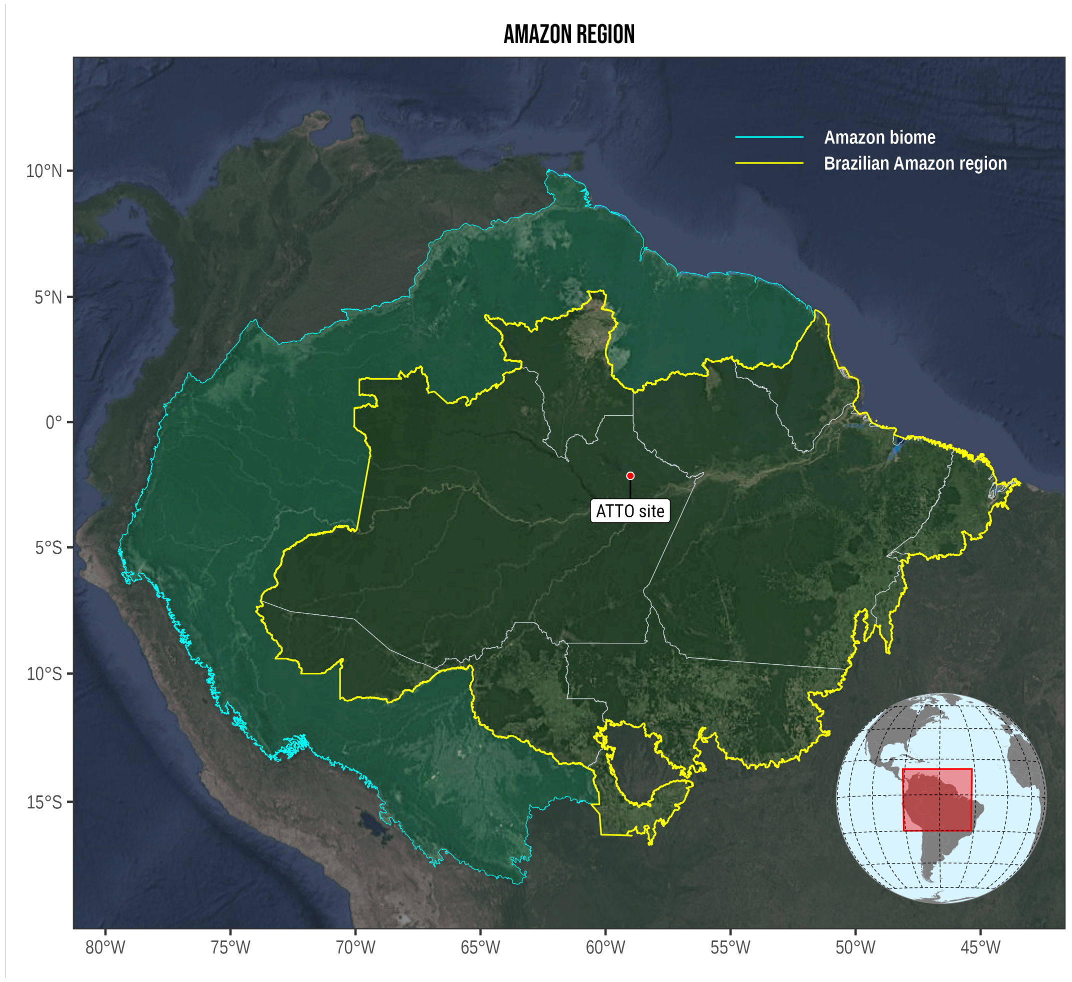
6° Congresso Nazionale AISAM
Brescia 10-12 Febbraio 2026

# A Co-Spectral Budget Approach: A New Perspective on Stratification Effects on Momentum Transfer over Tall Forested Canopies

Luca Mortarini, Gabriel Katul

ATTO
AMAZON TALL
TOWER OBSERVATORY

# The Amazon Tall Tower Observatory

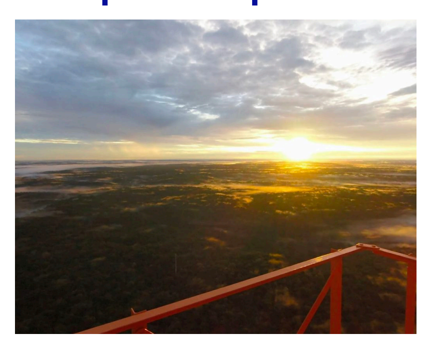



ATTO is located in the central Amazon rainforest of Brazil, in a pristine area, up till now mostly unaffected by deforestation or other human interference. In fact, it is situated within the Uatumã Sustainable Development Reserve. This ensures that it will remain undisturbed for some years to come.

