Complexity results on the Bi-level Interdiction Knapsack Problem with unit interdiction costs or unit weights

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Abstract. In this contribution we pursue the study of the Bi-level Interdiction Knapsack Problem, where a leader interdicts some knapsack items subject to a budget constraint in order to maximally reduce the total profit of the Knapsack Problem that the follower solves on the remaining items. We study the computational complexity of the problem when the leader's interdiction costs are unitary and prove that the problem remains Σ_2^p-complete using a reduction from the bi-level 3-SAT problem. Additionally, we prove that the version with unitary lower level weights but arbitrary interdiction costs is NP-complete.

Keywords: Bi-level Knapsack, Polynomial hierarchy, Interdiction, Stackelberg game

1 Introduction

Interdiction games, which are a subclass of *Stackelberg games*, represent the sequential action of two players who seek to optimise the same objective function but in opposite directions. These problems can be naturally modelled as bi-level optimisation programs, with one agent reacting to the decision of the other. In their basic version, an *attacker* has a limited amount of resources to disrupt or disable some elements of the problem (item to pack, edge or node of a network, etc...), after which a *defender* reacts by trying to perform a certain action as best they can, e.g., maximising a flow or finding a shortest path between an origin and a destination node on a network, selecting a set of items given some capacity constraint, etc...

The most classical interdiction problems are based on the attack of network elements to disrupt classic graph problems, such as maximising a quantity of flow (Maximum Flow Interdiction Problem or MFIP [17]) or minimising the length of the shortest path (Shortest Path Interdiction Problem or SPIP [9]) between an origin and a destination node. Such problems help uncover the weak points of, e.g., critical infrastructures and to assess the robustness of such infrastructures to worst-case failures or malevolent attacks. They have a wide range of possible applications, from the analysis of electric power grids [15] to the one of water distribution networks [20]. While these problems usually consider attacks on network arcs, other interdiction problems focus more on identifying the most vital nodes whose removal has the largest impact on network connectivity, such as the Critical Node Problem (CNP) [11]. Even though those problems are bi-level in nature, they are based on a lower level problem which is polynomially solvable. Hence, the decision version of these problems belongs to the class NP and is usually NP-complete in the general case [1, 9, 17].

More recently, harder interdiction problems have been considered, which have an NP-hard lower level and, therefore, do not necessarily belong to NP. This is the case of an interdiction version of the Knapsack Problem (KP), called Bi-level Interdiction Knapsack Problem (BIKP), where a leader can interdict a subset of items, subject to an attack budget, which cannot be packed by the follower when maximising the profit of his/her knapsack [3, 5, 6, 16]. A version of this problem where the lower level KP weights are equal to the items profits is proved to be Σ_2^p-complete, using a bi-level version of the Subset Sum Problem, along with other bi-level KPs, in [2]. Very few interdiction problems with an NP-hard lower level have been shown to be Σ_2^p-complete, e.g., the Maximum Clique Interdiction Problem [8] and the Facility Location Interdiction Problem [7]. Given its simple structure and its higher level extensions, the KP provides a useful basis for clarifying the complexity of further interdiction problems, which are often tackled with unit attack weights. Moreover, since in the single level KP, forcing either the profits or the weights to be unitary makes the problem polynomial, it is interesting to investigate what happens when some sets of costs, weights or profits are unitary in the BIKP. In the following, we propose to clarify the complexity status of the BIKP, first when the attack costs of the leader become unitary, and then when the weights of the follower become unitary. We will show that those two cases belong to different levels of the polynomial hierarchy.

