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Dancing a proof, or a dancing proof? Extended embodiment in a Pre-service geometry class

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How do we teach what a proof is?

Pre-service Mathematics teachers, even if—or especially if—enrolled in a Mathematics degree program, often do not learn how to *make* proofs but just as to replicate them: for most students, proofs are just a formal endeavour, the concept of mathematical investigation leading to proofs through reasoning being completely alien to them (Durand-Guerrier et al., 2012).

As part of a study on the use of large textile props to foster geometrical inquiry, I asked pre-service Master level students to determine which geometrical shape was represented by the props: *Convince yourselves and create an argument for convincing everyone else; and, once you are done, elaborate a choreography to show your proof*. The stated didactical objective was to rethink the meaning of "making a proof." In this poster we analyse the effectiveness of the two-way semiotic conversion between mathematical discourses and sensorimotor operatory schemes.

Textile as mathematical instrument for extended embodiment

Vogelstein et al. (2019) advocate the use of foraging and dissecting performances found in public media to create environments for *ensemble learning*, i.e., "learning things that you cannot do alone" (Ma & Hall, 2018); in particular, they present the reenaction of a choreographed performance where groups of four people explore symmetries and transformations of a large textile square. It is the size of the prop that makes it necessary for the participants to "collaboratively co-construct body movements" to "explore mathematical ideas deeply;" i.e., it defines the activity as an *extended embodiment*.

Here no performance is given, it is a metamathematical objective—*Convince everyone this is actually a square*—that leads the mathematical investigation. The *object* created by the textile *artifact* is a new concept image of proofs where the collaborative performance of motor actions enables the mathematical investigation that leads to the proof of a statement: "This [prop] is a square."

Dancing a formal proof

Mathematical discourses (proofs and argumentations) can be considered objects of a category PA whose maps are semiotic transformations; while embodied cognition can be defined as a category EC of sensorimotor schemes whose maps are defined functorially (figure 1) through a covariant map π: $obj(EC) \rightarrow obj(PA)$ given by *Action-Cognition Transduction* (Abrahamson, 2020).

E.g., if we apply the map π to the choreography of figure 2, we can recognise all steps of a *publishable* proof: (A) Consider a quadrilateral $ABCD$; (B) Fold it on the diagonal AC; (C) $ABCD$ is symmetrical with respect to AC; (D) Consider again $ABCD$; (E) Fold it on the line through the midpoints of AB and CD: again, it is symmetrical; (F) Hence $ABCD$ … (G) is a square.

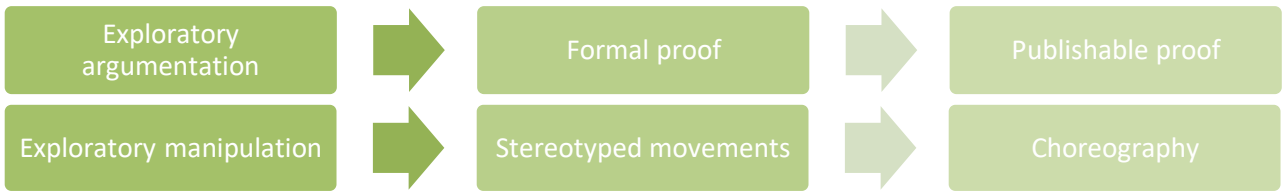

Figure 1: The functorial map from the category *PA* (1st line) to the category *EC* (2nd line)

In figure 3 we can observe another phenomenon: although no specific instruction was given, just a "Now you need to choreograph your proof," each group created a distinctive sensorimotor scheme to mark the end of the proof. Hence, we have a collection $\{\pi_\gamma^{-1}(Q.E.D.)\}$ where π_γ is the semiotic conversion specific to the group γ. What do these objects have in common? They *close* the movement, in the same way a final chord closes a piece of music; students stood still for a short while before folding the action (in (G) we see them holding a "Yes, we did it!" pose); the prop is often slanted towards the public (compare (A) with (G)).

Figure 2: A dancing proof

Figure 3: The choreographed transposition of *Q.E.D.*

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