org/Vol-3284/8511.pdf.

12 Caterina Feletti, Debasish Pattanayak, and Gokarna Sharma. Optimal uniform circle formation by asynchronous luminous robots. *CoRR*, abs/2405.06617, 2024. doi:10.48550/arXiv.2405.06617.

13 Paola Flocchini, Giuseppe Prencipe, and Nicola Santoro. *Distributed Computing by Oblivious Mobile Robots*. Synthesis Lectures on Distributed Computing Theory. Morgan & Claypool Publishers, 2012. doi:10.2200/S00440ED1V01Y201208DCT010.

14 Paola Flocchini, Giuseppe Prencipe, and Nicola Santoro, editors. *Distributed Computing by Mobile Entities, Current Research in Moving and Computing*, volume 11340 of *Lecture Notes in Computer Science*. Springer, 2019. doi:10.1007/978-3-030-11072-7.

15 Paola Flocchini, Giuseppe Prencipe, Nicola Santoro, and Giovanni Viglietta. Distributed computing by mobile robots: uniform circle formation. *Distributed Comput.*, 30(6):413–457, 2017. doi:10.1007/S00446-016-0291-X.

16 Giuseppe Antonio Di Luna, Paola Flocchini, Sruti Gan Chaudhuri, Nicola Santoro, and Giovanni Viglietta. Robots with lights: Overcoming obstructed visibility without colliding. In Pascal Felber and Vijay K. Garg, editors, *16th International Symposium on Stabilization, Safety, and Security of Distributed Systems, SSS 2014*, volume 8756 of *Lecture Notes in Computer Science*, pages 150–164. Springer, 2014. doi:10.1007/978-3-319-11764-5_11.

17 Marcello Mamino and Giovanni Viglietta. Square formation by asynchronous oblivious robots. In *CCCG*, pages 1–6. Simon Fraser University, Vancouver, British Columbia, Canada, 2016.

18 Moumita Mondal and Sruti Gan Chaudhuri. Uniform circle formation by mobile robots. In *ICDCN*, pages 20:1–20:2. ACM, 2018. doi:10.1145/3170521.3170541.

19 Moumita Mondal and Sruti Gan Chaudhuri. Uniform circle formation by fat robots under non-uniform visibility ranges. In *ICDCN*, pages 58:1–58:5. ACM, 2020. doi:10.1145/3369740.3372779.

20 Debasish Pattanayak and Gokarna Sharma. Time-color tradeoff on uniform circle formation by asynchronous robots. In *IEEE International Parallel and Distributed Processing Symposium, IPDPS 2024*, pages 987–997. IEEE, 2024. doi:10.1109/IPDPS57955.2024.00092.

21 David Peleg. Distributed coordination algorithms for mobile robot swarms: New directions and challenges. In *IWDC*, pages 1–12, 2005. doi:10.1007/11603771_1.

22 Gokarna Sharma, Ramachandran Vaidyanathan, and Jerry L. Trahan. Constant-time complete visibility for asynchronous robots with lights. In Paul G. Spirakis and Philippas Tsigas, editors, *19th International Symposium on Stabilization, Safety, and Security of Distributed Systems, SSS 2017*, volume 10616 of *Lecture Notes in Computer Science*, pages 265–281. Springer, 2017. doi:10.1007/978-3-319-69084-1_18.

23 Gokarna Sharma, Ramachandran Vaidyanathan, Jerry L. Trahan, Costas Busch, and Suresh Rai. O(log n)-time complete visibility for asynchronous robots with lights. In *IEEE International Parallel and Distributed Processing Symposium, IPDPS 2017*, pages 513–522. IEEE Computer Society, 2017. doi:10.1109/IPDPS.2017.51.

24 Giovanni Viglietta. Uniform circle formation. *Chapter 5 of [14]*, pages 83–108, 2019. doi:10.1007/978-3-030-11072-7_5.