2 Complexity of the Bi-level Interdiction Knapsack Problem with unit attack costs

As already stated earlier, the computational complexity of the BIKP (as well as a tri-level extension where the defender may act first by protecting some items) has been investigated in the literature, for a specific version where each item has an arbitrary attack cost, and in the lower level (classic KP) its packing weight is equal to the profit [2, 14]. We will consider the more general case where the lower level packing weights are not equal to the profits, but we will concentrate on the specific BIKP version where the attack costs are unitary. We stress that this version is interesting from a theoretical perspective, since in the single level KP, computational complexity tends to be strongly affected when a set of coefficients becomes unitary (as the problem goes from being weakly NP-hard to polynomial). It is therefore a legitimate concern to establish the consequences of adopting unit weights for the higher decision levels in interdiction versions of the KP. In this section, we will establish that even with fortification and attack costs, the BIKP is complete for the second level of the polynomial hierarchy. We stress that the Σ_2^p-hardness proof in [2] for the bilevel interdiction knapsack only holds when the interdiction costs are non-unitary. In their proof, the authors consider two sets of items, D (called *padding* and *dummy* items), with unit attack costs, and O (called *ordinary* items), which cannot be interdicted by the leader. By making the interdiction costs of the items in O unitary, the proof of *only if* does not hold: not only can the leader interdict items in O but there are instances where it has incentive to do so.

Our definition of the Bi-level Interdiction Knapsack Problem with unit attack costs is the following:

Unitary Bilevel Interdiction Knapsack (UBIK):
INSTANCE: A finite set of items I such that each $i \in I$ has a positive integer weight w_i and a positive integer profit p_i, a positive integer capacity W and a positive integer profit goal K for the follower, a positive integer attack budget A for the leader.
QUESTION: Is there a subset $I_l \subseteq I$ of items for the leader to select, with cardinality $|I_l| \leq A$, such that every subset $I_f \subseteq I \setminus I_l$ with $\sum_{i \in I_f} w_i \leq W$ that the follower can select has a total profit $\sum_{i \in I_f} p_i < K$?

In order to prove that the above problem is complete for the second level of the polynomial hierarchy, we will use the following problem, known to be Σ_2^p-complete (Corollary 6 of [18] and [10]):

2-Alternating Quantified Satisfiability ($B_2 \cap 3CNF$):
INSTANCE: Disjoint non-empty sets of Boolean variables X and Y, and a Boolean expression E over $U = X \cup Y$ in conjunctive normal form with at most 3 literals in each clause $c \in C$.
QUESTION: Is there a truth assignment for X such that for all truth assignments of Y, E is never satisfied?

In order to prove the completeness of UBIK, we will introduce a reduction from the $B_2 \cap 3CNF$ which draws inspiration from the reduction from 3-SAT to Subset Sum presented in [4, Theorem 34.15]:

- For each variable $u \in U$, we create two items i_u and $i_{\bar{u}}$, one for each possible truth assignment of u. We define $I_X = \{i_x, i_{\bar{x}} : x \in X\}$ and $I_Y = \{i_y, i_{\bar{y}} : y \in Y\}$. For variables $x \in X$, we introduce a third item j_x, which will serve to force the leader to choose one and only one item between i_x and $i_{\bar{x}}$. In order to ensure a sensible mapping of the variable assignment between Y and I_Y, we also introduce a fourth item j'_x for $x \in X$ to help saturate the budget of the follower for X-related digits (leaving therefore only the Y-related digits of the budget W available for selecting items from I_Y in I_f).

- For each clause $c \in C$, we create two items i^1_c and i^2_c. We designate by I_C the set of items associated with C.
- Weights, profits, capacity and profit goal will be expressed by integer numbers with $|C|+|Y|+2|X|+1$ digits in base 10. The least significant $|C|$ digits are labelled by the clauses, the next $|Y|$ digits by the variables Y, the next $2|X|$ digits by the variables X, two for each variable $x \in X$. Finally, the highest position is labelled as M.
  - For each $y \in Y$, the two corresponding items i_y and $i_{\bar{y}}$ have weights and profits (w_{i_y}, p_{i_y}, $w_{i_{\bar{y}}}$ and $p_{i_{\bar{y}}}$) with digit 1 in the position labelled by the variable y and 0 in the positions labelled by other variables.