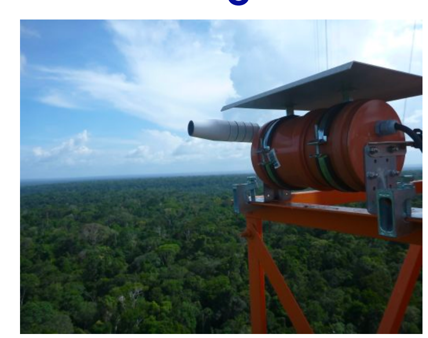
ATTO
AMAZON TALL
TOWER OBSERVATORY

www.attoproject.org

Atmospheric processes



Trace gases



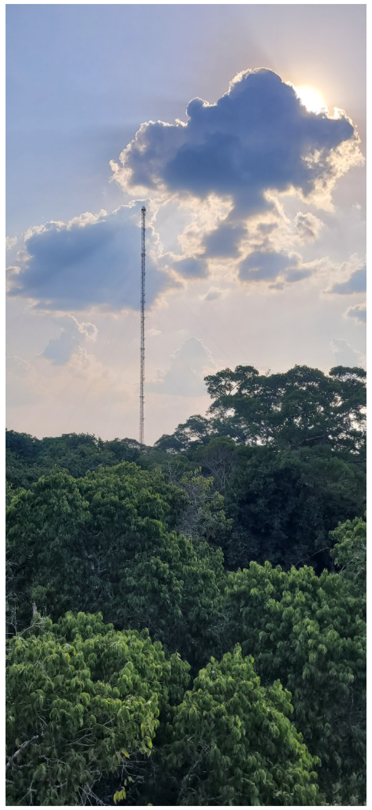
Aerosols and Clouds

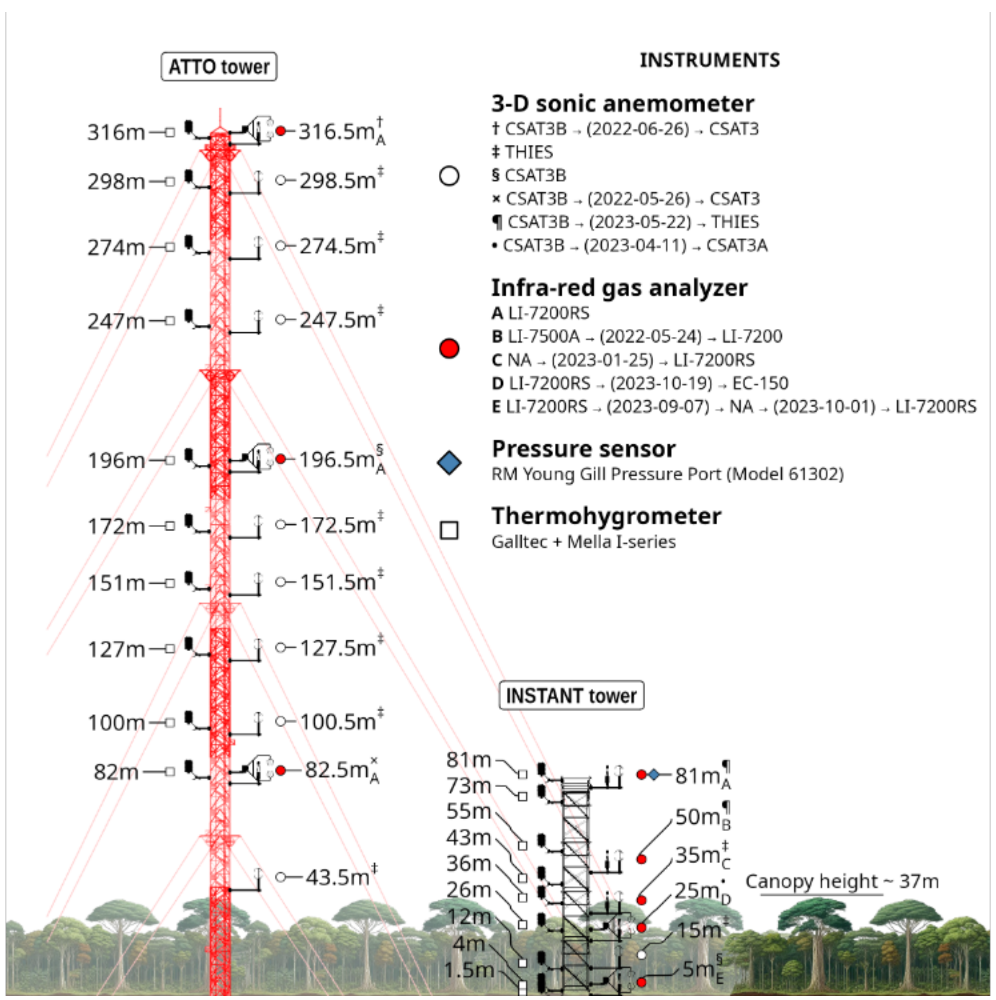
BVOCS

Forest Ecology

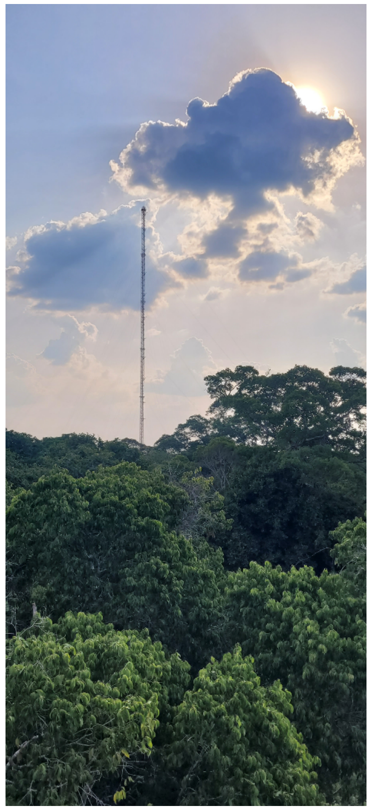
Cycles of Matter

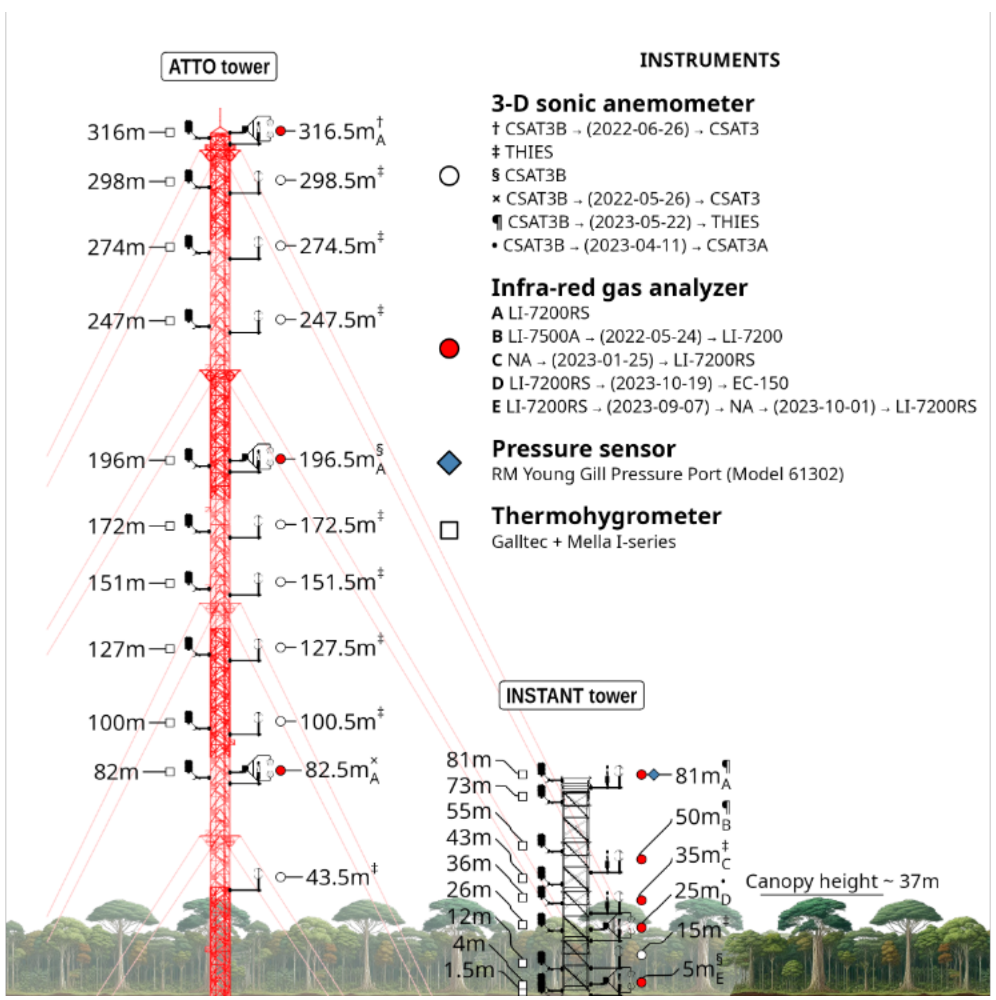
