

# Two-Loop QCD correction to massive spin-2 resonance $\rightarrow$ 3 gluons

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**ABSTRACT:** We present the  $\mathcal{O}(\alpha_s^3)$  virtual QCD corrections to the process  $h \rightarrow g + g + g$  due to interference of born and two-loop amplitudes, where  $h$  is a massive spin-2 particle and  $g$  is the gluon. We assume that the SM fields couple to  $h$  through the SM energy momentum tensor. Our result constitutes one of the ingredients to full NNLO QCD contribution to production of a massive spin-2 particle along with a jet in the scattering process at the LHC. In particular, this massive spin-2 could be a KK mode of a ADD graviton in large extra dimensional model or a RS KK mode in warped extra dimensional model or a generic massive spin-2. In addition, it provides an opportunity to study the ultraviolet and infrared structures of QCD amplitudes involving tensorial coupling resulting from energy momentum operator. Using dimensional regularization, we find that infrared poles of this amplitude are in agreement with the proposal by Catani confirming the factorization property of QCD amplitudes with tensorial insertion.

**KEYWORDS:** QCD Phenomenology, NLO Computations

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## 1 Introduction

Theoretical studies on production and decay of a spin  $J^P$  particle in the hadron colliders in the Standard Model (SM) and its extensions are underway due to a wealth of information available from ATLAS [1] and CMS [2] collaborations at the Large Hadron Collider (LHC). In particular, investigations involving particles with higher spin, namely  $J^P = 2^+$  bosons gained attention [3, 4] due to the discovery of a new boson at the LHC with a mass  $m \approx 125$  GeV as its spin is yet to be determined with no doubt. Searches of spin-2 particles earlier in Tevatron and recently in LHC were motivated due to their presence in theories with large extra dimensions, such as ADD [5–7] and warped extra dimensions such as RS [8] models. These beyond the SM (BSM) theories provide an alternate scenario to explain the hierarchy problem of the SM through the introduction of compact higher dimensions with factorisable (ADD) or non-factorisable (RS) geometries. The impact of these higher dimensions can be understood in four dimensional framework through the existence of a tower of massive spin-2 Kaluza-Klein (KK) gravitons. Their signatures at the colliders can provide valuable information on these models. For the production of scalar  $J^P = 0^+$

and vector  $J^P = 1^+$  particles in the SM and BSM, there are extensive studies available in literature. They include inclusive and semi-inclusive rates taking into account quantum chromodynamics (QCD) and electroweak (EW) radiative corrections to impressively high orders in perturbative expansion. Thanks to universal features of some of the perturbative corrections in QCD, one can safely use those computed in the SM for BSM processes. For spin-2 particles, searches in Drell-Yan (DY), di-photon and jet+missing energy at the LHC are already underway. Also the bounds on the scale of the new physics and the number of extra-dimensions are severely constrained from these searches. These studies use theoretical results computed in QCD at NLO level in perturbation theory. Such computations are inevitable due to various uncertainties resulting from renormalization and factorization scales as well as from the parton distribution functions. Needless to say that the corrections are also big giving large K-factors which are often sensitive to the observables and their physical scales. In ADD and RS models spin-2 KK gravitons contribute to physical processes through either exchange of virtual graviton state or production of a real graviton state. In the virtual mode, KK gravitons get exchanged between the SM particles and the summation over their high multiplicity leads to a compensation of small coupling strength. Hence, the cross section will be appreciable at collider energies, giving rise to non-resonant enhancement of the high invariant mass regions of a di-final state production [9–13] or final states involving more particles [14, 15]. Next to leading order QCD corrections were computed for the di-final state processes in the ADD model *viz.*  $\ell^+\ell^-$  [16–18],  $\gamma\gamma$  [19, 20] and  $ZZ$  [21, 22] and  $W^+W^-$  [23, 24], in addition they have been extended to NLO+PS accuracy [25, 26]. On-shell spin-2 particle production results in missing energy signal. Again large multiplicity of final states enhances the observable effects. Productions of on-shell spin-2 gravitons in association with (a) jet [27], (b) photon [28] and (c) electroweak gauge boson [29, 30] have been studied to NLO in QCD. Similar results taking into account NLO QCD effects for processes involving the resonant production of spin-2 graviton are available for di-lepton production [17, 18], di-photon production [20], di-neutral electroweak gauge boson production [22] and charged electroweak gauge boson production  $W^+W^-$  [24]. Since the NLO corrections and the associated scale uncertainties are not completely under control in most of the observable, an attempt [31] was undertaken to include soft and collinear contribution to NNLO accuracy for processes involving KK gravitons in both ADD and RS models. This was possible as the full two loop QCD matrix element of energy momentum tensor is now available [32]. The phenomenological results show significant reduction in dependence on renormalization and factorization scales and these results improve the stability of the perturbative predictions.

Observables with jet + missing energy are sensitive to new physics from many BSM scenarios. This missing energy could be due to a heavy resonance produced in the final state escaping the detector. The NLO QCD effects to this process in large extra dimensional model, namely ADD have been available for sometime and the importance of K-factor and scale uncertainties are documented in [27]. In this article, we compute  $\mathcal{O}(\alpha_s^3)$  virtual correction in massless QCD to the process  $h \rightarrow g + g + g$  due to interference of born and two-loop amplitudes, where  $h$  is a massive spin-2 particle. The full NNLO analysis also requires square of one loop amplitudes, real emission processes and appropriate mass

counter terms which we reserve for future work. We have assumed a minimal coupling between massive spin-2 field and the fields of the SM. Hence our results are applicable to scattering processes involving a massive spin-2 particle and three gluons such as a massive graviton production with a jet in ADD and RS models or production of a massive spin-2 Higgs like boson along with a jet after appropriate analytical continuation [33] of kinematical variables to the respective physical regions. Similar computations with massive spin-0 and spin-1 boson productions at two loop level in QCD exist in the literature for the processes  $Higgs \rightarrow g + g + g$  [34] and  $g + g \rightarrow V + jet, V = Z, \gamma$  [35] respectively.

Spin-2 field being a rank-2 tensor, we encounter for the first time the two loop amplitudes with higher tensorial integrals resulting from rank-2 derivative couplings of spin-2 fields with the SM ones. In addition, we encounter more than 2000 two loop Feynman amplitudes contributing due to the universal coupling of spin-2 field with all the SM particles. While these increase technical complexities at the intermediate stages of computation, the results confirm the universal infra-red structure of QCD amplitudes. In other words, we find that soft and collinear divergences not only factorise but also agree with the predictions from Catani's work [36] (see also [37]) on two loop QCD amplitudes for multi-leg processes. We also observe that there are no additional UV divergences as the interaction is through energy momentum tensor of the SM which is conserved. Hence, our present work is also useful to study the field theoretical structure of QCD amplitudes with tensor operator insertions, in particular with the energy momentum tensor of the SM.

In the next section, we describe the generic effective Lagrangian that describes coupling of spin-2 fields with those of the SM. Section 3 is devoted to the computational details. Section 4 and appendix contain our final results. In section 5, we conclude with our findings.

