

ORIGINAL ARTICLE

An embodied theorisation: Arend Heyting's hypothesis about how the self separates from the outer world finds confirmation

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Abstract

At the beginning of the twentieth century, among the foundational schools of mathematics appeared 'intuitionism' by Dutchman L. E. J. Brouwer, who based arithmetic on the intuition of time and all mental constructions that could be made out of it. His pupil Arend Heyting was the first populariser of intuitionism, and he repeatedly emphasised that no philosophy was required to practise intuitionism so that such mathematics could be shared by anyone. Still, stimulated by invitations to humanistic conferences, he wrote a series of notes, preserved in the State Archives, Haarlem, about solipsism. In them, he operated a series of theoretical reflections consisting of the stripping away of the patterns that are part of our consciousness and their subsequent progressive re-introduction, in order to understand the formation of our Self as distinct from the natural world and other humans. In 1996, following a stroke, neuroscientist Jill Bolte Taylor experienced the deprivation of certain abilities in her brain and their successive regaining. In her experience there are remarkable similarities with what Heyting had hypothesised about the formation of the self. The purpose of this article is to highlight them and point out how Heyting's theoretical construct was found to be embodied in the brain.

KEYWORDS

brain, epistemology, intuitionism, logic, mathematics

1 | INTRODUCTION

At the beginning of the twentieth century, among the foundational schools of mathematics appeared 'intuitionism' by Dutchman L. E. J. Brouwer, who based arithmetic on the intuition of

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time and all mental constructions that could be made out of it. A resounding consequence was the suspension of validity of the principle of the excluded third and of the proof by absurdity. These restrictions were unappealing to his contemporary mathematicians, who saw no good reason to accept them. In Brouwer's thinking, the reason was spiritual: the mathematician – like every man – can only be happy by remaining within his inner self: therefore, in his research activity, he must adhere to this indication, starting from what is available in the mind and proceeding within it, without application intent and without relying on language. His pupil Arend Heyting, who was the first populariser of intuitionism, offered a different motivation: to test how far mathematics, carried out all within the limits of human performance, extends. Moreover, he repeatedly emphasised that no philosophy was required to practise intuitionism and considered this a valuable element of his mathematics because it could be shared by anyone. Consequently, Heyting did not publish philosophical reflections. However, stimulated by invitations to humanistic conferences, at the end of his life, Heyting wrote a series of notes, preserved in the State Archives, Haarlem, about solipsism. In them, he operated a series of theoretical reflections consisting of the stripping away of the patterns that are part of our consciousness and their subsequent progressive re-introduction, in order to understand how one's Self becomes aware of its distinction from the world starting from a condition of indefiniteness associated with immense pleasure. In 1996, the neuroscientist Jill Bolte Taylor suffered a stroke and lost part of her brain functions, which she could later re-activate. After many years, she could describe each step of her recovery, starting from the condition of pleasurable indeterminacy experienced in her right hemisphere, when it was disconnected from the left due to the stroke, and ending with the (usual) separation of the self from a reality distinct from it, separation that could occur through the intervention of the left hemisphere. Reading her testimony, one notices remarkable similarities with what Heyting had hypothesised about the formation of the self. In this essay I will present Heyting's thought on the subject, comparing it with that of his teacher L. E. J. Brouwer. Finally, I will report Bolte Taylor's testimony, in order to determine how much of Heyting's (and Brouwer's) theorising has been confirmed by her.

2 | L. E. J. BROUWER

In 'Life, Art and Mysticism', the pamphlet he wrote in 1905, Brouwer described the strenuous achievement of the most hidden intimacy with oneself as follows:

And finally, you do know that very meaningful phrase turning-into-oneself. [...] If, however, you succeed in overcoming all inertia and proceed, you will find that passions will be silenced, you will feel dead to the old world of perception, of time and space, and all other forms of plurality; and your eyes, no longer blindfolded, will be opened to a scene of joyful quiescence. (Brouwer, 1996 [1905])

Brouwer pictured the content of this turning-into-oneself as a 'mixture of all kinds of worlds, a confluent sea of colors, without separation, without firmness and yet without movement, this chaos without disorder'.

He stressed that it is possible to fluctuate from one state to another: 'You will feel free to return when you so wish to the shackles of plurality, separation, time, space, and bodily consciousness. But you do not, or rather, you do and you do not at the same time.'

