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RESEARCH ARTICLE

# Group size as selection device

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**Abstract**
In a coordination game with multiple Pareto ordered equilibria and population uncertainty, we show that group size helps select a unique equilibrium, for reasons reminiscent of the global games literature. A critical mass phenomenon emerges at equilibrium. Group size has an emboldening effect on participants.

**Keywords** Poisson games · Coordination games · Equilibrium selection · Global games

**JEL Classification.** C72 · D82

## 1 Introduction

Consider the decision to subscribe to a new on-line platform for conversation in a foreign language. Subscription is costly: the cost is worth bearing if enough people subscribe to the platform but not otherwise; although there may be some enthusiasts who cannot wait to start the conversation, most people are willing to subscribe if a sufficient number of others do, but not otherwise. The number of potential subscribers is unknown.

The platform subscription dilemma can be represented formally as a coordination game[1] with population uncertainty. Modeling population uncertainty as in Myerson

[1] Precisely, a stag hunt, with a terminology that dates back to Jean Jacques Rousseau (see Binmore 1994, p. 120).

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(1998a) results in a Poisson game in which the number of potential subscribers is drawn from a Poisson distribution with a known expected number of subscribers. This game has a unique Nash equilibrium when the subscribers are expected to be few, in which case nobody, except the enthusiasts, subscribes to the platform, or many, in which case everyone subscribes, but has multiple Nash equilibria when the expected number of subscribers is midsize.

It turns out that, if the expected number of potential participants is also unknown but each person receives a slightly imprecise message about how large the participation is expected to be, a unique Nash equilibrium of the underlying game can be selected in all circumstances. In other words, a perturbation of the players' information about the expected number of players in the game enables equilibrium selection. The argument underlying this result resembles the one used in the global games literature à la Carlsson and Van Damme (1993a) and Morris and Shin (1998), where equilibrium selection is made possible by perturbing the players' information about payoffs. [2]

Given that the expected size of participation is not common knowledge among potential subscribers, the possibility that participants may be either few or many exerts an influence on actual situations through the hierarchy of mutual beliefs, helping select a unique equilibrium for any expected participation size. The equilibrium selected depends on the expected number of participants. Everybody subscribes when this expected number is sufficiently large, as the presence of a few enthusiasts exerts a pull on the undecided people, but not otherwise.

The platform subscription dilemma gives rise to a critical mass phenomenon of the type discussed by Schelling (1978). Critical mass phenomena have been examined in the theoretical literature under the rubric of regime change, using the terminology of Angeletos et al. (2006, 2007), as coordination games in which the status quo is abandoned if enough players take action against it. As in global games, a unique equilibrium can be selected when the strength of the status quo is not common knowledge among the players. In our model, it is the (actual and expected) number of participants in the game that is not common knowledge. Hence, the selection device is the overall size of the group, rather than the strength of the status quo. The contribution of this paper lies in the identification of a different source and interpretation of the uncertainty relative to the global games literature. In standard global games, noise affects players' information about payoffs, while in our framework the uncertainty concerns the size of participation. This is not a mere technicality, as it provides a novel channel through which coordination outcomes depend on an underlying fundamental with a different economic interpretation and new testable implications.

There is well-documented experimental evidence that group size matters for equilibrium selection in coordination games with multiple Pareto ordered equilibria, for example Van Huyck et al. (1990) and Weber (2006) for experiments on the minimum effort game à la Bryant (1983), Arifovic et al. (2023) on bank runs à la Diamond and Dybvig (1983) and Kim et al. (2026) on the classical stag hunt game. In our

[2] Originally, the selection result was proved in a stag hunt game with two players. Carlsson and Van Damme (1993b) have extended it to a stag hunt game with a finite number of players. Subsequently, the result has been extended further by Frankel et al. (2003) to a continuum of players and multiple actions with strategic complementarity. See (Morris and Shin 2003) for a comprehensive review of the literature on global games.

framework, what matters is not the actual group size per se, since it is indeed random and unobserved, but rather the role of uncertainty about group size, specifically how beliefs about the expected group size, and the uncertainty surrounding those beliefs, influence equilibrium selection.

In the following, we recast some critical mass phenomena, such as political protests of the type examined in Atkeson (2001) and Edmond (2013), as stag hunt Poisson games, finding an emboldening effect of group size. We also apply the framework to the generalized prisoner's dilemma à la Kalai (2024), finding that gangsters confess to the crime if the expected number of members of the gang who are being held for questioning is large but not otherwise.

The Poisson game framework, a standard tool for modeling population uncertainty, is especially suitable for an equilibrium selection theory based on the number of players, since the expected population size is a continuous variable and a sufficient statistic of the entire distribution. Moreover, Poisson games have several unique properties that make the analysis tractable (see Myerson 1998a). Poisson games have been applied to model scenarios with large but finite populations, such as general elections (e.g. Myerson 2000, 2002). Battaglini (2017) contains an application to political protests. [3] In a Poisson game with strategic complementarity, Makris (2008) has shown that the equilibrium is unique for a sufficiently small population. Here, instead, we show that, even when the underlying stag hunt Poisson game has multiple equilibria, a small amount of incomplete information about the expected number of players helps select a unique equilibrium for any expected population size.

Uncertainty about the expected population size in Poisson games was introduced by Myerson (1998b) within the framework of extended Poisson games and applied to the study of the Condorcet jury theorem. Ekmekci and Lauermann (2022) analyze a related framework in which uncertainty depends on a binary state of the world. They address the issue of multiple equilibria by studying their robustness to the ad hoc expectational stability refinement introduced by Fey (1997). [4]

The paper proceeds as follows. In Section 2, we present the model. In Section 3, we find the equilibria of the underlying stag hunt Poisson game. In Section 4, the selection procedure is carried out. The applications are in Section 5. Section 6 concludes. Appendix A contains a summary of Poisson games and their main properties. Appendices B, C and D contain the proofs of key results of Section 4.

## 2 The model

Consider a binary action Poisson game $\Gamma$. The number of agents is a Poisson random variable with mean $n \in \mathbb{R}_{++}$. We refer to the parameter $n$ as the *population size*. Given $n$, the actual number of agents is $m$ with probability

[3] Economic applications include Makris (2009) for the provision of public good; Ritzberger (2009) and De Sinopoli et al. (2023) for the Bertrand and Cournot competition, respectively; Lauermann and Speit (2023) for auctions.

[4] Within the standard Poisson framework, Bouton and Gratton (2015) tackle the multiplicity problem arising in the context of majority runoff elections using the solution concept of strict perfection introduced by Okada (1981). De Sinopoli et al. (2014) develop a general theory of strategic stability for Poisson games, following the approach of Kohlberg and Mertens (1986).

$$P(m \mid n) = \frac{e^{-n} n^m}{m!}.$$

The interaction resembles the classical "stag hunt" game. Players can choose between a safe action $S$ – corresponding to hunting the hare – which yields a certain payoff irrespective of what other agents do and a risky action $R$ – corresponding to hunting the stag – which yields a payoff that increases in the number of agents choosing it.

Agents can be of three types. Agents of type 1 are the actual players of the game with action set $A = \{S, R\}$. We will refer to them simply as *players*. Agents of type 2 are automata who always take the risky action, while agents of type 3 are automata who always take the safe action. [5] We will refer to them as *risky automata* and *safe automata*, respectively. A randomly sampled agent is a risky automaton with probability $r \geq 0$, a safe automaton with probability $s \geq 0$, and an actual player with probability $p = 1 - r - s > 0$.

