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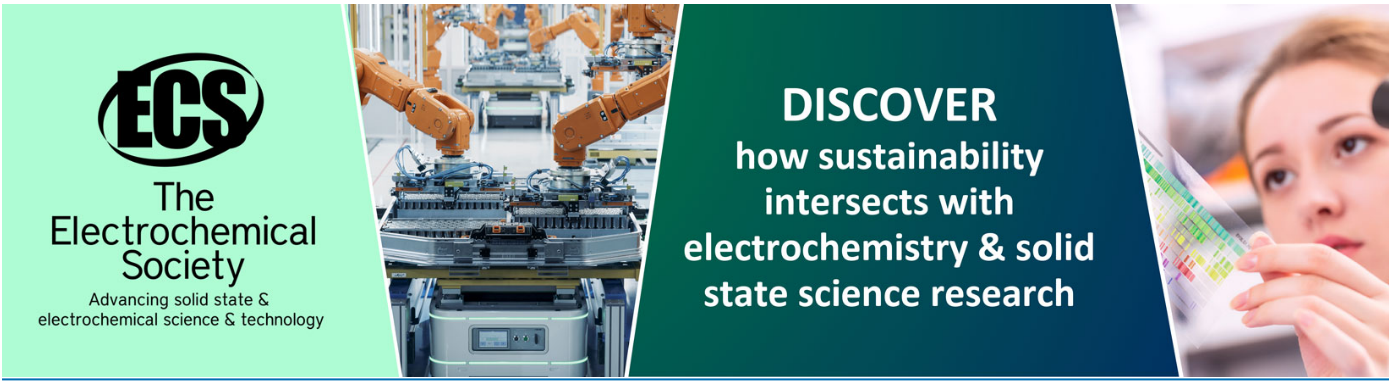

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# “The Elegance of Quantum Mechanics”: a didactic path for high school

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**Abstract**. This paper describes the work of design, testing and evaluation of the effectiveness of a pilot teaching-learning sequence on Quantum Mechanics (QM) presented to high school students and teachers. The aim is to construct a path starting from the discussion of real experiments to get to the mathematical basic structure of QM, and to test whether this path can be effectively proposed at high school level. The experimentation consisted of 10 Zoom meetings, between October 2021 and January 2022. At the end of each of the first nine meetings, each student was given a form aimed at bringing out the reasoning used, and the level of understanding achieved. At the end of the course a satisfaction survey was also given. The effectiveness of the activity was assessed by means of all homework and interviews with 13 students and 6 teachers.

## 1. Introduction

The knowledge and understanding of Quantum Mechanics (QM) are two essential aspects of every citizen’s life, since this theory has generated a revolution in the way we see ourselves and the world around us. In fact, although it seems very far from the way the world appears (or seems to appear) to us, QM influences practically every aspect of our life, from the most modern technologies to chemistry, from biology to medicine [1-3], playing the role of guiding theory in the construction of new knowledge, and constituting the theoretical paradigm of reference for the description of microscopic world.

Since one of the objectives of physics education is to build a conceptual framework of the discipline (as well as to convey its contents), and to address and disseminate culturally and socially relevant issues – facilitating a profound and meaningful understanding of the world in which we live and providing basic knowledge of the reality that surrounds us – there is no doubt that a teaching of this discipline is fundamental. To fulfil this purpose, profound reforms have been implemented within the school system on the entire international scene, starting from about 2000 [4-7].

Despite the numerous proposals put forward by the various research groups in almost 20 years of work [8-13], in general, at a school level, in common textbooks [14-21] and programs it is still presented the so-called “Old Quantum Theory”, that is a set of *ad hoc* models proposed between 1900 and 1925, in order to justify phenomenologies that could not be explained (or, at least, that was hard to explain) by means of classical physics (*i.e.*, black body spectrum, photoelectric effect, Compton effect, Bohr atomic model, etc.). These models are presented in a vaguely and predominantly chronological order, and constitute a set of unstructured information, explained in an uncoherent and didactically ineffective logical framework. In this way, we obtain what is usually defined as the “traditional” approach to QM, that is a fragmentary and not very consistent reconstruction of the discipline that makes quantum physics confusing, obscure and incomprehensible. The choice of limiting the discussion to these semiclassical models is often justified by the idea that the mathematics required for the presentation of the theory of QM is inaccessible to students. In our opinion, instead, this trend arises essentially from the lack of an adequate didactic reconstruction of the contents for secondary education, which is also reflected in the presentation proposed by school textbooks; added to this, there is often also an inadequate knowledge of the fundamental mathematical concepts and tools for QM owned by teachers (many of whom have a degree in mathematics, and have never faced these issues in their studies and training).

