Theoretical Computer Science 1036 (2025) 115153

Contents lists available at ScienceDirect

Theoretical Computer Science

journal homepage: www.elsevier.com/locate/tcs

Computational power of autonomous robots: Transparency vs. opaqueness ☆,☆☆

Caterina Feletti [a,*], Lucia Mambretti [b], Carlo Mereghetti [a,c], Beatrice Palano [a,c]

[a] Computer Science Department, Università degli Studi di Milano, Milan, Italy
[b] Milan, Italy
[c] Gruppo Nazionale Calcolo Scientifico-Istituto Nazionale di Alta Matematica (GNCS-INdAM), Italy

A R T I C L E   I N F O

Keywords:
Mobile robots
Look-compute-move
Computational complexity
Opaque robots
Obstructed visibility
Collision intolerance

A B S T R A C T

The research on distributed computing by robot swarms has formalized different models where robots act through a sequence of *Look-Compute-Move* cycles in the Euclidean plane. Models mostly under study differ for *(i)* the possibility of storing constant-size information, *(ii)* the possibility of communicating constant-size information, *(iii)* the synchronization mode, and *(iv)* the visibility of robots. By varying features *(i)* and *(ii)*, we obtain the noted four base models: $\mathcal{OBLOT}$ (silent and oblivious robots), $\mathcal{FSTA}$ (silent and finite-state robots), $\mathcal{FCOM}$ (oblivious and finite-communication robots), and $\mathcal{LUMI}$ (finite-state and finite-communication robots). Feature *(iii)* comprehends the three main synchronization modes: *fully synchronous*, *semi-synchronous*, and *asynchronous*. According to robot visibility *(iv)*, models can assume robots to be *transparent* (thus enjoying *complete visibility*) or *opaque* (thus experiencing *obstructed visibility* in case of collinearities). By combining features *(i-iv)*, we obtain 24 models. Extensive research has studied the *computational power* of the 12 transparent models, proving the hierarchical relations among them; to this regard, it is worth noticing that robots have been assumed to be collision-tolerant.
In this work, we assume our robots to be *collision-intolerant* and we lay down the computational hierarchy by considering all 24 models. Firstly, we study the relations between the transparent and the opaque framework, focusing on how obstructed visibility affects the computational power of a model. Then, we introduce five witness problems that prove most of the computational relations among the 24 models.

1. Introduction

In the far-ranging field of distributed computing, a significant area deals with *computing by mobile entities* [2,3], where tasks are required to be solved by swarms of computational, simple, and limited entities (also called *robots*) that can move in the environment. In this realm, multiple theoretical models have been introduced to formalize realistic scenarios (e.g. sensor or drone swarms, dynamic networks, software agents). One of the most studied is the *Look-Compute-Move* (LCM) model [2,3], where a robot, once activated, executes a *cycle* of three steps: it *looks* at the environment, it *computes* the next position executing a distributed algorithm whose sole input is the environment snapshot just taken, and it *moves* to the computed position.

☆ This article belongs to Section A: Algorithms, automata, complexity and games, Edited by Paul Spirakis.
☆☆ A preliminary version of this paper has been presented at the *3rd Symposium on Algorithmic Foundations of Dynamic Networks (SAND), 2024*, [1].

* Corresponding author.
E-mail addresses: caterina.feletti@unimi.it (C. Feletti), lucia.mambretti@studenti.unimi.it (L. Mambretti), carlo.mereghetti@unimi.it (C. Mereghetti), beatrice.palano@unimi.it (B. Palano).

https://doi.org/10.1016/j.tcs.2025.115153
Received 30 October 2024; Received in revised form 10 February 2025; Accepted 26 February 2025
Available online 27 February 2025

Under the umbrella of the LCM macro model, a vast combination of model features has been proposed to formalize different settings and to study how they affect the computational power of a model. In this respect, research considers robots with very limited and restricted features, aiming to find the minimal sets of capabilities that are required to perform a given task. Accordingly, robots are assumed to be *autonomous*, *indistinguishable*, *anonymous*, and *homogeneous*, namely: they act without any central control, they cannot distinguish themselves by external appearance or by internal ids, they possess the same features, they execute the same algorithm in a decentralized way. Moreover, most of the literature considers *punctiform* robots that cannot communicate with other robots (*silent*), without any persistent memory (*oblivious*), without any agreement on a global coordinate system, or chirality, or a unit of measure (*disoriented*). Traditionally, robots are assumed to be *transparent* and embedded with *unlimited visibility*, so that robots can completely sense all the other robots independently from collinearities and their mutual distance. Some contributions in the literature drop these impractical assumptions and consider *opaque* robots [4–7] (i.e. a robot cannot see beyond a collinear robot) or *myopic* robots [8–11] (i.e. robots can see up to a fixed distance). Besides robot capabilities, different *environment* models have been proposed to study diverse scenarios. The existing models can be mainly divided into two groups: the models where robots act on the Euclidean plane [4,5,12,13], and the models where robots act on discrete spaces (e.g. rings, lattices, or general graphs) [14–17]. Concerning the *activation* and *synchronization*, robots may adhere to different modes [18]. In general, robots may be synchronized or not: in the first case, time is globally divided into *rounds*. Specifically, three modes are mainly studied in the literature: the *fully synchronous* mode (FULLY), where all robots execute an LCM cycle synchronously in each round, the *semi-synchronous* mode (SEMI), where at each round an arbitrary subset of robots acts synchronously, and the *asynchronous* mode (ASYNCH), where robots act without any synchronization assumption.

The traditional problems studied for swarms of mobile entities include Pattern Formation [4–7,12,19–22], Gathering [14,16,23–25], Scattering [26,27], Flocking [28]. A common goal of the algorithmic investigation is to reduce the model capabilities required to solve a given problem or to prove the impossibility of solving it under a certain set of capabilities. This approach has led to studying the *computational power* of a given model (i.e. the set of problems it can solve) and outlining the hierarchical relations (dominance, equivalence, or orthogonality) among different models. In the last decade, multiple works [9,15,29–34] have inspected and compared the computational power of several models that differ in robot features and synchronization mode. Concerning the robot features, the impact of robot *communication* and *storage* capabilities on swarm computational power has been deeply investigated. Starting from the classical model where robots are both *oblivious* and *silent* (i.e. without any means of information storage and communication), researchers have investigated how possessing a persistent memory or communication means affects the power of such models. To implement these extra properties, researchers proposed to equip each robot with a *constant-size light* assuming a color among a fixed constant set of colors. Such a light is *persistent* (so the color is maintained until the next update), it can be updated at the beginning of a Move step of each cycle, and it can be internally or externally visible. Specifically, the literature focuses on four classes of robots: the $\mathcal{OBLOT}$ class, where robots are assumed to be *oblivious* and *silent*, the $\mathcal{FSTA}$ class, where each robot is equipped with an *internal light* (visible only to the robot itself, thus providing a constant-size persistent memory), the $\mathcal{FCOM}$ class, where each robot is equipped with an *external light* (visible only to the other robots, thus providing communication means), and the $\mathcal{LUMI}$ class, where each robot is equipped with an internal and external light. According to the synchronization mode, each class has been studied under the three modes: FULLY, SEMI, and ASYNCH.

Besides self-evident ones, several relations among models appear not so easy to be identified. This is particularly true for models characterized by completely different capabilities. In these cases, the literature has attempted either to design some *simulators* to prove the equivalence between models, or to provide some *witness problems* showing strict dominance or orthogonality. Specifically, the authors in [29,30,33,34] study the computational power of transparent and unlimited-visibility robots that can move on the Euclidean plane, assuming multiple robots can occupy the same positions at the same time (*multiplicity*). In [15,31,32], the same investigation is performed on robots moving on graphs. In [35], the authors consider *energy-constraint* robots, i.e. requiring an idle round to restore energy for a new cycle. Specifically, they show both the relations among the models under such energy constraint and the relations between energy-constraint and unrestricted models. Recently, in [9], a comparative analysis of models with myopic robots is proposed, focusing on how *short-sightedness* affects their computational power.

Related works and our contributions Our work is inspired by [29,30,33,34], where the authors exhibit the complete taxonomy of the computational power of the 12 models of *transparent* robots that can freely move on the Euclidean plane. Such models differ for the synchronization mode (FULLY, SEMI, and ASYNCH), and for the (im)possibility of memorizing and communicating ($\mathcal{OBLOT}$, $\mathcal{FSTA}$, $\mathcal{FCOM}$, $\mathcal{LUMI}$). However, we emphasize that these models are assumed to enjoy complete and unlimited visibility (i.e. they run *transparent* and *non-myopic* robots, respectively), and to be *collision-tolerant*, thus allowing robots to occupy the same position at the same time.

In this paper, we conduct a cross-model comparison among the 12 transparent models considered in [29,30,33,34] and the same 12 models in their *opaque* version, i.e. featuring robots that cannot see through collinearities. This cross-model comparison significantly extends the preliminary investigation in [1] where only the 12 opaque models were considered and studied in terms of their computational power. Besides deepening the analysis of opaque models, here we address the transparent vs. opaque issue by examining how opaqueness affects the power of robots.

Opaqueness introduces a remarkable difficulty in the design of correct algorithms to solve some classical problems [4–7]. In fact, the *obstructed visibility* leads to critical issues to be addressed in the algorithmic strategies: robots may not be aware of the swarm cardinality as well as of moving robots in the ASYNCH mode, robots may not know the complete topology of the current configuration, robots may (incorrectly) compute the next action based on partial information. Clearly, these issues may also be experienced by myopic robots; their computational power has recently been studied in [9], where the authors provide an initial analysis of the computational impact of *short-sightedness*. However, although many issues hold for both opaque and myopic robots, *ad hoc* techniques are separately needed to deal with these two different visibility limitations [8,10,25,36,37].

As in [1], in order to properly settle comparisons between transparent and opaque models, we felt it reasonable to drop a strong assumption often accepted in the literature. More precisely, besides the *opaqueness* feature, our models differ from those presented in [29,30,33,34] in that robots *do not tolerate collisions*, this implying dropping *multiplicity* assumption as well. The reason behind this choice is to avoid theoretical inconsistencies or ambiguities, although keeping coherency with the existing literature on opaque robots [4–7,36,37]. As a matter of fact, a multiplicity of two robots forms a "degenerate" collinearity with any other robot of the swarm, thus making the definition of the visibility relation among robots unnatural in this special case. Exactly for this multiplicity issue, some witness problems introduced in [29,33,34] cannot be used under our comparison, and a new study with specific witness problems is thus required. Moreover, the assumption of collision-intolerance is motivated by the ambition of providing theoretical models for more realistic systems, in perfect accordance with the spirit of assuming robot opaqueness.

Paper organization The paper is organized as follows. In Section 2, we firstly lay down the core features characterizing the robots in our swarms, and then provide some variable features attaining either memorization/communication capabilities, robot synchronization modes, and robot visibility. By tuning these latter variable features, we obtain the 24 robot models whose computational power we are going to investigate. We also provide a formalization of the notion of a problem for a robot swarm, and what we mean for a problem to be solved by robot swarms. This enables defining the computational power of a robot model as the class of problems solved under that model and allows a set-theoretic comparison of the computing power of different models. In Section 3, we expose a preliminary study of the relations between transparent and opaque models. Intuitively, a transparent model seems to computationally dominate the corresponding opaque model. Here, we formally prove this strict dominance: endowing a model with transparency increases its computational power, allowing the model to solve more problems. In addition, this result enables us to show that constant-size lights are *not always* sufficient to compensate for robots' obstructed visibility. In Section 4, we present five witness problems, showing the majority of the hierarchical relations among models of *collision-intolerant opaque robots*, thus yielding a first overview of their computational taxonomy. Such problems also lead to defining the cross-model relations among transparent and opaque models, providing us with an analysis of the impact of obstructed visibility on the computational power of robots. We then establish a sixth witness problem pointing out a peculiar issue occurring under `ASYNCH` and opaqueness. Finally, we focus on model relations still to be proved, suggesting some lines of attack. In Section 5, we summarize our results showing the *almost* complete hierarchy of the 24 models under study while, in Section 6, we draw some concluding remarks and address possible research outlooks.

2. Preliminaries

2.1. Models

In this work, we compare 24 robot models that differ in some features. We here introduce all the *core features* that such models share, and the *variable features* under study.

Core features We investigate swarms of autonomous computational mobile robots, which act in the Euclidean plane $\mathbb{R}^2$. Robots are *indistinguishable* (they cannot be distinguished by external appearance), *anonymous* (they are not provided with any id), *homogeneous* (they execute the same deterministic algorithm), and *punctiform* entities. We assume robots are *completely disoriented* i.e. they do not share a global common coordinate system (i.e. no agreement on origin, axis direction, chirality, or unit distance). Moreover, we assume that the local coordinate system of any given robot in the swarm may change from one activation to another (*variable disorientation*).

Each robot in a swarm $\mathcal{R} = \{r_1, \dots, r_n\}$ is equipped with a sensor system that allows perceiving the *visible* robots in $\mathcal{R}$, according to a *visibility relation* $\bowtie \subseteq \mathcal{R}^2$ whose meaning will be made clear below. We always assume that $\bowtie$ is reflexive and symmetric. Moreover, all robots in $\mathcal{R}$ are provided with the same deterministic algorithm: any robot executes such an algorithm every time the robot itself is activated. At each time, a robot can be either *idle* or *active*, according to the scheduler (see below for possible activation policies). Let $\mathbb{A}$ be the deterministic algorithm executed by $\mathcal{R}$ and let Ω be the *non-empty* $O(1)$-size palette (i.e. set of colors) that $\mathbb{A}$ is equipped with. We always assume that Ω contains at least $\not{b}$ which denotes the *null color*. Let $r \in \mathcal{R}$ be a robot activated at time t, and let $\Xi = (o, \theta_x, \theta_y, u)$ denote the local coordinate system of r at time t where o, θ_x, θ_y, u stand for the position of the origin, the direction of the positive x-axis and the positive y-axis, and the unit of measure, respectively, w.r.t. a fixed absolute coordinate system $\hat{\Xi}$ on $\mathbb{R}^2$. Note that r is unaware of $\hat{\Xi}$ and thus of the values o, θ_x, θ_y, u. So, r starts executing a *Look-Compute-Move* (LCM) cycle, consisting of the following steps:

- **Look:** let $\bowtie(r) = \{r_{i_0}, \dots, r_{i_l}\}$ with $0 \le l < n$ be the set of robots visible to r at time t. So r takes the instantaneous *snapshot* of the robots in $\bowtie(r)$, according to its local coordinate system Ξ. Precisely, this snapshot is the tuple[1]

$$\sigma = \langle (\underline{x}_{i_0}, c_{i_0}), \dots, (\underline{x}_{i_l}, c_{i_l}) \rangle \tag{1}$$

[1] We need a locally ordered data structure to manage snapshots so that r can always spot its state (i.e. position plus light color) among the states of the other robots.

where, for any $0 \le j \le l$, $\underline{x}_{i_j} \in \mathbb{R}^2$ is the position of r_{i_j} according to Ξ and $c_{i_j} \in \Omega$ is its color. We always assume that $(\underline{x}_{i_0}, c_{i_0})$ refers to the position and color of r itself.[2] We remark that *no other information* about the swarm (e.g. whether robots are idle/active, still/moving, ...) is displayed in the snapshot taken by r.