## 2 Theory

### 2.1 The effective Lagrangian

We consider the SM with an additional spin-2 field  $h^{\mu\nu}$ . We assume that the spin-2 field couples minimally with the SM ones through the SM energy momentum tensor  $T_{\mu\nu}^{SM}$ . Since we are interested only in the QCD effects of the process under study, we restrict ourselves to the QCD part of  $T_{\mu\nu}^{SM}$  and hence the action reads [5–8] as

$$\mathcal{S} = \mathcal{S}_{SM} + \mathcal{S}_h - \frac{\kappa}{2} \int d^4x T_{\mu\nu}^{QCD}(x) h^{\mu\nu}(x), \quad (2.1)$$

where  $\kappa$  is a dimensionful coupling and  $T_{\mu\nu}^{QCD}$  is the energy momentum tensor of QCD given by

$$\begin{aligned} T_{\mu\nu}^{QCD} = & -g_{\mu\nu} \mathcal{L}_{QCD} - F_{\mu\rho}^a F_{\nu}^{a\rho} - \frac{1}{\xi} g_{\mu\nu} \partial^\rho (A_\rho^a \partial^\sigma A_\sigma^a) + \frac{1}{\xi} (A_\nu^a \partial_\mu (\partial^\sigma A_\sigma^a) + A_\mu^a \partial_\nu (\partial^\sigma A_\sigma^a)) \\ & + \frac{i}{4} \left[ \bar{\psi} \gamma_\mu (\vec{\partial}_\nu - ig_s T^a A_\nu^a) \psi - \bar{\psi} (\overleftarrow{\partial}_\nu + ig_s T^a A_\nu^a) \gamma_\mu \psi + \bar{\psi} \gamma_\nu (\vec{\partial}_\mu - ig_s T^a A_\mu^a) \psi \right. \\ & \left. - \bar{\psi} (\overleftarrow{\partial}_\mu + ig_s T^a A_\mu^a) \gamma_\nu \psi \right] + \partial_\mu \bar{\omega}^a (\partial_\nu \omega^a - g_s f^{abc} A_\nu^c \omega^b) \\ & + \partial_\nu \bar{\omega}^a (\partial_\mu \omega^a - g_s f^{abc} A_\mu^c \omega^b). \end{aligned} \quad (2.2)$$

$g_s$  is the strong coupling constant and  $\xi$  is the gauge fixing parameter. The  $T^a$  are generators and  $f^{abc}$  are the structure constants of  $SU(3)$ . Note that spin-2 fields couple to ghost fields ( $\omega^a$ ) [38] as well in order to cancel unphysical degrees of freedom of gluon fields ( $A_\mu^a$ ).

## 2.2 Notation

We consider the decay of a massive spin-2 field into three gluons

$$h(q) \rightarrow g(p_1) + g(p_2) + g(p_3). \quad (2.3)$$

The associated Mandelstam variables are defined as

$$s \equiv (p_1 + p_2)^2, \quad t \equiv (p_2 + p_3)^2, \quad u \equiv (p_1 + p_3)^2 \quad (2.4)$$

which satisfy

$$s + t + u = M_h^2 \equiv Q^2 \quad (2.5)$$

where  $M_h$  is the mass of the spin-2 field. We also define the following dimensionless invariants which appear in harmonic polylogarithms (HPL) [39] and 2dHPL [40, 41] as

$$x \equiv s/Q^2, \quad y \equiv u/Q^2, \quad z \equiv t/Q^2 \quad (2.6)$$

satisfying

$$x + y + z = 1. \quad (2.7)$$

## 2.3 Ultraviolet renormalization

We describe here the ultraviolet (UV) renormalization of the matrix elements of the decay of a spin-2 resonance with minimal coupling up to second order in QCD perturbation theory. We regularize the theory in  $d = 4 + \epsilon$  dimensions and the dimensionful strong coupling constant in  $d$  dimensions is made dimensionless one ( $\hat{g}_s$ ) by introducing the scale  $\mu_0$ . We expand the unrenormalized amplitude in powers of  $\hat{a}_s = \hat{g}_s^2/16\pi^2$  as

$$|\mathcal{M}\rangle = \left(\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon\right)^{\frac{1}{2}} |\hat{\mathcal{M}}^{(0)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon\right)^{\frac{3}{2}} |\hat{\mathcal{M}}^{(1)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon\right)^{\frac{5}{2}} |\hat{\mathcal{M}}^{(2)}\rangle + \mathcal{O}(\hat{a}_s^3), \quad (2.8)$$

where  $S_\epsilon = \exp[\frac{\epsilon}{2}(\gamma_E - \ln 4\pi)]$  with Euler constant  $\gamma_E = 0.5772\dots$ , results from loop integrals beyond leading order.  $|\hat{\mathcal{M}}^{(i)}\rangle$  is the unrenormalized color-space vector representing the  $i^{th}$  loop amplitude. In  $\overline{MS}$  scheme, the renormalized coupling constant  $a_s \equiv a_s(\mu_R^2)$  at renormalization scale  $\mu_R$  is related to unrenormalized coupling constant  $\hat{a}_s$  by

$$\begin{aligned} \frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon &= \frac{a_s}{\mu_R^\epsilon} Z(\mu_R^2) \\ &= \frac{a_s}{\mu_R^\epsilon} \left[ 1 + a_s \frac{2\beta_0}{\epsilon} + a_s^2 \left( \frac{4\beta_0^2}{\epsilon^2} + \frac{\beta_1}{\epsilon} \right) + \mathcal{O}(a_s^3) \right], \end{aligned} \quad (2.9)$$

where

$$\beta_0 = \left( \frac{11}{3} C_A - \frac{4}{3} T_F n_f \right), \quad \beta_1 = \left( \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_F n_f - 4 C_F T_F n_f \right) \quad (2.10)$$

with  $C_A = N$ ,  $C_F = (N^2 - 1)/2N$ ,  $T_F = 1/2$  and  $n_f$  is the number of active quark flavors. Since, the spin-2 resonance couples to the SM particles through energy momentum tensor (eq. (2.1)) which is conserved, the coupling constant  $\kappa$  is protected from any UV renormalization. Hence, there will be no additional UV renormalization required other than the strong coupling constant renormalization. Using the eq. (2.9), we now can express  $|\mathcal{M}\rangle$  (eq. (2.8)) in powers of renormalized  $a_s$  with UV finite matrix elements  $|\mathcal{M}^{(i)}\rangle$

$$|\mathcal{M}\rangle = (a_s)^{\frac{1}{2}} \left( |\mathcal{M}^{(0)}\rangle + a_s |\mathcal{M}^{(1)}\rangle + a_s^2 |\mathcal{M}^{(2)}\rangle + \mathcal{O}(a_s^3) \right) \quad (2.11)$$

where

$$\begin{aligned} |\mathcal{M}^{(0)}\rangle &= \left( \frac{1}{\mu_R^\epsilon} \right)^{\frac{1}{2}} |\hat{\mathcal{M}}^{(0)}\rangle, \\ |\mathcal{M}^{(1)}\rangle &= \left( \frac{1}{\mu_R^\epsilon} \right)^{\frac{3}{2}} \left[ |\hat{\mathcal{M}}^{(1)}\rangle + \mu_R^\epsilon \frac{r_1}{2} |\hat{\mathcal{M}}^{(0)}\rangle \right], \\ |\mathcal{M}^{(2)}\rangle &= \left( \frac{1}{\mu_R^\epsilon} \right)^{\frac{5}{2}} \left[ |\hat{\mathcal{M}}^{(2)}\rangle + \mu_R^\epsilon \frac{3r_1}{2} |\hat{\mathcal{M}}^{(1)}\rangle + \mu_R^{2\epsilon} \left( \frac{r_2}{2} - \frac{r_1^2}{8} \right) |\hat{\mathcal{M}}^{(0)}\rangle \right] \end{aligned} \quad (2.12)$$

with

$$r_1 = \frac{2\beta_0}{\epsilon}, \quad r_2 = \left( \frac{4\beta_0^2}{\epsilon^2} + \frac{\beta_1}{\epsilon} \right). \quad (2.13)$$

## 2.4 Infrared factorization

Beyond leading order, the UV renormalized matrix elements  $|\mathcal{M}^{(i)}\rangle$ ,  $i > 0$  contain divergences arising from the infrared sector of massless QCD. They result from soft gluons and collinear massless partons present in the loops. They will cancel against similar divergences coming from real emission contributions in the infrared safe observables order by order in  $a_s$ , thanks to KLN theorem [42, 43]. The infrared divergence structure and their factorization property in QCD amplitudes have been well studied for long time. In [36], Catani predicted the infrared divergences of multi-parton QCD amplitudes precisely in dimensional regularization up to two loops excluding two loop single pole in  $\epsilon$ . In [37], Sterman and Tejeda-Yeomans provided a systematic way of understanding the structure of infrared divergences using factorization properties of the scattering amplitudes along with infrared evolution equations. They demonstrated the connection of single pole in  $\epsilon$  to a soft anomalous dimension matrix, later computed in [44, 45]. The structure of single pole term for the electromagnetic and Higgs form factors was first shown in [46, 47]. Using soft collinear effective field theory, Becher and Neubert [48] derived the exact formula for the infra-red divergences of scattering amplitudes with an arbitrary number of loops and legs in massless QCD including single pole in dimensional regularization. Using Wilson lines for hard partons and soft and eikonal jet functions in dimensional regularization, Gardi and Magnea also arrived at a similar all order result [49].