Let $x$ be the total number of other agents who choose action $R$. [6] The payoff to a player of choosing $S$ is independent of $x$, while the payoff of choosing $R$ increases in steps in $x$, with finitely many steps. [7] Let $k = (k_1, ..., k_I)$, where $k_i \in \mathbb{Z}_+$ for $i = 1, \ldots, I$. The payoffs of a player are normalized to

$$u(S, x) = v \quad \text{for every } x,$$

with $v \in (0, 1)$, and

$$\begin{aligned} u(R, x) &= 0 \quad \text{if } x \leq k_1, \\ u(R, x) &= u_i \quad \text{if } k_i < x \leq k_{i+1} \quad \text{for } i = 1, \ldots, I - 1, \\ u(R, x) &= 1 \quad \text{if } x > k_I, \end{aligned}$$

with $u_i \in [0, 1]$ for $i = 1, \ldots, I - 1$ and $u_{i'} \geq u_i$ for $i' > i$. We let $u_0 = 0$ and $u_I = 1$.

Population uncertainty implies that all players choose the same strategy $\sigma \in \Delta(A)$ (see Appendix A). A strategy $\sigma$ induces the population average behavior $\tau(\sigma) \in \Delta(A)$ which is defined by

$$\begin{aligned} \tau(\sigma)(S) &= p\sigma(S) + s, \\ \tau(\sigma)(R) &= p\sigma(R) + r = 1 - \tau(\sigma)(S). \end{aligned}$$

When players play according to $\sigma$, $\tau(\sigma)(a)$ is the probability that a randomly sampled agent takes the action $a \in A$. We will often avoid specifying the dependence of $\tau$ on

[5] Alternatively, one can think of automata as players who always strictly prefer one action over the other, so that action will be chosen with probability 1 in any Nash equilibrium.

[6] In the description of Poisson games in Appendix A, $x$ denotes the entire action profile. We slightly abuse notation since only the number of agents choosing $R$ is relevant.

[7] We assume finitely many steps for simplicity. We could assume infinitely many steps as long as utilities remain bounded.

$\sigma$, and we let $\tau_R = \tau(\sigma)(R)$. The probability that exactly $x$ agents choose the risky action is given by

$$P(x \mid n\tau_R) = e^{-n\tau_R} \frac{(n\tau_R)^x}{x!},$$

and the expected payoffs of a player are given by

$$U(S, \tau \mid n) = v,$$
$$U(R, \tau \mid n) = \sum_{x=0}^{\infty} P(x \mid n\tau_R) u(R, x).$$

We recall the standard concepts of dominated strategy and Nash equilibrium.

**Definition 1** Strategy $\sigma$ is *dominated* by strategy $\sigma'$ if $U(\sigma, \tau \mid n) \leq U(\sigma', \tau \mid n)$ for every $\tau \in \Delta(A)$ and $U(\sigma, \tau' \mid n) < U(\sigma', \tau' \mid n)$ for some $\tau' \in \Delta(A)$.

**Definition 2** The strategy $\sigma$ is a *Nash equilibrium* of the Poisson game $\Gamma$ if $U(\sigma, \tau(\sigma) \mid n) \geq U(\sigma', \tau(\sigma) \mid n)$ for every $\sigma' \in \Delta(A)$.

We say that the average behavior $\tau$ is an *equilibrium behavior* (or, simply, an *equilibrium*) if it is induced by a Nash equilibrium.

## 3 Equilibria

The space of population sizes $\mathbb{R}_{++}$ can be partitioned into intervals according to the equilibrium outcomes induced in the corresponding Poisson game $\Gamma(n)$. There are three intervals if the probability $r$ of risky automata is strictly positive and two intervals otherwise.

Let the Poisson cumulative distribution function be given by

$$F(\bar{m} \mid n) = \sum_{m=0}^{\bar{m}} P(m \mid n)$$

for every $\bar{m} \in \mathbb{Z}_+$. The expected payoff to a player of choosing $R$ can be written as

$$U(R, \tau \mid n) = \sum_{i=1}^{I-1} \big[F(k_{i+1} \mid n\tau_R) - F(k_i \mid n\tau_R)\big] u_i + 1 - F(k_I \mid n\tau_R)$$
$$= 1 - \sum_{i=1}^{I} F(k_i \mid n\tau_R)(u_i - u_{i-1}).$$

For every $i = 1, \ldots, I$, the function $F(k_i \mid n\tau_R)$ is continuous and strictly decreasing in $n\tau_R$ for $n\tau_R > 0$.[8] Consequently, $U(R, \tau \mid n)$ is strictly increasing in $n\tau_R$ in this region. Moreover, $P(0 \mid 0) = 1$ by convention and $\lim_{n\tau_R \to \infty} F(k_i \mid n\tau_R) = 0$.

[8] Recall that, given $n' > n$, the Poisson distribution with parameter $n'$ first order stochastically dominates the one with parameter $n$.

Let $\underline{n}$ be the value of the population size such that

$$1 - \sum_{i=1}^{I} F(k_i \mid \underline{n}(p+r))(u_i - u_{i-1}) = v,$$

that is, a player is indifferent between actions $S$ and $R$ when both the risky automata and all the other players play $R$. If $r > 0$, let $\overline{n}$ be the value of the population size such that

$$1 - \sum_{i=1}^{I} F(k_i \mid \overline{n}r)(u_i - u_{i-1}) = v,$$

that is, a player is indifferent between actions $S$ and $R$ when only the risky automata choose $R$. If $r = 0$, let $\overline{n} = \infty$. The next proposition summarizes the Nash equilibria of the Poisson games $\Gamma(n)$ for all possible values of $n$.

**Proposition 1**

*i) For $r \geq 0$ and $n < \underline{n}$, the game $\Gamma(n)$ has a unique Nash equilibrium, in which $\tau_R = r$.*

*ii) For $r \geq 0$ and $n = \underline{n}$, the game $\Gamma(n)$ has two Nash equilibria, in which, respectively, $\tau_R = r$ and $\tau_R = p + r$.*

*iii) For $r \geq 0$ and $\underline{n} < n < \overline{n}$, the game $\Gamma(n)$ has three Nash equilibria, in which, respectively, $\tau_R = r$, $\tau_R = p + r$, and $\tau_R = \underline{n}(p + r)/n$.*

*iv) For $r > 0$ and $n = \overline{n}$, the game $\Gamma(n)$ has two Nash equilibria, in which, respectively, $\tau_R = r$ and $\tau_R = p + r$.*

*v) For $r > 0$ and $n > \overline{n}$, the game $\Gamma(n)$ has a unique Nash equilibrium, in which $\tau_R = p + r$.*

***Proof*** i) If $n < \underline{n}$, a player always prefers to choose action $S$ rather than action $R$, even when all the other players play $R$.

ii) If $n = \underline{n}$, a player is indifferent between choosing $S$ and $R$ when all the other players play $R$, while she prefers to choose $S$ rather than $R$ for every other behavior. [9]

iii) If $\underline{n} < n < \overline{n}$, we have

$$1 - \sum_{i=1}^{I} F(k_i \mid nr)(u_i - u_{i-1}) < v$$

and

$$1 - \sum_{i=1}^{I} F(k_i \mid n(p+r))(u_i - u_{i-1}) > v,$$

[9] Thus, the equilibrium where $\tau_R = r$ is strict, while the equilibrium where $\tau_R = p + r$ is dominated.

that is, a player prefers to choose action $S$ (resp. $R$) rather than action $R$ (resp. $S$) if all the other players play $S$ (resp. $R$). If $\tau_R = \underline{n}(p+r)/n$, we have

$$1 - \sum_{i=1}^{I} F(k_i \mid n\tau_R)(u_i - u_{i-1}) = v,$$

that is, a player is indifferent between choosing $S$ and $R$ given $\tau_R$.

iv) If $r > 0$ and $n = \overline{n}$, a player is indifferent between choosing $S$ and $R$ when no other player plays $R$, while she prefers to choose $R$ rather than $S$ for every other behavior. [10]

iv) If $r > 0$ and $n > \overline{n}$, a player always prefers to choose $R$ rather than $S$, even when no other player plays $R$.