However, Brouwer noted, if you remain in the undifferentiated, you reach the divine, an idea compellingly expressed in a quote from Eckhart:

When all images have been removed from the soul, and she beholds the Only One, then the naked essence of the soul finds the naked formless Essence of Divine Unity, the presence of the Superior Being waiting in the self. (Brouwer, 1996 [1905])

and in another by Boehme:

When you are silent you are like God before He formed nature and creatures, including yours; you will then hear and see with what God saw and heard in you before your own willing, seeing, and hearing had begun. (Brouwer, 1996 [1905])

Man, however, is generally anxious about the future and tries to keep it under control through the means-effect relationship produced by the intellect. This is what gave rise to the sciences, which eventually became an end in themselves. Hence, the segmented world is a world of great suffering. Brouwer described this state as ‘decayed’ compared to the undifferentiated One. Still, man also recognises that life in the world of perceptions, with all its connections, is his inescapable karma. This leads him to accept his earthly bonds and be reconciled with them: ‘It is precisely this awareness, your humility, which will help you grow in the fullness of the Lord, who will protect you from desires and fears which are not part of the task given to you’ (Brouwer, 1996 [1905]).

Brouwer explicitly resumed this epistemological analysis of the content of consciousness many years later, in the 1948 lecture ‘Consciousness, Philosophy and Mathematics’. Here he described man’s exit from ‘consciousness’, depicted as a form of intimacy, without any mystical references¹:

Consciousness in its deepest home seems to oscillate slowly, will-lessly, and reversibly between stillness and sensation. The initial phenomenon is a *move of time*. [...] through this distinction between present and past, consciousness recedes from both and from stillness, and becomes *mind*. As *mind* it takes the function of a subject experiencing the present as well the past sensation as *object*. And by reiteration of this two-ity-phenomenon, the object can extend to a world of sensation of motley plurality. [...] [In measure of the irreversibility with which the subject has receded, mind becomes disposed to desire and apprehension] Among sensations, *causal attention* occurs. It performs identifications of different complexes of sensations, by creating *iterative complexes of sensations*. [...]

(Brouwer, 1948)

If the elements of these complexes have an invariable order of succession in time, the complexes are called *causal sequences*. If the elements of iterative complexes are permutable in time and are completely estranged from the subject, they are called *things* (among them, human bodies). The whole of egoic sensations indissolubly connected with an individual is called the *soul* of the corresponding human being.

(Brouwer, 1948)

Brouwer dismissed the possibility of affirming the existence of other minds,² since this would require elevating oneself to a second-order mind (i.e., experiencing other people’s minds as sensations) and would establish an infinite progression of meta-minds:

by the choice of this term [viz. mind] the subject in its scientific thinking is induced to place in each individual a mind with free-will dependent on this individual, thus elevating itself to a mind of second order experiencing incognizable alien consciousness as sensations. Quod non est. And which moreover would have the consequence that the mind of second order would causally think about the plurified

¹Heyting summarises this in Heyting (1974).

²Brouwer, unlike Heyting, distinguished between mind and soul, and called ‘intellect’ what Heyting would call ‘reason’. It is interesting to observe that in his unpublished notes ‘F5’, Heyting extensively quoted Popper-Eccles’ *The self and its brain: an argument for interactionism* (Popper & Eccles, 1977), but he was not at all concerned with the mind (ego)–brain relationship, which was at the centre of this text.

mind of first order, then cooperatively study the science of the plurified mind, and in consequence of this study assign a mind of second order with sensation of alien consciousness to other individuals, thus once more elevating itself, this time to a mind of third order. And so on. Usque ad infinitum. (Brouwer, 1948)

He concluded: ‘In default of a plurality of mind, there is no exchange of thought either. [...] By so-called exchange of thought with another the subject only touches the outer wall of an automaton. [...] Only through the sensation of the other’s soul sometimes a deeper approach is experienced’ (Brouwer, 1948).

