The opportunity cost of social relations: on the effectiveness of *small worlds*

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Abstract: *The aim of this paper is to extend the theoretical literature on knowledge and network structure by considering explicitly the choice to use the social network as a learning mechanism.*

In the model, we consider a set of actors that creates and diffuses knowledge. They are located on a lattice (identifying the social space) and they are directly connected with a small number of other individuals. Their aim is to increase their personal knowledge. We assume that individuals can learn in two ways: individually, by elaborating their personal knowledge; or socially, by interacting with other individuals in their social neighbourhood.

Given this framework, we compare network structures in terms of efficiency and equity. We find that networks characterized by low average distance perform well in the short run, when the opportunity cost of using the network is low, but cliquish networks are more efficient in the long run, when the opportunity cost is high. However, a "small world" structure, characterized both by low average distance and high cliquishness, is the most equal structure in terms of knowledge distribution.

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1. Introduction

The role of (social) networks in the economy has been a central concern in the recent development of economic theory (Kirman, 1998). The idea that agents typically interact with a subset of agents in the society, and that the structure of such interactions matters, has thrown new light on phenomena as different as criminal activity (Glaeser *et al.*, 1996), diffusion of economic and social behaviours (Bala and Goyal, 1998), unemployment and the search of new jobs (Topa, 2001).

In the field of economics of innovation, which, at an abstract level, is interested in understanding how agents create and diffuse knowledge, this role has been recognized as well. The "systemic" approach to innovation (Lundvall *et al*., 2002; Malerba, 2002), the notions of "networks as locus of innovation" (Powell *et al.*, 1996) and "collective invention" (Allen, 1983) all build on the idea that connections among agents matter. The empirical and policy interests on this issue has been accompanied by a theoretical literature studying the role of network structure in the process of knowledge creation and diffusion, building on the seminal work by Cowan and Jonard (2004a).

In this paper we make a contribution to this literature. We develop a model in which a population of (boundedly) rational and self-interested agents, active in an exogenously given social structure, can choose to use the social network, or not. In particular we study how the network structure affects its use, and the aggregate performance of the economy, and we compare our results with Cowan and Jonard (2003a), where no alternatives are given to the use of network.

The main result of our paper is that different network structures perform differently over the time horizon. Networks characterized by low average distance perform well in the short run, when the opportunity cost of using the network is low, but cliquish networks are more efficient in the long run, when the opportunity cost is high. However, a *small world* structure, characterized both by low average distance and high cliquishness, is the most equal one in terms of knowledge distribution. In terms of interpretation, we claim that the content of what is exchanged influences the optimal network structure: when the value of what is exchanged is

low, or when generic knowledge is exchanged, random networks, characterized mainly by weak ties in the terminology of Granovetter (1973), are socially optimal; conversely, when the value of what is exchanged is high, or when specific knowledge is exchanged, cliquish network (i.e. networks rich of strong ties) are preferable from a social point of view.

The rest of the paper is organized as follows: section 2 briefly reviews the literature, and stresses the element of novelty of our work. Section 3 is an exposition of the *small world* model by Watts and Strogatz (1998), which lies at the core of our comparative dynamics exercises. Section 4 describes the model, while section 5 discusses the results of numerical simulations. Finally, section 6 concludes.

2. Knowledge creation and diffusion, and social networks as opportunities

The literature on networks and knowledge, to which this paper belongs to, builds on two basic elements. First, it represents the economy as a *heterogeneous* population of agents involved in the creation and diffusion of knowledge. Second, agents are located on a network, which implies that each agent can "interact" (both in the process of knowledge creation and diffusion) only with a subset of other agents in the society (her "neighbourhood").

Existing models differ on several dimensions:

- On nature of networks. The network can be exogenous or endogenous. If endogenous, it can be a "virtual" network resulting from the historical sequence of one-period pairwise relationships, formed through a matching algorithm (Cowan *et al.*, 2002); or an "actual" network, where agents form and severe links following a kind of "reinforcement learning" mechanism (Morone and Taylor, 2004).

- On knowledge representation. Knowledge has been represented by a stock (Morone and Taylor, 2004), by a vector of real positive scalars (Cowan and Jonard, 2004a), a pair constituted by a scalar and an angle (Cowan *et al.*, 2004c), by a "tree" of activated nodes (Morone and Taylor, 2003).

- On diffusion process. Diffusion can occur though pairwise barter (Cowan and Jonard, 2004); through broadcasting, in which agents in turn share their knowledge with their neighbours (Cowan and Jonard, 2003a); through "narrowcasting", in which an agent shares her knowledge with only one agent each time (Morone and Taylor, 2004).

- On knowledge creation process. Creation can occur through recombination, when the receiver in an interaction recombines her knowledge with the new knowledge she has received, in order to create new knowledge (Cowan and Jonard, 2003a); or by innovation, when agents increase their knowledge (usually stochastically) through an unmodeled process of learning (Cowan *et al.*, 2004b).

- On other minor aspects. For instance the models can include further details to characterize the process of knowledge creation and diffusion in specific institutional contests, like the scientific community (Cowan and Jonard, 2003b); or being calibrated to reproduced specific historical and geographical cases (Morone and Taylor, 2003).

INSERT TABLE 1 ABOUT HERE

Given these differences, there is an important common element that all these models share: the use of the network has never been considered as a true *economic* choice. Network is considered as a distinct and *isolated* coordinating mechanism, and its properties are analyzed independently from the constraints individuals are subject to and from the alternative opportunities available to them.

For instance, none of the formal models listed above explicitly considers any kinds of interactions between network and one of the two other forms of coordinating economic exchange: hierarchies and market (Powell, 1990); or, in other terms, none of the models considers that "all forms of exchange contain elements of networks, markets and hierarchies".[1] Indeed, any economic agents, any node of the network, could be a member of a hierarchical

[1] This is Granovetter's critic to Powell's perspective. See Powell (1990). Note 9, p. 330.

organization or could exchange using price mechanism.[2] In order to clarify this point, let us consider the informal know-how exchange studied by von Hippel (1987). Any mini-mills steel engineer will exchange her know-how with a colleague employed in some rival firms, *only if* that exchange does not contrast her own firms' strategy. Otherwise, one firm could solve technical problems (i.e. acquiring a new piece of knowledge) stipulating a contract with a consultancy firm.