□

The multiplicity of equilibria in the region $(\underline{n}, \overline{n})$ cannot be addressed using standard strategic stability principles, as all three equilibrium points are stable sets as defined in De Sinopoli et al. (2014). In broad terms, a stable set of a Poisson game is a minimal subset of Nash equilibria such that every close-by game obtained through perturbations of the average behavior has a Nash equilibrium close to the stable set. As in standard games, also in Poisson games strict equilibria are robust to any possible perturbation. For the equilibrium in mixed strategy, any perturbation of the average behavior in the definition of stable set can be compensated by the players' mixed action.

As we will see in Section 4, the equilibrium multiplicity problem can be solved by introducing some uncertainty about the expected number of agents $n$.

### 3.1 Example: the on-line platform dilemma

Consider the on-line platform subscription dilemma illustrated in the Introduction. For the sake of symmetry, we will assume that there are people who are both very keen and very skeptical about subscription. Suppose that the probabilities of the different personalities are $p = 5/7$ and $r = s = 1/7$. Let the payoff of staying out be equal to 0.6, and the payoff of subscription be equal to 0 in case no one else subscribes and to 1 if at least another person does. If all other people subscribe except those who are skeptical, the payoff of subscribing is given by

$$1 - P\left(0 \,\middle|\, \frac{6}{7}n\right),$$

which is the probability that at least another person subscribes. The above expression is equal to 0.6 for $\underline{n} \approx 1.069$. Thus, whenever the expected number of participants is smaller than 1.069, an undecided person will always be better off staying out, as the probability of being alone on the platform even in the most favorable event is too high. On the other hand, in the least favorable event in which only people who are very keen to subscribe do so, the payoff of subscribing is

[10] Thus, the equilibrium where $\tau_R = r$ is dominated, while the equilibrium where $\tau_R = p + r$ is strict.

$$1 - P\left(0 \,\middle|\, \frac{1}{7}n\right).$$

This is equal to 0.6 for $\overline{n} \approx 6.414$. So, whenever the expected number of participants is greater than 6.414, an undecided person will always be better off subscribing, as the probability of meeting someone is always high enough. For intermediate participation sizes, the outcome is not determined. There is an equilibrium where all the people stay out of the platform, since

$$1 - P\left(0 \,\middle|\, \frac{1}{7}n\right) < 0.6,$$

an equilibrium where they all subscribe to the platform, since

$$1 - P\left(0 \,\middle|\, \frac{6}{7}n\right) > 0.6,$$

and an equilibrium where they choose to subscribe with probability $\sigma_R = \frac{\overline{n}-n}{5n}$, since

$$1 - P\left(0 \,\middle|\, \left(\frac{1}{7} + \frac{5}{7}\sigma_R\right) n\right) = 0.6.$$

At the end of the next section, it will be possible to determine a unique equilibrium outcome for any given participation size.

## 4 Equilibrium Selection

In this section, we provide an equilibrium selection criterion for the Poisson games in the indeterminacy region $(\underline{n}, \overline{n})$. The criterion is obtained perturbing the players' information about the population size $n$ and assuming that each player observes only a noisy signal of $n$. For the selection argument to apply, such a perturbed game must assign strictly positive probability to risky automata. Therefore, we also introduce a perturbation $\rho$ of $r$.[11] The perturbed game has, essentially, a unique equilibrium. Such a uniqueness is preserved in the original (unperturbed) game, which is recovered as the limit when, first, the noise in the observations vanishes and, then, $\rho$ approaches $r$.

### 4.1 The perturbed game

Assume that all the parameters of the Poisson game $\Gamma$ are common knowledge except the population size $n$, which is observed by the players only with some slight noise. Assume also that the probability that an agent is a risky automaton is strictly positive and equal to $\rho > 0$, and let $\overline{n}^{\rho}$ be the value of the population size such that

[11] As we will see, a strictly positive probability of risky automata is needed in the perturbed game for the existence of a region of signals where the risky action is strictly dominant. If the baseline Poisson game has $r > 0$, we could simply set $\rho = r$. If $r = 0$, we must instead consider some $\rho > 0$.

$$1 - \sum_{i=1}^{I} F(k_i \mid \overline{n}^{\rho}\rho)(u_i - u_{i-1}) = v.$$

The resulting incomplete information game is denoted $\Gamma_{\varepsilon,\rho}$ and can be described as follows:

1. Nature selects the size of the population $n$ according to the uniform density over the interval $[n', n'']$, where $n' < \underline{n}$ and $n'' > \overline{n}^{\rho}$.
2. When the population size is $n$, each player observes a signal that is drawn uniformly from the interval $[n - \varepsilon, n + \varepsilon]$ for some small $\varepsilon > 0$. [12] Conditional on $n$, the signals are identical and independent across players. [13]
3. Based on the observed signal, the players choose between safe and risky actions. Safe and risky automata choose, respectively, actions $S$ and $R$ with probability 1 independently of their observation.
4. Payoffs are determined by $\Gamma$ and the players' choices.

A strategy for the players in the game $\Gamma_{\varepsilon,\rho}$ is a function that assigns a mixed action to each observation. Given the strategy function $\tilde{\sigma}$, we denote by $\tilde{P}_{\tilde{n}}(x \mid \tilde{\sigma}, n)$ the probability for a player with signal $\tilde{n}$ that the number of other agents choosing $R$ is $x$, when the population size is $n$. A player's posterior of $n$ if she observes a signal $\tilde{n} \in [n' + \varepsilon, n'' - \varepsilon]$ will be uniform on $[\tilde{n} - \varepsilon, \tilde{n} + \varepsilon]$. In this case, the player's conditional expected payoffs when the other players play according to $\tilde{\sigma}$ are given by

$$\tilde{U}_{\tilde{n}}(S, \tilde{\sigma}) = v,$$
$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}) = \frac{1}{2\varepsilon} \int_{\tilde{n}-\varepsilon}^{\tilde{n}+\varepsilon} \sum_{x=0}^{\infty} \tilde{P}_{\tilde{n}}(x \mid \tilde{\sigma}, n) u(R, x) \mathrm{d}n.$$ [14]

**Definition 3** The strategy function $\tilde{\sigma}$ is a *Nash equilibrium* of $\Gamma_{\varepsilon,\rho}$ if $\tilde{U}_{\tilde{n}}(\tilde{\sigma}(\tilde{n}), \tilde{\sigma}) \geq \tilde{U}_{\tilde{n}}(\tilde{\sigma}', \tilde{\sigma})$ for every strategy $\tilde{\sigma}'$ and signal $\tilde{n}$.

### 4.2 Uniqueness

In this subsection, we show that the game $\Gamma_{\varepsilon,\rho}$ has, essentially, a unique Nash equilibrium.