- **Compute:** r executes the algorithm $\mathbb{A}$ with σ as the *sole* input. Specifically, r computes $\mathbb{A}(\sigma) = (\underline{x}', c')$, where $\underline{x}' \in \mathbb{R}^2$ is the position according to Ξ to be reached and $c' \in \Omega \cup \{-\}$ is the possible color to be assumed. We assume that $-$ is a special symbol never contained in Ω which indicates to r to not update its light.
- **Move:** if $c' \in \Omega$, r updates its light color to c'; then, r travels along a *straight* trajectory towards the computed destination $\underline{x}'$. If the computed destination position is equal to the current one, r is said to perform a *null movement*.

After the Move step, r becomes idle again. Throughout this paper, we will always be concerned with *rigid* models, i.e. in which no adversary can stop the motion of a robot.[3] Also, we will be dealing with *collision-intolerant* models, meaning that they do not tolerate either multiplicity (i.e., no robot can occupy the same location of another robot in the swarm at the same time) and overlapping trajectories (i.e., robots r and s have overlapping trajectories if *(i)* r is moving from point a to point a', *(ii)* s is moving from point b to point b', and *(iii)* the line segments $\overline{aa'}$ and $\overline{bb'}$ have points in common). We refer to both multiplicity and overlapping trajectories as *collisions*. Note that it is the responsibility of $\mathbb{A}$ to avoid collisions.

Variable features Let us now focus on four features: visibility, memory, communication, and synchronization. By varying them, we obtain the claimed 24 models.

Regarding the *visibility* of robots, we distinguish between *transparent* and *opaque* swarms. In the former case, all robots are assumed to be transparent so that they always enjoy *complete visibility* of the entire swarm in their Look step. Formally, we have the visibility relation set to $\bowtie = \mathcal{R}^2$, at any time. In the latter case, all robots are assumed to be opaque, so that for three collinear robots p, q, r, the endpoint robots p, r cannot see each other (we call this condition *obstructed visibility*). Formally, we have that $\bowtie \subseteq \mathcal{R}^2$ is defined as follows at any time: $r \bowtie p$ if and only if *(i)* $r = p$ or *(ii)* there does not exist a third robot sitting on the line segment between r and p.

Regarding the *memory* and *communication* features of robots, we consider the four models mainly proposed in the literature. In the $\mathcal{OBLOT}$ model, robots are assumed to be *oblivious* (i.e. they do not have any persistent memory to store data about past cycles) and *silent* (i.e. they do not have any means of communicating with other robots). In this model, robots are not equipped with any light or, equivalently, $\Omega = \{\not b\}$ for any algorithm under $\mathcal{OBLOT}$. Therefore, any snapshot under $\mathcal{OBLOT}$ will be in the form $\sigma = \langle(\underline{x}_{i_0}, \not b), \dots, (\underline{x}_{i_l}, \not b)\rangle$. In the $\mathcal{FSTA}$ model, robots are provided with a persistent *internally visible light* which can assume a color chosen from a constant-size palette Ω. Thus, under $\mathcal{FSTA}$, any snapshot in Equation (1) will be in the form $\sigma = \langle(\underline{x}_{i_0}, c_{i_0}), (\underline{x}_{i_1}, \not b), \dots, (\underline{x}_{i_l}, \not b)\rangle$, i.e. $c_{i_j} = \not b$ for any $1 \le j \le l$. This expresses the fact that a robot can only see its own light color. We remark that, for any robot, its internally visible light can play the role of a constant-size (namely, not depending on the cardinality of the swarm $\mathcal{R}$) persistent memory. In the $\mathcal{FCOM}$ model, robots are equipped with a persistent *externally visible light*, which can assume a color chosen in a constant-size palette Ω. Indeed, external lights can be exploited by the swarm to communicate information among visible robots. Under $\mathcal{FCOM}$, any snapshot in Equation (1) will be in the form $\sigma = \langle(\underline{x}_{i_0}, \not b), (\underline{x}_{i_1}, c_{i_1}), \dots, (\underline{x}_{i_l}, c_{i_l})\rangle$, i.e. $c_{i_0} = \not b$. This expresses the fact that a robot can only see the light color of the other robots visible to itself, but not its own light. Finally, the $\mathcal{LUMI}$ model encompasses the features of both $\mathcal{FSTA}$ and $\mathcal{FCOM}$. This model assumes *luminous* robots, which are equipped with a light that can be colored using a constant-size palette Ω. Such light is visible both to the robot itself (thus, working as an internal state) and to the other visible robots (thus, working as an external communication means). Therefore, under $\mathcal{LUMI}$, snapshots have the form in Equation (1), without any restriction on colors.

Regarding the *synchronization* of robots, we consider the three modes mainly studied in the literature. In the *fully synchronous* mode (FULLY), time is split into atomic rounds, within which all robots in the swarm are activated together, and all robots synchronously execute their LCM cycle. The *semi-synchronous* mode (SEMI) differs from FULLY only for the fact that at each round an arbitrary subset of robots is activated. In the *asynchronous* mode (ASYNCH), every robot is activated independently from the others, and every Compute and Move step in the LCM cycle lasts a finite but unpredictable amount of time. Instead, we always assume that the Look step is instantaneous. For the SEMI and ASYNCH modes, robots do not know which robots are activated at any given instant. Moreover, we always assume the *fairness condition*: for each time t and each robot r in a swarm, there exists a time $t' > t$ in which r will eventually be activated.

The selection of the subset of robots in the swarm $\mathcal{R}$ to be activated at every time is performed by an adversarial *scheduler*. Formally, let $\mathbb{T}$ be a time domain which can be discrete $\mathbb{N}$ (in the FULLY and SEMI modes) or continuous $\mathbb{R}_{\ge 0}$ (in the ASYNCH mode). An *activation scheduling* is a function $\mathfrak{S} : \mathbb{T} \to 2^{\mathcal{R}}$ establishing the subset of the robots in that swarm that is activated at a specific time. For example, for any $t \in \mathbb{N}$, the mapping $t \mapsto \mathcal{R}$ is the only activation scheduling under FULLY.

Notation for the 24 models Let us set $\mathcal{X} = \{\mathcal{OBLOT}, \mathcal{FSTA}, \mathcal{FCOM}, \mathcal{LUMI}\}$ and $\mathcal{Y} = \{\mathtt{F}, \mathtt{S}, \mathtt{A}\}$ (standing for FULLY, SEMI, ASYNCH, resp.). For $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$, we let $X^Y_{\mathcal{T}}$ (resp., $X^Y_{\mathcal{O}}$) denote the model for transparent (resp., opaque) robots that possess all the above core features, and that has X as communication-storage setting and Y as synchronization mode. Let $\mathcal{M}_{\mathcal{T}} = \{X^Y_{\mathcal{T}} \mid X \in \mathcal{X}, Y \in \mathcal{Y}\}$ and $\mathcal{M}_{\mathcal{O}} = \{X^Y_{\mathcal{O}} \mid X \in \mathcal{X}, Y \in \mathcal{Y}\}$: we collectively refer to these two classes of models as the *transparent* and *opaque framework*. In this work, we will consider the 24 models contained in the set $\mathcal{M} = \mathcal{M}_{\mathcal{T}} \cup \mathcal{M}_{\mathcal{O}}$.

[2] Since $\bowtie$ is reflexive, $\bowtie(r)$ contains at least r.

[3] In [29,30,33,34], the authors consider both rigid and non-rigid models.

For the sake of conciseness, we will compactly denote by $X_V^{Y_1,\ldots,Y_h}$ the set of models $\{X_V^{Y_1},\ldots,X_V^{Y_h}\}$, where $X \in \mathcal{X}$, $Y_j \in \mathcal{Y}$, and $V \in \{\mathcal{T},\mathcal{O}\}$. Moreover, we will sometimes use X_*^Y as a shorthand for the following notation: X_V^Y, where V can be both $\mathcal{T}$ and $\mathcal{O}$.

2.2. Problems

Robot swarms are distributed systems aimed at solving problems. For the LCM model, the typical problems proposed in the literature require swarms to form (a sequence of) geometric patterns, and/or to travel along specific trajectories.

Formally, let us assume a swarm of robots $\mathcal{R} = \{r_1,\ldots,r_n\}$ on the Euclidean plane $\mathbb{R}^2$. When no ambiguity arises, we indicate with r_i both the robot and the point in $\mathbb{R}^2$ where r_i is located at a given time. Given an absolute coordinate system $\hat{\Xi}$ on $\mathbb{R}^2$ (not known by the swarm), we define the *configuration* of $\mathcal{R}$ at time t as the multiset $C = \{(\underline{x}_1,c_1),\ldots,(\underline{x}_n,c_n)\}$, where $\underline{x}_i \in \mathbb{R}^2$ is the position of r_i according to $\hat{\Xi}$, and c_i is the light color of r_i, both at time t. The configuration C is *valid* if $\underline{x}_i \neq \underline{x}_j$, for $1 \le i \neq j \le n$. Let $\mathcal{C}$ be the set of all the configurations of $\mathcal{R}$ at any time.

We define[4] a *problem* for a swarm as a (finite or infinite) sequence $P = (\phi_0,\tau_0,\phi_1,\tau_1,\ldots,\phi_m,\tau_m,\ldots)$, where ϕ_i is a condition on the swarm configuration and τ_i is a condition that must be satisfied by every configuration assumed by the swarm while transitioning to a new configuration satisfying ϕ_{i+1}. The sequence $(\phi_0,\tau_0,\phi_1,\tau_1,\ldots,\phi_m,\tau_m,\ldots)$ is also said to be the *request of the problem* P. Since we deal with collision-intolerant models, all the ϕ_i,τ_i must impose that any configuration is valid. Moreover, the initial condition ϕ_0 must include the *color uniformity clause* requiring that all robots in the swarm have the same light color $\not{b}$. Except for this clause, the conditions ϕ_i and τ_i, with $i > 0$, cannot impose any restriction on light colors or the number of cycles, since P might in principle be solved under $\mathcal{OBLOT}$ (i.e. $\Omega = \{\not{b}\}$) and under any synchronization mode.

Let us now formally state the notion of solving a problem by a robot swarm. In that, we use the following notation: we write $C \models \phi$ to express the fact that the swarm configuration C satisfies the condition ϕ. So, let $\mathcal{R}$ be a robot swarm under a model $X_V^Y \in \mathcal{M}$ starting from an initial configuration $C_0 \in \mathcal{C}$ and executing an algorithm $\mathbb{A}$ which complies with memory-communication setting X and visibility V. Let $\mathfrak{S} : \mathbb{T} \to 2^{\mathcal{R}}$ be an activation scheduling under the Y mode, working on the appropriate time domain $\mathbb{T} \in \{\mathbb{N},\mathbb{R}_{\ge 0}\}$. We define the *evolution* induced by $\mathbb{A}$ w.r.t. $\mathfrak{S}$ and C_0 as the function $\mathcal{E} : \mathbb{T} \to \mathcal{C}$ such that $\mathcal{E}(0) = C_0$ and $\mathcal{E}(t)$ is the configuration reached by the swarm $\mathcal{R}$ at time $t > 0$ from the initial configuration C_0 upon executing $\mathbb{A}$ according to the activation scheduling $\mathfrak{S}$. Such an evolution is a *solving* evolution for the problem $P = (\phi_0,\tau_0,\phi_1,\tau_1,\ldots,\phi_m,\tau_m\ldots)$ whenever a sequence $0 = t_0 < t_1 < t_2 < \cdots < t_m < \cdots$ of time instants exists, along which $\mathcal{E}(t_i) \models \phi_i$ and $\mathcal{E}(t) \models \tau_i$ for every $t_i \le t < t_{i+1}$. More generally, we say that the problem P is *solved* under a given model $X_V^Y \in \mathcal{M}$ whenever there exists an algorithm $\mathbb{A}$ such that, for any robot swarm under the model X_V^Y:

- $\mathbb{A}$ works with the memory-communication and visibility setting X, V;
- for any $C_0 \in \mathcal{C}$ satisfying $C_0 \models \phi_0$ and any $\mathfrak{S}$ under Y, $\mathbb{A}$ induces a solving evolution for P.

If the request of the problem is *finite*, the last condition τ_m requires the swarm to stay still after entering a valid configuration satisfying ϕ_m.

2.3. Computational power

Given a robot model A, we indicate with $\mathcal{P}(A)$ its *computational power*, namely, the set of problems solved under A. Given two robot models A and B, we define the following relations:

- A is *computationally not less powerful* than B, formally $A \ge B$, if $\mathcal{P}(A) \supseteq \mathcal{P}(B)$, i.e. any problem solvable in B is solvable in A;
- A is *computationally more powerful* than B, formally $A > B$, if $\mathcal{P}(A) \supset \mathcal{P}(B)$, i.e. any problem solvable in B is solvable in A and there exists a problem solvable in A that is not solvable in B;
- A is *computationally orthogonal* to B, formally $A \perp B$, if $\mathcal{P}(A) \setminus \mathcal{P}(B) \neq \emptyset$ and $\mathcal{P}(B) \setminus \mathcal{P}(A) \neq \emptyset$, i.e. there exists a problem solvable in A (B, resp.) that is not solvable in B (A, resp.);
- A is *computationally equivalent* to B, formally $A \equiv B$, if $\mathcal{P}(A) = \mathcal{P}(B)$, i.e. A and B solve the same set of problems.

Indeed, the relations $<$ and $\le$ are defined as the converse relations of, respectively, $>$ and $\ge$.

3. Preliminary relations

In this section, we begin by establishing some basic facts concerning the computational power of our robot models.