In this paper, we simply take the perspective of considering the network as *one* among the learning options available to the individuals. We assume that individuals can learn in two ways: individually, by elaborating their personal knowledge; or socially, by interacting and exchanging knowledge with other individuals in their social neighbourhood. Quoting from von Hippel:

> "A firm's staff of engineers is responsible for obtaining or developing the know-how its firm needs. When required know-how is not available in-house, an engineer typically cannot find what he needs in publication either: much is very specialized and not published yet. He must either develop it himself or learn what he needs to know by talking to other specialists" (von Hippel, 1987, p.293).

With this assumption, it is possible to consider explicitly the opportunity cost of *using* the network, making the use of the network an economic decision: choosing to learn by talking to others (i.e. learning collectively) implies that some unit of effort and/or resources (e.g. time) cannot be used to learn individually, and *vice versa*.

With respect to the existing literature, we can locate our work at an intermediate position in the dimension exogenous vs endogenous networks.

We assume the existence of a social network which is exogenous and time-invariant, that is orthogonal to agents' incentives to barter knowledge. What we model here is a set of social connections that arise "accidentally", or in any case for reasons that are not directly related to the search of useful knowledge. This assumption seems consistent with evidence on informal

[2] "Treating only the strength of ties ignores, for instance, all the important issues involving their content. What is the relation between strength and degree of specialisation of ties, or between strength and hierarchical structure?" (Granovetter, 1973; p.1378).

collaboration between scientists and engineers: they are in contact with former colleagues, classmates and private friends (Dahl and Pedersen, 2004), but friendships have no significant effect on the probability that information is traded (Schrader, 1991). This view is close in spirit to a sociological perspective, in which economic agents are "embedded" in a social web of relations (Granovetter, 1985).

At the same time, agent's choices endogenously determine an actual network, which is given by the links that are activated by agents. Such choices are motivated by an economic, self-interested reasoning, because agents interact with their neighbours only if this maximizes their learning opportunities.

To perform our analysis, we modify Cowan and Jonard (2004a) in order to consider social network as an option, and then we answer to their same research question. Given the specific process of knowledge creation and diffusion, what is the effect of the network structure terms on the overall efficiency of the system, as measured by the level of average knowledge in the economy?

3. Network's structure: the Watt and Strogatz's model

Following the literature, the emphasis of our model is on the link between knowledge creation and diffusion and the network *structure*.

For that reason, network density (i.e. the number of total exiting links) is kept fixed, while we vary its topology that as a parameter in the simulation exercises. In our context, network structure is summarized by two properties: average path length and average cliquishness.

Average path length is a global concept, defined as the average number of steps separating two randomly chosen agents. Average cliquishness is instead a local concept, measuring the degree of link redundancy in an agent's neighbourhood.

More formally, Watts and Strogatz (1998) have proposed a model that is able to generate networks with different levels of average path length and average cliquishness as a function of a single parameter.

Consider a set of agents $I = \{1,...N\}$. For any *i,j* define the variable $\chi(i, j)$, which takes value 1 if the two agents are connected, and 0 otherwise. Then, the network $G = \{\chi(i, j); i, j \in I\}$ corresponds to the list of all pairwise relationships between agents. Then the neighbourhood of agent *i* is formally defined as $\Gamma_i = \{j \in I / \{i\} : \chi(i, j) = 1\}$. A path connecting two agents *i* and *j* is defined as a set of pairwise relationships $\{(i, i_i),...;(i_k, j)\}$ such that $\chi(i, i_i) = ... = \chi(i_k, j) = 1$; the distance between *i* and *j* *d(i,j)* is given by the number of steps in the shortest path between *i* and *j*. Given this notation we can define the average path length as:

$$L = \sum_{i,j \in I} \frac{d(i, j)}{(N(N-1)/2)} \tag{1}$$

The cliquishness of a set $S \subseteq I$ is defined as the proportion of pairwise relationships in S over the total number of possible number of relationships, formally:

$$cl(S) = \frac{\sum_{i,j \in S} \chi(i, j)}{\#S(\#S-1)/2} \tag{2}$$

In the model, average cliquishness is given by the average value of $cl(S)$ for $S = \Gamma_i, i \in I$, that is:

$$C = \sum_{i \in I} \frac{cl(\Gamma_i)}{N} \tag{3}$$

Watts and Strogatz consider a family of graphs with a given number of nodes *N* and average number of link *n*. Then, the total number of links is fixed, and equal to $\frac{n * N}{2}$. They start from a ring of *N* vertices, i.e. a regular periodic lattice with *n* nearest neighbours (*n* even). Loosely speaking, they consider *N* agents located on a circle, each of them connected (by undirected edges) with the *n* nearest neighbours.

The next step is to implement the following algorithm: choose a vertex and the edge that connects it to her nearest neighbour in a clockwise sense. With probability *p*, re-connect this edge to a vertex chosen uniformously at random over the entire ring, with duplicate edges forbidden; with probability 1-*p*, leave the edge in place. The process is repeated by moving clockwise around the ring, considering each vertex in turn until one lap is completed. Next, consider the edges that connect vertices to their second nearest neighbours clockwise. As

before, rewire randomly each of these edges with probability *p*, and continue the process, circulating around the ring and proceeding outward to more distant neighbours after each lap, until each edge in the original lattice has been considered once.

Varying *p*, it is possible to build graphs having different values for average path length and average cliquishness, *L(p)* and *C(p)*. Both *L(p)* and *C(p)* are monotonically decreasing in *p*. At one extreme, for *p=0*, we have a regular network, characterized by high average cliquishness and high average path length. At the other extreme, for *p=1*, we have a random network, where both average path length and average cliquishness are low. Indeed, the important contribution by Watts and Strogatz is to show that trade-off between the local concept of cliquishness and the global notion of average distance is not so severe as it could appear at first sight. For a non negligible region of parameter *p*'s space (approximately, the interval [0.01; 0.1]), they show that $L(p) \approx L(1)$, but $C(p) >> C(1)$. When in a regular network some "shortcuts" are created connecting distant parts of the graphs, average path length is dramatically reduced, but cliquishness is substantially preserved. Watts and Strogatz call this network structure *small world*. Figure 1 considers the three main typologies of networks we consider: a regular network, a *small world*, and a random network. Each graph has 16 nodes and 32 edges.