Brouwer also considered the issue of ‘language’, which will be important for our comparison with Bolte Taylor’s perspective. In his 1905 pamphlet, he described language as a *very fragile* instrument of communication: ‘The immediate companion of the intellect is language [...] never has anyone been able to communicate his soul by means of language’ (Brouwer, 1996 [1905]). He admitted that ‘in the very narrowly restricted domains of the imagination such as in the exclusively intellectual sciences [...] there can mutual understanding be maintained for some time. There is little scope for misunderstanding notions such as “equal” and “triangle”’, but, he added, ‘even then two different people will never feel them in exactly the same way’ (Brouwer, 1996 [1905]). Two years later, in his Ph.D. thesis *Over de grondslagen der wiskunde*, he claimed that ‘a logical construction of mathematics, independent of the mathematical intuition, is impossible – for by this method no more is obtained than a linguistic structure, which irrevocably remains separated from mathematics’ (Brouwer, 1907). Likewise, in 1930 (‘Die Struktur des Kontinuums’) he stated: ‘Nun gibt es für Willensübertragung, insbesondere für durch die Sprache vermittelte Willensübertragung, weder Exaktheit, noch Sicherheit ... Es gibt auch für die reine Mathematik keine sichere Sprache’ [Now there is neither exactness nor certainty for the transmission of will, especially for the transmission of will mediated by language ... There is no certain language for pure mathematics either.] (Brouwer, 1930).

In 1933 Brouwer stated that ‘for a human mind equipped with an unlimited memory, pure mathematics, practised in solitude and without using linguistic signs, would be exact, but the exactness would be lost in mathematical communication between human beings with an unlimited memory’ (Brouwer, 1933, 1948). Of course, he realised that there was no such human being, so he admitted the use of language to support memory (‘the language of daily intercourse [...] will only be suitable for its task of mnemotechnic, economy of thought and understanding in mathematical research and in mathematical intercommunication’ (Brouwer, 1948). Nonetheless, he stressed that ‘[e]ven when the formal system coincides with intuitive mathematics, or expressed more adequately, when they are parallel, then *exactness* lies in the intuition, never in the formula’ (Brouwer, 1948).

One can therefore understand Brouwer’s dislike of the use of formulas in his essays. It was further exacerbated by his awareness³ that all utterances have a content component and an emotional component, albeit to varying degrees (e.g., in symbolic language the emotional component is reduced to zero). The emotional component is essential in order to appeal to the interlocutor’s will. Consequently, a language that uses formulas (in which the emotional component is reduced to a minimum) does not lend itself to successful communication.

3 | A. HEYTING

Arend Heyting was always averse to connecting intuitionism with the mystical-moral justification given by its founder L.E.J. Brouwer. He was afraid that appealing to mysticism might put off those who did not have the sensitivity for that subject or even considered it ridiculous:

³This awareness was at least partly due to the influence of the Signific movement, imported into Holland from England, where it had been established by Victoria Welby.

Brouwer's explanations are incorporated in an exposition of his views on science in general and of an all-embracing conception of life and the world. Fortunately, in order to understand intuitionistic mathematics, it is not necessary to adhere to philosophic and psychological theories. (Heyting, 1958)

Therefore, he always stressed that no philosophical approach is needed to understand intuitionism, and that in order to grasp the original intuition from which mathematics can be developed, it is only necessary to observe one's own mental faculties. In this manner, we realise that we have the ability to 'distinguish', which lies at the basis of all our thinking (and indeed of the very possibility of thinking). This is the intuition from which natural numbers and 'indefinitely proceeding sequences' develop.

Nevertheless, in some surviving yet unpublished notes that Heyting drew up in different periods of his life – and which are described in Franchella (1995)⁴ – he outlined his general view of the origin of human knowledge, starting from the only possible foundation, the initial state of our consciousness: 'To follow these reflections, one has to move quite far away from one's daily habits of thinking, even to put them almost entirely aside' (F1.3).

These habits depend on the predominant action of what Heyting calls 'reason': 'The situation is such that reason (*Verstand*) flattens the emotional life and takes away its broad outlines' (F1.3).

If you manage to set apart habits – to put 'reason' aside – you will find that:

First, there is nothing 'specific', 'sharp' in it: the sensations flow into each other and have no individuality of their own. (F2)

[...] It is to me as if I were a fine fabric, but covered with a thick layer of paint, so that only the coarse folds remain visible. Where I manage to remove the layer, the pattern emerges, and it is a nondescript pattern. No one has ever seen anything more beautiful; its lines are meaningless: they fade away, yet in them lies all the power of the world, all beauty and emotion. (F1.3)

There is another feature of the life of the mind that I would like to highlight: its complete indeterminacy. The life of the mind is one and indivisible: it is our self and at the same time the universe. There are no separate units, no relations between them. (F1.4)

This 'original' state, freed from the 'habits of rationality', engenders ecstatic bliss: man can only live his life fully – in a state of ecstasy, when all rational considerations have been put aside. (F1.4)

Such ecstasy reminds Heyting of Nirvana: 'The most beautiful and purest state is imagined in Buddhist Nirvana, where the ultimate goal of human striving is Thought alone' (F1.4).

