A Network-Based Economic Growth Model with Endogenous Migration and Poverty Traps

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Abstract

We analyze a network-based macroeconomic framework with the objective to analyze the effects that endogenous migration choices may have on the mutual relation between population dynamics and capital accumulation. In our economy population size determines the labor input which, together with the available capital stock, shapes total output. Production takes place with a convex-concave technology allowing for a poverty trap. Migration depends on the origin-destination income differential and affects the fertility rate. Thus population growth ultimately turns out to be endogenously dependent upon economic conditions. Such feedback effects between population and capital dynamics give rise to possible heterogeneity in the patterns of economic development, allowing to explain the large variability in the level of development between regions we generally observe at world level. We show that a higher degree of economic interaction improves economic outcomes at global level by allowing poor economies to escape their poverty trap, suggesting thus that promoting the formation of tight relations between countries may be an important policy option to favor economic development.

Keywords: Economic Growth, Migration, Network-Based Analysis, Population Dynamics
JEL Classification: C60, J10, O40

1 Introduction

Along with the birth– and the death– rate, the migration–rate represents the last of the three components of demographic change. Migration (generally conceived as a move from an origin to a destination place) is probably even more important than fertility and mortality as a possible source of population growth because, unlike the first two, it is not a single unique event in time and space, but can repeat itself over the lifetime of an individual. The global migration system has recently changed with regard to the origins and destinations, as well as the volume and types of migrants. Countries that once were origins of migration have in the meanwhile become destinations of migrants, and vice versa. The shift of Europe from a major area of emigration (primarily towards the Americas and Australasia) to a major area of immigration over the course of the twentieth century is perhaps the most striking historical example in this regard. At the beginning of the twentieth century, indeed, a million migrants a year were leaving Europe mainly for Northern America (Hatton and Williamson, 2005). In 2010, the European Union absorbed 1.2 million 'permanent' migrants, more than the number of permanent migrants to the United States (OECD, 2012). The shift of Europe to net immigration was first evident in the 1950s in an area extending northwards and eastwards from France and Switzerland. By the early twenty-first century, most of the countries of southern and western Europe, as well as Turkey, had become areas of net immigration. Only a small number of countries extending from the Baltic Republics to the northern frontier of Greece experienced net emigration, which was mostly directed to western and southern Europe. In the light of these numbers and facts, the twenty-first century can be considered as 'the age of migration' (Castles and Miller, 2009), essentially because there are many more migrants in the world today than ever before. What is worth noting is also the fact that, while

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the proportion of the world's population identified as international migrants by the United Nations has not changed significantly since 1995 (at around 3The economic literature that studies the factors affecting the individual choice to migrate is extremely large and varied (Massey et al., 1993; Ghatak et al., 1996; Greenwood, 1997; Borjas, 1999; Bodvarsson and Van den Berg, 2009; for the most recent treatment of the determinants of the migration decision, see Kennan and Walker, 2013; Constant and Zimmermann, 2013, and Zaiceva and Zimmermann, 2014). In an attempt to summarize such a boundless bibliography, it is fair to conclude that wages– and employment–differentials between sending and receiving countries are now considered among the main economic reasons behind the decision to migrate. In this regard, the simplest neoclassical migration model postulates that the choice to move depends mostly on the wage differences between the two regions. This model has been extended to include also the expected (rather than the actual) wage differential between the two areas, along with the probability of finding employment in the destination one, as another possible major economic determinant of international migrations (Harris and Todaro, 1970). Hence, at the aggregate level, differences in earnings (wages), in (un-)employment rates, and in costs of living are all considered as fundamental factors behind the decision to move from one place to another [1]. According to demographers, instead, international migrations are mainly determined by the stage of the demographic transition that characterizes a specific country at a given point in time. As a matter of fact, the process of demographic transition (generally associated with falling mortality rates, followed by a decline in birth rates, and resulting in population aging and in a steady reduction of the labor-force) is causing unprecedented changes in population's size and age structure almost everywhere in the world, either in sending or in receiving countries. More concretely, demographers do believe that these changes will have two important consequences in the near future. First, for geographical and historical proximity to Africa, immigration pressures will likely be higher in Europe than in any other region of the world. This means that a prevailing share of population growth in Europe could soon be driven by the migration rate [2].Second, the inflow of immigrants, who generally are younger[3] and exhibit higher fertility relative to natives [4], will affect also the age composition of receiving European countries in a way that would likely lead to an increase in the ratio of working to retired people and in a greater sustainability of the pension and the health-care systems of these countries [5]. The overall conclusions from the demographic studies are therefore

[1] The microeconomic approach to migrations (Sjaastad, 1962) emphasizes that this choice is indeed a sort of investment-decision in human capital, so underlying the importance of expected returns and costs related to the decision to move. As in any investment-decision, in fact, an individual calculates the present discounted value of the expected lifetime stream of earnings, i.e. the returns to her human capital, in sending and receiving regions, and migrates only if the net return to migration is higher at the destination.

[2] Already during the 1990s, immigrants represented about 35% of the net population growth in Germany and the UK, and about the 40% of the net population growth in Sweden and Ireland. For some countries, whose native population is declining (Italy, for example), immigration seems to be the only component able to partially offset the population fall (see Docquier et al., 2010).

[3] Numerous empirical studies confirm that age negatively affects the decision to move (see, among others, Bauer and Zimmermann, 1999, for a review; Zaiceva and Zimmermann, 2008). In fact, when entered non-linearly into the migration equation, age generally shows an inverted U-shaped profile with the maximum migration likelihood manifesting at younger ages, mostly at 20 or 30 years old. In the European context, Zaiceva and Zimmermann (2008) analyze intentions to move abroad (for individuals from both new and old EU-member-states before and after the 2004 EU enlargement) and conclude that age has a strong negative impact. Hence, the larger the share of young individuals in the source country is, and the higher emigration from that country will be for a given wage differential, net of costs (Clark et al., 2002). Hatton and Williamson (2003) analyze the elasticities of emigration out of Africa. In particular, by instrumenting the share of young adults by the proportion of the population aged 10-14 five years earlier, they suggest that a rise in the share of the African young adult population by 5% increases net out-migration by 1.3 per thousand.

[4] According to Zaiceva and Zimmermann (2014), migrants typically show higher average fertility than natives (at least at the beginning of their time in the host country). Toulemon (2004) and Milewski (2007) argue that immigrants in industrialized countries have relatively higher fertility both because they usually come from high-fertility countries and because international (female) migration is often driven by marriage and family formation/re-unification reasons.

[5] One of the very few generalizations that can be made about international migrations is that the majority of those who move are young adults (Ravenstein, 1885). So, in general, migrants tend to move from areas of higher fertility (and usually lower education) to areas of lower fertility (and higher education), i.e. from relatively poorer to relatively richer areas. For this reason, a solution to the ageing of populations in the developed world is often considered the one that encourages the immigration of workers (especially those with high skill levels) who will not only fill vacancies in the local labor markets but also contribute to the tax-revenue to support the increasing number of retired people. However, research suggests caution when the overall economic contribution of immigration to receiving countries is evaluated. First, the increase in immigration needed to compensate for the declining native working-age population is expected to be unrealistically large. Second, there may also

that: (i) The demographic structure of the population is an important determinant of migrations and has the potential to change the population dynamics in both the sending as well as the receiving countries; and (ii) Younger cohorts are generally more likely to move. All this seems consistent with what we actually observe for the US, where the composition of migrants is shifting from fewer Mexicans to more Africans or Asians, with an important part of this change ascribable to demographic factors. In the light of all the things said above, the main objective of this paper is to shed a new light on the economic consequences that international migrations may have on long-term economic growth and development. At this aim, by employing a model where the choice to migrate depends on the origin-destination income differential (a proxy for the wage differential in the long-run), we highlight the mutual relation between population dynamics and economic growth. We do so by relying on a neural-network based dynamic economic growth model à-la Solow in which population growth and capital accumulation are reciprocally related: on the one hand, population size determines the amount of the labor input available in output-production; on the other hand, the capital stock, by driving output, determines migration-flows. Output is produced through a convex-concave technology that allows for the existence of poverty traps, and such poverty traps represent the activation threshold typically employed in a neural network framework to model the output produced by a neuron. To the best of our knowledge, this is the first attempt at analyzing migration-induced economic growth by integrating the macroeconomic and the neural-network literatures. Our setup eventually gives rise to heterogeneity in the patterns of economic development, allowing to explain the large variability between regions observed at world level.

This paper is organized as follows. Section 2 discusses related literature on the effects of migration on fertility and population dynamics. Section 3 describes our network based macroeconomic framework with feedback effects between capital accumulation and population dynamics. Section **??** presents the training stage in our specific model recalling some basic information of the general training process and rationale. Section 4 analyzes a simplified version of our model with two nodes, for which it is possible to derive the closed-form expression of the equilibria. Section 5 focuses on a more general model based on a larger 50-nodes network, presenting some numerical simulations to exemplify the implications of our modeling approach. Section 6 as usual concludes and suggests directions for future research.

Appendix A presents a particular case of our general framework in which the production function is Cobb-Douglas, ruling out thus the possibility for heterogeneity in the patterns of economic development, while appendix B determines explicitly the equilibria of our two-nodes model.