According to Catani's prediction, the renormalized amplitudes  $|\mathcal{M}^{(i)}\rangle$  for the process (eq. (2.3)) can be expressed in terms of the universal subtraction operators  $\mathbf{I}_g^{(i)}(\epsilon)$  as follows<sup>1</sup>

$$\begin{aligned} |\mathcal{M}^{(1)}\rangle &= 2 \mathbf{I}_g^{(1)}(\epsilon) |\mathcal{M}^{(0)}\rangle + |\mathcal{M}^{(1)fin}\rangle \\ |\mathcal{M}^{(2)}\rangle &= 2 \mathbf{I}_g^{(1)}(\epsilon) |\mathcal{M}^{(1)}\rangle + 4 \mathbf{I}_g^{(2)}(\epsilon) |\mathcal{M}^{(0)}\rangle + |\mathcal{M}^{(2)fin}\rangle \end{aligned} \quad (2.14)$$

where,

$$\begin{aligned} \mathbf{I}_g^{(1)}(\epsilon) &= \frac{1}{2} \frac{e^{-\frac{\epsilon}{2}\gamma_E}}{\Gamma(1+\frac{\epsilon}{2})} \mathcal{V}_g^{sing}(\epsilon) \left[ \left(-\frac{s}{\mu_R^2}\right)^{\frac{\epsilon}{2}} + \left(-\frac{t}{\mu_R^2}\right)^{\frac{\epsilon}{2}} + \left(-\frac{u}{\mu_R^2}\right)^{\frac{\epsilon}{2}} \right] \\ \mathbf{I}_g^{(2)}(\epsilon) &= -\frac{1}{2} \mathbf{I}_g^{(1)}(\epsilon) \left[ \mathbf{I}_g^{(1)}(\epsilon) - \frac{2\beta_0}{\epsilon} \right] + \frac{e^{\frac{\epsilon}{2}\gamma_E} \Gamma(1+\epsilon)}{\Gamma(1+\frac{\epsilon}{2})} \left[ -\frac{\beta_0}{\epsilon} + K \right] \mathbf{I}_g^{(1)}(2\epsilon) \\ &\quad + \mathbf{H}_g^{(2)}(\epsilon) \end{aligned} \quad (2.15)$$

and

$$\mathcal{V}_g^{sing}(\epsilon) = C_A \frac{4}{\epsilon^2} - \frac{\beta_0}{\epsilon}, \quad K = \left( \frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{10}{9} T_F n_f \quad (2.16)$$

$$\mathbf{H}_g^{(2)}(\epsilon) = \frac{3}{2\epsilon} \left\{ C_A^2 \left( -\frac{5}{12} - \frac{11}{24} \zeta_2 - \frac{1}{2} \zeta_3 \right) + C_A n_f \left( \frac{29}{27} + \frac{1}{12} \zeta_2 \right) - \frac{1}{2} C_F n_f - \frac{5}{27} n_f^2 \right\} \quad (2.17)$$

### 3 Calculation of two-loop amplitude

We now describe the computation of  $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle$  &  $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle$  matrix elements where all the Lorentz and color indices of external particles are summed over. The computation involves large number of Feynman diagrams. We need to perform various algebraic simplifications with Dirac, Lorentz and color indices before the loop integrals are evaluated. Due to tensorial coupling of spin-2 resonance with the SM fields, the loops contain higher rank tensor integrals as compared to the ones normally encountered in the SM processes. We have systematically automated this computation using various symbolic manipulation programs developed in house and few publicly available packages that use FORM [50] and Mathematica. In the following, we describe the method in detail.

#### 3.1 Feynman diagrams and simplification

We use QGRAF [51] to generate the Feynman diagrams. We find that there are 4 diagrams at tree level, 108 at one loop and 2362 at two loops, leaving out tadpole and self energy corrections to the external legs. The output of the QGRAF is then converted to the format that is suitable for further symbolic manipulation using FORM and Mathematica. A set of FORM routines is used to perform simplification of the squared matrix elements involving

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<sup>1</sup>The numerical coefficients 2 and 4 with  $\mathbf{I}^{(i)}$  come due to the different definition of  $a_s$  between ours and Catani.

gluon and spin-2 resonance polarization and color sums. We have used Feynman gauge throughout and for the external on-shell gluon legs, physical polarizations are summed using

$$\sum_s \varepsilon^\mu(p_i, s) \varepsilon^{\nu*}(p_i, s) = -g^{\mu\nu} + \frac{p_i^\mu q_i^\nu + q_i^\mu p_i^\nu}{p \cdot q} \quad (3.1)$$

where,  $p_i$  is the  $i^{th}$ -gluon momentum and  $q_i$  is the corresponding reference momentum which is an arbitrary light-like 4-vector. We choose  $q_1 = p_2$ ,  $q_2 = p_1$  and  $q_3 = p_1$  for simplicity. The spin-2 polarization sum in  $d$  dimensions is given by [16]

$$\begin{aligned} B^{\mu\nu;\rho\sigma}(q) = & \left( g^{\mu\rho} - \frac{q^\mu q^\rho}{q \cdot q} \right) \left( g^{\nu\sigma} - \frac{q^\nu q^\sigma}{q \cdot q} \right) + \left( g^{\mu\sigma} - \frac{q^\mu q^\sigma}{q \cdot q} \right) \left( g^{\nu\rho} - \frac{q^\nu q^\rho}{q \cdot q} \right) \\ & - \frac{2}{d-1} \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q \cdot q} \right) \left( g^{\rho\sigma} - \frac{q^\rho q^\sigma}{q \cdot q} \right) \end{aligned} \quad (3.2)$$

### 3.2 Reduction of tensor integrals

Beyond leading order, the resulting expressions involve tensorial one and two loop integrals which need to be reduced to a set of scalar integrals using a convenient tensorial reduction procedure. Tensorial reduction is quite straightforward at one loop level but not so at two loop level and beyond. In addition, finding a minimal set of integrals after the tensorial reduction is important to achieve the task with large number of Feynman integrals. A systematic approach to deal with higher rank tensor integrals and large number of scalar integrals is to use Integration by parts (IBP) [52, 53] and Lorentz invariant (LI) [54] identities.

The IBP identities follow from the fact that in the dimensional regularization, the integral of the total derivative with respect to any loop momenta vanishes, that is

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \int \frac{d^d k_L}{(2\pi)^d} \frac{\partial}{\partial k_i} \cdot \left( v_j \frac{1}{\prod_l \mathcal{D}_l^{n_l}} \right) = 0 \quad (3.3)$$

where  $n_l$  is an element of  $\vec{n} = (n_1, \dots, n_N)$  with  $n_l \in \mathbb{Z}$ ,  $L$  is the number of loops and  $\mathcal{D}_l$ s are propagators which depend on the loop and external momenta and also masses. The four vector  $v_j^\mu$  can be both loop and external momenta. Performing the differentiation on the left hand side and expressing the scalar products of  $k_i$  and  $p_j$  linearly in terms of  $\mathcal{D}_l$ 's, one obtains the IBP identities given by

$$\sum_i a_i J(b_{i,1} + n_1, \dots, b_{i,N} + n_N) = 0 \quad (3.4)$$

where

$$J(\vec{m}) = J(m_1, \dots, m_N) = \int \frac{d^d k_1}{(2\pi)^d} \cdots \frac{d^d k_L}{(2\pi)^d} \frac{1}{\prod_l \mathcal{D}_l^{m_l}} \quad (3.5)$$

with  $b_{i,j} \in \{-1, 0, 1\}$  and  $a_i$  are polynomial in  $n_j$ . The LI identities follow from the fact that the loop integrals are invariant under Lorentz transformations of the external momenta, that is

$$p_i^\mu p_j^\nu \left( \sum_k p_{k[\nu} \frac{\partial}{\partial p_k^{\mu]} \right) J(\vec{m}) = 0. \quad (3.6)$$



