"country", "place", "age". Examples are given: for "country", this reader suggests "England" or "France", for "place" he suggests "York" or "London", for "age" "adolescent" or "old man". As has been argued by others, the references to English place names may suggest that the glossator came from England. More in general, however, the use of the diagram shows how concepts were embedded in larger structures, and how they functioned to give other concepts a place in a large unit. They show a systematized, organised technique of understanding. The use of diagrams as a structuring and also mnemotechnical device is neatly illustrated.
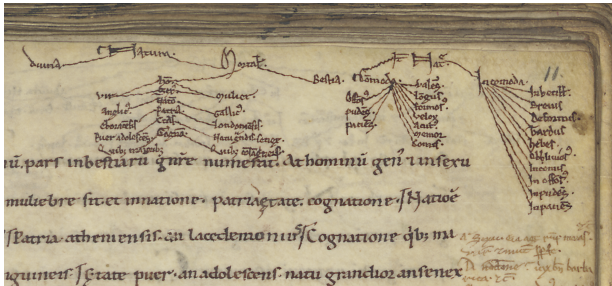

Figure 5: Leiden, UL, GRO 22, f. 11r (detail)

Manuscript Leiden, UL, BPL 88 was made in the third quarter of the ninth century. It contained the late-antique encyclopedia of the seven liberal arts (*trivium* and *quadrivium*) of the Latin author Martianus Capella, *On the marriage of Mercury and Philology* (see Teeuwen). In it, we find not only useful introductions to each of these disciplines, but also a neo-Platonic framework that gives meaning to the ancient knowledge tradition. The margins of medieval copies of this text are often filled with notes and remarks, which help the reader make sense of the difficult language and style of this author, and guide the reader to other, clearer authorities. These are not always contemporary with the making of the manuscript. In this ninth-century manuscript, for example, an author from the eleventh century added material on the flyleaves preceding the text of Martianus Capella (fols. 1v-2r): a set of diagrams which illustrate the method of creating affirmations, negations, oppositions, contradictions and consequences – the basic building blocks of argumentation. This material is, in fact, not taken from Martianus Capella's text, but rather a diagrammed version of a text from Aristotle, as commented upon by Boethius (see O'Daly). It must have been added here to provide means to understand the art of dialectic better and perhaps practice with it.
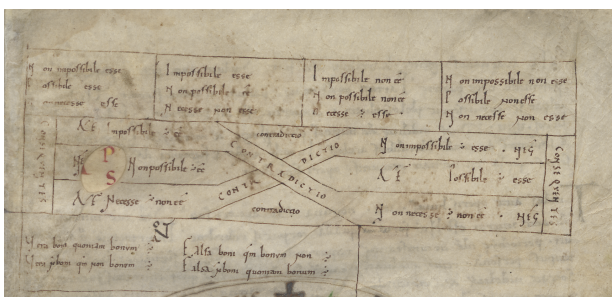

Figure 6: Leiden, UL, BPL 88, f. 2r (detail)

So, we can learn several things from these additions to the manuscript by a later reader: 1. the manuscript was still studied a few centuries later; 2. the reader was interested in logical

reasoning, and added material that was useful to learn and train this; 3. he used a method that did not only use text, but also the visual aid of the diagram; 4. he connected the authority which was already present in the manuscript (Martianus) with a different one (Aristotle, as translated by Boethius and 'diagramised' by an unknown medieval author). To conclude: medieval manuscripts are not only vessels of text, they are also fertile ground to find the traces of readers, their techniques of reading and learning, and their ways of transforming material into something that was their own. The voices in the margin, for a long time neglected as anonymous and therefore unimportant witnesses of readership and scholarship, cry out for a thorough exploration!

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Ecthesis, constructions and existential instantiation. A Kantian tale

According to Proclus (1992: A Commentary on the First Book of Euclid's Elements, Princeton University Press), the proof of a theorem and the solution of a problem in Euclid's *Elements* comprise six main parts. The second of them is called *ecthesis* and immediately follows the general enunciation of the proposition in question. It consists of the exhibition of a particular figure that sets out the geometrical entities with which the general enunciation deals. This step (together with further determinations and constructions) allows the geometer to carry out the proof proper or *apodeixis* on that particular figure and to conclude by extending the result to the general case, given that the particular determinations of the specific figure are utterly indifferent to the proof of the proposition.