This state of blissful indeterminacy, however, cannot be maintained for long:

But how is it that it cannot exist without the protective layer? No sooner has it been removed than the touch of the outside world causes it to wither and fade, so that I quickly cover it up again to save it from destruction. (F1.4)

If we verify the content of our consciousness, we realise at first that we perceive that other persons exist. The first step consists in recognising that other people exist (as minds). Heyting says that we have to believe in our perceptions, because it is very natural for us to do so. By focusing on our perceptions of other persons, we note that we perceive others as *minds* and not as bodies.

⁴The sheets grouped together and labelled as 'F1' and 'F2' date from 1930 to 1940. The sheets labelled as 'F3' were written in 1978; 'F5' were written between 1978 and 1980. No date is suggested for the F8 group.

In other words, another person will first come into contact with us through an exchange of thoughts and this contact enriches us. Therefore, we conclude that other people have a life of feelings and thoughts like our own. Still, ‘since Spirit and matter are not separate; my fellow human beings also appear to me materially’ (F3.2).

The second step consists in believing in the existence of the outer world, because it is one of the representations that we have in our consciousness. Certainty (or surety) is mixed with a sense of challenge and threat in the same perception, and this perception feeling is interpreted by us as a sign of the existence of the outer world. Once objects are identified, they need to be linked. This is where spatialisation and temporalisation come into play, for they are forms of ordering. The most frequent is temporalisation, which enables us to remember: since consciousness knows only the present, memories are actual sensations which we ourselves place in a past time. As for space, since some representations of sight and taste allow themselves to be inserted into a three-dimensional frame and this ‘works’ with respect to surviving, reason tries to generalise this frame.

Therefore, Heyting presents the various elements at play in the following order: content of consciousness; other minds; outer world; and abstraction.⁵

As for language, Heyting used symbolic language to express a provisional axiomatisation of intuitionist mathematics, because it seemed to him to be the only way to make his formalist or logicist colleagues understand the mathematics he was constructing. However, like his teacher, he was convinced that language was *not* an essential component of mathematics, but only served to support memory or communication – not to construct concepts themselves. Indeed, in the introduction to his 1931 article ‘Die intuitionistische Grundlegung der Mathematik’, Heyting wrote: ‘Die intuitionistische Mathematik ist eine Denктаetigkeit, und jede Sprache, auch die formalistische, ist fuer sie nur Hilfsmittel zur Mitteilung’ [Intuitionistic mathematics is a thinking activity, and every language, even the formalistic one, is for it only a means of communication.] (Heyting, 1931).

In the 1956 volume *Intuitionism: an Introduction*, within the fictitious dialogue between various representatives of foundational views on mathematics, the formalist states: ‘The only way to achieve absolute rigour is to abstract all meaning from the mathematical statements and to consider them for their own sake, as sequence of signs’ (Heyting, 1956). The intuitionist replies: ‘As the meaning of a word can never be fixed precisely enough to exclude every possibility of misunderstanding, we can never be mathematically sure that the formal system expresses correctly our mathematical thoughts’ (Heyting, 1956) and ‘intuitionism proceeds independently of the formalization, which can but follow after the mathematical construction’ (Heyting, 1956).

In his manuscripts Heyting states: ‘In every communication there is a factual content and an intention to influence’ (F8.32) and ‘language relates only, at least mainly, to social relations and is therefore not fundamental’ (F8.16).

He further explains that:

L[anguage is used to influence others. When I say: ‘the statement that there is an external world does not make clear sense’, I want to make others think about their belief in an external world. Relationships with other people play a special role in addition to the representation of the external world (F8.24).

With regard to the poverty of language, he specifies: ‘This belief [that there is an external world] is something in my mind. Not “Cogito ergo sum”, but “Cogito” or better still “I live”. There is no word for it, because language is not made to represent reflection’ (F8.30).