## 2. The reasons behind our approach

Classical Mechanics can be presented at school even without reading Newton's *Principia* and without knowing how to solve Cauchy problems, but by handling second degree equations. Electromagnetism can be treated without using complicated differential equations, as in Maxwell's treatise, but by passing through integrals and derivatives. In the same way, an appropriate educational reconstruction is also necessary for QM, *i.e.*, an adequate didactic reconstruction of the contents, aimed at simplifying the procedures, without however distorting in any way its spirit and meaning, and which, above all, takes into account not only the conceptual nodes of the discipline, but also those of students' learning. Different approaches have been adopted in various proposals (such as, for example, the Dirac "spin-first" approach [22] or that of Feynman's paths [23]); but there is probably no ideal one that addresses all learning problems in the simplest way.

The methodological and epistemological premise of the whole work of our research unit is based on the fact that what we call "reality" is subject to continuous changes, because the *status* of the fundamental entities that form reality itself is very flexible, and changes over time, in the same way that, for example, the concepts of time, space, ether, atom, etc… have changed [24]. In fact, thanks to theories – more or less structured – we can be aware of what reality is. More specifically, physical theories are mental constructions that help us find and define reality, and also use its resources [25]. This fact is valid for all physical theories, such as Mechanics, Thermodynamics, Electromagnetism, and, *a fortiori*, it is and must be true also for QM. Therefore, taking into account that it is the physical theory with its interpretations that provides our physical image of the world, it follows that, in the approach for didactic research in physics, we must primarily:

- choose a reference theory in one of its formulations;
- identify the concepts of this theory (what it talks about, what it can talk about, and therefore also what it cannot talk about);
- understand the relationships between concepts and their meaning within the theory;
- carry out an appropriate educational reconstruction.

The educational path that we present in this paper is the result of research that has made it possible to carry out a first educational reconstruction of quantum physics, intended as the set of formal principles that every quantum theory must satisfy. The primary purpose is essentially to understand and firmly anchor the meaning and reality of quantum physics within the framework of the theory itself, just as is the case of any other classical theory. The mathematical formalism of the theory, therefore, will not simply be seen as a trivial language to describe the elements of reality, but will become the context within which it is possible to interpret reality itself. This is why the objective of this path intends to be the construction of an axiomatic formal framework accessible to high-school students, starting from some crucial experiments that highlight the fundamental and specific characteristics of the theory of QM, and discussing its conceptual and didactic aspects with the teachers.

The aim of this paper is to present a comprehensible summary of our path, together with the research results that indicate its feasibility at high school level. The path presented here, starting from some clear experimental situations, therefore focuses on the motivations that lead to induce and introduce, one after the other, the axioms of QM. In summary, we therefore wonder:

A1) Why is a Hilbert space associated with each physical system and the state is represented by a unit vector?

A2) Why and how are quantum aspects linked to a precise probability theory?
A3) What does the measurement process involve?
A4) Why do we use self-adjoint operators to represent the observables?

## 3. The conceptual path

The path is divided into a series of key steps (from now on, the word "complex" will be referred only to complex numbers).