3.1. Model-based dominances

Theorem 1. *Given a visibility setting $V \in \{\mathcal{T},\mathcal{O}\}$ and synchronization modes $Y_1 = \mathtt{F}$, $Y_2 = \mathtt{S}$, $Y_3 = \mathtt{A}$, for each $i \le j \le k$, it holds that*

$$\mathcal{LUMI}_V^{Y_i} \ge \mathcal{FSTA}_V^{Y_j} \ge \mathcal{OBLOT}_V^{Y_k}, \quad \mathcal{LUMI}_V^{Y_i} \ge \mathcal{FCOM}_V^{Y_j} \ge \mathcal{OBLOT}_V^{Y_k}.$$

[4] For our purposes.

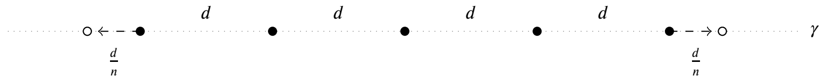


Fig. 1. `Line-Stretch` for $n = 5$ robots.

Proof. By model definition, for any $Y \in \mathcal{Y}$ and $V \in \{\mathcal{T}, \mathcal{O}\}$, we know that $\mathcal{LUMI}_V^Y \geq \mathcal{FSTA}_V^Y \geq \mathcal{OBLOT}_V^Y$ and $\mathcal{LUMI}_V^Y \geq \mathcal{FCOM}_V^Y \geq \mathcal{OBLOT}_V^Y$. Moreover, for any $X \in \mathcal{X}$ and $V \in \{\mathcal{T}, \mathcal{O}\}$, we have that $X_V^{\mathsf{F}} \geq X_V^{\mathsf{S}} \geq X_V^{\mathsf{A}}$. The other relations follow by transitivity. □

3.2. *Strict-dominance of transparency*

Theorem 2. *For any $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$, if $P \in \mathcal{P}\left(X_{\mathcal{O}}^Y\right)$ then $P \in \mathcal{P}\left(X_{\mathcal{T}}^Y\right)$.*

Proof. Let $\mathbb{A}_{\mathcal{O}}$ be an algorithm solving a problem P under $X_{\mathcal{O}}^Y$. We can easily construct an algorithm $\mathbb{A}_{\mathcal{T}}$ solving P under $X_{\mathcal{T}}^Y$, which works as follows. After taking its snapshot $\sigma_{\mathcal{T}}$ (where clearly all robots in the swarm are visible), any transparent robot r first computes the reduced snapshot $\sigma_{\mathcal{O}}$, obtained from $\sigma_{\mathcal{T}}$ by removing those robots which would be geometrically not visible to r due to opaqueness, and then runs $\mathbb{A}_{\mathcal{O}}(\sigma_{\mathcal{O}})$. Clearly, $\mathbb{A}_{\mathcal{T}}$ perfectly simulates $\mathbb{A}_{\mathcal{O}}$, thus correctly solving P for transparent robots. □

Corollary 1. *For any $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$,*

$$X_{\mathcal{O}}^Y \leq X_{\mathcal{T}}^Y .$$

Corollary 2. *For any model $M \in \mathcal{M}$, $\mathcal{OBLOT}_{\mathcal{O}}^{\mathsf{A}} \leq M$.*

Proof. It follows from Theorem 1 and Corollary 1. □

Problem 1 *(`Line-Stretch`)*. Let us consider an initial configuration where $n > 3$ robots are equally spaced along the same line, say γ. Let d be the distance between two adjacent robots. The problem asks the endpoint robots to move away from their adjacent robot and stop at a distance $d + \frac{d}{n}$ from them. The endpoint robots are allowed to travel only along γ. The other robots must stay still. See Fig. 1.

Lemma 1. `Line-Stretch` $\in \mathcal{P}\left(\mathcal{OBLOT}_{\mathcal{T}}^{\mathsf{A}}\right)$.

Proof. The problem is solved under the weakest model of the transparent framework. In fact, the endpoint robots can compute and head to their destination since they can count all the robots and spot the distance d among two adjacent robots. The final configuration is stable (i.e. no robot will move anymore). □

Lemma 2. `Line-Stretch` $\notin \mathcal{P}\left(\mathcal{LUMI}_{\mathcal{O}}^{\mathsf{F}}\right)$.

Proof. The problem cannot be solved under the strongest model of the opaque framework. By contradiction, suppose an algorithm $\mathbb{A}$ solves the problem under $\mathcal{LUMI}_{\mathcal{O}}^{\mathsf{F}}$ using the constant-size palette Ω. Consider an endpoint robot e that has not moved yet, and let a be its adjacent robot. Suppose e always has a local coordinate system such that the positions of e and a are respectively $(0,0)$ and $(1,0)$, regardless of the problem instance and the activation time. Thus, the snapshot taken by e will always have the form $\langle((0,0),c_e),((1,0),c_a)\rangle$ with $c_e, c_a \in \Omega$. Accordingly, the colors c_e, c_a are the only way for the swarm to code information needed to e to compute n. Let t be the time where e moves to its destination point at distance $1 + \frac{1}{n}$ from a (being $d = 1$ according to the coordinate system of e). Let σ be the snapshot taken by e at time t which has been used by $\mathbb{A}$ to compute the destination point. So, $\mathbb{A}$ must implement a function $f : \Omega^2 \to \mathbb{N}$ mapping the light combination in a snapshot (i.e. c_e, c_a) with the correct natural number n. However, since $\mathbb{A}$ solves the problem for any size of the swarm, f must be surjective. Contradiction. □

Theorem 3. *For any $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$, it holds*

$$X_{\mathcal{O}}^Y < X_{\mathcal{T}}^Y .$$

Proof. By Lemma 1 and Lemma 2, respectively, we get that `Line-Stretch` $\in \mathcal{P}\left(X_{\mathcal{T}}^Y\right)$ and `Line-Stretch` $\notin \mathcal{P}\left(X_{\mathcal{O}}^Y\right)$. Thus, the result follows from Corollary 1. □

Theorem 4. *Let P be a problem solved by an algorithm $\mathbb{A}$ under $X_{\mathcal{T}}^Y$, which guarantees that collinearities never occur along any solving evolution. Then, P can be solved under $X_{\mathcal{O}}^Y$.*

Proof. Running $\mathbb{A}$ under $X_{\mathcal{O}}^{Y}$ perfectly replicates the evolution of $\mathbb{A}$ by transparent robots. This is due to the fact that, by hypothesis, the snapshots taken in both the opaque and transparent frameworks are identical. □

3.3. Non-emptiness and non-disjointness

Problem 2 *(`Iso-Equi`).* Let three robots initially form a non-degenerate isosceles triangle. The problem requires the robots to form an equilateral triangle and terminate.

Lemma 3. `Iso-Equi` $\in \mathcal{P}\left(\mathcal{OBLOT}_{\mathcal{O}}^{\mathtt{A}}\right)$.

Proof. We provide this simple algorithm: firstly, the three robots can check whether they form an equilateral triangle and, in case, stand still. Otherwise, the problem can be easily solved by making the vertex robot move along the axis and stop to form an equilateral triangle with the base robots (which remain still). □

As a consequence of Lemma 3 and Corollary 2, we get `Iso-Equi` $\in \mathcal{P}(M)$ for each $M \in \mathcal{M}$. This proves the following properties of the computational power of the models under study in this paper:

Corollary 3 *(Non-emptiness). For any model $M \in \mathcal{M}$, we have $\mathcal{P}(M) \neq \emptyset$.*

Corollary 4 *(Non-disjointness). For any two models $M_1, M_2 \in \mathcal{M}$, we have $\mathcal{P}\left(M_1\right) \cap \mathcal{P}\left(M_2\right) \neq \emptyset$.*

3.4. Collinearities vs awareness

A natural question for the opaque framework is how collinearities may affect the computational power of the robots. The following observations highlight that a non-obstructed snapshot of an opaque swarm (i.e. taken from a collinearity-less configuration) is neither necessary nor sufficient for a robot to correctly perceive the whole current configuration.

Observation 1. *Let r be a robot under a model in $\mathcal{M}_{\mathcal{O}}$, activated in a configuration C. Then, C does not need to be collinearity-less for r to correctly perceive the whole C.*

This observation can be understood by considering the following example. Let us define the perpetual problem for a $\mathcal{OBLOT}_{\mathcal{O}}^{\mathtt{F}}$ swarm of three robots, starting from an equally spaced line configuration. The problem asks the endpoint robots to move along the line and double the previous distance from the center robot at every round. Even if the endpoint robots cannot see each other, at each activation they know the exact position of the hidden robot. On the other hand:

Observation 2. *Let r be a robot under a model in $\mathcal{M}_{\mathcal{O}}$, activated in a collinearity-less configuration C. This latter property of C does not guarantee that r is aware of enjoying a non-obstructed snapshot of the whole swarm.*

To understand this observation, let us consider the problem defined for a $\mathcal{OBLOT}_{\mathcal{O}}^{\mathtt{F}}$ swarm, which can start from two different configurations: *(i)* two robots or *(ii)* three equally spaced and aligned robots. From *(i)*, the problem asks the swarm to stand still. From *(ii)*, the problem asks the endpoint robots to double their distance from the center robot at every round, always moving along the original alignment line. This problem is unsolvable in $\mathcal{OBLOT}_{\mathcal{O}}^{\mathtt{F}}$, since each activated endpoint robot has no means to understand whether or not the current configuration describes a two or three-robot system. In particular, starting from *(i)*, the absence of collinearities does not suffice to guarantee swarm cardinality awareness.

3.5. Transparency vs opaqueness

Lemma 4. *Let $A_{\mathcal{O}} \in \mathcal{M}_{\mathcal{O}}$ and $B_{\mathcal{T}} \in \mathcal{M}_{\mathcal{T}}$. Then, neither $A_{\mathcal{O}} \equiv B_{\mathcal{T}}$ nor $A_{\mathcal{O}} > B_{\mathcal{T}}$ can hold.*

Proof. This trivially follows from Lemmas 1 and 2, showing the witness problem `Line-Stretch` is solved by any transparent model but by no opaque model. □

Theorem 5. *Let $A_{\mathcal{T}}, B_{\mathcal{T}} \in \mathcal{M}_{\mathcal{T}}$. If $A_{\mathcal{T}} \geq B_{\mathcal{T}}$ (i.e. either $A_{\mathcal{T}} > B_{\mathcal{T}}$ or $A_{\mathcal{T}} \equiv B_{\mathcal{T}}$), then $A_{\mathcal{T}} > B_{\mathcal{O}}$.*

Proof. This trivially holds since $B_{\mathcal{T}} > B_{\mathcal{O}}$ by Theorem 3. □

(a) Relation diagram in Theorem 6.

(b) Relation diagram in Theorem 7.

Fig. 2. Relation diagrams: a straight (dashed, resp.) line denotes a dominance (orthogonality, resp.) relation.

Theorem 6. *Let $A_{\mathcal{T}}, B_{\mathcal{T}} \in \mathcal{M}_{\mathcal{T}}$. If $A_{\mathcal{T}} \geq B_{\mathcal{T}}$ and $A_{\mathcal{O}} \geq B_{\mathcal{O}}$, and there exists a problem $P \in \mathcal{P}\left(A_{\mathcal{O}}\right) \setminus \mathcal{P}\left(B_{\mathcal{T}}\right)$, then it holds:*

$$A_{\mathcal{T}} > B_{\mathcal{T}}, \quad A_{\mathcal{T}} > B_{\mathcal{O}},$$
$$A_{\mathcal{O}} \perp B_{\mathcal{T}}, \quad A_{\mathcal{O}} > B_{\mathcal{O}}.$$

Proof. The claimed relations are depicted in Fig. 2a. The dominance relations $A_{\mathcal{T}} > B_{\mathcal{T}}$ and $A_{\mathcal{O}} > B_{\mathcal{O}}$ straightforwardly derives from the fact that $P \in \mathcal{P}\left(A_{\mathcal{T}}\right) \setminus \mathcal{P}\left(B_{\mathcal{T}}\right)$ and that $P \in \mathcal{P}\left(A_{\mathcal{O}}\right) \setminus \mathcal{P}\left(B_{\mathcal{O}}\right)$ by Theorem 3. The dominance relation $A_{\mathcal{T}} > B_{\mathcal{O}}$ holds by Theorem 5. The orthogonality relation $A_{\mathcal{O}} \perp B_{\mathcal{T}}$ holds true since $P \in \mathcal{P}\left(A_{\mathcal{O}}\right) \setminus \mathcal{P}\left(B_{\mathcal{T}}\right)$ and `Line-Stretch` $\in \mathcal{P}\left(B_{\mathcal{T}}\right) \setminus \mathcal{P}\left(A_{\mathcal{O}}\right)$ by Lemmas 1 and 2. □

Theorem 7. *Let $A_{\mathcal{T}}, B_{\mathcal{T}} \in \mathcal{M}_{\mathcal{T}}$. If there exists a problem $P \in \mathcal{P}\left(A_{\mathcal{O}}\right) \setminus \mathcal{P}\left(B_{\mathcal{T}}\right)$ and a problem $Q \in \mathcal{P}\left(B_{\mathcal{O}}\right) \setminus \mathcal{P}\left(A_{\mathcal{T}}\right)$, then it holds that $A_* \perp B_*$, i.e.:*

$$A_{\mathcal{T}} \perp B_{\mathcal{T}}, \quad A_{\mathcal{T}} \perp B_{\mathcal{O}}, \quad A_{\mathcal{O}} \perp B_{\mathcal{T}}, \quad A_{\mathcal{O}} \perp B_{\mathcal{O}}.$$

Proof. The claimed relations are depicted in Fig. 2b. By Theorem 3, we get that $P \in \mathcal{P}\left(A_{\mathcal{T}}\right) \setminus \mathcal{P}\left(B_{\mathcal{T}}\right)$, $P \in \mathcal{P}\left(A_{\mathcal{O}}\right) \setminus \mathcal{P}\left(B_{\mathcal{O}}\right)$ and $P \in \mathcal{P}\left(A_{\mathcal{T}}\right) \setminus \mathcal{P}\left(B_{\mathcal{O}}\right)$. The corresponding relations hold for the problem Q, so $Q \in \mathcal{P}\left(B_{\mathcal{T}}\right) \setminus \mathcal{P}\left(A_{\mathcal{T}}\right)$, $Q \in \mathcal{P}\left(B_{\mathcal{O}}\right) \setminus \mathcal{P}\left(A_{\mathcal{O}}\right)$ and $Q \in \mathcal{P}\left(B_{\mathcal{T}}\right) \setminus \mathcal{P}\left(A_{\mathcal{O}}\right)$. By combining these results, the four claimed orthogonality relations follow. □

4. Taxonomy of the 24 models

Firstly, we cite two main computational equivalences proved in [30,34] for transparent robots. Namely, the authors proved such equivalences through the construction of two simulators which allow to emulate a $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{T}}$ ($\mathcal{LUMI}^{\mathtt{F}}_{\mathcal{T}}$, resp.) algorithm in the $\mathcal{LUMI}^{\mathtt{A}}_{\mathcal{T}}$ ($\mathcal{FCOM}^{\mathtt{F}}_{\mathcal{T}}$, resp.) model. Since such simulators work regardless of the presence of collisions, they can be used to prove the same equivalences for our collision-intolerant models as well.