First, note that the probability for a player that the number of other players choosing $R$ is above a given threshold, when she expects them to play according to some given strategy function, increases in the set of signals for which the strategy function

[12] We need $2\varepsilon < \min\{\underline{n} - n', n'' - \overline{n}^{\rho}\}$.

[13] The assumption of uniformly distributed signals is made for the sake of concreteness. Our qualitative results remain valid as long as the signal a player receives does not provide her new information on her ranking within the population of signals. This is ensured by uniform priors on the population size and a distribution of signals that admits a continuous density.

[14] The formula must be appropriately adjusted if $\tilde{n} < n' + \varepsilon$ or $\tilde{n} > n'' - \varepsilon$.

prescribes action $R$.[15] Suppose a player observes a signal that is sufficiently small, i.e. $\tilde{n} < \underline{n} - \varepsilon$. Then, the player's conditional payoff of choosing $R$ is always lower than $v$, as it is lower for the strategy that prescribes players to choose $R$ for every possible observation. In fact, given such a strategy, we have $\tilde{P}_{\tilde{n}}(x \mid \tilde{\sigma}, n) = P(x \mid n(p+\rho))$ for every $x$, since every realized player chooses $R$ with probability 1, and $1 - \sum_{i=1}^{I} F(k_i \mid n(p+\rho))(u_i - u_{i-1}) < v$ for every $n \in [\tilde{n} - \varepsilon, \tilde{n} + \varepsilon]$. It follows that $S$ is conditionally strictly dominant at $\tilde{n}$. On the other hand, suppose that a player observes a signal that is sufficiently large, i.e. $\tilde{n} > \overline{n}^{\rho} + \varepsilon$. Then the action $R$ is strictly dominant at that observation, as the reward of choosing $R$ is higher than $v$ for every $n$ in the region. Now, consider a strategy that prescribes players to play $S$ for every observation below a given signal and to play $R$ for every observation above it. We can show that the payoff of choosing $R$ for a player at the margin of switching from $S$ to $R$ increases strictly and continuously in the cutoff signal. [16] Since that payoff is lower than $v$ for sufficiently small signals and higher than $v$ for sufficiently large ones, there exists a unique cutoff signal such that a marginal player is indifferent between the two actions. This signal characterizes the equilibrium of the game $\Gamma_{\varepsilon,\rho}$.

Given the signal $\tilde{n}$, let $\tilde{\sigma}_{\tilde{n}}$ be the strategy function that tells players to choose action $S$ for all observations smaller than $\tilde{n}$ and action $R$ for all observations larger than $\tilde{n}$. We call $\tilde{\sigma}_{\tilde{n}}$ the *cutpoint strategy* at $\tilde{n}$, and $\tilde{n}$ the *cutpoint*.

**Lemma 1** *$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}})$ is continuous and strictly increasing in $\tilde{n}$.*

The formal proof of this result is given in Appendix B. The intuition is the following. For a player with marginal signal equal to the cutpoint, the probability that exactly $x$ other players choose $R$ is equal to the probability that exactly $x$ other players have made larger observations. Uniform priors imply that for any realization of the number of other players, the probability that $x$ of them have made larger observations does not depend on the observed signal. As the signal increases, the player assigns higher values to the actual population size. Consequently, for any given threshold, the probability for a player that the population realization is above that threshold increases, and, therefore, the probability for a marginal player that the number of other players with larger observations is above that threshold also increases. Moreover, the probability for a player that the realization of risky automata is above the given threshold increases as well. It follows that the higher the cutpoint signal, the higher is the probability for the marginal player that the overall number of other agents choosing $R$ is above any given threshold, and hence the higher is her expected payoff of choosing $R$.

A standard argument based on the intermediate value theorem implies that there exists a unique cutpoint signal that defines an equilibrium. The next theorem, whose proof is given in Appendix C, establishes that no other equilibrium exists. [17] Let $\tilde{n}^*_{\varepsilon,\rho}$ be the unique solution to

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) = v.$$

[15] This fact will be extensively used in the proof of Theorem 1.

[16] At least if the signal lies at least $\varepsilon$ inside the support of $n$.

[17] Alternatively, we could have proved uniqueness of the equilibrium using an argument based on iterated deletion of dominated strategies, as in Carlsson and Van Damme (1993a, b).

**Theorem 1** *The game $\Gamma_{\varepsilon,\rho}$ has, essentially, a unique Nash equilibrium. In this equilibrium, a player with signal $\tilde{n}$ plays action $S$ if $\tilde{n} < \tilde{n}^*_{\varepsilon,\rho}$ and action $R$ if $\tilde{n} > \tilde{n}^*_{\varepsilon,\rho}$.*

The "essential" qualification arises from the fact that either action may be chosen when the signal exactly equals $\tilde{n}^*_{\varepsilon,\rho}$.

***Remark 1*** To see why the usual contagion argument from global games applies also to our setting, consider the following game.[18] A continuum of players chooses between two actions, $S$ and $R$. The payoff from action $S$ is constant and equal to $v$. The payoff from action $R$ depends on the proportion $l \in [0, 1]$ of other players who choose $R$ and is given by $\nu(R, l) = \sum_{x=0}^{\infty} P(x|n[pl + r])u(R, x)$. Each player receives a noisy signal about the value of $n$, an exogenous stochastic parameter with a smooth distribution on $\mathbb{R}_{++}$. This is a standard global game with strategic complementarities and dominance regions. As such, the standard contagion argument implies a unique equilibrium in the limit as the noise vanishes.

This game is strategically equivalent to ours. The players correspond to the strategic agents in our model. They expect a random finite number of players to be selected to participate, with the number of such players being Poisson distributed with mean $np$. They also expect a random number of risky automata to participate, with this number being Poisson distributed with mean $nr$. Each player in the continuum maximizes her expected utility conditional on being selected. Analogously, in a Poisson game, a player's participation does not affect her optimal choice conditional on being selected. This stems from the environmental equivalence property, which captures, in a finite setting, the "competitive" features characteristic of models with a continuum of players.

### 4.3 The selection

We can now let $\varepsilon$ go to zero, and then let $\rho$ go to $r$. In the limit, the equilibrium strategy can be characterized in a fairly simple way.

Let

$$U_n(R, \tilde{\sigma}_n) = 1 - \sum_{i=1}^{I} F_n(k_i \mid \tilde{\sigma}_n)(u_i - u_{i-1}),$$

where

$$F_n(k_i \mid \tilde{\sigma}_n) = \sum_{x=0}^{k_i} \sum_{z=0}^{x} \sum_{y=z}^{\infty} \frac{1}{y+1} P(y \mid np)P(x - z \mid n\rho),$$

and let $n^*_\rho$ be the unique solution to

$$U_n(R, \tilde{\sigma}_n) = v.$$ [19]

As the noise becomes negligible, players' observations become perfectly correlated and, in the limit, coincide with the value $n$ of the population size. We can show that,

18 We thank an anonymous referee for suggesting this observation.

19 It is possible to see that $U_n(R, \tilde{\sigma}_n)$ is strictly increasing in $n$, due to the first order stochastic dominance of the Poisson distribution and the fact that $1/(y + 1)$ decreases with $y$.

in this case, $U_n(R, \tilde{\sigma}_n)$ approximates the payoff of playing the risky action for the marginal player who expects the other players to play the cutpoint strategy at $n$. It follows that, as $\varepsilon$ goes to zero, the cutpoint signal $\tilde{n}^*_{\varepsilon,\rho}$ converges to $n^*_\rho$.

