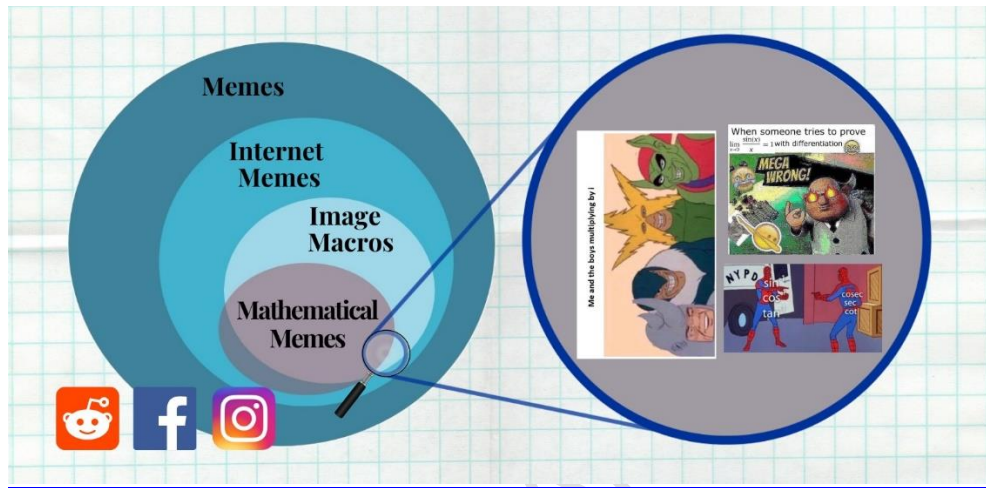# Maths in the time of social media: conceptualizing the Internet phenomenon of mathematical memes

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Mathematical memes are an Internet phenomenon with an epistemic potential noteworthy for the teaching and learning of mathematics. The aim of this paper is to conceptualize this phenomenon on an empirical base, elucidating its educational potential. To pursue this goal, a two-year ethnographic research has been conducted in a selection of social media communities that exchange mathematical memes. In the paper, a two-fold theoretical lens is used to investigate the phenomenon from a memetic perspective focused on the general characteristics of Internet memes, and from a cultural perspective considering the Web 2.0 environment. Analysis tools derived from the memetic and cultural theoretical background are applied to an instrumental case study, to reconstruct how mathematical memes activate the interaction among members of an online community. The investigation elicits key characteristics of the phenomenon, provides grounds to conceptualize mathematical memes as hybrid representations of mathematical statements, and sheds light on the traits of the epistemic culture they establish in the community.

> Though this be madness, yet there is method in it.
> *Hamlet Act 2, scene 2, 203–204*

Introduction: purpose and object of the study

Memes are humorous digital objects created by web users copying an existing image and overlaying a personal caption. Since their appearance in the late 90's, memes have been colonizing the web and their use spontaneously extended from the initial entertaining goals (Miltner, 2014) to encompass different subjects, including serious themes as politics (Huntington, 2017) and educational topics as mathematics (Benoit, 2018).

This research focuses on mathematical memes, an Internet phenomenon attracting communities of thousands of followers, still significantly under researched. The purpose of the study is to reach a conceptualization of the phenomenon that sheds light on its educational potentialities. To clarify the object of the study, we start with an example. The meme known as *Me and the boys* appeared in May 2019 on the popular social media website Reddit, when users started sharing posts with the same figurative element, a screenshot showing the villains from a 60's Spider-Man animated movie. This image, complemented by different captions all with the same *Me and the boys* incipit, was used to metaphorically represent a group of students doing things together, with a view to evoking nostalgic memories of schoolboys' lives (Figure 1a and 1b).

Figures 1(a) 1(b). *Me and the boys* mutations with different captions [source Reddit]

These posts were immediately widely liked and shared, and rapidly followed by others partaking the same evocative plot, differing in the textual part and in some alterations of the basic picture (Figure 2a and 2b).
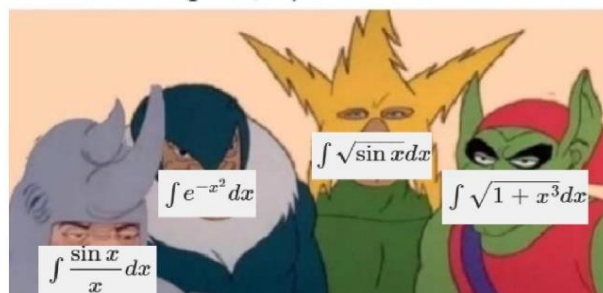
Figures 2(a) 2(b). *Me and the boys* mutations with edited images [source Reddit]

At the beginning of June 2019, two other users posted a couple of mathematical mutations of the *Me and the boys* post (Figure 3a and 3b), shifting the narrative from commonly shared schoolboys' memories to mathematical graduates' knowledge.

Figures 3(a) 3(b). *Me and the boys* first mathematical mutations [source Reddit and Facebook]

At the end of June 2019, another mathematical mutation (Figure 4) appeared. This mutation, although implying the acquaintance with the iconographic part (the original *Me and the boys'* horizontal image), distances itself from the initial storyline, bending it (or we could say *rotating it*) to convey a hidden meaning, hinted by the text and accessible only to mathematically savvy viewers.

Figure 4. *Me and the boys* second mathematical mutation [source Facebook]

In the example described, a popular image becomes a meme changing in the hands of web users who reinterpret it, adding a personal humorous touch. The process suddenly turns towards specialized mathematical contents when the memetic narrative device is edited to encode a mathematical idea. This mutation is further followed by users, who consequently become involved in the construction of a new form of representation of mathematical ideas, different from all other known representations, such as diagrams, graphs, number lines, manipulatives or formulas (Goldin, 2014). Reconsidering this process, we can grasp the gist of what memes are: a type of visual entertainment in the form of an indefinitely editable narrative device, composed of an image and a text. In the networked Internet environment, a meme spreads from user to user with changes in the text and possibly in the image, maintaining an overall recognizability. Mathematical memes are a type of memes in which the textual part refers to a mathematical idea, that can be identified combining the information carried by the text with those carried by the image: in other words, a mathematical meme is a kind of riddle that can be decoded with the help of mathematics. The encoding/decoding mechanism

mentioned above enlightens the educational potential of these objects for the learning of mathematics.

Rationale and research questions

Internet has produced a digital environment in which 21st-century students are immersed and where knowledge is easily accessible. In this culture, memes are recognized as building blocks (Milner, 2013; Marwick, 2013; Miltner, 2014; Shifman, 2014). Until now, they have been studied across different perspectives: digital culture (Stryker, 2011; Davison, 2012; Börzsei, 2013; Shifman, 2014; Danesi, 2019), genre studies (Wiggins & Bowers, 2015), semiotics (Cannizzaro, 2016; Osterroth, 2018), rhetoric and affect (Huntington, 2013, 2017). Their significance in the online culture is so widely established that, recently, research efforts have been aimed at developing a new technology to make memes accessible for people with visual impairment (Gleason et al., 2019). However, memes are greatly understudied in educational research written in English: we have found examples in new literacy (Lankshear & Knobel, 2003; Knobel & Lankshear, 2005, 2007, 2018; Silva, 2016; Harvey & Palese, 2018; Wells, 2018), racial discourse (Yoon, 2016) and visual literacy in language education (Romero & Bobkina, 2017). Portuguese-speaking academia seems more open to integrating memes into educational settings: an updated survey of Portuguese educational literature on memes, referencing 17 studies on the subject, can be found in Santos and Carvalho's work (2019). In the field of mathematics education, apart from Bini and Robutti's (2019a, 2019b, 2019c) conference papers and Ward-Penny's work (2011) that focuses on pre-Internet mathematical memes, we have found only five studies on mathematical memes. One in Spanish (Beltrán-Pellicer, 2016), three in Portuguese (Gonçalves & Gonçalves, 2015; Gonçalves, 2016; Friske, 2018) and one in English (Benoit, 2018). Benoit looks into mathematical memes to study the perception of mathematics in popular culture. Beltrán-Pellicer (2016) and Friske (2018) describe their class-use with students and prospective teachers. In Gonçalves & Gonçalves

(2015) and Gonçalves (2016), beyond a categorization according to teaching aspects (such as theme or purpose) of 30 mathematical memes, no further clarification of the phenomenon of mathematical memes themselves is revealed.

All cited studies agree on the fact that memes can activate educational situations where some knowledge, e.g. mathematical, is learned by users. However, they achieve their findings by focusing on memes only, separated from their natural habitat. This approach leaves two significant aspects aside: the study of the connections between mathematical memes and their ecological cultural environment, and the study of the interactions they initiate within online communities. Mathematical memes are *media*: they represent, and therefore communicate, mathematical ideas. According to Jenkins and Gitelman, media are defined through a two-level model "on the first, a medium is a technology that enables communication; on the second, a medium is a set of associated 'protocols' or social and cultural practices that have grown up around that technology" (Gitelman, 2006 as cited in Jenkins, 2006, pp. 13–14). Referring to these two levels, we want to deepen our understanding of the phenomenon of mathematical memes as representations of mathematical ideas and investigate how it is shaped and how it is used within online communities. To be more specificsorting we ask:

- How can the phenomenon of mathematical memes be characterized as a representation of mathematical ideas within the Internet culture?
- How do mathematical memes activate and guide interactions among members of online communities?

These research questions are answered by investigating mathematical memes with a two-fold theoretical lens: a *memetic* background, focused on characterizing mathematical memes as Internet artefacts; and a *cultural* background, dedicated to characterizing mathematical memes as cultural objects of the Web 2.0. Based on the innovative character of our research, we have entered the field with an ethnographical approach, deciding to collect mathematical memes and their threads of comments in social media communities. In the paper, we start our empirical

investigation by mapping the field in order to identify appropriate data sources to study the phenomenon in relation to its online communities. Then we identify paradigmatic cases for an instrumental case study to reconstruct in detail how a community deals with the mathematical idea encoded in a meme. We finally discuss the results with respect to the original questions, providing a conceptualization of the phenomenon and reflect on the educational potential of mathematical memes for the teaching and learning of mathematics.