Theorem 8 *([30]).* $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{T}} \equiv \mathcal{LUMI}^{\mathtt{A}}_{\mathcal{T}}$.

Theorem 9 *([34]).* $\mathcal{LUMI}^{\mathtt{F}}_{\mathcal{T}} \equiv \mathcal{FCOM}^{\mathtt{F}}_{\mathcal{T}}$.

We now present six witness problems to prove some strict dominance (>) and orthogonality (⊥) relations among the 24 models in $\mathcal{M}$. For each witness problem, we identify the *most limited* models under which the problem can be solved, and the *most powerful* models under which the problem cannot be solved.

The first witness problem we introduce, `Triangle Round-Trip`, is a restriction of a problem devised in [29] for the transparent framework. Thanks to Theorem 2 and Theorem 4, we can borrow it to prove some hierarchical relations hold in our opaque framework as well. However, other witness problems in [29,33,34] are not compliant with our collision-intolerant models. Thus, we present other four specific problems that fit our assumptions. Eventually, we introduce an *ad hoc* problem, `Pseudo-Polygon`, which raises a critical issue occurring under obstructed visibility.

4.1. Weakness of $\mathcal{OBLOT}$

Problem 3 *(`Triangle Round-Trip`).* Let C be a configuration with three robots, two of which lie on the vertices of an equilateral triangle, while the third robot lies on the triangle center. Let a be the empty triangle vertex. From C, the robot in the center has to move to a, forming the new configuration C'. Then, robots have to form C again, where a is again the empty vertex. Afterwards, the robots must stay still. See Table 1.

`Triangle Round-Trip` is a sub-case of the problem `N-gon Round-Trip` defined in [29], Definition 1.

Table 1
Configurations in `Triangle Round-Trip`.

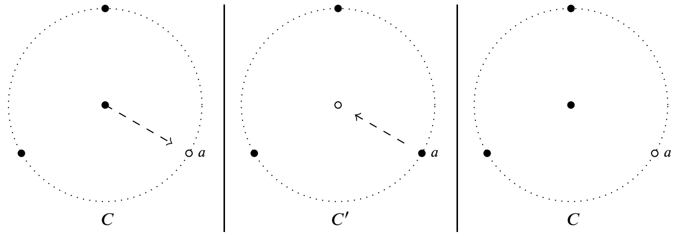
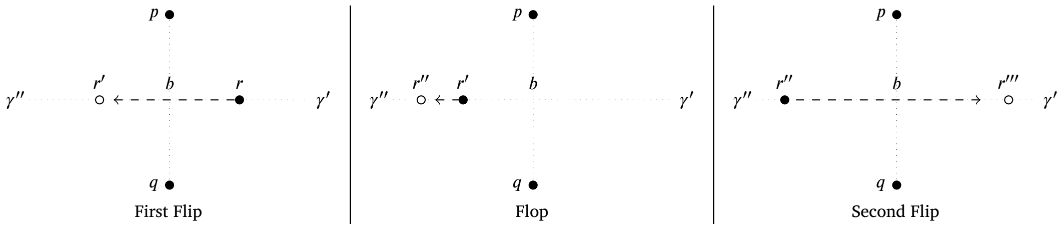


Table 2
Configurations in `Flip-Flop-Flip` where robot r reaches the points r', r'' and r'''.



Lemma 5. `Triangle Round-Trip` $\notin \mathcal{P}\left(\mathcal{OBLOT}_{\mathcal{T}}^{\mathrm{F}}\right)$.

Proof. The problem cannot be solved in $\mathcal{OBLOT}_{\mathcal{T}}^{\mathrm{F}}$ (see Lemma 3 in [29]). In fact, using oblivious and silent robots, there is no way to identify the former empty vertex a due to the full symmetry of C'. □

Lemma 6. `Triangle Round-Trip` $\in \left(\mathcal{P}\left(\mathcal{FSTA}_{\mathcal{O}}^{\mathrm{A}}\right) \cap \mathcal{P}\left(\mathcal{FCOM}_{\mathcal{O}}^{\mathrm{A}}\right)\right)$.

Proof. The problem has been solved in $\mathcal{FSTA}_{\mathcal{T}}^{\mathrm{A}}$ and $\mathcal{FCOM}_{\mathcal{T}}^{\mathrm{A}}$ by Lemmas 4 and 5 in [29], where solving algorithms never create collinearities or collisions. This enables us to apply Theorem 4 and obtain that `Triangle Round-Trip` can be solved both in $\mathcal{FSTA}_{\mathcal{O}}^{\mathrm{A}}$ and $\mathcal{FCOM}_{\mathcal{O}}^{\mathrm{A}}$ as well. □

Theorem 10. *Given the schedulers* $Y_1 = \mathrm{F}$, $Y_2 = \mathrm{S}$, $Y_3 = \mathrm{A}$ *and for any* $X \in \{\mathcal{FSTA}, \mathcal{FCOM}, \mathcal{LUMI}\}$, *the following relations hold for any* $1 \le i \le j \le 3$:

$$X_{\mathcal{T}}^{Y_i} > \mathcal{OBLOT}_{\mathcal{T}}^{Y_j}, \quad X_{\mathcal{T}}^{Y_i} > \mathcal{OBLOT}_{\mathcal{O}}^{Y_j},$$
$$X_{\mathcal{O}}^{Y_i} \perp \mathcal{OBLOT}_{\mathcal{T}}^{Y_j}, \quad X_{\mathcal{O}}^{Y_i} > \mathcal{OBLOT}_{\mathcal{O}}^{Y_j}.$$

Proof. Let us consider any $X \in \{\mathcal{FSTA}, \mathcal{FCOM}, \mathcal{LUMI}\}$. By Theorem 1, we know that $X_{\mathcal{T}}^{Y_i} \ge \mathcal{OBLOT}_{\mathcal{T}}^{Y_j}$ and $X_{\mathcal{O}}^{Y_i} \ge \mathcal{OBLOT}_{\mathcal{O}}^{Y_j}$. Moreover, we have that `Triangle Round-Trip` cannot be solved under $\mathcal{OBLOT}_{\mathcal{T}}^{\mathrm{F,S,A}}$ (by Lemma 5) but it can be solved under $X_{\mathcal{O}}^{\mathrm{A,S,F}}$ (by Lemma 6). Thus, the claimed relations follow from Theorem 6. □

4.2. Orthogonality between $\mathcal{FSTA}$ and $\mathcal{FCOM}$

Problem 4 *(`Flip-Flop-Flip`)*. Let p, q, r be three robots forming a strictly isosceles triangle such that $dist(p, r) = dist(q, r)$. Let γ be the perpendicular bisector to the line segment $\overline{pq}$ passing through the point $b \in \overline{pq}$. Let γ' (γ'', resp.) be the semi-line of γ starting from b and containing (not containing, resp.) r. The problem requires r to perpetually perform three subsequent actions (see Table 2), in an infinite loop: *(i)* r must reach its symmetric position r' on γ''; *(ii)* r must reach a different point r'' on γ'' such that $dist(r'', b) > dist(r', b)$; *(iii)* r must reach its symmetric position r''' on γ'. Along with these actions, r can never leave γ and can never stop in a configuration where robots p, q, r form an equilateral triangle. The robots p, q must always stay still.

Lemma 7. `Flip-Flop-Flip` $\in \left(\mathcal{P}\left(\mathcal{FSTA}_{\mathcal{O}}^{\mathrm{A}}\right) \cap \mathcal{P}\left(\mathcal{FCOM}_{\mathcal{O}}^{\mathrm{F}}\right)\right)$.

Proof. We solve the problem in both the models using the colors flip1, flop, and flip2, assuming w.l.o.g. all robots start with the color flip1. Note that the request of the problem guarantees that each robot can recognize its role by geometric conditions only, and that possible collinearities under $\mathcal{FSTA}^{\mathrm{A}}_{\mathcal{O}}$ do not affect the solvability of the problem. In $\mathcal{FSTA}^{\mathrm{A}}_{\mathcal{O}}$, the robot r moves along γ changing its internal color according to the perpetual scheme $(\text{flip1} - \text{flop} - \text{flip2})^{\infty}$, so that at each activation, r knows which is the current action to be performed. The robots p, q do not need to change their colors. In the $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}}$ model, all the robots synchronously update their external colors following the above scheme, so that at each round each robot knows what actions (color setting and move step) have to be accomplished by looking at the light of the other robots. □

Lemma 8. `Flip-Flop-Flip` $\notin \left(\mathcal{P}\left(\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{T}}\right) \cup \mathcal{P}\left(\mathcal{FCOM}^{\mathrm{S}}_{\mathcal{T}}\right)\right)$.

Proof. By contradiction, suppose an algorithm $\mathbb{A}$ solves `Flip-Flop-Flip` under $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{T}}$. Suppose that r always maintains the same local coordinate system Ξ. Let σ be the snapshot taken by r according to which $\mathbb{A}$ computes the first Flop action. Being in $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{T}}$, σ contains the positions of the three robots according to Ξ, while all the colors are set to $\not{b}$. Clearly, such a snapshot could perfectly describe an initial configuration for the problem. So, assume that `Flip-Flop-Flip` now starts from an initial configuration where the snapshot taken by r w.r.t. Ξ coincides with σ. Since $\mathbb{A}$ has no further information as input, its output is still a Flop, which causes r to perform an erroneous action. Contradiction.

By contradiction, suppose an algorithm $\mathbb{A}$ solves `Flip-Flop-Flip` under $\mathcal{FCOM}^{\mathrm{S}}_{\mathcal{T}}$, and let $\mathfrak{S}$ be a `SEMI` activation scheduling under which $\mathbb{A}$ solves the problem. We show that there exists a `SEMI` activation scheduling $\mathfrak{S}'$ such that `Flip-Flop-Flip` is not solved by $\mathbb{A}$. Let t be the first round in $\mathfrak{S}$ where r executes the first Flip. Let Ξ be a coordinate system having its origin in b and, w.l.o.g., the positive x-axis along the semi-line γ'. Assume the coordinate system of r for any activation time $t' \leq t$ is always Ξ, and let $\underline{x}_p$, $\underline{x}_q$, and $\underline{x}_r$ be the positions of p, q, and r at time t according to Ξ. So, at time t, r computes $\mathbb{A}(\sigma) = (-\underline{x}_r, c'_r)$ where $\sigma = \langle(\underline{x}_r, \not{b}), (\underline{x}_p, c_p), (\underline{x}_q, c_q)\rangle$ is the snapshot taken by r at time t, and it reaches its symmetrical position $-\underline{x}_r$ on γ''. Now, consider the scheduling $\mathfrak{S}'$ defined as $\mathfrak{S}'(t') = \mathfrak{S}(t')$, $\forall t' \leq t$. Clearly, r executes its first Flip at the t-th round under $\mathfrak{S}'$. Suppose that, in the $(t+1)$-th activation round under $\mathfrak{S}'$, r is the only robot that gets activated, namely $\mathfrak{S}'(t+1) = \{r\}$, and suppose that the coordinate system of r is the same as Ξ but now the positive x-axis is directed from b to r along γ''. Let σ' be the snapshot taken by r at time $t+1$. Being under $\mathcal{FCOM}$ and being the coordinate system mirror-symmetric w.r.t. Ξ, we have $\sigma' = \sigma$. As a consequence, r re-computes $\mathbb{A}(\sigma') = \mathbb{A}(\sigma)$ and makes again a Flip at time $t+1$. Contradiction. □

Theorem 11.

$$
\begin{array}{lll}
\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{T}} > \mathcal{FCOM}^{\mathrm{A}}_{*}, & \mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}} > \mathcal{FCOM}^{\mathrm{A}}_{\mathcal{O}}, & \mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}} \perp \mathcal{FCOM}^{\mathrm{A}}_{\mathcal{T}}, \\
\mathcal{LUMI}^{\mathrm{S}}_{\mathcal{T}} > \mathcal{FCOM}^{\mathrm{S,A}}_{*}, & \mathcal{LUMI}^{\mathrm{S}}_{\mathcal{O}} > \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{O}}, & \mathcal{LUMI}^{\mathrm{S}}_{\mathcal{O}} \perp \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}, \\
\mathcal{LUMI}^{\mathrm{F}}_{\mathcal{T}} > \mathcal{FCOM}^{\mathrm{S,A}}_{*}, & \mathcal{LUMI}^{\mathrm{F}}_{\mathcal{O}} > \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{O}}, & \mathcal{LUMI}^{\mathrm{F}}_{\mathcal{O}} \perp \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}, \\
\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} > \mathcal{FCOM}^{\mathrm{S,A}}_{*}, & \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} > \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{O}}, & \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} \perp \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}.
\end{array}
$$

Proof. By Theorem 1, we know that:

$$
\begin{array}{ll}
\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{T}} \geq \mathcal{FCOM}^{\mathrm{A}}_{\mathcal{T}}, & \mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}} \geq \mathcal{FCOM}^{\mathrm{A}}_{\mathcal{O}} \\
\mathcal{LUMI}^{\mathrm{S}}_{\mathcal{T}} \geq \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}, & \mathcal{LUMI}^{\mathrm{S}}_{\mathcal{O}} \geq \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{O}} \\
\mathcal{LUMI}^{\mathrm{F}}_{\mathcal{T}} \geq \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}, & \mathcal{LUMI}^{\mathrm{F}}_{\mathcal{O}} \geq \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{O}} \\
\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} \geq \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}, & \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} \geq \mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{O}}.
\end{array}
$$

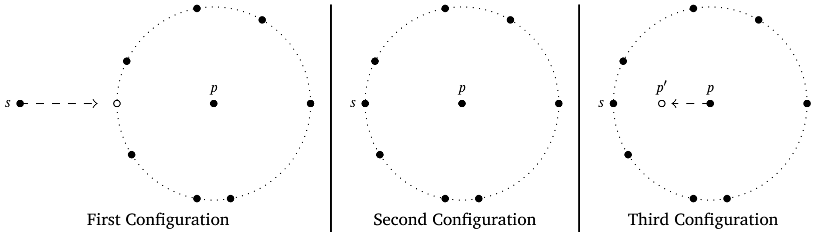
By Lemma 7, we have that `Flip-Flop-Flip` is solved under $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}}$ and $\mathcal{LUMI}^{\mathrm{A,S,F}}_{\mathcal{O}}$. In addition, by Lemma 8, `Flip-Flop-Flip` cannot be solved under $\mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}$. By using Theorem 6 with `Flip-Flop-Flip` as a separator, the claimed relations follow. □

Problem 5 *(`Newcomer Introducing`)*. Consider $n+2$ robots, with $n \geq 7$, of which n are placed on the same circle of radius of length ρ. Then, a robot p lies in the center of the circle, and the last robot s sits outside the circle so that no robot lies on the segment $\overline{sp}$. The problem requires sequentially the swarm to form two configurations and then stop. First, s must travel along the segment $\overline{sp}$ and stop on the boundary of the circle. Second, p must travel along the radius defined with s and stop at the position p' satisfying $dist(s, p') = \frac{1}{2}\rho$. All the other robots must stay still. See Table 3.

