Higgs production at RHIC and the positivity of the gluon helicity distribution

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We show that the negative polarized gluon distribution Δg found in a recent global next-to-leading-order QCD analysis of the nucleon helicity structure is incompatible with the fundamental requirement that physical cross sections must not be negative. Specifically, we show that the fact that this polarized gluon strongly violates the positivity condition $|\Delta g| \leq g$ in terms of the unpolarized gluon distribution g leads to negative cross sections for Higgs boson production at RHIC as a physical process, implying that this negative Δg is unphysical.

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Understanding the quark and gluon spin structure of the proton is a key focus of modern nuclear and particle physics. An important component of this endeavor is the precise determination of the proton helicity parton distribution functions (PDFs). The gluon helicity PDF $\Delta g(x, \mu)$, in particular, has received much attention in this context as its integral over all momentum fractions x measures the gluon spin contribution to the proton spin and hence could hold the key to decomposing the proton spin into its partonic contributions. A celebrated discovery was made in 2014, when it was shown [1,2] that data from the Relativistic Heavy Ion Collider (RHIC) [3] provided evidence for a nonvanishing and positive Δg in the region $0.05 \lesssim x \lesssim 0.2$. This finding was obtained on the basis of a global next-to-leading-order (NLO) QCD analysis of the world data on polarized (semi-)inclusive deep-inelastic scattering and polarized pp scattering. It was confirmed in additional studies [4,5] and later substantially corroborated when further sets of RHIC data became available [3].

The need for a positive Δg was recently called into question in Refs. [6–8]. Again in the context of an NLO analysis (in the $\overline{\mathrm{MS}}$ scheme) the authors found PDFs featuring a negative gluon helicity PDF, $\Delta g < 0$, thereby suggesting that negative gluon polarization is also possible. These PDFs are delivered as an ensemble of replicas, quantifying the PDF uncertainty. Figure 1 shows the 78 PDF replicas with negative Δg from Ref. [7], available at [9] in LHAPDF [10] format. We show results at factorization scales $\mu = \sqrt{10}$ GeV (left) and $\mu = 125$ GeV (right).

A striking feature of the PDFs with negative Δg proposed in [6–8] is that they strongly violate the positivity condition,

$$|\Delta g(x, \mu)| \leq g(x, \mu), \tag{1}$$

at momentum fractions $x \gtrsim 0.25$. Indeed, as the authors state, the negative Δg PDFs are obtained only when the positivity condition is relaxed in the analysis. The violation of the inequality (1) is evident from Fig. 1 where we also show in both panels the replicas for the corresponding unpolarized gluon distribution as obtained in the same analysis [7].

Condition (1) arises of course from the fact that $g = g_+ + g_-$ and $\Delta g = g_+ - g_-$ where g_+, g_- are the distributions for gluons with positive or negative helicity inside a proton with positive helicity, respectively. At leading order (LO) in perturbative QCD these can be regarded as number densities and hence positive,[1] so Eq. (1) strictly holds. This is a consequence of the fact that there exist physical processes for which at leading order the physically measurable cross section, which is a probability and thus positive, is proportional to the PDFs. However, beyond

[1]Here and elsewhere we use the word positive to mean non-negative.