1. From experiments to a linear, complex, and probabilistic theory. As a first step, analogies of behaviour between suitably prepared beams of matter (electrons, neutrons, fullerenes, etc.), mechanical waves and electromagnetic beams are analysed. From interference experiments it can reasonably be induced that the theory describing the behaviour of these electromagnetic and material beams should be linear and able to describe the behaviour of suitable electromagnetic and matter beams.
   Unlike what happens for mechanical waves or electromagnetic radiation, with matter beams the physical quantities do not show an oscillating trend (charge density, mass density, energy, etc., are in fact constant over time for the beams we are considering). In interference experiments, on the other hand, the wave aspects emerge clearly. Therefore, the wavelike aspects of matter beams are somewhat less evident than those of the electromagnetic ones; we might therefore think that such aspects are hidden in a complex description. In fact, the main physical quantities are generally obtained by means of the squares of the fields (for example the density of electromagnetic energy is given by the sum of the squares of the electric field and the magnetic field). In our description we presumably need to construct quadratic quantities that are independent of time, and which nevertheless allow us to describe the observed interference aspects. With evident symbology, we are induced to pass from expressions of the form $A\sin(kx-\omega t)$, usual for electromagnetic fields, to complex expressions of the type $Ae^{i(kx-\omega t)}$ which have a constant square modulus, but nevertheless allow a wave description.
   Very low intensity experiments show the "granularity" of the detected radiation (electromagnetic or material) and therefore introduce the need for a quantum description. The analysis of the distributions of the observed quanta also highlights the probabilistic nature of the theory.
   From the analysis of single quantum interference experiments (typically "which-path" experiments, such as double slit, Mach-Zehnder interferometer, experiments with birefringent crystals and Fresnel biprism) with electromagnetic and matter confirms in more depth the opportunity of a linear and probabilistic aspect of the theory.
2. Space, state and probability. At this point, we can choose whether to proceed with a theory of waves interacting through quanta or to construct a theory of quanta showing wave-like behaviour. With the first choice we will proceed towards quantum field theory; with the second, as we will do, we will proceed towards the construction of QM. The state of our system, *i.e.*, of our quantum, will have to comply with the linearity and complex-numbers requirements that we have highlighted. We will choose a linear space in which to set the theory; the state of a quantum will thus be represented by a vector.
   We have yet to understand what the other characteristics of this space are. For example: how many dimensions has it? What kind of mathematical structure, if any, does it have, besides linearity? What role do vector's components play? And how do we choose the bases in this space?
   To answer these questions, we need to specify how to calculate the probability. In fact, "which-path" experiments lead us to think that the "classic" way of calculating probability is not adequate. For example, in the double slit experiment, if an electron has a probability P1 of being detected in area A of the screen knowing that it passed through slit 1, and a probability P2 of always being detected in area A knowing that it passed through slit 2 (events that we spontaneously understand as independent), then from this it does not follow that the probability that the electron has of being detected in A is equal to P1 + P2.
   Since the probabilities must be positive numbers which add up to 1, a fairly natural way, but different from the classical one, is to consider the projections on orthogonal axes of a unit segment; the sum of the squares of these projections will then give 1. More formally, let us take an orthogonal basis. The number of possible events will match the number of space dimensions. In this way, the state of the

considered system will be given by a unit vector, whose projections on the axes (taken in square module) will correspond to the probabilities that each of the possible outcomes will occur. Independent events will then correspond to orthogonal segments (A2).

3. Scalar product. The need to consider the orthogonality between segments leads to the introduction of a scalar product. Since the space is complex, the only way is to introduce a sesquilinear form. Leaving aside questions of completeness that we deem unnecessary at this stage, a linear, complex space with a scalar product is what is called a Hilbert space (A1).
4. Collapse of the state. Single quantum polarization experiments [26] are taken as emblematic of the description of measurement in QM. Through these experiments, the postulate of state precipitation (A3) is introduced. Let us consider, for example, situations in which the state is changed – typically single quantum polarization experiments. If we will let photons with fixed linear polarization go through a horizontal polarizer followed by a detector, in general only the fraction $\cos^2\theta$ (where $\theta$ is the angle made by the direction of polarization with respect to the horizontal) will be revealed, *i.e.*, each photon will pass with a certain probability. The passing photons are polarized in the direction of the polarizer, horizontally in our case: their state has changed or, in jargon, has precipitated, *i.e.*, it has become the one dictated by the polarizer (A3).
   Measurements, beam splitters, etc. change the state of a system, hence we need to provide a mathematical representation of this fact. The concept of (linear) operator is therefore introduced as an object that to every vector of the space associates a vector, respecting the linearity of the structure. Operators associated with the action of some parts of the Mach-Zehnder interferometer are analysed (action of the beam splitter, interference of the beams, etc.).
5. Measurement and projection operator. We thus move on to the formalization of the measurement process. Let us consider an "observable" quantity G (relative to a certain state), which can provide results $g_1,...,g_n$, and the state of the system written on the basis identified by $g_1,...,g_n$. Measuring G consists in obtaining one of the possible values $g_i$ between $g_1,...,g_n$. The probability with which this result will be found is given by the square module of the state projection on the i$^{th}$ axis. Immediately after the measurement, the state of the system will be given by a unit vector along the i$^{th}$ axis.
   The idea of a projection operator is thus born. The fact that a measure provides a real-number result pushes us to consider linear combinations with real coefficients of projectors. In fact, a linear combination of orthogonal projectors is exactly the operator we are looking for: the one that projects onto the subspaces identified by the possibilities given by the measurement. If we identify the coefficients of the linear combination with the possible outcomes of the measurement, we obtain an operator that, starting from the state of the system, gives us both the probability of the single events – *i.e.*, the Born rule – of the possible results of the measurement (the spectrum of the operator) and of the state of the system after the measurement (the projection postulate).
6. Properties of projection operators. The concept of projector (projection operator) is explored by observing that projectors are idempotent. It is then observed that the projectors are self-adjoint operators, that is, they can be brought from one part of a scalar product to the other without altering the result. From this it follows that even a linear combination of them is symmetrical. We will define self-adjoint all operators who enjoy this property. Eigenvalues and eigenvectors of the self-adjoint operators are thus related to fundamental physical concepts: the eigenvalues provide the possible results of a measurement, while the eigenvectors the axes of the basis associated with the observable we are considering.
7. Self-adjoint operators. It is therefore natural to associate a self-adjoint operator to each observable of the system (A4). Of course, it is necessary to explicitly observe that not everything that is measurable becomes an observable (in this simple scheme, for example, mass is not, neither is time). At this point it may be appropriate to show how, from the knowledge of the operator associated with an observable, it is possible to obtain the results of the measurement and the states in which the system may be after the measurement (an inverted procedure with respect to what previously done in step 5).
8. Eigenvectors and eigenvalues. The concepts of eigenvector and eigenvalue are studied. The eigenvectors will provide the axes of the base identified by the self-adjoint operator.

