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Chapter 9: How Do We Transform Appearance into Experience?: Kant's Metaphysical Foundations of Phenomenology

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1. Introduction to the Phenomenology: Beyond Newton's *Principia*

The Phenomenology has not received much consideration from commentators, at least compared with the Dynamics and the Mechanics. For instance, Pollok's (2001) commentary to Kant's MAN devotes only around 35 pages (of a roughly 550 page book) to the Phenomenology. That said, Friedman's book *Kant's Construction of Nature* (2013) represents a recent exception to this trend, offering the most complete account that extensively discusses this chapter. Nonetheless, the Phenomenology is the most intriguing, yet under-explored part of MAN, not only with respect to its relationship with other parts of Kant's system, but also with respect to the natural science of Kant's time. This contribution builds upon and elaborates Friedman's reading. More specifically, it facilitates the comparison of the Phenomenology with both other parts of Kant's system and the science of his time, in particular with Euler's equation of motion for rigid bodies.

In *Kant's Construction of Nature* (2013), Michael Friedman proposed to read MAN as the result of Kant's attempt to find the a priori principles underlying Newton's laws and their application. Friedman argues that Kant's work provides "Leibnizean" metaphysical foundations for Newtonian physics (2013, 90). This inspired criticism from the side of historians of philosophy (Stan 2013; 2014a) who emphasize the limits of defining as 'Leibnizean' the metaphysics devised in MAN.[1] Friedman's work also stimulated further studies in the history and philosophy of science, such as Patton's (2017), and in the history of space-time theories (Hyder 2014; 2019). In this contribution, a) I extend Friedman's reading of the Phenomenology by including Euler in the picture and b) I propose a more radical thesis to the

[1] In Massimi and De Bianchi (2013), for instance, the legacy of Cartesians, such as De Mairan, is emphasized in order to show how the metaphysical concept of force in MAN cannot be defined neither as 'Leibnizean' nor as "Newtonian."

Phenomenology than Friedman. In particular, I offer a deeper analysis of time representation and emphasize the function of "reduction" (*Reduktion*) to be ascribed to the idea of absolute space in order to highlight the possible link of the Phenomenology to the development of Kant's thought, including his doctrine of measurement in KU. This move allows us to consider unexplored paths in Kant studies and contributes to a deeper understanding of the impact of the MAN on the development of Kant's system itself.

The rationale behind my reading is that in MAN Kant is not just assuming the perspective of transcendental philosophy, or that of determining the possibility of an object of experience in general. The perspective that Kant wants to develop is that of his metaphysics of nature, a metaphysics purified from errors and dogmatism. I therefore depart from Friedman's reading and attribute a central role to the faculty of reason in the Phenomenology. In my view, Kant's metaphysical foundations are not aimed at the conceptual foundation of Newton's physics only, but at any development of Newtonian mechanics. As shown in the next sections, Kant tried to attain this goal by means of the assumption of the law of antagonism in all community of matter through motion based on the principle of reciprocal interaction (*Wechselwirkung*), which corresponds to the function of reducing and unifying all appearances under an idea of reason. I do not contend that Friedman overlooks this; he instead offers a very detailed account of the Phenomenology and its connection to the first *Kritik* and Kant's cosmology (Friedman 2013, 509–608). However, Friedman looks at the correlative function of the Phenomenology as a link between KrV and MAN. With respect to Friedman's reading, I want to make a further step and frame the Phenomenology within a broader picture that includes not only KrV, but also the KU. In other words, my claim is that in the Phenomenology Kant had to face on fundamental questions related to his system and worldview, including the metaphysical foundations of any scientific knowledge, be it a priori or empirical, as well as to the a priori foundations of observational cosmology. Indeed, the conclusions of Phenomenology are related to the application of Newton's mechanics beyond the solar system and are in agreement with Kant's 1755 cosmology. They are related as such to the idea of the cosmos (*Weltganze*) as a whole (see Friedman 2013, 494–98). In this case, however, the object under discussion is not the whole of rational cosmology, i.e. the world (*die Welt*), but rather the physical universe. Thus, it seems to me crucial to read Kant's

Phenomenology within the construction of a metaphysics of nature, which includes the problem of harmonizing the mathematical and philosophical representation of (ideal) time, (material) space, and motion with both the foundations of cosmology and the end of natural science in general (see MAN, 4:477).

