*"Quaderni di Ricerca in Didattica", Numero speciale n. 13, 2024*
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# From chat to class: How online discussion framework enables to highlight critical elements of asynchronous class discussion

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**Abstract.** In this study, we investigate asynchronous mathematical discussions, a novel type of mathematical discussion that combines the features of Web-based online discussions and mathematical class discussions. Our aim is to evaluate the effectiveness of the digital educational environment designed for asynchronous mathematical discussions in promoting typical learning practices of Web-based online discussions. To achieve this goal, we employ a theoretical lens developed in the context of Web-based spontaneous online discussions. We analyse the data collected from an experiment conducted with a class group of 9th-grade students who engaged in an asynchronous mathematical discussion, mediated by an instant messaging platform and a collaborative digital board. Our analysis reveals that while spontaneous online discussions and asynchronous mathematical discussions share structural similarities, they engage participants in different ways. These results emphasise the crucial components of the current setting of the asynchronous mathematical discussion, which can inform the redesign of future experiments.

**Résumé.** Dans cette étude, nous examinons un nouveau type de discussion mathématique qui combine les caractéristiques des discussions en ligne et des discussions mathématiques en classe: les discussions mathématiques asynchrones. Notre objectif est d'évaluer l'efficacité de l'environnement éducatif numérique conçu pour les discussions mathématiques asynchrones pour promouvoir des pratiques d'apprentissage spécifiques à ces environnements. Pour atteindre cet objectif, nous utilisons un cadre théorique développé spécifiquement dans ce contexte. Nous analysons les données recueillies lors d'une expérience menée avec un groupe d'élèves de grade 9 qui se sont engagés dans une discussion mathématique asynchrone, médiée par une plateforme de messagerie instantanée et un tableau numérique collaboratif. Notre analyse révèle que si les discussions spontanées en ligne et les discussions mathématiques asynchrones présentent des similitudes structurelles, elles engagent les participants de manière différente. Ces résultats mettent l'accent sur les composantes cruciales du cadre actuel de la discussion mathématique asynchrone, ce qui peut éclairer la refonte pour les expériences futures.

## 1. Introduction and research problem: a tale of two discussions

Mathematical class discussion, defined as "a polyphony of articulated voices on a mathematical object" (Bartolini Bussi, 1996, p. 16), is a structured classroom practice effective in promoting students' learning as it supports the construction and sharing of knowledge between students and teachers. In this practice, the teacher plays a key role in nurturing and guiding students' interactions, eliciting hidden thinking, clarifying objectives, discussing strategies, interpreting results, and fostering students' reflections at a metacognitive level. Nevertheless, conducting a class discussion can be challenging for teachers due to several reasons: it can be hard to get students talking, the exchange can be dominated by a few individuals, or it may be difficult for the teacher to keep track of students' contributions.

At the opposite end of the discussion spectrum, we have the Web-based online discussion: a spontaneous unstructured practice in which users interact with each other on a particular topic through threads of written comments activated by content shared inside interest-driven communities (Jenkins, 2006). Even if these communities are not conceived as educational settings, they can become informal learning environments operating in a bottom-up, peer-to-peer knowledge-sharing mode, and these online discussions can provide learning opportunities for participants who can contribute at their own pace and convenience.

In this paper, we focus on a new kind of discussion that shares the features of both online and class discussions: asynchronous mathematical discussions. It is an online discussion, structured in threads of written comments relating to a common discussion topic, allowing students to make contributions on their own schedule (as in the Web-based online discussion), and it is characterised by the key role of the teacher (as in the mathematical class discussion) who communicates asynchronously with students (Andresen, 2009).

Our purpose is to investigate if and to what extent this new type of discussion is effective in intercepting the strengths of Web-based online discussion and the strengths of class discussion to foster mathematical learning. Specifically, we will reflect on data collected during an experiment with a group of 29 $9^{th}$-graders in Italy, where researchers explored the implementation of asynchronous mathematical discussions by means of integrating two different digital environments: an instant messaging platform (WhatsApp) and a collaborative digital board (Padlet).