While these identities are useful to express the tensorial integrals in terms of a set of master integrals, in practice, the computation becomes tedious due to the appearance of large variety of Feynman integrals involving different set of propagators each requiring a set of IBP and LI identities independently. We have reduced such varieties to a few by shifting the loop momenta suitably using an in-house algorithm which uses FORM. For one-loop diagrams, we can express each Feynman integral to contain terms from one of the following three sets:

$$\{\mathcal{D}, \mathcal{D}_1, \mathcal{D}_{12}, \mathcal{D}_{123}\}, \{\mathcal{D}, \mathcal{D}_2, \mathcal{D}_{23}, \mathcal{D}_{123}\}, \{\mathcal{D}, \mathcal{D}_3, \mathcal{D}_{31}, \mathcal{D}_{123}\} \quad (3.7)$$

where,

$$\mathcal{D} = k_1^2, \mathcal{D}_i = (k_1 - p_i)^2, \mathcal{D}_{ij} = (k_1 - p_i - p_j)^2, \mathcal{D}_{ijk} = (k_1 - p_i - p_j - p_k)^2 \quad (3.8)$$

In each set in eq. (3.7),  $\mathcal{D}$ 's are linearly independent and form a complete basis in the sense that any Lorentz invariant  $k_1 \cdot p_i$  can be expressed in terms of  $\mathcal{D}$ 's. At two loops, there are nine independent Lorentz invariants involving loop momenta  $k_1$  and  $k_2$ , namely  $\{(k_\alpha \cdot k_\beta), (k_\alpha \cdot p_i)\}, \alpha, \beta = 1, 2; i = 1, \dots, 3$ . After appropriate shifting of loop momenta, we can express each two loop Feynman integral to contain terms belonging to one of the following six sets:

$$\begin{aligned} &\{\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_{1;1}, \mathcal{D}_{2;1}, \mathcal{D}_{1;12}, \mathcal{D}_{2;12}, \mathcal{D}_{1;123}, \mathcal{D}_{2;123}\} \\ &\{\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_{1;2}, \mathcal{D}_{2;2}, \mathcal{D}_{1;23}, \mathcal{D}_{2;23}, \mathcal{D}_{1;123}, \mathcal{D}_{2;123}\} \\ &\{\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_{1;3}, \mathcal{D}_{2;3}, \mathcal{D}_{1;31}, \mathcal{D}_{2;31}, \mathcal{D}_{1;123}, \mathcal{D}_{2;123}\} \\ &\{\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_{1;1}, \mathcal{D}_{2;1}, \mathcal{D}_{0;3}, \mathcal{D}_{1;12}, \mathcal{D}_{2;12}, \mathcal{D}_{1;123}\} \\ &\{\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_{1;2}, \mathcal{D}_{2;2}, \mathcal{D}_{0;1}, \mathcal{D}_{1;23}, \mathcal{D}_{2;23}, \mathcal{D}_{1;123}\} \\ &\{\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_{1;3}, \mathcal{D}_{2;3}, \mathcal{D}_{0;2}, \mathcal{D}_{1;31}, \mathcal{D}_{2;31}, \mathcal{D}_{1;123}\} \end{aligned} \quad (3.9)$$

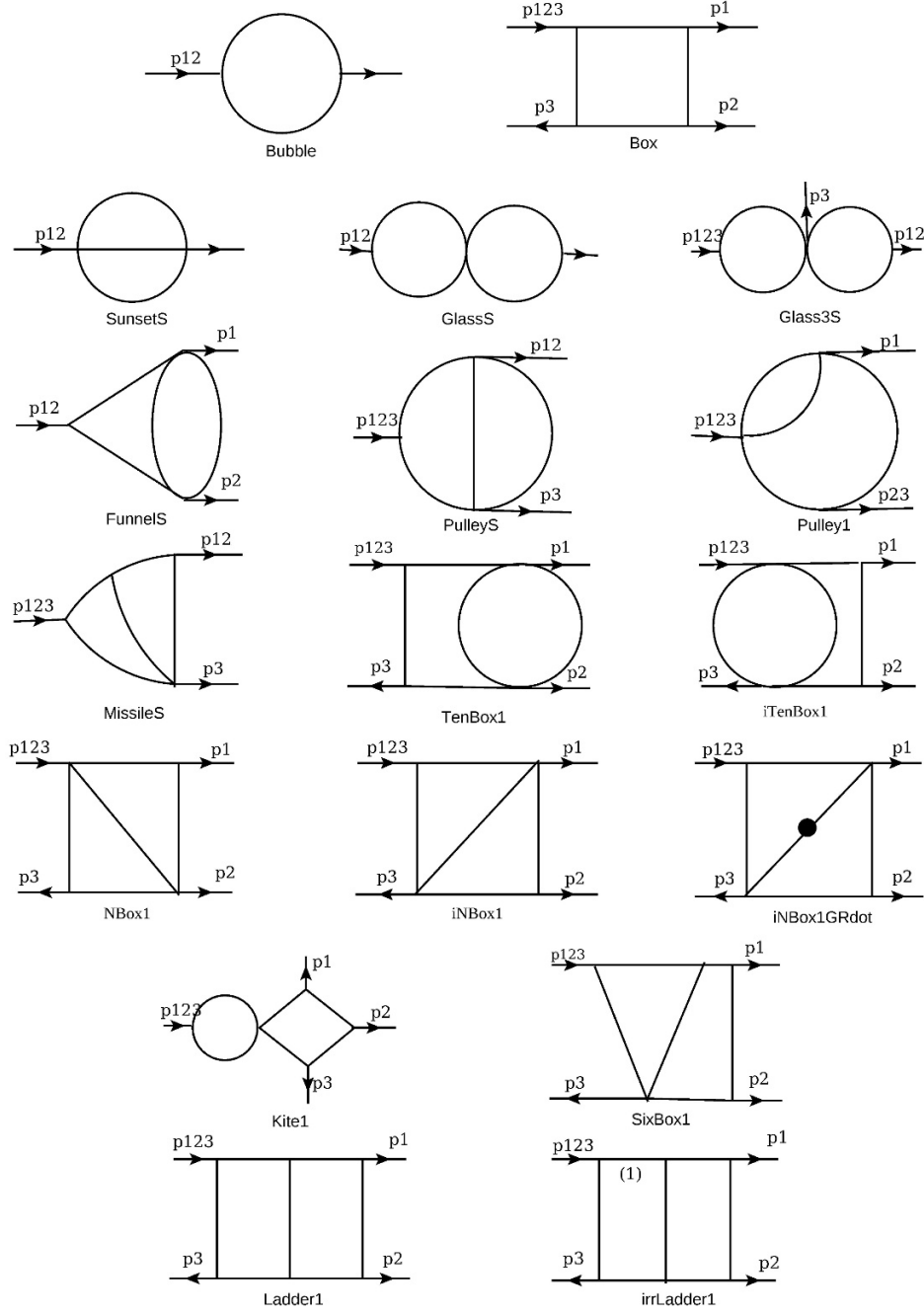
where,

$$\begin{aligned} \mathcal{D}_0 &= (k_1 - k_2)^2, \mathcal{D}_\alpha = k_\alpha^2, \mathcal{D}_{\alpha;i} = (k_\alpha - p_i)^2, \mathcal{D}_{\alpha;ij} = (k_\alpha - p_i - p_j)^2, \\ \mathcal{D}_{0;i} &= (k_1 - k_2 - p_i)^2, \mathcal{D}_{\alpha;ijk} = (k_\alpha - p_i - p_j - p_k)^2 \end{aligned} \quad (3.10)$$