**Proposition 2** *As $\varepsilon$ tends to zero, $\tilde{n}^*_{\varepsilon,\rho}$ converges to $n^*_\rho$.*

The formal proof of this result is contained in Appendix D. It derives from the fact that, under uniform priors, the signal $\tilde{n}$ received by a player conveys no additional information about her relative ranking among the population of signals. Consequently, for any given number $y$ of other players, a player assigns equal probability to each possible number of them having received a higher signal, that is, this number is uniformly distributed over $\{0, 1, \ldots, y\}$. If the player expects the other players to follow the cutpoint strategy at $\tilde{n}$, she will assign probability $\frac{1}{y+1}$ to the event that exactly $z$ of them choose the risky action for every $z \in \{0, 1, \ldots, y\}$. Moreover, as $\varepsilon$ tends to zero, the signal each player receives converges to $n$. Therefore, for a player, the probability that the number of other players equals some value $y$ tends to $P(y \mid np)$, while the probability that the number of risky automata equals some value $l$ tends to $P(l \mid n\rho)$.

The cutpoint value $n^*_\rho$ depends on the probability of risky automata $\rho$. We can now let $\rho$ approach $r$ and obtain the cut-off value $n^*$ as the limit point of $n^*_\rho$. This value solves the appropriate indifference condition $U_n(R, \tilde{\sigma}_n) = v$ of the marginal player, where $\rho = r$ in $U_n(R, \tilde{\sigma}_n)$.[20] We state this result without proof.

**Proposition 3** *As $\rho$ tends to $r$, $n^*_\rho$ converges to $n^*$.*

Thus, in the limit as we first take $\varepsilon \to 0$ and then take $\rho \to r$, all players will choose the safe action when the population size $n$ is below $n^*$ and the risky action when $n$ is above $n^*$. This provides an equilibrium selection criterion for the Poisson games $\Gamma(n)$ within the indeterminacy region $(\underline{n}, \overline{n})$, which are obtained with $\varepsilon = 0$ and $\rho = r$. This criterion selects the equilibrium in which every player chooses $S$ if $n < n^*$ and the equilibrium in which every player chooses $R$ if $n > n^*$.

***Remark 2*** In the stag hunt model we have considered, the payoff associated with the safe action is fixed. Our selection argument can be extended to a different payoff function as long as two main conditions hold. First, the initial class of games must contain games with different equilibrium structures, i.e. there must exist a value $\underline{n}$ such that

$$U(R, \tau \mid n) < U(S, \tau \mid n)$$

for all $n < \underline{n}$ and every $\tau$, and, if $r > 0$, a value $\overline{n}$ such that

$$U(R, \tau \mid n) > U(S, \tau \mid n)$$

for all $n > \overline{n}$ and every $\tau$. Second, the payoff gain of choosing the risky action rather than the safe action

$$U(R, \tau \mid n) - U(S, \tau \mid n)$$

must be continuous and strictly increasing in $n\tau_R$ for $n\tau_R > 0$.

[20] Recall that $P(l \mid 0) = 0$ for every $l > 0$ and $P(0 \mid 0) = 1$.

In particular, the payoff of choosing the safe action may depend on the number of agents choosing the risky action $x$, provided that

(1) $u(S, 0) > u(R, 0)$,
(2) $u(R, x) > u(S, x)$ for all $x \geq \bar{x} > 0$, and
(3) $u(R, x) - u(S, x)$ is weakly increasing in $x$.

Given that $u(R, x)$ increases with $x$, this last assumption is clearly satisfied whenever the payoff of choosing the safe action decreases with the number of agents choosing the risky action. [21] However, the payoff of choosing the safe action may also increase with $x$, as long as condition (3) holds. [22]

***Remark 3*** Our selection result can also be applied to the case in which the payoff yielded by the risky action decreases with the number of agents choosing the safe action. In this case, the Poisson baseline game has a unique equilibrium in which every player chooses $R$ if the expected number of players $n$ is sufficiently small, a unique equilibrium in which every player chooses $S$ if $s > 0$ and $n$ is sufficiently large, and three equilibria in all the other events. As above, we can solve the equilibrium multiplicity problem by perturbing players' knowledge of $n$ with vanishing noise. [23] Similar arguments allow us to select either the equilibrium in which every player chooses the risky action or the equilibrium in which every player chooses the safe action depending on whether the expected number of players $n$ is, respectively, below or above a given threshold.

### 4.4 Example (cont.)

We can now solve the indeterminacy problem of the on-line subscription dilemma. Recall that the indeterminacy region goes from $\underline{n} \approx 1.069$ to $\overline{n} \approx 6.414$. Suppose that every potential participant receives noisy information about the expected participation size $n$, as in the model described above, and let $\rho = r$. As the noise vanishes, the payoff of subscription for a participant who expects all the participants with higher signals to do so and all those with lower signals to stay out is approximated by

$$1 - P\left(0 \;\middle|\; \frac{1}{7}n\right) \sum_{x=0}^{\infty} P\left(x \;\middle|\; \frac{5}{7}n\right) \frac{1}{x+1},$$

which is the probability that at least one other person will subscribe in that event. The limit value of $n$ for which the person is indifferent between staying out and subscribing solves

[21] This is analogous to Kim (1996), where, precisely, the payoff of choosing the safe action increases with the number of players choosing it. In our model with population uncertainty, that precise assumption seems not sufficient to guarantee the existence of the dominance solvable regions of population sizes.

[22] This suggests that an extension of the model to multiple risky actions could be done as long as the payoff of each action depends on the number of agents choosing, e.g., the riskiest one, and the difference between the payoffs of a more and a less risky action is increasing in that number.

[23] In this case, the properly modified perturbed game must assign strictly positive probability to safe automata.

$$1 - P\left(0 \,\middle|\, \frac{1}{7}n\right) \sum_{x=0}^{\infty} P\left(x \,\middle|\, \frac{5}{7}n\right) \frac{1}{x+1} = 0.6$$

and is given by $n^* \approx 2$, which determines the outcome. All people in a dilemma will stay off the platform if the expected number of participants is at most 2, while all will subscribe for larger participation sizes.

The main freebie of a theory of equilibrium selection is the possibility to perform comparative-statics exercises. A natural exercise in our setting is to ask what the effects are on the equilibrium of the variations of the safe payoff $v$, the probability $r$ of having agents who single-mindedly take the risky action, and the thresholds $k_i$.

First, suppose that $v$ rises to 0.7. Not surprisingly, the group size threshold increases to $n^* \approx 2.711$, that is, the region in which all participants in a dilemma will stay out expands as the safe action becomes more attractive. This holds in general, as the equilibrium threshold solves the indifference condition $U_n(R, \tilde{\sigma}_n) = v$ and $U_n(R, \tilde{\sigma}_n)$ increases in $n$.

Suppose next that $r$ rises to 2/7, while $p$ falls to 4/7. As expected, the threshold falls to $n^* \approx 1.669$, that is, the region in which all participants subscribe expands when more enthusiasts of the platform are likely to show up. In fact, $U_n(R, \tilde{\sigma}_n)$ increases in $r$.

Finally, suppose that subscription requires at least two other people to subscribe instead of one. In this case, the threshold increases to $n^* \approx 4.437$, i.e. the region in which all people subscribe shrinks. This is intuitive since $U_n(R, \tilde{\sigma}_n)$ decreases in the minimum number of subscribers that attracts participation, everything else being equal.