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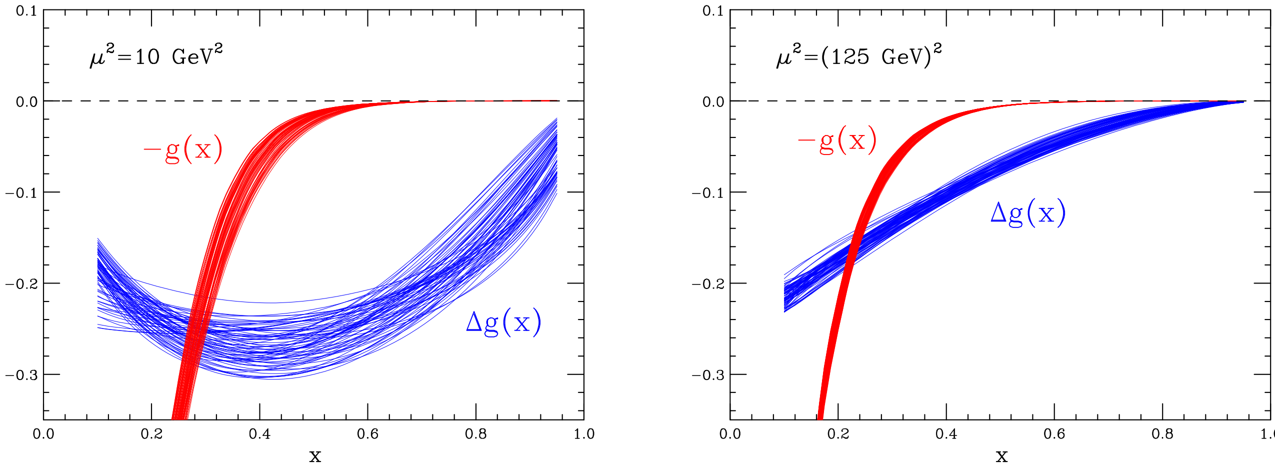

FIG. 1. PDF replicas of Ref. [7] for $-g(x,\mu)$ and $\Delta g(x,\mu)$ at $\mu = \sqrt{10}$ GeV (left) and $\mu = 125$ GeV (right).

LO the cross section is obtained by convoluting the PDF with a partonic cross section (coefficient function). The cross section remains, of course, positive but now the coefficient function and the PDF depend on the factorization scheme and hence are not necessarily separately positive, so strict positivity of g_+, g_- does not need to hold any longer [11–15]. As a result, solutions with $|\Delta g| \geq g$ are in principle formally possible.

Such violations of positivity however must have the size of the higher-order corrections, because the possible violation of positivity of PDFs must be compensated by the higher-order corrections to the coefficient functions so that the physical cross section remains positive. Indeed, In Ref. [15] NLO positivity bounds on polarized PDFs in the $\overline{\rm MS}$ scheme were derived by requiring positivity of NLO cross sections. These NLO bounds were used to derive a bound on the polarized gluon distribution in x space [14], which was compared to the naive LO bound Eq. (1) and found to differ from it at the percent level except at very small $x \lesssim 10^{-3}$. In contrast to this, the violation of positivity of the PDFs from Refs. [6–8] exhibited in Fig. 1 is much larger; in fact, $|\Delta g|$ exceeds g by a large factor. This suggests that these PDFs may lead to unphysical predictions.

To see how this may happen, we recall how positivity bounds can be derived at any perturbative order [15]. The derivation is based on the observation that physically observable cross sections—and theoretical predictions thereof—are proportional to the number of observed events, and thus cannot be negative. Using the spin-dependent cross section for some reaction in polarized pp scattering as an example, we must have

$$|A_{\rm LL}| \leq 1, \tag{2}$$

where

$$A_{\rm LL} \equiv \frac{\Delta\sigma}{\sigma} \equiv \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}}, \tag{3}$$

with σ_{++} (σ_{+-}) the cross section when the two colliding protons have the same (or opposite) helicities. (For simplicity, we are considering a parity-conserving interaction). The condition (2) must apply to any physical cross section, regardless of whether it has been measured, or even whether it is practically measured or measurable in an actual experiment. Positivity bounds on any polarized PDF at, say NLO can then be derived by imposing the condition Eq. (2) on a set of suitably chosen pairs of polarized and unpolarized NLO cross sections. For instance, the positivity bounds of Ref. [15] were obtained by imposing the condition on polarized and unpolarized deep-inelastic scattering, as well as for Higgs production in gluon-proton scattering.