Lemma 9. `Newcomer Introducing` $\notin \mathcal{P}\left(\mathcal{FSTA}^{\mathrm{F}}_{\mathcal{T}}\right)$.

Proof. By contradiction, let us assume there exists an $\mathcal{FSTA}^{\mathrm{F}}_{\mathcal{T}}$ algorithm $\mathbb{A}$ solving `Newcomer Introducing` with a constant-size color palette Ω, so that $|\Omega| = k$ for some $k \in \mathbb{N}$. Let us consider a swarm with $n = k+2$ robots, and let C be a second configuration consisting of the robot p at the center of $k+1$ robots $r_1, \ldots, r_{k+1}$. Specifically, C can derive from $k+1$ different types of configurations, say $C_1, \ldots, C_{k+1}$, where C_i denotes a type of initial configuration in which robot r_i is external to the circle. So, whenever in C, the robot p must detect which robot among $r_1, \ldots, r_{k+1}$ is the newcomer to move towards. Let us assume that p has a fixed coordinate system, and let σ be the snapshot p takes in C. For notation ease, we write this snapshot as $\sigma = \langle \underline{x}_p, \underline{x}_1, \ldots, \underline{x}_{k+1}, c_p \rangle$, where $\underline{x}_p$ and c_p

Table 3
Configurations in `Newcomer Introducing`.



are, respectively, the position and the internal color of p, while $\underline{x}_i$ is the position of r_i, for $1 \le i \le k+1$. Notice that the same positions $\underline{x}_p, \underline{x}_1, \ldots, \underline{x}_{k+1}$ would have been observed by p in its snapshot independently from the actual C_i which C derives from. So the only element in σ which could be used by p to distinguish the newcomer among the $k+1$ robots is its internal light color c_p. However, since there are $k+1$ robots and only k colors to be used, at least 2 robots, say r_j and r_h, would be associated with the same color, say c'. So, if by hypothesis we assume that $\mathbb{A}(\langle \underline{x}_p, \underline{x}_1, \ldots, \underline{x}_{k+1}, c' \rangle)$ correctly aims at the newcomer r_j when C derives from C_j, then the algorithm returns the wrong target position to p (i.e. r_j) when C derives from C_h. Contradiction. □

Lemma 10. *Newcomer Introducing* $\in \mathcal{P}\left(\mathcal{FCOM}_{\mathcal{O}}^{\mathtt{A}}\right)$.

Proof. We show an $\mathcal{FCOM}_{\mathcal{O}}^{\mathtt{A}}$ algorithm solving `Newcomer Introducing` using the palette $\Omega = \{\not{b}, \mathtt{S}\}$. All the robots are initially set to color $\not{b}$. Each robot can determine its role by the geometry of the configurations (p realizes to be at the center of $n \ge 7$ robots lying on a common circle, and spots an external robot, s sees at least four robots forming a circle with a robot at its center, the other robots can see they lie on a circle with at least $n-2 \ge 5$ other robots). When s is activated, it sets its light to S and starts to move. This color is maintained during its subsequent activations. When activated, upon seeing a robot with color S on the circle, p computes the correct destination and moves there. The last configuration is stable: no other robot will move. □

Theorem 12. *Given the schedulers* $Y_1 = \mathtt{F}$, $Y_2 = \mathtt{S}$, $Y_3 = \mathtt{A}$, *the following relations hold true for any* $1 \le i \le j \le 3$:

$$\mathcal{LUMI}_{\mathcal{T}}^{Y_i} > \mathcal{FSTA}_{\mathcal{T}}^{Y_j}, \quad \mathcal{LUMI}_{\mathcal{T}}^{Y_i} > \mathcal{FSTA}_{\mathcal{O}}^{Y_j}$$
$$\mathcal{LUMI}_{\mathcal{O}}^{Y_i} \perp \mathcal{FSTA}_{\mathcal{T}}^{Y_j}, \quad \mathcal{LUMI}_{\mathcal{O}}^{Y_i} > \mathcal{FSTA}_{\mathcal{O}}^{Y_j}.$$

Proof. Trivially, we know that $\mathcal{LUMI}_{\mathcal{T}}^{Y_i} \ge \mathcal{FSTA}_{\mathcal{T}}^{Y_j}$ and $\mathcal{LUMI}_{\mathcal{O}}^{Y_i} \ge \mathcal{FSTA}_{\mathcal{O}}^{Y_j}$. By Lemma 10, `Newcomer Introducing` is solved under $\mathcal{LUMI}_{\mathcal{O}}^{\mathtt{A,S,F}}$. By Lemma 9, `Newcomer Introducing` cannot be solved under $\mathcal{FSTA}_{\mathcal{T}}^{\mathtt{F,S,A}}$. Thus, the stated relations hold by applying Theorem 6. □

Theorem 13. $\mathcal{FSTA}_{*}^{\mathtt{F,S,A}} \perp \mathcal{FCOM}_{*}^{\mathtt{S,A}}$.

Proof. By Lemmas 7 and 8, `Flip-Flop-Flip` is solved in $\mathcal{FSTA}_{\mathcal{O}}^{\mathtt{F,S,A}}$ but not in $\mathcal{FCOM}_{\mathcal{T}}^{\mathtt{S,A}}$. By Lemmas 9 and 10, `Newcomer Introducing` is solved in $\mathcal{FCOM}_{\mathcal{O}}^{\mathtt{S,A}}$ but not in $\mathcal{FSTA}_{\mathcal{T}}^{\mathtt{F,S,A}}$. Thus, the stated relations hold by applying Theorem 7. □

4.3. The power of `FULLY`

Problem 6 *(Angle-Shift)*. Consider an initial configuration with three robots forming an acute and scalene triangle. Let a, b, c be the three robots, where a is placed on the greatest angle, say α, whereas c is placed on the smallest angle. Fixing a as the rotation center and following the direction given by $a \to b \to c$, the problem requires b to rotate by α and c to rotate by $\pi - \alpha$. Robot a must never change its position, while robots b and c, once left their initial positions, are not allowed to stop in any other positions but the target ones. Afterwards, the robots must stay still. See Table 4.

Lemma 11. *Angle-Shift* $\in \left(\mathcal{P}\left(\mathcal{OBLOT}_{\mathcal{O}}^{\mathtt{F}}\right) \setminus \mathcal{P}\left(\mathcal{LUMI}_{\mathcal{T}}^{\mathtt{S}}\right)\right)$.

Proof. `Angle-Shift` is solvable under any `FULLY` model: if b and c perform their cycles at the same time, then they correctly compute their target position. The final configuration is stable since it always forms an obtuse triangle (terminal condition).

Instead, the swarm can suffer from information loss under `SEMI`, making `Angle-Shift` unsolvable even under $\mathcal{LUMI}_{\mathcal{T}}^{\mathtt{S}}$. Suppose by contradiction that there exists an algorithm $\mathbb{A}$ that solves the problem in $\mathcal{LUMI}_{\mathcal{T}}^{\mathtt{S}}$ using a constant-size palette Ω. Consider a sequential activation scheduling (i.e. only one robot per round is activated). Suppose that b is the first robot that moves (the situation is analogous if c is the first to move). Let C' be the resulting configuration where the three robots are aligned. By problem request, b

Table 4
Angle-Shift.

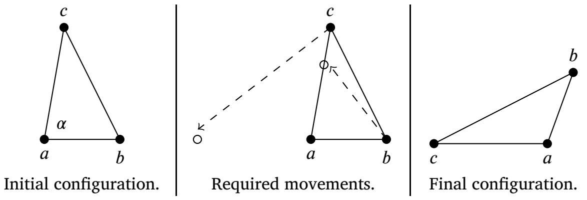
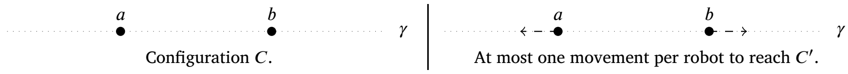


Table 5
Configurations in `Semi-Expansion`.



would have reached the same position as in C' by starting from an infinite range of initial configurations, where $\alpha \in (\frac{\pi}{3}, \frac{\pi}{2})$. So, from C', c infers the angle α getting information only from the light color combination in C'. Thus, the range of α must have at most the same cardinality as $|\Omega|^3$. Contradiction. □

Theorem 14. *For any $X \in \mathcal{X}$, it holds:*

$$X^{\mathrm{F}}_{\mathcal{T}} > X^{\mathrm{S,A}}_{\mathcal{T}}, \quad X^{\mathrm{F}}_{\mathcal{T}} > X^{\mathrm{S,A}}_{\mathcal{O}},$$
$$X^{\mathrm{F}}_{\mathcal{O}} \perp X^{\mathrm{S,A}}_{\mathcal{T}}, \quad X^{\mathrm{F}}_{\mathcal{O}} > X^{\mathrm{S,A}}_{\mathcal{O}}.$$

Proof. We know that $X^{\mathrm{F}}_{\mathcal{T}} \geq X^{\mathrm{S,A}}_{\mathcal{T}}$ and $X^{\mathrm{F}}_{\mathcal{O}} \geq X^{\mathrm{S,A}}_{\mathcal{O}}$. By Lemma 11, `Angle-Shift` is solved under $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{O}}$, and so any $X^{\mathrm{F}}_{\mathcal{O}}$, but it cannot be solved under $\mathcal{LUMI}^{\mathrm{S}}_{\mathcal{T}}$, and so any $X^{\mathrm{S,A}}_{\mathcal{T}}$. Thus, the stated relations hold by Theorem 6. □

Theorem 15.

$$\mathcal{OBLOT}^{\mathrm{F}}_{*} \perp \mathcal{FCOM}^{\mathrm{S,A}}_{*}, \quad \mathcal{OBLOT}^{\mathrm{F}}_{*} \perp \mathcal{FSTA}^{\mathrm{S,A}}_{*},$$
$$\mathcal{OBLOT}^{\mathrm{F}}_{*} \perp \mathcal{LUMI}^{\mathrm{S,A}}_{*}, \quad \mathcal{FSTA}^{\mathrm{F}}_{*} \perp \mathcal{LUMI}^{\mathrm{S,A}}_{*}.$$

Proof. These orthogonality relations follow by combining the previous lemmas with Theorem 7:

- $\mathcal{OBLOT}^{\mathrm{F}}_{*} \perp \mathcal{FCOM}^{\mathrm{S,A}}_{*}$ holds since `Angle-Shift` is solved in $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{O}}$ but not in $\mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{T}}$, and since `Newcomer Introducing` is solved in $\mathcal{FCOM}^{\mathrm{S,A}}_{\mathcal{O}}$ but not in $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{T}}$ (by Lemmas 9 to 11);
- $\mathcal{OBLOT}^{\mathrm{F}}_{*} \perp \mathcal{FSTA}^{\mathrm{S,A}}_{*}$ holds since `Angle-Shift` is solved in $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{O}}$ but not in $\mathcal{FSTA}^{\mathrm{S,A}}_{\mathcal{T}}$, and since `Triangle Round-Trip` is solved in $\mathcal{FSTA}^{\mathrm{S,A}}_{\mathcal{O}}$ but not in $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{T}}$ (by Lemmas 5, 6 and 11);
- $\mathcal{OBLOT}^{\mathrm{F}}_{*} \perp \mathcal{LUMI}^{\mathrm{S,A}}_{*}$ holds since `Angle-Shift` is solved in $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{O}}$ but not in $\mathcal{LUMI}^{\mathrm{S,A}}_{\mathcal{T}}$, and since `Triangle Round-Trip` is solved in $\mathcal{LUMI}^{\mathrm{S,A}}_{\mathcal{O}}$ but not in $\mathcal{OBLOT}^{\mathrm{F}}_{\mathcal{T}}$ (by Lemmas 5, 6 and 11);
- $\mathcal{FSTA}^{\mathrm{F}}_{*} \perp \mathcal{LUMI}^{\mathrm{S,A}}_{*}$ holds since `Angle-Shift` is solved in $\mathcal{FSTA}^{\mathrm{F}}_{\mathcal{O}}$ but not in $\mathcal{LUMI}^{\mathrm{S,A}}_{\mathcal{T}}$, and since `Newcomer Introducing` is solved in $\mathcal{LUMI}^{\mathrm{S,A}}_{\mathcal{O}}$ but not in $\mathcal{FSTA}^{\mathrm{F}}_{\mathcal{T}}$ (by Lemmas 9 to 11). □

4.4. *$\mathcal{OBLOT}$ and asynchrony*

Problem 7 *(`Semi-Expansion`)*. The problem is defined recursively without any stop conditions, for a swarm of two robots. Consider an arbitrary configuration C where such robots a, b are located in distinct positions. Let $u = dist(a, b)$ and let γ be the supporting line of a, b. Robots are required to move along γ *at most once*, and to stop to form a new static configuration C' where $dist(a, b) \in \{2u, \frac{3}{2}u\}$. Recursively, the problem demands the same request starting from C'. See Table 5.

Lemma 12. *`Semi-Expansion`* $\in \left(\mathcal{P}\left(\mathcal{OBLOT}^{\mathrm{S}}_{\mathcal{O}}\right) \cap \mathcal{P}\left(\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}}\right)\right)$.

