PROCEEDINGS OF SCIENCE

A new observable for W-mass determination

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In this contribution we discuss the properties of the jacobian asymmetry, the new observable introduced in [1] for a robust determination of the value and uncertainty of the W-boson mass at hadron colliders.

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1. Introduction

The determination of the W-boson mass m_W is of paramount importance for the precision programme of collider facilities such as the LHC [2–4]. In the Standard Model (SM), quantum corrections to the value of m_W are sensitive to other fundamental parameters of the theory, such as the top-quark and the Higgs-boson masses. Therefore, m_W is central to global fits of electroweak (EW) precision observables [5, 6], allowing for compelling tests of the SM itself [7, 8].

Measurements of the W-boson mass at different high-energy colliders span four decades, and the precision with which m_W is determined has steadily improved since discovery, owing to the wealth of available data and to many experimental advances, reaching nowadays the level of 10-20 MeV [2–4, 9], i.e. a relative accuracy at the permyriad level. Such a level of precision requires an exquisite control over all elements that feed into the extraction, including not only experimental calibrations, but also the robustness of the strategies adopted to infer m_W from data. The consideration that the most precise m_W measurement to date [9] is several standard deviations away from the SM expectations (and from the World average) further stimulates a careful assessment of the methodologies employed for m_W determination.

At hadron colliders, the value of m_W is primarily deduced from the charged-current Drell-Yan (CCDY) process, the hadro-production of a lepton-neutrino pair. Of particular relevance in this context are observables defined in the transverse plane with respect to the collision axis, such as the charged-lepton transverse momentum p_t^ℓ, or the lepton-neutrino transverse mass $M_t^{\ell\nu}$ [2–4, 9, 10]. The spectra of such quantities display a kinematical jacobian peak whose position directly depends on the value of m_W, hence the shape of these distributions in the peak region can be used as a privileged probe to extract the W-boson mass. A physical description of the shape of the jacobian peak requires to take into account a variety of theoretical and experimental effects. On the theory side, the peak arises at the boundary of the available charged-lepton phase space at leading order in QCD, whence soft QCD radiation causes an integrable singularity [11] in the spectrum around the peak in fixed-order perturbation theory, which calls for all-order resummation of infrared (IR) enhanced QCD effects. QED final-state radiation also significantly affects the shape of the peak, as well as mixed QCD-EW effects in the case of p_t^ℓ [12]. On the experimental side, the peak is relatively stable under detector effects for p_t^ℓ, while it gets significantly smeared by the latter in the case of $M_t^{\ell\nu}$, owing to neutrino reconstruction, see e.g. Figure 1 of [13].

2. Standard m_W determination

A common procedure to extract of the W-boson mass at hadron colliders is through template fitting. Theoretical template distributions for p_t^ℓ or $M_t^{\ell\nu}$ are computed with different hypotheses $m_{W,i}$ for the value of the W-boson mass, and then compared to experimental data. A measure χ_i^2 (be it a true χ^2 or a likelihood) is defined to quantify the distance between the templates and the experimental spectra, and the W-mass value is determined as the $m_{W,i}$ corresponding to the minimum χ_i^2. The main challenge with this strategy is that the shape of the jacobian peak needs to be controlled with a relative accuracy at the permille level, in order to resolve $\Delta m_W/m_W \sim 10^{-4}$ effects, while the current theoretical accuracy on p_t^ℓ and $M_t^{\ell\nu}$ spectra in CCDY is rather at the percent level [14–16].

The procedure is restored by leveraging the availability of high-precision p_t^Z data in neutral-current Drell Yan (NCDY). The tools used to produce the theoretical template distributions, typically flexible but low-accuracy parton-shower event generators, are calibrated to give the best description of such p_t^Z data, primarily by tuning the parameters of a non-perturbative (NP) model (e.g. the intrinsic k_t of partons in the proton, or the shower cutoff scale Q_0). The same tuning setup deduced in NCDY is then used to produce the templates of p_t^ℓ and $M_t^{\ell\nu}$ in CCDY, and, after tuning, the templates typically give rise to reasonably low minimum χ_i^2 values.