Because the bound Eq. (2) implies a bound on the polarized PDFs, it follows that PDFs that violate this positivity bound lead to unphysical negative cross sections. Given the enormous violation of positivity for the gluon PDF as seen in Fig. 1, one may immediately ask whether there could be a physical observable for which an NLO prediction based on this gluon density violates the condition Eq. (2). We will now show that this is indeed the case, rendering the solutions of [6–8] with negative Δg unphysical. A suitable candidate for this purpose is an observable that is gluon-driven at tree level, and probes the region $x \gtrsim 0.25$ of momentum fractions where the gluon distribution in Fig. 1 violates positivity, such as the Higgs production cross section in pp scattering. A dominant contribution to this process is gluon-gluon fusion, $gg \to H$, through a top quark loop. To lowest order in QCD no additional partonic channels involving incoming quarks contribute to this process.

This process is of course of paramount importance at the LHC, and its total cross section has been studied in this context in great theoretical detail (see Refs. [16–19]). Here we will instead consider Higgs production at RHIC energy, $\sqrt{S} = 510$ GeV, using a range of Higgs masses m_H between 100 GeV and 250 GeV. Although this is not relevant for canonical Higgs phenomenology, it will allow us to access a perturbative cross section in the kinematic regime where $x \gtrsim 0.25$, since the lowest x-value probed in the PDFs at leading order for a given Higgs mass is m_H^2/S, with $\sqrt{S}$ the pp center-of-mass energy.

The unpolarized and spin-dependent cross sections for $pp \to HX$ may, up to power corrections, be written in factorized form as

$$\sigma^{pp\to H} = \sigma_0 \sum_{i,j} \int_\tau^1 dx_1 \int_{\tau/x_1}^1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \times \omega^{ij\to H}\left(z = \frac{\tau}{x_1 x_2}, \alpha_s(\mu), \frac{\mu}{m_H}\right),$$

$$\Delta\sigma^{pp\to H} = \sigma_0 \sum_{i,j} \int_\tau^1 dx_1 \int_{\tau/x_1}^1 dx_2 \Delta f_i(x_1,\mu) \Delta f_j(x_2,\mu) \times \Delta\omega^{ij\to H}\left(z = \frac{\tau}{x_1 x_2}, \alpha_s(\mu), \frac{\mu}{m_H}\right), \quad (4)$$

where $\tau = m_H^2/S$ and $\omega^{ij\to H}, \Delta\omega^{ij\to H}$ are normalized hard-scattering functions that are computed in perturbation theory. They are defined as in Eq. (3) by $\omega^{ij\to H} \equiv \frac{1}{2}(\omega_{++}^{ij\to H} + \omega_{+-}^{ij\to H})$ and $\Delta\omega^{ij\to H} \equiv \frac{1}{2}(\omega_{++}^{ij\to H} - \omega_{+-}^{ij\to H})$, where $\omega_{\lambda_i\lambda_j}^{ij\to H}$ is the cross section for incoming partons i, j with helicities λ_i, λ_j. The normalization σ_0 is the same for $\sigma^{pp\to H}$ and $\Delta\sigma^{pp\to H}$ and is given by

$$\sigma_0 = \frac{\alpha_s^2 |A|^2}{256\pi v^2}, \quad (5)$$

with α_s the strong coupling and $v = 246$ GeV the Higgs vacuum expectation value. The factor $|A|^2$ results from the coupling of the two gluons to the Higgs boson via a heavy-quark loop. Ignoring contributions from charm and bottom quarks and keeping only the top quark of mass m_t, one has, at lowest order (see Refs. [20,21]),

$$A = \tau_q(1 + (1 - \tau_q)\arcsin^2(1/\sqrt{\tau_q})), \quad (6)$$

where $\tau_q \equiv 4m_t^2/m_H^2$, and beyond leading order the hard-scattering functions also depend on m_t. This expression was originally obtained for the spin-averaged cross section, but we have checked that it also holds for the spin-dependent one. One may further assume that the top quark is infinitely heavy. In the effective theory defined by this assumption, one has $|A|^2 = 4/9$. In any case, the factor $|A|^2$ cancels in the spin asymmetry. For simplicity, we have chosen the factorization and renormalization scales to be the same in Eq. (4) and denoted them by μ. For our numerical results further below, we will set $\mu = m_H/2$, a value that is known to lead to faster convergence of the perturbative expansion of the Higgs cross section [22]. However, none of our results depends qualitatively on this choice.

