

Contents lists available at ScienceDirect

Online Social Networks and Media



journal homepage: www.elsevier.com/locate/osnem

Characterizing growth in decentralized socio-economic networks through triadic closure-related network motifs

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ARTICLE INFO

Keywords: Blockchain Temporal network Triadic closure Network evolution Web3 Socio-economic networks

ABSTRACT

The emergence of the Web3 paradigm has led to more and more systems built on blockchain technology and relying on cryptocurrency tokens - both fungible and non-fungible - to sustain themselves and generate profit. The growth and success of these platforms are strongly dependent on the growth and evolution of the trade relationships among users. In this context, it is of paramount importance to understand the mechanism behind the evolution and growth dynamics of these economic ties: however, in these systems the trade relationships are strictly intertwined with social dynamics, posing significant challenges in the analysis. One of the most important mechanisms behind the evolution of social networks is the triadic closure principle: given the strict link between social and economic spheres, the mechanism emerges as a potential candidate among mechanisms in literature. Therefore in this work, we extend the existing methodology for triadic closure studies and adapt it to directed networks. We performed an analysis centered around 3-node subgraphs known as "triads" and statistically significant triads referred to as "triadic motifs", both from a static and temporal perspective. The methodology was applied to various decentralized socio-economic networks with distinct levels of social components. These networks include currency transfers from the blockchain-based online social media platform Steemit, trade relationships among NFT sellers and buyers on the Ethereum blockchain, and a blockchain based currency designed for humanitarian aid called Sarafu. Our measurements show how triadic closure is relevant during the evolution of these platforms and, for a few aspects, more impactful than centralized online social networks, where triadic closure is also incentivized by recommendation systems. Moreover, we are able to highlight both similarities and differences across networks with different levels of social components, both from a static and temporal standpoint. Overall our work presents strong evidence that triadic closure is an important evolutionary mechanism in decentralized socio-economic networks. Our findings provide a stepping stone in the study of decentralized socio-economic networks. Understanding the evolution of other decentralized networks, not following the same Web3 paradigm or with different social components will provide valuable insight into the understanding of dynamics in decentralized systems and potentially improve their design process.

1. Introduction

In the last years, the actual structure of the Web has been questioned by novel paradigms which are trying to reduce the over-centralization around a few big platforms and tech companies. The need for decentralization of online platforms has led to the development of decentralized counterparts of more established platforms [1]. In this scenario, one of the paradigms gaining momentum is Web3, i.e. the design of platforms and software systems built on blockchain technologies to promote a decentralized Web [2]. Blockchain technology has many features that are suitable for such a decentralization process, and one of the most important features is the option to create tokens. The most notable examples are *fungible tokens*, i.e. expendable tokens identical to each other, such as the popular cryptocurrencies like Bitcoin [3] and ETH ERC-20 tokens on Ethereum blockchain [4]. In the field of online social media, tokens are used to reward user participation and as payment for nodes that contribute to data validation [5]; they have also gained traction in many other sectors, including initiatives for social development and humanitarian aid [6]. Another type of token that has gained popularity is the *Non-fungible token (NFT)*, a token that acts as a certificate of ownership of digital objects, such as photographs, movies, and audio [7]. The exchange of these tokens – both fungible and non-fungible – has a key role in blockchain-based systems. In fact, the widespread circulation of tokens leads to the formation of trade relationships among users, which can be seen as a complex

https://doi.org/10.1016/j.osnem.2023.100266

Received 19 February 2023; Received in revised form 4 June 2023; Accepted 26 July 2023 Available online 14 August 2023

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network structure: in these networks, nodes are users/wallets and links represent the beginning of an exchange relationship. The key aspect is that in many Web3 systems relying on these tokens, exchanges are often strongly intertwined with the more social side of the platforms, making blockchain-based platforms very complex and interesting socioeconomic systems. However, there are only a few studies on them from a socio-economic network perspective, and their structure and growth dynamics have been partially studied only. In particular, there is no study focused on triadic closure, one of the main mechanisms driving social network evolution [8]. Such an evolution mechanism is present in online and offline social networks, and indeed it could be a driving factor in Web3 systems as well, where the social structure is strictly tied to the economical structure.

In this work, we analyze triadic closure in decentralized socioeconomic networks supported by blockchain technology. We extend the current methodology for triadic closure studies and adapt it to the analysis of decentralized networks. Moreover, we conduct an indepth analysis of network structure centered on *triads*, i.e. 3-node subgraphs, and *triadic motifs*, i.e. statistically significant triads, both from a static and temporal standpoint. We conduct our analysis on different decentralized socio-economic networks characterized by different levels of social components: the leading blockchain online social media Steemit [9], NFT trades on Ethereum [7], and a blockchain-based currency for humanitarian aid — Sarafu [10].

Our insights on triadic closure and triads from a static viewpoint have highlighted evident differences among decentralized socioeconomic networks mainly due to their main scopes and functionalities. Differences are so remarkable that the distribution of the closed triads may represent a footprint of the network since each socio-economic network has its specific distribution. In defining the footprint an important role is played by the "feed-forward" loop and by fully or almost fully reciprocated triangles. In fact, socio-economic networks where the social and economic traits are more intertwined are characterized by more reciprocal relationships and triads, while feed-forward loops are dominant where the interplay is weaker. The centrality of "feedforward" loops and reciprocity has been further confirmed by the analysis of the patterns forming closed triads. In fact, all the closing temporal triads forming a feed-forward loop are the most frequent in all the networks. Despite the importance of patterns related to the "feed-forward" loop, the distribution of the closing temporal triads is a further footprint of a network: NFT network is mainly built around patterns leading to "feed-forward" loops, while distributions of the closing temporal triads in Steemit and Sarafu are more uniformly spread over all the possible patterns, with temporal triads leading to the creation of fully reciprocal triangles frequent and significant. To sum up, both in a static and dynamic setting, each network has its own specific profile which depends on the nature of the socioeconomic actions it supports. Finally, we found that triadic closure has impacted the evolution and the growth of these platforms even more than in traditional and centralized online social platforms. The closure process is not stable, rather each network is characterized by its own dynamics, sometimes influenced by external conditions. However, there is a characteristic common to all these networks: the closure process is very fast, faster than in the centralized counterparts. So, even though in decentralized socio-economic networks social and economic relationships and interests mix up, the triadic closure, one of the main mechanisms behind the formation of social ties, emerges as an important factor contributing to the growth in trade relationships; even much faster than in centralized online social networks.

The paper is organized as follows. Section 2 provides a brief introduction to decentralization and network evolution mechanisms. In Section 3 we introduce the main research questions, we have on the evolution of these networks. The approach for modeling and analyzing the socio-economic networks is presented in Section 4. In Section 5 we describe the selected datasets and their preprocessing and details on the experimental setting. Section 6 reports the main findings regarding the impact of triadic closure. Finally, Section 7 concludes the paper, pointing out possible future works.

2. Background

2.1. Decentralized socio-economic networks

As the necessity of taking power and control away from the major centralized web platforms has become more evident, we observed the development of alternative platforms embracing decentralized and open principles [11]. This is especially evident in the field of online social media, where we have witnessed the development of decentralized online social networks [12]. Some of them are currently composing the Fediverse, led by Mastodon [13], and are based on public protocols supporting decentralization through federation. On the other side, decentralization can be reached through the Web3 paradigm [2], which is actually gaining momentum. This paradigm promotes decentralization through blockchain technology since it offers many design options, such as decentralized storage, consensus-based validation of stored data, and even the option to implement economic systems. Since blockchain technology provides systems to implement and manage multiple types of tokens with different purposes, Web3 platforms have at their disposal flexible financial instruments to support their growth and maintenance.

In fact, tokens have found important use in the field of online social media, where we have witnessed the rise of blockchain online social media (BOSM) [5], platforms which offer (i) a set of "social actions" - following, commenting, and voting - which facilitate online interactions among accounts; and (ii) whose core functions are rooted in an underlying blockchain that guarantees the persistence and validity of operations. One of the most interesting consequences of this architecture is the strong connection between economical aspects and online social behaviors. In fact, most of the current blockchain social media implement: (i) a token ecosystem based on blockchain technology for promoting high-quality content and users and validating social and economic operations; and (ii) a rewarding system for distributing the wealth of the platform. Within the landscape of Web3 social platforms, there are and have been many proposals, but most of the research studies have been focused on Steemit, Hive, and Mind. In this type of social media platform, since users rely on the very same blockchain for social interactions and financial operations, exchanges are often strongly influenced by social dynamics and vice-versa.