## 2. Theoretical and analytical framework

In this section, we present the frameworks used in the study. Since the purpose of the study is to connect mathematical class discussion and Web-based online discussion, we base our work on two different frameworks that are used in different stages of the study. The first framework, developed in the context of mathematical class discussion, is used to design the educational environment for the asynchronous mathematical discussion on the collaborative digital board. The second framework, which concerns Web-based online discussions, is used to develop the data analysis.

### *2.1 The M-$_{AE}$AB construct: a framework for the design of asynchronous mathematical discussions*

The Model of Aware and Effective Attitudes and Behaviours (M-$_{AE}$AB) is a theoretical construct by Cusi and Malara (2013) aimed at characterising the role played by the teacher during classroom discussions to foster students' aware and effective use of algebraic language as a thinking tool. The framework within which the M-$_{AE}$AB has been conceived is based on two sets of theoretical components: the first set focuses on the development of thinking processes through algebraic language, emphasising the importance of activating conceptual frames and suitable changes from one frame to another, appropriate anticipating thoughts, and coordination between different registers. The second set is comprised of those components that frame the investigation of teaching-learning processes and the role played by the teacher, drawing inspiration from Vygotsky's (1978) emphasis on expanding students' zone of proximal development, Leont'ev's (1978) idea of increasing students' awareness of the meaning of their learning processes, and Collins and colleagues' (1989) cognitive apprenticeship model.

The M-$_{AE}$AB construct provides transparent indicators to describe teachers' effective and aware actions aimed at making thinking visible to their students and at fostering metacognitive reflections during classroom discussions. These indicators are introduced by means of two groups of roles that the teacher can enact during mathematical discussions, as presented in table 1.

**Table 1.** Roles of a teacher as M-$_{AE}$AB

| **First group of roles** | | |
|---|---|---|
| Role as M-$_{AE}$AB | Characterization of the role | Indicators to code the role |

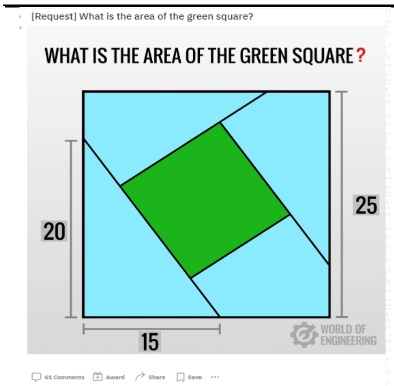
| | | |
|---|---|---|
| Investigating subject | Poses him/herself as part of the class group in the collective resolution of problems and stimulates a "inquiry attitude" towards the problems | Could you suggest to me what we can do now? |
| Practical-strategic guide | Poses him/herself as a model for students, by sharing thinking processes and reflections to make his/her reasoning visible to students | How can we interpret this question? What data can we use? Is this the only possible way to solve this problem? |
| Activator of anticipating thoughts | Asks questions and makes claims aimed at letting students focus on the objective of the activity or on the adopted strategy | What is our goal? Do we achieve our goal in this way? What expression/result do we expect to obtain? |
| Activator of interpretative processes | Asks questions aimed at fostering a continuous interpretation of the representations and results obtained when solving the problem | What does this representation mean? How can we represent this information? What interpretation can we give of this result? |
| **Second group of roles** | | |
| Role as M-$_{AE}$AB | Characterization of the role | Indicators to code the role |
| Guide in fostering a harmonised balance between syntactic and semantic levels | Helps students control the meaning and the syntactic correctness of the mathematical expressions and representations they construct and the reasons underlying the correctness of the transformations they perform | Is this transformation correct? Is it legitimate to simplify this expression? Why did you make this transformation? How have we obtained this result? Why have we obtained this result? |
| Reflective guide | Stimulates reflections on the effective approaches carried out during class activities to make students identify effective practical and strategic models | Could you explain your reasoning to your classmates? Is there someone who could explain your colleague's reasoning? Is it clear what your colleague said? |
| Activator of reflective attitudes and metacognitive acts | Stimulates and provokes meta-level attitudes, with particular focus on the control of the global sense of processes | Do you think it is an effective choice or strategy? Why? What are the differences/ commonalities between these answers? Was this task difficult for you? |