2 Related Literature: Migration, Fertility and Population Change

Understanding the possible effects that migration might have on the population growth rate of the receiving countries, and therefore on its economic growth prospects, is now important especially in the context of the changing demographics of developed, as well as developing countries [6]. To reach this objective, we integrate two different literatures (the Solow–type growth literature, on the one hand, and that based on neural–networks, on the other) that, for the most part, have developed separately in the last few years. As far as we may know, we are the first to do so, even though there already exists a rather rich economic geography literature that (since Krugman, 1991) has emphasized how rising levels of openness and, thus, larger migration flows across countries, are ultimately conducive to income polarization due to increasing returns to scale. The mechanism that Krugman (1991) describes in his seminal contribution can be summarized as follows. Consider two separate, although completely symmetric, regions among which the manufacturing labor-force is free to migrate. When trade frictions between the two regions are high (in the limit, infinite, so that there is autarky), the spatial equilibrium is unique and stable, and this equilibrium coincides with the symmetric equilibrium. However, when trade frictions decrease and reach a certain minimal threshold, a bifurcation does occur: the previous symmetric equilibrium, although still existing, becomes unstable

be some convergence in the fertility-behavior of migrants to that of natives over time. Last but not least, selective immigration policies (aiming at attracting especially young and high skilled migrants) may exacerbate the brain-drain in developing countries.

[6] Incidentally, in many growth models (especially those based on innovation and RD-activity), the size and the growth rate of the population play, indeed, a crucial role. Notable examples include, among many others, Kremer (1993), Acemoglu (1998), Ngai and Samaniego (2011), Doepke and Zilibotti (2014), Acemoglu and Restrepo (2018), Akcigit and Kerr (2018), Atkeson and Burstein (2019), and Buera and Oberfield (2020).

and there simultaneously emerges another equilibrium with full agglomeration of manufacturing in just one country (no matter which one, since the two are ex-ante identical). This mechanism is now known in the literature as 'symmetry-breaking' (see, for examples, Matsuyama, 1995, 2002, and 2008)[7]. Applications of the notion of symmetry-breaking to the growth and development literature include, among others, Krugman and Venables (1995), Matsuyama (1996 and 2004), and Boyd and Smith (1997). Indeed, in this literature cross-country differences in per capita income are usually attributed either to differences in total factor productivity or to differences in investment distortions among nations, with both sets of differences (total factor productivity vs. investment distortions, respectively) being generally left unexplained. As a matter of fact, just as low productivity or higher investment distortions may be a cause of low incomes, so too may low incomes be a cause for lower productivity or higher investment distortions. With such a two-way causality, cross-border movements of goods (as in Krugman and Venables, 1995, and Matsuyama, 1996), or capital (as in Boyd and Smith, 1997, and Matsuyama, 2004), can amplify even small differences across countries, which make the balanced development (i.e., spatial symmetry) unstable and the world economy evolve into a system of rich and poor countries (with a developed core characterized by high income, high TFP, low investment distortions, and a less-developed periphery characterized by low income, low TFP, high investment distortions). The fundamental implication of these seminal contributions is that the problem of a country's economic development/growth can never be analyzed in isolation, but rather it has to be examined as part of the interrelated whole, in the light of the fact that the rich may be rich also due to the presence of the poor and the poor may be poor also due to the presence of the rich. With respect to this interesting and fertile branch of the development/growth literature based on the working of a symmetry-breaking-mechanism, in our paper we focus on cross-border movements of people (migration), rather than of goods and capital, and in doing so we employ a neural-networks-based framework that the aforementioned papers do not use. In our setting, the choice to migrate hinges upon the origin-destination income differentials and affects the fertility rate, thus population growth is ultimately endogenous and depending on the flow of people that move from one area to a different one. In this sense, our paper is also related to that branch of the international-migrations literature that analyzes how the movement of individuals towards the most developed regions of the world does contribute to changes in the fertility rate of this side of the world. Such a literature is mainly empirical in nature, hence our paper can nicely complement it by providing a tractable theoretical framework of reference. In this regard, a recent paper by Bagavos (2019)[8] shows that between 2009 and 2015 the excess fertility of migrants (relative to native-women) ranged from a negligible level of 2% in the Netherlands to an impressive 73% in France, a fraction twice as high as that in the United States [9]. At the same time, Bagavos (2019) also demonstrates that there is significant heterogeneity in the shares of foreign-born women in the whole population of the countries being analyzed: in Switzerland, for instance, more than one in three women of reproductive age were born outside the country, compared to just one in ten in Finland. These differences (both in fertility and in the population shares of foreign-born women) are those that ultimately determine the magnitude of the authentic contribution of migration to the overall total fertility rate (TFR) of developed countries in the Bagavos (2019)'s study. On the whole, it is found that the so-called net effect of foreign-born women on a country's TFR is negative in Denmark and practically negligible in the Netherlands and Finland, while it reached non-negligible levels of 10% and 9% in France and Belgium, respectively, and 8% in Austria and Switzerland. However, due to the relatively low shares of foreign-born women, migration does not greatly affect the level of the overall TFR, even in those countries where migrants' fertility is significantly higher than that of natives. In conclusion, the effect

[7]According to the definition of Matsuyama (2008, p. 128): "*...Symmetry breaking creates asymmetric outcomes in a symmetric environment. It is the key concept for understanding self-organized (that is, endogenous) pattern formations... Similar questions about pattern formations also exist in economics. Why are there rich and poor countries? Why are industries clustered? Why are there booms and recessions? Why are some ethnic groups under-represented in certain jobs or neighborhoods?...*".

[8]The choice of the countries considered by the author is based on their experience as receiving countries, as well as on data availability for each year over the period 2009-2015. Thus, the United States, Australia, Switzerland, the United Kingdom, France, Belgium, the Netherlands, Germany, and Austria are analyzed as examples of longstanding destination countries with many settled migrants; Italy, Greece and Spain are considered new host countries with many recent migrants; Norway, Sweden, Denmark, and Finland are finally taken as examples of countries that have recently experienced significant levels of humanitarian migration.

[9]Notable exceptions are Denmark and Australia, where foreign-born women were less fertile than natives by 5% and 3%, respectively.

of migration on fertility seems to be more significant in Europe than in the United States and Australia, and in general it does not seem to be excessively high. The last fact can be explained either by a relatively low excess (or no excess at all) fertility of migrants (e.g., in Denmark and Australia), or by a relatively small share of foreign-born women in the population (e.g., Finland and France), or else by a combination of the previous two reasons. Yet, this should not lead to disregard the fact that migrations remain a decisive factor in current and, very likely, future population change (especially in more developed regions), both directly through positive net migration flows, and indirectly through the contributions of migrants to the fertility rate in the receiving country. For all these reasons, a thorough analysis of the consequences of international migration-flows on population and economic growth rates still remains essential for economists and policy-makers. The next sections of the paper are, thus, devoted to such a comprehensive analysis.

3 The Model

We aim at describing the evolution of a complex economy using a network-based model. The macroeconomic setting is simple and entirely characterized by the mutual interactions between capital accumulation and population growth. Capital accumulates according to the difference between saving and depreciation, where saving is given by the product between the saving rate and output (Solow, 1956). Output is produced through a convex-concave production function (Skiba, 1978) which takes the suitable form employed by La Torre et al. (2015), in which capital and labor are combined through a constant returns to scale technology characterized by decreasing returns to capital and labor but violating traditional Inada conditions (the marginal product at zero capital level is finite).[10] We abstract from unemployment, thus the population size and the labor force perfectly coincide. Population grows according to a logistic dynamics (Verhulst, 1838) augmented for migration flows, where migration depends on the origin-destination income differential which is a proxy for the long-run wage differential. Migration affects not only the population size but also its growth rate, meaning that the fertility rate in the destination is impacted by the patterns of migration (Kulu, 2005).

We introduce a network framework in such a basic macroeconomic setting. The entire network describes the global economy (i.e., the world or a single country), and each node within the network a single local economy (i.e., a country or a region) located in a specific geographic position. Edges represent existing trade and political relationships between local economies, which may make it more or less simple for people to move from one local economy to the next (i.e., bilateral trade or migration agreements). Each node interacts with the others through migration flows, and people move between nodes along the edges. More formally, the global economy is characterized by the means of a graph $G = (V, E)$ with M nodes, where V denotes the node set and $E \subset V \times V$ is the edge set. Given two nodes $V_i, V_j \in V$, $0 \leq \phi_{ij} \leq 1$ represents the "degree of economic interaction" from node i to node j, summarizing the intensity of their (reciprocal) trade and political relationships. The network is then described in compact form by the triplet $G = (V, E, \phi)$ where ϕ is a $M \times M$ weighted matrix with the property that $\phi_{ij} = \phi_{ji}$. The local economy of each node $i \in V$, is characterized by the macroeconomic framework earlier described: output of each node (in a single-perceptron framework) $Y_i(t)$ is produced through a production function combining several inputs such as the capital level $K_i(t)$ and the labor force $L_i(t)$. The production function Y_i takes an S-shaped or sigmoid form (characterized by a rational function), showing the existence of an intrinsic threshold which allows for the possibility of a poverty trap. Such a poverty trap plays then the role of the activation threshold in the classical neural network literature: the local economy at each node can be either activated if output $Y_i(t)$ at the time t is above the intrinsic threshold value, or not-activated otherwise. Therefore, activation represents a situation of successful economic development in which, as we shall see, the local economy is able in the long-run to converge to an equilibrium with a strictly positive (possibly high) capital level.