    If literal y appears in clause $c \in C$, then p_{i_y} and w_{i_y} have digit 1 in the position labelled as c, and 0 otherwise. Similarly, if the literal $\neg y$ appears in clause $c \in C$, $p_{i_{\bar{y}}}$ and $w_{i_{\bar{y}}}$ have digit 1 in the position labelled by c, and 0 otherwise.
  - For each $x \in X$, the two corresponding items i_x and $i_{\bar{x}}$ have weights w_{i_x} and $w_{i_{\bar{x}}}$ and profits p_{i_x} and $p_{i_{\bar{x}}}$ with digit 1 in the highest position labelled by the variable x and 0 in the lowest x position and in the positions labelled by other variables (remember that each X variable is associated to two consecutive digit positions).

    If the literal x appears in clause $c \in C$, then p_{i_x} and w_{i_x} have digit 1 in the position labelled as c, and 0 otherwise. Similarly, if the literal $\neg x$ appears in clause $c \in C$, $p_{i_{\bar{x}}}$ and $w_{i_{\bar{x}}}$ have digit 1 in the position labelled by c, and 0 otherwise. The weights and profits have digit 0 in all remaining positions.

    The item j_x corresponding to x has weight and profit with digit 2 in the highest position labelled as x (and 0 elsewhere).

    The item j'_x corresponding to x has weight $w_{j'_x}$ with digit 1 in the highest position labelled as x and profit p_{j_x} with digit 1 in the lowest position labelled as x (and 0 elsewhere).
  - For each $c \in C$, the first item has weight and profit with digit 1 in the position labelled as c and 0 elsewhere, while the second item has weight and profit with digit 2 in the position labelled as c and 0 elsewhere.
  - The attack budget A is equal to $|X|$.
  - The weight budget W has digit 1 in all positions labelled as variables in Y, 2 in all highest positions labelled as variables in X, 4 in all positions labelled as clauses in C, and digit 0 elsewhere. Hence, I_f can contain any item from I_X (as long as not interdicted).

- The profit goal K has digit $|X|$ at label M[4], 1 for all positions with labels in U and 4 for all positions with labels in C.

See Figure 1 for an illustration of the proposed reduction.

Analysing an instance from the above reduction, we can infer the following properties:

Property 1. If the leader does not interdict exactly one item between i_x and $i_{\bar{x}}$ for each $x \in X$, the follower can easily reach the profit goal K.

Proof. First of all, the attack budget forbids to interdict more than $|X|$ items. If the leader interdicts strictly less than $|X|$ items of I_X, the capacity of the follower allows to select all the uninterdicted items from I_X, reaching the profit goal K. Hence, only those solutions where the leader interdicts exactly $|X|$ items from I_X (and no item j_x or j'_x for $x \in X$) will be of interest for the rest of the proof. For such solutions, the follower has anyway a sufficient capacity to select all uninterdicted items from I_X (any other possibility leading to a failure to reach the profit goal K). Suppose that the two items i_x and $i_{\bar{x}}$ associated with the most significant digits whose label is $x \in X$ are taken simultaneously in I_l. In this case, the follower can select item j_x as well as all remaining uninterdicted items of I_X, achieving a profit of $|X|10^M + 2 \times 10^{|C|+|Y|+2|X|} > K$. Suppose instead that the leader selects neither i_x nor $i_{\bar{x}}$: since, as already stated, the follower has enough budget to select all remaining I_X items, it is possible for I_f to contain both i_x and $i_{\bar{x}}$ and the profit goal K is automatically achieved as a digit of 2 is achieved in the highest digit associated with x (as well as number $|X|$ in column M). The leader must therefore interdict one item between i_x and $i_{\bar{x}}$, after which, in order to be able to reach profit K, the follower can and must include item j'_x in I_f in order to reach digit 1 in the lowest digit associated to x (as not enough residual capacity is available to select item j_x). Consequently, the capacity associated to both digits associated to x is saturated (otherwise the follower cannot reach K).

This reasoning can then be recursively applied until we reach the lowest X-related digit positions, ensuring the above property.

Property 2. If the leader interdicts exactly one item between i_x and $i_{\bar{x}}$ for each $x \in X$, any solution of the follower which does not select the remaining $|X|$ uninterdicted items of I_X, all j' items and exactly one item between i_y and $i_{\bar{y}}$ for $y \in Y$ cannot reach the profit goal K.