# The Amazon Tall Tower Observatory

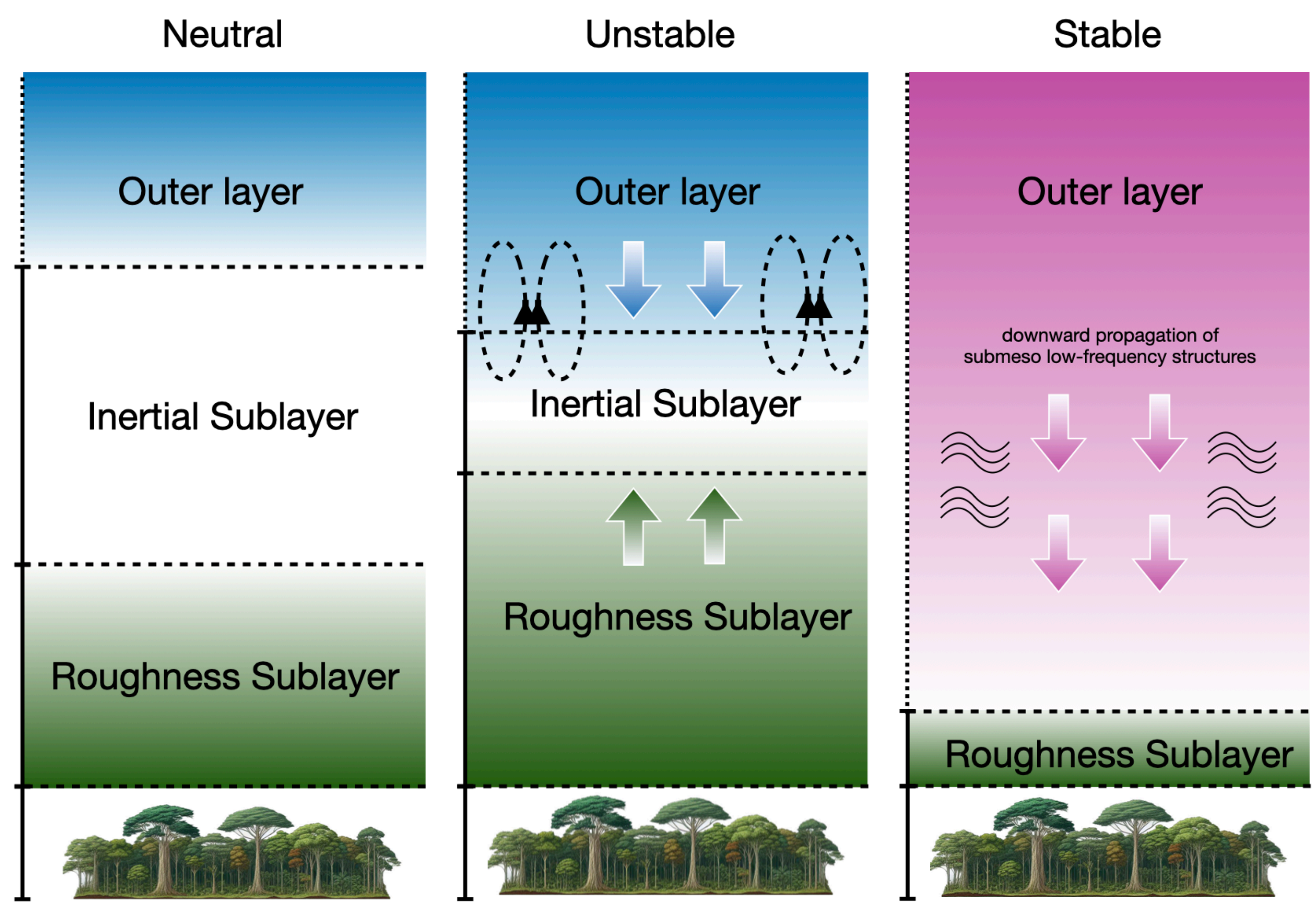




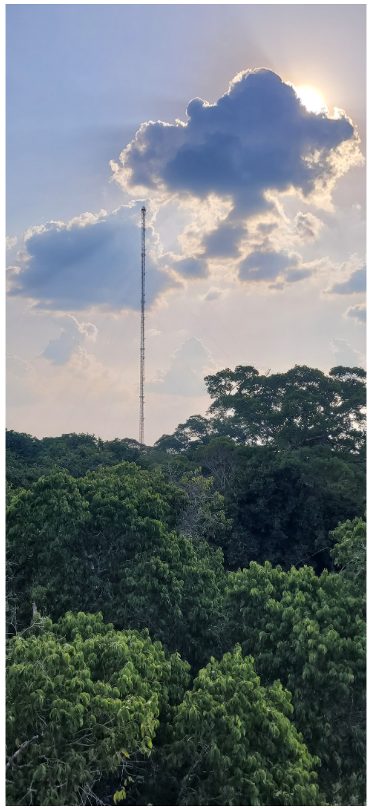
# The Amazon Tall Tower Observatory

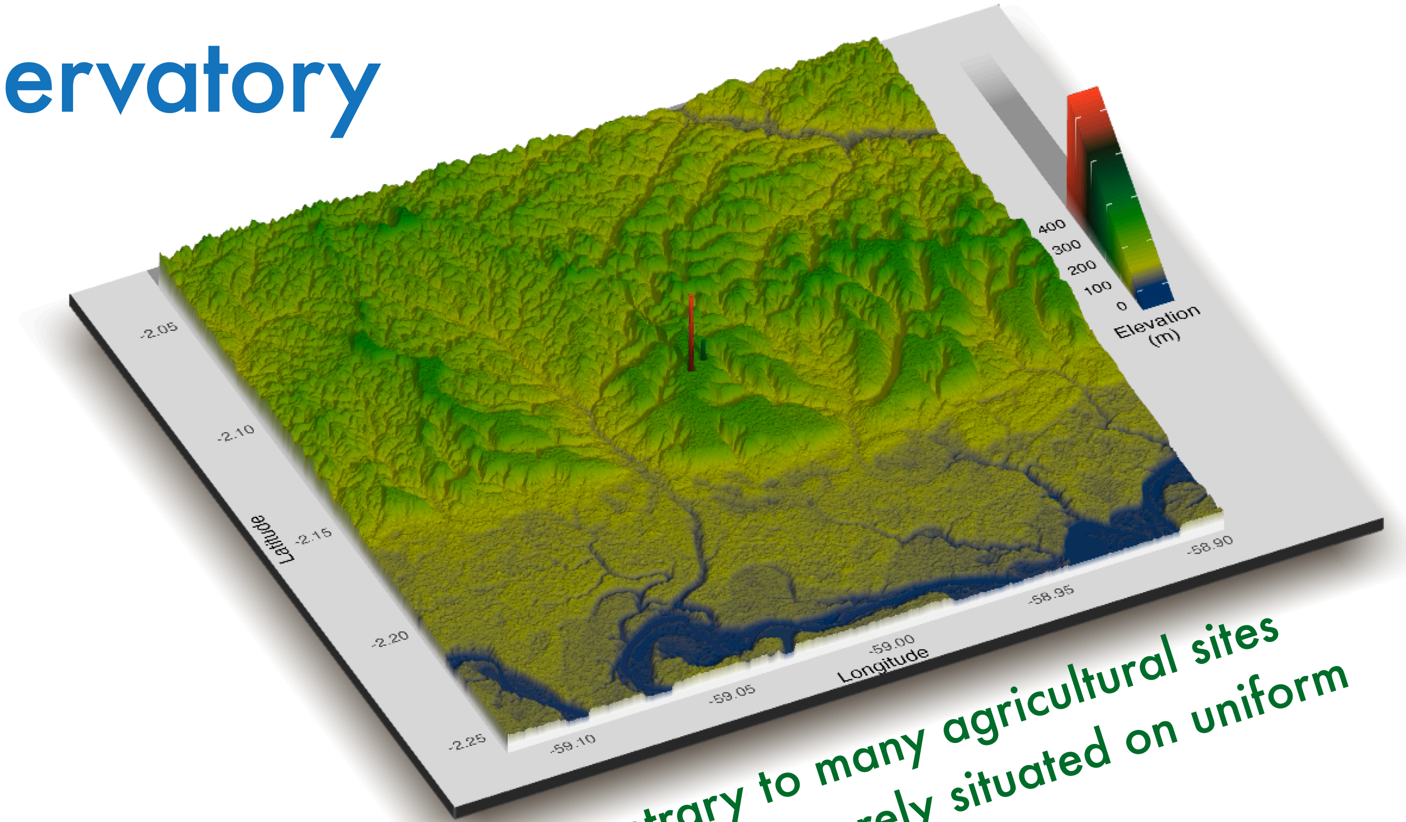




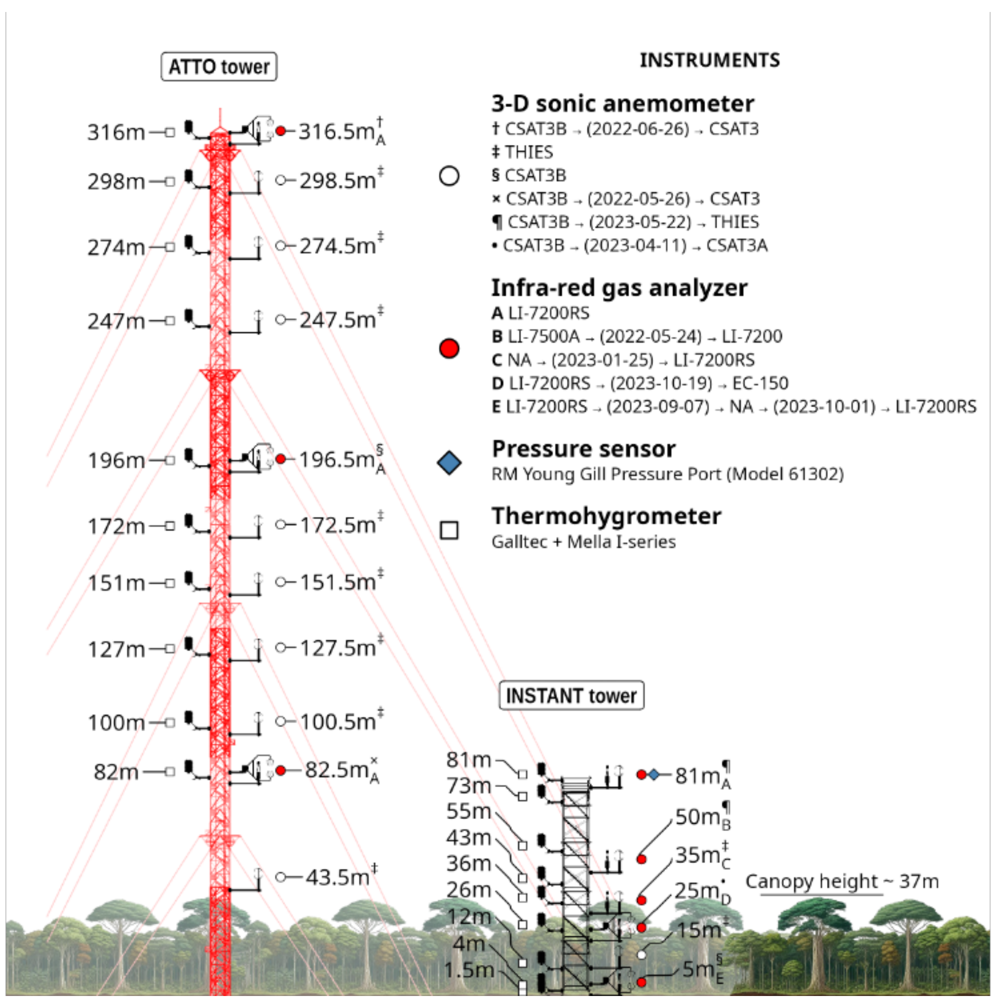



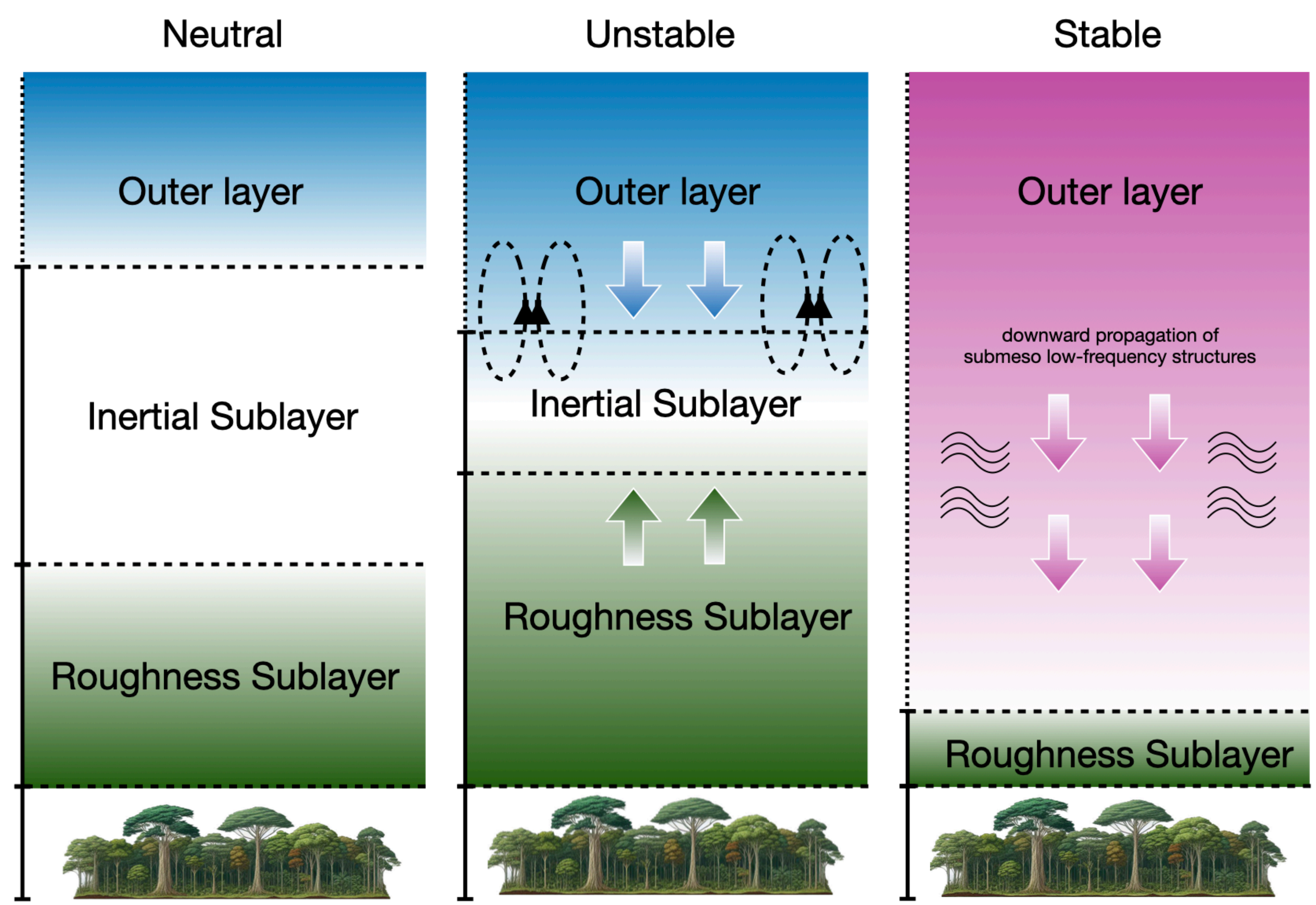
# The Amazon Tall Tower Observatory






Contrary to many agricultural sites forests are rarely situated on uniform and flat terrain.



