

# DO YOU SEE MATH? HOW BAYESIAN INFERENCE AND INTERNET MEMES CAN SHED LIGHT ON STUDENTS' UNDERSTANDING OF MATHEMATICAL CONCEPTS

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*This study examines mathematics students' engagement with visual resources, using Bayesian inference and Wittgenstein's "seeing as" concept to explore how they use meme templates to represent mathematical concepts. The analysis of memes created by two high-school students reveals the significant influence of their mathematical knowledge and conceptual understanding on their representations, uncovering strengths and weaknesses not easily captured by conventional tasks. The findings contribute a fresh viewpoint on students' understanding of mathematical concepts, broadening the conversation on the role of visual inputs in mathematics education.*

## INTRODUCTION: THE IMPORTANCE OF VISUAL INPUTS

The role of sensory perception in the construction of mathematical concepts is widely considered pivotal (Andrá et al., 2015; Arzarello et al., 2005). Among sensory perceptions, visual inputs are deemed crucial for the understanding of mathematics (Arcavi, 2003; Duval, 1999; Presmeg, 2006; Radford, 2010). This importance stems not only from the fact that the ability to process visual representations is considered a fundamental aspect of human cultural development since cave paintings (Cecchinato, 2009) but also from the “pictorial turn” of 21st-century culture (Mitchell, 1995, p. 15). This shift has overturned the long domination of written text in Western scholarly culture. Historically, images were considered a means to communicate with people with limited literacy, while written texts were the prerogative of a cultured elite. Fuelled by technology, facilitating the creation and diffusion of visual resources, the *pictorial turn* has promoted images to the centre of contemporary “communication and meaning-making” (Felten, 2008, p. 60).

## THEORETICAL FRAMEWORK: *SEEING* AND *SEEING AS*

While recognizing the significance of visual inputs in aiding communication and meaning-making in the mathematics classroom, the task of choosing appropriate visual resources to effectively support learners in constructing and understanding mathematical concepts is challenging. Cognitive science has shown that the way our brain organises, identifies, and interprets visual sensorial stimuli is subjective and strongly influenced by our previous knowledge and expectations (Bernstein, 2008; Serìes & Seitz, 2013). Indeed, research has shown that the processing of visual information involves both a *bottom-up* and a *top-down* progression. In the *bottom-up* phase, our brain processes inputs piecing them together to build up higher-level information (e.g., shapes for object recognition). In the *top-down* phase, our

expectations (informed by our prior knowledge) influence our perception. These two phases correspond to what Wittgenstein in his *Philosophical Investigations* (1953) calls *seeing* (the *bottom-up* input processing) and *seeing as* (the *top-down* personal interpretation). Wittgenstein illustrates his concept of *seeing as*, emphasizing the subjective nature of interpretation in visual experiences, through the study of ambiguous images, exemplified by the duck-rabbit illusion in Figure 1.

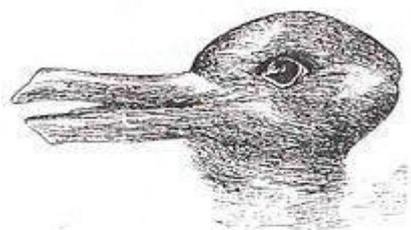


Figure 1: The duck-rabbit illusion

This visual puzzle, first published in the German magazine *Fliegende Blätter* on October 23rd, 1892, and subsequently investigated by the American psychologist Joseph Jastrow in 1899, presents an image that can be interpreted as either a duck or a rabbit, depending on the viewer's perception. Since its first appearance, the image has become paradigmatic to highlight how “we see with the mind as well as the eye” (Kihlstrom, n.d., par. 2), shedding light on the malleability and subjectivity inherent in our perceptual experiences. Indeed, it shows that perception is shaped not only by external stimuli but also by mental processes and expectations.

The influence of our expectations on the processing of visual stimuli is exemplified by Brugger and Brugger’s study (1993). The study shows how a sample of people presented with a stylised version of the duck-rabbit illusion tended to see it as a rabbit around Easter and as a duck (or a similar bird) in October. The testing place was at the main entrance of the Zurich Zoo in Switzerland; thus, we can imagine that the subjects’ expectations could be curved towards seeing an animal, but why a rabbit at Easter and a duck in October? We note that in Switzerland rabbit images at Easter and ducks in autumn are part of a shared folklore. Thus, Brugger and Brugger show us not only how much the subjects’ expectations shape their *seeing as* but also that, through this *seeing as*, we gain knowledge about the culture that infuses these expectations.