INSERT FIGURE 1 ABOUT HERE

The search for an "optimal", exogenously given network structure turns out to be equivalent to assess the relative roles of average cliquishness and average path length in the process of knowledge creation and diffusion under consideration.

It should be worth to clarify the relation existing between the concept of clique and that one of weak ties. The latter concept is used by Granovetter in his seminal article *The strength of weak ties* (1973). His argument is based on the following assumption: if an individual (say A) has strong ties[3] with other two persons (say B and C), then it is not possible for the tie between B and C to be absent. In less generic terms: if A has two *close* friends, named B and C necessarily

[3] The strength of a tie depends on a combination of time, emotional intensity, intimacy and reciprocal service. (Granovetter, 1973, p. 1361).

B and C know each other. This assumption, based on some empirical results, has a logical implication: only weak ties can bridge two subsets of the network. "If each person's close friends know one another, they form a closely knit clique. Individuals are then connected to *other* cliques through their weak rather than their strong ties. Thus, from an *aerial* view of social networks, if cliques are connected to one another, it is mainly by weak ties." (Granovetter 2005, p. 34). Therefore it seems plausible to consider links outside cliques as weak ties and links inside cliques to be strong ties.

In Cowan and Jonard (2004a), the authors consider a pure diffusion model, where agents barter knowledge at the pair level, and in this way the overall level of knowledge increases. In a pure diffusion model, the advantages of short distances are clear: knowledge flows more rapidly, and if we assume dispersion of knowledge when passing from one agent to the other, short average distances increase the overall efficiency of the system.

Cliquishness plays a positive role, too. Cliquishness is a way to circumvent the limitations imposed by the "double coincidence of wants" that barter implies. If agent i and agent j are linked, agent i possesses knowledge that is valuable to agent j, but the reverse is not true, cliquishness guarantees that there are several paths through which knowledge can indirectly flow from agent i to agent j. Then we need to have both high cliquishness and low average distances, and the small world is the optimal structure.

Consequently, our contribution is an assessment of the roles of network average distance and cliquishness, when the set-up is modified to include the notion of social networks as one among the learning opportunities. Next section describes the model more in detail, while section 5 discusses the simulation results.

4. The model

We consider an economy populated by N agents located on a graph G. For each agent, the graph identifies the subset of agents she can exchange knowledge with. The graph is exogenously given and it is kept fixed over time. Agents aim at increasing their own knowledge. This is done in two ways: through knowledge barter between directly connected agents, and through individual learning.

Knowledge is represented by a vector $V_i(t) \in \Re_+^K$, where the K components represent different categories of knowledge. $V_{i,k}(t)$ is the level of knowledge in category k for agent i at time t.

Agents pay-offs are represented by the utility function:

$$U(V_i(t)) = \frac{\sum_k V_{i,k}(t)}{K} \tag{4}$$

Agents aim at maximizing the average level of their knowledge in different categories, they are risk–neutral and they are not affected others' agent level of knowledge (i.e. there is no competition in this economy).[4]

Each agent starts with a given level of knowledge in each category at time t=0. Then each period one link (i.e. two directly connected agents) is picked at random with uniform probability. These two connected agents can choose to barter knowledge or engage in individual learning.

Opportunities for increasing agent i's knowledge, when she is involved in knowledge barter with j or individual learning respectively, are:

$$V_{i,k}^B(t+1) = V_{i,k}(t) + \alpha \max\{V_{j,k}(t) - V_{i,k}(t), 0\} \tag{5}$$

$$V_{i,k}^I(t+1) = V_{i,k}(t)\{1 + \beta_{i,k}(t+1)\} \tag{6}$$

In equation (5), $\alpha \in (0;1)$ measures the easiness of knowledge transfer through face-to-face interactions. Since $\alpha < 1$, we assume that knowledge is only partially assimilable, due to imperfect absorptive capacity. In equation (6), $\beta_{i,k}(t+1)$ is uniformously distributed on $[0, \overline{\beta}]$, where $\overline{\beta}$ is a measure of opportunities (i.e., easiness) for individual learning. Consistent with

[4] Agents engaged in the process of knowledge creation and diffusion are typically characterized by some degree of competition, both if operating within firms and in scientific communities. In the model, we implicitly assume that these effects are of second order with respect to knowledge accumulation. This may occur in situations of rapid technological or scientific change.

the literature on knowledge creation, we assume that the process of individual learning is cumulative, i.e. it is based on agents' existing knowledge.[5]

Knowledge barter occurs iff

$$\max_{k \in K} \{V^B_{r,k}(t+1) - V_{r,k}(t), 0\} \geq \max_{k \in K} E[V^I_{r,k}(t+1) - V_{r,k}(t)] \qquad (7)$$

where $r=i,j$.

In case of ties, the knowledge category for individual learning or barter is randomly chosen with uniform probability.

After knowledge barter or individual learning has occurred, the economy moves to the next period and all the previous steps are repeated.

The proposed decision rule has a number of implications. We assume that agents have a cognitive and time endowment constraint, in the sense that they can learn only in one category each period. They are rational, but myopic, in that they maximize only current period pay-offs. When mutually beneficial barter can occur, we select the Pareto superior outcome (which is such both for the agents involved and for the economy as whole), in which agents completely exhaust the knowledge trading opportunities.[6]

It should be noticed that when barter occurs, it is necessarily the Pareto superior option, since the consent of both players is required. This is not necessarily true in the opposite case: if the difference between the return from individual learning and the return from bartering is positive and small for one agent, and negative and large for the other, barter does not occur even if it is socially desirable[7].

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[5] In the long run, it is reasonable to assume that decreasing returns in learning prevail, so that the rate knowledge is accumulated through individual learning decreases. However, since we intend to model a situation of rapid technological or scientific change, we neglect this issue.

[6] In most real-world examples of knowledge trading, exchange of knowledge does not occur simultaneously. Typically, agent A releases a piece of knowledge to agent B *today* because she expects to receive useful knowledge from B *tomorrow*. Modelling the dynamic game through which agents build a reputation is beyond the scope of this paper, and it has never been considered in this literature. In our model, the double coincidence of wants *within the period* captures the idea of mutual convenience in the barter.