# A new perspective for the Roughness Sublayer: the Cospectral Budget Model

## INTRODUCTION

The scale-wise impact of thermal stratification on turbulent momentum fluxes is explored using a Co-spectral Budget (CSB) model applied to the Amazon Tall Tower Observatory (ATTO, Manaus, Brazil).

## NOVELTY

The CSB model addresses stratification scale-by-scale using the momentum budget instead of the TKE budget. **The idea is to describe the Roughness Sublayer starting from the energetics of turbulence whereby all eddy sizes contribute to momentum transport.**

## KEY FINDING

The momentum flux co-spectrum $F_{wu}(k_x)$ is impacted by the energy spectrum of the vertical velocity $E_{ww}(k_x)$ and the much less studied co-spectrum of the longitudinal heat flux $F_{u\theta_v}(k_x)$.

# The turbulent stress budget

Buoyancy contribution on momentum transfer - the turbulent stress budget.

In stationary and planar homogeneous high Reynolds number flow in the absence of subsidence and assuming the moving equilibrium hypothesis (Kader and Yaglom, 1978, Yaglom, 1979), the turbulent stress budget reduces to

$$\frac{\partial \overline{w'u'}}{\partial t} = 0 = -\sigma_w^2 \Gamma(z) + R_{u,w} + \beta_o \overline{u'\theta_v'} - \left[ \frac{\partial \overline{w'w'u'}}{\partial z} + 2\epsilon_{uw} \right]$$

# The turbulent stress budget

Buoyancy contribution on momentum transfer - the turbulent stress budget.

In stationary and planar homogeneous high Reynolds number flow in the absence of subsidence and assuming the moving equilibrium hypothesis (Kader and Yaglom, 1978, Yaglom, 1979), the turbulent stress budget reduces to

$$\frac{\partial \overline{w'u'}}{\partial t} = 0 = -\sigma_w^2 \Gamma(z) + R_{u,w} + \beta_o \overline{u'\theta_v'} - \left[ \frac{\partial \overline{w'w'u'}}{\partial z} + 2\epsilon_{uw} \right]$$

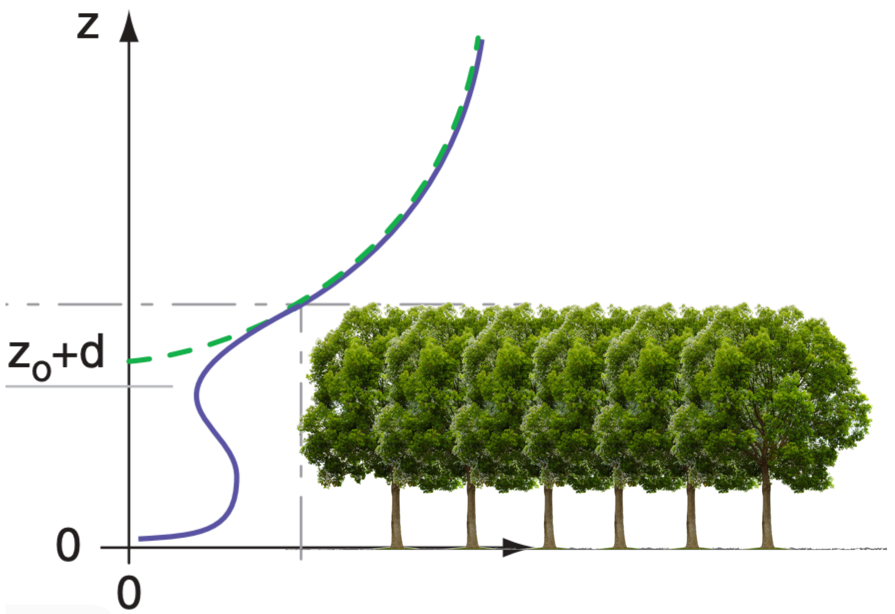
$$\frac{\partial \overline{e}}{\partial t} = 0 = -\overline{u'w'}\Gamma(z) + \beta_o \overline{w'\theta_v'} - \epsilon - \left[ \frac{\partial \overline{w'e}}{\partial z} + \frac{\partial \overline{w'p'}}{\partial z} \right]$$

The pressure-velocity interaction terms act differently in the budgets:

- In the TKE budget, they redistribute energy.
- In the momentum stress budget, they de-correlate $u'$ and $w'$.

$R_{u,w}$ **is way more efficient than** $\epsilon_{uw}$.

# The Momentum Budget



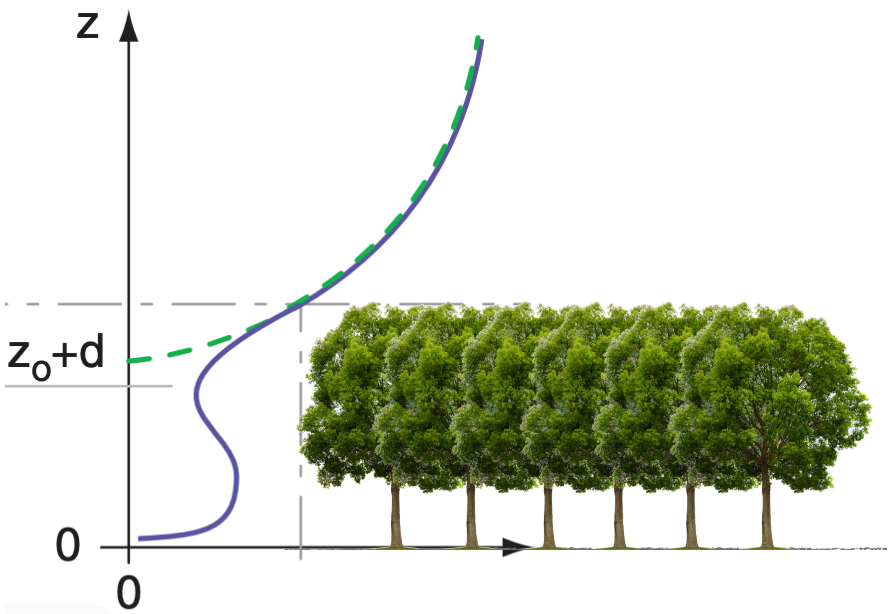

$$\frac{\kappa_v (z-d)}{u_*}\frac{dU}{dz} = \phi_{RSL}\left(z/z_*, d/h, \dots\right)$$

In stationary and planar homogeneous high Reynolds number flow and in the absence of subsidence, the turbulent stress budget reduces to

$$\frac{\partial \overline{u'w'}}{\partial t} = 0 = -\sigma_w^2 \frac{dU}{dz} - \frac{\partial \overline{w'w'u'}}{\partial z} + R_{u,w} - 2\epsilon_{uw},$$

Neutral Stratification

# The Momentum Budget




$$\frac{\kappa_v (z-d)}{u_*}\frac{dU}{dz} = \phi_{RSL}\left(z/z_*, d/h, \ldots\right)$$

In stationary and planar homogeneous high Reynolds number flow and in the absence of subsidence, the turbulent stress budget reduces to

$$\frac{\partial \overline{u'w'}}{\partial t} = 0 = -\,\sigma_w^2 \frac{dU}{dz} - \frac{\partial \overline{w'w'u'}}{\partial z} + R_{u,w} - 2\epsilon_{uw},$$

flux transport

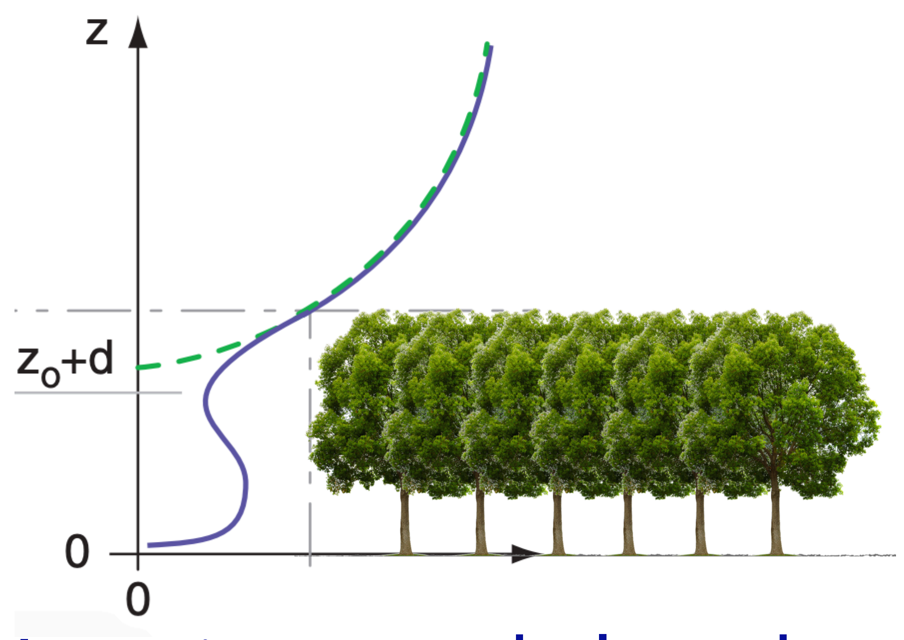
viscous de-correlation

mechanical production

pressure rate of strain de-correlation

Neutral Stratification

# The Momentum Budget




$$\frac{\kappa_v (z-d)}{u_*}\frac{dU}{dz} = \phi_{RSL}\left(z/z_*, d/h, \dots\right)$$