Research in cognitive neuroscience models the process of *seeing as* using Bayesian inference (Seriès & Seitz, 2013), which allows inferring how our brain evaluates the conditional probability that an interpretative hypothesis is true given some input data, taking into account a prior probability represented by existing knowledge. Specifically, if in Bayes formula  $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$  we consider A (Hypothesis) as the subject’s interpretative hypothesis of some visual sensory inputs B (Data), then Bayesian inference offers a mathematical model of how our cognitive system evaluates the reliability of an interpretation of visual sensory inputs  $P(\text{Hypothesis}|\text{Data})$ , based on

the *Likelihood* between data and hypothesis  $P(\text{Data}|\text{Hypothesis})$ , on *Prior* expectations  $P(\text{Hypothesis})$ , and on the *Clarity* of data  $P(\text{Data})$ , as detailed in Figure 2.

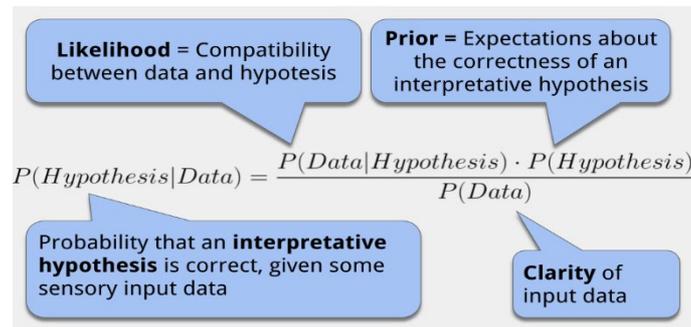


Figure 2: Bayesian inference on visual inputs interpretations

Thus, the subject's perceived probability that an interpretative hypothesis of some sensory input data is correct depends not only on the compatibility between data and hypothesis (Likelihood) but also on the ratio Prior/Clarity between the subject's expectations and the explicitness of input data. Focusing on this ratio, we see that its value gets bigger either when the Prior is very strong, or when the Clarity of the input data is very small. Thus, the more visually ambiguous the input data, the more the Prior influences the interpretation, as happens in Brugger and Brugger (1993). To sum up, our culture and existing knowledge constitute a Prior through which we interpret our visual perceptions, and their impact and visibility are all the more evident the vaguer is the information contained in the perceived visual stimuli (Esposito et al., 2023).

## INTERNET MEME TEMPLATES AS VISUALLY AMBIGUOUS STIMULI

Coming to the teaching and learning of mathematics, we can imagine gaining knowledge about students' *Mathematical Prior*, i.e., their knowledge and understanding of mathematical concepts, by investigating how they mathematically interpret images that can be considered visually ambiguous inputs from a mathematical standpoint. Examples of these images are meme templates (Figure 3): popular images diffused on the Web, that users utilise in creating Internet memes.

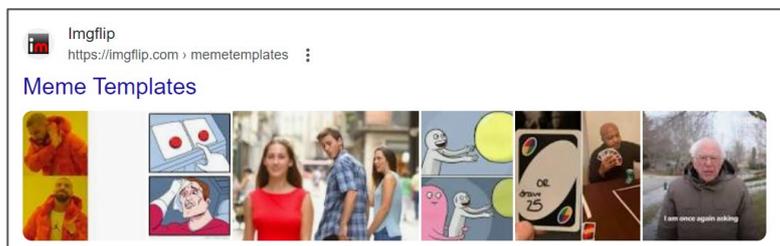


Figure 3: Samples of meme templates (source Imgflip)

Internet memes are a form of online cultural expression that rapidly spreads across the Web. They are typically relatable images accompanied by user-generated captions that evolve as users creatively modify and share them. Memes typically reflect current events, social trends or experiences, providing a shared language and cultural references within online communities (Bini et al., 2022). Meme templates are the

foundational elements from which various mutations of Internet memes are created. They serve as a framework for users to insert their own content while adhering to the established format (Bini et al., 2023). Templates contribute to the virality of memes by offering a familiar structure that users can adapt to convey their unique perspectives or reactions, overlaying original texts to express different messages, and fostering a sense of collective humour and connection in the landscape of digital culture.