Although the template-fitting procedure is long-established, with the aim of 10^{-4} relative accuracy on m_W, it is legitimate to raise methodological concerns about its reliability and robustness. First, as evinced from the above discussion, template fitting heavily relies on the tuning step of parton showers, i.e. the theoretical prediction is driven by NP physics, which is the least understood theoretically. In particular, tuned NP parameters effectively mimic the contributions of higher-order perturbative radiation, and the significant progress in the perturbative understanding of the Drell-Yan process [14–54] is not fully exploited (barring to a certain extent effects due to reweighing of event samples). Moreover, the universality of the underlying NP model [55], assumed when applying to CCDY the same NP parameters extracted from NCDY, can be spoiled by a variety of effects that differ in the two Drell-Yan processes [56–59], driven by the different parton flavour combinations they probe. Finally, the definition of χ^2 used as a distance measure does not include theoretical uncertainty, owing to the non-statistical nature of unphysical-scale variations. These features expose the template-fitting procedure to a potential severe underestimation of the real uncertainties associated with m_W extraction.

From the above considerations, it would be desirable to define a procedure of m_W determination that allows for a transparent discussion of theoretical and experimental uncertainties on the extracted W-boson mass value, and one which would ideally minimise the reliance on p_t^Z calibration/tuning when extracting m_W.

3. New strategy for m_W determination

To define the advocated novel procedure, we focus on the p_t^ℓ spectrum for definiteness. This is displayed in Figure 1 (left) at various perturbative accuracies, where one can appreciate the physical description of the jacobian peak at $p_t^\ell \sim m_W/2$ provided by QCD resummation (i.e. the absence of any integrable singularity), as well as the $O(2\%)$ theoretical accuracy achieved with state-of-the-art tools. The setup employed to obtain the plot is detailed in the caption of the Figure. The right panel of Figure 1 shows the remarkable feature that the ratio of p_t^ℓ spectra obtained with different m_W values is largely independent of the underlying QCD accuracy (provided resummation is included in the prediction), which can be understood since the sensitivity to the value of m_W stems from W-boson propagation and decay, and it is essentially factorised from QCD initial-state radiation. The sensitivity to a relative variation $\Delta m_W/m_W \sim 2 \cdot 10^{-4}$ is also well resolvable beyond the theoretical scale-uncertainty band (the latter being obtained as a 9-point envelope varying renormalisation, factorisation and resummation scales by factors of 2 around their central values).

In order to quantify which of the N bins σ_i carry most of the sensitivity to m_W, we can define a covariance matrix w.r.t. m_W variations as $C_{ij}^{(m_W)} = \langle \sigma_i\, \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$, where $\langle x \rangle = \frac{1}{p} \sum_{k=1}^{p} x_{(k)}$ denotes an average over the range of p different m_W hypotheses. By diagonalising $C_{ij}^{(m_W)}$ one gets N

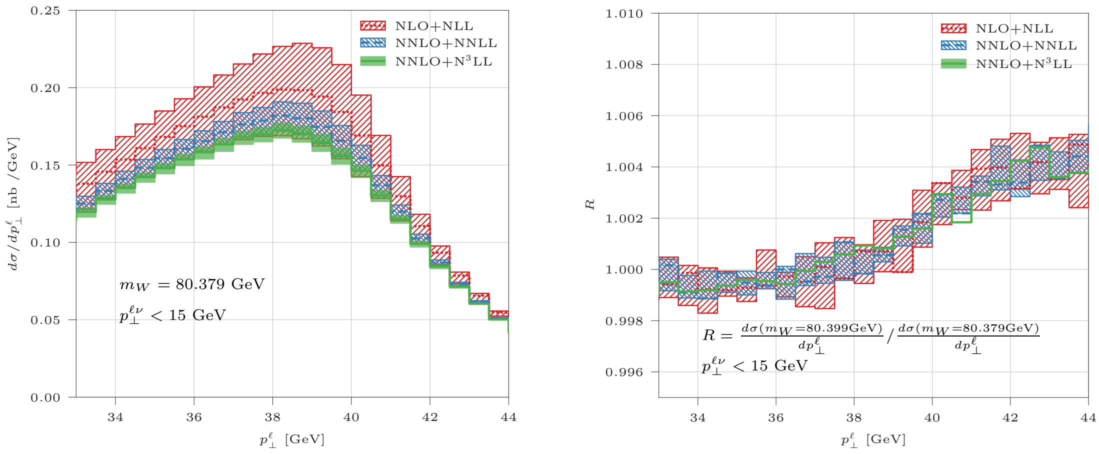
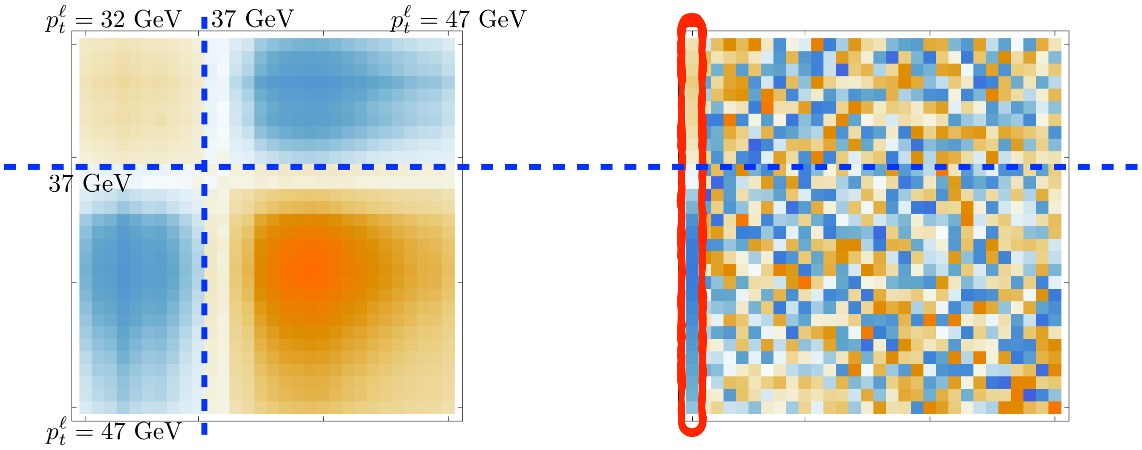