As mentioned, to lowest order, $gg \to H$ is the only contributing channel; because this is a $2 \to 1$ reaction, it is characterized by $\hat{s} = m_H^2$, corresponding to $z = 1$, where $\hat{s} = x_1 x_2 S$ is the partonic center-of-mass energy squared. Correspondingly, the partonic hard-scattering function is given by a Dirac delta $\delta(1-z)$ at this order. The NLO corrections to the partonic hard-scattering functions have been computed in the $\overline{\mathrm{MS}}$ scheme in Refs. [20,21] for the spin-averaged and in [15] for the spin-dependent cross section. For the gg-channel we have, up to corrections of order α_s^2:

$$\omega^{gg\to H}(z,\alpha_s,r) = \delta(1-z) + \frac{\alpha_s}{\pi}\left\{\delta(1-z)\left(\frac{11}{2}+\pi^2\right) - \frac{11}{2}(1-z)^3 + 6(1-z+z^2)^2\left[2\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{\ln(z)}{1-z} - \frac{\ln(r^2)}{(1-z)_+}\right]\right\},$$

$$\Delta\omega^{gg\to H}(z,\alpha_s,r) = \delta(1-z) + \frac{\alpha_s}{\pi}\left\{\delta(1-z)\left(\frac{11}{2}+\pi^2\right) + \frac{11}{2}(1-z)^3 + 6z(2-3z+2z^2)\left[2\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{\ln(z)}{1-z} - \frac{\ln(r^2)}{(1-z)_+}\right]\right\}, \quad (7)$$

where $r = \mu/m_H$, and where the $+$ distribution is defined in the usual way. Starting at NLO, there are also two new partonic channels, $qg \to Hq$ and $q\bar{q} \to Hg$. Their cross sections are also known at $\mathcal{O}(\alpha_s)$ from [15,20,21]:

$$\omega^{qg\to H}(z,\alpha_s,r) = \frac{\alpha_s}{\pi}\left\{-\frac{1}{3}(1-z)(7-3z) + \frac{2}{3}(1+(1-z)^2)\left[\ln\frac{(1-z)^2}{zr^2} + 1\right]\right\},$$

$$\Delta\omega^{qg\to H}(z,\alpha_s,r) = \frac{\alpha_s}{\pi}\left\{(1-z)^2 + \frac{2}{3}(1-(1-z)^2)\left[\ln\frac{(1-z)^2}{zr^2} + 1\right]\right\}, \quad (8)$$

and

$$\omega^{q\bar{q}\to H}(z,\alpha_s,r) = \frac{\alpha_s}{\pi}\frac{32}{27}(1-z)^3 = -\Delta\omega^{q\bar{q}\to H}. \quad (9)$$

We now compute the spin asymmetry $A_{\rm LL}$ in Higgs production at RHIC at $\sqrt{S} = 510$ GeV, as a function of the Higgs mass. We adopt the PDF set with the positivity-violating negative gluon distribution of Ref. [7], used already for Fig. 1. We also use the unpolarized PDFs of [7] for the denominator of the spin asymmetry; this allows us to effectively determine the individual helicity cross sections in Eq. (3) and thus check positivity on a replica-by-replica basis. As a cross-check we have verified that for the LO and NLO unpolarized cross sections we recover the results given in Table 7 of Ref. [22] when PDFs and parameters are chosen as in that paper. Figure 2 displays the asymmetry $A_{\rm LL}$ for the replicas shown in Fig. 1, on a linear (left) and on a logarithmic (right) scale, plotted as a function of the Higgs mass. For Higgs masses that deviate from the physical value this should be taken as the result obtained in a fictitious field theory in which the Yukawa couplings of quarks are readjusted so that their masses remain the same and the strongly interacting sector of the theory remains the same. A huge violation of the physical positivity condition (2) is observed. Already for Higgs masses around the physical value the asymmetry $A_{\rm LL}$ exceeds unity; at even larger masses it easily reaches values of 10 or even 100. Using instead the PDFs from Ref. [8], which also include lattice data, the positivity violation at the physical Higgs mass value would likely be reduced (see also Ref. [23]), but the trend of Fig. 2 suggests that it would again be very large as the Higgs mass increases. We are thus led to the conclusion that the PDF set with the positivity-violating negative gluon distribution cannot be regarded as physical as it leads to negative cross sections.