Overall, if staying out becomes less appealing or subscribing more appealing or having some platform enthusiasts becomes more likely, participants will subscribe for relatively smaller expected participation sizes.

## 5 Applications

Finally, we present some applications of our theory to economic and sociopolitical situations in which the assumption of population uncertainty seems natural.

### 5.1 Protest

A protest may turn into a full-fledged riot, as discussed in Atkeson (2001).[24] The Poisson game that represents this situation has the following features. The demonstrators decide whether to riot or not. A person who does not join the riot gets nothing. If the police are overwhelmed, the rioters win and enjoy the loot, with payoff $\mathcal{W} \equiv 1 - v > 0$; if the police contain the riot, the rioters lose and are arrested, getting payoff $\mathcal{L} \equiv -v < 0$. The riot succeeds, and the police are overwhelmed if enough people join the protest; otherwise the riot is contained. Nobody knows how many peo-

[24] According to Granovetter (1978) outcomes of protests often follow threshold dynamics, that depend on how many people physically gather together.

ple will show up at the protest. The number is drawn from a Poisson distribution. The expected number of demonstrators is not common knowledge among the players who receive slightly disturbed signals about its value. There is a group of activists who will definitely riot, while the rest decide strategically what to do. Adding $v$ to all payoffs, we have the same setting considered above with a single-step function. The uniqueness result derived above applies directly to this setting. In the unique equilibrium, strategic participants join the riot if the protest is expected to be sufficiently large, but not otherwise. Hence, there is an emboldening effect of group size, due to the presence of some activists who exert an indirect influence on the undecided participants. Modulo a relabeling of the variables, this setting can be interpreted as a currency attack à la Morris and Shin (1998).

### 5.2 Generalized prisoner's dilemma

Finally, we apply our framework to the generalized prisoner's dilemma of Kalai (2024), which is based on actual multi-defendant Racketeer Influenced and Corrupt Organizations (RICO) trials in the US. A mastermind might have recruited some conspirators to commit a crime. The prosecutor's objective is to impose a 5 year jail sentence on the mastermind if the crime was committed but to let her go otherwise. Simultaneously and privately, prior to the start of the trial, the prosecutor offers each suspect two options: confess the crime (providing some convincing evidence) or deny it. If all conspirators deny the crime, then they are all declared innocent – and so is the mastermind – and get payoff 1. If some conspirator confesses, he gets a minimal penalty with corresponding payoff 0, while each denier is sentenced to 5 years – and so is the mastermind – with payoff $-5$. The conspirators don't know exactly how many suspects are offered the two options, and the number of participants $n$ is drawn from a Poisson distribution. This situation fits the setting in Remark 3 with a single threshold $k$. Confessing the crime is the safe action, which guarantees $v \in (0, 1)$, while denying the crime is the risky action, which ensures payoff 1 if the number of confessing suspects is equal to $k = 0$, and 0 otherwise. Our selection argument maintains the equilibrium in which every conspirator confesses to the committed crime whenever $n$ is sufficiently large.

This result is in line with Kalai (2024) and Kalai and Kalai (2024), whose critical mass criterion for ranking equilibria (in settings with deterministic and commonly known populations) considers the equilibrium in which all conspirators confess to be increasingly more robust than the one in which all conspirators deny, as the number of conspirators increases. [25] Despite the differences in the information structures, the similarities between our results and theirs reflect the same economic intuition

[25] The notion of "critical mass" captures the number of players needed for the adoption of a strategy profile as equilibrium play. Kalai (2024) refers to the equilibrium in which every conspirator confesses the crime as *contagious*, because the choice of confessing by one suspect creates strong incentives to confess for the others, and to the equilibrium in which every suspect denies the crime as *fragile*, because any defection induces the other suspects to also defect. Kalai and Kalai (2024) offer several examples, where equilibria can be ranked according to their critical mass, which fit either our baseline stag hunt setting or the one in Remark 3. In the first case, as the number of players increases, the advantage of the risky equilibrium over the safe one increases, while in the second case the opposite holds (as above).

underpinning both frameworks. As Kalai (2024) puts it, it is not surprising that the prosecutors prefer a large number of conspirators.

## 6 Concluding remarks

We have shown that modeling some economic and political situations that require the participants to coordinate their decisions on whether to alter the status quo or not as Poisson games in which the expected number of players is not common knowledge helps select a unique equilibrium in a way that parallels what happens in global games, allowing to interpret these situations as critical mass phenomena in which the overall expected number of participants plays a key role in determining whether the status quo is abandoned or not. Our result suggests an important role for the information concerning the size of the group that participates in the interaction.

There is experimental evidence that the size of the group matters in deciding among multiple equilibria in coordination settings. In the minimum effort game, Van Huyck et al. (1990) show that large groups of players coordinate in the minimum effort equilibrium with the lowest payoff, while pairs of subjects may coordinate in the Pareto optimal equilibrium. Using the same game, Weber (2006) also shows that there is a strong negative relationship between a group's size and the ability of its members to coordinate efficiently; however, efficient outcomes may be maintained when the size is gradually increased if new members are informed about the group history. Arifovic et al. (2023) consider a bank run game where the payoff of the risky action decreases in the fraction of players choosing the safe action and find that, if the safe payoff is high enough, small and large groups behave differently, the former coordinating on the risky Pareto dominant equilibrium while the latter on the safe Pareto dominated one. In the classical stag hunt game with a fixed threshold, Kim et al. (2026) show that the probability that subjects play the risky action varies with the number of participants and decreases as the fraction of participants required to reach the threshold increases. Their game is a particular case of our baseline setting, while minimum effort and bank-run games have features similar to the setting described in Remark 3. Our theory offers a possible explanation for this evidence, showing how the informational environment players face regarding the scale of interaction affects coordination behavior.

## APPENDIX A. Poisson games

This appendix summarizes the basic structure of Poisson games and their main properties. [26]

A Poisson game is given by a tuple $\Gamma = (n, \mathcal{T}, q, A, u)$. The number of players is a Poisson random variable with parameter $n \in \mathbb{R}_{++}$, so the actual number of players is $m$ with probability

$$P(m \mid n) = \frac{e^{-n} n^m}{m!}.$$

[26] We refer to Myerson (1998a).

The set $\mathcal{T} = \{1, \ldots, T\}$ is the set of players' types. The probability that a randomly selected player is of each type is given by the vector $q = (q_1, \ldots, q_T) \in \Delta(\mathcal{T})$. That is, a player is of type $t \in \mathcal{T}$ with probability $q_t$. A type profile $y \in \mathbb{Z}_+^T$ is a vector that specifies for each type $t \in \mathcal{T}$ the number of players $y_t$ of that type. The finite set of actions is $A$. An action profile $x \in Z(A) = \mathbb{Z}_+^{|A|}$ specifies for each action $a \in A$ the number of players $x(a)$ that choose that action. Players' preferences are summarized by the vector $u = (u_1, \ldots, u_T)$, where $u_t : A \times Z(A) \to \mathbb{R}$ for every $t \in \mathcal{T}$. That is, $u_t(a, x)$ is the payoff for a type $t$ player when she chooses action $a$ and the realization resulting from the rest of the population's behavior is the action profile $x \in Z(A)$.