In stationary and planar homogeneous high Reynolds number flow and in the absence of subsidence, the turbulent stress budget reduces to

$$\frac{\partial \overline{u'w'}}{\partial t} = 0 = -\sigma_w^2 \frac{dU}{dz} - \frac{\partial \overline{w'w'u'}}{\partial z} + R_{u,w} - 2\epsilon_{uw},$$

flux transport

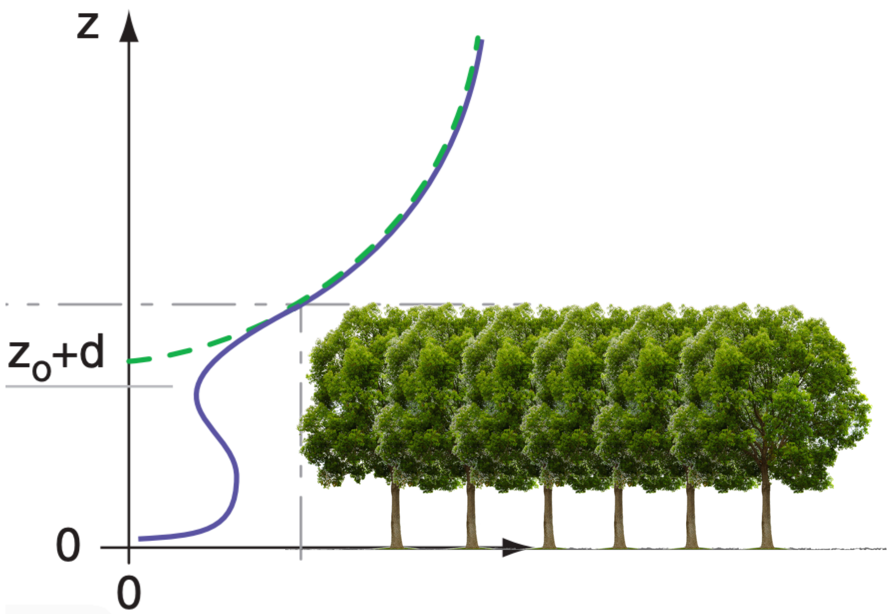
viscous de-correlation

mechanical production

pressure rate of strain de-correlation

Neutral Stratification

# The Momentum Budget




$$\frac{\kappa_v(z-d)}{u_*}\frac{dU}{dz} = \phi_{RSL}\left(z/z_*, d/h, \dots\right)$$

Neutral Stratification

In stationary and planar homogeneous high Reynolds number flow and in the absence of subsidence, the turbulent stress budget reduces to

$$\frac{\partial \overline{u'w'}}{\partial t} = 0 = -\sigma_w^2 \frac{dU}{dz} - \frac{\partial \overline{w'w'u'}}{\partial z} + R_{u,w} - 2\epsilon_{uw},$$

flux transport

viscous de-correlation

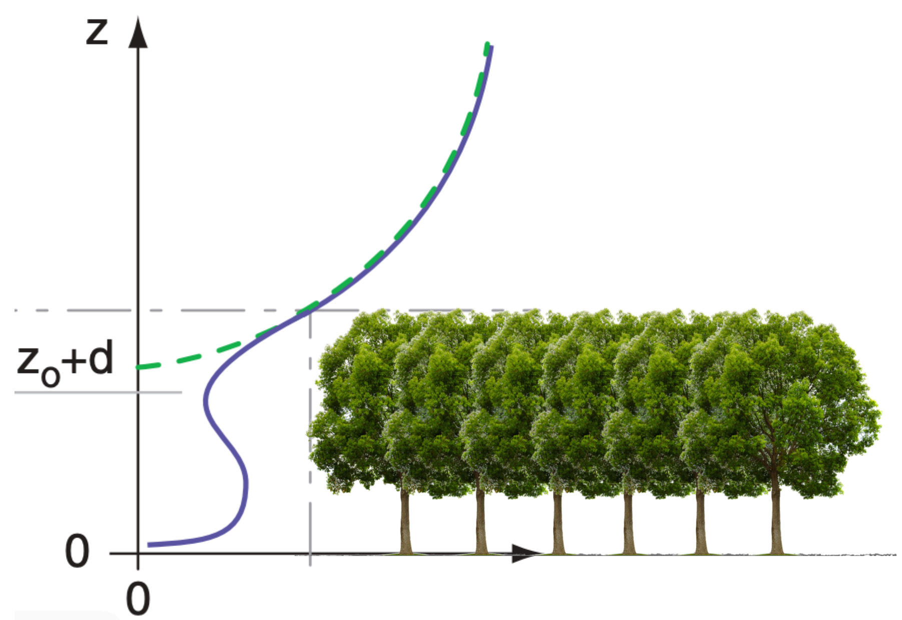
mechanical production

pressure rate of strain de-correlation

$R_{u,w}$ can be parameterised using the LRR-IP model (Rotta return-to-isotropy closure scheme corrected for the isotropization of the production):

$$-(1-C_I)\sigma_w^2\frac{dU}{dz} - C_R\frac{\overline{w'u'}}{\tau} = 0$$

# The Momentum Budget




$$\frac{\kappa_v (z-d)}{u_*}\frac{dU}{dz} = \phi_{RSL}\left(z/z_*, d/h, \dots\right)$$

In stationary and planar homogeneous high Reynolds number flow and in the absence of subsidence, the turbulent stress budget reduces to

$$\frac{\partial \overline{u'w'}}{\partial t} = 0 = -\,\sigma_w^2 \frac{dU}{dz} - \frac{\partial \overline{w'w'u'}}{\partial z} + R_{u,w} - 2\epsilon_{uw},$$

flux transport

viscous de-correlation

mechanical production

pressure rate of strain de-correlation

**Neutral Stratification**

$R_{u,w}$ can be parameterised using the LRR-IP model (Rotta return-to-isotropy closure scheme corrected for the isotropization of the production):

$$-(1-C_I)\sigma_w^2 \frac{dU}{dz} - C_R \frac{\overline{w'u'}}{\tau} = 0$$

$C_R = 1.8,\ C_I = 0.6$

$A = \frac{C_R}{1-C_I} = 4.5$

de-correlation timescale $\tau = \frac{2\sigma_w^2}{\epsilon}$

# The CSB - Neutral Stratification

$$\overline{w'u'} = -\frac{\tau}{A}\sigma_w^2\frac{dU}{dz} \longrightarrow \phi_{RSL} = \frac{L_{BL}}{u_*}\frac{dU}{dz} \longrightarrow \phi_{RSL} = -A\frac{\overline{u'w'}}{u_*^2}\frac{u_*}{\sigma_w}\frac{L_{BL}}{\tau\sigma_w}$$

The CSB model suggests that the RSL introduces deviations from $\phi_{RSL} = 1$ through 2 key mechanisms:

- an $\overline{u'w'}/u_*^2$ and $\sigma_w/u_*$ dependency on $z$ presumably due to presence of **complex topography** distorting $\overline{P}$ from its idealized ISL budget expectations

- a $\tau\sigma_w$ that no longer scales with $L_{BL} = \kappa_v(z-d)$.

$$\phi_{RSL} = -\frac{A}{2}\frac{\overline{u'w'}}{u_*^2}\left(\frac{u_*}{\sigma_w}\right)^4\frac{L_{BL}}{L_d}$$

$\phi_{RSL} \neq 1 \longrightarrow$ **deviaton from the law of the wall**

# The CSB - Neutral Stratification

$$\overline{w'u'} = -\frac{\tau}{A}\sigma_w^2\frac{dU}{dz} \longrightarrow \phi_{RSL} = \frac{L_{BL}}{u_*}\frac{dU}{dz} \longrightarrow \phi_{RSL} = -A\frac{\overline{u'w'}}{u_*^2}\frac{u_*}{\sigma_w}\frac{L_{BL}}{\tau\sigma_w}$$

The CSB model suggests that the RSL introduces deviations from $\phi_{RSL} = 1$ through 2 key mechanisms:

- an $\overline{u'w'}/u_*^2$ and $\sigma_w/u_*$ dependency on $z$ presumably due to presence of **complex topography** distorting $\overline{P}$ from its idealized ISL budget expectations

- a $\tau\sigma_w$ that no longer scales with $L_{BL} = \kappa_v(z-d)$. $\tau = \frac{2\sigma_w^2}{\epsilon}$

$$\phi_{RSL} = -\frac{A}{2}\frac{\overline{u'w'}}{u_*^2}\left(\frac{u_*}{\sigma_w}\right)^4\frac{L_{BL}}{L_d}$$

$\phi_{RSL} \neq 1$ $\longrightarrow$ **deviaton from the law of the wall**

# The CSB - Neutral Stratification

$$\overline{w'u'} = -\frac{\tau}{A}\sigma_w^2\frac{dU}{dz} \longrightarrow \phi_{RSL} = \frac{L_{BL}}{u_*}\frac{dU}{dz} \longrightarrow \phi_{RSL} = -A\frac{\overline{u'w'}}{u_*^2}\frac{u_*}{\sigma_w}\frac{L_{BL}}{\tau\sigma_w}$$