Two- and three-level systems are studied. In two-level systems, the energy operator and also the interaction energy operator are introduced.

9. Position, momentum, energy and angular momentum operators. At this point, if we are measuring only one observable, we are able to define the corresponding operator starting from the experimental results and, *vice versa*, to calculate the results starting from the knowledge of the operator. Otherwise, in different cases, the situation is more complicated. In Classical Mechanics, once the initial state of the system (position and momentum) is defined, all the quantities associated with the system (energy, angular momentum, etc.) are automatically defined as well. In QM, on the contrary, the state contains only information on the probability of obtaining a certain result, and state and observable are two separate concepts. We start by studying the observables position and momentum. We then define the position and momentum operator, bearing in mind that, in QM, the physical link will no longer be $p = m\dot{x}$ (as in the classical case), not even between the results of the measurements.
Since the definition of the position operator involves dealing with states with infinite components, a graphical method is proposed to view vectors in a space of infinite size. The construction above, along with the strategy employed to construct the operator corresponding to a certain measurement, leads immediately to consider the position operator as the multiplication operator by "$x$".
Subsequently, it is highlighted that the square modulus of the function resulting from this representation – called the wave function – corresponds to the expression we used to describe the intensity of the interference figure created in the double-slit experiment. Finally, the momentum operator, the energy operator and the angular momentum operator are defined. The time-evolution operator is introduced, and the Schrödinger equation is written.

## 4. The didactic methodology

The conceptual path presented in Section 3, born as the development and implementation of an educational path proposed in a training course for teachers, organized in 2019-2020, was translated into the activity "The Elegance of Quantum Mechanics", organized in 2021-2022 within the Scientific Degree Plan (Piano Lauree Scientifiche – PLS) of Physics, of the University of Milan.

This activity was proposed jointly to a group of teachers and students from the last three years of high school (for a total of 120 participants, 30 of which were teachers) and took place over a period of four months (from October 2021 to January 2022), through 10 weekly appointments of one hour and a half each. Given the situation linked to the Covid-19 pandemic and the large number of participants, it was decided to deliver the course remotely, through the *Zoom* platform, integrating lessons with slides, questions with *Kahoot!* (useful to take stock of the situation and verify the correct understanding of the topics by the students), and interactive graphic examples created with *GeoGebra* (visually very effective and intuitive; an example is available at the link: https://www.geogebra.org/m/aqf2dgn3).

At the end of each of the first 9 lessons, a *Google Form* was sent to the students (an example, translated in English, can be found at: https://forms.gle/e6o78B4Pq932Sani7) with questions and exercises related to the topics covered, to be carried out and delivered before the start of the next meeting. To stimulate an active participation from students during the meetings, as well as a constant commitment to completing the modules proposed at the end of each lesson, the course was also delivered as a PCTO (Percorsi per le Competenze Trasversali e l'Orientamento – Paths for Transversal Skills and Orientation) activity, compulsory for Italian high-school students, for a total of 25 hours.