⁵In this regard, Heyting recalls Popper-Eccles’ distinction between ‘three worlds’: (1) the physical world, which includes the brain itself; (2) the mental world; and (3) the world of products of thought considered as objects in their own right. This was originally proposed in opposition to the viewpoint that reduced worlds (2) and (3) to the brain (materialism) and to the viewpoint that reduced worlds (1) and (3) to the mind alone. Heyting contrasted this with his own new subdivision into four worlds: (1) the world of the contents of consciousness; (2) the world of other minds; (3) the external world; and (4) the world of abstract entities.

There is one text, however, which seems to open up the possibility that language might play a significant role in the existence of numbers. In a 1974 article ('Intuitionistic views on the nature of mathematics'), describing how it is possible to construct mathematics without philosophical foundations, Heyting sets out from the fundamental function of our mind (i.e., the isolation of objects):

In reality, what we isolate mentally are not objects, but perceptions. I can fix my attention on a certain impression, in most cases visual. In practice this impression is immediately associated with innumerable memories, impressions, and images to form the notion of an object in the general sense of the world. But for counting it is inessential what there is isolated, it is the mental act of isolating that matters. (Heyting, 1974)

Then the question arises: 'How is it possible that an entity that has been isolated maintains its entity and how can it be distinguished from another entity?' (ibid.). Heyting answers that the fact is enough for us:

Again this is a reasoning post factum: Anybody can experience for himself that he is able to fix his attention on a perception, and then on another perception, keeping the first in his memory. This is the basis of counting (ibid.).

Still, he adds: 'It is clear that in constructing the number five, say, the nature of the entities which constitute the number is completely irrelevant. *As soon as the numerals*⁶ were introduced, people have learnt to abstract from the content of the perceptions which are isolated and to consider them as pure entities' (Heyting, 1974).

Thus, in referring to numerals, that is to the names for numbers as the basis for the formation of the corresponding concepts, Heyting attributed a role to language in the formation of concepts, but neither grasped nor emphasised it as such. He only seemed interested in highlighting how – to use Kantian terminology – reference to a *noumenon* would be useless for doing intuitionist mathematics.

Later, in the twilight of his life, he lamented: 'I regret that my name is known to-day mainly in connection with these papers [...] They diverted the attention from the underlying ideas to the formal system itself' (Heyting, 1974).

In any case, notwithstanding his theoretical cautiousnesses on the issue of successful communication, Heyting declared his conviction that contact with other human beings enriched him spiritually. Indeed, he stated that, in his theoretical journey from the undifferentiated content of consciousness to other human beings and to objects in the 'external' world, he knew human beings first as minds and only subsequently as bodies – precisely because he felt spiritually enriched by them (and this presupposed successful communication).

This trust in exchanges between human beings meant that Heyting did not base intuitionism on the need to reconcile mathematical research with the practice of remaining as closed within one's own self as possible, by avoiding sinful contact with others; rather, he simply based it on the original human function of 'distinguishing', of finding unities. Perhaps this is why Heyting did not refer to any of Brouwer's writings as a starting point for his reconstruction of how man attains knowledge of himself, others and the world, even though there are remarkable similarities between their views. Heyting did not quote Brouwer, but only the philosophers he had read at the time of writing his notes (Wittgenstein, Popper-Eccles, Mithoff, De Pater, Russell, Hume, Wisdom, and Ryle), noting any relevant and contrasting ideas.

⁶Emphasis added.

4 | A COMPARISON BETWEEN BROUWER/HEYTING AND BOLTE TAYLOR

If one reads Jill Bolte Taylor's text with Heyting's notes in mind, one is astonished at how Heyting's epistemological framework – which takes as its starting point the undifferentiated from which 'reflection' then draws the distinction between self and world – finds its counterpart in the two hemispheres of the human brain. Bolte Taylor describes the sensations she experienced after a stroke in her left hemisphere as follows:

Devoid of language and linear processing, I felt disconnected from the life I had lived, and in the absence of my cognitive pictures and expansive ideas, time escaped me. [...] In the absence of my left hemisphere's analytical judgement, I was completely entranced by the feelings of tranquillity, safety, blessedness, euphoria, and omniscience. (Bolte Taylor, 2009)