Proof. Suppose the leader does interdict exactly one item between i_x and $i_{\bar{x}}$ for each $x \in X$. The follower has enough capacity to select all remaining uninterdicted I_X items and must do so, otherwise it is impossible to reach value $|X|$ at

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[4] Since $|X|$ can be larger than 10, the contribution of this digit to the value of K should be understood as the number $|X|10^M$. In order to facilitate the exposition, we continue to refer to this contribution to K as a "digit" in base 10.

I			X				Y		C		
		M	a		b		c	d	c_1	c_2	c_3
i_a	w_{i_a}	0	1	0	0	0	0	0	1	0	1
	p_{i_a}	1	1	0	0	0	0	0	1	0	1
$i_{\bar{a}}$	$w_{i_{\bar{a}}}$	0	1	0	0	0	0	0	0	1	0
	$p_{i_{\bar{a}}}$	1	1	0	0	0	0	0	0	1	0
j_a	w_{j_a}	0	2	0	0	0	0	0	0	0	0
	p_{j_a}	0	2	0	0	0	0	0	0	0	0
j'_a	$w_{j'_a}$	0	1	0	0	0	0	0	0	0	0
	$p_{j'_a}$	0	0	1	0	0	0	0	0	0	0
i_b	w_{i_b}	0	0	0	1	0	0	0	1	0	1
	p_{i_b}	1	0	0	1	0	0	0	1	0	1
$i_{\bar{b}}$	$w_{i_{\bar{b}}}$	0	0	0	1	0	0	0	0	1	0
	$p_{i_{\bar{b}}}$	1	0	0	1	0	0	0	0	1	0
j_b	w_{j_b}	0	0	0	2	0	0	0	0	0	0
	p_{j_b}	0	0	0	2	0	0	0	0	0	0
j'_b	$w_{j'_b}$	0	0	0	1	0	0	0	0	0	0
	$p_{j'_b}$	0	0	0	0	1	0	0	0	0	0
i_c	w_{i_c}	0	0	0	0	0	1	0	0	0	1
	p_{i_c}	0	0	0	0	0	1	0	0	0	1
$i_{\bar{c}}$	$w_{i_{\bar{c}}}$	0	0	0	0	0	1	0	1	0	0
	$p_{i_{\bar{c}}}$	0	0	0	0	0	1	0	1	0	0
i_d	w_{i_d}	0	0	0	0	0	0	1	0	1	0
	p_{i_d}	0	0	0	0	0	0	1	0	1	0
$i_{\bar{d}}$	$w_{i_{\bar{d}}}$	0	0	0	0	0	0	1	0	0	0
	$p_{i_{\bar{d}}}$	0	0	0	0	0	0	1	0	0	0
$i^1_{c_1}$	$w_{i^1_{c_1}}$	0	0	0	0	0	0	0	1	0	0
	$p_{i^1_{c_1}}$	0	0	0	0	0	0	0	1	0	0
$i^2_{c_1}$	$w_{i^2_{c_1}}$	0	0	0	0	0	0	0	2	0	0
	$p_{i^2_{c_1}}$	0	0	0	0	0	0	0	2	0	0
$i^1_{c_2}$	$w_{i^1_{c_2}}$	0	0	0	0	0	0	0	0	1	0
	$p_{i^1_{c_2}}$	0	0	0	0	0	0	0	0	1	0
$i^2_{c_2}$	$w_{i^2_{c_2}}$	0	0	0	0	0	0	0	0	2	0
	$p_{i^2_{c_2}}$	0	0	0	0	0	0	0	0	2	0
$i^1_{c_3}$	$w_{i^1_{c_3}}$	0	0	0	0	0	0	0	0	0	1
	$p_{i^1_{c_3}}$	0	0	0	0	0	0	0	0	0	1
$i^2_{c_3}$	$w_{i^2_{c_3}}$	0	0	0	0	0	0	0	0	0	2
	$p_{i^2_{c_3}}$	0	0	0	0	0	0	0	0	0	2
	W	0	2	0	2	0	1	1	4	4	4
	K	2	1	1	1	1	1	1	4	4	4