It is important to note that the violation occurs in a kinematic region corresponding to momentum fractions x where the PDFs are generally known best. For instance, at $m_h = 150$ GeV and with $\sqrt{S} = 510$ GeV we have $x_1 x_2 \approx 0.09$ so at central rapidity $x_1 = x_2 \approx 0.3$. Hence, the violation of physical positivity of $A_{\rm LL}$ depends on the behavior of the PDF replicas in a central x region, and furthermore, it is clear from Fig. 2 that it is a bulk property the PDF replica distribution. Hence, it cannot be attributed to outliers, or to the PDF behavior in very small x or large x extrapolation regions; it is not a consequence of statistical fluctuations or large uncertainties.

Moreover, in this region unpolarized PDFs, and even the gluon PDF, are known rather accurately. Indeed, the uncertainty on the unpolarized gluon, taking the conservative PDF4LHC21 combination, is about 5% [24], so the violation of positivity cannot be reasonably attributed to imperfect knowledge of the unpolarized gluon and its uncertainty. Indeed, we have checked that replacing the unpolarized PDFs of [7] in the computation of the asymmetry with the PDFs of the PDF4LHC21 set [24] and always taking the largest of the 100 PDF4LHC21 replicas, which corresponds to a more than three-σ interval about the central gluon, we still get positivity violation for $m_H \gtrsim 140$ GeV, exponentially increasing with Higgs mass with a pattern analogous to the curves shown in Fig. 2.

For comparison, we also show in Fig. 2 (lower bands) the double-spin asymmetries obtained for the sets of [7] with *positive* Δg, which show a strikingly different behavior. In this case, about 1000 PDF replicas are available and plotted. The vast majority of them satisfies positivity. Violations of positivity are either in the tail of the distribution, or in kinematic regimes dominated by very