Example 3.1. *Consider a global economy of six countries described by the directed graph $G = (V, E)$, with*

[10] Since the goal of our paper consists of characterizing heterogeneity in the patterns of economic development, we need the production function to leave room for multiple equilibria and thus we cannot rely on a traditional globally concave production function (such as the Cobb-Douglas technology). Indeed, as we shall show in appendix A under a globally concave production function the whole network will converge to the same the long run outcome describing thus a situation of long run homogeneity between economies (i.e., nodes).

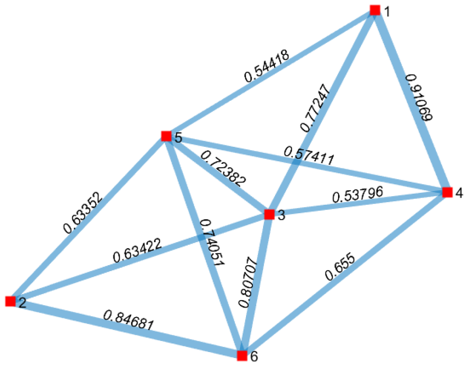

Figure 1: The graph G of a six countries network.

six nodes $V := [1, 2, 3, 4, 5, 6]$ *and edges* E *described by the weighted matrix:*

$$\phi = \begin{bmatrix} 0 & 0 & 0.77 & 0.91 & 0.54 & 0 \\ 0 & 0 & 0.63 & 0 & 0.63 & 0.85 \\ 0.77 & 0.63 & 0 & 0.54 & 0.72 & 0.81 \\ 0.91 & 0 & 0.54 & 0 & 0.57 & 0.65 \\ 0.54 & 0.63 & 0.72 & 0.57 & 0 & 0.74 \\ 0 & 0.85 & 0.81 & 0.65 & 0.74 & 0 \end{bmatrix}$$

Each number in the matrix represents the migration rate ϕ_{ij} *from country* i *to country* j *(directed). The geometric representation of the graph is shown in Figure 1.*

The local economy at each node $i \in V$ is described by the capital and population dynamic equations. Capital accumulates as follows: $\dot{K}_i(t) = s_i Y_i(t) - \delta_{K_i} K_i(t)$, where $0 < s_i < 1$ is the saving rate and δ_{K_i} the depreciation rate. Output is produced competitively by firms through the following production function: $Y_i(t) = \frac{K(t)^{\alpha_i} L_i(t)^{1-\alpha_i}}{1+\gamma_i K_i(t)^{\alpha_i} L_i(t)^{-\alpha_i}}$, where $\alpha_i > 1$ is a productivity parameter and $\gamma_i > 0$ measures eventual diseconomies.[11] Population dynamics is described by the following logistic-type equation:[12] $\dot{L}_i(t) = \left(r_i - C_i L_i(t) + \sum_{j=1, j\neq i}^{M} \phi_{ij} L_j(t)(y_i(t) - y_j(t))\right) L_i(t)$, where r_i represents the net growth rate (capturing both fertility and mortality), C_i the carrying capacity, and $\sum_{j\neq i}^{M} \phi_{ij} L_j(t)(y_i(t) - y_j(t))$ migration-induced population change. This latter term depends on the intensity of the connection between nodes, $\phi_{ij} \geq 0$, the population size at nodes different from i, $L_j(t)$, and the per-capita income differential between nodes, $y_i(t) - y_j(t)$.[13] Per-capita income, $y_i(t) = \frac{Y_i(t)}{L_i(t)}$, is a proxy for standards of living and thus migration

[11] Note that the characteristics of such a production function critically depend on the value of the parameter α_i. If $\alpha_i < 1$, the production function turns out to be globally concave and the marginal productivity of capital at zero capital levels infinite, as in a standard neoclassical neoclassical framework; moreover, in such a case, if $\gamma_i = 0$ the marginal productivity of capital at infinite capital levels is zero, restoring the classical Cobb-Douglas production function. If $\alpha_i = 1$, the production function turns out to be globally concave but with finite marginal productivity at zero capital levels. If instead $\alpha_i > 1$ the production function turns out to be convex for lower capital levels and concave for higher capital levels with positive and finite marginal productivity of capital at zero capital levels, as in Skiba (1982). Only in this latter scenario, which is the one we focus on in our analysis, multiple equilibria (and thus heterogeneity between local economies) may exist.

[12] Despite the traditional view in macroeconomic theory is that human population follows an exponential dynamics, several works discuss that such an assumption is unrealistic as resource constraints may limit population growth which thus cannot continue forever yielding an infinite number of individuals on our planet (Verhulst, 1838; Marsiglio and La Torre, 2012). The importance of such a skepticism is confirmed by the fact that even the UN in their population projections assume that the growth rate of the human population will gradually decrease over time, consistent with a logistic equation, to lead to a stabilization of the human population size by 2100. As a matter of realism, it seems convenient thus to adopt this approach also in our analysis.

[13] It is possible to show that in the very long-run (i.e., when k_t goes to infinity), per capita income equals the competitive wage rate, so that per capita income differences between any two countries are equivalent to wage differentials between the same two places.

decisions may be thought to depend directly on per-capita income differences. In such a setting migration occurs from a local economy with the lowest income level to all local economies it is connected to with higher incomes, and migration flows from one local economy to the next ones are simply proportional to the income differential.

To summarize, at each node $i \in V$ and at each instant of time t, given the initial level of capital and population, $K_i(0)$ and $L_i(0)$, the local economy evolves accordingly to the following system of differential equations:

$$\dot{K}_i(t) = s_i Y_i(t) - \delta_{K_i} K_i(t) \tag{1}$$

$$\dot{L}_i(t) = \left(r_i - C_i L_i(t) + \sum_{j \neq i}^{M} \phi_{ij} L_j(t) \left(\frac{Y_i(t)}{L_i(t)} - \frac{Y_j(t)}{L_j(t)} \right) \right) L_i(t) \tag{2}$$

where the above defined $Y_i(t) = \frac{K_i(t)^{\alpha_i} L_i(t)^{1-\alpha_i}}{1+\gamma_i K_i(t)^{\alpha_i} L_i(t)^{-\alpha_i}}$ has been introduced in the equations in order to simplify the notation. The same approach will be followed in the remainder of the paper.

4 The $M = 2$ Case

We start our analysis by focusing on a particular case of the above model in which only two nodes are involved. With two nodes only, the topology of the network is completely irrelevant and thus this case represents a suitable benchmark to understand in the simplest possible way the meachanisms at work in our neural-network-type model economy. In particular, this case allows to determine in closed-form all equilibria and to clearly understand the impact of the degree of economic connection on long-run economic dynamics. We will show that the parameter ϕ_{12} plays an important role by determining the basin of attraction of the long-run stable equilibrium, and in particular the larger ϕ_{12} the larger the attractor which implies that it will be easier for economies to escape their poverty traps and converge to a long-run stable equilibrium characterized by a positive (and possibly high) capital levels. In the case of two nodes, the above dynamic system boils down to:

$$\begin{cases} \dot{K}_1(t) = s_1 Y_1(t) - \delta_{K_1} K_1(t), \\ \dot{K}_2(t) = s_2 Y_2(t) - \delta_{K_2} K_2(t), \\ \dot{L}_1(t) = \left(r_1 - C_1 L_1(t) + \phi_{12} \left(\frac{Y_1(t)}{L_1(t)} - \frac{Y_2(t)}{L_2(t)} \right) L_2(t) \right) L_1(t), \\ \dot{L}_2(t) = \left(r_2 - C_2 L_2(t) + \phi_{12} \left(\frac{Y_2(t)}{L_2(t)} - \frac{Y_1(t)}{L_1(t)} \right) L_1(t) \right) L_2(t), \\ K_1(0), L_1(0) \text{ are given} \\ K_2(0), L_2(0) \text{ are given} \end{cases} \tag{3}$$

Without losing generality, let us suppose that $\phi_{12} > 0$. The above system of four equations displays nine steady states (i.e., solutions of the set of algebraic equations $\{\dot{K}_1 = 0; \dot{K}_2 = 0; \dot{L}_1 = 0; \dot{L}_2 = 0\}$), whose analytical expressions turn out to be particularly cumbersome and are reported in appendix A. Through numerical simulations, we explore how these 4-dimensional system's steady states and their basins of attractions change with the degree of economic interaction, and, as we are going to see, how variations in ϕ_{12} may lead to dramatic differences in the long-run economic performance of the two local economies. In order to visualize the results of our analysis in the simplest and most intuitive way, we project the system's phase portrait on the plane (K_1, K_2), and in order to do so consistently, we set the initial conditions on L_i as follows: $L_1(0) = 1, L_2(0) = 1$. Moreover, in order to stress the importance of the degree of economic interaction, ϕ_{12}, we perform the numerical simulations by focusing on a situation in which the two local economies are perfectly symmetric, and their parameters take on the values reported in Table 1.

s_1	s_2	α_1	α_2	γ_1	γ_2	δ_{K_1}	δ_{K_2}	r_1	r_2	C_1	C_2
0.2	0.2	2	2	1	1	0.05	0.05	1	1	1	1

Table 1: Parameter values employed in the analysis of the $M = 2$ case.