Meme templates are born outside the mathematical context and are visually ambiguous from a mathematical standpoint, i.e. they do not have an intrinsic mathematical content. Nevertheless, they can be *seen as* representations of mathematical concepts and, in this sense, they are used within dedicated online communities to create memes representing mathematical statements (Bini et al., 2022).

This research is therefore guided by the following research question: What prior mathematical knowledge and understanding can we infer from students' seeing meme templates as representations of mathematical concepts?

## METHODOLOGY AND METHODS

Data come from a school experiment conducted in May 2018 with a group of 27 12th-grade learners in a scientific-oriented high-school in Italy. The task, to be completed individually at home as an end-of-the-year recap activity, was to create a mathematical meme on one of the year's maths topics and record an explanatory video of the addressed mathematical concept. Students shared memes and videos through a collective digital space, using the free Web-app Padlet, and subsequently their productions were the focus of a class discussion.

Assigning this task sets a cultural scenario that we can expect to activate students' mathematical priors influencing the act of *seeing as* meme templates as representations of mathematical concepts. This cultural scenario is in turn influenced by the way of schooling mathematics in the geographical environment that constitutes the setting for the experiment.

In the following July 2018, two of the students were interviewed, Mario and Luca (pseudonyms), selected because they created memes that were particularly interesting as they used templates to carry part of the mathematical meaning and not simply to emotionally reinforce the meme's message. Students were interviewed together, and they were both asked to explain how they got their idea for the meme. Interviews have been audio-video recorded, transcribed and subsequently analysed utilising a qualitative methodology and adopting an interpretative approach as outlined by Cohen et al. (2007), focussed on eliciting how Mario and Luca's mathematical priors influenced their view of meme templates as representations of mathematical concepts.

## DATA AND ANALYSIS

### Mario's mathematical meme

Mario's meme (Figure 4, centre) focusses on the study of a function, specifically on the process of plotting the first derivative sign chart to find a function's local maximum. In the cultural context where the experiment was conducted, it is customary to complete this sign chart with slanted arrows, pointed upward for increasing intervals and downward for decreasing ones. Body shapes in the template used by Mario (Figure 4, left) evoke the outline of these slanted arrows, that also appear in his explanatory video (Figure 4, right).

Researcher: How did you get your idea for the meme?

Mario: I was scrolling through various images [on Instagram] and I came across this picture [Figure 4, left], obviously without text, with these three people put in strange positions and the mind of a normal person says *ah that's nice, they are in strange positions*, while the mind of a math student says *this could represent a function* and therefore be effectively seen as a function following the lines of the bodies and the vertices formed by the heads and feet look like the maximum and the minimum of the function, so I said it could be a creative way to find the maximum and minimum of a function

Researcher: Ok, so it was the image that evoked a mathematical thought [...] while scrolling, I get a mathematical idea

Mario: exactly!

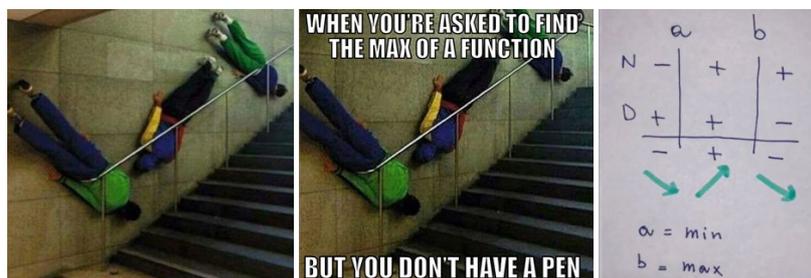


Figure 4: Mario's template (left) finished meme (centre) and video explanation (right)