The CSB model suggests that the RSL introduces deviations from $\phi_{RSL} = 1$ through 2 key mechanisms:

- an $\overline{u'w'}/u_*^2$ and $\sigma_w/u_*$ dependency on $z$ presumably due to presence of **complex topography** distorting $\overline{P}$ from its idealized ISL budget expectations

- a $\tau\sigma_w$ that no longer scales with $L_{BL} = \kappa_v(z-d)$. $\tau = \frac{2\sigma_w^2}{\epsilon}$

$$\phi_{RSL} = -\frac{A}{2}\frac{\overline{u'w'}}{u_*^2}\left(\frac{u_*}{\sigma_w}\right)^4\frac{L_{BL}}{L_d}$$

$\phi_{RSL} \neq 1 \longrightarrow$ **deviaton from the law of the wall**

$L_d = \frac{u_*^3}{\epsilon(z)}$ is the dissipation length scale, an integral scale much larger than the Kolmogorov microscale:

$$Re_d = \frac{u_* L_d}{\nu} \longrightarrow \frac{L_d}{\eta} = Re_d^{3/4}$$

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\bar{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\bar{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

Local rate of change

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\bar{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\bar{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

Local rate of change

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\bar{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\bar{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

Molecular destruction

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

Local rate of change

Transfer

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\bar{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\bar{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

Molecular destruction

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

Local rate of change

Transfer

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\bar{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\bar{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

Molecular destruction

Generation by mean flow

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

Local rate of change | Molecular destruction | Transfer | Generation by mean flow | Buoyancy Contribution

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\overline{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\overline{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

Local rate of change | Molecular destruction | Transfer | Generation by mean flow | Buoyancy Contribution | Pressure decorrelation

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\overline{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\overline{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

# The CoSpectral Budget - diabatic stratification

The cospectrum of the coupled momentum and heat fluxes in diabatic conditions accommodates all eddy sizes.

The co-spectral budgets at any wavenumber $k_x$ read as:

Local rate of change — Transfer — Buoyancy Contribution

$$\frac{\partial F_{wu}(k_x)}{\partial t} + 2\nu k_x^2 F_{wu}(k_x) = T_{wu}(k_x) + P_{wu}(k_x) + \frac{g}{\overline{\theta}_v} F_{u\theta_v}(k_x) + R_{u,w}(k_x)$$

$$\frac{\partial \overline{w'u'}}{\partial t} + 2\epsilon_{uw} = -\frac{\partial \overline{w'w'u'}}{\partial z} - \sigma_w^2 \Gamma(z) + \frac{g}{\overline{\theta}_v} \overline{u'\theta_v'} + R_{u,w}$$

Molecular destruction — Generation by mean flow — Pressure decorrelation

Neglected — Derived from Data — Parameterized

# The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),

II) stationary planar homogeneous flow (only vertical gradients considered)

III) standard closure for $R_{u,w}(k)$ using the Rotta scheme

IV) the flux transfer terms across scales is ignored

Simplified CSB:

# The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),

II) stationary planar homogeneous flow (only vertical gradients considered)

III) standard closure for $R_{u,w}(k)$ using the Rotta scheme

IV) the flux transfer terms across scales is ignored

Simplified CSB:

Momentum Flux

$$-F_{wu}(k) = A^{-1}\tau(k)\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

# The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),
II) stationary planar homogeneous flow (only vertical gradients considered)
III) standard closure for $R_{u,w}(k)$ using the Rotta scheme
IV) the flux transfer terms across scales is ignored

Simplified CSB:

Momentum Flux

$$-F_{wu}(k) = A^{-1}\tau(k)\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

$\tau(k) = \alpha\epsilon^{-1/3}k^{-2/3}$ is the relaxation time at scale $k$ associated with turbulent stress de-correlation

# The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),

II) stationary planar homogeneous flow (only vertical gradients considered)

III) standard closure for $R_{u,w}(k)$ using the Rotta scheme

IV) the flux transfer terms across scales is ignored

<u>Simplified CSB:</u>

Momentum Flux

$$-F_{wu}(k) = A^{-1}\tau(k)\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

Neutral Stratification
Mortarini et al. (2023)

$\tau(k) = \alpha\epsilon^{-1/3}k^{-2/3}$ is the relaxation time at scale $k$ associated with turbulent stress de-correlation

# The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),

II) stationary planar homogeneous flow (only vertical gradients considered)

III) standard closure for $R_{u,w}(k)$ using the Rotta scheme

IV) the flux transfer terms across scales is ignored

Simplified CSB:

Momentum Flux

$$-F_{wu}(k) = A^{-1}\tau(k)\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

Neutral Stratification
Mortarini et al. (2023)

$\tau(k) = \alpha\epsilon^{-1/3}k^{-2/3}$ is the relaxation time at scale $k$ associated with turbulent stress de-correlation

**NEW TERM:**
Buoyancy Contribution

# The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),

II) stationary planar homogeneous flow (only vertical gradients considered)

III) standard closure for $R_{u,w}(k)$ using the Rotta scheme

IV) the flux transfer terms across scales is ignored

Simplified CSB:

Momentum Flux

$$-F_{wu}(k) = A^{-1}\tau(k)\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

Neutral Stratification
Mortarini et al. (2023)

$\tau(k) = \alpha\epsilon^{-1/3}k^{-2/3}$

is the relaxation time at scale $k$ associated with turbulent stress de-correlation

**NEW TERM:**
Buoyancy Contribution

$$\overline{w'u'} = \frac{\tau}{A}\left[-\sigma_w^2\frac{dU}{dz} + \frac{g}{\overline{\theta}_v}\overline{u'\theta_v'}\right]$$

Neutral Stratification

# The CSB - diabatic stratifications

Assuming:

I) high Reynolds number (viscous destruction ignored relative to $R_{u,w}(k_x)$),

II) stationary planar homogeneous flow (only vertical gradients considered)

III) standard closure for $R_{u,w}(k)$ using the Rotta scheme

IV) the flux transfer terms across scales is ignored

<u>Simplified CSB:</u>

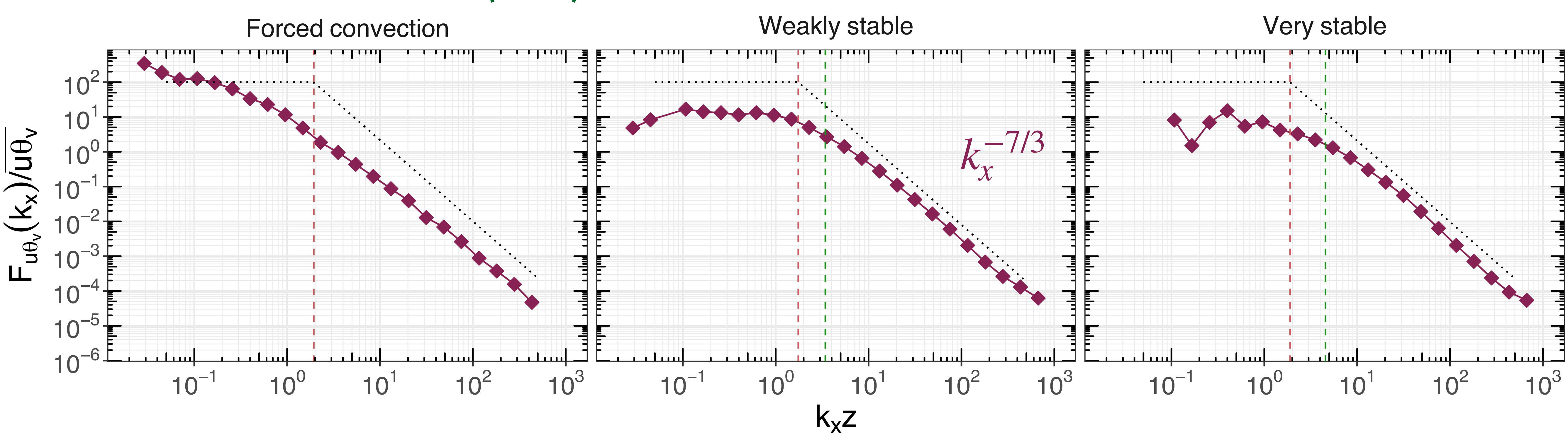
Momentum Flux

$$-F_{wu}(k) = A^{-1}\tau(k)\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

Neutral Stratification
Mortarini et al. (2023)

$\tau(k) = \alpha\epsilon^{-1/3}k^{-2/3}$ is the relaxation time at scale $k$ associated with turbulent stress de-correlation

**NEW TERM:**
Buoyancy Contribution



# The CSB - corollaries (I - II)

$$\overline{w'u'} = \frac{\tau}{A}\left[-\sigma_w^2\frac{dU}{dz} + \frac{g}{\overline{\theta}_v}\overline{u'\theta_v'}\right] \qquad -F_{wu}(k) = \alpha A^{-1}\epsilon^{-1/3}k^{-2/3}\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

I) Gradient-diffusion break down:

$$\overline{w'u'} = -\nu_T\frac{dU}{dz} + \left[\frac{\tau}{A}\beta_o\overline{u'\theta_v'}\right] \qquad \nu_T = A^{-1}(\tau\sigma_w^2) = A^{-1}\int\tau(k)E_{ww}(k)dk$$

This cannot be attributed to the flux transport terms.
Instead, it is entirely driven by thermal stratification.