Another interesting example of token systems supporting social systems can be found in the field of blockchain for good, where many initiatives have started relying on blockchain-based currency systems to promote social development and local economies [6]. For example, tokens can be used to provide humanitarian aid, such as in Sarafu blockchain-based digital currency [10]. In these systems, financial and economic operations are the main instruments to promote social cooperation and support social groups or individuals in need; there is a clearly intertwined nature of social and economic relationships.

We also observe socio-economic relationships mediated by Nonfungible tokens (NFTs) i.e. tokens that provide a certificate of ownership of a digital object [7]. An NFT is linked to a given digital asset to attest to its uniqueness and non-transferability: in practice, an NFT can represent a variety of digital items, including photographs, movies, and audio. As a consequence, several fields, such as art, gaming, and sports collectibles, utilize NFTs to regulate and control digital objects. Most NFTs follow the same standard ERC-721 [14], for the Ethereum blockchain [4]. However, users do not need to interact directly with the blockchain, as several web platforms, known as *NFT marketplaces*, act as intermediaries between users and blockchains facilitating the exploration of existing NFTs, their sales, and ownership transfers. Even in NFT trade relationships, where there is not an explicit social media platform, studies show that hype and community talk on other platforms influence prices and exchanges among other users [15].

In short, in all these decentralized systems, albeit at different levels, there is also an important social component, besides the economic aspects. In this context, network-based modeling has emerged as an effective methodology to analyze the growth and dynamics of trade



Fig. 1. The 13 possible triads in a directed network (if isomorphous subgraphs are counted only once). They can be divided into 2 categories: open triads (a) and closed triads (b).

relationships in these complex economic systems. Indeed, the exchange of tokens in Web3 systems is often modeled through networks where users/wallets are treated as nodes and links represent money transfers between them.

2.2. Network evolution through triadic closure

Many models, mechanisms, and measures describing network growth from a link formation perspective have been proposed. Among them, triadic closure has emerged as one of the most important mechanisms [16]. The main assumption of triadic closure is that individuals with a common friend have a higher chance to become friends themselves at some point in the future [8]. Although the triadic closure has been recognized as one of the fundamental mechanisms driving the formation of dense groups and communities [17] in social networks, their properties and laws are still scarcely studied at a large scale, due to the limited availability of temporal-annotated datasets capturing the growth of large social networks.

From a static standpoint, triadic closure influences graph structure on the level of *triads*, i.e. 3-node directed subgraph. Specifically, in a directed network, we have 13 possible triads (if isomorphous subgraphs are counted only once) that can be divided into the 2 categories of closed and open triads: there are 6 possible open triads (see Fig. 1(a)) and 7 closed triads (see Fig. 1(b)). Indeed, the structure of a network can be characterized by the distribution of these triads: for example Milo et al. [18] rely on triads and other subgraphs to characterize networks in different domains, showing that similar networks have similar characteristic subgraphs. For example, focusing on triads in the field of online social networks, Huang et al. [19] confirmed some similarities among centralized online social networks such as Twitter and Weibo.

While frequency is an important indicator of the importance of a triad, it could be frequent simply because of the size of the network. Therefore many studies focus on the analysis of *motifs* i.e. classes of isomorphic induced subgraphs whose frequency is higher in the data than in a reference null model [18]. There are many ways to test whether a subgraph is a motif [20], however the most common in literature are the significance tests based on the z-*score* and the p-*value* [18]. The

idea is that the count of each subgraph in the original network should be compared with the same counting in a randomized version of the original network (the reference model or null model): a subgraph could be (*i*) over-represented i.e. its frequency is higher in the original dataset than in the reference model, (*ii*) under-represented i.e. its frequency is significantly lower in the original network than in the null model, or (*iii*) similarly represented, which corresponds to a not significative subgraph. In the literature, the most common approach is to consider a subgraph *g* as significant when |z(g)| > 2.0, i.e. the absolute value of its z-*score* is greater than 2 [20]. So, the combination of triad frequencies and motifs could be used to characterize decentralized socio-economic networks highlighting common traits and differences in their network structure and in their evolution.

In fact, while static structure already provides some insights into the effects of the triadic closure process, leveraging temporal information is essential to obtain a more complete analysis and characterization of the network evolution. Zignani et al. [21] proposed some temporal metrics to quantify triadic closure in undirected networks. The first one is the *triangles/link ratio*, i.e. the fraction of triangles produced over the links. Monitoring the ratio at regular intervals, like daily observations, provides an overview of how much the links tend to form closed triads. A further important measure is the *triadic closure delay*, a measure quantifying the "eagerness" of users in building social structures. The value of delay provides insight into the speed at which users act in building and extending their social neighborhoods by closing triangles. Both measures are able to capture and quantify the presence and dynamics of the triadic closure mechanism in a network, and they can also be used to compare different networks.

Furthermore, temporal information favors the study of the evolution of network structure from a temporal standpoint. In this case, networks are modeled as temporal networks, a representation that combines both topology and time. From the triadic closure viewpoint, we can therefore focus on temporal triads i.e. 3-node temporal subgraphs. A subset of temporal triads that represent triadic closure are displayed in Fig. 2. The identification of such temporal subgraphs is less straightforward than in the static case since the introduction of the temporal dimension has led to different definitions of temporal subgraphs and motifs. One of the most important works on the subject is by Kovanen et al. [22]. In their work, they consider a subset of temporal subgraphs in which (i) the time difference of consecutive events is less than an input interval Δc , and (ii) the events in the subgraphs are all consecutive. A further definition is in Paranjape et al. [23], where they use as a starting point the previous definition [22] but they remove the constraint on consecutive events, as it allows to study more subgraphs that tend to occur in short bursts. They also use a time window Δw to bound the time difference between the last and the first events in a subgraph. There are a few other models in the literature [24], but the key aspect is that the distribution of temporal subgraphs can be used for comparison and characterization of networks [25], similarly to the static scenario. And similarly to the static setting, we can also detect temporal motifs [22], i.e. temporal subgraphs that result as statistically significant compared to a null model. However, among the works studying temporal subgraphs and temporal motifs, the term motif may be found even for not statically significant subgraphs. Indeed, not all the works actually perform a statistical significance test, both for computational reasons (exact temporal subgraph counting is expensive, and performing it multiple times may not be computationally feasible) or because of the difficulty of selecting a meaningful null model. In fact, as noted in different works [22,24], the selection of a null model for temporal networks is not trivial. In general, there are many possible reference models for temporal networks, and each model randomizes certain parts of the network, with the goal of preserving some features of the original one. Among the many classes of models presented in the survey by Gauvin et al. [26], the most frequently used model in many fields are the "topology-constrained link shuffling" methods, also known as edge randomization or link shuffling. Indeed, it preserves most of the characteristics of the original temporal network: it preserves the original graph structure while eliminating all causal correlations between events taking place on adjacent links.



Fig. 2. Closing temporal triads capturing the triadic closure. Blue links are established before time *t*, green links are established at timestamps t' > t to form the closed triad. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3. Research questions

There are few studies that deal with decentralized socio-economic systems from a network and evolutionary dynamics perspective. And currently, there are no works exploring the mechanism of triadic closure and the presence of triadic network motifs in decentralized networks. Specifically, here our hypothesis is that the intertwined nature of social and economical relationships in blockchain-based social networks should lead to an evolution of the economic relationship networks with traits similar to social networks. On the other side, we also investigate the specificity of each economic network asking whether different socio-economic networks are characterized by different network characteristics or patterns, from a microscopically and triadic closure-related perspective. In particular, in this work, we would answer the following research questions:

Research question RQ1: When dealing with the triadic closure process, triads and their census are the fundamental building blocks for describing the actual state of a network (closed triads) and for identifying where closures may occur (open triads). From this perspective, and in a static setting, we ask whether decentralized socio-economic networks are similar in terms of triadic-based structures, or whether each network is characterized by specific triadic-based patterns depending on its nature.