### *2.2 The cultural analysis: a framework for the analysis of online discussions*

Jenkins' ideas of convergence and participatory culture (2006) are powerful theoretical lenses to use to understand how online discussion unfolds on the Web. Convergence culture refers to the phenomenon where different media platforms and technologies merge, creating new forms of cultural expression and engagement. It represents a "cultural shift" (p. 3) where users are actively participating in the creation and circulation of content and rely on social interactions for collective meaning-making. Jenkins argues that convergence culture, with its blurring of boundaries between producers and consumers, has given rise to a participatory culture, in which individuals are actively involved in creating and sharing content rather than just consuming it. This has facilitated the transformation of the way people interact online, leading to the emergence of online communities where users can collaborate and contribute to the production of content. In the context of Jenkins' participatory culture, the concept of spontaneous mentorship - introduced by Lave and Wenger (1991) in their theory of situated learning - is particularly relevant as it reflects the collaborative and peer-driven nature of the culture. Members of the community are encouraged to learn from each other and share their expertise, contributing to a dynamic and supportive environment. This mentorship is characterised by the informal sharing of knowledge, skills, and experience between community members, without any formal structure or hierarchy. The result is a vibrant community of learners actively engaging in online discussions about shared interests and committed to helping each other grow and develop through these discussions.

Reading online discussions through Jenkins' lens (2006), we can provide insight into how they incorporate aspects of convergence culture and participatory culture, applying the analytical framework identified by Bini (2023) as cultural analysis. Bini's cultural analysis elucidates the typical learning practices of Web-based online discussion: the emergence of epistemic needs, collective meaning-making and spontaneous informal mentorship. In online discussions, shared digital content activates users' desire to develop a thorough understanding of the topic addressed and this desire is akin to what Kidron and colleagues (2011) identify as an epistemic need. Users experiencing epistemic needs address the community for collective meaning-making (reflecting convergence culture), and users reacting to others' epistemic needs provide spontaneous informal mentorship (reflecting participatory culture). To find examples of online discussion, we explore r/theydidthemath (https://www.reddit.com/r/theydidthemath/), a community inside the social media Reddit where users take on maths problems, calculations, and other quantitative challenges. In the threads of comments activated by images shared inside this community, users showcase the typical learning practices identified by the cultural analysis (Bini, 2023). Table 2 illustrates an instance of an online discussion and its associated cultural analysis. We present this example with a triple purpose: (1) to exemplify a spontaneous online discussion on a mathematical topic, (2) to demonstrate the genuine activation of the learning practices under discussion, and (3) to showcase the implementation of the cultural analysis.

**Table 2.** Example of online discussion and cultural analysis (C1, C2 commenters)

| Original post | Excerpts from the comments (unredacted) | Cultural analysis |
|---|---|---|
|  | C1: There are too many degrees of freedom in this problem with the information given. I tried an assisted algebraic solution and it simply doesn't snap into place due to these freedoms. | C1 shows epistemic needs and addresses the community for meaning-making (convergence culture) |
| | C2: There are two assumptions that have to be made to make a single numerical answer:<br>1. The large figure binding it all is a 25 unit-sided square.<br>2. The green square is positioned in a specific way. If the inner square is created by 4 identical overlapping 20x15 triangles like the one in the lower left corner, the answer as many have said, is 121 units.<br>What I haven't seen anyone produce is the range of answers it could be if assumption 2 is not made. This would be the problem I assume engineers would have to solve in an actual exam | C2 responds to C1's epistemic needs and offers a spontaneous mentorship, highlighting the assumptions to solve the problem (participatory culture)<br><br>C2 also shows epistemic needs in relation to the role of assumption #2 (convergence culture) |

## 3. Design of the educational environment and research question

We report here an experiment conducted with a 9th-grade class group of 29 students attending a science-oriented high school in the north of Italy. The mathematics teacher of the class is presently involved in the local instantiation of Liceo Matematico (Mathematical High School, https://liceomatematicomilano.unimi.it/), a national professional development project for secondary school teachers, aimed at enhancing the quality of mathematics education in Italian secondary schools.