# The CSB - corollaries (I - II)

$$\overline{w'u'} = \frac{\tau}{A}\left[-\sigma_w^2\frac{dU}{dz} + \frac{g}{\overline{\theta}_v}\overline{u'\theta_v'}\right] \qquad -F_{wu}(k) = \alpha A^{-1}\epsilon^{-1/3}k^{-2/3}\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\overline{\theta}_v}F_{u\theta}(k)\right)$$

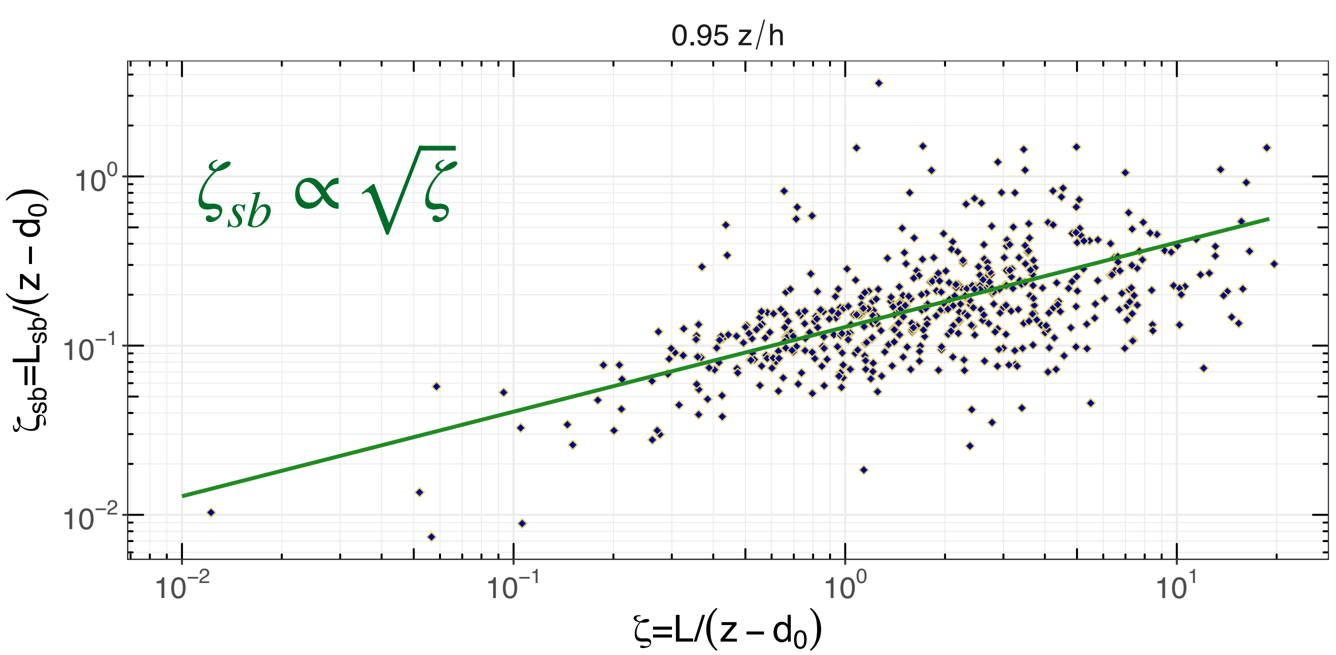
I) Gradient-diffusion break down:

$$\overline{w'u'} = -\nu_T\frac{dU}{dz} + \left[\frac{\tau}{A}\beta_o\overline{u'\theta_v'}\right] \qquad \nu_T = A^{-1}(\tau\sigma_w^2) = A^{-1}\int\tau(k)E_{ww}(k)dk$$

This cannot be attributed to the flux transport terms.
Instead, it is entirely driven by thermal stratification.

II) Emergence of a length scale that reflects the contribution of mechanical production and buoyancy production or destruction terms to the turbulent stress budget

$$L_{sb} = \frac{\sigma_w^2 u_*}{\kappa\beta_o\overline{u'\theta_v'}}\phi_m(\zeta)$$



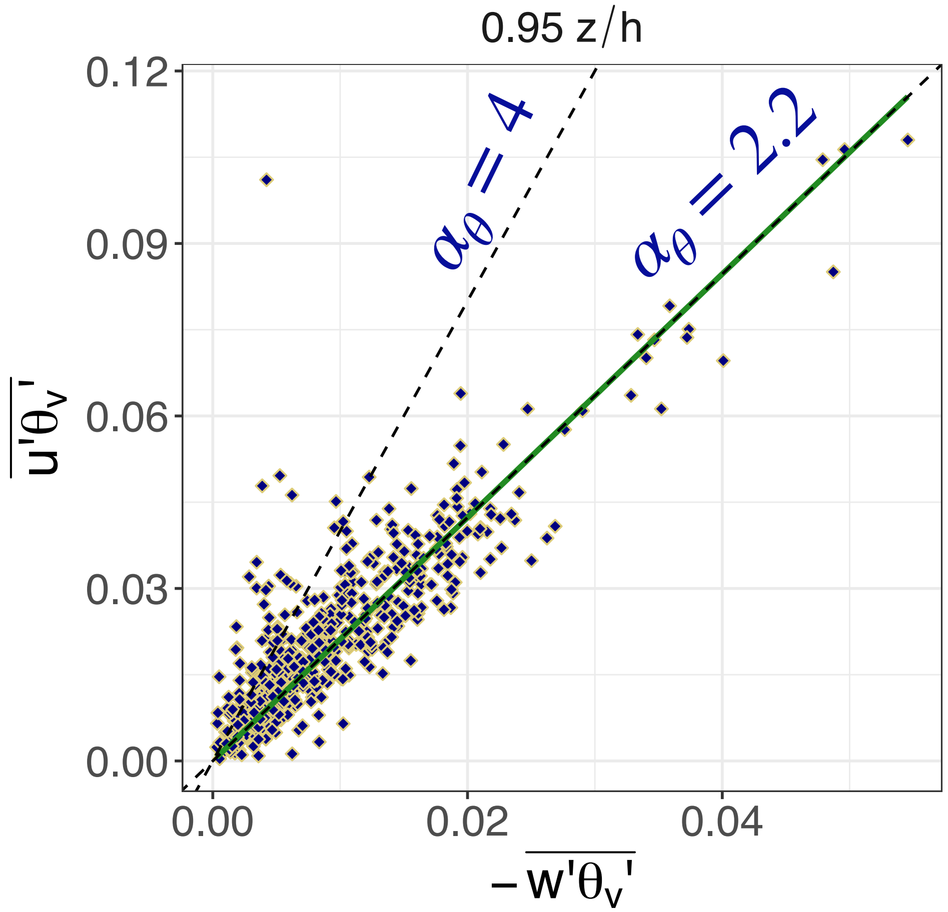
# The CSB - corollaries (III)

$$\overline{w'u'} = \frac{\tau}{A}\left[-\sigma_w^2\frac{dU}{dz} + \frac{g}{\bar{\theta}_v}\overline{u'\theta_v'}\right] \qquad -F_{wu}(k) = \alpha A^{-1}\epsilon^{-1/3}k^{-2/3}\left(\frac{dU}{dz}E_{ww}(k) - \frac{g}{\bar{\theta}_v}F_{u\theta}(k)\right)$$

III) Maximum sustainable heat flux in stable stratification:

Assuming a negative momentum flux and $\overline{u'\theta'} = -\alpha_\theta\overline{w'\theta'}$ results in

$$-\sigma_w^2\frac{dU}{dz} + \frac{g}{\bar{\theta}_v}\overline{u'\theta_v'} \leq 0 \longrightarrow \overline{u'\theta'} \leq \frac{\bar{\theta}_v}{g}\,\sigma_w^2\frac{dU}{dz} \longrightarrow \overline{w'\theta'} \geq -\frac{1}{\alpha_\theta}\frac{\bar{\theta}_v}{g}\,\sigma_w^2\frac{dU}{dz}$$




The concept of maximum sustainable heat flux was introduced by Van de Wiel et al. (2007, 2012a, b). Here, this flux is:

$$\overline{w'\theta'}_{max} = -\frac{1}{\alpha_\theta}\frac{\bar{\theta}_v}{g}\,\sigma_w^2\frac{dU}{dz}$$

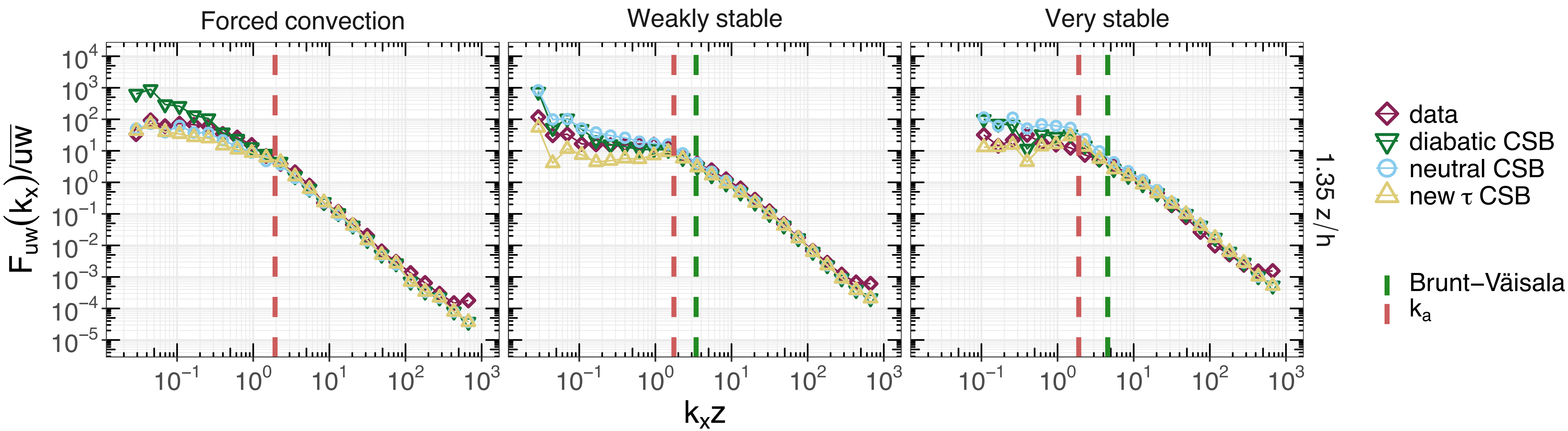
And represents the maximum heat flux the flow can provide for a given shear.