Figure 1: Left: p_t^ℓ distribution in CCDY, computed with different QCD approximations and reference $m_W = 80.379$ GeV. Right: ratio of p_t^ℓ distributions computed with two m_W values differing by 20 MeV. Setup employed: W^- production at the 13-TeV LHC with acceptance cuts $p_t^\ell > 20$ GeV, $M_t^{\ell\nu} > 27$ GeV, $|\eta_\ell| < 2.5$, 66 GeV $< M^{\ell\nu} <$ 116 GeV, $p_t^{\ell\nu} < 15$ GeV (η_ℓ and $M^{\ell\nu}$ being the charged-lepton rapidity and the lepton-pair invariant mass, respectively); central replica of the NNPDF4.0 NNLO proton PDF set [60] with strong coupling constant $\alpha_s(m_Z) = 0.118$ through the LHAPDF interface [61]; $p_t^{\ell\nu}$ resummation and matching provided by RadISH [62–65], fixed-order predictions provided by MCFM [66].

orthogonal p_t^ℓ-bin combinations whose eigenvalues represent the sensitivity of such eigenvectors to m_W variations. The left panel of Figure 2 displays for illustrative purposes the covariance $C_{ij}^{(mw)}$ for NNLO+N^{3}LL predictions with central scales, from which one can appreciate a clear anti-correlation between the p_t^ℓ regions at the left and at the right of the jacobian peak (orange bins are positive, blue bins are negative, their magnitude being proportional to colour intensity). The right panel of Figure 2 shows the matrix of eigenvectors of $C_{ij}^{(mw)}$, sorted from left to right by decreasing eigenvalues. Only the leftmost eigenvector displays a clear pattern of coefficients (positive for p_t^ℓ below 37 GeV, negative above). Its corresponding eigenvalue e_1 is by far the dominant one, with $e_1/\text{tr}[C_{ij}^{(mw)}] \sim 0.99$. This means that most of the sensitivity to m_W variations hiding in the p_t^ℓ spectrum is captured by the sole first bin combination.

On physical grounds, this pattern lends itself to a relatively straightforward interpretation: a variation Δm_W in the W-boson mass has the sole (or at least by far dominant) effect of inducing a rigid shift of the p_t^ℓ spectrum by $\Delta m_W/2$. A single p_t^ℓ-bin combination is thus sufficient to entirely capture the effect of m_W variations. Such a combination is the one corresponding to the translational mode of the spectrum: indeed its coefficients are proportional to the derivative of the spectrum w.r.t. p_t^ℓ (i.e. $\mathrm{d}^2\sigma/\mathrm{d}p_t^{\ell 2}$), as expected from the generator of p_t^ℓ translations.

