# Synergies and dissonances between designers' and students' perspectives on key features of digital mathematical discussion

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*In this paper, we showcase students' perspectives on their experiences with Digital Mathematical Discussion (DMD) and we compare them with the designers' perspectives. After presenting the design of the DMD and the key features of its design in terms of the timing and communication modalities, we report an analysis of the responses given by grade 9 students through online questionnaires following their experience with DMD. We aim at looking for insights into students' interpretations and assessments of how a DMD is structured and operates, potentially revealing areas of strength and opportunities for improvement in future redesign.*

*Keywords: Digital mathematical discussion, asynchrony, written communication.*

## Introduction and theoretical background

In recent years, digital technologies have played an increasingly central role in educational environments, particularly by offering new opportunities for extending collaboration and communication beyond the traditional classroom setting (Ball & Barzel, 2018). According to Ball and Barzel (2018), there is still significant, understudied potential in utilising social networks and other communication technologies to connect students learning mathematics from different locations. Moreover, when considering communication through technology, the nature of technology use transforms how students engage with, learn, and think about mathematics (Ball & Barzel, 2018). In this perspective, Digital Mathematical Discussion (DMD) has been introduced to allow students to collaborate, discuss, and co-construct mathematical knowledge also outside the physical space of the classroom (Gagliani Caputo et al., 2023). DMD is framed within mathematical discussion (Bartolini Bussi, 1991) and asynchronous discussion (Wang, 2023). Specifically, DMD relies on the concept of mathematical balance discussion, in which students socialise and evaluate strategies set up in their individual or small-group work on a mathematical problem (Bartolini Bussi, 1991), and it is realised in asynchronous modality with support of specific digital environments that facilitate its unfolding. The discussion is guided by the teacher organising and sequencing excerpts from students' solutions for comparing similar or contrasting strategies (Cusi et al., 2017). The teacher orchestrates the discussion acting as a model of aware and effective attitudes and behaviours (Cusi & Malara, 2013) aiming at making thinking visible to students and at fostering metacognitive reflections.

In this contribution, we consider students' and designers' perspectives on the main features of DMD with the aim of highlighting synergies or dissonances between them. When referring to features, we consider Trouche's (2004, p. 290, footnote 6) distinction between tools' constraint (obliging or impeaching the user), enablement (making the user able to do something), potentiality (virtually opening possibilities), and affordance (favouring particular gestures). In the case of the features of DMD, we interpret Trouche's definitions referring to *constraints* as the norms and rules that shape

how the discussion unfolds, to *enablement* as the conditions or resources that allow participants to contribute effectively in the discussion, to *potentialities* as the range of possibilities that the discussion opens up for participants, and to *affordances* as the features that favour certain types of involvement in the discussion.

## Design of Digital Mathematical Discussion and its key features

DMD has been designed by the authors (Gagliani Caputo et al., 2023) and consists of three phases:

1. The first phase involves students in small-group work (4-5 students per group) for developing strategies for solving mathematical problems. The mathematical problems can vary depending on the objectives the teacher aims to achieve through the discussion. This phase takes place over some days (3-4 days in our study) within chats on an instant messaging platform (e.g., WhatsApp). The teacher is present in the group chats but only intervenes if explicitly requested by students. At the end of this phase, one student from each group submits the collective solution to the teacher via an institutional platform (e.g., Google Classroom).
2. The second phase is an asynchronous whole-class discussion orchestrated by the teacher with the aim to guide students in analysing, comparing, and reflecting on the strategies proposed by the groups in the first phase (Cusi et al., 2017). The discussion occurs over multiple days (3-4 days in this study) on a collaborative web platform (Padlet). The teacher structures columns in the Padlet asking questions on selected excerpts from the group solutions (Gagliani Caputo et al., 2023). Students and the teacher can add posts containing text, images, or attachments, and can insert comments and react to posts through upvotes or downvotes.
3. The third phase, following the whole-class discussion on Padlet, is a one-hour in-person discussion in the classroom to summarise and consolidate what emerged on previous phases.

The first two phases of DMD are conducted online and asynchronously as homework over several days. Students are asked to make all thought processes explicit, including problem-solving steps, doubts, new ideas, difficulties, and individual explorations, so that all participants can see the reasoning. Moreover, they are asked to interact on the activity only through the digital environment provided for the discussion (chat and Padlet) taking advantage of the technical features provided by the environments (e.g., sending pictures or attaching documents) and to maintain communication in written form. The digital environment was selected by the designers through an analysis to ensure it closely matched the desired settings for the DMD and could adequately support its implementation.