2. Structure of the *Phenomenology*

At the very beginning of the chapter, the Explication and its Remark give the definition of motion as appearance and show in which sense one needs to objectively represent motion in order to transform appearance into experience:

> "Hence the movable, as such a thing, becomes an object of experience, when a certain object (here a material thing) is thought of as determined with respect to the predicate of motion. But motion is change of relation in space. There are thus always two correlates here, such that either, first, the change can be attributed in the appearance to one just as well as to the other, and either the one or the other can be said to be moved, because the two cases are equivalent; or, second, one must be thought in experience as moved to the exclusion of the other; or, third, both must be necessarily represented through reason as equally moved. In the appearance, which contains nothing but the relation in the motion (with respect to its change), none of these determinations are contained. But if the movable, as such a thing, namely with respect to its motion, is to be thought of as determined for the sake of a possible experience, it is necessary to indicate the conditions under which the object (matter) must be determined in one way or another by the predicate of motion. At issue here is not the transformation of semblance into truth, but of appearance into experience" (MAN, 4:554–55)

Proposition 1 is based on the definition of motion and change offered in the Remark to the Explication. It states that the rectilinear motion of matter with respect to an empirical space is a merely possible predicate. The same rectilinear motion, when thought in no relation at all to matter external to it, that is, as absolute motion, is impossible. The Proof (4:555) goes as follows. Relative motion, be it of a body or a space that moves with same speed in opposite direction and the other is at rest, is always determinable with respect to its relation to the representation of the subject. Therefore, relative motion is always appearance and never experience. This is the case because the subject a) either collocates himself in the space at rest and the body appears in motion to him or b) if he locates in another space comprehending the first one relative to which the body is likewise at rest, then that relative space counts as moving. More specifically, in case b) an external framework or space is needed in order to establish whether a space is in relative motion to a body, but this always implies a standpoint external to the system, e.g. the body at rest and the relative space in motion, in such a way that the objective determination of the predicates of matter (as being at rest or moving) cannot be actually assessed but it is a mere possibility.

Subsequently comes the interesting part of Kant's argument insofar as he gives a definition and a mixed proof to support this position. Kant writes:

> "Thus in experience (a cognition that determines the object validly for all appearances) there is no difference at all between the motion of the body in the relative space, and the body being at rest in absolute space, together with an equal and opposite motion of the relative space" (4:555–56).

In this mixed proof Kant appeals to principles that do not strictly pertain to metaphysics but that serve to help metaphysics to ground natural science. Namely he refers to logical rules that make possible the determination of the object for all appearances. Whereas in the proof of the second Proposition Kant is more explicit and explicitly defines this principle, here we can extrapolate a negative rule, namely that relative motion always brings with it the concept of underdetermination with respect to which of two opposed predicates that can be attributed to it. In particular, appearance gives rise to judgments that use one of the predicates that differ only in regard to the subject and its mode of representation. These are alternative judgments, not disjunctive judgments proper. Alternative judgments allow an arbitrary choice between the two predicates. Both can be applied depending on where the subject is located in the system, and therefore we can infer only the possibility of rectilinear motion.

The chapter then proceeds to Proposition 2, which derives from Lambert's treatment of motion in the Phenomenology Chapter of his *Neues Organon* (1764).[2] Kant reinterprets it as follows:

> "The circular motion of a matter, as distinct from the opposite motion of the space, is an actual predicate of this matter; by contrast, the opposite motion of a relative space, assumed instead of the motion of the body, is no actual motion of the latter, but if taken to be such, is mere semblance (*Schein*)." (MAN, 4:556–57)

In the Proof of Proposition 2, Kant employs the conceptual dichotomy between internal and external and deploys again a mixed proof. The proof displays, I suggest, his adoption of Euler's laws of motion, because he portrays circular motion as a change of a change in external relations in space and as continuous arising of new motions. This definition is compatible with Euler's laws of motion. Indeed, Kant grounds this definition of circular motion on the law of inertia and a concept of moving force that is heavily linked with his metaphysics of causality.[3] He then appeals again to general logic, arguing that in the case of circular motion we are appropriately, i.e. universally and necessarily determining the object when we are effectively using a disjunctive judgment in which one of the predicates that we attribute to matter is actual, whereas the one attributed to space is excluded. This logical underpinning helps in identifying any source of error, at least in Kant's view, when attributing circular motion to space, because phoronomically it has no moving force, therefore this predicate is not determining the object validly for all appearances. Only a physical body can possess circular motion.[4] I shall come back to this point later, but for the time being we should notice that from the strict physical standpoint, Kant is here following upon Euler, who also attributed circular motion to the body rather than to space. In his work on the dynamics of rigid bodies, Euler did not endorse

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[2] Friedman (2013, 421–31), among others, rightly put it in connection to Lambert. In his *Neues Organon*, indeed, the fourth section is called "Phenomenology," in a quite similar way to Kant's work. However, Kant investigates the difference between true and apparent motion, and, in a sense, he is echoing Lambert's view of Phenomenology in Proposition 2 where the distinction between apparent and true motion is associated with a disjunctive judgment.
[3] For a clearer account of the relationship between semblance (*Schein*), appearance (*Erscheinung*) and causality in connection with transcendental idealism of space and time, see FM (20:269). I am very thankful to Claudia Laos for pointing out to me this passage.
[4] For this reason, I think that one should compare Euler's equations of motion with the Phenomenology rather than the Phoronomy, which deals with the kinematics of points rather than physical bodies. About this point see also section 4.1 below.