Theoretical background

The memetic perspective: mathematical memes as Internet artefacts

As a first step to understand mathematical memes, we address the fact that they are part of the general Internet phenomenon of memes, following a line of investigation aimed at eliciting *what* constituent elements of Internet memes occur in mathematical memes and *how* these elements are edited to encode a mathematical idea.

The first definition of memes as a contemporary Internet phenomenon is given by Davison (2012, p. 122): "an Internet meme is a piece of culture, typically a joke, which gains influence through online transmission". A more recent and detailed definition, frequently referred to in literature, is the online Oxford Dictionary entry[1] (n.d. para. 2) where a meme is described as "an image, a video, a piece of text, etc. that is passed very quickly from one Internet user to another, often with slight changes that make it humorous". Memes are generally shared in social media platforms and dedicated websites (meme aggregators): these environments, identified in literature with the terms *memesphere* (Stryker, 2011) or *memescape* (Wiggins & Bowers, 2015), are governed by collectively established and shared rules (Osterroth, 2018), and constitute the virtual and cultural habitat where Internet memes are created and consumed.

Although memes have become common ground in the Internet era, the term meme dates back to the mid 7's, when evolutionary biologist Dawkins derived it from the Greek word

μίμημα, meaning *imitation*, to identify "a unit of culture that is passed from one generation to another and can be understood as the cultural equivalent of a gene […] examples of memes are tunes, ideas, catch-phrases, clothes fashions, ways of making pots or of building arches" (Dawkins, 1976, p. 249). Merging the various definitions, we can say that Internet memes are comprised of the following elements: a digital image and/or video or text, a typically humorous nature, a very high changeability and spreadability (Jenkins, 2013), and a capacity of being a unit of culture. Due to their digital nature, Internet memes spread across space and cultures at an extremely faster pace than their offline ancestors, and have reached a massive worldwide diffusion (Figure 5, data collected by the first author).
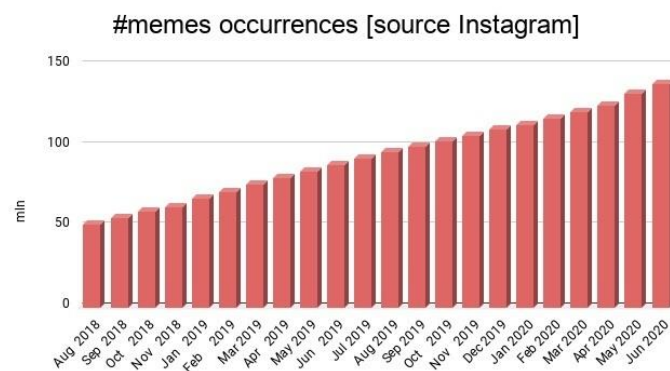

Figure 5. Diffusion of the hashtag #memes in millions [source Instagram]

Artefacts corresponding to the hashtag #memes belong to all categories listed in the dictionary definition: they can be images, videos or texts. In fact, Shifman (2014, pp. 99-117) classifies nine meme genres between video and image-based memes. Video-based memes in Shifman's classification are (1) "flash mobs", videos depicting strangers who gather in a public place to perform a particular act, (2) "lip-synch", videos in which a popular song or movie clip is dubbed with a user-created soundtrack, (3) "misheard lyrics", captioned videos with deliberately erroneous translations of spoken sounds, and (4) "recut trailers", reinterpreted movie trailers. Image-based memes are (5) "photo fads", staged photos in which the subject imitates a specific action, (6) "reaction photoshops", user-created images incorporating a

specific recurring element, (7) "LOLCats", images of cats with misspelled captions, (8) "rage comics", amateur-looking comics with particularly expressive characters, and (9) "stock character macros" (now known as *image macros*), existing digital images wittily reinterpreted with superimposed captions. Image macros are by far the most diffused kind of meme, to the extent that, in a process of semantic narrowing (Wilkins, 1996), the term *meme* is now commonly used as a synonym for image macro, and the original wider meaning is almost forgotten.

Mathematical memes come also mostly in the form of image macros, so we focus on this genre of memes, exemplified by the *Me and the boys* opening example. In Figure 6 the relation between mathematical memes and other memetic artefacts is represented, showing a nearly nested structure.
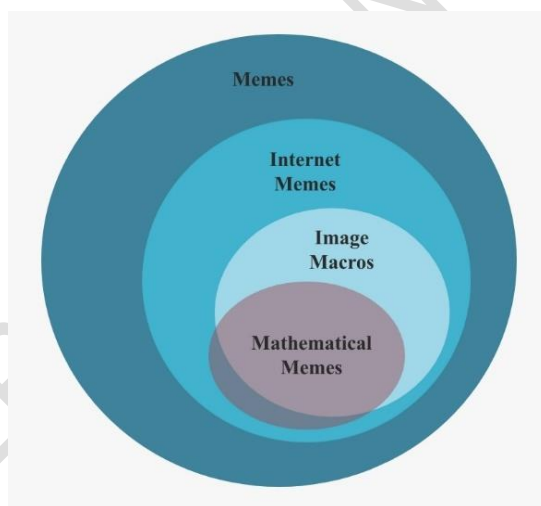

Figure 6. Embedding mathematical memes into the general phenomenon of memes

As we now take image macros as our core focus, we give a description of their constituent elements and of the process that originates them, based on literature (Davison, 2012; Shifman, 2014; Wiggins & Bowers, 2015) and on the first author's two years' experience observing online communities dedicated to mathematical memes and collecting around 1000 mathematical memes. As already noted, memes are basically jokes made up of a pictorial and a textual part. The pictorial element is usually an eye-catching picture, which appeared on the

web in a different context: it can be the portrait of a celebrity or a common person, a famous logo, a snapshot from a video or a comic strip, as in the *Me and the boys* example. The creator of the meme extracts it from its original context and encodes a new meaning in it, adding a coherent humorous caption and possibly editing the image, and uploads it to a sharing website. The meme, thus, begins its virtual roaming in the memesphere, where it can inspire new mutations. Some images prompt only a few personalisations, while others become established *meme bases*, acquiring names and codified meanings, with the initial figure clearly recognizable as a characteristic trait. Here is where the meme acquires its value as a unit of culture, where the image becomes able to transmit meanings for those who have the knowledge to read it. The cultural level achieved is confirmed by the circumstance that established memes and their meanings are categorized in the Know Your Meme website (www.knowyourmeme.com), an online meme encyclopaedia created and edited by volunteer contributors around the world. In a nutshell, the core characteristics of these memes are *recognizability* of the meme base and *sharpness* of the joke, attained with a right match of humour and subject knowledge. Capturing the meaning of the meme base and connecting it to the text allows to decode the meme, grasp the joke and laugh, as happens in the *Me and the boys* example.

Memes' constituent elements are maintained in mathematical memes, with the textual and pictorial part edited to convey a particular mathematical idea. This allows us to summarize the constituent elements in the following compositional structure. Mathematical memes are artefacts where a meme base and a mathematical idea are merged through a humorous/emotional link (Figure 7) that embodies the mathematical content in a rigorous sense. This merger creates a new kind of encoded representation of a mathematical idea that can be decoded only if the mathematics and the meme base are understood, and the mathematical meme is consequently interpreted through the lens of this understanding.
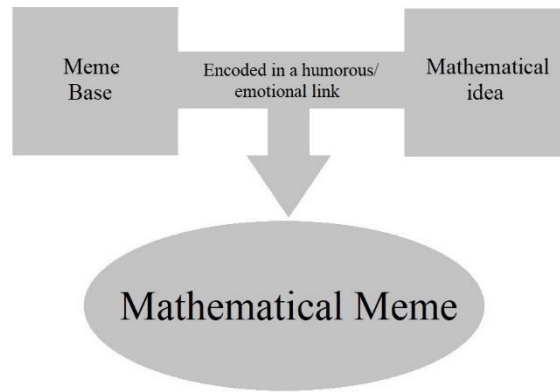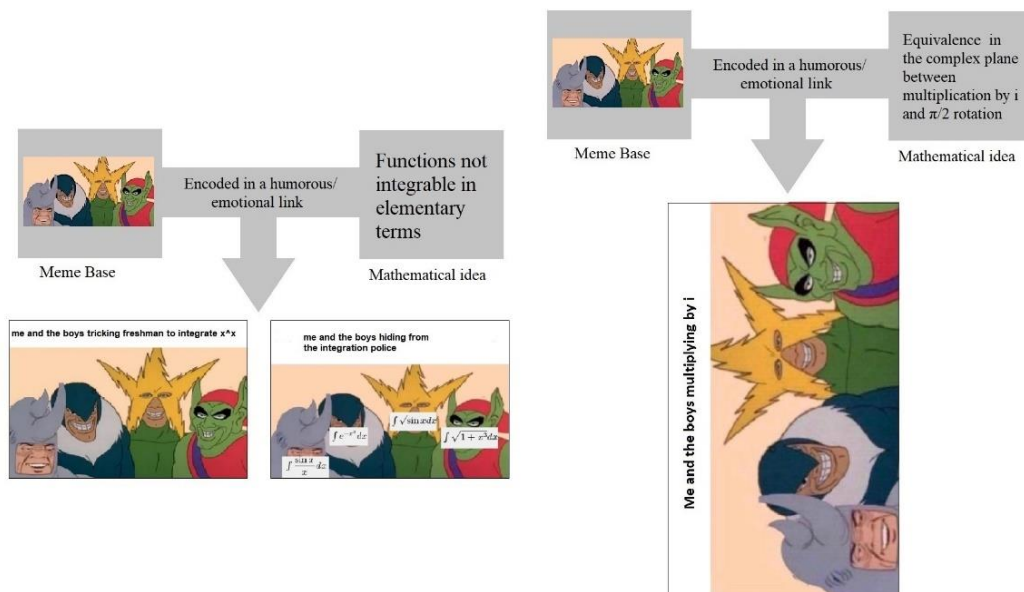
Figure 7. The compositional structure of a mathematical meme

To understand how these constituent elements are edited to encode a mathematical idea, we apply the compositional structure in Fig. 7 to some cases of popular mathematical memes, starting with the initial *Me and the Boys* examples.