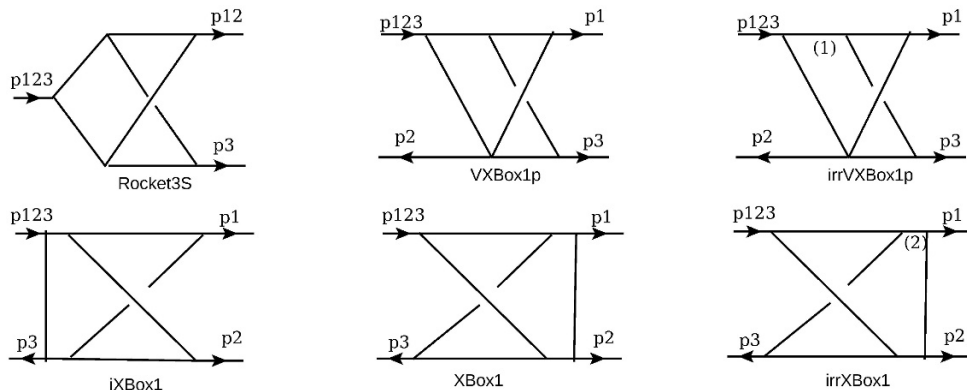
Given the fewer number of sets (eqs. (3.7) and (3.9)), it is easier to use IBP and LI identities using Laporta algorithm [55]. These identities can be generated using publicly available packages such as AIR [56], FIRE [57], REDUZE [58, 59], LiteRed [60, 61] etc. For our computation, we use a Mathematica based package LiteRedV1.51 along with MintV1.1 [62]. This package has the option to exploit symmetry relations within each set and also among different sets.

### 3.3 Master integrals

Using these IBP and LI identities, we reduce all the integrals that appear in our computation to a minimal set of master integrals. For one loop, we get two topologically different



**Figure 1.** Planar topologies of master integrals.



**Figure 2.** Non-Planar topologies of master integrals.

master integrals namely box and bubble, see figure 1 and we find that the master integrals with three propagators are absent. For two loops, we encounter 16 planar and 5 non-planar topologies of master integrals. These master integrals can be related to those that were computed by Gehrmann and Remiddi in their seminal papers [40, 41]. In particular, our set of master integrals does not contain integrals with irreducible numerator, instead we have higher power of propagators. We use IBP and LI identities to express our set of master integrals to those of [40, 41]. All the master integrals are drawn in figure 1 and figure 2 up to different permutations of the external momenta  $p_1, p_2$  and  $p_3$ . We also observe that some topologies like iXBox1 given in figure 2 are absent in our final result and find some new topologies namely Glass3S and Kite1 given in figure 1 which are absent in the [40, 41] and those are simply a product of two one loop topologies.

Substituting the master integrals computed by Gehrmann and Remiddi [40, 41] in terms of HPLs and 2dHPLs, we obtain the unrenormalized matrix elements  $\langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(1)} \rangle$  and  $\langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(2)} \rangle$ . We use shuffle algebra to express product of lower weight HPLs as a sum of higher weight HPLs. In the next section we present our results.

## 4 Results

The UV renormalized matrix elements  $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle$  and  $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle$  are computed using the unrenormalized counter parts through

$$\begin{aligned} \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle &= \left( \frac{1}{\mu_R^\epsilon} \right)^2 \left[ \langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(1)} \rangle + \mu_R^\epsilon \frac{r_1}{2} \langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(0)} \rangle \right], \\ \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle &= \left( \frac{1}{\mu_R^\epsilon} \right)^3 \left[ \langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(2)} \rangle + \mu_R^\epsilon \frac{3r_1}{2} \langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(1)} \rangle \right. \\ &\quad \left. + \mu_R^{2\epsilon} \left( \frac{r_2}{2} - \frac{r_1^2}{8} \right) \langle \hat{\mathcal{M}}^{(0)} | \hat{\mathcal{M}}^{(0)} \rangle \right]. \end{aligned} \quad (4.1)$$

Using eq. (2.14), we can also express the renormalized matrix elements in terms of  $\mathbf{I}_g^{(i)}(\epsilon)$ , given by

$$\begin{aligned}\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle &= 2 \mathbf{I}_g^{(1)}(\epsilon) \langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)fin} \rangle, \\ \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle &= 2 \mathbf{I}_g^{(1)}(\epsilon) \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + 4 \mathbf{I}_g^{(2)}(\epsilon) \langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)fin} \rangle.\end{aligned}\quad (4.2)$$

Expanding the right hand side of equations (4.1) and (4.2) in powers of  $\epsilon$  and comparing their coefficients of  $\mathcal{O}(\epsilon^0)$ , we obtain  $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)fin} \rangle$  and  $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)fin} \rangle$ .

We find that all the poles in  $\epsilon$  resulting from the soft and collinear partons in eq. (4.1) are in agreement with those of eq. (4.2). This serves as an important check on our result. In addition, it establishes the universal structure of infrared poles in QCD amplitudes involving tensorial operator insertion. We also observe that the contributions resulting from the gauge fixing term in eq. (2.1) cancel exactly with those of ghosts confirming the gauge independence of our result. As we anticipated, the eq. (4.1) does not require any over all operator renormalization constant due to the conservation of energy momentum tensor and it can be made UV finite through strong coupling constant renormalization (eq. (2.9)) alone. Below we present our final results

$$\begin{aligned}\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle &= \mathcal{F}_h \mathcal{A}^{(0)}, \\ \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)fin} \rangle &= \mathcal{F}_h \left\{ -\frac{\beta_0}{2} \mathcal{A}^{(0)} \ln \left( -\frac{Q^2}{\mu^2} \right) + \left( \mathcal{A}_1^{(1)} \zeta_2 + \mathcal{A}_2^{(1)} \right) \right\}, \\ \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)fin} \rangle &= \mathcal{F}_h \left\{ \frac{3\beta_0^2}{8} \mathcal{A}^{(0)} \ln^2 \left( -\frac{Q^2}{\mu^2} \right) \right. \\ &\quad \left. + \left( -3 C_A^2 \zeta_3 \mathcal{A}^{(0)} + \mathcal{A}_1^{(2)} \zeta_2 + \mathcal{A}_2^{(2)} \right) \ln \left( -\frac{Q^2}{\mu^2} \right) \right. \\ &\quad \left. + \left( \mathcal{A}_3^{(2)} \zeta_2^2 + \mathcal{A}_4^{(2)} \zeta_3 + \mathcal{A}_5^{(2)} \zeta_2 + \mathcal{A}_6^{(2)} \right) \right\}\end{aligned}\quad (4.3)$$

where,

$$\begin{aligned}\mathcal{F}_h &= 16\pi^2 \kappa^2 N(N^2 - 1), \\ \mathcal{A}^{(0)} &= \frac{4}{stu} \left( s^4 + 2s^3(t+u) + 3s^2(t^2+u^2) + 2s(t^3+u^3) + (t^2+tu+u^2)^2 \right), \\ \mathcal{A}_\alpha^{(1)} &= \mathcal{A}_{\alpha;C_A}^{(1)} C_A + \mathcal{A}_{\alpha;n_f}^{(1)} n_f, \\ \mathcal{A}_\alpha^{(2)} &= \mathcal{A}_{\alpha;C_A^2}^{(2)} C_A^2 + \mathcal{A}_{\alpha;C_A n_f}^{(2)} C_A n_f + \mathcal{A}_{\alpha;C_F n_f}^{(2)} C_F n_f + \mathcal{A}_{\alpha;n_f^2}^{(2)} n_f^2.\end{aligned}\quad (4.4)$$

The coefficients  $\mathcal{A}_{\alpha;\mathcal{C}}^{(i)}$  are given in the appendix except  $\mathcal{A}_{6;C_A n_f}^{(2)}$ ,  $\mathcal{A}_{6;C_F n_f}^{(2)}$  and  $\mathcal{A}_{6;n_f^2}^{(2)}$  which can be found in the files A6Canf, A6Cfnf and A6nfn2 respectively attached with this arXiv submission.