Research question RQ2: From a temporal standpoint, are different socio-economic networks characterized by specific evolution patterns of the triads or do they follow a common growth mechanism?

Research question RQ3: From a dynamic viewpoint, do the different types of triads resulting from a triadic closure process form at the same speed? Is the dynamic of triad formation stable along the evolution of these networks?

4. Methods

4.1. Modeling

In general, transactions can be modeled as a set of tuples $I = \{(u, v, a, t)\}$ where *u* and *v* are users that "moved" tokens: user *u* transferred to user *v* an amount *a* of tokens at time *t*. Our focus is on the relationships between users determined by token transfers, by modeling them as a network: transactions over a time interval $[t_0, t_1]$ can be modeled as a temporal network [27]. More precisely, the transaction data over time can be represented as a temporal network $G_{[t_0,t_1]} = (V, E)$, where:

- V is the set of users,¹
- *E* is a set of timestamped directed links $(u, v, t) \in E$ where $u, v \in V, t \in [t_0, t_1]$; in other words, links represent a transfer/trade relationship: two users are linked if they performed at least a transfer/trade in the time interval $[t_0, t_1]$, and $t \in [t_0, t_1]$ is the timestamp of the first transaction between *u* to *v*.

It is worth noting that the direction of links captures the flow of money from a source to a destination – in the case of transfer – or from a buyer to a seller in the case of NFT trading. As for NFT trade, it is a complementary modeling approach w.r.t. the seminal work on NFT trade networks by Nadini et al., where links are directed from the seller to the buyer. In this work, we do not consider the amounts *a* of each transfer/trade but the model could be extended to include them as edge attributes. The evaluation of network statistics can give us an insight into the similarity of the datasets.

4.2. Frequent triads and triadic motifs

For RQ1 we need to analyze the structure of decentralized socioeconomic networks. As detailed in Section 2, we can compare the structure of different socio-economic networks from a static standpoint, by studying the frequency of *triads* i.e. 3- node directed subgraphs. Therefore, we consider $G_{[t_0,t_1]}$ as a static network, in this case, discarding the temporal information from the structure. For each triad, we obtain g_i the frequency $N(g_i)$. Then, we can compare the distributions of triads to assess the similarity between the two networks. We separate open and closed triads for an easier comparison so that each network is assigned to two distributions: the distribution of open triads and the distribution of closed triads.

Then, we study whether frequent triads are also statistically significant and if there are differences across the selected networks. We consider a *triadic motif* to be a triad that is also statistically significant. As introduced in Section 2, to assess the significance of triads, we have to define a proper null model. Here we adopted the null model defined in [28] and since we do not have a closed formula for the null model, we rely on bootstrap by performing N times the randomization of the original network, obtaining for each triad g_i , N outcomes $N_{rand}(g_i)$, corresponding to the counting of g_i in the N realizations of the random model. These counts are confronted with the count of each triad in the original network. We evaluate the statistical significance of the countings both through the z - score and the p - value [18]. For the former, denoted $\bar{N}_{rand}(g_i)$ as the average count with standard deviation σ_{rand} , we can compute the z - score of a triad g_i w.r.t. the null model as:

$$z(g_i) = \frac{N(g_i) - \bar{N}_{rand}(g_i)}{\sqrt{\sigma_{rand}^2}}.$$
(1)

Finally, a triad can be regarded as statistically significant in a network if its associated p - value is less than 0.01 and the absolute value of its z - score is greater than 2, $|z(g_i)| > 2$.

¹ In economy, they are referred to as economic agents.

4.3. Temporal subgraphs and temporal motifs

Answering RQ2 asks for studying how network structure evolves, more precisely how an open triad becomes a closed one, i.e. what is the sequence of link insertion operation transforming an open triad into a closed one? Here, we focus on a special case of *temporal triads* – temporal subgraph of 3 nodes – denoted as *closing temporal triads* $g_{i\rightarrow j}$, i.e. temporal triads that represent the transition from an open triad g_i to a closed one g_j , as shown in Fig. 3 on the right. We count the closing temporal triads in the different socio-economic networks, obtaining for each of the possible closing temporal triads the value $N(g_{i\rightarrow j})$. We can compare the distribution of closing temporal triads, for each network. This way, we are able to assess the similarity of the networks in terms of how the triadic closure process has closed open triads.

We also assess how significant each temporal triad is by identifying *closing temporal triadic motifs*, i.e. temporal triads that are statistically significant w.r.t. a null model for temporal networks [22]. We obtain the frequency of each temporal triad $(g_{i\rightarrow j})$, denoted as $N_{rand}(g_{i\rightarrow j})$, one for each of the *N* randomized versions of the network. Their average $\bar{N}_{rand}(g_{i\rightarrow j})$ and standard deviation σ_{rand} are used for computing the *z*-score and *p*-value tests. Similarly to the static case, the *z*-score of a closing temporal triad $g_{i\rightarrow j}$ is:

$$z(g_{i \to j}) = \frac{N_{(g_{i \to j})} - N_{rand}(g_{i \to j})}{\sqrt{\sigma_{rand}^2}}$$
(2)

Finally, we evaluate which temporal triads can be considered closing temporal triadic motifs, i.e. as statistically significant in the selected network. Similarly to the static case, we need to evaluate if the associated p - value is less than 0.01 and if $|z(g_{i \rightarrow j})| > 2$.

4.4. Measuring triadic closure

For RQ3, we analyze the triadic closure as a temporal process by leveraging the temporal information of the edges. Specifically, to understand how impactful triadic closure is, we leverage a few temporal metrics for triadic closure [21]. First, we study the impact of closure focusing on the number of triads that become closed (*n_closed_triads*), compared to the formation of new links (*n_links*) and monitoring their *ratio* over time as:

$$ratio = \frac{n_closed_triads}{n_links}$$
(3)

In short, the above measure indicates the overall contribution of new links in the formation of new triangles, and it is strictly related to the densification of the network as time goes on. However, it only returns a general trend in the evolution of the network, since it is counting-based.

A more specific measure based on the temporal information of the links forming a triangle is the triadic closure delay [21], a property characterizing each temporal triangle in a network. Viewing the triadic closure as a dynamic process, it measures the speed of the formation of closed triads. Through triadic closure delay we can capture the nature of the triadic closure process acting in online social networks: for instance, if only fast closed triads are forming, or if latent triangles are woken up by external mechanisms, such as seasonal events or recommendation systems [21]. The measure has been defined only for undirected graphs. In the undirected setting, we deal with triangles, i.e. an undirected closed triad of vertices u, w, z, where each edge u, w has a timestamp $\tau(u, w)$. So, a triad g will move from an open triad with two links – for example, (u, w) and (w, z) – to a closed triad (triangle) when the last pair ((u, z) in the example) connects. Consequently, undirected close triads are characterized by opening and closing times. The delay accounts for the time the triad g needs to close, namely:



Fig. 3. On the left, a static close directed triad among the vertices *u*, *w* and *z*. Number 7 corresponds to the ID assigned to each kind of triad. On the right, the corresponding closing temporal triad. From the open triad (blue link) by the insertion of the green link (*u*, *z*) we move to the close triad 7. $1 \rightarrow 7$ indicates that me move from the open triad with ID 1 to the closed triad with ID 7. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



Fig. 4. Example of open and close triads. On the left, an open triad where the blue link forms before the red one, which reciprocates the relationship between u and w: both links may be considered for defining the opening time. On the right, a closed triad where both links (blue and red) can be considered for the definition of the closing time of the directed triad. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where $\tau(u, z)$ is the closing time and $max(\tau(u, w), \tau(w, z))$ is the opening time.