Proof. `Semi-Expansion` can be trivially solved as follows under $\mathcal{OBLOT}^{\mathrm{S}}_{\mathcal{O}}$: when a robot is activated, it computes γ and the current distance u, and it travels along γ in order to form a new distance $u + \frac{1}{2}u$ with the other robot. If only one robot is activated, the new distance between the robots is $\frac{3}{2}u$. If both robots are activated, the new distance is $2u$.

We can exploit the first part of the lemma to state that Semi-Expansion is solved under $X^{\mathtt{S}}_{\mathcal{O}}$, and thus under $X^{\mathtt{S}}_{\mathcal{T}}$, for any $X \in \mathcal{X}$. By applying Theorem 8 (which mentions the valuable result $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{T}} \equiv \mathcal{LUMI}^{\mathtt{A}}_{\mathcal{T}}$ proved in [12]), we get that Semi-Expansion can be solved also under $\mathcal{LUMI}^{\mathtt{A}}_{\mathcal{T}}$. Being only two robots, the swarm never suffers from obstructed visibility, thus allowing us to conclude that Semi-Expansion $\in \mathcal{P}\left(\mathcal{LUMI}^{\mathtt{A}}_{\mathcal{O}}\right)$. □

Lemma 13. *Semi-Expansion* $\notin \left(\mathcal{P}\left(\mathcal{FSTA}^{\mathtt{A}}_{\mathcal{T}}\right) \cup \mathcal{P}\left(\mathcal{FCOM}^{\mathtt{A}}_{\mathcal{T}}\right)\right)$.

Proof. Semi-Expansion cannot be solved under $\mathcal{FSTA}^{\mathtt{A}}_{\mathcal{T}}$. The impossibility derives from the fact that a robot cannot distinguish between a static configuration and a transient one (i.e. where the other robot is moving), so an activated robot has no means to compute the correct distance it has to travel.

Semi-Expansion cannot be solved under $\mathcal{FCOM}^{\mathtt{A}}_{\mathcal{T}}$. By contradiction, let us assume that there exists an algorithm $\mathbb{A}$ under $\mathcal{FCOM}^{\mathtt{A}}_{\mathcal{T}}$ solving Semi-Expansion. Suppose the local coordinate system of each activated robot is always so that the origin corresponds to the position of the robot, and the other robot is placed at position $(1,0)$. By definition, $\mathbb{A}$ must solve the problem even under a FULLY scheduler: being completely symmetric, both robots synchronously move at each round to form a new configuration where the distance is doubled w.r.t. the previous round, and they always set the same color. In fact, they both compute $\mathbb{A}(\sigma)$ where $\sigma = \langle((0,0),\not{b}),((1,0),c)\rangle$ and c is the color of the two robots at a given round.

Let us now consider a scheduler that works as FULLY until a time t, and let C be the static configuration obtained at time t where the two robots a,b have the same color and are at a distance $u = dist(a,b)$ according to a global coordinate system. Suppose at time t both robots perform the instantaneous Look step where they get the same snapshot, say σ'. Now, let us suppose robot b computes the next position and moves there, whereas a is still executing its Compute step. Let C' be the static configuration obtained after the movement of b, where now $dist(a,b) = \frac{3}{2}u = u'$. Suppose b is activated again while a is still computing. Since a has the same color as at time t, then b takes the same snapshot σ' and deterministically re-executes the same action as before, moving a distance $\frac{1}{2}u' = \frac{3}{4}u$. Let C'' be the static configuration obtained after the second movement of b, where now $dist(a,b) = \frac{3}{2}u + \frac{3}{4}u = \frac{9}{4}u = u''$. At this time, suppose b stays still and a eventually executes the Move step w.r.t. the snapshot at time t. So a travels a distance $\frac{1}{2}u$, obtaining a static configuration C''' where $dist(a,b) = \frac{9}{4}u + \frac{1}{2}u = \frac{11}{4}u = u'''$. By request, u''' must be in $\{2u'', \frac{3}{2}u''\}$. However, since $\frac{11}{4}u \neq 2 \cdot \frac{9}{4}u$ and $\frac{11}{4}u \neq \frac{3}{2} \cdot \frac{9}{4}u$, the request of the problem has not been fulfilled. Contradiction. □

Theorem 16. *For each* $X \in \{\mathcal{OBLOT}, \mathcal{FSTA}, \mathcal{FCOM}\}$*, it holds:*

$$X^{\mathtt{S}}_{\mathcal{T}} > X^{\mathtt{A}}_{\mathcal{T}}, \quad X^{\mathtt{S}}_{\mathcal{T}} > X^{\mathtt{A}}_{\mathcal{O}}$$
$$X^{\mathtt{S}}_{\mathcal{O}} \perp X^{\mathtt{A}}_{\mathcal{T}} \quad X^{\mathtt{S}}_{\mathcal{O}} > X^{\mathtt{A}}_{\mathcal{O}}.$$

Proof. Trivially, we know that $X^{\mathtt{S}}_{\mathcal{T}} \geq X^{\mathtt{A}}_{\mathcal{T}}$ and $X^{\mathtt{S}}_{\mathcal{O}} \geq X^{\mathtt{A}}_{\mathcal{O}}$. By Lemmas 12 and 13, Semi-Expansion can be solved in $X^{\mathtt{S}}_{\mathcal{O}}$ but not in $X^{\mathtt{A}}_{\mathcal{T}}$. By applying Theorem 6, the claimed relations hold. □

Theorem 17.

$$\mathcal{FSTA}^{\mathtt{A}}_{*} \perp \mathcal{OBLOT}^{\mathtt{S}}_{*},$$
$$\mathcal{FCOM}^{\mathtt{A}}_{*} \perp \mathcal{OBLOT}^{\mathtt{S}}_{*}.$$

Proof. By Lemmas 12 and 13, we know that Semi-Expansion is solved under $\mathcal{OBLOT}^{\mathtt{S}}_{\mathcal{O}}$ but under neither $\mathcal{FSTA}^{\mathtt{A}}_{\mathcal{T}}$ nor $\mathcal{FCOM}^{\mathtt{A}}_{\mathcal{T}}$. By Lemmas 5 and 6, we know that Triangle Round-Trip is solved under both $\mathcal{FSTA}^{\mathtt{A}}_{\mathcal{O}}$ and $\mathcal{FCOM}^{\mathtt{A}}_{\mathcal{O}}$ but not under $\mathcal{OBLOT}^{\mathtt{S}}_{\mathcal{T}}$. By applying Theorem 7, the orthogonalities follow. □

4.5. Opaqueness and asynchrony

We now introduce the Pseudo-Polygon problem which shows a peculiar issue occurring in case of obstructed visibility and asynchrony.

Definition 1. *A* safe zone *of a regular polygon* $\mathcal{N}$ *is the locus of all points* x *in the plane such that:*

- *x does not belong to the closed region delimited by $\mathcal{N}$;*
- *x is not aligned with any pair of vertices of $\mathcal{N}$;*
- *x does not lie on the bisector of any edge of $\mathcal{N}$ (equivalently, x is not equally distanced from any two adjacent vertices);*
- *if ℓ is the length of the edge of $\mathcal{N}$, then the distance between x and any vertex of $\mathcal{N}$ is at least ℓ.*

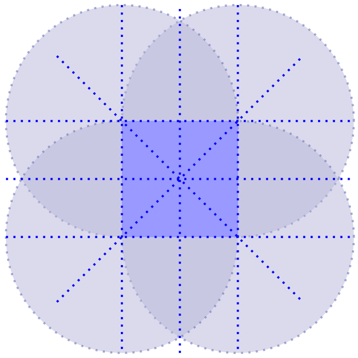


Fig. 3. The safe zone of the square comprehends all the points not belonging to the blue-colored regions and (infinite) lines. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

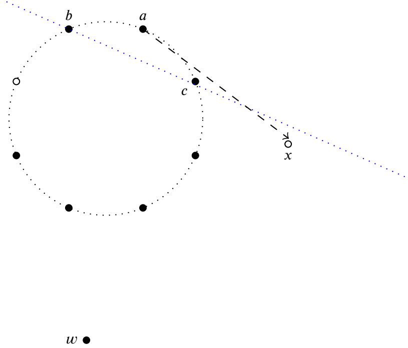


Fig. 4. An instance of the `Pseudo-Polygon` problem built on an octagon.

The blue regions plus infinite lines in Fig. 3 depicts the complement of the safe zone of a square.

Definition 2. *Given a regular n-gon $\mathcal{N}$, for any $n \geq 4$, a* pseudo-polygon *Q is a subset of vertices of $\mathcal{N}$, such that $|Q| \geq \lfloor \frac{n}{2} \rfloor + 1$. We call $\mathcal{N}$ the* polygon associated *with Q.*

Given a pseudo-polygon Q, it is always possible to determine the associated polygon, which is unique. In fact, as Q contains at least three vertices, the circumscribed circle is univocally defined. Moreover, since Q contains more than half of the vertices of the associated n-gon $\mathcal{N}$, there always exist at least two vertices in Q that are adjacent in $\mathcal{N}$, thus enabling the reconstruction of all the edges of $\mathcal{N}$.

Problem 8 *(`Pseudo-polygon`).* Let $\mathcal{N}$ be a regular n-gon with $n \geq 6$ vertices. Let Q be a pseudo-polygon of $m \geq \lfloor \frac{n}{2} \rfloor + 2$ vertices, with which $\mathcal{N}$ is associated. Consider a swarm of $m+1$ robots, where m robots lie on Q and let the last robot, w, lie in the safe zone of $\mathcal{N}$. Let a be the farthest robot from w. Let b be the first robot encountered while moving from a along the perimeter of $\mathcal{N}$ in one direction, and c be the first robot encountered while proceeding in the opposite direction. Assume $dist(b,w) > dist(c,w)$. Note that the two distances are always different since w belongs to the safe zone of $\mathcal{N}$. The problem requires a to move away from b towards a point x such that *(i)* x belongs to the safe zone of $\mathcal{N}$, *(ii)* x belongs to the halfplane delimited by the line $\overline{bc}$ that does not contain a, and *(iii)* x must not lie on any line passing through w and any other robot on Q. Note that the requests *(i)* and *(iii)* are imposed in order to have x visible by every robot in the swarm. See Fig. 4.

Lemma 14. `Pseudo-Polygon` $\notin \mathcal{P}\left(\mathcal{FSTA}_{\mathcal{O}}^{\mathrm{A}}\right)$.

Proof. `Pseudo-Polygon` cannot be solved in the `ASYNCH` opaque model, only using internal lights. Let us consider the problem instance shown in Fig. 4, where the pseudo-polygon of the initial configuration consists $\lfloor \frac{n}{2} \rfloor + 3$ vertices, with $n = 8$. Let us assume b is activated for the first time during the movement of a towards x, when a is hidden by c (i.e. b, c, a are collinear). When b looks at its snapshot, it recognizes a feasible initial configuration (it sees a pseudo-polygon with $\lfloor \frac{n}{2} \rfloor + 2$ robots, and the robot w). According to this configuration, b erroneously elects itself as the robot that has to move away from the pseudo-polygon. It has no means to understand whether or not a exists.[5] On the other hand, a cannot postpone its movement and wait for b to memorize that it is not the elected robot to move. In fact, a has no means of knowing if b has updated its internal light. □

False election The impossibility of solving Pseudo-Polygon with opaque and asynchronous robots with only internal lights (Lemma 14) derives from a critical issue that is peculiar for swarms with obstructed visibility. This critical issue can be described as the *false election* phenomenon. Such phenomenon can be informally explained as follows: from a stable configuration, a given problem requires using a *leader election* routine to single out the unique robot (the *true leader*) which has to execute a non-null movement to reach the next configuration. All the other robots have to stay still. In ASYNCH, a robot r executes its Look step while the true leader is moving and is hidden from r. Thus, if r cannot detect the presence of the true leader from its snapshot, r might elect itself as the (*false*) leader by applying the same leader election routine. This clearly would start a non-requested movement.

We observe that attention must be paid to the false election phenomenon, e.g., when trying to adapt a SEMI algorithm to the ASYNCH framework. In particular, the use of lights must be considered as a possible method to avoid false elections. As we have shown in Lemma 14 for Pseudo-Polygon, internal lights are not sufficient to cope with them. Instead, we are now going to prove that external lights are required (and sufficient) to correctly solve the Pseudo-Polygon problem in ASYNCH.

Lemma 15. *Pseudo-Polygon* $\in \left(\mathcal{P}\left(\mathcal{OBLOT}^{\mathtt{S}}_{\mathcal{O}}\right) \cap \mathcal{P}\left(\mathcal{FCOM}^{\mathtt{A}}_{\mathcal{O}}\right)\right)$.

Proof. Pseudo-Polygon is solvable in $\mathcal{OBLOT}^{\mathtt{S}}_{\mathcal{O}}$ (i.e. in any synchronous model), since complete visibility is guaranteed at any activation time and all the movements (null and non-null) are univocally determined by geometric conditions. In fact, each robot can determine Q, the watcher w, and the robot a (the farthest from w). The robot a can compute its final destination and move there. If a robot is not the farthest from the watcher, or if it sees two robots that are not part of the pseudo-polygon, then it stands still.

Pseudo-Polygon needs at least external lights to be solvable in the ASYNCH mode by opaque robots. We show here an algorithm that uses 4 colors: off (assuming it as default, w.l.o.g.), on, a, b. In the first phase, every robot updates its color according to its role: robot a turns into a, robot b turns into b, whereas the remainder turns into on. Afterward, let r be an activated robot that sees no off robots and that notes there is only one robot (the watcher) out of the pseudo-polygon. Let V_r be the set of colors r can see.

- if $V_r = \{\mathsf{a}, \mathsf{b}, \mathsf{on}\}$, r turns into on and stays still;
- if $V_r = \{\mathsf{a}, \mathsf{on}\}$, r turns into b and stays still;
- if $V_r = \{\mathsf{b}, \mathsf{on}\}$, and if r is the farthest robot from w, it turns into a and starts moving;
- if $V_r = \{\mathsf{on}\}$, it means r is b and stays still (robot a is hidden).

If a robot r sees two robots not belonging to the pseudo-polygon, then r does not move (the final configuration is already formed or is about to be formed). □

4.6. Unknown relations

So far, all the theorems exhibited in this section prove, for each considered pair $A = X^Y$ and $B = Z^W$ with $X, Z \in \mathcal{X}$ and $Y, W \in \mathcal{Y}$, the four relations $A_{\mathcal{T}}$ vs. $B_{\mathcal{T}}$, $A_{\mathcal{T}}$ vs. $B_{\mathcal{O}}$, $A_{\mathcal{O}}$ vs. $B_{\mathcal{T}}$, $A_{\mathcal{O}}$ vs. $B_{\mathcal{O}}$, thus providing the complete hierarchy for the four models obtained by tuning the visibility setting. Notably, each relation $A_{\mathcal{T}}$ vs. $B_{\mathcal{T}}$ here proved is identical to the corresponding relation $A_{\mathcal{O}}$ vs. $B_{\mathcal{O}}$.