## 5. Course effectiveness for students learning: analysis and results

The evaluation of path effectiveness was carried out by collecting and analysing different types of data deriving from:

- an anonymous satisfaction survey (given at the end of the course),
- 9 *Google Forms*, containing a total of 38 open questions and 24 exercises (about 2000 answers overall),
- 19 individual interviews: 13 with students and 6 with teachers.

In order to be able to conduct an analysis based on meaningful data, for each module, the answers provided by students who did not continuously follow or answer to the previous lessons and modules, or who did not follow the lesson regarding the topic of the form, were excluded (our analysis was thus limited to 1972 answers over 3018 given). The only exceptions concerned: the first module, of which we took into account all the answers; the second module (in which the first question was misinterpreted by all students, therefore we decided to exclude the whole form from our analysis) and the fifth module (corresponding to the lesson on complex numbers), for which we decided to consider the answers of all the participants, being an independent topic.

The analysis of the open answers was done independently by each of the three authors of this work, cataloguing each answer in one of the following categories:

1) the student used the formalism and concepts presented, in an appropriate way;
2) the student did not use the formalism and concepts presented, or used them only to a very little extent;
3) the student used the formalism presented, but improperly mixing together the concepts discussed;

and, in addition to that, even if the numerical answers provided in problems were right or wrong.

By the analysis of the answer given by students, several interesting aspects emerged. We recall some of them.

Module 1: When dealing with a new topic in physics, and above all in modern physics, it is important to know how students imagine the entities they are going to talk about. We must not underestimate these pre-existing ideas that can derive both from a formal context, such as schools, and from informal contexts. A significant path must therefore take into account previous knowledge and start from it, and then, naturally and gradually, lead the student to a more mature scientific point of view. The purpose of the first questions proposed was therefore to observe how students imagine entities (photons, electron, protons, etc.) involved in the path. These questions do not have a "right" answer from a scientific point of view, as the visualization of the entities in question is substantially impossible (or at least problematic). Nevertheless, it is natural for students to make a pictorial representation of what they study and, therefore, it is useful to know the most common types of representations. From the answers provided, it emerged that almost all the students imagine the electron (95%) and the proton (98%) as spheres. Conversely, only 12% imagine the photon only as a sphere; in fact, for 47% of students the photon is a small sphere moving with a sinusoidal motion. Many students drew the electron (53%) and the proton (47%) not alone, but only inside the atom; for others (49%), the proton is a ball of larger dimensions than the electron. Referring to the atom, for about 95% of the students it is like a planetary system. Finally, as regards the answers relating to the number of energy levels of the hydrogen atom, only 9% correctly recognize that they are infinite; for 65% there is only one level, presumably the one occupied by the only electron in the ground state; and for 10% there are 7 levels, an answer that might probably derive from previous knowledge of chemistry, as 7 shells are enough to make the periodic table. These aspects are in accordance with research findings already present in the literature [27-32].

Module 4: Most of the students expressed themselves with reasonable words regarding the complex (64%) and probabilistic (80%) nature of quantum theory. The most problematic point concerned linearity: in fact, some students, were uncertain about what a linear space is, and what this entails. It also emerged how the modelling process is deficient: about 30% of the students explicitly believe that it was the quanta themselves that added up (like waves), not the states. It thus emerged a confusion between the state of the quantum and the quantum itself. This aspect has already appeared in the literature: some articles on students' understanding of models have shown how they have difficulty distinguishing between a model and the reality described by that model [33], as some of them often consider the models as exact representations, magnified or resized, of the "real thing" [34].

Module 6: Concerning the concept of state evolution, it seemed a widespread idea (45% of students) that only in Classical Mechanics is it possible to follow the evolution of a state, while in QM it is not. There may also be a confusion between the concept of object trajectory and the concept of state evolution: in fact, the concept of state evolution in the context of the Hilbert space, and of how the evolution in the absence of measures is deterministic (analogously to the classical case) was dealt in little detail and in a too-short time in our path. Much greater attention must be paid to this point in future experimentation. As far as the Hilbert space is concerned, 78% of the students were able to list its characteristics; as far as the concept of scalar product is concerned, it was noted that, from an operational point of view, most of them (82%) make correct use of it, but from a conceptual point of view; only few students recognize its importance/utility in QM (18%). Moreover, 37% of students faced some difficulties in using Dirac's notation; in reality, this result was not a surprise, since there are several studies [35-36] in the literature, especially at the university level, which highlight these students' difficulties, and the inconstancy students show in its use.