The above definition does not hold for the directed case, since the time of opening and closing is not straightforward as in the undirected case: they can be interpreted in different ways because of the presence of bidirectional links. The presence of bidirectionality means that the creation of two links does not imply the presence of an open triad, as we could observe a bidirectional link and an unconnected node. Similarly, the addition of a link, may not lead to a closure: as displayed in Fig. 4, in the case of opening time, when a link to an open triad is added, we may not have a closure, because the new link may reciprocate an existing link, hence we have more opening times. Whereas for the case of closed triads (see the example in Fig. 4 on the right) that form by bidirectional links, we may be interested in either the earliest (t_3) or the latest (t_4) closing time. This is an important limitation for the analysis of decentralized networks: the importance of tokens in these systems means that we need to distinguish the sender or seller of the token/s from the receivers or buyers. Therefore it is of paramount importance to extend the current approach for directed graphs. In general, to measure the triadic closure delay in directed networks, we have to adapt the formulation to include the direction of links. Here, to measure the delay we consider the earliest opening time and the earliest closing time. Formally, given a closed triad g, with vertices u, w, z, and where each edge u, w has a timestamp $\tau(u, w)$ denoting its creation time and $\tau(u, w) = \infty$ for non existing links, we denote the *earliest closing time* $\tau_c(g)$ as:

$$\tau_c(g) = \min(\tau(z, u), \tau(u, z))$$
(5)

In this case, we assume that u, z is the last pair to form a link. Given the assumption that $\tau(u, w) = \infty$, by definition, in an open triad, the *min(*) always returns a real number. In the same setting, the *earliest opening time* $\tau_a(g)$ is defined as:

$$\tau_{o}(g) = max(min(\tau(u, w), \tau(w, u)), min(\tau(w, z), \tau(z, w)))$$
(6)

(4)

where we assume an existing at least a link between u, w and at least one between w, z, formally $min(\tau(u, w), \tau(w, u)) \neq \infty$, $min(\tau(u, w), \tau(w, u)) \neq \infty$. Then, the directed triadic closure delay can be extracted as:

$$directed_delay(g) = \tau_c(g) - \tau_o(g) \tag{7}$$

Once the triadic closure delay is defined for each closed triad, we can study its distribution and compare it to other online social networks to assess similarities and differences in the dynamic aspects of the closure process. With our proposed approach we can now analyze every directed graph as it is the case with most of the decentralized networks.

5. Datasets

In this section, we present the datasets selected for our study. We focused on three blockchain-based systems with different social components.

5.1. Steemit blockchain online social network

Launched in 2016, Steemit [5] has been one of the most widespread decentralized online social platforms and is considered a pioneer for the Web3 ecosystem since it introduced the seminal concepts of rewarding system and delegated proof-of-stake (DPoS) consensus algorithm for block validation in social media apps. The platform is hosted on a blockchain called Steem and implements three different tokens: STEEM, Steem Dollar - (SBD), and Steem Power - SP; where the last one is on the basis of the internal rewarding system and the first two tokens are tradable on exchange markets [9]. Steemit has gathered the interest of researchers for its characteristics and has been dissected in many aspects. However, only some works have studied network structure and evolution. For example, a few studies have focused on the features of different types of social networks resulting from diverse interactions or specific subsets of accounts. Guidi et al. have studied "follow" network and other operations in Steemit [29] and have delved into a study of the follower-following graph and the token transfer graph [30]. Other works focused on economic aspects and network structure: for instance, Li et al. [31] have analyzed the rewarding system in Steemit from a network perspective, while Ba et al. discussed the interplay between cryptocurrency price and the link creation process [9], the impact of user migration on the social networks [32], the role of groups network structure in migration [33], and the bursty dynamics of the link creation process [34]. Also Tang et al. [35] model voting and currency transfer data to study user collusion behavior. Moreover, Galdeman et al. [36] studied the network growth using transfer operations, subgraphs of up to 4 nodes, in a span of 3 months. They highlighted that in Steemit, network structure is characterized by rules which increase network transitivity and reciprocity. In this work, we rely on the transaction dataset used in [9]. We consider Steemit's transfer operations, the most common type of financial action, that allows the exchange of the two main tokens, STEEM and SBD; covering four years of user activity, for a total of 55 033 746 transactions. For a transfer operation, we consider the users involved, and the action timestamp.

5.2. NFT network

Following their gain of popularity, there has been an increasing amount of studies on NFTs [15]. For instance, Nadini et al. [7] have conducted a comprehensive quantitative overview of the NFTs market, including a network-based analysis. Franceschet et al. [37] focused on the creators–collectors network, while Galdeman et al. [36] highlighted the presence of frequent trading chain patterns. In this work, we rely on the dataset of NFTs sales collected and analyzed in [7]. The dataset aggregates NFT trades from different marketplaces (APIs): Cryptokitties, Atomic, Opensea, Gods-unchained, and Decentraland. The data collection is composed of 6.1 million trades of 4.7 million NFTs in 160 cryptocurrencies, primarily Ethereum and WAX, covering the period from June 23, 2017, to April 27, 2021. We consider, for each transaction, the id of sellers and buyers, as well as the time of sale/transfer.

5.3. Sarafu community currency

Sarafu [38] ("currency" in Kiswahili) is a digital community currency token created by the Grassroots Economics (GE) Foundation, a humanitarian aid foundation. Complementary or community currencies (CCs) are currency systems, often born out of cooperation among members that face a period of crisis and introduced in a certain community, with the objective of creating bonds of reciprocity and integrating social networks among people [39]. In both cases, there is a strong interplay between social and economical aspects. In the case of Sarafu, users may perform payments using mobile phones to transfer Sarafu digital tokens to other registered users [40]. Sarafu relies on blockchain technology for enhancing transparency, as transaction data allows contributors to fully disclose the impact of their donations. Furthermore, data analysis can lead to more informed decision-making processes regarding, for example, future investments and project functioning. Further, it also helps the GE Foundation to improve user welfare and minimize potential misuse. There are currently few studies on Sarafu from a network standpoint. The GE organization realized a dataset [38] which includes detailed and anonymized information on token transactions. Ussher et al. [10] presented an accurate description of complementary currencies, the Sarafu project history, and an analvsis of the dataset. Mattsson et al. [41] proposed an analysis modeling the entire dataset through a static network structure: their analysis highlights that money circulation is highly modular, geographically localized, and occurring among users with diverse jobs. While Ba et al. [42] model the dataset as a sequence of temporal networks to study currency flows and cooperation patterns.

In this work, we rely on the same dataset [38]. The data span the period from January 2020 to June 2021, totaling 412 050 economic transactions involving 53 277 users. Each economical transaction specifies its source and its target as anonymized IDs of the sender and receiver of the cryptocurrency token. Alongside that, we have important additional information for this study: one being the timestamp, i.e. the date and time of when a transaction happened, with a granularity of ms.

5.4. Preprocessing and experimental setting

Before delving into the identification of the triads of interest, we proceeded with a data preparation step. For Steemit we limit the analysis to the first 2 years (2016 and 2017), both due to computational constraints as well as to obtain a number of transactions similar to the other datasets. We limit to 8 327 832 operations. For the NFT trades dataset, we consider all the 6 071 027 transactions in the original dataset. Finally, for Sarafu we utilize the same preprocessing steps as in [42], overall getting 412 050 operations.

For the computation of the frequencies of triads, we implemented a parallelized version of the triad census algorithm presented by Batagelj et al. [43]. It is a sub-quadratic algorithm for large and sparse networks able to not enumerate every possible 3-node sub-graph in the network, and whose complexity is O(m), where *m* is the number of links. As for the evaluation of the significance of the triad frequencies through a null model, we proceeded with a network structure randomization of the static network done using the greedy algorithm of Havel and Hakimi, which was extended to directed graphs by Erdos et al. [28]. Instead, when we deal with temporal triads, we implemented a strategy based on the topology-constrained link shuffling method, a randomization method for temporal networks presented by Gauvin et al. [26].

Table 1

Overview on socio-economic network properties. For each network we report number of nodes (|V|), number of links (|E|), density ($x10^5$) (de), diameter (di), average local clustering coefficient (cc), and reciprocity (r).

	V	E	de	di	cc	r
Sarafu	40 343	143239	8.80	22	0.16	0.52
NFT	532 944	2991601	1.05	53	0.05	0.02
Steemit	200 913	1356011	3.36	14	0.17	0.25

6. Results

In the following sections, we report and discuss the main outcomes resulting from applying the methodology discussed above to the selected datasets. Transaction networks are modeled as temporal networks, where a trade/transfer relationship is established when the first exchange happens: we have a link between users if they exchanged a token or non-fungible token, with the source being who is sending or selling the token/s and the target of the link will be the receiver. The main network characteristics are displayed in Table 1.