[7] Suppose for instance that there are 5 knowledge categories, $\alpha = 0.5$ and $\overline{\beta} = 0.02$. For agent *i*, $V_{i,1}(t) = 10$ and $V_{i,k}(t) = 8$ for all the other *k*. For agent *j*, $V_{i,k}(t) = 8$ for all *k*. Total gains from bartering is 1; expected total gains from individual learning is 0.18. Then, bartering does not occur, even if it is socially desirable. Side-payments are not allowed in this model. The agent who gains more from the barter cannot compensate through monetary transfer the agent who gains less. To justify this, one can think of agents who are liquidity constrained.

The degree of rationality required to agents is significantly increased with respect to the assumption of Cowan and Jonard. Agents are fully able to evaluate other's agents' knowledge, or, if we see the model as a "mean field approximation", they do not commit systematic errors in such an evaluation. In our model, we fully exploit the representation of knowledge as a vector of real numbers, since the cardinality of $V_i(t)$ plays an important role in agents' decisions. This increases the *hiatus* between the appreciative discussions on the nature of "knowledge" (see for instance, Cowan *et al*., 2000) and its formal representation, but it helps in the keeping the model simple, and it lets us focusing on the key aspect of the model: the notion of social network as an option.

5. Simulation results

This section describes the design and results of the numerical implementation of the model.

Most parameters of the model are not varied in the numerical exercises we performed. In all the simulations we ran, we considered a population of 500 agents ($N=500$), each having on average 10 links ($n=10$); there are five categories of knowledge ($K=5$)[8]. The value of parameter α, i.e. the easiness of knowledge transmission is equal to 0.5. Initial value in each knowledge category is 1 for all agents. The simulation number of periods is fixed to T=100000[9].

Two parameters are varied. The first parameter is p, the re-wiring probability; we consider three values of p, in order to build graphs with features of a regular, small world and random networks. The values of p are 0.007, 0.07 and 0.7 respectively.

The second parameter is $\overline{\beta}$, that is a measure of opportunities for individual learning, i.e. the opportunity cost of using network. We consider two values of $\overline{\beta}$: "low" opportunities for individual learning ($\overline{\beta}$ = 0.01) and "high" opportunities for individual learning ($\overline{\beta}$ = 0.03).

[8] This parameterization is the same adopted in Cowan and Jonard (2004a).

[9] In terms of interpretation of the time horizon, we note the following. Since each agent has a probability 1/250 to be drawn each period, in expected value every agent will have an opportunity to learn every 250 periods. Then, it is plausible to define this as the basic unit of time. If the empirical counterpart of this basic unit is one week, the time horizon considered will correspond to approximately 8 years.

Overall, we got six combinations for the two parameters. For each of the six combinations, we ran fifty simulations with different random seeds. Here we report the average of the fifty runs.[10]

For each case, we consider the patterns of four statistical measures, catching different dimensions: efficiency, equity, specialization and use of the network.

In analogy with Cowan and Jonard (2004a), we consider two statistics in order to measure the efficiency and equity of networks in creating and diffusing knowledge. These two measures are average level of knowledge and the coefficient of variation, respectively.

Agent *i*'s average knowledge level is:

$$\overline{V}_i(t) = \frac{\sum_k V_{i,k}(t)}{K} \tag{8}$$

The average level of knowledge in the economy at time *t* is:

$$\mu(t) = \frac{\sum_i \overline{V}_i(t)}{N} \tag{9}$$

The variance in knowledge levels is:

$$\sigma^2(t) = \sum_{i \in I} \overline{V}_i^2(t) / N - \mu^2(t) \tag{10}$$

from which the coefficient of variation $\frac{\sigma(t)}{\mu(t)}$ is derived.

As measure of average specialization of individuals we consider the average Herfindhal index computed as follows. $s_{ik}(t) = \frac{V_{i,k}(t)}{\overline{V}_i(t)}$ is the share of total knowledge that agent *i* possesses in category *k*. Then

$$H_i(t) = \sum_{k \in K} s^2{}_{i,k}(t) \tag{11}$$

[10] We ran other numerical experiments, but without other economic insights besides those reported in this paragraph. We also computed confidence intervals for the statistics reported in the paper, but since these turned out to be very small we omit them for graphs readability.

and

$$H(t) = \frac{\sum_i H_i(t)}{N} \tag{12}$$

This is the statistics we report here.

Finally, we consider another indicator to measure the use of the network. For each period *t*, we indicate with *S(t)* the number of times that the network is used up to *t*. We report:

$$RS(t) = S(t)/t \tag{13}$$

This measure is relevant for the exercise because it captures the effects of our original assumption on networks as option.

Before commenting the numerical results, we need to clarify a point. Differently from Cowan and Jonard (2004a), our model does not reach any steady state in the long run. Our model is not a pure model of diffusion, but of knowledge creation too. In the long run, average knowledge diverges. This difference has clear implications for commenting simulation results. *In our exercise what is relevant is the process and not the level reached.* What is relevant is the interaction between social and individual learning, or in other terms use or not use of the network. So our comments have to be focused on the *dynamics* of the process.

We start considering jointly the efficiency of the three different network structures (Figure 2), the use of network (Figure 3) and knowledge specialization (Figure 4). In order to make easily comparable the results with respect to different values of parameters, the Figures 2 and 4 report the data using the regular network as benchmark. In particular, for average knowledge levels and average knowledge specialization, the value for the three series (regular network, small world, random network) is divided by the value of the regular network.

In terms of average knowledge levels, the dynamics of the ordering in the three networks is the same for both values of $\overline{\beta}$. In a first phase, the random network is initially the most efficient network, followed by the small world and the regular network. In a second phase, the small world maximizes average knowledge. In a third phase, the more regular network becomes more efficient. We will define the first phase as "short run", the second phase as "medium run", the

third phase as "long run". This timing is common to both value of $\overline{\beta}$, but when $\overline{\beta}$ is higher the changes in the most efficient network occur earlier.