the necessity of determining through an external moving force circular motion as true motion, rather he pointed out how the inner state of the body and the law of inertia could explain this thanks to the introduction of external frames of reference (Bartoloni-Meli 1993; Maronne and Panza 2014).[5] Just as Euler did, Kant appeals to the conceptual distinction between internal state of the body and external force. However, the difference between Euler's and Kant's respective takes on force must be highlighted. Kant believes that force is in need of a definition, a metaphysical one, because it is a fundamental concept of natural science, whereas Euler thought that impenetrability was the fundamental idea from which the concept of force could be derived. Also consider the following passage:

> "Thus the circular motion of a body, as distinct from that of space is an actual motion so that the latter, even though it agrees with the former according to the appearance, nevertheless contradicts it in the context of all appearances, that is of a possible experience, and so is nothing but mere semblance" (MAN, 4:557).

Here Kant remarks on the complex character of possible experience: we never transform appearance (in singular) into experience, but the latter always presupposes a collective interaction of a plurality of appearances. The fundamental concept of force serves as a bridging-concept between metaphysics, mathematics, and physics and grounds the mutual interaction among bodies. In this respect, impenetrability is just a consequence of repulsive force.[6]

This passage also marks the transition to the third Proposition: "In every motion of a body, whereby it is moving relative to another, an opposite and equal motion of the latter is necessary" (4:558). Its Proof is based on the third Law of Mechanics:

[5] As Maronne and Panza (2014, 16) underscored: "A new fundamental change occurs when extrinsic reference frames, typically constituted by triplets of orthogonal fixed Cartesian coordinates, are introduced and when the relativity of motion is conceived to be the invariance of its laws with respect to different frames submitted to uniform retailer motions".
[6] For studies on Kant's notion of repulsive force, see Warren (2001), Friedman (2013), Massimi and De Bianchi (2013).

> "The communication of motion of bodies is possible only by the community of their original moving forces, and the latter only by mutually opposite and equal motion. The motion of both is therefore actual." (MAN, 4:558)

Thus Kant proceeds to the conclusion that since this motion does not rest on the influence of external forces, as was in the case of circular motion, this motion is necessary. This is the case because the concept of the relation of the moved in space to anything else movable determines the object validly for all appearances and in unavoidable way. However, also in this case, Kant provides a mixed proof, but the argument is stated in the General Remark to the Phenomenology. In particular, Kant there adds that Proposition 3 is clearly a distributive judgment, in which the two bodies equally share the mutual attribution of motion. The General Remark clarifies that Proposition 3 determines the modality of motion with respect to the Mechanics and adds that all three Propositions of the Phenomenology determine the motion of matter with regard to its possibility, actuality, and necessity; that is, they determine it with respect to all three categories of modality.

3. The General Remark to Phenomenology and the Role of Reason

In the General Remark to Phenomenology Kant asserts that this Chapter provides the general use of three concepts in natural science:

1) Motion in relative (movable) space
2) Motion in absolute (immovable) space
3) Relative motion in general as excluding absolute motion

The basis (*Grund*) of these three concepts is the idea of absolute space.[7] The relationship between these concepts of motion and the idea of absolute space should be understood as a ground-consequence (*Grund-Folge*) relation. That is, absolute space as an idea of reason is necessary to ground the three kinds of motion:

> "Absolute space is therefore necessary, not as a concept of an actual object, but rather as an idea, which is to serve as a rule for considering all motion therein as merely as relative;

[7] For a recent study on absolute space and rotational motion, see Stan (2016).

> and all motion and rest must be reduced (*reducirt*) to absolute space, if the appearance thereof is to be transformed into a determinate concept of experience (which unite all appearances)" (MAN, 4:560)

We are facing here a new positive function of the idea of reason, that of reduction (*Reduktion*).[8] How do we have to interpret this function? Should it be distinguished from the mere unifying capacity of the ideas of reason, as such? How does reduction contribute to the objective determination in metaphysics and the possibility of transforming appearance into experience? I offer an answer to these questions in the remaining part of this section.