First, we observe that in Steemit we have more repeated transactions between the same users. Indeed Steemit network has a size less than NFT one even though there are more transactions in the former. Further, Steemit and Sarafu differ from NFT trades in terms of density: they are much denser and likely their structure may be characterized by more cohesive structures than in the NFT networks. Sarafu and Steemit also differ from NFT trade networks for other properties: they are characterized by a higher level of reciprocity than NFT trades. These last two features are coherent with the nature of the platforms: Steemit and Sarafu are more social by nature since they revolve around social media or cooperation groups, so more connected structures and reciprocal exchanges are to be expected; while in NFT trade networks there is a distinction between buyers and sellers, and it is unlikely that an account has both roles since there is only a single type of asset to trade. A further consequence of the different nature of NFT trade networks is reflected in the diameter of the networks. They all have larger diameters compared to established OSNs, but the more social Steemit has the lowest value, followed by Sarafu, while the NFT is by far the largest. A similar trait is also observable when considering connected components: both weakly and strongly largest connected components in the NFT trade network span only a subset of the network, while in Sarafu and Steemit the network has a huge largest connected component (>95%). Finally, the separation between sociallike networks, such as Steemit and Sarafu, and NFT trade networks has been also captured by the average clustering coefficient, computed on an undirected version of the graph. Indeed, we observe higher values for Steemit and Sarafu, while in the NFT trade network, it is less likely to observe clustered neighborhoods.

In short, from a network-level standpoint, socio-economic networks such as Sarafu and Steemit express characteristics more resembling online social networks than the NFT trade network; the latter being less clustered, less connected, and probably characterized by more chainlike structures. As for the triadic closure process, the results on the average clustering coefficient offer of first hint at the diversity of how the closure process acts, and its impact on the structure of the network.

6.1. Triadic structure to characterize socio-economic networks

Addressing RQ1 – i.e. to what extent decentralized socio-economic networks are similar in terms of static triads – asks for an enumeration of all the possible triads making the structure of the decentralized socio-economic networks; and an evaluation of their statistical significance. Then, we can compare the structure of different socio-economic networks from a static standpoint, by focusing on the most frequent and significative *triads*, i.e. 3-node directed subgraphs, common to all networks, or specific for one network only. Our analysis of open and closed triads and significative triads has highlighted the following main findings:

- Open and close triad distributions are very different among the socio-economic networks. The main scopes and functionalities of the platforms, these networks have been derived from, largely determine the formation of characterizing patterns. For instance, in the case of open triads, the high or low frequency of "collector" or "spreader" patterns (triads 0 and 3) depends on the nature of the socio-economic network, e.g. buying from creators is very common in the NFT trade network. Moreover, open and closed triads are also influenced by the level of reciprocity, i.e. a trait merely linked to more social behaviors of the accounts.
- The distribution of the closed triads represents a footprint of the network since each socio-economic network has its specific distribution. In particular, the main discriminative characteristics are the frequencies of "feed-forward" loops and fully or almost fully reciprocated triangles. Socio-economic networks where the interplay between social and economic traits is stricter are characterized by more reciprocal relationships and triads, while where the interplay is weaker, such as NFT networks, feed-forward loops are dominant.
- All patterns are significative, thus not explainable by a random behavior of the accounts. In particular, the tendency of reciprocating impacts the formation of fully reciprocated open triads, especially in socio-economic networks where the interplay between social and economic actions is stricter. The significance of closed triads is a further discriminative element of the type of network, indeed there is a pronounced difference for under- and over-represented close triads between Steemit and the remaining networks.

From now on, we separately consider open and closed triads (*i*) to highlight similarities and differences both in terms of these two types of triads; and (*ii*) because of the skewness of the triad distribution (see Table 2) toward open triads, which would make the visual exploration of closed triads harder.

Open triads. First, we report the distribution of the frequencies $N(g_i)$ of open triads (triads with index from 0 to 5 in Fig. 1(a)) in Fig. 5. As discussed above, the distribution is limited to the possible open triads only. At first glance, we can observe that each network has its own profile, i.e. open triad distributions are different from one another. So, we can comment triad by triad, in order to highlight specific differences but even similarities.

Triad 0 is the most frequent triad in Steemit. This open triad can be seen as an "out-flow" triad, where tokens are only transferred to two other users, or as a "buying" triad, representing a user buying to two different users, in the case of NFT trades. This triad is very frequent in Steemit and the second most frequent in NFT, while it is very rare in Sarafu. In blockchain social media, this pattern is typical of accounts that are sources of resources: in the case of Steemit those might be content creators with money to spend or "whales", i.e. the richest accounts, who used their money as an influence mean; while in the case of NFTs it should be a triad where the resource spreaders are likely NFT collectors. In contrast, in Sarafu, this kind of triad is much less frequent but it is expected due to the cooperative nature of the platform. Indeed, most of the accounts in Sarafu are targets of micro-credit transactions or donations, while there are only a few donors or cooperative groups lending crypto-tokens. This observation also impacts the level of reciprocity of the Sarafu network.

A further evident difference involves triad 1. This triad represents a chain of currency transfers o sales. The frequency of this triad differentiates between Sarafu/Steemit, and the NFT trade network. In fact, in the latter, it is more relevant — 3rd most frequent, than in the former networks. Even for this triad, the difference in frequencies is due to the nature of the NFT network. Triad 1 mirrors a typical chain of sales, especially in the case of "wash trading" a.k.a. the practice of selling among coordinated users to inflate the price of an NFT. Such a trait is less frequent in networks more affine to social

Table 2

Significance of the 13 possible directed triads, in all three socio-economic networks. For each directed triad, we compute the *z*-score (*z*) and report the scores with p-value < 0.01, while the rest are not significant (NS). In each cell, the first line reports the count of the pattern, the second one its frequency, and the third line reports the *z*-score, between parenthesis.

Pattern	Steemit	NFT	Sarafu
0	2978073772	327981715	197348
\square	57.29%	28.18%	3.07%
0 Ö	(-85.74)	(-64.55)	(-64.79)
0	343201721	238232822	581008
	6.60%	20.47%	9.05%
	(-135.27)	(-78.15)	(-66.97)
0	379895328	18235194	466798
	7.31%	1.57%	7.27%
2 ⁰	(-100.07)	(32.60)	(-38.26)
Q	627116917	540106984	1780132
	12.06%	46.40%	27.72%
^o ₃ ^o	(-181.06)	(-100.39)	(-63.57)
Q	448251409	25306422	1662086
\sum	8.62%	2.17%	25.88%
	(-11.77)	(71.12)	(9.05)
Q	417169005	1094385	1460816
\sum	8.03%	0.09%	22.75%
5 ⁰	(236.97)	(52.51)	(45.40)
Q	2017335	9094191	64221
	0.04%	0.78%	1.00%
	(-129.44)	(75.88)	(-20.73)
Q	44441	405589	8218
	0.00%	0.03%	0.13%
	(-46.06)	(-35.84)	(-36.19)
Q	495953	996757	31942
\angle	0.01%	0.09%	0.50%
	(-34.22)	(73.56)	(23.31)
Q	503554	688699	29076
$\langle \rangle$	0.01%	0.06%	0.45%
9	(-49.22)	(108.03)	(-21.17)
Q	428919	1232177	33677
\sum	0.01%	0.11%	0.52%
10	(-86.03)	(159.98)	(12.39)
Q	834229	507460	62277
\sum	0.02%	0.04%	0.97%
	(-64.32)	(112.64)	(31.55)
Q	157261	82705	43590
	0.00%	0.01%	0.68%
	(-14.57)	(130.42)	(70.29)

networks, where other patterns characterized by a higher degree of reciprocity are to be expected. Even the distribution of Triads 4 and 5 are very specific to each socio-economic network. In fact, these types of triads are very frequent in Sarafu, less in Steemit, and rare in the NFT network. Both triads are characterized by the presence of reciprocal





(b) NFT



Fig. 5. The distribution of open triads in the three datasets. On the *y*-axis: percentage of each open triad. Each open triad, along with its index, is reported below its color bar. The distributions have been computed on the set of all the open triads. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

links, which can justify the low frequency in NFTs, where users tend to be either sellers or buyers. In particular, triad 5 captures an interesting situation where there is an open triad composed of two users strongly connected by reciprocal links; and yet, the two unconnected nodes end up not forming any link among them. According to the triadic closure principle, this situation should resolve in a closed triad; or an eventual breaking of the triangle whereas the two unconnected nodes



Fig. 6. The distribution of closed triads in the three datasets. On the *y*-axis: percentage of each closed triad. Each closed triad, along with its index, is reported below its color bars. The distributions have been computed on the set of all the closed triads. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

are actually not on friendly terms. In Sarafu, this triad may be also representative of good practice in cooperation and microcredit-based systems: the lender is the central node and the two unconnected nodes have been able to repay the loan to the lender.