The students participating in the experiment are not used to engaging in mathematical class discussions and, when involved, they show difficulties in being effectively engaged and tend to poorly interact. This can be ascribed to two main reasons: first, that, in Italy, grade 9 is the first year of high school, so students are presented for the first time with a "new way" of studying mathematics, involving argumentation and proof, and, second, that the mathematics teacher of this class rarely stimulates mathematical class discussions. The class was involved in the experiment as part of the Liceo Matematico experimentations, for the purpose of allowing students to experience mathematical discussions.

The experiment is structured in four tasks aimed at making students experience the use of algebra as a thinking tool by means of activities that focus on numerical explorations, conjecture, argumentation, and proof through the means of the methodology of mathematical discussion. The four tasks are designed to pursue different goals in order to support students in the gradual development of algebraic proofs, and are structured as follows: (task 1) transformation between registers of semiotic representations (natural language and algebraic notation), exploration and interpretation of expressions written in algebraic notation; (task 2) formulation of algebraic conjectures and their justifications; (task 3) analysis of algebraic proofs; (task 4) construction of proofs of given algebraic statements. This article focuses on the second task (figure 1), and the conjecture expected from students is that the result is always a multiple of 6.

| |
|---|
| What can you say about the difference between the cube of a natural number and the number itself? Give some numerical examples to support you in the formulation of the requested conjecture. Could you prove what you state? |

*Figure 1. Task 2 as assigned to students*

The experimental environment is set with the teacher creating homogeneous groups of 4-5 students with diverse profiles and configuring the different online platforms for the asynchronous discussion: an instant messaging platform (WhatsApp chats, one for each group) and a collaborative digital board (Padlet wall, one for the whole class). The researcher and the teacher have access to both digital environments to follow the asynchronous discussions, but they are involved differently in the WhatsApp chats and the Padlet wall. They do not intervene within the WhatsApp chats to let the groups discuss the task freely, and they intervene on the Padlet wall to foster the whole class discussion according to the roles envisioned by the M-$_{AE}$AB construct (table 1).

The unfolding of the discussion is structured in different phases, presented in figure 2 and detailed below.