Fig. 1. Example of construction of an instance of UBIK from an instance $B_2 \cap 3CNF$ with $E = (a \vee b \vee \neg c) \wedge (\neg a \vee \neg b \vee d) \wedge (a \vee b \vee c)$, where $X = \{a, b\}$, $Y = \{c, d\}$ and the clauses are labelled from left to right.

position M. Following the reasoning in the proof of Property 1, the follower must also select all j'_x items, in order to reach digit 1 in all the lowest digit associated to each $x \in X$, therefore saturating the capacity associated to X variables and forbidding to select any j item. In order to be able to reach the profit goal, the follower must also select a subset of items from I_Y:

- The two items associated with the most significant digit whose label is in Y cannot be taken simultaneously in I_f as it would violate the weight capacity W (remember that all non-interdicted items from I_X must be included in I_f, otherwise the profit K cannot be reached). In fact, exactly one of these items must be taken, as otherwise the follower cannot achieve a profit of K.
- The two items associated with the second most significant digit whose label is in Y cannot be taken simultaneously, since we already know that one of the items associated with the most significant digit in Y is taken which would result in a violation of the weight capacity W. Hence, as before, I_f will include exactly one of the items associated with the second most significant digit in Y.
- The reasoning above propagates until the least significant digit labelled in Y. We conclude that the best set I_f will include exactly one of the items i_y or $i_{\bar{y}}$ for $y \in Y$.

Theorem 1. *UBIK is Σ_2^p-complete.*

Proof. The statement of UBIK is of the form $\exists I_l \quad \forall I_f \quad Q(I_l, I_f)$ where Q can be tested in polynomial time, directly implying that it is in Σ_2^p. As mentioned above, we use the previously introduced reduction from the $B_2 \cap 3CNF$. In order to prove the completeness of UBIK, we need to prove that the $B_2 \cap 3CNF$ instance is a *Yes* instance if and only if the corresponding UBIK instance is a *Yes* instance.

Let $B_2 \cap 3CNF$ be a *Yes* instance. We form a solution to UBIK by packing in I_l the items i_x such that $x \in X$ is 1 and the items $i_{\bar{x}}$, otherwise. Clearly, the cardinality of I_l is equal to $|X|$. By construction, given I_l, the best I_f takes all $|X|$ items associated with X not packed in I_l, plus all j'_x items for $x \in X$, as it is necessary if the follower wants to reach a profit of at least K. Furthermore, following Property 2 the optimal I_f for the follower will also take exactly one of the items i_y or $i_{\bar{y}}$ for $y \in Y$. Finally, I_f can be completed by items from I_C. However, since the $B_2 \cap 3CNF$ instance is a *Yes* instance, there is at least a clause for which no item from I_X and I_Y with a non-zero value in the associated digit can be selected. Therefore, for this specific digit, set I_f cannot reach value exactly 4 with only items of I_C while respecting the budget W and the UBIK instance is a *Yes* instance.

Next, suppose that the UBIK instance is a *Yes* instance. According to Property 1 and 2, I_l must contain exactly one of the items i_x and $i_{\bar{x}}$ for $x \in X$ and I_f will contain exactly one item between i_y and $i_{\bar{y}}$ for each $y \in Y$ otherwise the follower cannot achieve profit K. However, since the UBIK instance is a *Yes* instance, no choice of the follower allows to reach a value of 4 for each digit associated to C. This means that for at least one digit associated to $c \in C$, none of the items of X and Y with a digit 1 in position c can be packed, otherwise it is still possible to reach exactly value 6 for this specific digit. Assign 1 to $x \in X$ such that $i_{\bar{x}} \in I_l$, and 0 otherwise. For any follower's packing I_f: assign 1 to $y \in Y$ if $i_{\bar{y}} \in I_f$, 0 if $i_y \in I_f$. Since, by hypothesis, the UBIK instance is a *Yes* instance, there is no Y variables assignment such that E is satisfied in the corresponding $B_2 \cap 3CNF$ instance which is therefore is a *Yes* instance.