However, there exist other pairs of models A, B for which the relation $A_{\mathcal{O}}$ vs. $B_{\mathcal{O}}$ is unknown to date and, in particular, we point out the relations $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{O}}$ vs. $\mathcal{LUMI}^{\mathtt{A}}_{\mathcal{O}}$ and $\mathcal{LUMI}^{\mathtt{F}}_{\mathcal{O}}$ vs. $\mathcal{FCOM}^{\mathtt{F}}_{\mathcal{O}}$. Both these relations, in their transparent version, boil down to equivalences. This has been proved in [30,34] by designing model simulators. However, such simulators cannot be used *as they are* in the related opaque models due to the obstructed visibility. So, the only facts we can state up to now are $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{O}} \geq \mathcal{LUMI}^{\mathtt{A}}_{\mathcal{O}}$ and $\mathcal{LUMI}^{\mathtt{F}}_{\mathcal{O}} \geq \mathcal{FCOM}^{\mathtt{F}}_{\mathcal{O}}$, without discerning between $>$ or $\equiv$.

For other pairs of models, two possible relations can exist: $>$ or $\perp$. For instance, for $\mathcal{FCOM}^{\mathtt{F}}_{\mathcal{O}}$ and $\mathcal{FSTA}^{\mathtt{F}}_{\mathcal{O}}$, we have built the witness problem Newcomer Introducing $\in \left(\mathcal{P}\left(\mathcal{FCOM}^{\mathtt{F}}_{\mathcal{O}}\right) \setminus \mathcal{P}\left(\mathcal{FSTA}^{\mathtt{F}}_{\mathcal{O}}\right)\right)$. To prove the orthogonality relation, we should further design a witness problem to be settled within $\left(\mathcal{P}\left(\mathcal{FSTA}^{\mathtt{F}}_{\mathcal{O}}\right) \setminus \mathcal{P}\left(\mathcal{FCOM}^{\mathtt{F}}_{\mathcal{O}}\right)\right)$. Instead, to obtain a strict dominance, we should prove that any problem in $\mathcal{P}\left(\mathcal{FSTA}^{\mathtt{F}}_{\mathcal{O}}\right)$ can be solved in the model $\mathcal{FCOM}^{\mathtt{F}}_{\mathcal{O}}$ as well. This could be achieved, e.g., by showing how to simulate any $\mathcal{FSTA}^{\mathtt{F}}_{\mathcal{O}}$ algorithm under the model $\mathcal{FCOM}^{\mathtt{F}}_{\mathcal{O}}$.

For the sake of completeness, we here list all the relations A_* vs. B_* for which the relation between $A_{\mathcal{O}}$ vs. $B_{\mathcal{O}}$ is so far unknown.

Theorem 18.

$$\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{T}} \equiv \mathcal{LUMI}^{\mathtt{A}}_{\mathcal{T}}, \quad \mathcal{LUMI}^{\mathtt{S}}_{\mathcal{T}} > \mathcal{LUMI}^{\mathtt{A}}_{\mathcal{O}},$$
$$\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{O}} < \mathcal{LUMI}^{\mathtt{A}}_{\mathcal{T}}, \quad \mathcal{LUMI}^{\mathtt{S}}_{\mathcal{O}} > \textit{ or } \equiv \mathcal{LUMI}^{\mathtt{A}}_{\mathcal{O}}.$$

[5] This is true since a may form a collinearity with b and c for a finite but unpredictable time range.

Proof. The equivalence $\mathcal{LUMI}^{\mathrm{S}}_{\mathcal{T}} \equiv \mathcal{LUMI}^{\mathrm{A}}_{\mathcal{T}}$ (see Theorem 8) has been proved in [30]. The two strict dominances directly follow from this equivalence and Theorem 5. Finally, we know that $\mathcal{LUMI}^{\mathrm{S}}_{\mathcal{O}} \geq \mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}}$: whether it is a strict dominance or an equivalence remains to be proved. □

Theorem 19.

$$\mathcal{LUMI}^{\mathrm{F}}_{\mathcal{T}} \equiv \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}}, \quad \mathcal{LUMI}^{\mathrm{F}}_{\mathcal{T}} > \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}},$$
$$\mathcal{LUMI}^{\mathrm{F}}_{\mathcal{O}} < \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}}, \quad \mathcal{LUMI}^{\mathrm{F}}_{\mathcal{O}} > \textit{ or } \equiv \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}}.$$

Proof. The equivalence $\mathcal{LUMI}^{\mathrm{F}}_{\mathcal{T}} \equiv \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}}$ (see Theorem 9) has been proved in [34]. The other relations follow from the same reasoning as the proof of the previous theorem. □

Theorem 20. *For any $X \in \{\mathcal{OBLOT}, \mathcal{FSTA}, \mathcal{FCOM}\}$, we have*

$$\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{T}} > X^{\mathrm{S}}_{\mathcal{T}}, \quad \mathcal{LUMI}^{\mathrm{A}}_{\mathcal{T}} > X^{\mathrm{S}}_{\mathcal{O}},$$
$$\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}} \perp X^{\mathrm{S}}_{\mathcal{T}}, \quad \mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}} > \textit{ or } \perp X^{\mathrm{S}}_{\mathcal{O}}.$$

Proof. Let us proceed considering $X = \mathcal{OBLOT}$. Using the equivalence $\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{T}} \equiv \mathcal{LUMI}^{\mathrm{S}}_{\mathcal{T}}$ and `Triangle Round-Trip` as separator (see Lemma 6), we can conclude that $\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{T}} > \mathcal{OBLOT}^{\mathrm{S}}_{\mathcal{T}} > \mathcal{OBLOT}^{\mathrm{S}}_{\mathcal{O}}$ (by Theorem 5). The orthogonality $\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}} \perp \mathcal{OBLOT}^{\mathrm{S}}_{\mathcal{T}}$ follows using `Triangle Round-Trip` and `Line-Stretch` as separators (remember that `Line-Stretch` is solved under any transparent model but by no opaque model). Between $\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}}$ and $\mathcal{OBLOT}^{\mathrm{S}}_{\mathcal{O}}$ we only know that `Triangle Round-Trip` is solved under $\mathcal{OBLOT}^{\mathrm{S}}_{\mathcal{O}}$ but not under $\mathcal{LUMI}^{\mathrm{A}}_{\mathcal{O}}$: whether they are related by a strict dominance or an orthogonality remains to be proved.

The proof for $X = \mathcal{FSTA}$ ($X = \mathcal{FCOM}$, resp.) has the same structure, using `Newcomer Introducing` (`Flip-Flop-Flip`, resp.) as separator, instead of `Triangle Round-Trip`. □

Theorem 21. *For any $X \in \{\mathcal{FSTA}, \mathcal{LUMI}\}$, we have*

$$\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} > X^{\mathrm{S,A}}_{\mathcal{T}}, \quad \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} > X^{\mathrm{S,A}}_{\mathcal{O}},$$
$$\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} \perp X^{\mathrm{S,A}}_{\mathcal{T}}, \quad \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} > \textit{ or } \perp X^{\mathrm{S,A}}_{\mathcal{O}}.$$

Proof. We know that $\mathcal{LUMI}^{\mathrm{F}}_{\mathcal{T}} \equiv \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}}$ [34], thus concluding that $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} \geq X^{\mathrm{S,A}}_{\mathcal{T}}$. By Lemma 11, we know that `Angle-Shift` is solved under $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}}$ but not under $X^{\mathrm{S,A}}_{\mathcal{T}}$, thus making self-evident the dominance $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} > X^{\mathrm{S,A}}_{\mathcal{T}}$. The second dominance $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} > X^{\mathrm{S,A}}_{\mathcal{O}}$ follows by Theorem 5. The orthogonality $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} \perp X^{\mathrm{S,A}}_{\mathcal{T}}$ follows using `Angle-Shift` and `Line-Stretch` as separators. Between $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}}$ and $X^{\mathrm{S,A}}_{\mathcal{O}}$ we only know that a problem (`Angle-Shift`) is solved under $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}}$ but not under $X^{\mathrm{S,A}}_{\mathcal{O}}$: whether they are related by a strict dominance or an orthogonality remains to be proved. □

Theorem 22.

$$\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} > \mathcal{FSTA}^{\mathrm{F}}_{\mathcal{T}}, \quad \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{T}} > \mathcal{FSTA}^{\mathrm{F}}_{\mathcal{O}},$$
$$\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} \perp \mathcal{FSTA}^{\mathrm{F}}_{\mathcal{T}}, \quad \mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}} > \textit{ or } \perp \mathcal{FSTA}^{\mathrm{F}}_{\mathcal{O}}.$$

Proof. The proof has the same structure as in Theorem 21, using `Newcomer Introducing`, instead of `Angle-Shift`, as separator between $\mathcal{FCOM}^{\mathrm{F}}_{\mathcal{O}}$ and $\mathcal{FSTA}^{\mathrm{F}}_{\mathcal{T}}$ (Lemmas 9 and 10). □

5. Relation map

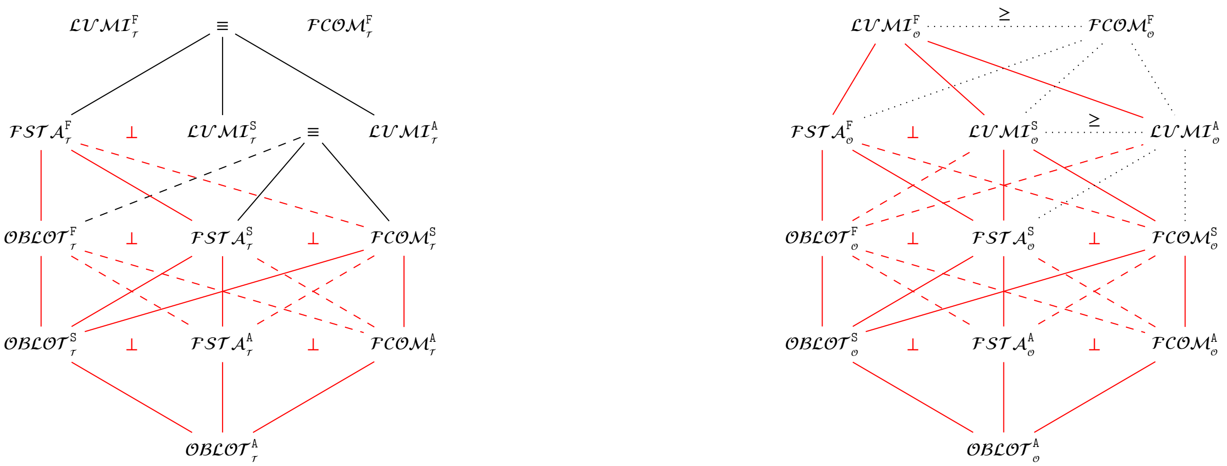
Table 6 summarizes the results proved in this work, showing the relations ($>$, $<$, $\perp$, and $\equiv$) holding between any two models from the set $\mathcal{M}$ of the 24 models here under study. Specifically, the cell related to the pair of models A_* and B_* shows the relation existing between $A_{\mathcal{T}}, B_{\mathcal{T}}$ and $A_{\mathcal{T}}, B_{\mathcal{O}}$ (in the first row) and between $A_{\mathcal{O}}, B_{\mathcal{T}}$ and $A_{\mathcal{O}}, B_{\mathcal{O}}$ (in the second row). To avoid redundancy, we have specified the relation $\perp$ for the blue cells only once, since it holds for all four combinations. The relation map also shows which witness problems have been used to prove dominances and orthogonalities: `TRT` stands for `Triangle Round-Trip`, `FFF` for `Flip-Flop-Flip`, `NWC` for `Newcomer Introducing`, `ASH` for `Angle-Shift`, `SEP` for `Semi-Expansion`.

Fig. 5 shows the *intra-framework* hierarchies of the models in $\mathcal{M}_{\mathcal{T}}$ (Fig. 5a) and $\mathcal{M}_{\mathcal{O}}$ (Fig. 5b), depicting the relations among them. The unknown relations shown in Section 4.6 are highlighted using the gray cells in Table 6 and the dotted lines in Fig. 5b.

Table 6

Relation map. Each cell shows the relation holding between the $\mathcal{T}-\mathcal{T}$ and $\mathcal{T}-\mathcal{O}$ models (first row), and $\mathcal{O}-\mathcal{T}$ and $\mathcal{O}-\mathcal{O}$ models (second row). The acronyms refer to problems witnessing dominances and orthogonalities: `TRT` for `Triangle Round-Trip`, `FFF` for `Flip-Flop-Flip`, `NWC` for `Newcomer Introducing`, `ASH` for `Angle-Shift`, `SEP` for `Semi-Expansion`.