Figures 8(a) 8(b). *Me and the boys* mathematical memes [see Figures 3 and 4]

Both memes in Figure 8a use the metaphorical meaning of the *Me and the boys* meme base as shared school experiences to connect a goliardic feeling to the mathematical idea of functions that are not integrable in elementary terms. In the meme in Figure 8b the conventional meaning is overruled, for the image is iconically used to encode the mathematical idea of the rotation in the complex plane, hinted by the caption. However, the link between the meme base

and the mathematical idea draws its humorous strength from the popularity of the original horizontal image.
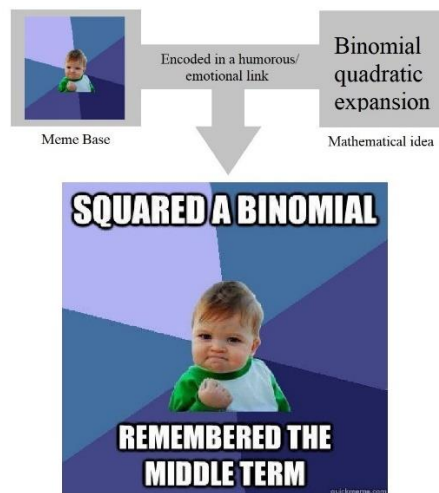
Figure 9. *Success kid* mathematical meme [source Facebook]

In the example in Figure 9, the metaphorical meaning of the *Success kid* meme base to boast about something good is used to connect a positive feeling to the correct computation of the binomial quadratic expansion. This meme encodes the mathematical idea in a didactical way, focusing on the recurring mistake of forgetting the middle term. The result is an artefact that can be decoded only if the viewer knows the binomial expansion, is aware that the middle term is often forgotten and thus can relate the mathematical message to the meme base metaphorical meaning.
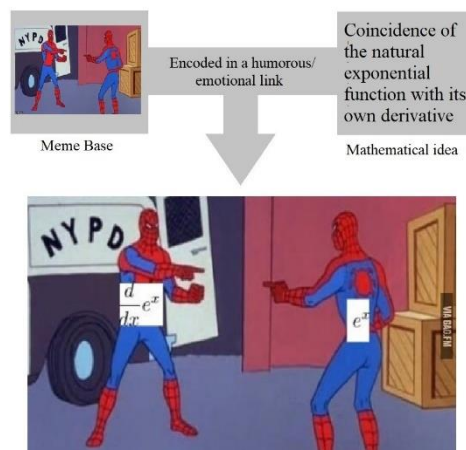

Figure 10. *Spiderman pointing at Spiderman* mathematical meme [source Reddit]

In the example in Figure 10 we see how the metaphorical meaning of the *Spiderman pointing at Spiderman* meme base, i.e. the meeting of two similar elements, is used to convey the mathematical idea of the coincidence between the exponential function and its derivative. In this case the meme base is used to encode part of the mathematical idea, namely the equal sign that links the two mathematical expressions.

Going back to the line of investigation at the beginning of this paragraph, the memetic theoretical background allows to understand that humour and emotions, spreadability and diffusion, and image macros structural and compositional rules are the constituent elements of Internet memes that occur in mathematical memes, and that, in mathematical memes, images are edited to encode mathematically rigorous effects (tangible or emotional) or elements of the mathematical idea hinted by the text.

The cultural perspective: mathematical memes as Web 2.0 cultural objects

As a second step to understand the phenomenon of mathematical memes, we address the fact that they are part of the culture that typify the Web 2.0, the development of the World Wide Web that emphasizes collaboration between users, social networking and user-generated content. This is done following a line of investigation aimed at eliciting *what* features of the Web 2.0 culture relate to the production and use of mathematical memes and *how* the perception of mathematical memes is shaped by these features.

The culture nurtured in the Web 2.0 environment is characterized by Jenkins as being *participatory* (2009) and *convergent* (2006, a term which has to be interpreted in a non-mathematical way). Participatory culture is typified by a "strong support for *creating* and *sharing creations* with others, [and] some type of *informal mentorship* whereby what is known by the most experienced is passed along to novices" (Jenkins, 2009, pp. 5-6, emphasis added). Building on Jenkins' idea, Bruns (2008) introduces the hybrid term *produsage*, a portmanteau of the words *production* and *usage,* to summarize the participatory environment nurtured by

social software, whose protagonists not only *use* digital content, but also *produce* it, as happens for example with online encyclopaedias like Know Your Meme or Wikipedia.

In this environment, "Internet users seemed to have grasped […] that *the meme is the best concept to encapsulate some of the most fundamental aspects of the Internet* in general, and of the so-called participatory or Web 2.0 culture" (Shifman, 2014, p. 18, emphasis in original) because they "harness the participatory potential of the Internet and typify modern popular culture" (Marwick, 2013, p. 13). So, the participatory culture inspires Internet users to create memes in general (Wiggins & Bowers, 2015) and mathematical memes in particular. It fuels the creative activity, which reveals itself both in the making of the mathematical meme and in the interaction around it, through which people with good math skills offer their "informal mentorship" (Jenkins, 2009, p. 6) to less skilled individuals. The qualifying value of participation in the Web 2.0 ecosystem explains also the role that memes have in this culture. In an environment where users "care what other people think about what they have created" (p. 6), memes are exchanged as "passcodes" (Schmidt, 2014, 00:08:55) to regulate the entrance to the memesphere or the "status in the community" (Stryker, 2011, p. 76).

The first author's observation of online communities dedicated to mathematical memes, revealed that these communities form a sub-region of the memesphere, whose citizens are proficient in memes and mathematics. Symbolic admission is granted only by sharing OC (original content), i.e. mathematical memes not copied from elsewhere, that have to be appropriate at both memetic and mathematical level. We take as an example the meme in Figure 11 and an excerpt of its comments[2]:

Figure 11. Checking the mathematics behind a meme [source Facebook]

1-C1: if your math is wrong in a meme, the repercussions are a lot more severe lol

[…]

2-C2: Well that's because the teacher is a single person who will ridicule you in private if you're wrong. Lol. I haven't shared anything to this group because the small pool of things I could come up with have already been done

3-C3: It's only right. You're risking your good name throwing math memes out there

This is a meme *about* mathematical memes (i.e. a *meta*-meme), that is taken as a source to gain information about the cultural practices (in the sense of Gitelman, 2006) grown up around mathematical memes. It describes the users' contrasting attitudes with respect to school mathematics and meme mathematics. Produsers of mathematical memes show a responsible attitude in checking their mathematical content, as opposed to a more naïve attitude shown towards school mathematics. Comments indicate the reasons why this checking has to be thorough (#1), that the content has to be original (#2), and that mathematical memes are perceived as personal identifiers in the community, acting as passcodes or as status qualifiers (#3). This example is one of many from which we can infer that mathematical memes are perceived as qualifying proofs of produsers' mathematical knowledge, because memes in these

communities are publicly scrutinized by a cultured, merciless and potentially worldwide audience.

Besides participation, the second typifying characteristic of the Web 2.0 culture is *convergence*. Convergence is not simply the wide availability of information funnelled in a single device, but is "a cultural shift [in which] consumers are encouraged to seek out new information and make connections among dispersed media" (Jenkins, 2006, p. 3). In a convergent perspective, academic and popular culture are perceived as a continuum. For example, a recent discovery in number theory has been inspired by an episode of the popular TV series The Big Bang Theory (Pomerance & Spicer, 2019), and an anonymous post on the social platform 4chan has given an essential contribution in solving a 25-year-old problem in combinatorics (Anonymous 4chan Poster et al., 2018). In the memesphere, convergence is "a central pattern" (Danesi, 2019, p. 57): this implies, among other things, that the traditional boundaries of education are challenged. Thus, Web 2.0 users are ready to learn even if they are not at school (Thomas & Seely Brown, 2011; Ito et al., 2013), in a process of "*collective meaning-making* [...] within the brains of individual consumers and through their *social interactions with others*" (Jenkins, 2006, pp. 3–4, emphasis added). Convergence describes how mathematical memes act as "cultural activators, setting into motion their decipherment, speculation, and elaboration" (p. 95). They spark a process of collective meaning-making, providing opportunities for non-curricular learning experiences, a practice which is widely confirmed by evidences shared on the web. In Figure 12 we see two meta-memes expressing appreciation for the use of mathematical memes to support the understanding of mathematical concepts (12a) and to foster engagement (12b).

Figures 12(a) 12(b). Mathematical memes as learning objects

However, the meta-meme in Figure 13 shows that mathematical memes produsers are also aware that, to build proper mathematical knowledge non-curricular, meme-induced learning is not enough, but it has to be put together with curricular learning, in a truly convergent perspective where "old and new media collide" (p. 2).

Figure 13. Don't forget to read some actual maths [source Facebook]

To sum up, the cultural theoretical background provides evidence to characterize mathematical memes as cultural objects, showing that participatory culture inspires the production of mathematical memes and shape their perception as membership identifiers, and that convergence culture inspires ways of using them as learning opportunities.