## 5 Conclusion

The computation of one and two loop QCD results for the process  $h \rightarrow g+g+g$  is presented. We use dimensional regularization to regulate both UV and IR divergences. Our result is very general in the sense that it can be used for any scattering process involving production of a massive spin-2 particle that has a universal coupling with the SM fields. We can use it to study the production of a jet with missing energy due to KK graviton escaping the detector or a process with resonant massive spin-2 particle in association with a jet. Since the spin-2 field couples with the SM ones through rank-2 tensor, we not only encounter large number of Feynman diagrams but also the formidable challenge of reducing one and two loop Feynman integrals with high powers of loop momenta to scalar ones. The IBP and LI identities reduce all such integrals to only few master integrals that were already computed by Gehrmann and Remiddi. This computation is the first of the kind involving four point function at two loop level in QCD with tensorial insertion and one massive external state. We find that no overall UV renormalization is required due to the conservation of energy momentum tensor. We also find that our results exhibit the right IR structure confirming the factorization property of QCD amplitude even with tensorial insertion.

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## A Harmonic polylogarithms

In this section, we briefly describe the definition and properties of HPL and 2dHPL. HPL is represented by  $H(\vec{m}_w; y)$  with a  $w$ -dimensional vector  $\vec{m}_w$  of parameters and its argument  $y$ .  $w$  is called the weight of the HPL. The elements of  $\vec{m}_w$  belong to  $\{1, 0, -1\}$  through which the following rational functions are represented

$$f(1; y) \equiv \frac{1}{1-y}, \quad f(0; y) \equiv \frac{1}{y}, \quad f(-1; y) \equiv \frac{1}{1+y}. \quad (\text{A.1})$$

The weight 1 ( $w = 1$ ) HPLs are defined by

$$H(1, y) \equiv -\ln(1-y), \quad H(0, y) \equiv \ln y, \quad H(-1, y) \equiv \ln(1+y). \quad (\text{A.2})$$

For  $w > 1$ ,  $H(m, \vec{m}_w; y)$  is defined by

$$H(m, \vec{m}_w; y) \equiv \int_0^y dx f(m, x) H(\vec{m}_w; x), \quad m \in 0, \pm 1. \quad (\text{A.3})$$

The 2dHPLs are defined in the same way as eq. (A.3) with the new elements  $\{2, 3\}$  in  $\vec{m}_w$  representing a new class of rational functions

$$f(2; y) \equiv f(1 - z; y) \equiv \frac{1}{1 - y - z}, \quad f(3; y) \equiv f(z; y) \equiv \frac{1}{y + z} \quad (\text{A.4})$$

and correspondingly with the weight 1 ( $w = 1$ ) 2dHPLs

$$H(2, y) \equiv -\ln\left(1 - \frac{y}{1 - z}\right), \quad H(3, y) \equiv \ln\left(\frac{y + z}{z}\right). \quad (\text{A.5})$$

**Properties.** Shuffle algebra: A product of two HPL with weights  $w_1$  and  $w_2$  of the same argument  $y$  is a combination of HPLs with weight  $(w_1 + w_2)$  and argument  $y$ , such that all possible permutations of the elements of  $\vec{m}_{w_1}$  and  $\vec{m}_{w_2}$  are considered preserving the relative orders of the elements of  $\vec{m}_{w_1}$  and  $\vec{m}_{w_2}$ ,

$$H(\vec{m}_{w_1}; y)H(\vec{m}_{w_2}; y) = \sum_{\vec{m}_w = \vec{m}_{w_1} \uplus \vec{m}_{w_2}} H(\vec{m}_w; y). \quad (\text{A.6})$$

Integration-by-parts identities: The ordering of the elements of  $\vec{m}_w$  in an HPL with weight  $w$  and argument  $y$  can be reversed using integration-by-parts and in the process, some products of two HPLs are generated in the following way

$$\begin{aligned} H(\vec{m}_w; y) &\equiv H(m_1, m_2, \dots, m_w; y) = H(m_1, y)H(m_2, \dots, m_w; y) \\ &\quad - H(m_2, m_1, y)H(m_3, \dots, m_w; y) \\ &\quad + \dots + (-1)^{w+1}H(m_w, \dots, m_2, m_1; y). \end{aligned} \quad (\text{A.7})$$

## B One-loop coefficients

$$\begin{aligned} \mathcal{A}_{1;C_A}^{(1)} &= -\frac{4}{stu} \left( s^4 + 2s^3(t+u) + 3s^2(t^2+u^2) + 2s(t^3+u^3) + t^4 + u^4 \right) \\ \mathcal{A}_{1;n_f}^{(1)} &= -\frac{2}{s} (t^2 + u^2) \\ \mathcal{A}_{2;C_A}^{(1)} &= \left\{ -6(t+u)^4 \left( 11s^8 + 22(3t+u)s^7 + (187t^2 + 64ut + 33u^2)s^6 + (330t^3 + 60ut^2 + 78u^2t + 22u^3)s^5 + (396t^4 + 14ut^3 \right. \right. \\ &\quad + 51u^2t^2 + 36u^3t + 11u^4)s^4 + 2t(165t^4 + 7u^3t^3 + 6u^2t^2 + 7u^3t + 10u^4)s^3 + t^2(187t^4 + 60ut^3 + 51u^2t^2 + 14u^3t + 24u^4)s^2 \\ &\quad + 2t^3(33t^4 + 32ut^3 + 39u^2t^2 + 18u^3t + 10u^4)s + 11t^4(t^2 + ut + u^2)^2 \Big) H(0, y)(s+u)^4 - 36(s+t)^4(t+u)^4(s^4 + 2(t+u)s^3 \\ &\quad + 3(t^2+u^2)s^2 + 2(t^3+u^3)s + t^4 + u^4) H(0, y)H(0, z)(s+u)^4 + 6(s+t)^4 \left( (11t^4 + 20ut^3 + 24u^2t^2 + 20u^3t + 11u^4)s^4 \right. \\ &\quad + 2(11t^5 + 18ut^4 + 7u^2t^3 + 7u^3t^2 + 18u^4t + 11u^5)s^3 + 3(t+u)^2(11t^4 + 4ut^3 - 2u^2t^2 + 4u^3t + 11u^4)s^2 + 2(t+u)^3(11t^4 \\ &\quad - ut^3 - u^3t + 11u^4)s + 11(t+u)^4(t^2 + ut + u^2)^2 \Big) H(1, z)(s+u)^4 + 36(s+t)^4(t+u)^4(s^4 + 2ts^3 + 3t^2s^2 + 2t^3s + (t^2 + ut \\ &\quad + u^2)^2) H(0, y)H(1, z)(s+u)^4 + 6(s+t)^4 \left( (11t^4 + 20ut^3 + 24u^2t^2 + 20u^3t + 11u^4)s^4 + 2(11t^5 + 18ut^4 + 7u^2t^3 + 7u^3t^2 \right. \\ &\quad + 18u^4t + 11u^5)s^3 + 3(t+u)^2(11t^4 + 4ut^3 - 2u^2t^2 + 4u^3t + 11u^4)s^2 + 2(t+u)^3(11t^4 - ut^3 - u^3t + 11u^4)s + 11(t+u)^4(t^2 \\ &\quad + ut + u^2)^2 \Big) H(2, y)(s+u)^4 + 36(s+t)^4(t+u)^4(s^4 + 2us^3 + 3u^2s^2 + 2u^3s + (t^2 + ut + u^2)^2) H(0, z)H(2, y)(s+u)^4 \\ &\quad - 36(s+t)^4(t+u)^4(2s^4 + 2(t+u)s^3 + 3(t^2+u^2)s^2 + 2(t^3+u^3)s + 2(t^2 + ut + u^2)^2) H(1, z)H(3, y)(s+u)^4 \\ &\quad - 36(s+t)^4(t+u)^4(s^4 + 2ts^3 + 3t^2s^2 + 2t^3s + (t^2 + ut + u^2)^2) H(0, 1, z)(s+u)^4 + 36(s+t)^4(t+u)^4(s^4 + 2ts^3 \\ &\quad + 3t^2s^2 + 2t^3s + (t^2 + ut + u^2)^2) H(0, 2, y)(s+u)^4 - 36(s+t)^4(t+u)^4(2s^4 + 2(2t+u)s^3 + 3(2t^2 + u^2)s^2 + 2(2t^3 \\ &\quad + u^3)s + 2t^4 + 2u^4 + 2tu^3 + 3t^2u^2 + 2t^3u) H(1, 0, y)(s+u)^4 - 36(s+t)^4(t+u)^4(s^4 + 2(t+u)s^3 + 3(t^2 + u^2)s^2 + 2(t^3 \\ &\quad + u^3)s + t^4 + u^4) H(1, 0, z)(s+u)^4 + 36(s+t)^4(t+u)^4(s^4 + 2ts^3 + 3t^2s^2 + 2t^3s + (t^2 + ut + u^2)^2) H(2, 0, y)(s+u)^4 \\ &\quad - 36(s+t)^4(t+u)^4(2s^4 + 2(t+u)s^3 + 3(t^2 + u^2)s^2 + 2(t^3 + u^3)s + 2(t^2 + ut + u^2)^2) H(3, 2, y)(s+u)^4 \end{aligned}$$