The above triads and their frequencies represent distinctive elements among the various networks. However, we also observe open triads which are among the most frequent in all networks. In fact, triad 3 is a very important triad across all networks. In this triad we have well-defined roles: a node is a target or "collector" of token transfers while the remaining two nodes are not connected to each other but send tokens to the collector. This triad is very frequent in all networks: indeed, for Steemit, it could be a content creator receiving money, a service provider receiving a payment, or even a content promoter or a whale being reached by other users in need of visibility for their posts. In Sarafu this pattern is probably caused by the presence of "group accounts", special accounts handled by more users, that are saving up money. Finally, in the case of NFTs, the target node may represent an NFT creator or an owner of interesting NFTs. A further common trait among all networks revolves around triad 2: it is not very frequent in all of them, with a small increase in Steemit and Sarafu. This triad can be seen as a chain with some reciprocity, and since the difference across the networks is not large, the difference could be simply a byproduct of the higher levels of reciprocity of Steemit and Sarafu compared to the NFTs.

Closed triads. In Fig. 6 we report the distribution of closed triads, i.e. triads with index from 6 to 13 in Fig. 1(b). Similarly to open triads, we can observe that each network is characterized by a different profile, a plausible consequence of the diverse nature of the networks. However, in the case of closed triads, it is more difficult to semantically characterize the overall pattern as it strongly depends on how they are formed — an aspect we shall focus on in the following sections. Nevertheless, we can still highlight similarities and differences triad by triad, as discussed above.

Starting from closed triad 6, we can observe it is the most frequent in all scenarios, even if there are significant differences in its frequency: in the NFT trade network, it is very frequent - about 70%, quite important in Steemit (45%), and less frequent and comparable to other closed triads in Sarafu (about 25%). It is worth noting that Triad 6 corresponds to the well-known "feed-forward loop" pattern, characterizing diverse types of networks, such as biological and regulatory networks [44] or land trade networks [45]. Closed triad 7, the loop, is rare in all networks: it is the least present in Sarafu and Steemit and among the least frequent in the NFT network. In the case of financial networks, the 3-node cycle is strictly related to suspicious money laundering activities [46]. Further, there is a strong similarity across the networks with regard to triads 8, 9, and 10. These triads tend to be in the middle of the pack in terms of frequency, with very similar rankings across the three networks. While the ranking and the frequency associated with the above triads are traits common to the three networks, the frequencies of triads 11 and 12 are specific to each network. For instance, triad 11 is very frequent in Steemit and Sarafu - the second most frequent - while marginal in NFT. Even in this case, the high frequency in Steemit and Sarafu is a consequence of the high degree of reciprocity. This is also confirmed in the case of triad 12: very rare in the NFT network and quite frequent in Sarafu, otherwise.

Triadic motifs. Finally, we deepen the study of triads by focusing on their significance and identifying triadic motifs, i.e. statistically significant triads. Here, we discuss each socio-economic network separately, then highlight similarities and differences. In Table 2, we observe the z-scores (see Eq. (1)) for both open and closed triad motifs. We first observe that all the triads can be considered statistically significant with regard to the selected null model since most of the z-scores are greater than 10 (absolute values). However, there are differences in the *z*-scores throughout the different networks. For open triads (0 to 5) we can observe that shuffled graphs (random models) end up containing more open triads. Indeed, open triad motifs 0, 1, 2 and 3 are under-represented, except triad 2 in the NFT network. Differences are more evident for open triad motifs 4 and 5. For instance, in Steemit, even open triad motif 4 is under-represented, while in NFT and Sarafu networks we actually have more open triad motif 4 compared to random networks. Finally, there are more open triad motifs 5 in all three networks, where in Steemit the z-score is particularly higher. In short, the tendency of reciprocating relationships in Steemit and Sarafu is far from being the outcome of random behaviors: in socio-economic networks, such as Steemit and Sarafu, where the interplay between social and economic actions is stricter, the reciprocity impacts the formation of fully reciprocated open triads. The tendency of reciprocating links even impacts the significance of reciprocated open triads (2 and 4) in the NFT scenario.

A structural difference in terms of the significance of closed open triads separates Steemit from Sarafu and the NFT network. In fact, for Steemit, all closed triad motifs are actually underrepresented w.r.t. the randomized networks. Given the nature of the network it is quite an unexpected outcome since one would have expected over-represented triadic closure structures. A possible explanation of this outcome is twofold: (i) the period covered by the dataset captures the early stages of the network where accounts mostly joined other accounts without any attempts to consolidate their neighborhoods through closing triads; and (ii) open triad motifs 5 are over-represented according to its zscore, and when randomized, those triads tend to turn into closed triad motifs 11 and 12, so increasing the average frequency of triangles in the random model. On the contrary, the NFT trade and Sarafu networks are characterized by over-represented closed triad motifs. Specifically, all of the closed triads in the NFT network are more frequent than in the null model, while Sarafu has only some actually more present (8, 10, 11 and 12), those characterized by the presence of bidirectional links. In short, various kinds of triangles in NFT and Sarafu are not the outcome of random actions of the accounts, rather users are more likely to form a close triad. In particular, in Sarafu the tendency toward reciprocating links and the formation of triangles act together.

In a nutshell, to answer the first research question RQ1, each decentralized socio-economic network is different from the others. In a static setting, each network has its own specific profile based on the distribution of open and closed triads.

6.2. Closing temporal triads and triadic motifs

Although the analysis of triads on the static network representation highlighted how triad distributions differentiate a network from another one, our comprehension of the mechanisms leading to the formation of these specific patterns is only partial since we lose the sequentiality of the formation process provided by the temporal dimension. For this reason, herein we cope with temporal triads in order to answer RQ2, i.e. how triadic structures evolve and change over time, and whether there are growth patterns common to all the socioeconomic networks. The main findings, detailed and discussed in the following, highlight:

- the central role of triadic closure processes leading to the formation of "feed-forward" loops, fundamental directed closed triads characterizing many directed networks in different domains. In fact, all the closing temporal triads ending into a feed-forward loop are the most frequent in all the networks; even if in Sarafu and Steemit some of these patterns are not statistically significant;
- the distribution of the closing temporal triads is a footprint of these socio-economic networks: distributions are different from one another, especially excluding the three most frequent closing temporal triads. For instance, the NFT network is mainly built around patterns leading to "feed-forward" loops while other patterns are irrelevant. On the contrary, the distributions of the closing temporal triads in Steemit and Sarafu are more uniformly spread over all the possible patterns. In particular, the temporal triads leading to the creation of fully reciprocal triangles are frequent and significant. In short, even from the closing temporal triad standpoint, each network has its own specific profile which depends on the nature of the socio-economic actions it supports.

In the first instance, we look at the distribution $N(g_i)$ of closing temporal triads as reported in Fig. 2 and in Fig. 7. Overall, the three most frequent closing temporal triads are common to all three socioeconomic networks, with slightly different rankings or frequencies. Specifically, all three temporal patterns lead to the formation of the "feed-forward" loop (identifier 6). In this pattern there is a specific hierarchy where a node is an "initiator" - it is only a source of token transfers, a node is a "target" - it is only a destination of transfers and an "intermediate" node which is both source and destination. In the most frequent temporal triad $3\rightarrow 6$ the initiator and the intermediate accounts transferred money to the same account - the target - and, after that, the initiator transfers money to the intermediate one. So, in this case, the target is immediately identified by both the remaining nodes. On the contrary, in 1+6 transfers between the initiator and the target are not immediate at the beginning, rather there is a twohop connection passing through the intermediate node. Finally, in $0 \rightarrow 6$ the initiator transfers tokens to the remaining nodes and later the intermediate node interacts with the target. Observing the frequencies of the three most frequent temporal patterns we note that in Steemit and Sarafu patterns are almost equiprobable, while in NFT the gap between $3 \rightarrow 6$ and the other two temporal subgraphs is more evident. Indeed, in the NFT context, the pattern $3\rightarrow 6$ may represent a collector behavior of the initiator which first collects and buys NFTs from a target creator and then collects other NFTs produced by the same creator but bought by the intermediate node, i.e. a third account.