The results can be rephrased in terms of the two main properties of the graphs considered: average distance and average cliquishness. Low average distance is the key property for efficiency at the beginning (when the average level of knowledge is sufficiently low), while high cliquishness becomes crucial later (when the average level of knowledge is high). In the short run, the most efficient graph is the more cliquish network; in the medium run, the most efficient network is the graph with relatively high cliquishness and relatively low average distance; in the long run, the most efficient network is the graph with lowest average distance.

INSERT FIGURE 2 ABOUT HERE
INSERT FIGURE 3 ABOUT HERE
INSERT FIGURE 4 ABOUT HERE

Also in terms of use of the network (Figure 3), the three network structures present different dynamics, which resemble the behaviour of average knowledge series. Since we plot average use of the network until period *t*, tendency in the use of the network in a given period of time has to be seen in the *slope* of the curve. The random network exhibits the highest use of network in the short run; the small world is the most used in the medium run; the regular network is the most used in the long run. The three networks also differ in the way their use vary over time: the random network is intensively used in the short run, but early the use declines at a quite rapid rate. The regular graph, instead, presents a lower use of the network at the beginning, but then the value remains quite stable over the simulation horizon. The *small world* graph is an intermediate case. The peak in the use of the network is lower than in the random graph case, but the decline in the use is the less severe.

Finally, the average Herfindhal index (Figure 4) is very similar in the short run for the different network structure, but at a closer inspection for $\overline{\beta} = 0.01$ and more clearly for $\overline{\beta} = 0.03$ it can be shown the specialization is higher for the regular network, followed by the small world and the random graph. As time goes, the random network emerges as the structure with the highest level of specialization, even if we observe evidence of convergence with the small world network at the end of the simulation time horizon.

It is possible to account for the joint dynamics of these variables, distinguishing among short, medium and long run.

In the short run, the average knowledge level is low. The level of knowledge is the key determinant of the opportunity cost of using the network: from equation (6) we see that the return from individual learning is increasing in agent's knowledge levels. This means that when the average knowledge level is low, also the level of opportunity cost of using network is low. In this situation, networks with short path lengths are really effective in terms of knowledge accumulation, because they diffuses globally, with low dispersion, the successfully innovative episodes that occur in an economy where the knowledge levels are very similar among agents. Networks with low average distances are intensively used for this reason, and the use of the network, by diffusing knowledge, leads to low levels of specialization.

As the average knowledge increases, the opportunity cost of using the network becomes higher and higher. The return from individual learning increases, and this is the alternative that agents have to the use of the network. Small world and regular network become more efficient (respectively in the medium and long run) due to the higher cliquishness of these networks. As we mentioned before, cliquishness is a way to circumvent the double coincidence of wants. When the opportunity cost increases, this coincidence becomes less likely, so that cliquishness becomes crucial for sustaining the use of the network. This is socially important because, while barter is always a Pareto superior option when it is used, this is not the case for individual learning. In the long run, the regular network is the most efficient network because cliquishness becomes crucial in diffusing knowledge through the social network.

We noticed also that in graphs with low average distance the level of specialization is relatively low at the beginning, because of the intense use of the network. Since barter implies some degree of heterogeneity to occur, this reinforces the tendency towards individual learning in random graphs and small world. Then, the low use of network in random graphs (and to a lesser extent, in *small worlds*) tends to favour specialization, and consequently the increase in the Herfindhal index, since agents learn in the category they are already specialized in.

We consider now the equity properties of the three different network structures (Figure 5). In this case, the order among different networks changes several times over the simulation horizon, and it depends on $\overline{\beta}$.

In principle, both individual learning and barter tends to increase the variance among agents, so what matters is the particular sequence of frequencies in the use of the network. It turns out that a small world structure emerges as the most equal structure in the long run for both $\overline{\beta}$. For low $\overline{\beta}$ the regular network is at *T=100000* the most unequal structure (while being also the most efficient in the long run), while the coefficient of variation is maximal at a random graph for high $\overline{\beta}$.

INSERT FIGURE 5 ABOUT HERE

In conclusion, differently from Cowan and Jonard (2004a), the small world does not emerge unambiguously as the most efficient network structure when the opportunity cost of using network is considered. As the opportunity cost (both in level and in proportion) increases, more cliquish networks are preferable from a knowledge accumulation point of view. These results could be rephrased in different terms. By assumption, in a regular network any agent has connections only within her clique (i.e. she has exclusively strong ties), while any random network agent is very rarely a member of a clique (i.e. she has exclusively weak ties). Therefore, the difference observed in the use of network in time shows both the increasing role of the cliques over time and give an insight of the growing importance of strength of ties.[11]

Interestingly, the small world is the most equal structure, because it is able to spread globally the knowledge accumulated locally. This is the same result as Cowan and Jonard (2004a), when

[11] It is worth mentioning that the different role over time of average distance and cliquishness is already present in Cowan and Jonard (2004a). They show that at the beginning short distances are conducive to higher knowledge levels, while cliquishness plays a positive role only after a significant period of time. In their model of pure diffusion, however, both properties must be present in an optimal network when the steady state is reached. In our model, which does not have a steady state, cliquishness emerges as the most important properties in the long run.

agent heterogeneity is measured by the coefficient of variation. In our model, then, a trade-off emerges between efficiency and equity, which is absent in Cowan and Jonard[12].

In the literature on knowledge and networks, it is interesting to compare our results with those of Cowan *et al.* (2004). Their model is of knowledge creation and diffusion through broadcasting. Their result is that cliquish networks are optimal when technological opportunities are high, or, in an other interpretation, when innovation is relatively more important compared to imitation. Our results are substantially similar, even if different is the rationale that leads to them. In their model, the redundancy of links in the clique favors a self-reinforcing mechanism in which knowledge is created and diffused at the local level. In our model, when opportunities *at the individual* level are high (both because the knowledge levels are high and because expected β is high) cliquish networks becomes more efficient.

Our result could be also discussed in light of the debate about the better strategy of network positioning: social capital vs structure holes.

The first view derives from the work of Coleman (1988) and its basic argument expressed is that being embedded in a very dense, interconnected, highly cohesive and redundant network (e.g. inside a clique) brings benefits by enhancing trust among individuals and thereby encouraging joint activities and the sharing of tacit and complex knowledge.