The general Proof relies on a hidden premise, namely that external relative space can be generated *ad infinitum*. From this follows the general rule that all motion or rest can be relative only and not absolute, that is, matter can be thought as moved or as at rest only in relation to matter and never with respect to mere empty space. Therefore, absolute motion is impossible without any relation of one matter to another. In other words, this implies that no concept of motion or rest valid for all appearance is possible in relative space:

> "Rather, one must think a space in which the latter can itself be thought as moved, but which depends for its determination on no further empirical space, and thus is not conditioned in turn – that is, an absolute space to which all relative motions can be referred, in which everything empirical is movable, precisely so that in it all motion of material things may count as merely relative with respect to one another, as alternatively mutual, but none as absolute motion or rest (where, while one is said to be moved, the other, in relation to which it is moved, is nonetheless represented as absolutely at rest)." (MAN, 4:559–60)

Thus, the unconditioned idea of absolute space becomes the reference frame through which not only all relative motions, but also reciprocal empirical interactions are determined. We have therefore to conclude

[8] Studies on reduction and unity of science in Kant's system and beyond, include Stan (2014b), Buchdahl (1986), and Morrison (2008).

that the function of reduction attributed to the idea of absolute space is intimately connected to the operations of unifying all relative motions and of ensuring the objective determination of phenomena.

At this stage, it is important to verify whether the idea of absolute space introduced in this chapter is different from Newton's. Kant made a distinction between absolute and relative space which clearly refers to Newton's Scholium on space, time, and motion at the beginning of the *Principia*:

> "Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space." (Newton 1999, 408–9)

In MAN, Kant claims that "absolute space is *in itself* nothing and no object at all," but refers to an indefinite process of considering ever more extended relative spaces (4:481–82). Moreover, Kant states in the Phenomenology that absolute space is not an existing external object but rather is to serve as the rule for considering all motion and rest therein merely as relative (4:560). This conception resembles Euler's (1752, 186) and once again points to the necessity of distinguishing when Kant is referring to Newton's work and when he is rather trying to harmonize his system more broadly with the physics and the astronomy of his time.

> "Nevertheless, the first difference to notice with respect to Newton consists in the application of the concept of central force to the whole hierarchical structure of the universe; second the transcendental status of absolute space; third the fact that according to Kant gravitation is only one case of the concept of central force and the universal law that Kant has in mind is the one from which Newton's third law is derived, i.e. 'the law of antagonism in all community of matter through motion.'" (MAN, 4:563)

This means that we must be cautious before accepting a perfect overlap between the principles of MAN and Newton's laws. Kant was far more ambitious: he wanted to explore the foundations of the generalizations of Newton's physics, including Euler's, and was aware of the limits of Newton's physics to explain some astronomical observations, such as those made by William Herschel or Johann Hieronymus Schröter.[9]

We should also note the use that Kant makes of measurement in order to support his view. In the General Remark to Phenomenology (MAN, 4:561), for instance, in order to show that true circular motion is not semblance and that circular motion is a continuous dynamical change in the relation of matter within space, Kant claims that we must be able to measure acceleration. In other words, we must measure how the direction of bodies' motion deviates within the same reference system that is comoving. In order to obtain the objective determination of a moving body,[10] we have to include its internal state in this determination,[11] not only the external forces or states acting upon it. This means that to prove that the Earth is revolving upon its axis or completing its revolution around the Sun we do not have to observe the starry heavens, but rather the measurement apparatus must be posited within the system of which we want to determine the motion. For example, to prove the axial rotation of the Earth Kant references the example of throwing a stone within a deep hole descending to the center of the planet and observing the continuous deviation of its perpendicular trajectory Likewise to prove the Earth's revolution about the Sun, one has to consider the Sun-Earth system as two body problem. Friedman understands these passages to imply that the law of universal gravitation falls under the category of necessity. According to this reading, Kant's reconstruction of Newton's "deduction" of the law of universal gravitation from the initial Keplerian "analogies" provides a perfect illustration of the three-step procedure, described in the Postulates of Empirical Thought, by which a mere "empirical rule" is transformed into a "necessary and universally valid" objective law. However, I contend that Friedman's interpretation overlooks the role of measurement (and therefore the link to KU) as well as Kant's attempt to ground Euler's physics from a metaphysical standpoint (see section 4.1).

For the time being, let me return to my analysis of the General Remark to Phenomenology, which is fundamental for two reasons. First, it illuminates how Kant connects the application of formal conditions

[9] For more details on this topic see De Bianchi (2013; 2019).
[10] Recall what Kant writes in the MAN: "Hence, the movable, as such a thing, becomes an object of experience, when a certain object (here a material thing) is though as determined with respect to the predicate of motion" (4:554).
[11] In other words, the equations must contain the internal state of motion of the body à la Euler.

to the concept of "the reciprocal interaction in all community of matter through motion," rather than to the object of the possible experience in general. Second, the General Remark implies the necessity of determining this reciprocal interaction by means of the application of mathematics to physics, i.e. measurement. This enables us to establish a connection between this part of Kant's system and the third *Kritik* where Kant makes explicit his doctrine of measurement (see Section 4.2).[12]