$$\begin{aligned}
 & -2(s+t)(t+u)\left(203(t+u)^3 s^{10} + 2\left(506t^4 + 1988ut^3 + 2973u^2t^2 + 1988u^3t + 506u^4\right) s^9 + 3\left(807t^5 + 3709ut^4 + 7123u^2t^3 \right. \right. \\
 & + 7123u^3t^2 + 3709u^4t + 807 u^5\left. \right) s^8 + 3\left(1208t^6 + 6186ut^5 + 13887u^2t^4 + 17836u^3t^3 + 13887 u^4t^2 + 6186u^5t + 1208u^6\right) s^7 \\
 & + \left(3624t^7 + 21596ut^6 + 54318u^2 t^5 + 80567u^3t^4 + 80567u^4t^3 + 54318u^5t^2 + 21596u^6t + 3624u^7\right) s^6 + 3\left(807t^8 + 6186ut^7 \right. \\
 & + 18106u^2t^6 + 29776u^3t^5 + 34116u^4 t^4 + 29776u^5t^3 + 18106u^6t^2 + 6186u^7t + 807u^8\left. \right) s^5 + \left(1012 t^9 + 11127ut^8 + 41661u^2t^7 \right. \\
 & + 80567u^3t^6 + 102348u^4t^5 + 102348u^5 t^4 + 80567u^6t^3 + 41661u^7t^2 + 11127u^8t + 1012u^9\left. \right) s^4 + \left(203 t^{10} + 3976ut^9 + 21369u^2t^8 \right. \\
 & + 53508u^3t^7 + 80567u^4t^6 + 89328u^5 t^5 + 80567u^6t^4 + 53508u^7t^3 + 21369u^8t^2 + 3976u^9t + 203u^{10}\left. \right) s^3 + 3tu\left(203t^9 + 1982ut^8 \right. \\
 & + 7123u^2t^7 + 13887u^3t^6 + 18106u^4 t^5 + 18106u^5t^4 + 13887u^6t^3 + 7123u^7t^2 + 1982u^8t + 203u^9\left. \right) s^2 + t^2u^2(t+u)^2\left(609t^6 \right. \\
 & + 2758ut^5 + 5002u^2t^4 + 5796u^3t^3 + 5002 u^4t^2 + 2758u^5t + 609u^6\left. \right) s + t^3u^3(t+u)^3\left(203t^4 + 403u t^3 + 603u^2t^2 + 403u^3t \right. \\
 & + 203u^4\left. \right)(s+u) - 6(s+t)^4(t+u)^4\left(11s^8 + 22(t+3u)s^7 + \left(33t^2 + 64ut + 187u^2\right) s^6 + \left(22 t^3 + 78ut^2 + 60u^2t + 330u^3\right) s^5 \right. \\
 & + \left(11t^4 + 36ut^3 + 51u^2t^2 + 14 u^3t + 396u^4\right) s^4 + 2u\left(10t^4 + 7ut^3 + 6u^2t^2 + 7u^3t + 165 u^4\right) s^3 + u^2\left(24t^4 + 14ut^3 + 51u^2t^2 + 60u^3t \right. \\
 & + 187u^4\left. \right) s^2 + 2u^3\left(10t^4 + 18ut^3 + 39u^2t^2 + 32u^3t + 33u^4\right) s + 11u^4\left(t^2 + ut + u^2\right)^2\left. \right\} H(0, z) \Big/ \left(9st(s+t)^4u(s+u)^4(t+u)^4\right) \\
 \\
 \mathcal{A}_{2;n_f}^{(1)} = & \left\{ -18u(s+u)^4(t+u)^4\left(s^2 - ts + t^2 + u^2\right) H(1, 0, y)(s+t)^5 + 6(t+u)^4\left(2s^8 + 4(t+3u)s^7 + 2\left(3t^2 + 5ut + 17u^2\right) s^6 \right. \right. \\
 & + \left(4t^3 + 15ut^2 + 6u^2t + 60u^3\right) s^5 + 2\left(t^4 + 12u^2t^2 - 2 u^3t + 36u^4\right) s^4 + 2u\left(t^4 - 2ut^3 + 15u^2t^2 - 2u^3t + 30 u^4\right) s^3 \\
 & + 2u^2\left(3t^4 - 2ut^3 + 12u^2t^2 + 3u^3t + 17u^4\right) s^2 + \left(12u^7 + 10tu^6 + 15t^2u^5 + 2t^4u^3\right) s + 2u^4\left(t^2 + u t + u^2\right)^2\left. \right\} H(0, z)(s+t)^4 \\
 & - 18tu(s+u)^4(t+u)^4\left(t^2 + u^2\right) H(0, y)H(0, z)(s+t)^4 - 6(s+u)^4\left(2\left(t^4 + u t^3 + 3u^2t^2 + u^3t + u^4\right) s^4 + 4\left(t^5 - u^2t^3 \right. \right. \\
 & - u^3t^2 + u^5\left. \right) s^3 + 3(t+u)^2\left(2t^4 + ut^3 + 4u^2t^2 + u^3t + 2u^4\right) s^2 + 2(t+u)^3\left(2t^4 - ut^3 - u^3t + 2u^4\right) s + 2(t+u)^4\left(t^2 + u t \right. \\
 & + u^2\left. \right)^2\left. \right\} H(1, z)(s+t)^4 + 18su(s+u)^4(t+u)^4\left(s^2 + u^2\right) H(0, y)H(1, z)(s+t)^4 - 6(s+u)^4\left(2\left(t^4 + u t^3 + 3u^2t^2 + u^3t \right. \right. \\
 & + u^4\left. \right) s^4 + 4\left(t^5 - u^2t^3 - u^3t^2 + u^5\right) s^3 + 3(t+u)^2\left(2t^4 + ut^3 + 4u^2t^2 + u^3t + 2u^4\right) s^2 + 2(t+u)^3\left(2t^4 - ut^3 - u^3t + 2u^4\right) s \\
 & + 2(t+u)^4\left(t^2 + u t + u^2\right)^2\left. \right\} H(2, y)(s+t)^4 + 18st\left(s^2 + t^2\right)(s+u)^4(t+u)^4H(0, z)H(2, y)(s+t)^4 - 18s(s+u)^4(t+u)^5\left(s^2 \right. \\
 & + t^2 + u^2 - t u\left. \right) H(1, z)H(3, y)(s+t)^4 - 18su(s+u)^4(t+u)^4\left(s^2 + u^2\right) H(0, 1, z)(s+t)^4 + 18su(s+u)^4(t+u)^4\left(s^2 \right. \\
 & + u^2\left. \right) H(0, 2, y)(s+t)^4 - 18tu(s+u)^4(t+u)^4\left(t^2 + u^2\right) H(1, 0, z)(s+t)^4 + 18su(s+u)^4(t+u)^4\left(s^2 + u^2\right) H(2, 0, y)(s+t)^4 \\
 & - 18s(s+u)^4(t+u)^5\left(s^2 + t^2 + u^2 - tu\right) H(3, 2, y)(s+t)^4 + 2(s+u)(t+u)\left(35(t+u)^3 s^{10} + 2\left(86t^4 + 335ut^3 + 507u^2t^2 \right. \right. \\
 & + 335u^3t + 86u^4\left. \right) s^9 + 3\left(135t^5 + 607ut^4 + 1177u^2t^3 + 1177u^3t^2 + 607u^4t + 135u^5\right) s^8 + 3\left(200t^6 + 990ut^5 + 2211u^2t^4 \right. \\
 & + 2860u^3t^3 + 2211u^4t^2 + 990u^5 t + 200u^6\left. \right) s^7 + \left(600t^7 + 3428ut^6 + 8436u^2t^5 + 12593u^3 t^4 + 12593u^4t^3 + 8436u^5t^2 \right. \\
 & + 3428u^6t + 600u^7\left. \right) s^6 + 3\left(135 t^8 + 990ut^7 + 2812u^2t^6 + 4612u^3t^5 + 5328u^4t^4 + 4612u^5t^3 + 2812u^6 t^2 + 990u^7t + 135u^8\right) s^5 \\
 \\
 & + \left(172t^9 + 1821ut^8 + 6633u^2t^7 + 12593 u^3t^6 + 15984u^4t^5 + 15984u^5t^4 + 12593u^6t^3 + 6633u^7t^2 + 1821u^8 t + 172u^9\right) s^4 \\
 & + \left(35t^{10} + 670ut^9 + 3531u^2t^8 + 8580u^3 t^7 + 12593u^4t^6 + 13836u^5t^5 + 12593u^6t^4 + 8580u^7t^3 + 3531u^8t^2 + 670 u^9t \right. \\
 & + 35u^{10}\left. \right) s^3 + 3tu\left(35t^9 + 338ut^8 + 1177u^2t^7 + 2211 u^3t^6 + 2812u^4t^5 + 2812u^5t^4 + 2211u^6t^3 + 1177u^7t^2 + 338u^8t + 35 u^9\right) s^2 \\
 & + t^2u^2(t+u)^2\left(105t^6 + 460ut^5 + 796u^2t^4 + 918u^3 t^3 + 796u^4t^2 + 460u^5t + 105u^6\right) s + t^3u^3(t+u)^3\left(35t^4 + 67u t^3 + 99u^2t^2 \right. \\
 & + 67u^3t + 35u^4\left. \right)(s+t) + 6(s+u)^4(t+u)^4\left(2 s^8 + 4(3t+u)s^7 + 2\left(17t^2 + 5ut + 3u^2\right) s^6 + \left(60t^3 + 6u t^2 + 15u^2t \right. \right. \\
 & + 4u^3\left. \right) s^5 + 2\left(36t^4 - 2ut^3 + 12u^2t^2 + u^4\right) s^4 + 2t\left(30t^4 - 2ut^3 + 15u^2t^2 - 2u^3t + u^4\right) s^3 + 2t^2\left(17t^4 + 3ut^3 + 12u^2t^2 \right. \\
 & - 2u^3t + 3u^4\left. \right) s^2 + \left(12t^7 + 10u t^6 + 15u^2t^5 + 2u^4t^3\right) s + 2t^4\left(t^2 + ut + u^2\right)^2\left. \right\} H(0, y) \Big/ \left(9st(s+t)^4u(s+u)^4(t+u)^4\right)
 \end{aligned}$$