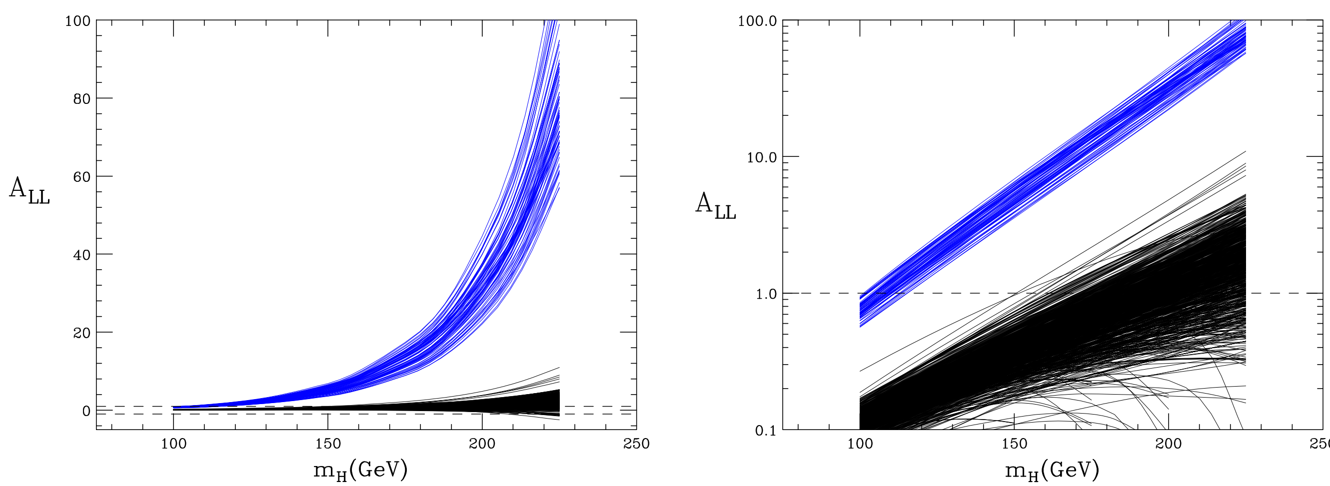

FIG. 2. Double-helicity asymmetry for Higgs production at RHIC ($\sqrt{s} = 510$ GeV) plotted as a function of the Higgs mass, with a linear (left) or logarithmic (right) scale on the vertical axis. The upper bands show $A_{\rm LL}$ as obtained for the gluon distribution shown in Fig. 1, while the lower bands provide the corresponding result for the sets of [7] with $\Delta g \geq 0$. In both plots, the dashed lines show the physical limit given by $|A_{\rm LL}| = 1$.

large values of x. In these regions, the unpolarized gluon PDF is poorly known, and it is in fact extrapolated from information at smaller x, so the positivity violation could be reabsorbed in a change of unpolarized gluon PDF.

As mentioned, in a consistent quantum field theory negative cross sections cannot occur for any process, regardless of whether it is measurable in practice or even in principle. However, in this case it is interesting to observe that it is a physically measurable hadronic cross section that is predicted to be negative. Indeed, the gluon fusion process dominates the Higgs production cross section at all energies (see e.g. [25,26]). We have in fact checked explicitly, using MadGraph5_aMC@NLO [27] that the vector-boson fusion process, which dominates Higgs productions in the quark channel $qq' \to qq'H$, despite the presence of initial-state valence quarks, gives a contribution to Higgs production in proton-proton collisions that is more than an order of magnitude smaller than that by gluon fusion also at RHIC energy. Hence the positivity violation seen in Fig. 2 leads to the prediction of a negative cross section for inclusive Higgs boson production in proton-proton collisions.

Along the same lines, one may wonder whether, given the large size of higher-order corrections to Higgs production in gluon fusion, the violation of physical positivity seen in Fig. 2 could be due to these NLO corrections, or perhaps be alleviated by higher-order corrections. However, NLO corrections in fact cancel to a very large degree in the spin asymmetry. Furthermore, the channels with incoming quarks, $qg \to Hq$ and $q\bar{q} \to Hg$, although nominally favored for very high x thanks to the participation of a valence quark PDF, remain subdominant and hence cannot re-instate positivity. It would be straightforward to further improve the perturbative framework by carrying out threshold resummation for the Higgs cross section, following the lines of [28]. This would, in fact, be required for an accurate phenomenological study of Higgs production at RHIC. However, this, too, is irrelevant for positivity since both the spin averaged and the polarized cross section receive the same QCD corrections near partonic threshold $\hat{s} \approx m_H^2$. In fact, the latter observation can be made more general; the fact that the region of interest here is $x \gtrsim 0.25$ means for a pp collider that any relevant process is probed close to partonic threshold. Given that at threshold the QCD corrections are dominated by soft emission and that soft-gluon emission is spin-independent, for such a kinematic regime the dominant QCD corrections will be very similar for general polarized and unpolarized cross sections, and even identical in some cases as for color-singlet $2 \to 1$ Higgs production. Therefore, spin asymmetries are given by their LO expression to high accuracy. Thus a significant violation of the LO positivity condition such as Eq. (1) in a dominant partonic channel will automatically lead to the violation of positivity in the physical hadronic cross section and invalidate the corresponding parton distributions.

In conclusion, by considering Higgs boson production at RHIC we have shown that previously proposed scenarios for the proton's polarized parton distribution functions with a large negative gluon polarization lead to unphysical negative cross sections. Reassuringly, such scenarios appear to be disfavored by RHIC data for direct-photon [29] and dijet [3] production not included in the analyses of Refs. [6–8], as well as by the currently most advanced lattice study of Δg [30]. Amusingly, the Higgs production process that we have considered is in fact not hypothetical at all: based on our results, we estimate that about half a dozen Higgs bosons should have been produced at RHIC during its lifetime with 510 GeV running.

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