The second view derives instead from the works of Burt (e.g. 1992) on structural holes. The basic argument here is that when the objective is to access new knowledge being embedded in a very dense and strongly cohesive network may indeed harm a individual learning process, by producing high levels of *redundancy* and thereby reducing the amount and quality of accessible information. According to this view efficiency in accessing knowledge is instead achieved by limiting the number of redundant contacts and positioning across structural holes, i.e. linking to individuals that are not connected to each other. Individuals positioned in structural holes are able to broker knowledge flows across unconnected groups.

[12] In figure 5 it is shown that for high $\overline{\beta}$ the coefficient of variation of small world is increasing compared to the coefficient of variation for the regular network, then in the very long run the trade off presumably vanish. What we claim here is that for a significant period of time such a trade-off exist.

Our results suggest the most appropriate strategy of network positioning depends on the content of what is exchanged.

We propose two interpretations. The first one refers to the value of what is exchanged. As consequence of our assumption, the opportunity cost of using network depends positively on average knowledge: the cost of using the network increases over time, i.e. the system of incentives is endogenously modified. Then, if the network continues to be used as time elapses, this implies that what is exchanged is of great value. Then, as the *value* of what is exchanged becomes higher and higher, cliques (i.e. strong ties) become more and more important with respect to weak ties for knowledge accumulation.

The second interpretation refers to the type of knowledge which is exchanged. At the beginning, agents are not specialized in a particularly category of knowledge. Knowledge levels across categories, and among agents, are relatively similar, and barter occurs involving an amount of knowledge that is relatively small. In this case, we claim that generic knowledge is exchanged. Conversely, as time elapses, specialization increases.[13] Typically, each agent is knowledgeable in one category (the one in which the agent performs individual learning), and when agents engage in bartering, it is more likely that they release knowledge in the category in which in they are specialized. In this case, we claim that specific knowledge is exchanged, i.e. knowledge for which an agent has great advantage over the others. Then, when specific knowledge is exchanged, cliques (i.e. strong ties) are more important with respect to weak ties for knowledge accumulation, while the opposite is true for generic knowledge.

These interpretations are consistent with Chwe (1999). In a different context (i.e. a model aims to explain different political commitment), he shows that weak ties are effective in diffusing a piece of information or signaling opportunities, but strong ties are relevant if a collective action is involved.

6. Conclusions

In this paper we give a contribution to the theoretical literature studying network structure and creation and diffusion of knowledge. In all the papers belonging to this program of research, the use of the network has never been considered as a true economic choice. Network is

[13] This can be seen in the graphs of the non normalized Herfindhal index, not reported here.

considered as a distinct and isolated coordinating form, and its properties are analyzed independently from the constraints individuals are subject to and from the alternative opportunities available to them.

We propose a simulation model that explicitly considers the choice to use the social network in order to learn. Social network is one available option to individuals. Given this assumption, we analyse the optimal network structure in terms of overall efficiency of the system and in terms of its equity in diffusing knowledge.

We find that the optimal network structure depends on the time horizon. Networks characterized by low average distance perform well in the short run, when the opportunity cost of using the network is low, but cliquish networks are more efficient in the long run, when the opportunity cost is high. However, a *small world* structure, characterized both by low average distance and high cliquishness, is the most equal one in terms of knowledge distribution. As long as the interpretation is concerned, we claim that content of what is exchanged influences the optimal network structure: when the value of what is exchanged is high, or when specific knowledge is exchanged, cliquish networks (i.e. networks rich of strong ties in the Granovetter's terminology) are preferable from a social point of view; conversely, when the value of what is exchanged is low, or when generic knowledge is exchanged, random networks (characterized mainly by weak ties) are socially optimal.

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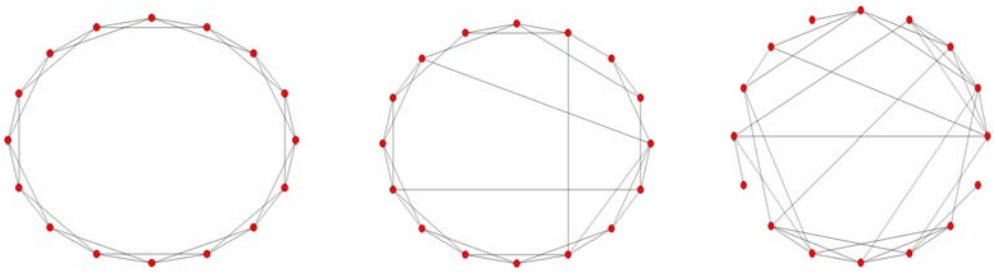
Table 1. Taxonomy of the theoretical literature

Characteristics	Assumption
Nature of networks	- Exogenous - Endogenous, as virtual network from the historical sequence of one-period pairwise relationship - Endogenous, real network with "reinforcement learning"
Knowledge representation	- Stock - Vector - Scalar and an angle - A tree of "activated" nodes
Knowledge diffusion	- Pairwise barter - Broadcasting - "Narrowcasting"
Knowledge creation	- Innovation (stochastic increase of the knowledge) - Recombination: the receiver in a interaction recombines the knowledge she received with her existing knowledge
Other aspects	- Specific institutional contexts - Specific geographical and historical cases