In his book *Logic, Language-games and Information* (1973: OUP), Jaakko Hintikka suggests that the geometrical method described by Proclus, and *ecthesis* in particular, acts as a model for Immanuel Kant's conception of the synthetic *a priori*. In the *Critique of Pure Reason*, Kant introduces his celebrated analytic-synthetic distinction in these terms: "In all judgments in which the relation of a subject to the predicate is thought [...] this relation is possible in two different ways. Either the predicate B belongs to the subject A as something that is (covertly) contained in this concept A; or B lies entirely outside the concept A, though to be sure it stands in connection

with it. In the first case I call the judgment analytic, in the second synthetic" (Kant, 1998: Critique of Pure Reason, CUP, A6-7/B10).

As this quotation clarifies, the concept of the predicate of synthetic judgments is not contained in the concept of the subject. The connection between the two concepts involved, which is necessary for grounding the truth of the judgment, cannot be but indirect in that it must link the two concepts to one another by connecting them to a third element. The third element that is always necessary for the truth of synthetic judgments is an object, and the relation between concepts and objects must always be mediated by intuitions, which can only be sensible for all human beings. As a result, in Kant's picture, intuition dependence turns out to be the main feature of synthetic judgments as opposed to the analytic ones.

According to Hintikka's reading, synthetic methods might be seen, for Kant, as a kind of generalisation of geometrical methods. This conclusion is reached through the following interpretational steps: 1. judgments are synthetic if synthetic methods must be essentially used in their proof; 2. methods are synthetic if they make use of constructions (thus, Kant is "an heir to the constructional sense of analysis", cf. Hintikka 1973, 205); 3. constructions are a priori exhibition of intuitions; 4. intuitions are singular representations. Although many of them might been challenged, these claims lead directly to the conclusion that, for Kant, the truth of *a priori* synthetic judgments is shown through a generalisation of the reasoning pattern for proving geometrical propositions. The latter are demonstrated using *ecthesis*, namely the introduction into the argument of geometrical figures, which are singular and individual objects; similarly, *a priori* synthetic judgments in general are proved using constructions, namely the introduction into the argument of intuitions, which are singular representations.

But Hintikka pushes this similarity further and applies (his interpretation of) Kant's analytic-synthetic distinction to the logic of quantifiers. The result is the following definition: a step of a derivation in first-order logic is said to be synthetic if it introduces new individuals into the argument, that is, individuals that are not thought of in grasping the premises of that argument. The Kantian flavour of this notion should be clear enough: new individuals introduced into the argument are analogous to *a priori* intuitions understood as singular representations exhibited in constructions.

A new individual usually turns up in a derivation by applying the rule of existential instantiation. By this rule, a free individual symbol is introduced to replace the occurrences of a certain bound variable: this rule allows to infer from an existentially quantified sentence $\exists x\, F$ a sentence instantiating it, e.g. $F(x/a)$, where a is a free individual symbol and $F(x/a)$ the result of replacing x by a in F. The individual introduced in the proof through an application of this rule is new because of the requirement that the instantiating symbol a must be different from all the free individual symbols occurring earlier in the proof.

Thus, in Hintikka's work, *ecthesis* turns out to be the paradigmatic reasoning pattern not only for the use of constructions in Kant's synthetic judgments, but also for the application of the rule of existential instantiation in modern first-order logic. Hintikka devised this Kantian tale for the purpose of challenging the logical empiricists' thesis, which soon became traditional after Quine's *Two Dogmas of Empiricism*, that logic is analytical and tautological. Although his work did not en-

joy a considerable critical fortune, it has been recently resumed and provided inspiration for the construction of depth-bounded first-order logics (see D'Agostino, Larese, Modgil, 2021: Towards Depth-bounded Natural Deduction for Classical First-order Logic, Journal of Applied Logics, 8,2, 423-51), a new proof-theory for classical first-order logic that allows for a natural characterization of a notion of inferential depth.

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