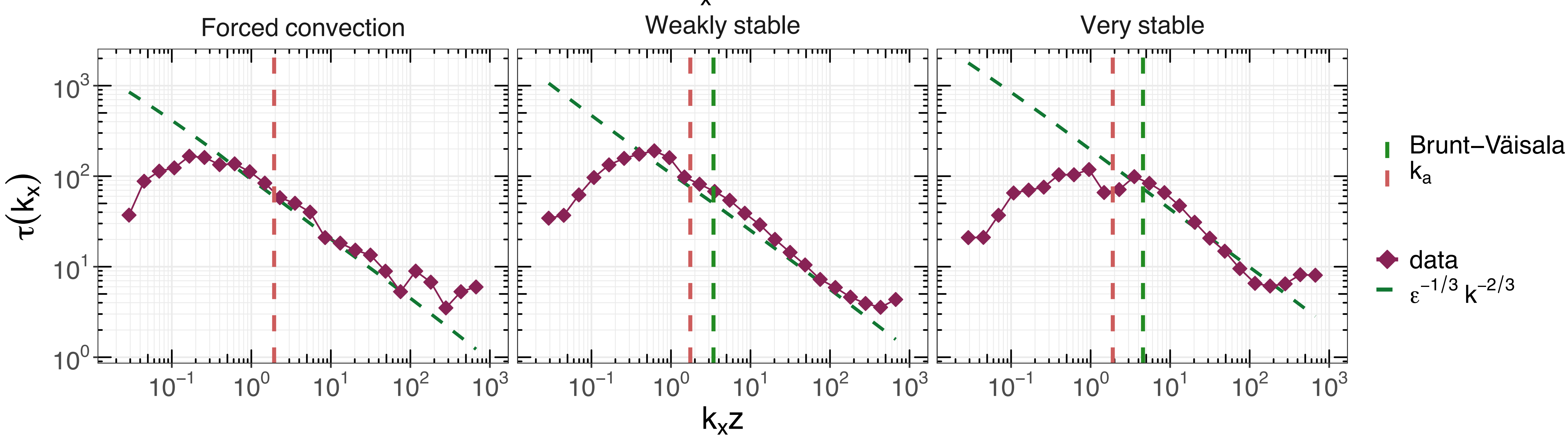
# The de-correlation time-scale

$$\tau(k_x) = -\frac{AF_{wu}(k_x)}{\Gamma(z)E_{ww}(k_x) - \frac{g}{\overline{\theta}_v}F_{u\theta_v}(k_x)}$$

$$\tau(k_x) = \epsilon^{-1/3}k_x^{-2/3} \quad \forall k_x$$

$$\begin{cases} \tau(k_x) = \epsilon^{-1/3}k_a^{-2/3} & k_x \leq k_a \\ \tau(k_x) = \epsilon^{-1/3}k_x^{-2/3} & k_x > k_a \end{cases}$$



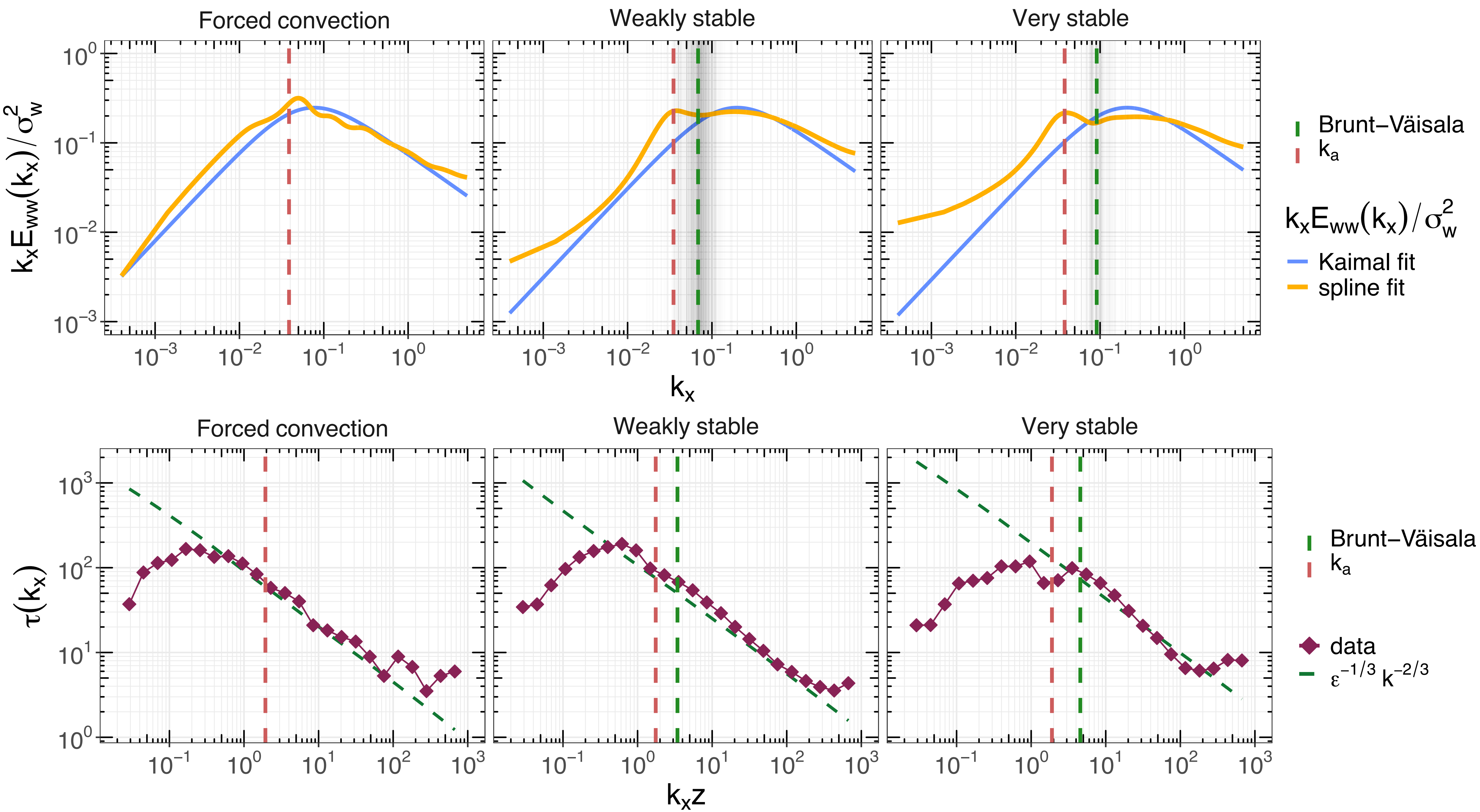



# The de-correlation time-scale

$$\tau(k_x) = -\frac{A F_{wu}(k_x)}{\Gamma(z) E_{ww}(k_x) - \frac{g}{\bar{\theta}_v} F_{u\theta_v}(k_x)}$$

$$\tau(k_x) = \epsilon^{-1/3} k_x^{-2/3} \quad \forall k_x$$

$$\begin{cases} \tau(k_x) = \epsilon^{-1/3} k_a^{-2/3} & k_x \leq k_a \\ \tau(k_x) = \epsilon^{-1/3} k_x^{-2/3} & k_x > k_a \end{cases}$$



# Conclusions

1. Newly proposed CSB model shows that thermal stratification on the turbulent momentum flux is **direct** and **indirect**:

**Direct:** through the longitudinal heat flux

**Indirect:** through the effects of thermal stratification on the vertical velocity energy spectrum, the mean velocity gradient, and the turbulent kinetic energy dissipation rate through a de-correlation time.

2. In presence of buoyancy $\overline{w'u'}$ may no longer be explained by the mean velocity gradient and gradient-diffusion breaks down.

3. The analysis shows that stability characterization is not only through the Obukhov length. The latter was developed from considerations of the turbulent kinetic energy budget, not the turbulent stress budget.

ATTO

AMAZON TALL TOWER OBSERVATORY

# Thank you!

luca.mortarini@unimi.it

This study is part of the Amazon Tall Tower Observatory (ATTO), funded by the German Federal Ministry of Education and Research (BMBF, contracts 01LB1001A and 01LK1602A), the Brazilian Ministry of Science, Technology and Innovation (MCTI/FINEP, contract 01.11.01248.00) and the Max Planck Society (MPG). ATTO is also supported by the Fundação de Amparo à Pesquisa do Estado do Amazonas (FAPEAM), Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Universidade do Estado do Amazonas (UEA), Instituto Nacional de Pesquisas Amazônia (INPA), Programa de Grande Escala da Biosfera-Atmosfera na Amazônia (LBA) and the SDS/CEUC/RDS-Uatumã.