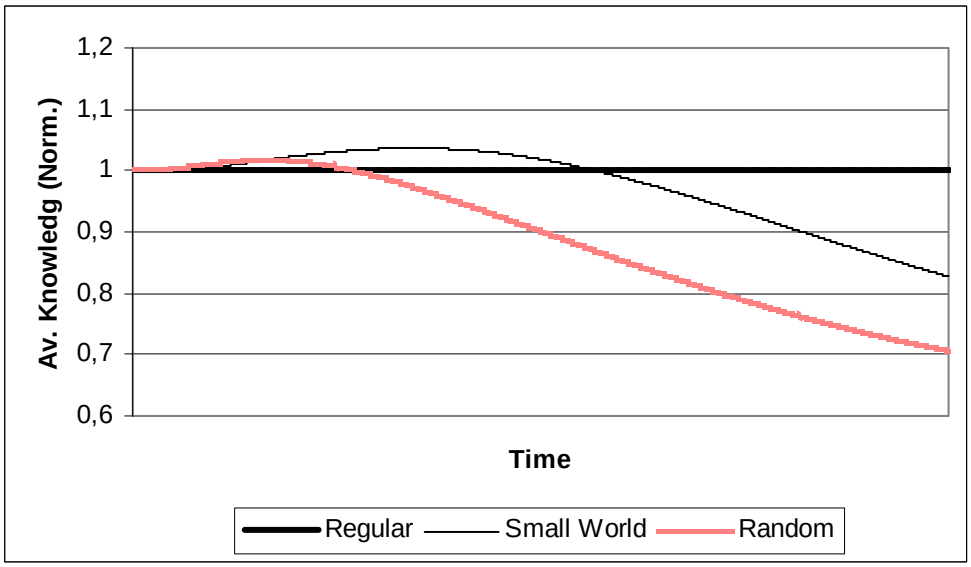
Figure 1. The three social structures



regular network ***"small world"*** ***random network***

Figure 2. Simulation results: Average Knowledge Levels

$\overline{\beta} = 0.01$



$\overline{\beta} = 0.03$

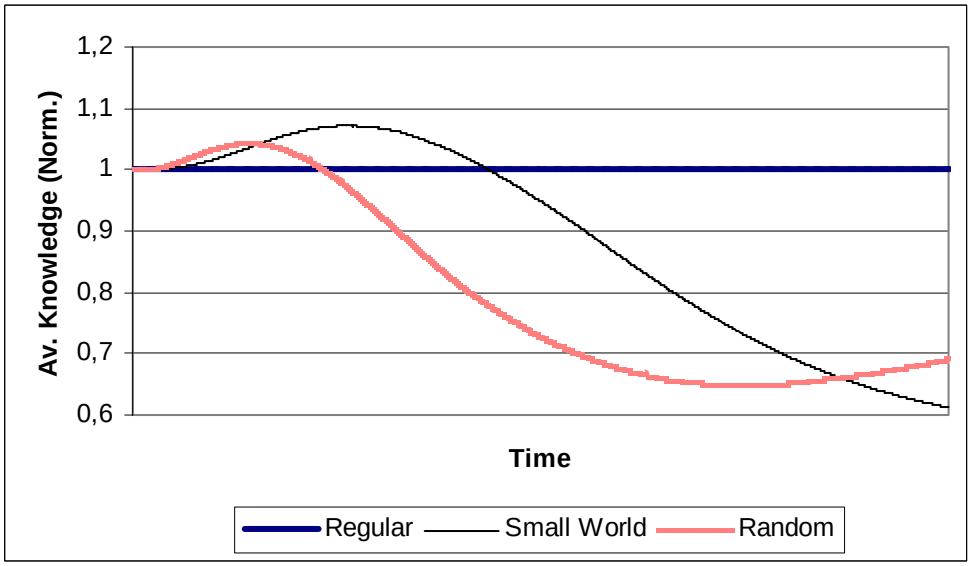
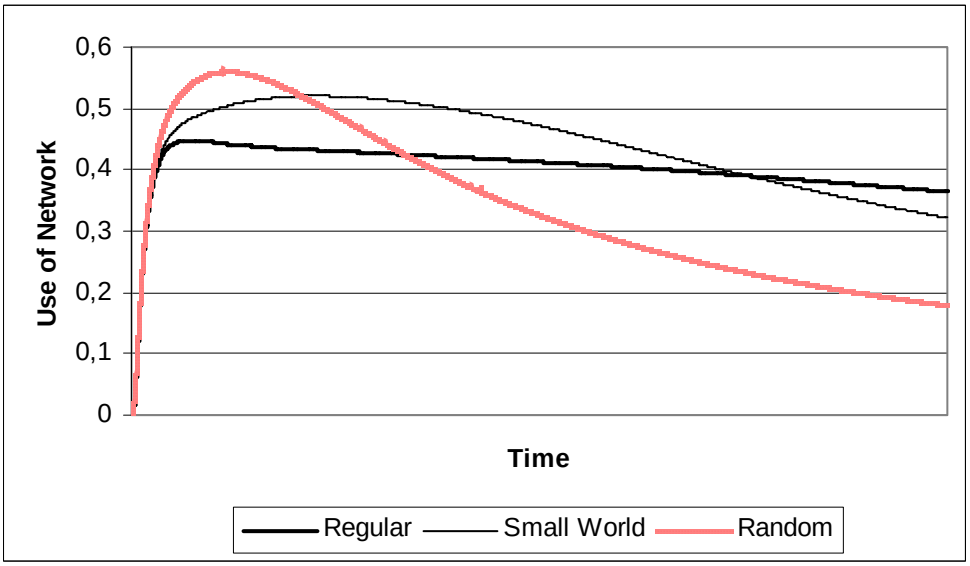