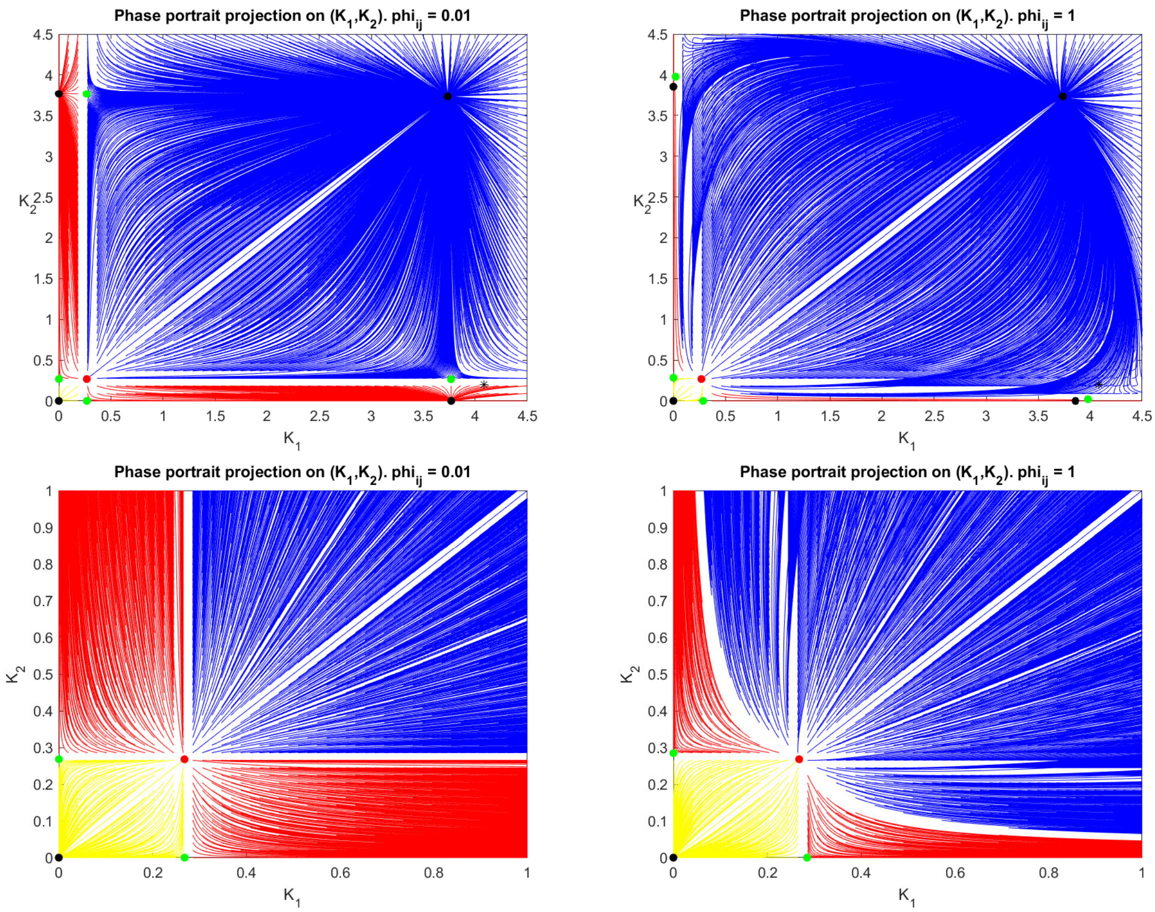

Figure 2: Phase portraits for two values of ϕ_{ij}: $\phi_{ij} = 0.01$ (left) and $\phi_{ij} = 1$ (right). Wider region (top) and details around the solid red circle ($K_1 = 0.268, K_2 = 0.268$).

Since exploiting the explicit expressions for the nine steady states results not possible, we will proceed numerically. The results of our numerical analysis are summarized in Figure 2, which shows the phase portrait of the above model under two different parametrizations for ϕ_{12}. The left top panel represents a situation with low degree of economic interaction (i.e., $\phi_{ij} = 0.01$), and it shows the nine steady states with solid circles of different colors. The four solid black circles represent attractive steady states. Three of them lie on the (K_1, K_2) axes, and in particular: the origin, $(\bar{K_1} = K_1(+\infty) = 0, \bar{K_2} = K_2(+\infty) = 0)$, corresponds to the dire steady state for both the local economies that have initial conditions $(K_1(0), K_2(0))$ in its basin of attraction (yellow trajectories); the two solid black circles in position $(\bar{K_1} = 3.766, \bar{K_2} = 0)$ and $(\bar{K_1} = 0, \bar{K_2} = 3.766)$ represent the symmetric situations in which one of the two economies reaches a positive long-run economic outcome, while the other does not escape the poverty trap (initial conditions that give rise to the red trajectories). The fourth attractive steady state belongs to the strictly positive orthant $(\bar{K_1} = 3.732, \bar{K_2} = 3.732)$, and its basin of attraction is visualized via the blue trajectories: from an economic point of view this is the best outcome for the two local economies jointly since both of them reach a positive long-run economic outcome. The solid red circle in position $(\bar{K_1} = 0.267, \bar{K_2} = 0.267)$ is repulsive: any initial condition $(K_1(0), K_2(0))$ no matter how close (but not coincident) to this point is doomed to create a diverging trajectory falling into the basins of attraction of (one of) the black circles earlier described. The four solid green circles have a saddle-like behavior, that is they are repulsive except for one direction that works as a separatrix (i.e. a border between adjacent basins of attraction): the green circles $(\bar{K_1} = 0.268, \bar{K_2} = 0)$ and $(\bar{K_1} = 0, \bar{K_2} = 0.268)$ are the asymptotic destinations of two heteroclinic orbits (trajectories) that originate from the red circle and divide the basin of attraction of the origin from the two symmetric basins of the black circles on the axes; while the attractive direction of the green circles $(\bar{K_1} = 3.764, \bar{K_2} = 0.265)$ and $(\bar{K_1} = 0.265, \bar{K_2} = 3.764)$ separates the basin of $(\bar{K_1} = 3.732, \bar{K_2} = 3.732)$

from the basins of ($\bar{K_1} = 3.766, \bar{K_2} = 0$) and ($\bar{K_1} = 0, \bar{K_2} = 3.766$). The right top panel represents a situation with high degree of economic interaction (i.e., $\phi_{ij} = 1$), and the steady states have qualitatively the same nature, but the positions of some of them is substantially different with respect to what seen earlier, simply because of the increase in ϕ_{12}. It is immediate to see that the region crossed by the red trajectories substantially shrinks, making room for the blue ones: in other words, the basin of attraction of the most-desirable steady state increases. This in turn means that it is possible to choose a set of initial conditions $(K_1(0), K_2(0))$ whose long run behavior will be diverted from one attractor to the other as we increase the parameter ϕ_{12}. The bottom panels corroborate the previous results by offering a zoom in the regions around the solid red circle, in a situation with low (left panel) and high (right panel) degree of economic interaction.

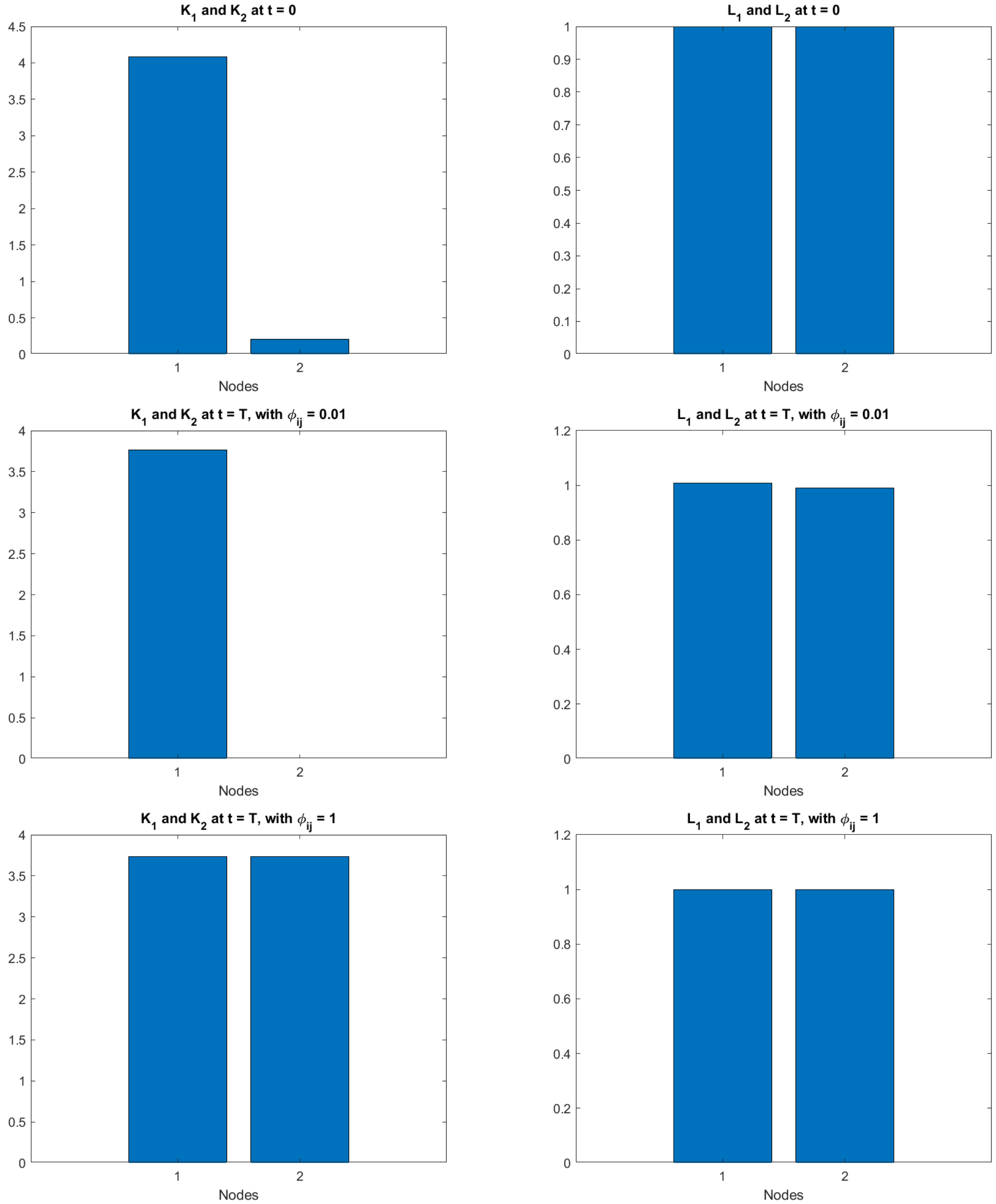

Figure 3: Evolution of K_i (left) and L_i (right), starting from their initial value (top), to their final value in the case in which $\phi_{ij} = 0.01$ (middle) and $\phi_{ij} = 1$ (bottom).