Mario's attention is seized by the template (Figure 4, left), which he *sees as* (his own words!) the graphic representation of a function. He neatly captures the role of the culture in interpreting the image, picturing that *a normal person* would not see in it more than people *in strange positions*, while a mathematically infused observer (*a math student*) is reminded of the graph of a function with a maximum and a minimum. However, his understanding appears confused: he shifts from referring to the cartesian graph of the function itself to referring to the derivative sign chart needed *to find the maximum and minimum of a function*, which is the idea apparently addressed by the meme (Figure 4, centre) and developed in the explanatory video (Figure 4, right). We can hypothesise that Mario's confusion is induced by the similar appearance between the possible cartesian graph of a function *following the lines of the bodies* in the template (such as  $y = x - x^3$ ) and the arrows in its derivative sign chart. Arrows in his video match the bodies in the template but show another misconception: they describe the increasing and decreasing trend of a quotient (N and D stand for numerator

and denominator, and the chart corresponds to the sign rule for division), but the value  $b$  where the denominator of the derivative changes its sign is not necessarily a maximum for the function. This analysis reveals that Mario's mathematical prior is deeply influenced by the iconic aspect of the mathematical concept he wanted to represent, a fact that hinders his understanding and produces the confusion.

### Luca's mathematical meme



Figure 5: Luca's template (left) finished meme (centre) and video explanation (right)

Luca's meme (Figure 5, centre) addresses the fact the derivative of the exponential function  $y = e^x$  is the function itself. Luca's template (Figure 5, left) is a two-panel image to be read from top to bottom: in the upper frame a bartender is throwing a customer out the bar, and in the lower frame the customer reappears undisturbed behind the bartender. This template is habitually used to represent recurring events: in this case, the mathematical fact that  $\frac{d(e^x)}{dx} = e^x$ , which is subsequently proved by Luca in his explanatory video (Figure 5, right).

Researcher: The same question for you: how did you get your idea for the meme?

Luca: I did not have any idea yet... so I was looking among the various templates to see if any could particularly inspire me and then I saw this template [Figure 5, left], which by the way I had actually already thought of, that is, I had already seen that template and something about it had already occurred to me before doing this work on memes... so as soon as I found the template it was an epiphany... because after having worked on it so much it was automatic, that is, the image itself unleashed...

Researcher: So, it's the same thing ...

Luca: Yes, it is the image that brings [the mathematical concept] to your mind... [...] it is precisely the image and the mathematical topic that are connected, almost naturally

Similar to Mario, Luca's attention is captivated by a template (Figure 5, left) discovered while scrolling the Internet, activating a mathematical idea. Luca acknowledges the role of the culture in his vision (*after having worked on it so much it was automatic*) but it's important to highlight that Luca's *epiphany* is sparked by the metaphorical value of the image, representing recursion, rather than by its iconic value, as is the case with

Mario's meme. This reveals that Luca's mathematical prior involves a conceptual understanding of the mathematical idea represented by the meme, that attains to the deep mathematical structure and not simply to its surface appearance. This conceptual understanding is confirmed in the video explanation where he faultlessly proves that the “derivative of  $e^x$  is  $e^x$ ” (Figure 5, right).

## RESULTS AND DISCUSSION

The analysis of the mathematical memes created by Mario and Luca provides valuable insights into how students' mathematical prior influences the way they see templates as representations of mathematical concepts. Comparing Mario's and Luca's cases, it becomes evident that students' mathematical priors, constituted of conceptual understanding of the topic, cultural context, and educational experiences, play a crucial role in shaping their interpretation of visual stimuli. Borrowing Etkind and Shafrir's terminology (2013, p. 5347), we can affirm that Luca is a *good conceptual thinker* and his mathematical prior allows him to recognise the *meaning equivalence* between the template and the mathematical topic, while Mario is a *poor conceptual thinker* and is misled by the *surface similarity* between the template and contrasting mathematical concepts. Acknowledging the limitation of the sample cases, these examples illustrate the significance of analysing how students interact with visually ambiguous stimuli, such as meme templates, to gain insights into their knowledge and understanding of mathematical concepts, which is something that is not so straightforward to achieve with traditional tasks. The analysis also uncovers students' strengths and weaknesses, such as misconceptions and cultural influences on students' understanding of mathematical concepts. The findings from this research have implications for educators in planning interventions to address misconceptions and select further attuned visual inputs to support students' meaning-making.

In conclusion, the analysis of meme templates as visually ambiguous stimuli offers a unique lens through which to explore students' understanding of mathematical concepts. The insights gained from this study can contribute to the broader conversation on the role of visual inputs in mathematics education and provide practical considerations for educators seeking to enhance students' mathematical understanding.

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