Figure 3. Simulation results: Use of the Network

$\overline{\beta} = 0.01$



$\overline{\beta} = 0.03$

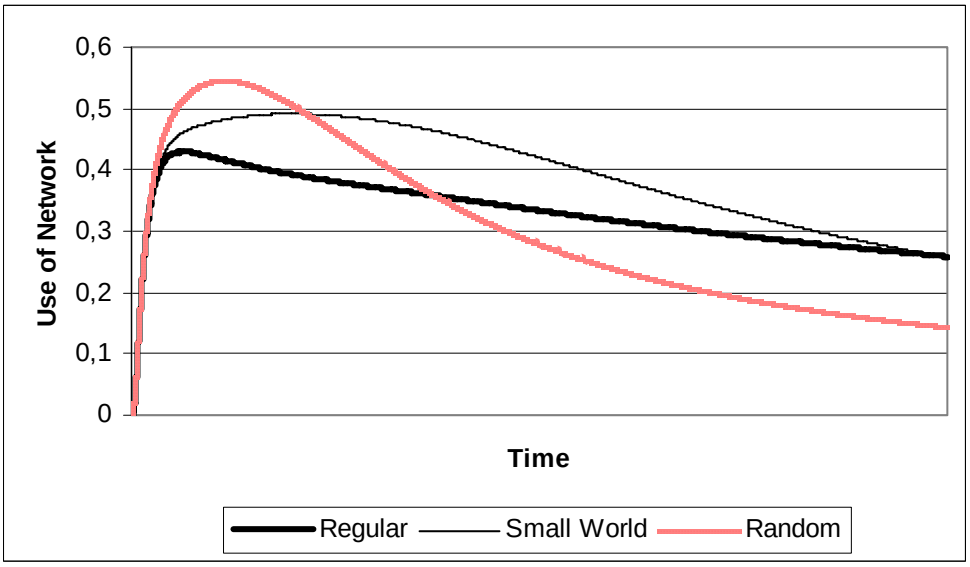
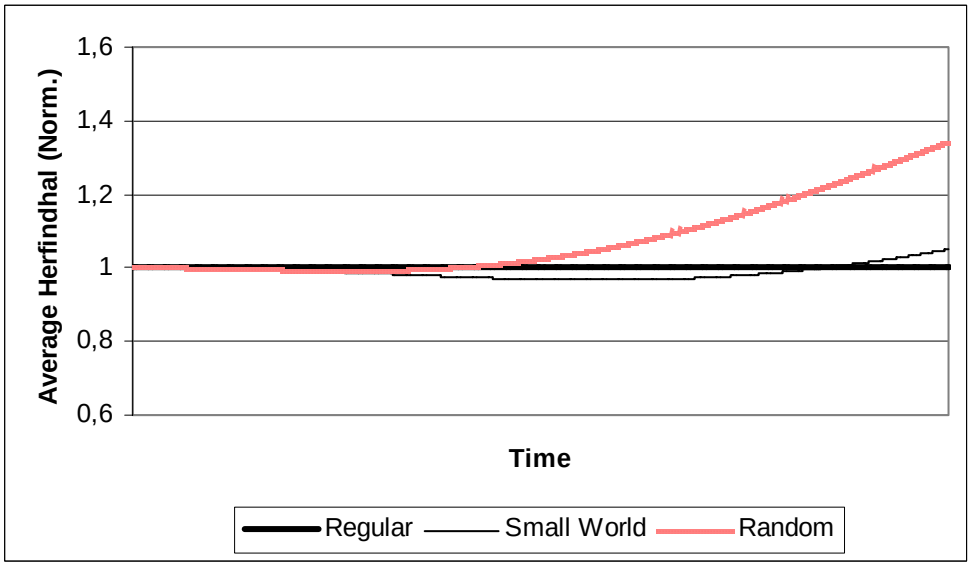


Figure 4. Simulation results: Herfindhal Index

$\overline{\beta} = 0.01$



$\overline{\beta} = 0.03$

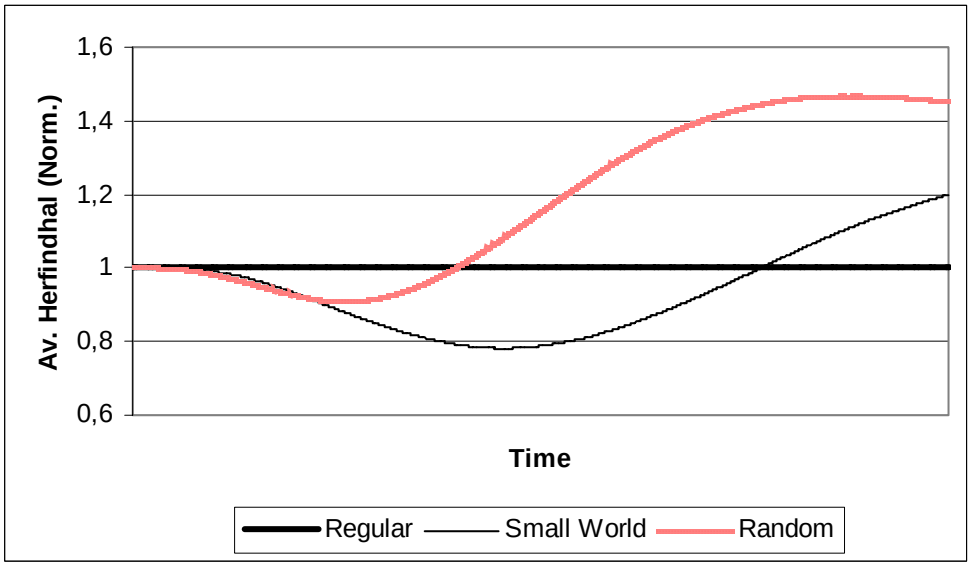
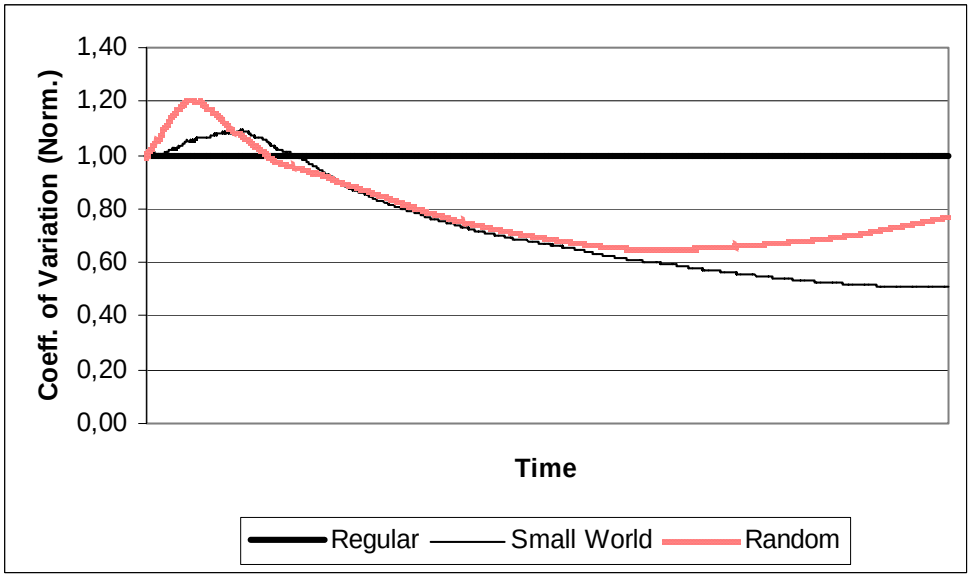


Figure 5. Simulation Results: Coefficient of Variation of Knowledge Levels

$\overline{\beta} = 0.01$



$\overline{\beta} = 0.03$