Figure 3 further visualizes this outcome for local economies starting from some specific initial conditions. The figure shows the initial conditions (top panels) for capital (left panel) and population (right panel), along with and their steady states associated with low and high degrees of economic interaction: $\phi_{ij} = 0.01$ (middle panels) and $\phi_{ij} = 1$ (bottom panels). The initial conditions ($K_1(0) = 4.082, K_2(0) = 0.202$) ensure that in the absence of economic interaction (i.e., $\phi_{12} = 0$) the two economies show very different dynamics, that is $K_1(t)$ converges to $\bar{K_1} > 0$ and $K_2(t)$ to $\bar{K_2} = 0$. Exactly as discussed earlier by referring to the phase portrait, we can see that, ceteris paribus, an increase in the degree of economic interaction pushes node 2 beyond the poverty trap: comparing the left panels it is straightforward to observe that the economy in node 2 would have been doomed to a collapse (top) without a tight interaction with node 1 (bottom). Therefore, tight interactions between local economies allows to remove the eventual heterogeneity in their level of economic development that would otherwise occur with loose interactions.

These results show that the long-run outcome that each local economy converges to depends on the degree of economic interaction which quantifies the intensity of the (reciprocal) trade and political relationships between economies. A higher degree of interaction increases the chances of a poor economy to escape its poverty trap and to start a smooth process of economic development, suggesting thus that promoting the formation of tight relations between countries may be an important policy to achieve globally desirable outcomes. Similar conclusions have been derived in completely different settings, for example in the context of spatial externalities (La Torre et al., 2015, 2019). For example, in a similar Solow-type model with convex-concave production function where local economies are interconnected globally via the spatial diffusion of capital which gives rise to a spatial externality, La Torre et al. (2015) show that poor economies may escape their poverty traps thanks to the effects of capital diffusion: the possibility to trade capital from the rich and the poor locations may allow for an economic takeoff to occur in poor economies. In our network framework, population migration allowed by the existence of economic interaction between nodes works in the same way as capital diffusion. The main difference between these two alternative mechanisms is the direction of flows: while capital flows from rich to poor economies, migration flows from poor to rich ones. In reality we observe both firms relocations from rich to poor regions of the world compensated by workers migration in the opposite directions, suggesting that the two mechanisms are not substitutable but complementary explanations of real world dynamics.

5 The $M > 2$ Case

We now build on the previous two-nodes analysis to show that our main results hold true even for an arbitrary large number of nodes and for different network configurations (see Barabási, 2016, for an introduction to network science). With a larger number of nodes it is not possible to characterize explicitly the steady states and thus we necessarily need to rely on a numerical approach. Our numerical analysis is based on large but not too large network with 50 nodes in order to understand how our results extend in more complicated setups and, at the same time, to preserve the possibility to visualize the results. We focus on four sets of simulations, whose common characteristic is that they consider undirected networks, that is we focus on the existence and the strength of the bilateral connection between two economies.

In the first two sets of simulations we assume that the degree of economic interaction and most parameters including initial conditions are homogeneous across nodes in order to focus in particular on the effects of low and high degrees of interaction to mimic what discussed in the previous section. Specifically, the first set of simulations refer to a random network à-la Erdos-Renyi, where each node is connected to any other one by virtue of a fixed probability p: given the number of nodes, there is a probability p for the node i to be connected to the node j, with $i, j \in 1, 2..50$ and $i \neq j$. Networks of this sort have a normal degree distribution, that is the numbers of connections reaching to or starting from node i, $\forall i$ in the network, are normally distributed, and the mean degree distribution (i.e., the average number of connections for a typical node) is a representative property of the network, constituting its "scale" or characteristic dimension. A random network can be used as a first, crude approximation of a system of interconnected economies, because the underlying hypothesis that all economies share more or less the same number of connections on average appear quite far away from real world observations, where instead a small number of economies have a number of connections exceeding by many standard deviations those of others. Indeed, in reality there are economies whose number and intensity of connections are orders of magnitude larger than the average, making the mean degree irrelevant to many purposes. Therefore, in the second set of simulations we focus on a "scale-free" network, with a power law degree distribution such that the presence of hubs is perfectly plausible. We can imagine the genesis of a scale-free network as a process in which the nodes are included in the network one at a time, with a mechanism of preferential attachment [14] to the nodes that already have the highest degrees of connection: the n-th node joining the network has greater probability to connect to an extant node with a high degree than to a node with low degree. The third set of simulations is still based on a scale-free network but now the hypothesis of homogeneous degree of economic interaction is dropped, as we will explain in detail later on. Finally, in the last set of simulations we relax our earlier hypotheses that the initial conditions and most parameters (in particular, r_i and C_i) are homogeneous across nodes in order to explore the full implications of our model.

Our simulations in the case of a random network à-la Erdos-Renyi with homogeneous degree of economic interaction are reported in Figure 4. The figure shows the network layout (top panels) for capital (left panel) and population (right panel), along with couples of barplots (middle and bottom panels) depicting the initial and steady states of capital (left panels) and population (right panel) associated with the same two values of ϕ_{ij} considered in the previous section: $\phi_{ij} = 0.01$ (middle panels) and $\phi_{ij} = 1$ (bottom panels). In particular, the barplots show how the initial conditions $K_i(0)$ and $L_i(0)$ are mapped into the corresponding steady states. From the top panels we can observe that the dimension and color of each node i are proportional to the initial conditions. In line with the 2-dimensional simulations, half of the nodes have $K_i(0) = 4.082$ and half $K_i(0) = 0.202$, while the population has been set $L_i(0) = 1, \forall i = 1..50$. From the middle and bottom panels we can see the effects of different values of the degree of economic interaction on the steady states of capital and population. By comparing the two couples of panels in the middle left position to the ones in the bottom left one, we can conclude that increasing the intensity of the degree of economic interaction from $\phi_{i,j} = 0.01$ to $\phi_{i,j} = 1$, the number of activated nodes, N^+, that is the number of nodes escaping the poverty trap, increases from $N^+ = 25$ to $N^+ = 50$. Our simulations in the case of a scale-free random network with homogeneous degree of economic interaction are reported in Figure 5. The layout of the networks is represented in the top panels. The network has been generated with the Julia Language function *static_scale_free(50,100,3)*, that produces a graph with 50 vertices, 100 edges and an expected power-law distribution of exponent 3. This Figure clearly shows that the above results are robust with respect to the topology of the network, as exactly the same qualitative conclusions apply also in this case. The initial conditions are exactly the same as in the previous set of simulations, and the steady states show analogous an outcome.

In the previous two set of simulations we have used the rather restrictive hypothesis that the degree of economic interaction, captured by the parameter ϕ_{ij}, is the same across all the connections ij. Now we let ϕ_{ij} be a symmetric matrix, with entries proportional to the weighted average of node $i's$ and $j's$ degree - number of links coming to or starting from those nodes, namely deg_i and deg_j. This allow us to link the random network generating process that shaped the layout of the network in the first place, to the very dynamics of K_i and L_i: it's reasonable to think that the forces that molded the network during its formation have their inertial effects on dynamics of the systems. The degree of economic interaction follows now a power-law distribution that mimics the structure of the network. In other words, dropping the hypothesis of a homogeneous degree of economic interaction across nodes means that we now study the behavior of our economies in an actual weighted scale-free network: the average number of connections of the nodes is not informative, because the presence of a huge hub cannot be ruled out, and the intensity of the connections is space-dependent, meaning that each couple of economies (ij) has its particular ϕ_{ij}. In particular, we define

$$\phi_{ij} = c\frac{(deg_i + deg_j)}{2} \tag{4}$$

where c has the role of a proportionality constant directly impacting the strength of the economic interaction. The corresponding simulations are reported in Figure 6. The figure is organized similarly to the previous

[14] In this section's simulations we used the preferential attachment algorithm outlined in Goh et al. (2001) and codified in the Julia Language package LightGraphs, specifically the random network generating function static_scale_free(n, m, α), that generates a random graph with n vertices, m edges and expected power-law degree distribution with exponent α. Available at https://github.com/sbromberger/LightGraphs.jl

ones: network layout on the top panels, comparison between initial and final values of K_i and L_i in the middle and bottom ones. This time though, what changes going from the middle panels to the bottom ones is the value of the proportionality constant: $c = 0.01$ and $c = 1$ in the middle and bottom panels respectively. By comparing the the middle and the bottom left panels for the different values of c, we can observe that increasing ϕ_{ij} via the proportionality constant c, the number of economies reaching a positive long run equilibrium goes from $N^+ = 25$, to $N^+ = 49$. This confirms our previous results: an increase in economic interaction helps more economies to escape from the poverty trap. For completeness, Table 2 reports the number of economies crossing the poverty trap for three different values of the proportionality constant c.

c	0.01	0.1	1
N^+	25	26	49

Table 2: Number of activated nodes for different values of the proportionality constant c.