My entire self-concept shifted as I no longer perceived myself as a single, a solid, an entity with boundaries that separated me from the entities around me. I understood that at the most elementary level, I am a fluid. (Bolte Taylor, 2009)

My right mind is all about the richness of this present moment. It is filled with gratitude for my life and everyone and everything in it. It is content, compassionate, nurturing, and eternally optimistic. (Bolte Taylor, 2009)

What we find in Bolte Taylor, then, is a state analogous to the initial state described by Heyting: undifferentiated and blissful. Furthermore, Bolte Taylor, too, compares this state to Buddhist Nirvana: 'I'm no authority, but I think the Buddhists would say I entered the mode of existence they call Nirvana' (Bolte Taylor, 2009).

Still, such a state, according to Heyting, is unsustainable for a human being (making it necessary to abandon the original state of consciousness), whereas Boyte Taylor had a more specific reason for wanting to reconnect the right hemisphere – the hemisphere of bliss – to the left one: 'The memories from my past were no longer available for recollection, leaving me cloaked from the bigger picture of who I was and what I was doing here as a life form' (ibid.). She also attributes intuition to the right hemisphere (in a sense very close to Poincaré's, i.e., as a capacity for global vision⁷) and wishes to see a balance between the two sides.

My right mind does not perceive or give heed to territories or artificial boundaries like race or religion [...] In the consciousness of my right mind, we are laced together as the universal *tapestry* of human potential. And life is good and we are all beautiful [...] My right mind character is adventurous, celebrative of abundance, and socially adept. [...] [I]t is open to the eternal flow whereby I exist *at one* with the universe. It is the seat of my divine mind, the knower, the wise woman, and the observer. It is my intuition and higher consciousness. [...] [I]t is highly creative in its willingness to try something new. (Bolte Taylor, 2009)

⁷Poincaré repeatedly examined the concept of intuition in an effort to better appreciate its indispensable contribution to mathematical discovery. In *La valeur de la science*, he listed three types of intuition (Poincaré, 1905): 'First, the appeal to the senses and imagination; then, generalization by induction, based, so to speak, on the procedures of the experimental sciences; finally we have the intuition of pure number'. However, he repeatedly described intuition as something fundamental and possessed – at least to some extent – by all mathematicians, even those he called 'analysts': mathematicians who like to proceed by means of logic, without any visual support. Indeed, their reasoning by recurrence (i.e., mathematical induction), 'contains, as it was, condensed into a single formula, an infinity of hypothetical syllogisms'; therefore, it is a kind of unitary view. Poincaré seems to attribute a further function to this 'unitary' intuition, that of checking the adequateness of a definition with respect to the real object it refers to: 'This shows us that logic is not sufficient, that the science of demonstration is not the whole of science, and that intuition must retain its role as a complement, but I would also say as a counterweight or antidote to logic' (Poincaré, 1905).

My left mind is responsible for taking all of that energy, all of that information about the present moment, and all of those magnificent possibilities perceived by my right mind, and shaping them into something manageable [...] Just as my right mind thinks in collages of images, my left mind thinks in language and speaks to me constantly. Through the use of brain chatter, it not only keeps me abreast of my life, but also manifests my identity. Via my left brain language center's ability to say, 'I am', I become an independent entity separate from the eternal flow. As such, I become a single, a solid, separate from the whole. [...] its ability to categorize, organize, describe, judge, and critically analyze everything. It thrives in its constant contemplation and calculation. [...] it is a magnificent multi-tasker [...] Because it thinks sequentially, it is great at mechanical manipulation [...]. (Bolte Taylor, 2009)

Bolte Taylor stresses the social character of the left mind, while Heyting expresses his social sensibility by distinguishing the self from 'the rest', that is what should be the domain of the left mind; in other words, he first perceives others' minds and then the outer world (and, with it, bodies) because he feels enriched by others. For his part, Brouwer did not share such sociability at all. When he presented his profession of faith at the age of 17, he did not like other people at all: 'The human spectres around me are the ugliest part of my world of images' (van Stigt, 1990).