In this paper, we focus exclusively on the first two phases of DMD, as we are interested in investigating asynchronous modality and written communication. The asynchronous nature of DMD allows participants to engage with discussions at their own pace, affording flexibility that enables students to reflect, process information, and contribute thoughtfully without the constraints of synchronous timing (Wang, 2023). This flexibility is particularly beneficial for accommodating diverse learning styles and schedules, as students can revisit discussion threads, formulate responses over time, and manage their commitments. Similarly, written communication supports clarity and permanence, and provides a transcript of the discussion that can be referenced later (Wang, 2023). Moreover, as highlighted by Santos and Semana (2015), effective communication through writing is crucial for developing a deeper understanding of mathematics. Writing aids students in clarifying and refining their ideas, and in presenting and justifying their problem-solving methods formally.

Asynchrony and written communication can be characterised as key features of DMD as presented in Table 1. The characterisation is based on the design of DMD presented above and introduces the designers' perspective on asynchrony and written communication as key features of DMD.

**Table 1: Designers' characterisation of asynchrony and written communication as features of DMD**

| | **Constraints** | **Enablement** | **Potentialities** | **Affordances** |
|---|---|---|---|---|
| **Asynchrony** | It is required not to interact with others in synchronous modality<br>Time to submit contributions and to refresh to see new contributions | The duration of the discussion is set over multiple days<br>Engagement of participants who feel less comfortable in real-time interaction | Exploration of concepts over time<br>Introduction of ideas that open up new directions for the discussion | Flexibility in participation |
| **Written communication** | Impossibility of exploring functionalities that do not involve written communication<br>Lack of body language for non-verbal communication | Making thinking processes explicit for both the one writing a contribution and those who read it<br>Engagement of participants who feel writing as support for developing their reasoning or reflecting on others' contributions | Control over the whole discussion thanks to the availability of its complete transcript and consequent potential activation of metacognitive reflections on the structure of the discussion and on the links between its parts | Encourage structured contributions<br>Encourage the use of mathematical symbols |

## Context, research question and methods

The study presented in this paper is part of the 2nd and 3rd cycles of a wider design-based research project (Cobb et al., 2003). These cycles involved two Italian science-oriented high schools and took place between February and April in 2023 and 2024. The experiments were conducted with two grade 9 classes and their teachers for each cycle. The mathematical problems students faced during the activities were focused on using algebraic language as a tool to construct proofs in arithmetic.

In this paper, we aim to investigate synergies and dissonances between designers' and students' perspectives on asynchrony and written communication as key features of DMD, in order to highlight relevant aspects of DMD design that should be reconsidered for re-design. The study is guided by the following research question: How do students perceive asynchrony and written communication as key features of DMD in their articulation as constraints, enablement, potentialities and affordances?

In order to answer the research question, we collected students' responses to an online written questionnaire with open-ended questions that highlighted their reflections on the role of the asynchronous modality and written communication in DMD after completing the four activities of the experimentation. The questions aimed to investigate students' perspectives on the challenges they

encountered during the asynchronous phases of the DMD, the usefulness of having access to their peers' written messages, the benefits and weaknesses of working asynchronously, and their reflections on the differences between working in-person and in the different digital environments involved in the DMD. Specifically, to gather students' perspectives on the asynchronous phases of DMD, their reflections were analysed with a focus on the perceived usefulness of having time to read and respond to messages, as well as the value they attributed to accessing written contributions from group members. Although the latter aspect focuses specifically on the availability of peers' written contributions during group work, students' answers were general enough to allow us to analyse their broader opinions about the written communication modality in DMD. We collected 65 students' responses to the questionnaire and we analysed only the 63 responses containing answers to both questions. The analysis of students' answers to the questionnaire is conducted through a thematic analysis. The analysis is conducted reading all students' answers and grouping them according to common themes emerging from the content of their answers with respect to asynchrony and written communication in DMD. The emerging themes are articulated as constraints, enablement, potentialities and affordances recognised by students. In particular, a theme concerns constraints if it highlights students' perceived difficulties or limitations, enablement if it refers to what actually allows them to participate in the DMD, potentialities if it concerns new possibilities DMD offers to students, and affordances if it relates to what students experience in the DMD in relation to their modes of involvement.

## Analysis and results

The analysis of the answers given by students to the questionnaire allow us to identify themes of students' perceptions concerning the asynchronous modality and the written communication in DMD as constraints, enablement, potentialities, and affordances. In this section we present the results of the analysis that consist in the identification of themes based on students' reflections and the percentage of answers pertaining to each theme (the total percentage does not add up to 100% because each response may fall into multiple themes).

Concerning the asynchronous modality of DMD, four perceived themes emerged in students' answers: (1) flexibility in work management (22% of answers); (2) availability of time to reflect on one's contribution (60% of answers); (3) availability of time to reflect on others' contributions (18% of answers); (4) lack of pressure to respond immediately (3% of answers).