This analysis suggests two possible strategies. One would be that of measuring and template-fitting directly $\mathrm{d}^2\sigma/\mathrm{d}p_t^{\ell 2}$, as opposed to $\mathrm{d}\sigma/\mathrm{d}p_t^\ell$: the former essentially distills all of the sensitivity to m_W variations, without being blurred by other concurring effects, chiefly QCD radiation (cfr. independence of the spectrum derivative from QCD in the right panel of Figure 1). Whether this strategy is more robust and resilient to tuning than the standard fitting procedure is subject of future developments. Another strategy, pursued in [1], is to encode (as much as possible of)



Covariance @ NNLO+N3LL central scales

Covariance eigenvectors

Figure 2: Left: covariance matrix $C_{ij}^{(mw)}$ at NNLO+N^{3}LL with central scales, see main text for its detailed definition. Right: matrix of $C_{ij}^{(mw)}$ eigenvectors as columns, sorted from left to right by decreasing eigenvalue magnitude.

the information carried by the translational eigenvector into the definition of a simple observable enjoying good features both from the experimental and from the theoretical point of view.

4. Jacobian asymmetry

The dominant translational eigenvector of $C_{ij}^{(mw)}$ collects the bins below (above) the peak with positive (negative) coefficients. An observable encoding this information is thus the *jacobian asymmetry* $\mathcal{A}_{p_t^\ell}$, defined as

$$\mathcal{A}_{p_t^\ell}(p_{t,\mathrm{min}}^\ell, p_{t,\mathrm{mid}}^\ell, p_{t,\mathrm{max}}^\ell) = \frac{L-U}{L+U},$$

$$L = \int_{p_{t,\mathrm{min}}^\ell}^{p_{t,\mathrm{mid}}^\ell} \mathrm{d}p_t^\ell \, \frac{\mathrm{d}\sigma}{\mathrm{d}p_t^\ell}, \qquad U = \int_{p_{t,\mathrm{mid}}^\ell}^{p_{t,\mathrm{max}}^\ell} \mathrm{d}p_t^\ell \, \frac{\mathrm{d}\sigma}{\mathrm{d}p_t^\ell}, \tag{1}$$

where three values $p_{t,\mathrm{min}}^\ell$, $p_{t,\mathrm{mid}}^\ell$, and $p_{t,\mathrm{max}}^\ell$ define two adjacent windows in the p_t^ℓ spectrum, selecting bins below and above the peak, respectively. Crucial is that $p_{t,\mathrm{mid}}^\ell$ be close to 37 GeV, in order to match the change of sign in the coefficients of the covariance eigenvector (see Figure 2) [1].

A feature one can immediately notice in this definition is that $\mathcal{A}_{p_t^\ell}$ is constructed as a combination of fiducial rates in relatively wide p_t^ℓ windows (imagining $p_{t\,\mathrm{min}}^\ell \sim 30$ GeV, and $p_{t\,\mathrm{max}}^\ell \sim 50$ GeV), whence $\mathcal{A}_{p_t^\ell}$ is a single scalar number experimentally measurable by means of an inclusive counting of events in the two windows.

Figure 3 displays $\mathcal{A}_{p_t^\ell}$ in various QCD approximations, as a function of m_W. Its linearly decreasing behaviour stems from the fact that an m_W shift by $+\Delta m_W$ induces a shift in the position

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[1] Alternatives to $\mathcal{A}_{p_t^\ell}$ could be devised: for instance, the two p_t^ℓ windows might not be exactly adjacent, or one could give relative weights to L and U, in order to better align $\mathcal{A}_{p_t^\ell}$ to the dominant covariance eigenvector.

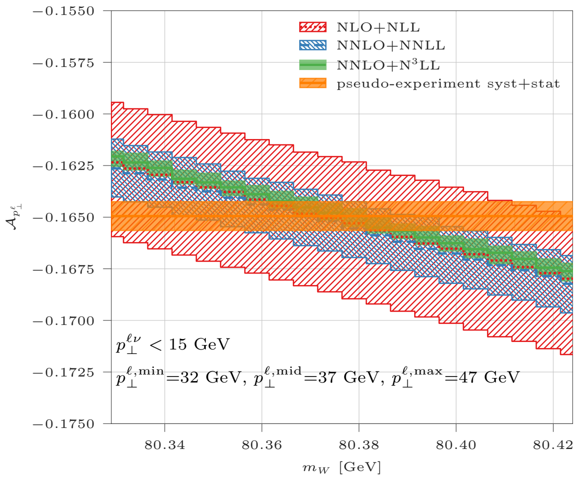

Figure 3: Jacobian asymmetry as a function of m_W at different perturbative QCD orders. Window edges are specified in the plot. See text for details.