Module 7: Most of the students expressed themselves in fairly correct terms regarding the characteristic aspects of the measurement process in QM, what the measurement process involves (88%), and what an operator is (81%). These results were also found by other research groups which developed paths starting from the discussion of key experiments [37]. With regard to matrix calculus, on the other hand, although for almost all the students (92%) it was a complete novelty, they were generally able (69%) to manage it without any difficulties.

Module 9: The use of *GeoGebra* during the explanation allowed students to have a graphical vision of the concept of eigenvector and eigenvalue and this certainly improved the general understanding: in fact, everyone managed, at least graphically, to recognize the eigenvectors and eigenvalues of simple operators. However, it was interesting to note that more than half of the students (55%) recognized vectors having the same direction but opposite orientation as different eigenvectors: this is an aspect to be taken into consideration for a future proposal. As regards the study of physical systems, given a state written as a superposition and the observable written as a matrix, all the students were able to identify the probability of obtaining a specific value; 82% also found those values; and 91% also calculated the mean value correctly. These results show how the use of software for visualizing abstract mathematical aspects plays a significant role in learning. Indeed, there are numerous projects, such as the Physics Education Technology (PhET) project [38], the Quantum Interactive Learning Tutorials [39] and the Physics Applets [40], which aim to create useful tools to help students become more familiar with abstract concepts.

This analysis allowed us to identify: some strengths of the activity, some limits of this work, and some problems that were not foreseen during the preparation and the organization of the course. The detailed analysis of the results transcends the scope of this work. However, to give an idea of what was achieved on the "students" side, we provide a very brief summary here.

- The mathematical formalism (complex linear spaces, matrices, Dirac notation, etc.) did not prove to be an obstacle to understanding, but, rather, it was seen by students as a supporting and a reassuring aspect, and it was found to be an aid towards conceptual understanding. However, this result was achieved only when the presentation of mathematical concepts made use of interactive geometric manipulation tools (such as *GeoGebra*).
- Students of the last three years of high school followed the activities; as was to be expected, the ability to solve exercises turned out to be better (at least on average) with increasing schooling. However, the understanding of some conceptual aspects (*i.e.*, explaining why is necessary to have a complex space to describe a quantum system) also with regard to their formal writing, was on average better for the youngest students than for those of the other two years.
- We were able to identify some problems that we had not foreseen in preparing and organising the course, such as, for example, the fact that almost all the students were not familiar with complex numbers and matrices.
- All the main problems encountered by students regarding the issues dealt during the course, were clarified during the following meeting, such as, *i.e.*, the concept of linearity and self-adjoint operator.
- Finally, by evaluating students' homework with criteria similar to those usually used at school, we have found that the knowledge and skills acquired were completely in line with those achieved in the "classic" subjects of high school physics.

In the interviews with teachers – conducted in a separate way and moment with respect to those with students, and focused on what teachers thought about the actual feasibility of the proposal in the classroom – it was also highlighted that:

- in compiling *Google Forms*, the effort of the students was certainly lower than that which would have been obtained in an ordinary school context, through tests, oral examinations and assessments;

- students who attended the course and who filled all forms were generally interested in the topics covered (even if they did not necessarily have high marks in physics, as emerged during the interviews with students). However, they belonged to different classes (from the 11th to the 13th grade); therefore, their mathematical knowledge was not uniform at all.

Finally, in order to more easily compare students' answers to the proposed path, we have selected among *Google Forms*' questions only those – so to speak – with a more standard structure (basically problems and exercises). 15 students among those who had completed all the 9 modules were then randomly selected, and, with the help of the teachers involved, we prepared an evaluation grid traditionally used in school, in order to see which were the marks obtained and compare them with those achieved on average in classical standard topics (Mechanics, Thermodynamics, Electromagnetism, etc.). The grid was based on 4 categories: a) knowledge of the contents; b) logical development and technical skills; c) correctness, clarity of procedures, use of specific language; 4) completeness and originality of the resolution. Answers (which were made anonymous, only keeping track of the school grade attended) were thus individually read and evaluated by each researcher (using the grid, but without teachers' support, in order to be as much homogeneous and impartial as possible); then, researchers compared and negotiated all the discordant cases (the scores reported in the paper are the one agreed upon after the comparison). The marks obtained in the individual tests (which in Italy range from 1 to 10, and with the sufficiency corresponding to 6/10) revealed to be comprised between 4.0 and 8.0, with an overall average of 6.2 (as reported in Table 1), in line with the average trend found at school level in physics. This fact helped a lot to make teachers perceive the path as actually usable in school.