Methodology and methods

Based on the theoretical background, we were able to characterize mathematical memes as memetic artefacts encoding mathematical ideas, and as cultural objects in dedicated participatory and convergent online communities. These characterizations give some important information to start building up our answers to the research questions. What is still missing is a characterization of mathematical memes as *representations of mathematical ideas* and a deeper investigation of how they *activate and guide interactions* within the community. Since such a representation not only differs from all usual mathematical representations but is also in constant change in the mathematical memesphere, it can only be characterized from inside the meme community, for example analysing the meme's decoding process in the situated interactions among its members.

To address the missing information, we have approached the field ethnographically (Eisenhart, 1988; Harwati, 2019), considering practices as cultural habits shown and re-established in the interaction among the members of the community (Gitelman, 2006). This ethnographic approach is chosen because it is aligned with our aim to investigate "what people's opinions are about a particular phenomenon occurred in their social context and what their action is". (Harwati, 2019, p. 152). The study began in February 2018 and is still in progress. It is conducted by participant observation of the first author being a read-only (a *lurker* in the Internet jargon) member of the communities without interacting with them. This role allowed the first author to become "part of the situation being studied in order to be able to feel the way people do in that situation" (p. 152), taking care not to perturb the environment while gathering data.

Data collected in the ethnographic research are chronological field notes, memes (constituting a *meme pool*) and their threads of comments (constituting a *comment pool*). Focusing on the comment pool, we reconstruct the interaction activated by the memes from

inside the observed communities, considering the meaning-making processes. Because of the novelty of our topic, an instrumental case study that aims at theorizing through comparing cases is an appropriate approach (see Grandy, 2010), coherently combined with the ethnographic approach (Harwati, 2019). This case study follows the principle of comparison between two contrasting empirical cases, which represent polar types of mathematical memes, one being mathematically right and the other mathematically wrong. Data are analysed based on the tools provided by the theoretical background. The final step is theorizing the results with respect to the original aim of reaching a conceptualization of the phenomenon of mathematical memes. Since our study addresses an under-researched field of a fluid nature with a huge amount of available data sources on the web, our first step is to systematically map the field to identify appropriate data sources that could provide paradigmatic cases to investigate the community from the inside.

Mapping the field and identifying appropriate data sources

Memes are everywhere on the web, from blogs to meme aggregator websites and social media, but mathematical memes are shared mainly in social media websites. Therefore, to organize the research, we referred to the shared methodologies of qualitative research in social media as exposed by the Social Media Research Group (2016), the SAGE Handbook of Social Media Research Methods (Sloan & Quan-Haase, 2016) and, in particular, to Hand's work onimpairment researching images in social media, included in the latter. According to Hand (2016), the primary methodological issue in investigating images in social media is due to the large amount of available data, which necessarily implies a selection that will affect the research and therefore must be made knowingly. This selection is conducted beginning with websites, funnelling towards particular communities as data sources: Choices were made applying combined criteria of popularity and relevance for the research as described below.

Selecting social media websites

Within social media websites, the choice fell on Reddit (www.reddit.com), Facebook (www.facebook.com) and Instagram (www.instagram.com). Reddit (355 million users) was chosen for its relevance for the meme culture, being the commonly acknowledged birthplace of most memes (as happened to the *Me and the boys* example in the opening section). Facebook (2.26 billion users) and Instagram (1 billion users) were chosen for their popularity, being respectively first and second in the global social networks ranking (Statista[3], 2019).

Selecting communities

In social media websites, we looked for active (i.e. sharing new content at least weekly) communities interested in mathematics *and* memes, using both the intern search engines with keywords *meme+mathematics* or *meme+math/maths* and the *similar pages* suggestion option. Among them, the second level of selection was again made according to popularity, based on the number of followers, expected to imply deeper, wider and more meaningful interactions. Thus, communities with more than 10,000 (10k) followers were chosen (Table 1).

Table 1. Observed communities ordered according to the number of followers (June 2020)

Reddit		Instagram		Facebook	
r/dankmemes	3.7million	@bad science jokes	388k	Educational Memes	597k
r/math	1.2million	@juicy mathematical memes	128k	Mathematical Mathematics Memes [group]	226k
r/theydidthemath	601k	@meme for mathematicians	113k	I love Mathematics 2.0	110k
r/mathmemes	82.6k	@mathsforyou	62.6k	L'Agorà Del Superuovo	97.6k
r/physicsmemes	77k	@maths.meme	58.2k	Math Memes	72k
r/mathpics	21.4k	@memes educational	46.6k	Matematica del Suicidio	30k
		@mathematical jokes	27.6k	Matemáticas y Café	30k

		@puremaths memes	22.8k	>implying we can discuss mathematics	26k
		@calculuscious	21k	The Name Of This Group Is Left As An Exercise For Its Members	12k

Selecting data

Communities in Table 1 were followed daily by the first author to collect data for the research. Considering the prospective idea of using the results of this research to inform teaching in school, we selected mathematical memes encoding mathematical ideas on 8th to 13th grade mathematical topics according to the Italian national curriculum[4], and meta-memes, i.e. memes *about* mathematical memes. Memes adhering to these criteria were downloaded and archived using a double tagging criterium (meme base name + brief math reference). In the period of observation (February 2018–current) this process produced a meme pool of around 1400 items, filtering reposts. This meme pool was complemented by a comment pool of saved active links connected to the first author's personal accounts on the three websites to follow real-time comments engagement. Occasionally these communities share other image-based humorous digital objects addressing mathematical themes, like memes dealing with emotions connected to doing mathematics and mathematical cartoons. These other objects were also inspected to gather information for comparison that could contribute in characterizing mathematical memes.

Selecting cases

As noted in the participant observation and described by Jenkins (2006), mathematical memes communities are coherent in the way they interact with memes. Therefore, our selected cases are present not for themselves but as representative characteristics of a class, in a sense echoing that of Balacheff's "generic examples" (1988, p. 57). Further, there is no theoretical knowledge about the nature of this kind of interaction in the context of mathematical memes communities.

In order to disclose such theoretical knowledge we, therefore, have chosen the instrumental case study approach built on the "particularly important theoretical sampling strategy" (Eisenhardt & Graebner, 2007, p. 27) of polar types, "in which a researcher samples *extreme* [...] cases in order to more easily observe contrasting patterns in the data", [relying of the fact that] this sampling leads to very clear pattern recognition of the central constructs, relationships, and logic of the focal phenomenon" (p. 27, emphasis added).

Communities usually share correct mathematical memes, while wrong memes are extremely rare. For that reason, we choose a mathematically correct and a mathematically wrong meme as the two extreme polar cases in our sampling strategy.

The choice of the environment of the polar cases was based on a quantitative criterion of popularity, i.e. the number of followers. Selected cases were chosen from Facebook, confirmed as the most popular social media among teens (13-17 y.o.) with 113 million users (Hootsuite – We are Social Global Digital Report[5], 2019; Edison Research, The Infinite Dial[6], 2019). Within Facebook, we chose the most popular among the communities in Table 1: Educational Memes (597k users) and Mathematical Mathematics Memes (226k users). Among mathematical memes shared within these communities, we applied a qualitative criterium of choice, selecting the two memes among those where mathematics is deeply used, a characteristic expected to imply richer threads of comments and where highly relevant aspects are discussed, embracing Pettigrew's suggestion recalled by Eisenhardt (1989) that "if the phenomena to be observed is to be contained within a single or relatively small number of cases, then choose cases where the progress is transparently observable" (Pettigrew, 1988/1990, p. 275). In particular, our memes stood out because they dealt with two key issues for the teaching and learning of mathematics: the ability to identify circular arguments (Freudenthal, 1971; Hanna & de Villiers, 2008) in the correct meme, and the misconceptions that could arise from the polysemy of mathematical symbols (Durkin & Shire, 1991; Gray & Tall, 1994; Priss, 2018) in the wrong one. We excluded

a quantitative selection centred on the number of likes or comments because not fitting with the qualitative criterium of finding memes with deep mathematical content. The selection was carried out using a sorting criterion, to identify the two polar types, so that the emerging characterization is expected to be well grounded in their comparison (Eisenhardt, 1989; Eisenhardt & Graebner, 2007).

Analysing data

Data in the meme and comment pools have been explored with an ethnographical attitude, taking as a unit of analysis the memes referring to their threads of comments. Mathematical memes were taken as a source to gain information about the interaction with the mathematical idea represented in the meme, and meta-memes were taken as a source to gain information about the cultural value of mathematical memes and of the related interaction inside the community.

In the empirical case study, theoretical concepts previously provided in the paper are merged in the analyses. In the first step, the encoding of the mathematical idea in each case is shown according to the compositional structure presented in Figure 7. In the second step, a sequential analysis of the thread of comments is conducted, following Jenkins's (2009) idea that the interaction in a community is a collective process where each contribution is related to the others. In the analysis, the process of meaning-making is reconstructed following the interactive threads of comments, where each comment is taken as an interpretation of previous comments and of the meme itself. Comments convey two levels of information: an explicit information, carried by the text openly expressing something, and an implicit information, carried by the way the comments are articulated, conveying information about the commenters themselves, their relationship with others and with the meme. These two levels of information are traced through the excerpts by re-interpreting comments in a turn-by-turn-analysis (Jungwirth, 2003).

We begin with a case whose mathematical content is correct (and recognized as such by the commenters), and then we analyse the mathematically incorrect case. As the culture in the community is re-shaped in the interaction around the mathematical meme, the cultural codes *participatory* and *convergence* are added in the analyses when evidenced. The analyses with the coded excerpts are then used to identify and synthesize the emergent mathematical themes. Separate analyses of each of the two polar types are subsequently compared and contrasted to answer the research questions. This will show how the meme drives the interaction and how the interaction keeps going while decoding the mathematical idea hereby encoded.