3 Complexity of the Bi-level Interdiction Knapsack Problem with unit follower's weights

When considering the BIKP with unit attack costs, should one set of parameters (either the profits or the follower's weights) become unitary, not only the follower's problem becomes polynomial but the bi-level problem itself also becomes polynomial. In this section, we propose to study the case where the attack costs a_i as well as the profits are arbitrary integers while the lower level weights are all unitary. The attack budget constraint now becomes $\sum_{i \in I_l} a_i \leq A$. In order to prove that this version of BIK is NP-complete, we will use a reduction from Subset Sum:

Subset Sum:
INSTANCE: Given a set $S = \{s_1, ..., s_n\}$ of n integers whose sum is $\sum_i s_i = 2\Sigma$.
QUESTION: Is there a subset $S' \subseteq S$ such that $\sum_{s \in S'} s = \Sigma$?

Theorem 2. *BIK with unit lower level weights is NP-complete.*

Proof. The statement of BIK with lower level weights is of the form $\exists I_l \quad Q(I_l)$ where Q can be tested in polyomial time, since the lower level KP is solvable in polynomial time (we simply choose the W items with higher profit not interdicted by the leader), directly implying that it is in NP.

We introduce the following reduction from Subset Sum:

- For each integer s_i, we add an item i to BIK with attack cost and profit $a_i = p_i = s_i$ and unitary knapsack weight $w_i = 1$.
- We set $A = \Sigma$, $K = \Sigma + 1$ and $W = n$, so that potentially the follower can pack all items at the lower level.

From the structure of the BIK instance created, we can see that the only way to ensure that this instance is a *Yes* instance is to interdict a set of items with a total attack cost of Σ, since the profit which can be packed at the lower level is exactly $2\Sigma - \sum_{i \in I_l} a_i$.

Suppose first that the Subset Sum instance is a *Yes* instance. Define a solution of BIK by interdicting all the items corresponding to the integers of S', i.e. $I_l = \{i : s_i \in S'\}$. Since the Subset instance Sum is a *Yes* instance, there exists a solution of the leader in BIK with a total attack cost of Σ, therefore the total profit that can be gathered by the follower is $\Sigma < K$ and the BIK instance is a *Yes* instance.

Conversely, suppose that the BIK instance is a *Yes* instance: in order to have a solution that does not allow a total profit of $\Sigma + 1$, the leader needs to interdict at least a profit of Σ and therefore to interdict a set of total attack cost $\sum_{i \in I_l} a_i = \Sigma$. Define a solution of Subset Sum $S' = \{s_i : i \in I_l\}$, since it has $\sum_{s \in S'} s = \Sigma$, the Subset Sum instance is a *Yes* instance, which proves that BIK with unit lower level weights is NP-complete.

4 Conclusion

We have refined existing complexity results about the Bi-level Interdiction Knapsack Problem, which was proven to be Σ_2^p-complete in [2] with arbitrary attack costs. Given the fact that the Knapsack Problem becomes polynomial when one set of parameters becomes unitary, we investigated the case where one set of parameters of BIKP becomes unitary. We proved that this problem is still Σ_2^p-complete with unitary attack costs and arbitrary lower level weights and profits. When the lower level knapsack weights are unitary, we have proved that the problem, which trivially belongs to NP (since the follower's problem becomes polynomial), remains NP-complete. These results may be used in future works to help prove more straightforwardly the computational complexity of interdiction and fortification problems, which otherwise need more convoluted reductions from multi-level 3-SAT problems [7].

The interesting avenues of research to extend our results include the investigation of the complexity of higher level Knapsack Interdiction Problems with alternating fortification (an item becomes immune to interdiction) and interdiction rounds above the final KP round. The Tri-level IKP, for example has been proved to be Σ_3^p-complete with arbitrary fortification and attack costs in [14], but no other weight configuration has been studied yet. Given that almost no result exists for tri-level fortification-interdiction problem (except those of [14]), we could also use the existing complexity results for the BIKP to prove the Σ_2^p-completeness result for many existing fortification problems with polynomial lower level, such as those of [12, 13, 19].

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