of the jacobian peak by $+\Delta m_W/2$ (i.e. linear in Δm_W), depleting L and populating U if $p^\ell_{t\,\mathrm{mid}}$ is at the left of the peak. The slope of $\mathcal{A}_{p_t^\ell}$ as a function of m_W is independent of the QCD approximation and of the scale choice, which again reflects the factorisation of QCD initial-state radiation from the m_W-sensitive propagation and decay; this feature carries over to NP QCD effects as well [1], which will just result in a rigid shift of $\mathcal{A}_{p_t^\ell}$. The slope itself is related to the magnitude of the first covariance eigenvalue, and depends on the value of the chosen window edges.

From Figure 3 one can appreciate the excellent perturbative-QCD convergence properties of the observable, where predictions for $\mathcal{A}_{p_t^\ell}$ at higher orders perfectly lie within lower-order uncertainty bands, with a residual theoretical uncertainty steadily decreasing while including more accurate predictions. This ultimately highlights the importance of state-of-the-art results for high-accuracy Drell Yan predictions. The perturbative merits of $\mathcal{A}_{p_t^\ell}$ are not unexpected, since the observable is inclusive over radiation in wide p_t^ℓ windows. This very feature has also evident experimental advantages. On one side, the measurement of L and U should be relatively simple, with both systematic and statistical uncertainties under good control; on the other side, the usage of wide fiducial windows should be beneficial towards unfolding detector effects, allowing for a combination of different m_W determinations [67, 68]. The orange band in Figure 3 reports the results of a putative experimental measurement of $\mathcal{A}_{p_t^\ell}$: the central value is arbitrary, while the uncertainty band is obtained by propagating in quadrature a realistic 0.1% systematic error on L and U, and assuming no correlations (the statistical error is already negligible with a luminosity $\mathcal{L} = 140$ fb^{-1}).

In the context of $\mathcal{A}_{p_t^\ell}$, the W-boson mass would just be extracted as the intersection of two non-parallel straight bands, making it straightforward both to include new theoretical and experimental refinements, as they become available, and to interpret robustly the effect and the uncertainty of each of the various contributions to the observable, e.g. the impact of different PDF choices, of NP QCD contributions, of EW corrections, of detector effects, and so on. As a consequence, the procedure of extracting m_W through $\mathcal{A}_{p_t^\ell}$ would not be entirely driven by tuning to NCDY data, as

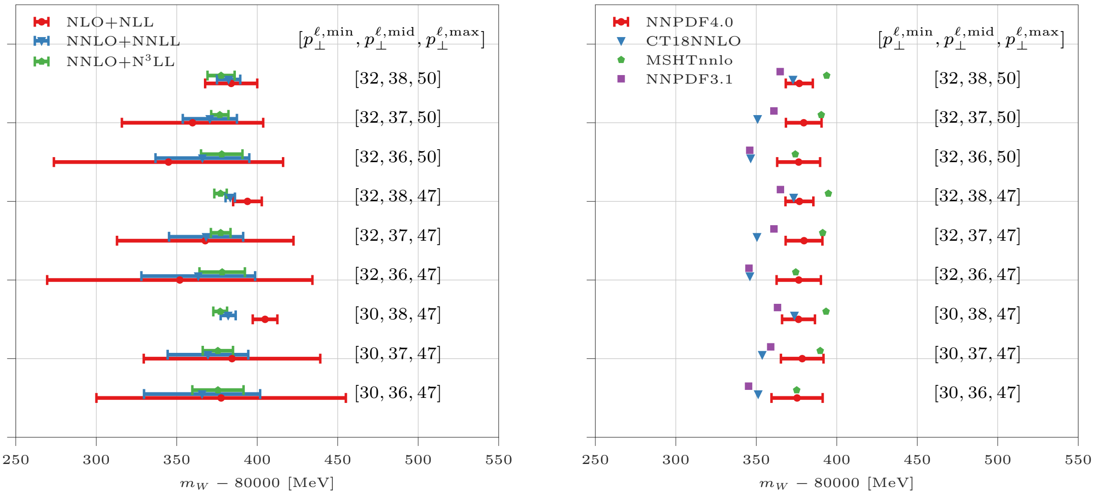

Figure 4: Left: jacobian asymmetry with various choices for window edges. Right: study of the impact of PDFs on the jacobian asymmetry.

the NP contribution would just become one of the many ingredients (and most probably of modest impact) concurring to an accurate prediction of $\mathcal{A}_{p_t^\ell}$, thus of the W-boson mass.