Second, the General Remark to Phenomenology is fundamental for Kant's cosmology. It is at this point of the text that Kant denies the possibility of the universe moving or expanding, for this would presuppose a space external to it, which is impossible. However, Kant was aware that, according to his 1755 cosmology, it could have been perfectly possible for the universe to expand in all directions as a sphere. Nonetheless, at this stage Kant seems still prudently avoiding any reference to his cosmological system. But we know that Kant republished his cosmology at least partially in 1791 and that he discusses it extensively again in the *Opus postumum*. Therein he endorses the idea of an all penetrating and oscillating cosmic matter, the aether, endowed with repulsive force and responsible for the expansion of the universe, being additionally itself the whole hypostatized space and therefore the internal ground responsible for the expansion and oscillatory motion of the whole universe (see De Bianchi 2013). In that context, absolute space coincides with matter and creates through its oscillatory motion the expansion of the universe and its self-generation. However, we have no explicit trace of such a conception in Kant's 1786 work.

4. Phenomenology: Measurement and Foundations of Mathematical Physics

In this section I underscore both Kant's debt to Euler and advance a new reading of the Phenomenology as being meant to ground the equations of Euler's mechanics of solid bodies. This shows that to transform appearance into experience by using absolute space as an idea of reason means to connect and measure the plurality of comoving bodies, thereby embodying in practice the law of antagonism in all community of

[12] See Sutherland (2006) for Kant's view of arithmetic and proportions in KrV.

matter through motion. In the second subsection, I show the consequences of such a view and connect the Phenomenology with Kant's view of measurement as developed in the third *Kritik*.

4.1. Phenomenology and the Foundations of Euler's Equations

The Phenomenology is the laboratory in which Kant not only tests the third law of mechanics in its capacity to ground the concept of actual motion but also deploys the conceptual apparatus for the foundations of Euler's equations of solid bodies. In a sense, the Phenomenology also results in a conceptual reconstruction of Newton's a posteriori derivation of the law of universal gravitation from the observable phenomena described by Kepler's "rules" or analogies. These phenomena are considered as appearances (*Erscheinungen*), which are to be transformed into experience (*Erfahrung*). This is attained by using the law of inertia to mathematically derive inverse-square accelerations of satellites directed towards every primary body in the solar system. Further, by identifying the accelerations in question as effects of a central force, it follows that these accelerations must hold immediately between each part of matter and every other part of matter and are also directly proportional to the mass. However, in order to transform appearance into experience, that is, in order to obtain experience of matter through the empirical concept of motion, we need to measure acceleration. This provides the ground for determining other two elements which give us a necessary universal determination of bodily motion, i.e. the translation of the center of mass and the rotation about the center of mass. Both requirements are present in Euler's equations of motion of rigid bodies. Furthermore, in Kant's view, to metaphysically construct matter as object of experience also means to exhibit the relationship between the idea of absolute space and the three types of motion, which are not possible, actual, or necessary objects with respect to the understanding (as it would be the case if only the postulates of empirical thought in general were at stake), but with respect to reason. Now, in reviewing Friedman's (2013), Stan correctly noticed that when talking about Newton we have carefully to distinguish it from Newtonianism. In turn, I want to stress a similar point that Massimi and De Bianchi (2013) made about Descartes and Cartesianism in the 18th century, namely one has to distinguish Newton or Descartes from Newtonianism or Cartesianism given the quasi-syncretistic nature of these positions in the first half of the 18th century. In other words, when approaching the MAN, we have to take into account those physicists and mathematicians that confronted the most fundamental and still-open questions raised by Newton; by his predecessors, such as Descartes and Galilei; and by his contemporaries, like Leibniz and Huygens. More emphasis should additionally be placed on the representatives of the Berlin Academy of Sciences, such as Euler and D'Alembert, as they inherited the burden of extending and formalizing Newton's physics.

Newton's *Principia* indeed required much more than a reformulation in analytic terms, if it was to provide the basis for the comprehensive mechanics that was developed in the eighteenth century. According to Stephen Gaukroger (1982), there were also a number of areas crucial to the unification of mechanics that Newton did not deal with at all in the *Principia*: particularly the dynamics of rigid, flexible, and elastic bodies and the dynamics of several bodies with mutual interactions. Euler's contribution to the development of these topics, and hence to the unification of mechanics, was immense. Building upon Buchdahl's (1969b) interpretation of MAN, Harman (1983) focused on the link between Euler's physics and Kant's notion of force, but he did not attempt to demonstrate that in the Phenomenology Kant meant to refer to the advancements of Newtonian physics embodied by Euler's equations. This cost him the sharp criticism of Okruhlik (1983) who rightly suggested that one must be careful in establishing an overlap between Kant's and Euler's notion of force. However, in what follows, I shall show how Kant's Propositions of the Phenomenology are meant to explore the metaphysical underpinnings of Euler's equations of the dynamics of rigid bodies, even if the meaning that they attributed to "force" is not the same. Leonhard Euler gave a formalization of the extension of Newton's laws to rigid bodies, considered from the standpoint of continuous mechanics.[13] Today this extension is designated through two equations or axioms that are complemented by a third one the Newton-Euler law. Historically, Euler achieved the result of extending Newton's laws of motion for point particles to rigid bodies in a series of papers,[14] by further developing the law of inertia and the definition of impenetrability and abandoning the idea of conceptually defining force.