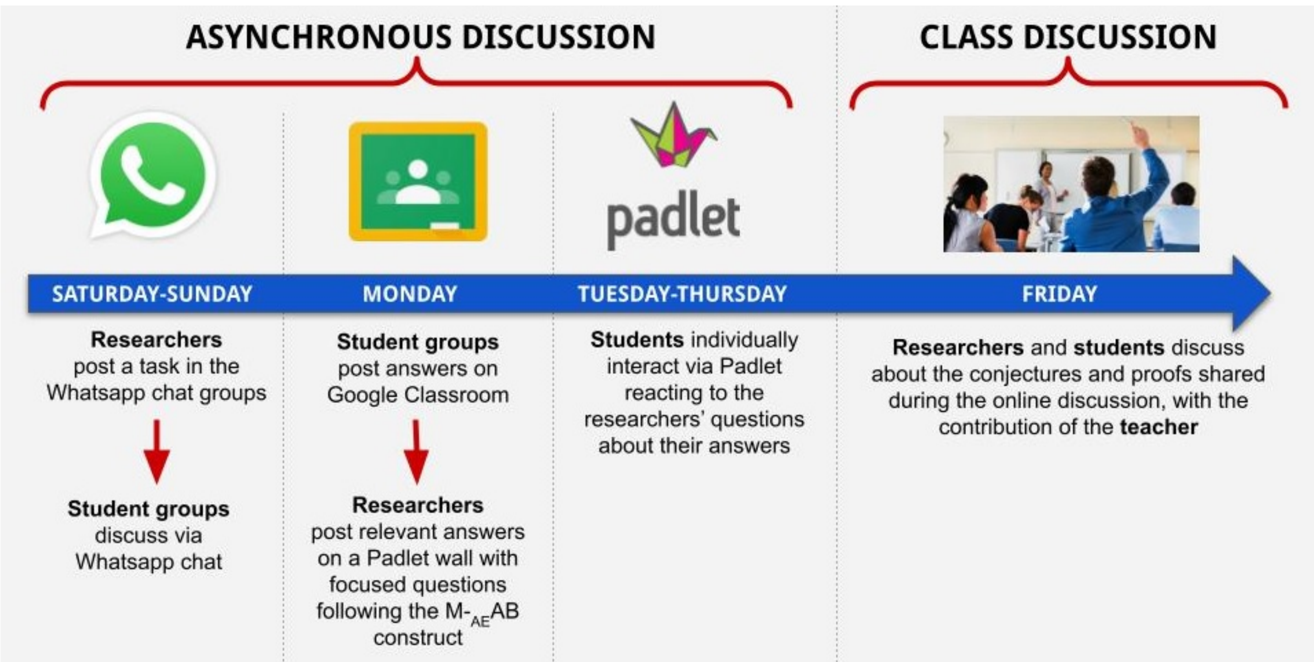



*Figure 2. Timeline and structure of the experiment*

The first phase begins with the teacher posting the task on each WhatsApp chat, and students then engage in asynchronous small group discussions to collaboratively solve it. This phase is scheduled across the weekend (Saturday and Sunday) to limit additional interactions to the ones on the chat. On the following Monday, after the small group discussions are complete, all groups submit their solutions, which become the object of the subsequent asynchronous discussion involving the whole class on the Padlet wall. The researcher initiates the Padlet wall discussion threads by posting selected screenshots of relevant excerpts from the groups' solutions, following Cusi and colleagues (2017). Examples of these solutions, presented in figure 3, were selected as representative of two emblematic cases: in Padlet discussion thread 1, students manipulate the algebraic expressions but they do not formulate any actual conjecture; in Padlet discussion thread 2, students only produce a partial conjecture.

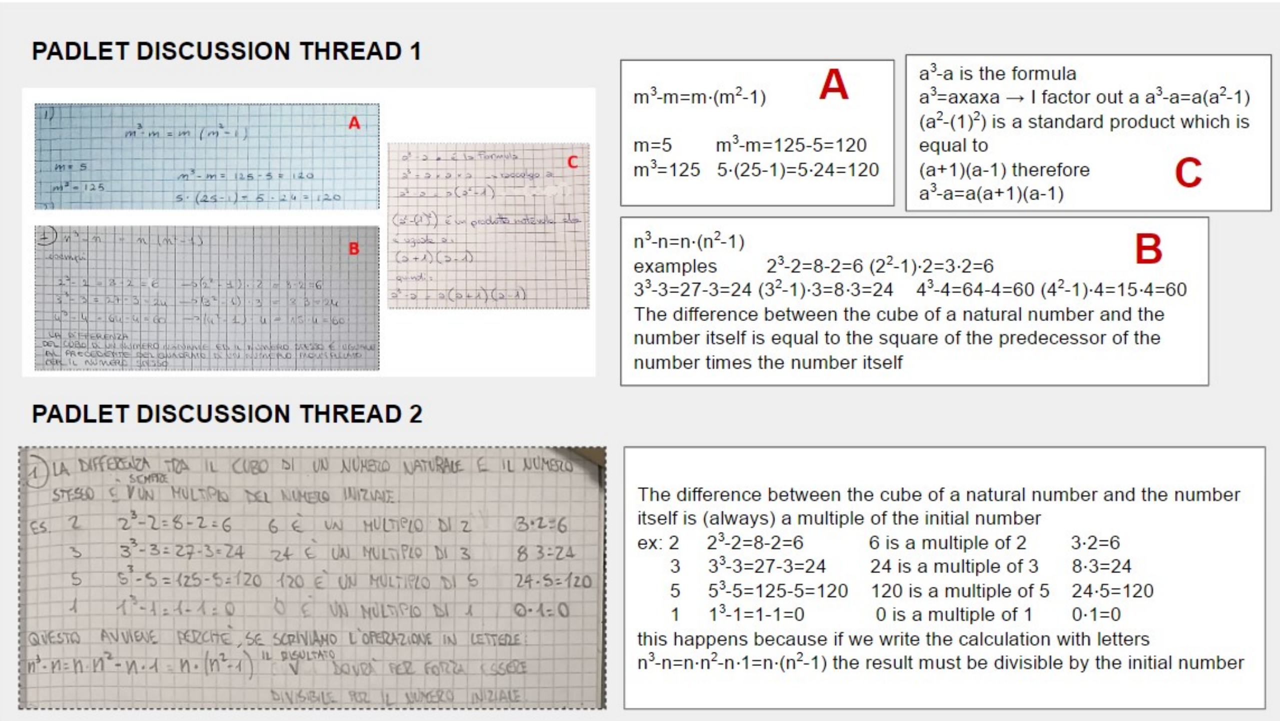



*Figure 3. Screenshots of group solutions from Padlet discussion threads with translations*