In the left panel of Figure 4 we display results for $\mathcal{A}_{p_t^\ell}$ for different choices of the window edges $p_{t\,\mathrm{min}}^\ell$, $p_{t\,\mathrm{mid}}^\ell$, and $p_{t\,\mathrm{max}}^\ell$. In general the perturbative convergence is very well behaved across different values of the edges, and we stress the importance of N^{3}LL resummation also for assessing the quality of the perturbative convergence, and to check it beyond the mere level of scale variations. We notice a certain trade-off between sensitivity to m_W variations, improving at higher $p_{t\,\mathrm{mid}}^\ell$, and perturbative stability, improving at lower $p_{t\,\mathrm{mid}}^\ell$. Given the general convergence pattern, we conclude that an m_W determination with a perturbative-QCD accuracy $\Delta m_W \sim \pm 5$ MeV seems achievable by means of $\mathcal{A}_{p_t^\ell}$.

The right panel of Figure 4 shows the impact on $\mathcal{A}_{p_t^\ell}$ due to considering the envelope of the PDF replicas within our default set, or to employing alternative PDF sets [69–71]. In the former case, a spread of $\Delta m_W \sim \pm 12$ MeV is induced using the NLO+NLL result as a baseline; varying central PDF set conversely results in an $O(30$ MeV$)$ effect on m_W at NNLO+N^{3}LL. Such effects can however be reduced to few MeV via PDF profiling, employing further information not included in $\mathcal{A}_{p_t^\ell}$, such as additional p_t^ℓ bins [72], the anti-correlation of different rapidity windows [73, 74], the combination of W^+ and W^- production channels [2].

As a concluding remark, if the exercise leading to the left panel of Figure 4 is repeated in the operating conditions of the CDF II experiment, the theoretical-QCD uncertainty associated to m_W by means of the jacobian asymmetry is $\sim \pm 30$ MeV ($\sim \pm 10$ MeV) using p_t^ℓ ($M_t^{\ell\nu}$) at NLO+NNLL, which is the accuracy of the theoretical tools [75–77] employed by CDF II. Such figures should be compared to the $O(\pm 2$ MeV$)$ perturbative-model uncertainties quoted by the CDF measurement [9].

5. Outlook

The study of theoretical uncertainties in the context of W-mass extraction is crucial, aiming at 10^{-4} relative accuracy, as well as particularly delicate. The standard procedure of template fitting makes this study involved, especially owing to a necessary step of calibration of the predictions to neutral-current Drell-Yan data, which are to a certain extent extraneous to the W-boson production process: tuning to data certainly improves the accuracy of the description, but not the precision of the underlying physics model.

The proposed jacobian asymmetry $\mathcal{A}_{p_t^\ell}$, described in this contribution, represents an attempt to cast the discussion of m_W uncertainties on more solid theoretical grounds. While capturing most of the sensitivity to m_W variations, thereby representing a good candidate observable for m_W determination, $\mathcal{A}_{p_t^\ell}$ displays remarkable perturbative-QCD properties of accuracy and stability. It also allows good control over statistical and systematic experimental errors, and it allows for a simple unfolding of detector effects, in view of a global combination of different experimental m_W measurements.

A solid prediction for m_W through $\mathcal{A}_{p_t^\ell}$ necessarily hinges upon a reliable description of all effects concurring to the latter, such as PDF variations, non-perturbative modelling, EW and mixed QCD-EW radiation, higher orders in QCD, together with a concrete assessment of the associated uncertainties. While the predictions for $\mathcal{A}_{p_t^\ell}$ presented above are still partial, in that they lack most of these input theoretical ingredients, a new roadmap for m_W measurement is set up where each of these effects can be separately included and assessed in detail, helping reaching a general theory-experiment consensus on the value and the uncertainties associated to the W-boson mass.

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