↱	$\mathcal{LUMI}^{F}_{*}$	$\mathcal{FCOM}^{F}_{*}$	$\mathcal{FSTA}^{F}_{*}$	$\mathcal{OBLOT}^{F}_{*}$	$\mathcal{LUMI}^{S}_{*}$	$\mathcal{FCOM}^{S}_{*}$	$\mathcal{FSTA}^{S}_{*}$	$\mathcal{OBLOT}^{S}_{*}$	$\mathcal{LUMI}^{A}_{*}$	$\mathcal{FCOM}^{A}_{*}$	$\mathcal{FSTA}^{A}_{*}$	$\mathcal{OBLOT}^{A}_{*}$
$\mathcal{OBLOT}^{A}_{*}$	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< ASH	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< SEP	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< TRT	≡,> <,≡
$\mathcal{FSTA}^{A}_{*}$	<,⊥ <,< NWC	<,⊥ <,< or ⊥ ASH	<,⊥ <,< ASH	⊥ TRT, ASH	<,⊥ <,< NWC	⊥ NWC, FFF	<,⊥ <,< SEP	⊥ TRT, SEP	<,⊥ <,< NWC	⊥ NWC, FFF	≡,> <,≡	
$\mathcal{FCOM}^{A}_{*}$	<,⊥ <,< FFF	<,⊥ <,< FFF	⊥ NWC, FFF	⊥ NWC, ASH	<,⊥ <,< FFF	<,⊥ <,< SEP	⊥ NWC, FFF	⊥ TRT, SEP	<,⊥ <,< FFF	≡,> <,≡		
$\mathcal{LUMI}^{A}_{*}$	<,⊥ <,< ASH	<,⊥ <,< or ⊥ ASH	⊥ NWC, ASH	⊥ TRT, ASH	≡,> <,≡ or <	>,> ⊥,> or ⊥ FFF,	>,> ⊥,> or ⊥ NWC,	>,> ⊥,> or ⊥ TRT,	≡,> <,≡			
$\mathcal{OBLOT}^{S}_{*}$	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< ASH	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< TRT	≡,> <,≡				
$\mathcal{FSTA}^{S}_{*}$	<,⊥ <,< NWC	<,⊥ <,< or ⊥ ASH	<,⊥ <,< ASH	⊥ TRT, ASH	<,⊥ <,< NWC	⊥ NWC, FFF	≡,> <,≡					
$\mathcal{FCOM}^{S}_{*}$	<,⊥ <,< FFF	<,⊥ <,< FFF	⊥ NWC, FFF	⊥ NWC, ASH	<,⊥ <,< FFF	≡,> <,≡						
$\mathcal{LUMI}^{S}_{*}$	<,⊥ <,< ASH	<,⊥ <,< or ⊥ ASH	⊥ NWC, ASH	⊥ TRT, ASH	≡,> <,≡							
$\mathcal{OBLOT}^{F}_{*}$	<,⊥ <,< TRT	<,⊥ <,< TRT	<,⊥ <,< TRT	≡,> <,≡								
$\mathcal{FSTA}^{F}_{*}$	<,⊥ <,< NWC	<,⊥ <,< or ⊥ NWC	≡,> <,≡									
$\mathcal{FCOM}^{F}_{*}$	≡,> <,≡ or <	≡,> <,≡										
$\mathcal{LUMI}^{F}_{*}$	≡,> <,≡											



(a) Hierarchy of the transparent models.

(b) Hierarchy of the opaque models.

Fig. 5. Relation maps for the 12 models, in the transparent framework (Fig. 5a) and in the opaque framework (Fig. 5b). *Straight* (*dashed*, resp.) lines represent dominances (orthogonalities, resp.). *Dotted* lines join models whose relation is still unknown. The *red* relations hold both in case of transparency and opaqueness. The *black* relations are currently proved for the transparent models only.

6. Conclusions

We have investigated the computational power of 24 models of *collision-intolerant* robots, obtained by tuning four features: communication and storage ($\mathcal{OBLOT}$, $\mathcal{FSTA}$, $\mathcal{FCOM}$, and $\mathcal{LUMI}$), synchronization (FULLY, SEMI, and ASYNCH), and visibility (*transparent* and *opaque*). We have been inspired by [29,30,33,34], where the authors provide the complete map of the relations among the 12 *transparent* models, but considering *collision-tolerant* robots. In this work, we have extended their investigation by considering also opaque robots and showing the relations between the transparent and the opaque framework. We have introduced six witness problems to prove the dominance or orthogonality relations between models: such problems work as separators both in the transparent and opaque models, showing that the relations existing in the opaque models still hold for the same transparent models. Thus far, the relations proved here in our opaque framework are the same as in the corresponding transparent framework. So, the natural question arises, of whether the relation map for the opaque turns out to be identical to that for the transparent models.

To answer this question, future research should address the missing relations among the 12 opaque models, in order to obtain the complete hierarchy in the opaque framework. Among others, the yet unknown relation between $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{O}}$ and $\mathcal{LUMI}^{\mathtt{A}}_{\mathcal{O}}$ is worth mentioning. In the transparent framework, these two models were proved in [30] to be computationally equivalent by designing a simulator which, thanks to extra light colors, turns any SEMI algorithm into an equivalent ASYNCH algorithm. This simulator cannot be directly used to prove the same relation for opaque robots, mainly because of the obstructed visibility. With the Pseudo-Polygon problem, we have presented the *false election* phenomenon whose formalization and investigation will be preparatory to answer this interesting open question: is it possible to simulate a $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{O}}$ algorithm in the ASYNCH mode, thus proving that $\mathcal{LUMI}^{\mathtt{S}}_{\mathcal{O}}$ and $\mathcal{LUMI}^{\mathtt{A}}_{\mathcal{O}}$ are two equivalent models also in the opaque framework? Are constant-size lights sufficient to always avoid the phenomenon of false elections? In addition, it would be crucial to formalize and study all the critical issues caused by obstructed visibility: such formalizations may be essential for the correct investigation of the missing relations.

In conclusion, to obtain an even more complete analysis of the computational power of models under different visibility settings, it would be well worth extending this line of research to *myopic* models [9]. Specifically, future work may focus on how the combination of transparency/opaqueness with unlimited visibility/short-sightedness can affect the computational power of robots.

CRediT authorship contribution statement

Caterina Feletti: Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization. **Lucia Mambretti:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization. **Carlo Mereghetti:** Writing – review & editing, Methodology, Formal analysis. **Beatrice Palano:** Writing – review & editing, Methodology, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] C. Feletti, L. Mambretti, C. Mereghetti, B. Palano, Computational power of opaque robots, in: 3rd Symposium on Algorithmic Foundations of Dynamic Networks, SAND, in: LIPIcs, vol. 292, 2024, pp. 13:1–13:19.
[2] P. Flocchini, G. Prencipe, N. Santoro, Distributed Computing by Oblivious Mobile Robots, Synthesis Lectures on Distributed Computing Theory, Morgan & Claypool Publishers, 2012.
[3] P. Flocchini, G. Prencipe, N. Santoro (Eds.), Distributed Computing by Mobile Entities, Current Research in Moving and Computing, LNCS, vol. 11340, Springer, 2019.
[4] K. Bose, M.K. Kundu, R. Adhikary, B. Sau, Arbitrary pattern formation by asynchronous opaque robots with lights, Theor. Comput. Sci. 849 (2021) 138–158, https://doi.org/10.1016/J.TCS.2020.10.015.
[5] C. Feletti, C. Mereghetti, B. Palano, $O(\log n)$-time uniform circle formation for asynchronous opaque luminous robots, in: 27th International Conference on Principles of Distributed Systems, OPODIS, in: LIPIcs, vol. 286, 2023, pp. 5:1–5:21.
[6] D. Pattanayak, G. Sharma, Time-color tradeoff on uniform circle formation by asynchronous robots, in: 38th IEEE International Parallel and Distributed Processing Symposium, IPDPS, IEEE, 2024, pp. 987–997.
[7] C. Feletti, D. Pattanayak, G. Sharma, Brief announcement: optimal uniform circle formation by asynchronous luminous robots, in: 38th International Symposium on Distributed Computing, DISC, in: LIPIcs, vol. 319, 2024, pp. 46:1–46:7.
[8] Q. Bramas, H. Kakugawa, S. Kamei, A. Lamani, F. Ooshita, M. Shibata, S. Tixeuil, Stand-up indulgent gathering on lines for myopic luminous robots, in: 38th International Conference on Advanced Information Networking and Applications, AINA, in: LNDECT, vol. 200, Springer, 2024, pp. 110–121.
[9] A. Das, S. Ghosh, A. Sharma, P. Goswami, B. Sau, The computational landscape of autonomous mobile robots: the visibility perspective, in: 20th International Conference on Distributed Computing and Intelligent Technology, ICDCIT 2024, in: LNCS, vol. 14501, Springer, 2024, pp. 85–100.
[10] S. Kamei, S. Tixeuil, An asynchronous maximum independent set algorithm by myopic luminous robots on grids, Comput. J. 67 (1) (2024) 57–77, https://doi.org/10.1093/COMJNL/BXAC158.
[11] S. Nagahama, F. Ooshita, M. Inoue, Ring exploration of myopic luminous robots with visibility more than one, Inf. Comput. 292 (2023) 105036, https://doi.org/10.1016/J.IC.2023.105036.
[12] P. Flocchini, G. Prencipe, N. Santoro, G. Viglietta, Distributed computing by mobile robots: uniform circle formation, Distrib. Comput. 30 (6) (2017) 413–457, https://doi.org/10.1007/S00446-016-0291-X.
[13] P. Kling, F.M. auf der Heide, Continuous protocols for swarm robotics, in: Distributed Computing by Mobile Entities, Current Research in Moving and Computing, in: LNCS, vol. 11340, Springer, 2019, pp. 317–334.

[14] S. Das, R. Focardi, F.L. Luccio, E. Markou, M. Squarcina, Gathering of robots in a ring with mobile faults, Theor. Comput. Sci. 764 (2019) 42–60, https://doi.org/10.1016/J.TCS.2018.05.002.

[15] M. D'Emidio, D. Frigioni, A. Navarra, Synchronous robots vs asynchronous lights-enhanced robots on graphs, in: 16th Italian Conference on Theoretical Computer Science, ICTCS, Elsevier, 2015, pp. 169–180.

[16] S. Dobrev, P. Flocchini, G. Prencipe, N. Santoro, Multiple agents rendezvous in a ring in spite of a black hole, in: 7th International Conference on Principles of Distributed Systems, OPODIS, in: LNCS, vol. 3144, Springer, 2003, pp. 34–46.

[17] G.A.D. Luna, Mobile agents on dynamic graphs, in: Distributed Computing by Mobile Entities, Current Research in Moving and Computing, in: LNCS, vol. 11340, Springer, 2019, pp. 549–584.

[18] X. Défago, M. Potop-Butucaru, S. Tixeuil, Fault-tolerant mobile robots, in: Distributed Computing by Mobile Entities, Current Research in Moving and Computing, in: LNCS, vol. 11340, Springer, 2019, pp. 234–251.

[19] G. Prencipe, Pattern formation, in: Distributed Computing by Mobile Entities, Current Research in Moving and Computing, in: LNCS, vol. 11340, Springer, 2019, pp. 37–62.

[20] K. Sugihara, I. Suzuki, Distributed algorithms for formation of geometric patterns with many mobile robots, J. Field Robot. 13 (3) (1996) 127–139, https://doi.org/10.1002/(SICI)1097-4563(199603)13:3 < 127::AID-ROB1 > 3.0.CO;2-U.

[21] I. Suzuki, M. Yamashita, Distributed anonymous mobile robots: formation of geometric patterns, SIAM J. Comput. 28 (4) (1999) 1347–1363, https://doi.org/10.1137/S009753979628292X.

[22] M. Yamashita, I. Suzuki, Characterizing geometric patterns formable by oblivious anonymous mobile robots, Theor. Comput. Sci. 411 (26–28) (2010) 2433–2453, https://doi.org/10.1016/J.TCS.2010.01.037.

[23] G. D'Angelo, G.D. Stefano, R. Klasing, A. Navarra, Gathering of robots on anonymous grids and trees without multiplicity detection, Theor. Comput. Sci. 610 (2016) 158–168, https://doi.org/10.1016/J.TCS.2014.06.045.

[24] P. Flocchini, Gathering, in: Distributed Computing by Mobile Entities, Current Research in Moving and Computing, in: LNCS, vol. 11340, Springer, 2019, pp. 63–82.

[25] S. Kamei, A. Lamani, F. Ooshita, S. Tixeuil, K. Wada, Gathering on rings for myopic asynchronous robots with lights, in: 23rd International Conference on Principles of Distributed Systems, OPODIS, in: LIPIcs, vol. 153, 2019, pp. 27:1–27:17.

[26] T. Izumi, D. Kaino, M.G. Potop-Butucaru, S. Tixeuil, On time complexity for connectivity-preserving scattering of mobile robots, Theor. Comput. Sci. 738 (2018) 42–52, https://doi.org/10.1016/J.TCS.2018.04.047.

[27] M. Mondal, S.G. Chaudhuri, Uniform scattering of robots on alternate nodes of a grid, in: 23rd International Conference on Distributed Computing and Networking, ACM, 2022, pp. 254–259.

[28] D. Canepa, M.G. Potop-Butucaru, Stabilizing flocking via leader election in robot networks, in: 9th International Symposium on Stabilization, Safety, and Security of Distributed Systems, SSS, in: LNCS, vol. 4838, Springer, 2007, pp. 52–66.

[29] K. Buchin, P. Flocchini, I. Kostitsyna, T. Peters, N. Santoro, K. Wada, Autonomous mobile robots: refining the computational landscape, in: 35th International Parallel and Distributed Processing Symposium Workshops, IPDPS, IEEE, 2021, pp. 576–585.

[30] S. Das, P. Flocchini, G. Prencipe, N. Santoro, M. Yamashita, Autonomous mobile robots with lights, Theor. Comput. Sci. 609 (2016) 171–184, https://doi.org/10.1016/J.TCS.2015.09.018.

[31] M. D'Emidio, D. Frigioni, A. Navarra, Characterizing the computational power of anonymous mobile robots, in: 36th International Conference on Distributed Computing Systems, ICDCS, IEEE Computer Society, 2016, pp. 293–302.

[32] M. D'Emidio, G.D. Stefano, D. Frigioni, A. Navarra, Characterizing the computational power of mobile robots on graphs and implications for the Euclidean plane, Inf. Comput. 263 (2018) 57–74, https://doi.org/10.1016/J.IC.2018.09.010.

[33] P. Flocchini, N. Santoro, Y. Sudo, K. Wada, On asynchrony, memory, and communication: separations and landscapes, in: 27th International Conference on Principles of Distributed Systems, OPODIS, in: LIPIcs, vol. 286, 2023, pp. 28:1–28:23.

[34] P. Flocchini, N. Santoro, K. Wada, On memory, communication, and synchronous schedulers when moving and computing, in: 23rd International Conference on Principles of Distributed Systems, OPODIS, in: LIPIcs, vol. 153, 2019, pp. 25:1–25:17.

[35] K. Buchin, P. Flocchini, I. Kostitsyna, T. Peters, N. Santoro, K. Wada, On the computational power of energy-constrained mobile robots: algorithms and cross-model analysis, in: 29th International Colloquium on Structural Information and Communication Complexity, SIROCCO, in: LNCS, vol. 13298, Springer, 2022, pp. 42–61.

[36] G. Sharma, R. Vaidyanathan, J.L. Trahan, Constant-time complete visibility for robots with lights: the asynchronous case, Algorithms 14 (2) (2021) 56, https://doi.org/10.3390/A14020056.

[37] G. Sharma, R. Vaidyanathan, J.L. Trahan, C. Busch, S. Rai, Complete visibility for robots with lights in O(1) time, in: 18th International Symposium on Stabilization, Safety, and Security of Distributed Systems, SSS, in: LNCS, vol. 10083, 2016, pp. 327–345.