Complying with the ethical principles of social media research (Social Media Research Group, 2016), we do not include nor analyse produsers' personal information, replacing identifiers in all quoted excerpts (A = author of the mathematical meme, C1 C2 C3… = commenters) and adopting an inclusive and bias-free language in our analysis. All comments are unredacted and originally in English. To follow the development of the interaction across time, the excerpts are reported in chronological order with the timestamp of the meme and of the first and last comment, and the indication, whenever present, of who is replying to who. If needed, slang terms are interpreted using the Urban Dictionary website (www.urbandictionary.com).

Analyses and results

To understand the phenomenon of mathematical memes with respect to their use as representations of mathematical ideas inside the community and investigate how they activate and guide interactions among members, we first summarize the results of our participant observation while collecting the meme and comment pools.

Participant observation: memes and mathematical ideas

The ethnographic research, documented by the ethnographic representative in the author group with chronological notes, has provided a data pool organised for the specific purpose of this part. From this data pool, we have extracted information about the practices around mathematical memes in the inspected communities.

Our observation revealed that almost all mathematical memes are followed, in the same social media page, by threads of comments where the meme's mathematical content is discussed by the meme's author and other users. It revealed also that the practice of exchanging mathematical comments is specific of mathematical memes: in fact, in the case of memes about emotions connected to doing mathematics (Figure 14a) there are usually no mathematical comments, but just common emojis or friends tagging. The same applies to cartoons about mathematics (Figure 14b) which are different digital objects having, sometimes, a proper mathematical content.
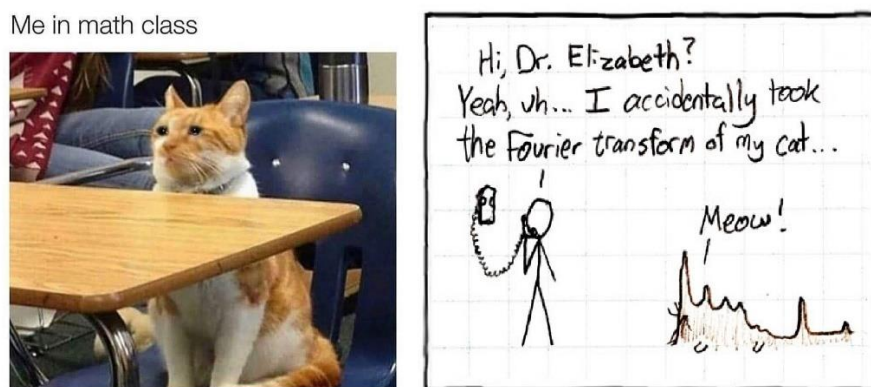
Figures 14(a) 14(b). Emotional memes and cartoons [sources Facebook and xkcd.com]

Thus, we can say that, among image-based humorous digital objects exchanged within the observed communities, threads of *explicated mathematics* stand out as a characterizing practice of the interaction with mathematical memes as representations of mathematical ideas.

The importance of mathematical comments in the community is confirmed by the high value given to this habit, as shown for example in the meta-meme in Figure 15.

When you post a math meme and people comment about the theory behind it

I am so proud of this community

Figure 15. The importance of mathematical comments [source Facebook]

The observation of these interactions proves that threads of explicated mathematics are not simply openings for more educated members of the community to show off some knowledge. In fact, they allow the collective meaning-making of the mathematical idea encoded in the meme to take place. This is clearly acknowledged in the comments to the meta-meme in Figure 16.

When someone posts a university level mathematics meme in the university level mathematics memes facebook page

I really really wish I understood this
3 d   Like   Reply

If (o)nly people would (c)onsider the high school students in MMM

2 DEC 2019

1.9k   149 comments

Figure 16. The importance of understanding mathematical memes [source Facebook]

This meme, posted in the Mathematical Mathematics Memes (MMM) page, addresses the issue of high school students' understanding of university level mathematical memes. The image of a hand moving fast towards a blue button is a popular meme base used to symbolise situations that trigger impulsive reactions. In this case the triggering situation is described by

the top text "*When someone posts a university level mathematics meme in the university level mathematics meme facebook page*" and the impulsive reaction is represented by the caption superimposed on the button "*I really really wish I understood this*". The author's note accompanying the meme "*if (o)nly people would (c)onsider the high school students in MMM*" and the number of likes (1,9k) tell us how much this feeling is experienced within the community, while the following excerpt from the 149 comments gives us insight into how the community relies on the threads of explicated mathematics to overcome these difficulties.

> 1-C1: Every time I've seen someone actually explain the mathematics behind a meme, it's been a great experience. … [I] feel like going and learning more. Thank you to those wonderful mathmemeaticians who help others appreciate the joy of glimpsing new realms of mathematics.
>
> […]
>
> 2-C2: I think this group is the very reason why i am even more interested in mathematics than i was before, even though i dont understand more than half of the stuff here...i m still learning and its really fun!

Here we see that explaining *the mathematics behind a meme* is accredited to activate the learning mechanisms (#1) and that mathematical memes are recognized as resources to foster learning and interest in mathematics (#2).

This characterizing practice is connected to both features of the Web 2.0 culture: it is participatory in the sense that it gives commenters the opportunity to contribute offering the characterizing informal mentorship, and it is convergent because the interplay between mathematical memes and comments allow the collective meaning-making to happen. Indeed, these communities operate as spontaneous communities of practice (Wenger, 1998), and build group meanings of mathematical concepts, acting as a "sphere[s] of practice" where "mathematical meanings are constructed" (Kilpatrick et al., 2005, p. 10).

The educational value of mathematical memes and their threads of mathematical comments is not only recognized by members of the community, but it may also be the reason

for the very existence of the community. This is the case of MEME4000W, two twin mathematical memes pages created by graduate students on Facebook and Instagram, to teach mathematical concepts using memes as a medium. The following interview excerpt illustrates the origin of the project:

> At the start of last year, I joined a mathematics meme group and realized that *I learn so much from the interactions on the group, through the memes and the comments* [emphasis added]. I wanted to share this experience with others and show them that maths isn't as threatening as people make it out to be: It can be experienced in a very fun and conducive environment. (founder of the pages, interviewed by the first author via chat, April 2019).

All these evidences indicate that threads of explicit mathematical comments connected to mathematical memes are exactly the loci where the characteristics of mathematical memes as representations of mathematical ideas become evident, and where the educational potentialities of these objects can be observed. In the next paragraph we deepen our understanding of the phenomenon by analysing the interaction in two contrasting cases selected according to the criteria explained in the methodology section.

Instrumental case study: Threads of comments on mathematical memes

Case 1: the correct meme

- Meme base name: *Mega Wrong!*
- Meme base use: stigmatize situations where something significantly wrong occurs
- Source: Facebook Total comments: 96
- Topic: Calculus
- Subtopic: Circular reasoning, Limits, Derivatives proofs, L'Hôpital's rule
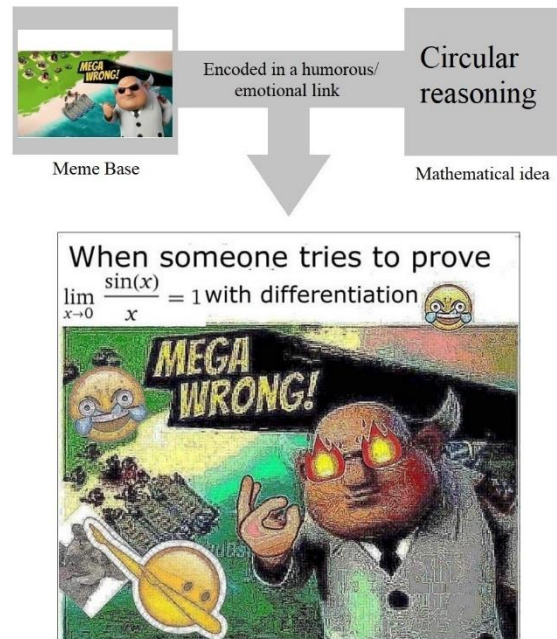- Publication year: 2019

Figure 17. Case 1: the memetic analysis

Contextual description: The meme in this example (Figure 17) is used to label as *Mega Wrong* the proof of the indicated limit using differentiation (i.e. L'Hôpital's rule). The proof is considered *Mega Wrong* because it is circular, involving a logical fallacy that occurs if we assume that the derivative of the sine function is obtained using the result of the $\lim\limits_{x \to 0} \frac{sinx}{x}$, which is the usual development of the topic in high school. This same procedure would be logically sound if, for example, we accept power-series definitions for trigonometric functions, and obtain the derivative of the sine function from these premises. This knowledge is mastered and disseminated by the author of the meme and by several commenters, as will be shown in the analysis. We chose key parts of the exchange taking a large section from the beginning where the engagement is established. We are taking further parts following the same mathematical topic. The selection is also made according to indicators related to the analysis perspective. The development of the comments goes in various directions, several proofs are offered, only one is included in the analysed excerpt (Table 2).