A comparison among the overall profiles of the closing temporal triad distribution let emerge an important difference: the frequencies of closing temporal triads excluding the top three in Steemit and Sarafu are higher compared to the NFT network, where the gap between the top three and the other temporal triads is much more evident. More precisely, in NFT, besides the three most frequent subgraphs, only a few closing temporal triads are notable in terms of frequency: $1 \rightarrow 7$ a directed closing loop, $2 \rightarrow 10$ and $2 \rightarrow 8$; where the last two are strictly related to the feed-forward loop as triads 8 and 10 are "feed-forward" loops where either the link between the initiator and the intermediate or the link between the intermediate and the target is reciprocated. On the other side, Steemit is characterized by a more varied distribution, where all the remaining temporal triads are more frequent, especially those involving open triads 4 and 5 as starting points (leftmost side of the distribution in Fig. 7a), i.e. open triads containing reciprocal links. This characteristic is even more evident in the closing temporal triad distribution for Sarafu (see Fig. 7c), where the temporal pattern $5 \rightarrow 12$, made by reciprocal links only, is among the most frequent items. Even in this case, the cooperative nature of the Sarafu socio-economic network impacts how open triads close, especially when reciprocal links are involved in the pattern.

Closing temporal triadic motifs. So far we individuated some differences in the frequencies of temporal triads. However, as in the static case, closing temporal triads with high frequency may not be statistically significant. Therefore, we move on to the study of closing temporal triadic motifs, i.e. statistically significant closing temporal triads w.r.t a null model. We compute the z-score (Eq. (2)) for all the possible closing temporal triads and report the values in Table 3. Overall, we can observe important differences in the set of closing temporal triadic motifs. An interesting result concerns the statistical significance of the most frequent closing temporal triad 3+6. In fact, in Steemit and Sarafu, it is not statistically significant, i.e. we would find it similarly in a randomized network. Not being statistically significant does not mean it is not an impacting pattern during the evolution of these socioeconomic networks, rather it raises some doubts on the willingness of such trait since it may be a consequence of random behavior. On the contrary, the same closing temporal triad is largely over-represented in the NFT network, a further signal that the purchasing strategy of the initiator in "feed-forward" loops has a certain level of intentionality. As for the remaining two most frequent closing temporal triads, they are under-represented in all networks, indicating that these patterns do not result from random behaviors. Furthermore, in Sarafu many closing temporal triads have failed the significance test as motifs, i.e. they occur in a comparable manner in randomized versions of the network. Finally, the analysis of the statistical significance further supports the

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Fig. 7. Distribution of the closing temporal triads in the three socio-economic networks. On the *y*-axis the frequency of the temporal subgraphs. Each temporal subgraph, along with its index, is reported below its color bar. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

findings about closing temporal triads involving reciprocal links; in fact, we observe that also the closing temporal triads starting from open triad 5 ($5 \rightarrow 11$ and $5 \rightarrow 12$), tend to be significant and overrepresented in Sarafu and Steemit. This result emphasizes the importance of reciprocal links in the creation of fully or almost fully reciprocal closed triads (identifiers 11 and 12).

In summary, to answer the second research question RQ2, a common growth pattern involves only the formation of "feed-forward" loops, while each network is characterized by specific creation patterns for closed triads.

6.3. Measuring triadic closure

Finally, we address RQ3, i.e. we focus on the stability of the triadic closure process as the network grows, and we assess how fast closing temporal triads form. To these aims, we measure a few dynamic aspects of triadic closure by leveraging the temporal information of the edges and computing different temporal metrics for triadic closure. Here, we find that each network has its own specific closure process trend, but all trends are unstable and sometimes connected to external conditions. Moreover, the triadic closure process is fast, i.e. half of the closed triads have formed in ten days. In general, from a dynamic viewpoint, these decentralized socio-economic networks are more unstable, more dynamic, and faster than centralized online social networks.

First, we study the impact of closure focusing on the number of triads that become closed (*n_closed_triads*), compared to the formation of new links (*n_links*) and their *ratio* over time. This metric highlights the average contribution of a new link in closing open triads. The obtained measurements are reported in Fig. 8, and they can be also confronted with those from previous studies on not-decentralized on-line social networks [21]. In more detail, in the Steemit network (see Fig. 8a), we have 730 days with at least a new link formed, with an average of 1858 links per day and a peak of 23009 new links established on the same day. As for triadic closure, it is worth noting that in a few days we did not observe any closing temporal triads. In fact, in the very beginning of the decentralized platform – the bootstrap

Table 3

Significance of the possible closing temporal triads for all three socio-economic networks. For each motif, we compute the *z*-score (*z*) and report the scores with p-value < 0.01, while the rest are not significant (NS). In each cell, the first line reports the count of the pattern, the second one its frequency, and the third line reports the *z*-score, between parenthesis.

Pattern	Steemit	NFT	Sarafu
0	137829	192243	14394
	3.01%	1.48%	5.27%
0→10	(NS)	(-24.10)	(NS)
0	785803	2834845	29521
\square	17.16%	21.79%	10.81%
	(-11.44)	(-75.09)	(-4.64)
0	710914	3079939	26072
	15.52%	23.68%	9.55%
	(-24.35)	(-59.07)	(-21.63)
	59399	478230	12181
X	1.30%	3.68%	4.46%
d d	(-21.94)	(-39.07)	(-19.54)
	129140	147972	12722
\mathcal{R}	3 02%	14/0/2	12122
1 de la	(25.02%)	(24.85)	4.00%
1→9	(-23.33)	(-34.03)	(-33.03)
Q	196684	61866	14953
\sum	4.30%	0.48%	5.48%
0_2→11	(32.13)	(-5.90)	(NS)
0	315517	533712	18672
	6.89%	4.10%	6.84%
0 2→8	(5.48)	(3.33)	(NS)
0	175873	192721	10297
	3.84%	1.48%	3.77%
0 2→9	(2.74)	(-5.15)	(NS)
0	853664	3968946	32630
\square	18.64%	30.51%	11.95%
0 3→6	(NS)	(156.49)	(NS)
0	123287	254909	14827
	2.69%	1.96%	5.43%
A	(-8.38)	(20.98)	(7.95)
3-78	289478	779546	21629
X	6 32%	5 99%	7 92%
1 de la	(11.80)	(41.91)	(3.48)
4→10	(11.00)	(11.)1)	(5.16)
\mathcal{A}	18//3/	90723	10200 5.060
	4.10%	(27.20)	5.96%
4→11	(20.12)	(27.30)	(7.10)
Q	182410	260942	10887
	3.98%	2.01%	3.99%
O _{4→9} O	(6.45)	(49.41)	(3.18)
Q	317065	113449	17988
\sum	6.92%	0.87%	6.59%
0 _{5→11} 0	(31.38)	(11.39)	(20.28)
Q	105461	17633	19968
	2.30%	0.14%	7.31%
5→12	(47.69)	(3.25)	(56.86)

period – is very common that most of the new links have involved new accounts reducing the chance of closing an open triad. However, after the bootstrap period, we observe an increase in the number of new







(b) NFT



Fig. 8. Measurements of links and triads. On the *x*-axis: days. On the left *y*-axis (log scale): the daily number of new links (orange) and triads (blue) formed during the growth of the three socio-economic networks. Trends have been smoothed by a moving average on a week sliding window. On the right *y*-axis (linear scale): the daily triad/link ratio between the triads and the links (red). The trend has been smoothed by a moving average. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

daily triads resulting in an average number of daily closures equal to 6384, with a peak of 137 414 on the same day, for a total of 4 481 692 closing patterns. This leads to an average ratio of 1.88 triad/link and a peak of 8.45 triad/link. Note that while the number of links and triangles are both rising, the ratio is actually growing, indicative that the links forming are actually making the structure more cohesive. In a comparison with not-decentralized online social networks, the average ratio resembles the measurements on the RenRen online social network,



Fig. 9. Triadic closure delay in days. On the y-axis: CDF of triadic closure delays. On

the y-axis: number of days of closure before the triad closed

but the peak clearly surpasses the mainstream social networks. In fact, the values are similar to the peak values observed in Facebook after the introduction of the friend recommendation system, namely the "People you may know" (PYMK) service [21]. So, in Steemit, especially after the summer of 2017, the average contribution of links toward closing triads is naturally more important than in platforms that had introduced algorithms incentivizing the formation of triads.