The set of mixed actions is $\Delta(A)$. A strategy function $\sigma$ maps each type to the set of mixed actions.[27] It induces the average behavior $\tau(\sigma) \in \Delta(A)$ which is defined by $\tau(\sigma)(a) = \sum_{t \in \mathcal{T}} q_t \sigma_t(a)$. When players play according to $\sigma$, $\tau(\sigma)(a)$ is the probability that a randomly sampled agent chooses action $a$. Since a player's payoff depends only on the number of other players who choose each action, independently of their specific types, $\tau$ is a sufficient statistic for the analysis of players' optimal behavior.

A Poisson game is characterized by the following properties.

***Decomposition property*** Let each player be independently assigned some characteristic in a set $S$ according to the probability distribution $(\theta(s))_{s \in S}$. Let $w(s)$ be the number of players with characteristic $s$. For every $s \in S$, the random variables $w(s)$ are mutually independent, and each $w(s)$ is Poisson distributed with mean $n\theta(s)$.

***Aggregation property*** Any sum of independent Poisson random variables is also a Poisson random variable.

***Independent actions property*** For every strategy function $\sigma$ and action $a$, the random variables $x(a)$ are independent.

***Environmental equivalence property*** A player of any type assesses the same probability distribution for the type profile of the other players as an external observer assesses for the type profile of the whole game.

The decomposition and independent actions properties imply that, when the population's aggregate behavior is $\tau(\sigma)$, the number of players who choose action $a$ is a Poisson random variable with mean $n\tau(\sigma)(a)$ and is independent of the number of players who choose any other action. Then, the probability that the action profile $x \in Z(A)$ is realized is equal to

$$\mathbf{P}(x \mid \tau(\sigma)) = \prod_{a \in A} \left( e^{-n\tau(\sigma)(a)} \frac{(n\tau(\sigma)(a))^{x(a)}}{x(a)!} \right).$$

Environmental equivalence implies that $\mathbf{P}(x \mid \tau(\sigma))$ is also the probability that each player assigns to the event that the action profile resulting from the other players'

[27] Population uncertainty requires the use of symmetric strategies (Myerson 1998a, p. 377). The reason is that, in a Poisson game, a player is aware only of the possible types of the other players, but not of their specific identities. As a result, two distinct individuals of the same type possess no commonly known characteristics by which one can assess a different behavior for them and, therefore, must be treated symmetrically.

behavior is $x$. Hence, the expected payoff to a player of type $t$ who plays $a \in A$ is given by

$$U_t(a, \tau(\sigma) \mid n) = \sum_{x \in Z(A)} \mathbf{P}(x \mid \tau(\sigma)) u_t(a, x).$$

## APPENDIX B. Proof of Lemma 1

**Lemma 1** *$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}})$ is continuous and strictly increasing in $\tilde{n}$.*

***Proof*** Given a cutpoint strategy $\tilde{\sigma}_{\tilde{n}}$, the probability for a player with signal $\tilde{n}$ that the overall number of other agents choosing $R$ is equal to $x$ is the probability for the player that the realizations of risky automata and of other players with signal larger than $\tilde{n}$ sum up to $x$.

Let $P_{\tilde{n}}(z \mid np)$ be the probability for the player that $z$ other players have made larger observations, if the population size is $n$. We have

$$P_{\tilde{n}}(z \mid np) = \sum_{y=z}^{\infty} P(y \mid np) B(y, z \mid \tilde{n}, n),$$

where $P(y \mid np)$ is the probability for the player that there are $y$ other players in the game when the population size is $n$, and $B(y, z \mid \tilde{n}, n)$ is the probability that $z$ out of $y$ other players have observed a signal larger than $\tilde{n}$. The assumption of uniform priors on $n$ implies that, given $y$ and $z$, $B(y, z \mid \cdot)$ depends only on the difference between the player's signal and the actual population size, independently of the signal. [28] In particular, we have

$$B(y, z \mid \tilde{n}, n) = B(y, z \mid \tilde{n} - n) = \binom{y}{z} \mathbb{P}(\tilde{n} - n)^{y-z} \, [1 - \mathbb{P}(\tilde{n} - n)]^z,$$

where

$$\mathbb{P}(\tilde{n} - n) = \frac{\tilde{n} - n + \varepsilon}{2\varepsilon}.$$

The decomposition property of the Poisson distribution implies that, for the marginal player, the number of other players who have made larger observations is Poisson distributed with parameter $np[1 - \mathbb{P}(\tilde{n} - n)]$. That is, for every $z \in \mathbb{Z}_+$,

$$P_{\tilde{n}}(z \mid np) = P(z \mid np[1 - \mathbb{P}(\tilde{n} - n)]).$$

Let $\tilde{P}_{\tilde{n}}(x \mid \tilde{\sigma}_{\tilde{n}}, n)$ be the probability for a player with signal $\tilde{n}$ that the overall number of other agents choosing $R$ is equal to $x$, given the cutpoint strategy $\tilde{\sigma}_{\tilde{n}}$ and if the population size is $n$. Since types are independent, we have

$$\tilde{P}_{\tilde{n}}(0 \mid \tilde{\sigma}_{\tilde{n}}, n) = P(0 \mid np[1 - \mathbb{P}(\tilde{n} - n)]) P(0 \mid n\rho),$$

[28] At least if the signal lies at least $\varepsilon$ inside the support of $n$.

$$\tilde{P}_{\tilde{n}}(1 \mid \tilde{\sigma}_{\tilde{n}}, n) = P(0 \mid np[1 - \mathbb{P}(\tilde{n} - n)])P(1 \mid n\rho) + P(1 \mid np[1 - \mathbb{P}(\tilde{n} - n)])P(0 \mid n\rho),$$

$$\ldots$$

$$\tilde{P}_{\tilde{n}}(x \mid \tilde{\sigma}_{\tilde{n}}, n) = \sum_{z=0}^{x} P(z \mid np[1 - \mathbb{P}(\tilde{n} - n)])P(x - z \mid n\rho).$$

The aggregation property of the Poisson distribution implies that, for the marginal player, the number of other agents choosing $R$ is Poisson distributed with parameter $np[1 - \mathbb{P}(\tilde{n} - n)] + n\rho$. That is, for every $x \in \mathbb{Z}_+$,

$$\tilde{P}_{\tilde{n}}(x \mid \tilde{\sigma}_{\tilde{n}}, n) = P(x \mid np[1 - \mathbb{P}(\tilde{n} - n)] + n\rho).$$

It follows that the utility of choosing $R$ for the marginal player is

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) = 1 - \frac{1}{2\varepsilon} \int_{\tilde{n}-\varepsilon}^{\tilde{n}+\varepsilon} \sum_{i=1}^{I} F(k_i \mid np[1 - \mathbb{P}(\tilde{n} - n)] + n\rho)(u_i - u_{i-1}) \, \mathrm{d}n.$$

Given that $\mathbb{P}(\cdot)$ depends only on the difference $\tilde{n} - n$, independently of $\tilde{n}$, we can set $\delta = \tilde{n} - n$ and express the above function as

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) = 1 - \frac{1}{2\varepsilon} \int_{-\varepsilon}^{\varepsilon} \sum_{i=1}^{I} F(k_i \mid (\tilde{n} - \delta)p[1 - \mathbb{P}(\delta)] + (\tilde{n} - \delta)\rho)(u_i - u_{i-1}) \, \mathrm{d}\delta.$$