Table 2. Case 1 – the cultural analysis

Comments	Explicit information	Implicit information	Environmental culture	Emerging mathematical themes
1-C1: [Day 1; 16:29] How is lhopitals not correct	Asks for reasoning (*need*) about what is labelled as "mega wrong" in the meme	*How* means "how come", an *informal* way to ask about the reason, (*need for reason*). C1 shows knowledge (recognizing the limit and that differentiation refers to L'Hôpital's rule); C1 is open for broadening knowledge (why this procedure is incorrect)	C1 addresses the community as a knowledge holder for the meaning-making (convergence culture)	
[…]				
2-C3: That would be a circular argument	Gives a key word that it is "mega wrong" because of a logical fallacy	C2 leaves deeper reasons unexplained, acknowledges the expertise shown by C1	Informal mentorship (participatory culture)	Why using differentiation to solve the limit is incorrect
3-C3: um why?	Asks for reasons why (*need for reasons*)	The expression "*um*" indicates uneasiness, probably due to the *lack of reason* in C2's answer, thus C3 explicitly asks "why"	C3 needs help from the community for the meaning making (convergence culture)	
4-C4: You usually use sin(x)/x -> 1 to prove sin'=cos	Provides a missing reason, a brief *explanation* (reacts to the need) about the circularity	C4 shows experience and assumes C3 knows that applying L'Hôpital's rule to the limit means using the derivative of y=sinx, recognizes sin' as the derivative	C4 explains, hence, practices informal mentorship (participatory culture)	
5-C3: Oh, right. Yeah that	Acknowledgment	The acknowledgement without further questions indicates that C3 was already familiar with all information shared, but C4's comment helped to connect them	Meaning-making is complete (convergence culture)	

[…]				
6-C5: but i thought we used taylor series to prove that sin'=cos tho	Asks about the use of Taylor series instead of the limit for the proof of the derivative of sinx	C5 indicates a lack of information (*need*) C4's answer does not satisfy C5, C5 does not (seem to) recognize the role of the limit in proving sin'=cos	C5 expresses the *need* for more information (convergence culture)	
7-C4: [replying to C5] you begin with their Taylor series definitions?	Asks about the first step of the topic (*need for clarity*)	C4 is not evaluating C5's comment, but changes the focus of the discussion to the steps of the topic	Informal (didactic) mentorship (participatory culture)	
8-A: As long as you explicitly show that d/dx sinx = cosx then you can use l'hopitals rule	A intervenes to *clarify* (to an implicit *need*) which are the mutual dependencies of the steps	A brings the community back to the original problem Here the adverb "*explicitly*" referred to the proof of the derivative is to be read as independently of the limit in the meme	Informal mentorship giving a rule (participatory culture)	Digging deeper into the usual steps of proofs and what makes the reasoning in the meme circular
9-C5: ok nice ! 😊	Acknowledgement and acceptance of the explanations	C5 reacts with appreciation	C5 completes the meaning-making (convergence culture)	
10-C4: [replying to A] yeah, but in rigor, depending on how you begin, you *might need* first to prove sin(x)/x. In the usual development of the topics I suppose it's the case	Replies to A referring to the established steps of proofs: first the limit and then the derivative (as is implied by the meme)	C4 focuses again on the *usual* steps of proofs with the limit as the first step of the *usual development*. C4 connects this interpretation to the meme (*"I suppose it's the case"*), meaning this is the case where the "*mega wrong*" circularity occurs (clarifies the usual context of learning the maths of the meme, reacts to an implicit *need to clarify*)	C4 (coming back to the meme) acknowledges the fact that the meme is constructed according to what is mathematically consistent in the usual way of proving (convergence culture)	

[…]				
11-A: [replying to C4] that's what makes it circular reasoning because you'll use sinx/x to show sin'x = cosx and then go use that to prove sinx/x notice how that reasoning goes in circles	A explains in details the logical steps of the circular reasoning occurring if the steps of proofs starts with the limit (reacts to *need for reasoning)*	A comes back to the original meme and acknowledges C4's explanation of the fallacy ("*that's what makes it circular reasoning")* and gives a detailed explanation ending with a didactical observation "*notice how that reasoning goes in circles"*	A is self-positioning in the community as an expert, owning the meme acting toward a *need* to decode (participatory culture)	
12-C4: Yeah, I said that in my very first comment, lol [laugh out loud]	Acknowledgement	C4 points out that the same clarification had been already provided and endorses A's	C4 is self-positioning at the same level of expertise (participatory culture)	
[…]				
13-C6: How is it proved??	Clarification request about the proof of the limit sinx/x	Based on the previous comments and on the meme, C6 expresses the *need for a proof* of the limit in the meme. It may be part of the sentence: if it is "mega wrong" to prove it with differentiation, then *how is it proved*?	C6 addresses the community as a knowledge holder (convergence culture)	Proof of the limit using the sandwich theorem
14-A: [replying to C6] Geometric argument is the easiest way	A replies hinting to the geometrical proof of the limit	A reacts to the *need for proof*, stressing: there are other not mentioned proving techniques and A is able to select the *easiest* one	A is self-positioning in the community as an expert, offering informal mentoring (participatory culture)	

15-C4: You can geometrically prove sin(x) <= x <= tan(x), divide by sin(x) and use the reciprocal inequalities: cos(x) <= sin(x)/x <= 1 and then the sandwich theorem (plus geometrical demonstration that cos(x) -> 1)	Description of the steps of the geometric proof using the sandwich (squeeze) theorem to prove the limit (reacts to *the need to prove* by offering one)	The explanation is consistent although many elements are left to the reader to fill in: how to "*geometrically prove*" the first inequality and how the sandwich theorem actually works	Informal mentorship is worked out (participatory culture)	
[…]				
16-C7: So the reason why this doesn't work is dx/dy sinx=cosx is proven by lim x->0 sinx/x =1?	Questions about the wrap up of the reasoning (reacts to a *need for certainty*)	C7 summarizes the reasoning towards the others to confirm, showing that C7 followed the discussion and correctly grasped "*why this doesn't work*"	C7 addresses the community as a knowledge holder (convergence culture)	Final consideration about the correctness of the mathematical idea represented by the meme
17-C4: [Day 1, 17:11] Generally	Acknowledgement according to the usual way of proving	C4 does not notice C7 final typo on the derivative (dx/dy instead of dy/dx) possibly because the focus is on the wrap up of the reasoning	Final informal mentorship (participatory culture)	

The convergence culture is driven by different *epistemic needs* (Kidron et al., 2011) towards the understanding of the mathematics encoded in the meme: need for reasons (#1, #3), need for more information (#6), need for clarity (#7), need for argumentation premises (#10), need for proof (#13), need for certainty (#16), where the latter seems to be the main one that underlies all the others. Kidron et al. (2011) focus on the epistemic orientation of needs, Drodge and Reid (2000) point to their emotional orientation in a mathematical activity, both indicate

the relevance of the social when (epistemic) needs appear in a mathematical activity as in the participatory culture shown in the thread of comments.

The participatory culture is driven by the informal mentorship activated by the epistemic needs, even if sometimes implicitly: reacting to a need for explanation (#2, #4), clarity (#8, #10, #16), and reasoning (#11). Doing this, the commenters also show, keep, and address expert levels of mathematics according to the meme, offering as much information as needed, but no more, unless someone expresses an epistemic need (#2). Sometimes commenters explain without someone asking for an explanation, in this case they play the role of mentors towards an implicit need for explanation in the group (#8, #11), similarly they clarify the argumentation spontaneously and refer to the original meme (#10). We are aware that some of the users might be people with good math skills or even mathematics teachers, willingly offering their knowledge. What is new here is that less-skilled individuals do not hold back from interacting with more competent people, spontaneously showing epistemic needs and taking advantage of the informal opportunity to learn something.

Throughout the exchanges, the mathematical idea encoded in the meme is reconstructed producing and using information and knowledge by commenters, naturally enacting the different roles of *teacher* (A C2 C4) and *learner* (C1 C3 C5 C6 C7). A meaning-making process emerges, activated by the meme and kept by the epistemic culture, revealing an argumentation progression with different mathematical themes. The process is summarized in C7's final comment: applying L'Hôpital's rule to solve the limit means taking the derivative of sinx, *AND* the proof of the derivative of sinx is based on the knowledge of the same limit (in the high school developing of the topic), *HENCE* the reasoning is circular. *THEREFORE,* applying L'Hôpital's rule to solve the limit is *mega wrong*. It is interesting to note that C7 was not among the commenters who asked the initial questions, nonetheless the final remark shows us that C7 read the preceding comments, and elaborated the information shared to produce some

mathematical knowledge. In the progressing of the reasoning, some side knowledge is also produced, stemming from the acquired correctness of the meme in a mathematically consistent way: *IF* applying L'Hôpital's rule is *mega wrong*, *THEN* what is the correct solving procedure?

Drawing together, the meaning-making process takes the form of an argumentation nurtured by an epistemic culture. This epistemic culture is established and re-established by the interplay between the convergence culture driven by epistemic needs and the participatory culture where the community members react towards explicit or implicit epistemic needs by explaining, clarifying, and reasoning. In this epistemic culture, the negotiation of knowledge is expert-like, recognizing the other as experts in the community, mutually offering and asking for mentorship.

Case 2: the wrong meme

- Meme base name: *Spiderman pointing at Spiderman*
- Meme base use: describe situations in which similar elements meet
- Source: Facebook Total comments: 131
- Topic: Functions
- Subtopic: Direct and inverse trig functions
- Critical point: misinterpreted equivalence between $f^{-1}(x)$ and $\frac{1}{f(x)}$
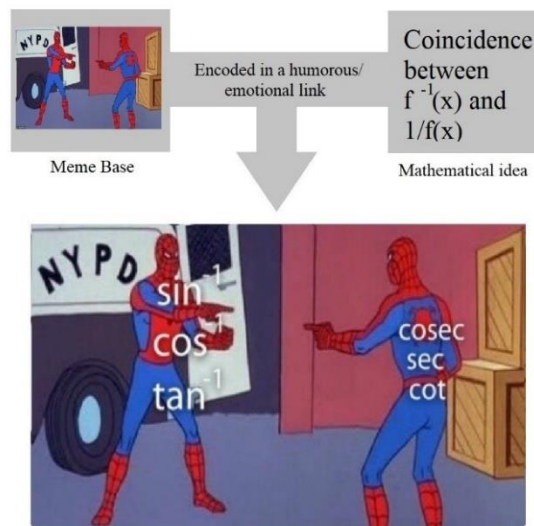- Publication year: 2018

Figure 18. Case 2: the memetic analysis

Contextual description: The image of this meme in Figure 18 is commonly used to describe situations in which similar elements meet: in this case, the intent of the author is to represent the equivalence between the triplets of functions *sin^{-1} cos^{-1} tan^{-1}* and *cosec sec cot.* This equivalence is, of course, incorrect: the excerpt (Table 3) shows how swift and enduring the online community is in singling out and discussing the mistake.