In our last set of simulations, presented in Figure 7, we build on the previous one, as far as the relation 4 is concerned: ϕ_{ij} is still proportional the the weighted average of the degree of node i and j via the constant c. Moreover we drop the other structural homogeneity hypotheses and assume that both the initial conditions $(K_i(0), L_i(0))$ and the parameters r_i and C_i are proportional to the degree of the corresponding node. In particular, for consistency with respect to the previous simulations, we adopt the following relations to select the parameters:

$$K_i(0) = \frac{deg_i}{\max\limits_{i \in V}\{deg_i\}}(4.082 - 0.202) \tag{5}$$

$$L_i(0) = \frac{deg_i}{\max\limits_{i \in V}\{deg_i\}} \tag{6}$$

$$r_i = \frac{deg_i}{\max\limits_{i \in V}\{deg_i\}} \tag{7}$$

$$C_i = \frac{deg_i}{\max\limits_{i \in V}\{deg_i\}} \tag{8}$$

This re-scaling allows us to unleash the full implications of our network analysis: since ϕ_{ij} is a random symmetric matrix - it inherits the randomness from the network generating process, and both the initial conditions and most of the parameters are analogously selected, we can imagine that the network parameters we use in our analysis are actually taken from the training phase of a neural network and are ready to be employed in the forecasting phase. Our steady state analysis unveils the underlying dynamics, with some local economies greatly outperforming the others. The main argument from our previous simulations still applies, as increasing the value of ϕ_{ij} brings greater number of economies above the poverty trap and toward a positive long-run equilibrium, as shown both in the left and right panels – to read exactly as in the the previous figures – and in Table 3. In Table 3 we additionally provide the capital and population levels in the global economy corresponding to increasing values of c: from a global perspective the higher the degree of economic interaction, the higher the economic performance, both in levels and in per capita terms.

c	0.01	0.1	1
N^+	19	23	47
$\sum_i K_i$	72.5	95.4	175.4
$\sum_i L_i$	49.6	45.3	47.0

Table 3: Number of activated nodes, global capital and global population for different values of c.

6 Conclusion

The importance of migration on the economic development of modern economies is under everyone's eyes. In order to shed some light on this, we analyze a neural network-based macroeconomic framework to understand the possible effects that endogenous migration choices may have on the mutual relation between population dynamics and capital accumulation. In our setting population size determines the labor input which, together with the available capital stock, drives total output; production takes place with a convex-concave technology allowing for a poverty trap, which plays the role of the activation function in neural networks. Migration depends on the origin-destination income differential and affects the fertility rate, thus population growth depends endogenously upon economic conditions. The global economy is represented by the whole network while each local economy by a within the network. In this context, the long-run outcome of each local economy depends on its chances to escape the poverty trap and converge to a long-run equilibrium with a strictly positive capital level. We show that this possibility strictly depends on the degree of economic interaction which quantifies the intensity of the (reciprocal) trade and political relationships between economies. This type of conclusion is robust with respect to the topology of the network, as it similarly applies in a network with two nodes and with fifty nodes, in a random network and in a scale-free network. Therefore, favoring tight relations between countries may allow poor countries to escape their poverty trap giving rise to a smooth process of economic development. Moreover, such interactions may give rise to possible heterogeneity in macroeconomic outcomes allowing to explain the large variability observed in the real world.

To the best of our knowledge this is the first paper analyzing endogenous migration in a neural-network-type macroeconomic setting. In order to make our conclusions as clear as possible we have relied on a simplified framework abstracting from agent's optimization and network dynamics. Clearly, the choice to migrate is in reality the outcome of an individual decision process, thus by treating local economies as aggregate entities we have not been able to capture such choices. Also, the intensity of interactions between local economies in reality tends to change over time with ties becoming stronger or weaker according to the specific economic conditions in connected economies, thus by assuming that the network structure is given we have not been able to account for such dynamic effects. Extending the analysis along these directions is left for future research.

A A Special Case with Cobb-Douglas Production Function

We now show that in the special case of a Cobb-Douglas production function the analysis becomes to a large extent trivial, as the absence of a poverty trap implies that in the long run every local economy will converge to the same long run equilibrium. The Cobb-Douglas production function case corresponds to the situation in which $\alpha_i < 1$ and $\gamma_i = 0$, and under such a parametrization at each node i the local economy evolves accordingly to the following system of differential equations:

$$\dot{K}_i(t) = s_i K(t)^{\alpha_i} L_i(t)^{1-\alpha_i} - \delta_{K_i} K_i(t) \quad (9)$$

$$\dot{L}_i(t) = \left(r_i - C_i L_i(t) + \sum_{j \neq i}^{M} \phi_{ij} L_j(t) \left(\frac{Y_i(t)}{L_i(t)} - \frac{Y_j(t)}{L_j(t)} \right) \right) L_i(t) \quad (10)$$

For the sake of simplicity, we focus on the simple two-nodes framework, but our results straightforwardly extend to any given number of nodes. Figure 8 shows the phase portrait of the above model under two different parametrizations for ϕ_{12}, clearly showing that independently of the degree of economic interaction each local economy (provided that its initial capital and population levels are strictly positive) will naturally converge to the same nontrivial equilibrium in which capital and population are strictly positive. This result is intuitive given our previous discussion in the convex-concave production function: since in the Cobb-Douglas case the poverty trap coincides with a zero capital level, any economy with positive capital has already exceed this critical threshold and this does not leave any room for heterogeneity in the pattern of economic development between local economies.

Figure 9 shows exactly the same result by representing the initial condition and the steady state values associated with low and high degrees of economic interaction. The degree of economic interaction only determines the state capital values in the local economies, but the steady state achieved is always the same for all local economies, confirming thus that with a Cobb-Douglas production function each local economy will converge to the same long run equilibrium, characterizing a homogeneous pattern of economic development between economies.

B Explicit Equilibria

In this appendix we provide the analytical expressions of the nine steady states in the two-nodes case, mentioned in section 4. The nine steady states, $E^{(j)} = (\bar{K_1}, \bar{K_2}, \bar{L_1}, \bar{L_2})^{(j)}$ for $j \in 1, 2, ...9$ are reported below:

$$
\begin{aligned}
\bar{K_1}^{(1)} &= 0 \\
\bar{K_2}^{(1)} &= 0 \\
\bar{L_1}^{(1)} &= \frac{r_1}{C_1} \\
\bar{L_2}^{(1)} &= \frac{r_2}{C_2} \\
\bar{K_1}^{(2)} &= \Omega_1^+ \bar{L_1}^{(2)} \\
\bar{K_2}^{(2)} &= 0 \\
\bar{L_1}^{(2)} &= \frac{\gamma_1\, s_1\, \left(\Omega_1^+\, \delta_{K1}\, r_2 \phi_{12} + C_2\, r_1\, s_1\right)}{C_1\, C_2\, \gamma_1\, {s_1}^2 + (\delta_{K1} s_1\, \Omega_1^+ - {\delta_{K1}}^2){\phi_{12}}^2} \\
\bar{L_2}^{(2)} &= \frac{\gamma_1\, s_1\, \left(-\Omega_1^+\, \delta_{K1}\, r_2 \phi_{12} + C_2\, r_1\, s_1\right)}{C_1\, C_2\, \gamma_1\, {s_1}^2 + (\delta_{K1} s_1\, \Omega_1^+ - {\delta_{K1}}^2){\phi_{12}}^2} \\
\bar{K_1}^{(3)} &= \Omega_1^- \bar{L_1}^{(3)} \\
\bar{K_2}^{(3)} &= 0 \\
\bar{L_1}^{(3)} &= \frac{\gamma_1\, s_1\, \left(\Omega_1^-\, \delta_{K1}\, \phi_{12}\, r_2 + C_2\, r_1\, s_1\right)}{C_1\, C_2\, \gamma_1\, {s_1}^2 + \delta_{K1}\, {\phi_{12}}^2 s_1\, \Omega_1^- - {\delta_{K1}}^2 {\phi_{12}}^2} \\
\bar{L_2}^{(3)} &= \frac{\gamma_1\, s_1\, \left(-\Omega_1^-\, \delta_{K1}\, \phi_{12}\, r_2 + C_2\, r_1\, s_1\right)}{C_1\, C_2\, \gamma_1\, {s_1}^2 + \delta_{K1}\, {\phi_{12}}^2 s_1\, \Omega_1^- - {\delta_{K1}}^2 {\phi_{12}}^2} \\
\bar{K_1}^{(4)} &= 0 \\
\bar{K_2}^{(4)} &= \Omega_2^+ \bar{L_1}^{(4)} \\
\bar{L_1}^{(4)} &= \frac{\gamma_2\, s_2\, \left(-\Omega_2^+\, \delta_{K2}\, r_2 \phi_{12} + C_2\, r_1\, s_2\right)}{C_1\, C_2\, \gamma_2\, {s_2}^2 + (\delta_{K2}\, s_2\, \Omega_2^+ - {\delta_{K2}}^2){\phi_{12}}^2} \\
\bar{L_2}^{(4)} &= \frac{\gamma_2\, s_2\, \left(\Omega_2^+\, \delta_{K2}\, r_2 \phi_{12} + C_2\, r_1\, s_2\right)}{C_1\, C_2\, \gamma_2\, {s_2}^2 + (\delta_{K2}\, s_2\, \Omega_2^+ - {\delta_{K2}}^2){\phi_{12}}^2} \\
\bar{K_1}^{(5)} &= 0 \\
\bar{K_2}^{(5)} &= \Omega_2^- \bar{L_1}^{(5)} \\
\bar{L_1}^{(5)} &= \frac{\gamma_2\, s_2\, \left(-\Omega_2^-\, \delta_{K2}\, \phi_{12}\, r_2 + C_2\, r_1\, s_2\right)}{C_1\, C_2\, \gamma_2\, {s_2}^2 + \delta_{K2}\, {\phi_{12}}^2 s_2\, \Omega_2^- - {\delta_{K2}}^2 {\phi_{12}}^2} \\
\bar{L_2}^{(5)} &= \frac{\gamma_2\, s_2\, \left(\Omega_2^-\, \delta_{K2}\, \phi_{12}\, r_2 + C_2\, r_1\, s_2\right)}{C_1\, C_2\, \gamma_2\, {s_2}^2 + \delta_{K2}\, {\phi_{12}}^2 s_2\, \Omega_2^- - {\delta_{K2}}^2 {\phi_{12}}^2} \\
\bar{K_1}^{(6)} &= \Omega_1^+ \bar{L_1}^{(6)} \\
\bar{K_2}^{(6)} &= \Omega_2^+ \bar{L_2}^{(6)} \\
\bar{L_1}^{(6)} &= \frac{\gamma_1\, \gamma_2\, s_1\, s_2\, \left(\delta_{K1}\, \phi_{12}\, r_2\, s_2\, \Omega_1^+ - \delta_{K2}\, \phi_{12}\, r_2\, s_1\, \Omega_2^+ + C_2\, r_1\, s_1\, s_2\right)}{\Theta - 2\, \delta_{K1}\, \delta_{K2}\, \gamma_1\, \gamma_2\, {\phi_{12}}^2 s_1\, s_2\, \Omega_1^+\, \Omega_2^+ + \delta_{K1}\, \gamma_2\, {\phi_{12}}^2 s_1\, {s_2}^2 \Omega_1^+ + \delta_{K2}\, \gamma_1\, {\phi_{12}}^2 {s_1}^2 s_2\, \Omega_2^+} \\
\bar{L_2}^{(6)} &= \frac{\gamma_1\, \gamma_2\, s_1\, s_2\, \left(-\delta_{K1}\, \phi_{12}\, r_2\, s_2\, \Omega_1^+ + \delta_{K2}\, \phi_{12}\, r_2\, s_1\, \Omega_2^+ + C_2\, r_1\, s_1\, s_2\right)}{\Theta - 2\, \delta_{K1}\, \delta_{K2}\, \gamma_1\, \gamma_2\, {\phi_{12}}^2 s_1\, s_2\, \Omega_1^+\, \Omega_2^+ + \delta_{K1}\, \gamma_2\, {\phi_{12}}^2 s_1\, {s_2}^2 \Omega_1^+ + \delta_{K2}\, \gamma_1\, {\phi_{12}}^2 {s_1}^2 s_2\, \Omega_2^+}
\end{aligned}
$$

$$
\begin{aligned}
\bar{K_1}^{(7)} &= \Omega_1^+ \bar{L_1}^{(7)}\\
\bar{K_2}^{(7)} &= \Omega_2^- \bar{L_2}^{(7)}\\
\bar{L_1}^{(7)} &= \frac{\gamma_1\,\gamma_2\,s_1\,s_2\,\left(\delta_{K1}\,\phi_{12}\,r_2\,s_2\,\Omega_1^+ - \delta_{K2}\,\phi_{12}\,r_2\,s_1\,\Omega_2^- + C_2\,r_1\,s_1\,s_2\right)}{\Theta - 2\,\delta_{K1}\,\delta_{K2}\,\gamma_1\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2\,\Omega_1^+\,\Omega_2^- + \delta_{K1}\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2{}^2\Omega_1^+ + \delta_{K2}\,\gamma_1\,\phi_{12}{}^2 s_1{}^2 s_2\,\Omega_2^-}\\
\bar{L_2}^{(7)} &= \frac{\gamma_1\,\gamma_2\,s_1\,s_2\,\left(-\delta_{K1}\,\phi_{12}\,r_2\,s_2\,\Omega_1^+ + \delta_{K2}\,\phi_{12}\,r_2\,s_1\,\Omega_2^- + C_2\,r_1\,s_1\,s_2\right)}{\Theta - 2\,\delta_{K1}\,\delta_{K2}\,\gamma_1\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2\,\Omega_1^+\,\Omega_2^- + \delta_{K1}\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2{}^2\Omega_1^+ + \delta_{K2}\,\gamma_1\,\phi_{12}{}^2 s_1{}^2 s_2\,\Omega_2^-}\\
\bar{K_1}^{(8)} &= \Omega_1^- \bar{L_1}^{(8)}\\
\bar{K_2}^{(8)} &= \Omega_2^+ \bar{L_2}^{(8)}\\
\bar{L_1}^{(8)} &= \frac{\gamma_1\,\gamma_2\,s_1\,s_2\,\left(\delta_{K1}\,\phi_{12}\,r_2\,s_2\,\Omega_1^- - \delta_{K2}\,\phi_{12}\,r_2\,s_1\,\Omega_2^+ + C_2\,r_1\,s_1\,s_2\right)}{\Theta - 2\,\delta_{K1}\,\delta_{K2}\,\gamma_1\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2\,\Omega_1^-\,\Omega_2^+ + \delta_{K1}\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2{}^2\Omega_1^- + \delta_{K2}\,\gamma_1\,\phi_{12}{}^2 s_1{}^2 s_2\,\Omega_2^+}\\
\bar{L_2}^{(8)} &= \frac{\gamma_1\,\gamma_2\,s_1\,s_2\,\left(-\delta_{K1}\,\phi_{12}\,r_2\,s_2\,\Omega_1^- + \delta_{K2}\,\phi_{12}\,r_2\,s_1\,\Omega_2^+ + C_2\,r_1\,s_1\,s_2\right)}{\Theta - 2\,\delta_{K1}\,\delta_{K2}\,\gamma_1\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2\,\Omega_1^-\,\Omega_2^+ + \delta_{K1}\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2{}^2\Omega_1^- + \delta_{K2}\,\gamma_1\,\phi_{12}{}^2 s_1{}^2 s_2\,\Omega_2^+}\\
\bar{K_1}^{(9)} &= \Omega_1^- \bar{L_1}^{(9)}\\
\bar{K_2}^{(9)} &= \Omega_2^- \bar{L_2}^{(9)}\\
\bar{L_1}^{(9)} &= \frac{\gamma_1\,\gamma_2\,s_1\,s_2\,\left(\delta_{K1}\,\phi_{12}\,r_2\,s_2\,\Omega_1^- - \delta_{K2}\,\phi_{12}\,r_2\,s_1\,\Omega_2^- + C_2\,r_1\,s_1\,s_2\right)}{\Theta - 2\,\delta_{K1}\,\delta_{K2}\,\gamma_1\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2\,\Omega_1^-\,\Omega_2^- + \delta_{K1}\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2{}^2\Omega_1^- + \delta_{K2}\,\gamma_1\,\phi_{12}{}^2 s_1{}^2 s_2\,\Omega_2^-}\\
\bar{L_2}^{(9)} &= \frac{\gamma_1\,\gamma_2\,s_1\,s_2\,\left(-\delta_{K1}\,\phi_{12}\,r_2\,s_2\,\Omega_1^- + \delta_{K2}\,\phi_{12}\,r_2\,s_1\,\Omega_2^- + C_2\,r_1\,s_1\,s_2\right)}{\Theta - 2\,\delta_{K1}\,\delta_{K2}\,\gamma_1\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2\,\Omega_1^-\,\Omega_2^- + \delta_{K1}\,\gamma_2\,\phi_{12}{}^2 s_1\,s_2{}^2\Omega_1^- + \delta_{K2}\,\gamma_1\,\phi_{12}{}^2 s_1{}^2 s_2\,\Omega_2^-}
\end{aligned}
$$

where, for the sake of readability, we have introduced two following accessory aggregate parameters, namely Ω_1^+, Ω_1^-, Ω_2^+, Ω_2^- and Θ: $\Omega_1^+ = \frac{1}{2}\,\frac{s_1+\sqrt{-4\,\delta_{K_1}{}^2\gamma_1+s_1{}^2}}{\delta_{K_1}\,\gamma_1}$, $\Omega_1^- = \frac{1}{2}\,\frac{s_1-\sqrt{-4\,\delta_{K_1}{}^2\gamma_1+s_1{}^2}}{\delta_{K_1}\,\gamma_1}$, $\Omega_2^+ = \frac{1}{2}\,\frac{s_2+\sqrt{-4\,\delta_{K_2}{}^2\gamma_2+s_2{}^2}}{\delta_{K_2}\,\gamma_2}$, $\Omega_2^- = \frac{1}{2}\,\frac{s_2-\sqrt{-4\,\delta_{K_2}{}^2\gamma_2+s_2{}^2}}{\delta_{K_2}\,\gamma_2}$, $\Theta = C_1\,C_2\,\gamma_1\,\gamma_2\,s_1{}^2 s_2{}^2 - \delta_{K1}{}^2\gamma_2\,\phi_{12}{}^2 s_2{}^2 - \delta_{K2}{}^2\gamma_1\,\phi_{12}{}^2 s_1{}^2$.