When writing to his friend C. S. Adama van Scheltema on 23 May 1903, he expressed his belief in their own superiority: 'feel ourselves joined by "knowing each other to be King"' (van Dalen, 2011). In 1905, he expressed regret over the loss of the happy world of original man, before sin, by painting the following picture of it:

Originally man lived in isolation. Supported by nature, every individual sought to maintain his equilibrium between sinful temptations. That filled the whole of his life; there was no involvement with others, nor was there any worry about the future. As a result hard work did not exist, nor did sorrow, hatred, fear, or lust. But man was not content; he started to assert control over his fellow men and to search for certainty about the future. (Brouwer, 1996 [1905])

Furthermore, as we have seen above, he was very sensitive to the difficulty of communication and did not theoretically acknowledge that other people have a mind. Finally, when describing Brouwer's life in 1968, Heyting characterised his thought as 'solipsistic', adding, however, that solipsism did not exhaust all aspects of Brouwer's thought and that it was not essential for his theory of mathematics (Heyting, 1968, 1980). In his unpublished notes, Heyting shares Jan Mitthof's opinion that solipsism is a premise for any philosophy; still, he adds that solipsism cannot be accepted:

The expression: 'thinking of an outer world is an illusion' is not a good expression of solipsism, since it presupposes that there should exist an outer world, or at least that we know what it means. (F8.32)

So Brouwer's poor sociability represents a difference between him and the other two thinkers.

Furthermore, Brouwer did not base intuitionism on a Poincaré-like notion of intuition – as an overall view – but rather on a temporal kind of intuition, which should therefore be located in the left hemisphere, like Heyting's intuition, the 'faculty of individuation'. Still, in most human beings this hemisphere is logical-linguistic, whereas, for the two mathematicians, intuitionism proceeds independently of logic, by finding new paths, which would suggest more the

involvement of the right hemisphere. Nevertheless, if we look back at the fact that Bolte Taylor puts language at the basis of the stability of the ego, and hence of objects, through continuous internal dialogue, we can see a connection with the task that the two intuitionists assigned language, namely to be an instrument of communication: internal dialogue is communication with oneself. We can say that Brouwer admitted this when he acknowledged that only a being with an unlimited memory could do without language. Man, not having an unlimited memory, must remind himself through language of the constructions he has made.

Finally, if we look at the 1912 text 'Intuition and formalism', in which Brouwer sought to define the opposition between formalists and intuitionists, we find that to the question 'Where is exactness found?', he answered 'in the mind' in the intuitionists' case, 'on paper' in the formalists' case (Brouwer, 1913 [1912]). So, what characterises the intuitionists is their view that mathematics is a mental activity, drawing on nothing external to the mind. And this is absolutely in line with what has been described concerning the two hemispheres.

5 | CONCLUSIONS

In the previous paragraph we have considered similarities and differences among the three authors. The most striking similarity has emerged from Bolte Taylor's description of the right hemisphere as the source of emotion, a place where there is no classification, no correlation between discrete elements, no linguistic expression; where we do not perceive our body as distinct from our surroundings through our skin, but are embedded in a marvellous universal whole floating in an overall vision. This suggests that the original state described by Brouwer and Heyting is rooted in the human brain – more precisely, in its right side. In this regard, a caution is needed. When referring to the laterality of functions in the human brain, we must always use adverbs like 'in general', 'mostly' and so on, because in the literature there are many studies showing *differences* in the distribution of functions between the hemispheres *across individuals* and looking for what causes such differences.⁸ Bolte Taylor herself has recently recognised in her 2021 book *Whole Brain Living* that our emotional limbic tissue is divided between our two hemispheres and consequently that each hemisphere has both an emotional brain *and* a thinking brain. She has presented these four distinct modules of cells as four *characters* that make up who we are (Character 1, Left Thinking; Character 2, Left Emotion; Character 3, Right Emotion; and Character 4, Right Thinking). She has also suggested 'how to identify and relate them to one another, and choose the right one in any situation' (Bolte Taylor, 2021). Therefore at the end of this paper, we have to stress that what is relevant for the subject at issue is that Bolte Taylor's description of her stroke experience lets us believe that the original state described by Brouwer and Heyting is rooted in the human brain (no matter where) and that a theoretical construct has turned out to be truly embodied in the human brain.

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⁸Since Knecht et al. (2000) showed that 'in healthy subjects there is a consistent and almost linear relationship between the degree of handedness and the direction of language dominance' (Heyting, 1980) a lot of studies have been devoted to point out relationships between brain function lateralisations and specific characteristics of the subjects. For instance, see Beking et al. (2018) and Packheiser et al. (2020).

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