Table 3. Case 2 - the cultural analysis

Comments	Explicit information	Implicit information	Environmental culture	Emerging mathematical themes
1-C1 [Day 1, 9:22]: This is mathematically wrong	C1 declares the meme as incorrect	By using the adverb *mathematically* C1 makes clear that the problem is a mathematical one and not a memetic one	C1 addresses A as a teacher, pointing to the mistake (participatory culture)	
[…]				
2-A: Some people are trying to be buzzkills by saying sin^-1 is angle and 1/sin is the actual ratio(cosec). However, those smart alecks don't realise the first mistake of the meme is that I didn't even mention the angle unit or anything lol. Try scrutinizing a little harder.	A defends the position, sharing information about what is considered as *the first mistake in the meme* which was not captured by the community	A attacks C1 making fun of C1's mathematical smartness (*smart aleck* = geek) The use of the slang word *buzzkill* (someone who ruins something enjoyable) tells us that A perceives memes as jokes rather than as mathematical objects	A refuses to be treated as a student and enters a competition about mathematical knowledge (participatory culture)	Difference between $\frac{1}{f(x)}$ and $f^{-1}(x)$
3-C1: nice comeback after the mistake 😃😃	Ironically confirms the mistake	C1 gives a patronizing comment that positions A at a lower level of expertise The emoji says "don't take it too seriously"	C1 is self-positioning as an expert (participatory culture)	
4-C2: Arcsin can be 0 while csec can not. same for the arccos and sec	Observes that the value 0 is in the range of the function arcsin (or arcos) while it is not in the range of the function csec (or	C2 offers an explanation of why the meme is *mathematically wrong*: if the codomains of the two functions are different *therefore*	Informal mentorship (participatory culture)	

	sec) (reacts to an implicit *need for a disprove*)	the functions cannot be in the relation implied by the meme (one example is enough)		
5-C3: The ^-1 doesnt necessarily means that they are a ratio it a sign that its inverse of that trig fn	Explains the misunderstanding connected to the different meanings of the notation ^-1	The use of the term *necessarily* gives the idea that C3 acknowledges the common meaning of the ^-1 sign for ratios (building on A's comment), but wants to make clear that there is another meaning	Informal mentorship expressly directed towards A (participatory culture)	
[…]				
6-C4: [replying to A] Whatever u say it's wrong even if there's no angle. An inverse trig function is not the same as raising it to -1 making it a reciprocal of itself. This meme is wrong, accept it!!! 😫 [in a separate comment] Also you can't raise a function to -1 and say that it's a reciprocal. You only do that to variables especially the independent ones like (x). Hahahha just commenting in behalf of the dumb alecks 😆 […]	C4 confirms the mistake and explains that the notation ^-1 means reciprocal only when applied to variables	C4 replies to A's defense confirming the mistake *even if there's no angle* C4'sconclusion about these comments being for *dumb alecks* (nice but not very clever guys) implies that C4's observations are well known mathematical facts that A does not know.	Mathematical memes as membership identifiers, thus they must follow the norm to be mathemati-cally correct (participatory culture)	
7-C5: [replying to A] Imagine trying to justify the conflation of different mathematical identities by saying you weren't using units. Nice try, but have you ever heard of a functions domain? Trig functions have domains from -1 to 1 (except tangent) unless you're using complex analysis to define them in some other way, which most people aren't. Just accept that you're wrong	Reply to A's defense about the missing angle units, confirming the mistake	C5 supports A's comment bringing in a not well related argument about functions range (erroneously referred to as domain)	C5 is self-positioning as an expert, allowed to mentor (participatory culture)	Domain and codomain of a function

8-A: [Day 3, 17:10] I did admit I was wrong lol. If you think I don't know anything about trigonometric ratios and angle, then hear this: sin=1/cosec cos=1/sec tan=1/cot Whenever there's an inverse (^-1), it becomes the value of an angle. Also, (sin or cos or tan)^-1 =arc(sin or cos or tan), which I learnt after making the mistake. Moreover, another admin made this meme. The idea was just mine. And idk [I don't know] what you are saying about domain. You probably wanted to say codomain. I hope I don't have to justify myself anymore. I am just in school NIBBA. [a slang word for "guys"]	A acknowledges the mistake, shows what has been learned and identifies C5's mistake about the domain	Given the time span between the initial comment and this, A likely uses external resource to straighten the issue. A comes back to recognize the mistake and show off the newly acquired knowledge, *learnt after making the mistake*, scoring a point towards C5's domain/ codomain jumble. The new knowledge is exposed with emphasis, both verbal (*hear this*) and structural, in the layout of the writing (as opposed to the usual in line comments) to make concepts stand out. Dilemma: A wants to be acknowledged vs. uses the school background as an excuse for the mistake	A accepts the norm and achieves the meaning-making (convergence culture) A generalizes the norm to the behaviour in the community addressing C5's mistake (participatory culture)	Final consideration about the incorrectness of the mathematical idea represented by the meme
9-C5 [Day 4, 04:22] Understandable	C5's closing comment replies only to A's excuse for the mistake and accepts it	When A shows some willingness to learn and reveals as a high school student, C5 recognizes A the right to make mistakes	Final informal mentorship (participatory culture)	

C1 shows the need to check the correctness of the meme mathematically. This initiates a negotiation on how a meme and the encoded mathematics should be perceived, either as a mathematical object or as a joke (#2, #3). A's defence is initially not accepted (#3, #6, #7) as A's comment does not show any epistemic need to reason or ask for reasons necessary when something is incorrect. In the following interactions the epistemic culture is re-established by the participatory culture which is usually implemented in the community. The comments

express actions towards needs aimed at disproving the mathematical idea encoded in the meme (#4), giving reasons (#4, #5, #6, #7), explaining (#5, #6), and mentoring others (#6, #7). The community is strongly committed to the fact that the mathematics encoded in the meme must be mathematically correct (#6, 7#). The commenters' participation can be taken as expressing what experts do with incorrect mathematical ideas: they take them in and disprove them, showing a need for proof and its underlying need for certainty.

The incorrectness of the meme is finally recognized by A (#8). By finding mistakes in the experts' comments, too, A appears not fully acquainted with the epistemic culture established in the community, which tolerates mistakes in the epistemic interaction but not in the meme. By stressing that the meme was another admin's creation, A refuses to take responsibility for the mistake (apologizing), showing a need to explain the background situation rather than an epistemic need. A uses the declared status of high school students as an excuse for the incorrectness of the meme. However, as soon as A shows some willingness to learn, the mistake is accepted (#9).

The convergent meaning-making process about the incorrectness of the mathematical idea represented by the meme comes about only in the end, hindered by A's initial reluctance (#2). This can be interpreted as an evidence of the perceived value of these digital objects as *identifiers of the belonging to the community*. Three commenters (C2 C3 C4) provide diversified and mathematically consistent information, from counterexamples to notation conventions, aimed at sustaining the first negative comment (C1). Finally, some mathematical knowledge is constructed: A acknowledges the mistake made but at the same time tries to reaffirm the position in the group by making in turn note C5's domain/codomain error.

Drawing all together, we have reconstructed the practices of convergence and participation in this community. Community members share the norm that the mathematics in the meme must be correct. Epistemic needs related to the memes drive the meaning-making

(convergence), from there members act fulfilling the needs based on the norm of keeping a certain expert level but being also open to include those who express epistemic needs. We call this the *meme epistemic culture*, fostering a process of argumentation, driven by epistemic needs, resulting in proofs and clarifications around the mathematical idea in the meme.

These two polar types epitomize a practice that our participant observation revealed as characteristic for the interaction with mathematical memes in dedicated online communities. Indeed, our comment pool investigation showed that, apart from emojis and friends tagging, comments to mathematical memes are always at the mathematical level, aimed at proving or disproving the mathematical idea encoded in the meme, and never at the memetic level (e.g. asking explanations about the meaning of a particular image). We take this as evidence of the fact that users are familiar with the rules of the memesphere and therefore do not need support in this field. However, this indicates also that these communities perceive mathematical memes as mathematical objects rather than as entertaining objects, as suggested by name choices as Mathematical *Mathematics* Memes. Another relevant point is that, even if the understanding of a mathematical meme requires the decoding of the encoded link, comments do not make this decoding process explicit. E.g., in case 2, C1 does not write "the equalities $\sin^{-1}=\text{cosec}$, $\cos^{-1}=\sec$ and $\tan^{-1}=\cot$ are mathematically wrong", but simply "the meme is mathematically wrong" (#1), assuming that everybody gets the equalities encoded in the meme. Comments are all directed towards giving arguments to prove or disprove the mathematical idea expressed by the meme, explaining *why* the functions are different or explaining *why* using L'Hôpital's rule to solve the limit is incorrect.

To sum up, interacting with a mathematical meme implies a two-step process: the first step is the decoding of the mathematical idea, and the second step is the argumentation needed to determine whether this mathematical idea is correct or incorrect. The first step requires memetic and mathematical knowledges: this step is not externalized in the comments; thus, we

assume that users achieve it on their own. The second step requires mathematical knowledge only and is what the community actively engages in: a collective meaning-making process of the correctness of the mathematical idea, activated by an epistemic culture. This meme epistemic culture even enriches the epistemic process with further clarifications about the mathematical theme, for example offering more than just one proof although one would be enough or explaining concepts at the periphery of the meme.