**Table 1.** Marks obtained by 15 students.

| | Form 1 | Form 3 | Form 4 | Form 5 | Form 6 | Form 7 | Form 8 | Form 9 | **Mean** |
|---|---|---|---|---|---|---|---|---|---|
| S1 | 5.0 | 5.0 | 7.0 | 5.5 | 7.0 | 5.5 | 6.5 | 6.5 | **6.0** |
| S2 | 4.5 | 5.0 | 8.0 | 6.5 | 7.0 | 4.5 | 6.5 | 6.5 | **6.1** |
| S3 | 6.0 | 7.5 | 7.5 | 7.0 | 7.0 | 6.5 | 7.5 | 7.5 | **7.1** |
| S4 | 4.5 | 7.5 | 6.0 | 6.5 | 4.5 | 4.5 | 4.5 | 4.5 | **5.3** |
| S5 | 5.0 | 7.5 | 7.7 | 5.5 | 7.5 | 5.5 | 6.5 | 6.5 | **6.5** |
| S6 | 5.0 | 7.5 | 7.5 | 7.0 | 7.5 | 5.5 | 7.5 | 7.5 | **6.9** |
| S7 | 4.0 | 7.5 | 8.0 | 5.5 | 6.0 | 5.5 | 6.0 | 6.0 | **6.1** |
| S8 | 5.0 | 6.0 | 6.5 | 5.0 | 7.5 | 4.5 | 5.5 | 5.5 | **5.7** |
| S9 | 4.5 | 5.5 | 5.5 | 6.0 | 6.5 | 5.5 | 7.5 | 7.5 | **6.1** |
| S10 | 6.0 | 7.5 | 8.0 | 7.0 | 7.5 | 6.5 | 7.5 | 7.5 | **7.2** |
| S11 | 5.0 | 6.0 | 7.0 | 5.5 | 7.5 | 5.0 | 5.0 | 5.0 | **5.8** |
| S12 | 4.0 | 7.5 | 7.5 | 5.5 | 7.5 | 5.5 | 6.0 | 6.0 | **6.2** |
| S13 | 4.5 | 5.0 | 8.0 | 6.5 | 7.0 | 5.0 | 6.5 | 6.5 | **6.1** |
| S14 | 4.5 | 7.5 | 7.5 | 6.0 | 5.0 | 5.0 | 5.5 | 5.5 | **5.8** |
| S15 | 4.0 | 6.5 | 7.0 | 7.0 | 5.5 | 5.0 | 6.0 | 6.0 | **5.9** |
| | | | | | | | | **Total:** | **6.2** |

## 6. Conclusions and future projects

The developed path is intended to be a first step towards the creation of a complete path on QM, concretely achievable in Italian high school, which, on the one hand, is respectful of the disciplinary contents and, on the other hand, is accessible and meaningful for students. The information obtained from the experimentation of this path satisfied the objectives set. Overall, therefore, we believe that the experimented path gives positive indications that a meaningful and non-popular teaching of QM is possible, at least hypothetically, already in today's Italian school.

The goal for the current A.Y. was, first of all, to be able to propose the course again, integrated with the information obtained from this first experimentation. A small group of teachers declared themselves available for experimentations in classes in the A.Y. 2022-2023; a first experimentation has already started in presence in a high school of Northern Italy, held by prof. M. Porta of Liceo Palli (Casale Monferrato), and involving about 20 students of the 13th year.

From the planned experimentations it is expected to obtain new research data for building an active learning path to be experimented for a better suited and effective educational reconstruction of QM that leads also to the development of new material:

- the creation of worksheets for students and a specific tutorial for teachers;
- new representation methods (applets, etc.) for many dimensional spaces and self-adjoint operators;
- the preparation of new questions with *Kahoot!*;
- the creation of new interactive exercises with *GeoGebra*;
- the revision of the course notes, updated and integrated with the new aspects that emerged from the analyses conducted;
- the creation of appendices with the necessary mathematical contents (in particular relating to the knowledge of matrices and complex numbers), in order to provide a brief theoretical summary of support to students.

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