[13] See Suisky (2008, 29, 41, 129ff.).
[14] The works are Euler (1736, 1752, 1765a, 1765b, 1765c, 1776).

In 1765, Euler published *Theoria motus corporum solidorum seu rigidorum*. This work was improved by Euler's son Johann Albrecht in a new edition in 1790. In the introduction to the original work Euler confirmed the principles stated in his *Mechanics* (1736). He defines the main characteristic of a rigid body by the invariability of distances between any two points belonging to a body. In addition, Euler defined a "mass centrum" or an "inertiae centrum" for every and each body. In this work, Euler points out that the "center of gravity" of a rigid body implies a more restricted concept than a "mass center" or an "inertia center." The last two concepts are better defined by the inertia itself when the system of forces acting on the rigid body is neglected. In addition, he adopted a reference frame attached to the rigid body and identified the principal axes of inertia. Euler's study further included the rigid body dynamics by means of the decomposition of motion in two elements: the translation of the center of mass and the rotation about the center of mass.[15] In this context he considered what we now call 'Euler angles' and studied rotational problems motivated by the precession of the equinoxes. In particular, Euler distinguished between internal and external states of bodies; the internal state is described in terms of relations between different parts of the bodies, whereas the external state is obtained from the relation of the whole body (including its internal parts) to other bodies. In my view, Kant's emphasis regarding the fundamental law of mutual interaction among bodies, as well as the appeal in Proposition 2 to the conceptual dichotomy of internal and external state of a body are meant to provide the metaphysical foundation of Euler's equations. Furthermore, the definition of the states of rest or uniform motion was presented in *Reflexions sur l'espace et le tems* and is based on the external relations to other bodies and on the relations between the bodies and the space (Euler 1750). Euler claimed that the preservation of direction of a uniformly moving body cannot be explained by Leibniz's relational theory of space and time (1750, 332–33). Therefore, Euler accepted absolute time and space but rejected absolute motion.[16]

[15] For more details on Euler's external frames of reference, see Maronne and Panza (2019).

[16] See Suisky (2008, 129ff.). Kant rejected the idea of absolute time whereas accepted that of absolute space in the Phenomenology, a move that is not in contradiction with the doctrine of transcendental idealism as long as absolute space is a regulative idea of reason.

These sources shed considerable light on Kant's Phenomenology and his rejection of absolute motion stated in Proposition 1 and in the General Remark. In *Découverte d'un nouveau principe de la mécanique*, Euler states:

> "Among the infinity of motions to which a solid body is susceptible, the first thing to consider is that in which all parts persist constantly directed toward the same points of absolute space. In other words, if we conceive of a straight line passing through any of two points of a body, this line will always conserve the same direction or, which is the same, it will perpetually persist parallel to itself. This sort of motion is called purely progressive…"[17] (Euler 1752, 186)

This passage helps to clarify Kant's statement that all three types of motion analyzed in the three Propositions of the Phenomenology presuppose the concept of absolute space. Nevertheless, Kant distances himself from Euler insofar as he assumes repulsion and attraction as fundamental properties of matter in dynamical terms. That said, Kant completely follows upon the mathematician when stating:

> "There is thus no absolute motion, even when a body in empty space is thought as moved with respect to another; their motion here is not considered relative to the space surrounding them, but only to the space between them, which considered as absolute space, alone determines their external relations to one another and is in turn only relative" (MAN, 4:562).

Euler's explanation for the assumption of absolute space is grounded on mathematical considerations by means of the representation of projected trajectories, whereas Kant aims to harmonize Euler's equations with transcendental philosophy. Thus, he defines absolute space as an idea of reason with a necessary use, capable of encompassing all possible (relative) motions and therefore all appearances according to the universal and necessary law of antagonism in all community of matter through motion.