Results and theorizing towards a conceptualization of the phenomenon

Our research questions aimed at characterizing the Internet phenomenon of mathematical memes as a new form of representation of mathematical ideas and at exploring how they activate interactions among members of online communities. We have investigated these issues applying a twofold theoretical background to a participant observation of the field and an instrumental case study. Results are summarized in Table 4.

Table 4. A summary of the findings

Theoretical background	Memetic	The memetic constituent elements of mathematical memes are: humour and emotions, spreadability and diffusion, and image macros structural and compositional rules. In mathematical memes images are used to represent effects (tangible or emotional, always mathematically rigorous) or elements of the mathematical idea hinted by the text
	Cultural	The cultural features of the Web 2.0 that inspire the production and use of mathematical memes are participatory culture and convergence culture. These features shape the perceptions of mathematical memes as membership identifiers and as learning opportunities inside the communities
Empirical study	Participant observation	Threads of comments with explicated mathematics are a characterising practice of the interaction with mathematical memes as representations of mathematical ideas. This practice is highly respected inside the community, where mathematical memes and the accompanying comments are recognised for their educational value.
	Instrumental case study	The interaction with mathematical memes as representations of mathematical ideas revealed in the threads of comments focuses on discussing the correctness of the mathematical idea encoded in the meme. In these interactions, epistemic needs are activated. They drive the discussion, handled with rigor, resulting in a process of transfer of mathematical knowledge, where engagement is highly valued

The memetic and cultural perspectives provide grounds to characterize mathematical memes as humorous and spreadable artefacts where images and texts are combined to represent mathematical ideas. They are cultural objects, created respecting both the laws of the memesphere and those of mathematics, and shared within dedicated communities to acquire a status. Participant observation and a case study illustrate mathematical memes' educational potential, provided by the epistemic culture activated by the meme itself and leading to a collective meaning-making (cf. Kruglanski, 2010). The meme epistemic culture in the communities is (re-)established by an embedding participatory culture of acting epistemically, and by a convergence culture driven by epistemic needs. Members of the communities spontaneously tend to organize themselves according to a social order depending on mathematical skills, and interact following shared sociomathematical as well as social norms (Yackel & Cobb, 1996). The key sociomathematical norm requires mathematical memes to be correct, a key social norm implies, among others, an assumption of expertise, which manifests itself in the fact that, unless epistemic needs are expressed, commenters never overexplain.

These results allow us to move further in conceptualizing the phenomenon. We have seen that comments focus mainly on the rigorous assessment of the correctness or incorrectness of the mathematical idea encoded in the meme. Our explanation is that mathematical memes represent mathematical ideas by their properties that can be logically proved or disproved. Therefore, we hypothesize through abduction (Peirce, 1931, CP 5.189; cf. Arzarello et al., 1998; Douven, 2011) that mathematical memes represent *mathematical statements* in the sense of propositional logic (Durand-Guerrier, 2008; Cook, 2009), that is declarative propositions whose truth-value can be determined as either true or false through logically valid arguments (i.e. arguments that originate from premises assumed as true and that develop in a truth-preserving way).

From this, we propose a *conceptualization* of the observed phenomenon:

Mathematical memes represent mathematical statements. Elements that compose the statement can come from the textual part, from the image or from a combination of the two, in a new hybrid language which merges mathematical elements with memetic elements, rigorously abiding by the rules of both fields. The need to determine the truth-value of the statement evokes a negotiation shaped by an epistemic culture on the mathematics encoded.

To validate this hypothesis, we tested it on our meme pool. For each and all of the collected memes we were able to express the matching statement. Due to space constraints, we will only show here the correspondence between mathematical memes previously presented in this paper and the related statements, along with their truth-value (Table 5).

Table 5. Mathematical memes as statements

Fig.	Meme	Statement	Truth-value
3a	Me and the boys	$y = x^x$ cannot be integrated in elementary terms	True
3b	Me and the boys	$y = \frac{sinx}{x}, y = e^{-x^2}, y = \sqrt{sinx},\ y = \sqrt{1 + x^3}$ cannot be integrated in elementary terms	True
4	Me and the boys	In the complex plane, the effect of multiplying by i is a $\frac{\pi}{2}$ rotation	True
10	Success kid	$(a + b)^2 = a^2 + 2ab + b^2$	True
11	Spiderman pointing at Spiderman	$\frac{d}{dx}(e^x) = e^x$	True
17	Mega wrong	Evaluating $\lim_{x \to 0} \frac{sinx}{x}$ with L'Hôpital's rule is a logical fallacy	True (assuming certain premises)
18	Spiderman pointing at Spiderman	$sin^{-1}(\alpha) = cosec\alpha; cos^{-1}(\alpha) = sec\alpha; tan^{-1}(\alpha) = cot\alpha$	False

This conceptualization fits with all the characteristics that emerged from our empirical analysis: it explains that

- mathematical memes are rigorously formulated from the memetic and mathematical standpoint because they are mathematical statements,
- they are taken as membership identifiers as they display mathematical expertise,
- the discussion focuses on their truth-value and, thus, mobilizes mathematical information and knowledge as prerequisites for (dis-)proving and understanding.

It also explains why memes are not explicitly decoded in the comments, as the community members are aware that paraphrasing a statement does not count in the process of argumentation.

Given that mathematical memes represent mathematical statements, we can differentiate what is and what is not a manifestation of the phenomenon, distinguishing these objects from other digital objects shared within these communities, as emotional memes and cartoons. In fact, the emotional meme seen in Figure 14a conveys the message *math is puzzling*, which is not a mathematical statement, because its truth-value is subjectively evaluated. While the cartoon in Figure 14b, offers a *cartoonised* version of the effect of the Fourier transform that cannot be translated into a provable or disprovable mathematical statement, as opposed to the *Me and the boys multiplying by i* meme in Figure 4.

Going back to Gitelman's two-level model for media cited at the beginning (2006), we can say that a mathematical meme is firstly a technology that enables the communication of mathematical statement. Secondly, a mathematical meme is embedded in a set of social practices and cultural norms, implying that the mathematical statement must be true, and shaping an epistemic culture where the meme activates and guides epistemic processes about the mathematics addressed in the statement.

Discussion, reflection, concluding remarks

Ordinary mathematical representations can take the form of numerical, algebraic, graphical, verbal mathematical expressions, sometimes including logic operators (Goldin, 2014). They are formal, relieved from any possible emotion that mathematics can spark. Not so in the representation of mathematical memes, where mathematical statements are taken as rigorously as in mathematics but their decoding requires additional knowledge from the memesphere. Only a deep understanding of the encoded mathematics unfolds the power of the joke in the meme. Such a hybrid representation of a mathematical statement is no longer dry, it addresses

emotions although being rigorous with respect to mathematics. This may provide widened access to mathematics for an enlarged range of learners.

Our conceptualization resulted in the insight that mathematical memes represent statements and activate an epistemic culture in the online community. This is an important finding pointing to their educational potential for constructing mathematical knowledge. It expands and updates our semiotic capital to represent mathematics. Following O'Halloran's idea that "a culture may be understood as a typical configuration of choices from a variety of semiotic resources" (2005, p. 6), this can open new research paths of investigating hybrid representational systems in mathematics. Another research path may address learning in everyday classrooms, investigating there how mathematical memes activate and drive mathematical discussions and how this fosters interest in mathematics.

Our findings can also be a first step to figure out how mathematical memes can be used for teaching in traditional educational settings. In fact, statements are the building blocks of mathematics and proving statements is the building block of any mathematical activity. Thus, mathematical memes could be used in argumentation-based learning scenarios, where their manifestation as objects closer to students' experience than to classical mathematical registers can leverage on learners' popular culture to engage students in the essential mathematical practice of justification.

Naturally, this work has some limitations. As a case study, although taken from and supported by a big meme pool, our findings do not offer information about the scope of the study, e.g. how other online communities treat these memes or if there are online communities creating mathematical memes that cannot be taken as statements. Further, we do not know the members of these communities and the time they invest in making the communities work as shown in the investigation. But our study has also a certain strength: our conceptualizing is ecologically valid, because it is rooted in the ethnographic approach which made us gain data

of high ecological validity. As a lurker, the observer knew what was relevant for community members but also was controlling this by comprehensive data collections and analyses. Ecological validity was further substantiated by mapping the field with a systematic funnelling procedure to gain paradigmatic cases for our theorizing, which added ecologically significant data to be analysed on a micro level from inside the interaction in the communities.

This empirically founded conceptualizing could provide a well-established basis for further investigations of the memesphere and testing of our findings. Since we have gained a new kind of representation, semiosis related to discussing the meme' mathematical ideas could reveal deeper insight into the mechanisms of knowledge construction activated by the meme, and semiosis related to meme mutations could reveal scientific as well as educational knowledge of the encoding mechanisms, which may inform teaching.

Observing that Internet culture is in constant change, we conclude that we cannot predict if, a few years from now, mathematical memes will still have the same characteristics as those described in this study or will exist altogether. Nonetheless, even if memes, as we know them, will go out of fashion or morph into something different, the present work aspires to be a first step in the investigation of the evolving epistemic mechanisms of the new generation of convergent and participant learners, with a value that may go beyond memes and their possible transience.

Footnotes

1.https://www.oxfordlearnersdictionaries.com/definition/english/meme
2.To protect the privacy of commenters, personal identifiers have been substituted with codes throughout the study.
3.https://www.statista.com/statistics/272014/global-social-networks-ranked-by-number-of-users/

4.http://www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_Liceo%20scientifico.pdf
5.https://wearesocial.com/global-digital-report-2019
6.https://www.edisonresearch.com/wp-content/uploads/2019/03/Infinite-Dial-2019-PDF-1.pdf

Conflict of interest statement

All authors declare that they have no conflict of interest.

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