Students' answers in theme 1 concern how the asynchronous modality supports them at the organisational level both as enablement and affordance. Indeed, they noted that the flexibility in time allows them to effectively taking part in the discussion (enablement) but also that it shapes their time of interaction within the discussion (affordance), as they could choose when to participate in discussions without restrictions on their peers' timing ("I didn't have any problems coordinating with others, and I could write in the group whenever I had time"), and manage their participation alongside other school's activities ("I was able to organise myself better with the rest of my homework") and personal commitments ("With my personal commitments, I wouldn't have always been able to write"). Themes 2 and 3 show students' exploitation of the potentiality identified by the designers of exploring concepts over time. Students reported that the possibility to take their time allowed them

to reason more thoroughly ("[taking my own time] gave me the opportunity to have all the time to reason") both revisiting their own reasoning multiple times before submission ("one can review their ideas more calmly") and reading and understanding others' contributions more easily due to the ample time available ("I had the opportunity to read and understand others' contributions more easily, given that I had a lot of time available [for the activity]"). Theme 4 includes students' interventions who state explicitly that they "didn't have the pressure to respond immediately". Even though theme 4 was identified in only a few students' contributions, it is still relevant to be considered, as it highlights a potentiality of DMD recognized by the students but only partially addressed in the design. With respect to asynchrony, students were generally positive in their comments, nevertheless some have identified some constraints. For theme 1, students noted that the freedom to organise their timing sometimes led to procrastination and close proximity to deadlines ("You could work on the activity whenever you wanted, but sometimes, there was a tendency to always postpone the work, thus delaying it until almost the deadline"), or to forgetting to respond ("Sometimes they [peers' contributions] would arrive while I was working on other assignments, and I needed to focus, and then I would forget to respond"), or losing the line of reasoning ("If you take time to respond you risk losing the line of reasoning, which in turn results in wasting more time").

Regarding the use of written communication, three themes were identified. According to students, written communication in DMD supports: (A) read and reflect on others' reasoning while discussing (25% of answers); (B) compare others' reasoning with one's own (34% of answers); (C) re-read old contributions (42% of answers).

In theme A, students reflected on the enablement of written communication for accessing others' contributions more easily within the discussion. Examples include statements such as "[having access to others' written contributions] was helpful for comparing and seeing the different types of solutions" and "I was able to better understand what my classmates wanted to communicate". In theme B, students noted that having access to written contributions enabled them to revise their own opinions ("I was able to have direct comparisons between my opinions and those of my classmates, and on many aspects that I thought were clear, I changed my mind") and to find new insights ("[others' written contributions] were tools for gaining insights, but it was also challenging to read them and sometimes not understand them"). Theme C concerns students' exploitation of the potentiality identified by the designers concerning the availability of the transcript of the discussion. Students highlighted the possibility to re-read contributions once the discussion has moved on or finished to gain a better understanding of others' interventions ("since they [others' contributions] were written, they could be reviewed multiple times and reflected upon more deeply"), revisit one's own reasoning ("[having access to written contributions was useful] to revisit my reasoning even after days"), avoid forgetting important points during the discussion ("I didn't risk forgetting them [others' contributions]"), recover from missed parts of the conversation ("It was useful because I sometimes lost track of the conversation when others were writing in my absence, but by re-reading the messages, I was still able to understand what had been discussed"), or reconstruct the entire work ("It was easier to trace the development of the work"). However, few students expressed a perceived constraint that was not completely considered in the design, that the challenge to deal with long written contributions ("Having so much written in a single message is really annoying").

Students' comments focus also on combined features concerning asynchrony and written communication, such as the possibility to use the written contributions at their own pace to develop individual reasoning, and the availability of the written transcript of the entire workflow and discussion to all participants at any time. Regarding the first aspect, some students highlighted a potentiality of written communication, stressing that taking their own time to reflect on their own contribution was possible because they were writing their reasoning and that it wouldn't have been possible otherwise, such as using vocal messages ("before sending any message, there is time to review and correct any possible errors (which is not the case when sending a voice message)") or calls ("it was useful because I could respond when I had time. Therefore, it was more convenient than a phone call and offered more flexibility."). Regarding the second aspect, students shared their views on the availability of transcripts in response to the question "Do you think having access to the messages (both on chats and Padlet) even after the activities ended was helpful? Why? Did you find yourself going back to re-read messages (both on chats and Padlet) after the activities ended?". Forty students said that having access to the transcripts was beneficial and that they used them for various reasons: to clarify initially unclear issues ("I found myself going back to re-read the messages at the end of the activity to try to reflect on and understand some questions that were initially unclear"), to support subsequent class work ("In my opinion, keeping the messages was useful because, for example, while we were discussing in class, we could review our written reasoning from the chat"), to review the entire activity ("The messages show the progression of the entire activity, allowing one to 'retrace' it"), or to review parts of the activity when transitioning between phases of the discussion ("I found myself re-reading the passages when we moved from the chat to Padlet to understand our reasoning" or "[it was helpful] to compile the summary of the discussion that I then submitted on Classroom"). On the other hand, 29 students reported that they did not take advantage of the opportunity to revisit the messages. Many of them (21) do not recognise the enablement and affordances of written communication considered in the design, as they declare that having the transcripts was not useful at all ("I did not revisit the messages after the end of the activity because, once the posts were read, the reasoning was understood and memorised, making it unnecessary to go back"). Other 4 students, instead, acknowledged the potential future utility of having all contributions saved ("I did not review the messages after the end of an activity, but it is useful to have them saved in case they might be needed in the future"), even if at the moment they experienced DMD they have not recognised the availability of the transcript of the discussion as a potentiality.