[17] The original text reads : "Entre l'infinité des mouvements, dont un corps solide est susceptible, le premier, qu'il faut considérer, est celui, ou toutes les parties demeurent constamment dirigées vers les mêmes points de l'espace absolu. C'est-à-dire, si nous concevons une ligne droite tirée par deux points quelconques du corps, cette ligne conservera toujours la même direction, ou ce qui revient au même, elle demeurera perpétuellement parallèle a elle-même. Un tel mouvement est nommé purement progressif…" English translation is mine.

There are now two questions I would like to mention before concluding. First, I strongly agree with Friedman (2013) that the notion of reference frame is fundamental in order to understand the Propositions of the *Phenomenology* and more generally his concepts of relative motion and rest. And precisely the notion of the coordinate system (x_1 x_2, x_3) as an inertial frame of reference is what Euler presupposed for his equations. Second, it is worth mentioning that the Newton-Euler equation fits nicely with the observations made by Kant in the General Remark to Phenomenology. In classical mechanics, the Newton–Euler equations describe the combined translational and rotational dynamics of a rigid body. This formalism relates the motion of the center of gravity of a rigid body with the sum of forces and torques (or moments) acting on the rigid body.[18] In other words, it embodies in a very sophisticated way the scope of objectively determining material bodies through actual relative motions, by transforming the latter from appearances into experience, namely actual relative motions extend Newton's laws to all appearances, such as comoving bodies, be they elastic, rigid or fluid. Therefore, Kant's Phenomenology aimed at encompassing the scope of Euler's Mechanics. In distinction to Euler, however, Kant dismisses the idea of absolute time and removes it from the picture, even if he clearly thinks that the objective representation of time as a formal intuition is fundamental to ground measurement and therefore to transform appearance into experience, by connecting the plurality of comoving bodies.[19]

4.2. Measurement and the Asymmetry of Space and Time: Phenomenology as the Laboratory of Pure Theoretical Reason

[18] Stan (2015a) underscores the necessity for Kant to ground Euler's equations and identifies in his early physical monadology the key to make transcendental philosophy compatible with Euler's mechanics. In my view, there is no need to include Kant's monadological theory of matter with the Phenomenology to show the latter's compatibility with Euler's mechanics. Indeed, what is at stake in both cases is the discussion of the torque law applied to comoving bodies. Of course, Stan's observation on the necessary presupposition of a certain theory of matter excluding its infinite divisibility holds if one focuses on the kinematics discussed in the Phoronomy, because in that chapter Kant is considering points rather than bodies.

[19] It is worth noticing the relevance of investigating in a deeper fashion the relationship between the Phenomenology and the Phoronomy with respect to Kant's view of measurement and the foundations of mathematics.

This subsection is devoted to show in which sense we could interpret the Phenomonology as a laboratory for the empirical use of theoretical reason. In order to substantiate my claim, it is necessary to investigate the connection between the conception of time as formal intuition and Kant's view of measurement.

In KrV we find the distinction between space as formal intuition and as form of intuition:

> "Space, represented as object (as is really required in geometry), contains more than the mere form of intuition, namely the comprehension of the manifold given in accordance with the form of sensibility in an intuitive representation, so that the form of intuition merely gives the manifold, but the formal intuition gives unity of the representation. In the Aesthetic I ascribed this unity merely to sensibility, only in order to note that it precedes all concepts, though to be sure, it presupposes a synthesis . . ." (KrV, B160n–61n)

This passage concerns the objective representation of space, but it is silent with respect to the results of the objective representation of time. Is there a science in which time is represented as object? Even if it is not possible to give a final answer to this question, because there could be more than one science apt to do that, it is remarkable that Phoronomy seems to be a candidate for this scope.[20] The mathematical construction of motion in the Phoronomy accomplishes the task of measuring the object at the same time in which it is drawn, however what is drawn technically speaking is velocity, therefore what is objectively represented is a certain time taken by a point to move from A to B. In the Phoronomy thus we find a sort of objective representation of time. Contrary to the geometrical representation of space, time must undergo to an indirect objective representation through motion (see KrV, B154–56).[21] In other words, only by means of measurement does time acquire an objective meaning. However, things become subtler and more intriguing when comparing the objective representation of time of the Phoronomy with that of the Phenomenology. The connection between these two sections is stated by Kant himself in the text (MAN, 4:557). The major

[20] The debate on this topic is rich and marked by conflicting interpretation on the role of schematism for the objective representation of time. See for instance Longuenesse (1998, 2001) and the debate with Friedman (2000a, 2000b).

[21] Dunlop (2009) offers an interpretation of Kant's passages in the B-Deduction (B154ff.) that I find convincing and able to reconcile Friedman's and Longuenesse's views.

difference between the Phoronomy and the Phenomenology is that the former only concerns the motion of space and is devoid of any reference to a moving force, i.e. of a dynamic account.