The first order stochastic dominance of the Poisson distribution implies that $F(k_i \mid (\tilde{n} - \delta)p[1 - \mathbb{P}(\delta)] + (\tilde{n} - \delta)\rho)$ is strictly decreasing in $\tilde{n}$. Since $u_i - u_{i-1} \geq 0$ for every $i = 1, \ldots, I$, the payoff $\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}})$ is strictly increasing in $\tilde{n}$. Continuity derives from the fact that the Poisson probability functions are continuous in the parameter. □

## APPENDIX C. Proof of Theorem 1

**Theorem 1** *The game $\Gamma_{\varepsilon,\rho}$ has, essentially, a unique Nash equilibrium. In this equilibrium, a player with signal $\tilde{n}$ plays action $S$ if $\tilde{n} < \tilde{n}^*_{\varepsilon,\rho}$ and action $R$ if $\tilde{n} > \tilde{n}^*_{\varepsilon,\rho}$.*

***Proof*** Let $\tilde{\sigma}_a$ be the strategy function that prescribes the choice of action $a \in A$ with probability 1 for every observed signal. For every $\tilde{n} < \underline{n} - \varepsilon$ we have

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) \leq \tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_R) < v,$$

while for every $\tilde{n} > \overline{n}^{\rho} + \varepsilon$ we have

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) \geq \tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_S) > v.$$

Lemma 1 implies that there exists a unique value $\tilde{n}^*_{\varepsilon,\rho}$ such that

$$\tilde{U}_{\tilde{n}^*_{\varepsilon,\rho}}(R, \tilde{\sigma}_{\tilde{n}^*_{\varepsilon,\rho}}) = v.$$

Note that for every $\tilde{n} < \tilde{n}^*_{\varepsilon,\rho}$ we have

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}^*_{\varepsilon,\rho}}) \leq \tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) < v,$$

while for every $\tilde{n} > \tilde{n}^*_{\varepsilon,\rho}$ we have

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}^*_{\varepsilon,\rho}}) \geq \tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) > v,$$

so $\tilde{\sigma}_{\tilde{n}^*_{\varepsilon,\rho}}$ is an equilibrium of $\Gamma_{\varepsilon,\rho}$.

Suppose that there exists another equilibrium $\tilde{\sigma}$ that is not a cutpoint strategy. Let $\tilde{n}_1$ be the largest signal such that $\tilde{\sigma}(\tilde{n})(S) = 1$ for all $\tilde{n} < \tilde{n}_1$ and let $\tilde{n}_2$ be the smallest signal such that $\tilde{\sigma}(\tilde{n})(R) = 1$ for all $\tilde{n} > \tilde{n}_2$.[29]

Clearly, $\tilde{n}_1 < \tilde{n}_2$. By continuity of the payoff function in the signal, we have

$$\tilde{U}_{\tilde{n}_1}(R, \tilde{\sigma}) = \tilde{U}_{\tilde{n}_2}(R, \tilde{\sigma}) = v.$$

Note that

$$\tilde{U}_{\tilde{n}_1}(R, \tilde{\sigma}) \leq \tilde{U}_{\tilde{n}_1}(R, \tilde{\sigma}_{\tilde{n}_1})$$

and

$$\tilde{U}_{\tilde{n}_2}(R, \tilde{\sigma}) \geq \tilde{U}_{\tilde{n}_2}(R, \tilde{\sigma}_{\tilde{n}_2}).$$

But, by Lemma 1, we have

$$\tilde{U}_{\tilde{n}_1}(R, \tilde{\sigma}_{\tilde{n}_1}) < \tilde{U}_{\tilde{n}_2}(R, \tilde{\sigma}_{\tilde{n}_2}),$$

which leads to a contradiction. □

## Proof of Proposition 2

**Proposition 2** *As $\varepsilon$ tends to zero, $\tilde{n}^*_{\varepsilon,\rho}$ converges to $n^*_\rho$.*

***Proof*** Let $\delta = \tilde{n} - n$, $\mathbb{P}(\delta) = (\delta + \varepsilon)/2\varepsilon$, and fix $y \in \mathbb{Z}_+$. For every $z \in \{0, 1, \ldots, y\}$, the probability for a player with signal $\tilde{n}$ that $z$ other players have observed a signal larger than $\tilde{n}$ when their number is $y$ is given by

$$\int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon} \binom{y}{z} \mathbb{P}(\delta)^{y-z} \, [1 - \mathbb{P}(\delta)]^z \, \mathrm{d}\delta.$$

[29] Note that the initial arguments of the proof imply that constant strategies cannot be equilibria.

Integrating by parts, we obtain

$$\begin{aligned}\int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon} \binom{y}{z} \mathbb{P}(\delta)^{y-z} \, [1-\mathbb{P}(\delta)]^{z} \, \mathrm{d}\delta &= \binom{y}{z-1} \int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon} \, \mathbb{P}(\delta)^{y-(z-1)} \, [1-\mathbb{P}(\delta)]^{z-1} \, \mathrm{d}\delta \\ &= \ldots \\ &= \int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon} \, \mathbb{P}(\delta)^{y} \, \mathrm{d}\delta \\ &= \frac{1}{y+1}.\end{aligned}$$

That is, conditional on the realization of the number of other players, a player has uniform beliefs over the number of them with larger observations.

As seen in the proof of Lemma 1, given the cutpoint strategy $\tilde{\sigma}_{\tilde{n}}$, the utility of choosing $R$ for a player with marginal signal $\tilde{n}$ is given by

$$\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}}) = 1 - \int_{-\varepsilon}^{\varepsilon} \frac{1}{2\varepsilon} \sum_{i=1}^{I} F(k_i \mid (\tilde{n}-\delta)p[1-\mathbb{P}(\delta)] + (\tilde{n}-\delta)\rho)(u_i - u_{i-1}) \, \mathrm{d}\delta,$$

where

$$\begin{aligned}&F(k_i \mid (\tilde{n}-\delta)p[1-\mathbb{P}(\delta)] + (\tilde{n}-\delta)\rho) = \\ &= \sum_{x=0}^{k_i} \sum_{z=0}^{x} \sum_{y=z}^{\infty} P(y \mid (\tilde{n}-\delta)p) \binom{y}{z} \mathbb{P}(\delta)^{y-z} \, [1-\mathbb{P}(\delta)]^{z} \, P(x-z \mid (\tilde{n}-\delta)\rho).\end{aligned}$$

Moreover,

$$\tilde{U}_{\tilde{n}^*_{\varepsilon,\rho}}(R, \tilde{\sigma}_{\tilde{n}^*_{\varepsilon,\rho}}) = v.$$

As $\varepsilon$ goes to zero, the signal $\tilde{n}$ that each player receives converges to the value $n$ of the population size and $\delta$ shrinks to zero. The probabilities for the marginal player that $y$ other players have realized and that $x-z$ risky automata have realized converge, respectively, to $P(y \mid np)$ and $P(x-z \mid n\rho)$. In this case, the utility $\tilde{U}_{\tilde{n}}(R, \tilde{\sigma}_{\tilde{n}})$ converges to

$$U_n(R, \tilde{\sigma}_n) = 1 - \sum_{i=1}^{I} \sum_{x=0}^{k_i} \sum_{z=0}^{x} \sum_{y=z}^{\infty} \frac{1}{y+1} P(y \mid np) \, P(x-z \mid n\rho) \, (u_i - u_{i-1}).$$

Given that

$$U_{n^*_\rho}(R, \tilde{\sigma}_{n^*_\rho}) = v$$

and given the continuity of the employed probability functions, the result follows. □

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## Declarations

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