Screenshots are accompanied by guiding questions, formulated considering the roles introduced through the M-$_{AE}$AB construct presented in table 1. Following the choice of the emblematic cases, in thread 1 questions prompt the students to elicit the missing conjectures; in thread 2 questions stimulate students' reflections in order to complete the partial conjecture. Table 3 presents examples of the researcher's guiding questions related to the selected excerpts from the groups' solutions as reported in figure 3.

While the discussion unfolds, the researcher and the teacher intervene with further attuned questions.

**Table 3.** M-$_{AE}$AB construct to design the Padlet wall for the asynchronous mathematical discussion

| Thread | Researcher's guiding questions | Roles as M-$_{AE}$AB |
|---|---|---|
| 1 | Some groups presented the following expressions. Are they the requested conjectures? | R poses herself as an activator of interpretative processes and as an activator of reflective attitudes fostering the interpretation of the representations constructed by the groups and the comparison between the requested conjecture and the expressions proposed by some groups |
| 2 | One of the groups presented the following conjecture. What do you think about it? | R's intervention is in tune with the role of reflective guide since she asks students to give sense to the conjecture proposed by one of the groups |

Finally, on the following Friday, an in-person mathematical class discussion is carried out to summarise and deepen the reflections that emerged in both asynchronous discussions (WhatsApp and Padlet), at a mathematical level and at a metacognitive level, further discussing the mathematical content of the task and reflecting on the role of the online environments within the asynchronous discussion. Since the attention of this study is on asynchronous mathematical discussion, this final in-person discussion will not be furtherly addressed.

Focusing on the asynchronous mathematical discussion phases, we hypothesise that the task and guiding questions provided in online environments can activate students' epistemic needs, which are typical catalysts for learning practices in Web-based online discussions. Therefore, we will use the online discussion analytical framework (Bini, 2023) to analyse students' interventions within WhatsApp chats and the Padlet wall. Our investigation will be guided by the following research question: how and to what extent does the design of the digital educational environment in asynchronous mathematical discussion facilitate the emergence of typical learning practices of Web-based online discussion?

## 4. Data and analysis

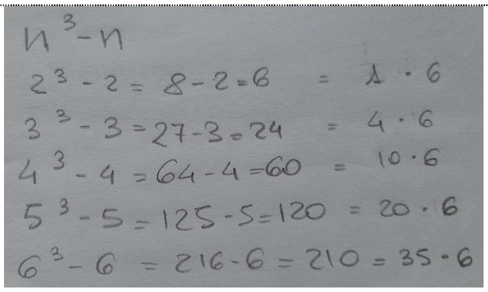
In this section, we present excerpts (translated) of the asynchronous mathematical discussions as unfolded on WhatsApp and Padlet with the related cultural analyses. More specifically, in tables 4 and 5 we report excerpts from two different groups' WhatsApp chats and in tables 6 and 7 we report excerpts from two discussion threads on the Padlet, corresponding to lines 1 and 2 of table 3.

**Table 4.** WhatsApp chat group 1 (S1, S2, S3 students)

| Excerpt from the WhatsApp chats | Cultural analysis |
|---|---|
| 1 - 13/02, 17:13 S1: I found a similar answer, but I didn't understand how you expressed it through letters, why is the result $n \cdot (n^2 - 1)$? | S1 shows epistemic needs and addresses the community for meaning-making (convergence culture) |
| 2 - 13/02, 17:14 S2: I took $n$ as a common factor and I divided it in each term in order to factor out, so you can notice that the result can be divided by the initial number $n$ | S2 responds to S1's epistemic needs and offers spontaneous mentorship, explaining his reasoning and his symbolic representation (participatory culture) |
| 3 - 13/02, 17:14 S3: That's it | S3 concisely expresses his agreement |
| 4 - 13/02, 17:15 S3: Do you understand now, S1? | S3 offers a spontaneous mentorship (participatory culture) |
| 5 - 13/02, 17:19 S1: Yes, thanks | Meaning-making is complete |