## Discussion and conclusion

The analysis of students' answers to the online questionnaire following their experience with DMD allow us to answer the research question by characterising their reflections on the asynchronous modality and the written communication in DMD and highlighting alignment and divergence in how the key features of DMD were characterised by designers. Students' responses reveal a more complex interaction with these features, introducing new insights that were only partially anticipated by the designers. Concerning asynchrony, for designers, flexibility (theme 1) is seen primarily as an affordance, allowing participants to engage in the discussion at their own pace. This feature aligns with students' perspectives, but according to them, flexibility functions as both an enablement and an affordance. It enables participation by allowing students to engage in the discussion according to their

own schedules, making it possible for them to manage their time effectively and participate despite other commitments. This aligns with the designers' intentions but goes further as students perceive it as actively shaping their interaction with the discussion. However, this flexibility also introduces a constraint that was not fully considered by designers: some students experience a loss of control over their work, leading to procrastination, forgotten responses, and difficulties maintaining the flow of reasoning. This reveals a dissonance between the designers' idea of flexibility and the practical challenges students face when this flexibility is not well managed. Moreover, designers emphasised the potentiality of asynchrony for exploring concepts over time. This potentiality is exploited by students, as reflected in their recognition of the benefits of having time to reflect on both their own and others' contributions (themes 2 and 3). Students describe how this extended time allows them to carefully reason, revisit, and refine their ideas before submission, as well as to understand others' contributions more deeply. This alignment between designers' intentions and students' experiences suggests that the potential of asynchrony to enhance reflective thinking is being effectively realised in practice. A key difference between students' and designers' perspectives emerges around the emotional dimension of participation, particularly the lack of pressure to respond immediately (theme 4). While designers considered the emotional aspects of asynchrony only partially, they did not specifically identify this as a significant feature. However, for students, the reduced pressure is perceived as a new potentiality of asynchronous discussions. This emotional benefit, which allows students to engage without the stress of immediate responses, fosters a more relaxed and thoughtful approach to participation, in line with Wang's (2023) vision of online discussion as a "relaxing environment" (p. 3) in which students can participate at their convenience. This aspect adds a new dimension to the designers' understanding of asynchrony and suggests that the emotional impact of flexible timing should be given more consideration in future design iterations.

Concerning written communication, designers identify it as a key affordance that facilitates reflective and structured engagement and thinking within discussions (as in Wang, 2023), but students' experiences introduce additional insights and challenges that were not fully anticipated. Designers characterise the ability to read and reflect on others' contributions (theme A) as an enablement, intending to make thinking processes more explicit. This aligns with students' perspectives, as they view this feature as an enablement that allows them to easily access others' reasoning within the discussion. Also concerning theme B, students' and designers' perspective align since written communication serves as an enablement that supports participants in comparing others' reasoning with their own, fostering deeper understanding and self-reflection. Themes A and B also align with the use of written communication experienced by Santos and Semana (2015) to introduce and justify reasonings in a clearer way to themselves and to others. Designers identify the potentiality of re-reading old contributions (theme C) as a feature of written communication, allowing participants to engage in metacognitive reflection. This potentiality is actively exploited by students, who highlight the advantages of being able to re-read contributions once the discussion has moved on or concluded. However, some students perceive a constraint in managing long written contributions, finding them challenging to navigate, an issue that was not fully considered in the design. This suggests a need for more user-friendly ways to manage extensive written content in future redesign.

Overall, the comparison reveals that while there are significant synergies between designers' intentions and students' experiences, some dissonances and new insights also arise. Students highlight new potentialities regarding the emotional benefits of reduced response pressure, but also new constraints, suggesting that further refinements could enhance the usability and effectiveness of asynchrony and written communication in DMD. Concerning the potentiality of having a transcript of the discussion identified by designers, just some students used it for further work even if most of them recognised the importance of having it for possible future uses. The ability to go back, review, and rethink is an attitude promoted through discussion and students should be taught how to exploit the potentiality of having at disposal the transcript of the discussion. In future experimentations this can be realised asking students to re-read and comment on some excerpts of their previous asynchronous discussion from the transcripts.

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