As for daily new links and closing triads in the NFT network, shown in Fig. 8b, we have 1252 days with at least a new link, with an average of 2389 new daily links and a maximum of 103486 links formed in one day. Every day has at least a triadic closure, with an average of 10389 new daily closing temporal triads and a peak of 288827 on the same day, for a total of 13007578 closures. This leads to a high average ratio of 5.94 triad/link and a peak of 19.30 triad/link. The ratio is actually in a larger range than Steemit, and the average ratio is actually quite large. Over the entire observation period, the trend of the triad/link ratio is characterized by two phases of higher closing activities (from November 2017 to July 2019 and from November 2020 to February 2021) and a central period of low closing activity - from July 2019 to November 2020. This trend is generally different from the Steemit trend, where the triad/link ratio has almost always grown. By comparing these outcomes with other social platforms, the measured values are indeed in line with traditional online social networks, with peak values actually higher than the ones observed in the initial growth of RenRen and Facebook.

Finally, the measurements on Sarafu offer another different trait of the dynamic of the triadic closure process. In Sarafu we have 507 days with at least a new link, a total of 143 239 links, an average of 283 new daily links, and a peak of 1370 new daily links created. Only in one day, we did not observe the formation of any triads. We record an average of 540 and a peak of 7328 new daily closing triads, leading to a total of 273001 closures. In Sarafu we observe an average triad/link ratio of 1.73 and a large peak of 15. The average ratio is indeed similar to Steemit, but the peaks are larger and closer to the NFT network. Unlike the previous networks, triadic closure seems to have an important impact in only a portion of the observation period (see Fig. 8c): the triad/link ratio started to grow only around July 2020, with the largest spikes occurring during the central period, from September 2020 to January 2021, while in the last period the triad/link ratio has reached a closing activity similar to the initial period: a low and stable average contribution of the new links to closed triads. In Sarafu, the overall trend is strongly connected to conditions external to the decentralized network, indeed, it had huge growth during the pandemic period, given its important role in supporting economic activities during the COVID-19 pandemic [10,42]. Moreover, when compared to traditional OSNs, the values are still similar, and the peak is actually large, confirming the importance of triadic closure even in Sarafu.

To assess how fast is the triadic closure process in the three decentralized socio-economic networks, we analyze the triadic closure delay to understand if the triadic closure is a relevant factor. In fact, triangle closing speed compared to social networks would be another strong indicator of the importance of triadic closure. In Fig. 9, we report the Cumulative Distribution Function - CDF - of triadic closure delays, for the three networks. We can observe an interesting result: the distributions of delay have similar shapes, with a significant amount of triadic closures happening fast. More precisely, we focus on triads that close in less than a day: in Steemit 18%, in the NFT network 21%, and in Sarafu 23%. In a comparison with not-decentralized social networks, the triadic closure process is much faster, in fact, in both Facebook and RenRen those values are actually much lower, in the range of 5% [21]. In particular, in those OSNs, half of the triads close in 25 days, while we find even higher values in these decentralized networks: in Steemit and NFT network 64% of closing triads are closed in less than 25 days, and a similarly high value characterizes Sarafu (61%). In all the networks, we record very fast closures, as most of them are closed in less than 3 months (90 days): respectively, Steemit 91%, NFT 88%, Sarafu 89%. In the centralized counterparts, the values are similar, around 80%.

To answer the third question RQ3; from a dynamic and longitudinal perspective, in decentralized socio-economic networks, the triadic closure has impacted the evolution and the growth of these platforms even more than in traditional and centralized online social platforms. The process is not stable at all, rather each network, as already discussed in the previous sections, is characterized by its own dynamics. However, there is a characteristic common to all these networks: the closure process is very fast, faster than in the centralized online social networks.

7. Conclusion

In this work, we analyzed how triadic closure, one of the primary mechanisms underlying the formation of social ties, affects decentralized socio-economic networks, where social and economic interactions are strongly intertwined. We extended the existing methodology for triadic closure studies to generalize with directed networks, making it suitable to cope with the characteristics of decentralized networks, such as directionality a key component in economic transactions. We conducted an analysis of network structure centered on triads, i.e. 3-node subgraphs, and triadic motifs, i.e. statistically significant triads while considering both a static and dynamic viewpoint. The methodology was applied to three distinct decentralized socio-economic networks (Steemit, Sarafu, NFT trades) with varying degrees of influence from social ties. The main takeaways are:

- From both a static and dynamic perspective, each network has a distinctive profile depending on the nature of the socio-economic activity it facilitates.. From a static viewpoint, the analysis shows that networks, where the interplay between social and economic traits is stricter, are characterized by more reciprocal relationships and triads, whereas networks where the interplay is weaker, such as NFT networks, are characterized by a predominance of feed-forward loops. Moreover, although all triadic closure patterns bear significance, rendering them inexplicable through random behavior, we have observed variations among networks regarding the prevalence of both underrepresented and overrepresented close triads. From a temporal perspective, the distribution of closing temporal triads serves as an indicative representation of these socio-economic networks. The distributions exhibit variations among each other, particularly excluding the three commonly occurring frequent closing temporal triads. For instance, the NFT network is mainly built around patterns leading to feed-forward loops while other patterns are unimportant. In contrast, the distributions of the closing temporal triads in Steemit and Sarafu are more evenly dispersed across all the possible patterns. In particular, the temporal triads that result in the creation of fully reciprocal triangles are frequent and significant.
- Triadic closure has impacted the evolution and the growth of these platforms even more than in traditional and centralized online social platforms. The analysis of the stability of the process over time shows how the triadic closure process is not stable at all, rather each network is characterized by its own dynamics. While the measurement of how fast closing temporal triads form, through the directed triadic closure delay, showed how there is a characteristic common to all these networks: the closure process is very fast, faster than in the centralized online social networks.

Overall our work presents strong evidence that triadic closure is an important evolutionary mechanism in the selected networks. Our analysis through temporal motifs highlighted similarities and differences across decentralized networks with different levels of social components. And indeed those observations make sense when we consider that the method highlighted both differences and similarities between systems where native cryptocurrencies are used for social-economic purposes and the maintenance of the platform (Steemit and Sarafu), from systems where exchanges of cryptocurrency still have a social component but are also tied to the trade of the NFT tokens, created for specific purposes (NFT market). This highlights the expressivity of the footprints based on temporal motifs. Indeed, our findings suggest that the social component cannot be ignored for a better comprehension of network growth of decentralized socio-economic networks.

Future works include the analysis of other Web3 systems with more or less of a social component. Understanding the growth of other decentralized online social networks not following the Web3 paradigm is also an important open issue. It would also be interesting to analyze trade relationships in other economic networks, to understand the differences in their structure. Moreover, we could leverage user features [47] to study the interplay with triadic closure. The evaluation of other established growth mechanisms would also be an important step toward the comprehension of the growth of these innovative systems. The results could be leveraged to improve the design process and functionalities of these systems — influencing various aspects such as consensus protocols, security, privacy, and usability.

CRediT authorship contribution statement

Cheick Tidiane Ba: Conceptualization, Methodology, Software, Validation, Formal analysis, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Matteo Zignani:** Conceptualization, Methodology, Resources, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition. **Sabrina Gaito:** Conceptualization, Methodology, Resources, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

This work was partially supported by project SERICS (PE00000014) under the NRRP MUR program funded by the EU.

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