Following the discussion in section 4, the equilibria $E^{(1)}$, $E^{(2)}$, $E^{(4)}$ and $E^{(6)}$ are described by the solid black circle. In particular the origin, $E^{(1)}$, $E^{(2)}$ and $E^{(4)}$ lie on the axes of the projection of (K_1, K_2, L_1, L_2) on the plane (K_1, K_2). As for the saddle-like solid green circles, the two on the axes are $E^{(3)}$ and $E^{(5)}$ while $E^{(7)}$ and $E^{(8)}$ lie within the strictly positive (K_1, K_2) plane. Finally, the fully unstable solid red circle is described by $E^{(9)}$.

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RN_K0_50_phi_001.png

RN_L0_50_phi_001.png

RN_KT_50_phi_001.png

RN_LT_50_phi_001.png

RN_KT_50_phi_1.png

RN_LT_50_phi_1.png

Figure 4: Random network with homogeneous degree of economic interaction. Evolution of K_i (left) and L_i (right), starting from their network layout (top), to their initial and final barplot distribution in the case in which $\phi_{ij} = 0.01$ (middle), and $\phi_{ij} = 1$ (bottom).

SC_K0_50_phi_001.png

SC_L0_50_phi_001.png

SC_KT_50_phi_001.png

SC_LT_50_phi_001.png

SC_KT_50_phi_1.png

SC_LT_50_phi_1.png

Figure 5: Scale-free network with homogeneous degree of economic interaction. Evolution of K_i (left) and L_i (right), starting from their network layout (top), to their initial and final barplot distribution in the case in which $\phi_{ij} = 0.01$ (middle), and $\phi_{ij} = 1$ (bottom).

SC_K0_50_phi_001.png

SC_L0_50_phi_001.png

SC_KT_50_random_phi_ij_c_100.png

SC_LT_50_random_phi_ij_c_100.png

SC_KT_50_random_phi_ij_c_1.png

SC_LT_50_random_phi_ij_c_1.png

Figure 6: Scale-free network with heterogeneous degree of economic interaction. Evolution of K_i (left) and L_i (right), starting from their network layout (top), to their initial and final barplot distribution in the case in which $c = 0.01$ (middle), and $c = 1$ (bottom).

full_K0_c_100.png

full_K0_c_100.png

full_KT_c_100.png

full_LT_c_100.png

full_KT_c_1.png

full_LT_c_1.png

Figure 7: Scale-free network with heterogeneous degree of economic interaction, initial conditions and structural parameters. Evolution of K_i (left) and L_i (right), starting from their network layout (top), to their initial and final barplot distribution in the case in which $c = 0.01$ (middle), and $c = 1$ (bottom).

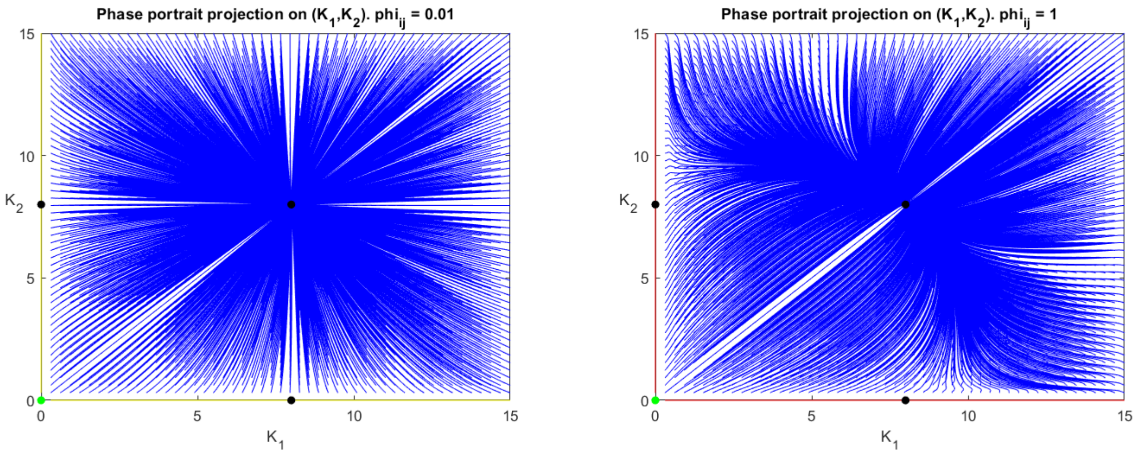

Figure 8: Phase portraits for two values of ϕ_{ij}: $\phi_{ij} = 0.01$ (left) and $\phi_{ij} = 1$ (right) in the Cobb-Douglas production function case.

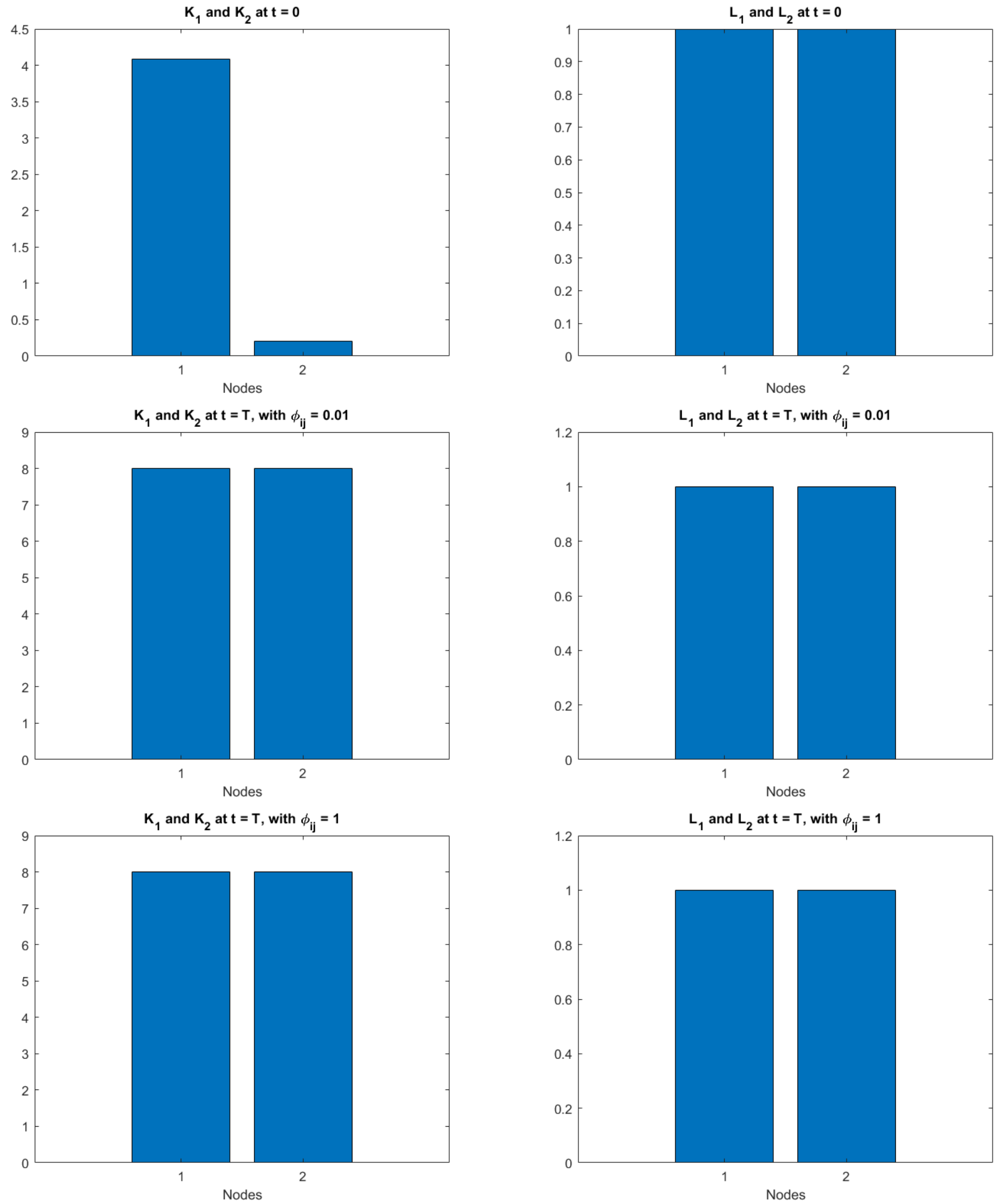

Figure 9: Evolution of K_i (left) and L_i (right), starting from their initial value (top), to their final value in the case in which $\phi_{ij} = 0.01$ (middle) and $\phi_{ij} = 1$ (bottom), in the Cobb-Douglas production function case.