**Table 5.** WhatsApp chat group 2 (S4, S5 students)

| Excerpt from the WhatsApp chats | Cultural analysis |
|---|---|
| 6 - 13/02, 16:53 S4: Some examples for the first [task]: 8-2=6, 27-3=24, 64-4=60, 125-5=120, 216-6=210 | S4 addresses the community offering a first clue for meaning-making (convergence culture) |
| 7 - 13/02, 16:57 S5: It can somehow be related to the divisibility by 6 | S5 interprets S4's comment as the manifestation of an epistemic need and offers spontaneous mentorship, explaining her reasoning and her symbolic representation (participatory culture) |

| | |
|---|---|
| <br>8 - But I don't know what it could mean, do you* have any ideas? *Translator's note: in the original language (Italian), S5 uses the 2nd person plural [voi] | S5 also shows epistemic needs and addresses the whole group for meaning-making (convergence culture) |
| 9 - 13/02, 16:58 S4: No | S4 empathises with S5's epistemic needs but is unable to complete the meaning-making |

The cultural analyses of these two WhatsApp excerpts reveal that asynchronous discussions in small groups lead to the emergence of the learning practices typical of Web-based online discussions. Students' epistemic needs are activated by the assigned task (# 1 #6 #7), prompting spontaneous mentorship (#2 #4 #7) aimed at collective meaning-making, whether reached (#5) or not reached (#9).

We find it significant that S5 reacts first to S4's comment offering a possible explanation (#7) and subsequently moves from replying to S4 to addressing the whole group when she realises that neither she nor S4 can find an explanation for the observed regularity: in other words, S5 relies on the whole group for collective meaning-making.

**Table 6.** Padlet wall: excerpt of discussion thread 1 (T=teacher, Si=student)

| Excerpt from the comments | Cultural analysis |
|---|---|
| 1 - 14/02, 14:53 Researcher's guiding question: Some groups presented the following expressions. Are they the requested conjectures? | R tries to activate the students' epistemic needs by posing herself as a $M\text{-}_{AE}AB$ (see Table 3) |
| 2 - 15/02, 20:50 S6: To me, expression C seems the most complete since it refers to the rule used to factorise polynomials | S6 reacts to the epistemic need activated by the researcher, but she does not answer R's question |
| 3 - 16/02, 9:50 T: So does the expression C represent the requested conjecture? | T relaunches the initial question trying to activate the same roles played by the researcher, referring to expression C mentioned by S6 |
| 4 - 16/02, 17:56 S7: I agree with S6 | S7 concisely expresses his agreement |
| 5 - 16/02, 17:56 S8: According to me, C is the requested conjecture | S8 concisely answers R and T's questions |

**Table 7.** Padlet wall: excerpt of discussion thread 2 (T=teacher, Si=student)

| Excerpt from the comments | Cultural analysis |
|---|---|
| 6 - 14/02, 14:53 Researcher's guiding question: One of the groups presented the following conjecture. What do you think about it? | R tries to activate students' epistemic needs by posing as a $M\text{-}_{AE}AB$ (see Table 3) |

| | |
|---|---|
| 7 - 16/02, 15:58 S9: I believe that the conjecture the group proposed is complete and, in my view, correct. It is complete because they wrote "concrete" examples, if we can define them this way, moreover they added a "formula" to be used with whatever number we decide to consider. It is correct because my group and I reasoned more or less in the same way | S9 reacts to the epistemic need activated by the researcher: she answers R's question, but the justifications that she provides remain at a superficial level |
| 8 - 16/02, 17:22 T: The conjecture here is "the result is divisible by the initial number", is it the only conclusion that emerged addressing the first problem? | T intervenes in the discussion with a comment in tune with the role of a practical-strategic guide aimed at activating students' epistemic needs with respect to the identification of new conjectures related to the addressed problem |
| 9 - 16/02, 18:00 S10: My group and I worked in a different way, that is: the result is as doing the number times its predecessor and its successor | S10 reacts to the epistemic need activated by the teacher expressing the conjecture elaborated by his group |
| 10 - 16/02, 18:04 S11: I thought for example $4^3$-4=64-4=60 and 60 is a multiple of 4 (4x15=60) | S11 reacts to the epistemic need activated by the teacher presenting an example in tune with the conjecture reported by T |