How does the Phenomenology represent time objectively? And why does Kant eventually need a second way in which time can be objectively represented through motion? Part of the answer lies in the previous passage: the Phenomenology shows us the dynamics underlying the reciprocal relationship among bodies and encodes within itself all three modes of judging motion as presented in the previous Chapters of the MAN. Within this enlarged picture, time is represented as object through comoving reference frames and not by means of an abstract motion of a space. However, if comoving reference frames are at stake, it means that they can be measured relatively to one another and that their relative magnitude can be established. In other words, time is objectively represented, but not, for instance, as a single cosmic time. Time is rather parametrized and represented as object only relatively to each reference frame.

Now, this position is not simply intriguing with respect to the science of Kant's time. What is extremely interesting is to compare this conclusion of the Phenomenology with Kant's view of measurement as portrayed in KU, in particular in the passages devoted to the sublime and the foundations of measurement (see KU, 5:248). In the third *Kritik* the foundations of measurement are to be found in the capacity of reason of providing ideals to intuitions, rather than in the principle of the pure understanding. Once again, this operation, albeit in a different fashion, brings ideas and their faculty back in view. In the case of MAN, the idea of absolute space is the actor that can make sense of magnitudes. And what is the objective representation of time, if not that of an infinite given magnitude, a continuous succession? And now comes the critical part of my reflection and perhaps a puzzling aspect not only of Kant's philosophy, but of philosophy of time in general. Whereas the idea of absolute space is able to fully justify the structure and aim of the Phenomenology, the concept of absolute time is not. And indeed it is not mentioned in the last chapter. What is even more striking is what we read in the KU regarding time and measurement:

> "The measurement of a space (as apprehension) is at the same time the description of it, thus an objective movement in the imagination and a progression; by contrast, the comprehension of multiplicity in the unity not of thought but of intuition, hence the

comprehension in one moment of that which is successively apprehended, is a regression, which in turn cancels the time-condition in the progression of the imagination and makes simultaneity intuitable. It is thus (since temporal succession is a condition of inner sense and of an intuition) a subjective movement of the imagination, by which it does violence to the inner sense, which must be all the more marked the greater the quantum is which the imagination comprehends in one intuition. Thus the effort to take up in a single intuition a measure for magnitudes, which requires an appreciable time for its apprehension, is a kind of apprehension which, subjectively considered, is contrapurposive, but which objectively, for the estimation of magnitude, is necessary, hence purposive; in this way, however, the very same violence that is inflicted on the subject by the imagination is judged as purposive for the whole vocation of the mind." (KU, 5:258–59)

The asymmetric treatment of space and time in Kant's philosophy is not something new, but here we see that in order to obtain a temporal formal intuition, reason (and neither the imagination nor the understanding) must exert violence on the manifold of representation in intuition. In the case of the Phenomenology, something quite similar to what is described in KU is outlined, although with more accuracy and by means of the idea of absolute space. This move conforms Kant's theory of matter to Euler's equations but creates an asymmetry between space and time. This is due to the fact that an idea of absolute time would lead to a concept of absolute acceleration and the impossibility of comparing reference frames. In connection to the results of the Phenomenology regarding the three types of motion this would also lead to consequences contrary both to the universal law of antagonism in all community of matter through motion and to Kant's own cosmology. Indeed, to accord absolute time a role in the last chapter of the MAN would have meant to admit that there are times to be calculated in a different manner with respect or relative to various star systems. A fascinating conclusion, but it was not Kant's.[22]

[22] Kant conserved the structure of his 1755 cosmology when publishing an excerpt again in 1791. In the NTH he talks about spaces in plural, but not about times in the universe. The problem of absolute time is still present and object of enquiry in the manuscript of the *Opus postumum*, when Kant has to resort the idea of aether as the all-penetrating and oscillating cosmic matter to account for the measurable acceleration of the universe expansion, but to my knowledge

5. Conclusion

In the previous sections, I highlighted the importance of considering the Phenomenology in connection with the advancements of Newtonian physics and with Euler's equations of motion of rigid bodies in particular. I emphasized the relevance of the function of reduction attributed to the idea of absolute space and how it articulates in a more precise way the transcendental function of unity attributed to the ideas of reason in the first *Kritik*. Another crucial point highlighted in this contribution is the fundamental role played by the connotation of time as forms of intuition and formal intuition to determine acceleration and make it compatible with Euler's mathematical and conceptual standpoint. The most relevant point that I would like to convey to the reader is that the Phenomenology represents Kant's attempt to embody the empirical use of reason which is identified with "transforming appearance into experience" and enabling the application of mathematics to natural science through measurement. In other words, the last Chapter of MAN constitutes a unique laboratory to test the necessary empirical use of pure theoretical reason.

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he never refers to the idea of absolute time to make sense of both Herschel's observations and the applications of Newton's and Euler's equations in the late writings.