The cultural analyses of these two excerpts from the Padlet wall indicate that, within the whole class asynchronous mathematical discussion, the researcher's guiding questions and the teacher's interventions stimulate students' epistemic needs (#1 #3 #6 #8). However, students' responses to these epistemic needs vary: some contribute by commenting on the Padlet without referring to the researcher or teacher's prompts (#2), while others express agreement with previous interventions in a concise manner (#4 #5) or offer their own ideas (#7 #9 #10). Despite these diverse responses, none of them demonstrates the emergence of the typical learning practices of Web-based online discussions of collective meaning-making and spontaneous mentorship. Moreover, students' contributions to the whole-class discussion on Padlet do not meet the expected objectives set forth by the guiding questions and by the subsequent interventions. In discussion thread 1, students do not formulate the missing conjecture, and in discussion thread 2, students expand on the proposed partial conjecture but still fall short of providing the complete conjecture (i.e., the difference between the cube of a natural number and the number itself is always a multiple of 6).

## 5. Results and conclusion

The aim of this study is to investigate the potential effectiveness of a novel discussion format that combines the strengths of Web-based online discussion and mathematical class discussion for enhancing learning.

The cultural analyses of the WhatsApp and Padlet interactions highlight that to shed light on the emergence of the learning practices typical of Web-based online discussion it is important to distinguish between the level of the whole discussion (macro level) and the level of specific exchanges between participants (micro level).

At the macro level, when observing the entire discussion on both WhatsApp and Padlet, it becomes evident that students' epistemic needs are only activated through the researcher's and teacher's interventions. However, these needs are not subsequently internalised by the students themselves. This is evident from the brief exchanges between students on both platforms and the absence of a collaborative solution being constructed in the chats. We see this as the first critical element of asynchronous mathematical discussion, probably due to a lack of devolution of the task which hinders students from spontaneously engaging in the discussion. At the micro level, we can distinguish different scenarios by observing specific exchanges in WhatsApp and Padlet. On the one hand, we have the WhatsApp chats, where students ask for (and obtain) advice from groupmates to complete the task: using the cultural analysis terminology, they rely on the community for collective meaning-making and provide spontaneous mentorship. On the other hand, the threads of comments on Padlet reveal that students mostly react to the teacher's prompt and only marginally interact with their classmates: generally, they concisely express agreement with another student's opinion, or they do not consider and do not relaunch on previous comments.

Considering the context of the experiment as presented in section 3, we observe that the difficulties that arise in in-presence mathematical class discussions (e.g., few students actively participate within the discussion, few students relaunch on their classmates' interventions) transfer to asynchronous mathematical discussions in both digital environments (WhatsApp chats and Padlet wall). The small number of students' interventions on the Padlet wall can be seen as the second critical element of asynchronous mathematical discussion: it hinders the whole class discussion because the teacher does not feel like pressing students with written messages when they are not texting back. In this way, it prevents the researcher and the teacher from moving forward with the implementation of further M-$_{AE}$AB roles and from reaching the expected objectives set for the different discussion threads of the whole class discussion on the Padlet.

Although spontaneous online discussions and asynchronous mathematical discussions share some structural similarities, our analyses show that they engage participants in significantly different ways. Simply providing students with an online environment for mathematical discussion is not enough to foster typical learning practices of Web-based discussions, especially if students are not already accustomed to participating effectively in mathematical class discussions. This suggests that for the redesign of future experiments, we can consider rethinking the teacher's M-$_{AE}$AB roles, figuring out how to promote the emergence of epistemic needs and foster task devolution, and encourage collective meaning-making and spontaneous mentorship to enliven the discussion.

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