## C Two-loop coefficients

$$\begin{aligned}
 \mathcal{A}_{1;C_A^2}^{(2)} &= \frac{11}{stu} \left( s^4 + 2s^3(t+u) + 3s^2(t^2+u^2) + 2s(t^3+u^3) + t^4 - 2t^3u - 3t^2u^2 - 2tu^3 + u^4 \right) \\
 \mathcal{A}_{1;C_A n_f}^{(2)} &= -\frac{1}{stu} \left( 2s^4 + 4s^3(t+u) + 6s^2(t^2+u^2) + 4s(t^3+u^3) + 2t^4 - 15t^3u - 6t^2u^2 - 15tu^3 + 2u^4 \right) \\
 \mathcal{A}_{1;C_F n_f}^{(2)} &= 0 \\
 \mathcal{A}_{1;n_f^2}^{(2)} &= -\frac{2}{s} (t^2 + u^2)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{2;C_A^2}^{(2)} = & \left\{ 33(t+u)^4 \left( 11s^8 + 22(3t+u)s^7 + (187t^2 + 64ut + 33u^2)s^6 + (330t^3 + 60ut^2 + 78u^2t + 22u^3)s^5 + (396t^4 + 14ut^3 + 51u^2t^2 \right. \right. \\
 & + 36u^3t + 11u^4) s^4 + 2t(165t^4 + 7u^3t^3 + 6u^2t^2 + 7u^3t + 10u^4) s^3 + t^2(187t^4 + 60ut^3 + 51u^2t^2 + 14u^3t + 24u^4) s^2 + 2t^3(33t^4 + 32ut^3 \\
 & + 39u^2t^2 + 18u^3t + 10u^4) s + 11t^4(t^2 + ut + u^2)^2 \Big) H(0, y)(s+u)^4 + 198(s+t)^4(t+u)^4 \left( s^4 + 2(t+u)s^3 + 3(t^2 + u^2)s^2 + 2(t^3 \right. \\
 & + u^3)s + t^4 + u^4 \Big) H(0, y)H(0, z)(s+u)^4 - 33(s+t)^4 \left( (11t^4 + 20ut^3 + 24u^2t^2 + 20u^3t + 11u^4) s^4 + 2(11t^5 + 18ut^4 + 7u^2t^3 + 7u^3t^2 \right. \\
 & + 18u^4t + 11u^5) s^3 + 3(t+u)^2(11t^4 + 4ut^3 - 2u^2t^2 + 4u^3t + 11u^4) s^2 + 2(t+u)^3(11t^4 - ut^3 - u^3t + 11u^4) s + 11(t+u)^4(t^2 + ut \\
 & + u^2)^2 \Big) H(1, z)(s+u)^4 - 198(s+t)^4(t+u)^4 \left( s^4 + 2ts^3 + 3t^2s^2 + 2t^3s + (t^2 + ut + u^2)^2 \right) H(0, y)H(1, z)(s+u)^4 - 33(s+t)^4 \left( (11t^4 \right. \\
 & + 20ut^3 + 24u^2t^2 + 20u^3t + 11u^4) s^4 + 2(11t^5 + 18ut^4 + 7u^2t^3 + 7u^3t^2 + 18u^4t + 11u^5) s^3 + 3(t+u)^2(11t^4 + 4ut^3 - 2u^2t^2 + 4u^3t \\
 & + 11u^4) s^2 + 2(t+u)^3(11t^4 - ut^3 - u^3t + 11u^4) s + 11(t+u)^4(t^2 + ut + u^2)^2 \Big) H(2, y)(s+u)^4 - 198(s+t)^4(t+u)^4 \left( s^4 + 2us^3 \right. \\
 & + 3u^2s^2 + 2u^3s + (t^2 + ut + u^2)^2 \Big) H(0, z)H(2, y)(s+u)^4 + 198(s+t)^4(t+u)^4 \left( 2s^4 + 2(t+u)s^3 + 3(t^2 + u^2)s^2 + 2(t^3 + u^3)s \right. \\
 & + 2(t^2 + ut + u^2)^2 \Big) H(1, z)H(3, y)(s+u)^4 + 198(s+t)^4(t+u)^4 \left( s^4 + 2ts^3 + 3t^2s^2 + 2t^3s + (t^2 + ut + u^2)^2 \right) H(0, 1, z)(s+u)^4 \\
 & - 198(s+t)^4(t+u)^4 \left( s^4 + 2ts^3 + 3t^2s^2 + 2t^3s + (t^2 + ut + u^2)^2 \right) H(0, 2, y)(s+u)^4 + 198(s+t)^4(t+u)^4 \left( 2s^4 + 2(2t+u)s^3 \right. \\
 & + 3(2t^2 + u^2)s^2 + 2(2t^3 + u^3)s + 2t^4 + 2u^4 + 2tu^3 + 3t^2u^2 + 2t^3u \Big) H(1, 0, y)(s+u)^4 + 198(s+t)^4(t+u)^4 \left( s^4 + 2(t+u)s^3 \right. \\
 & + 3(t^2 + u^2)s^2 + 2(t^3 + u^3)s + t^4 + u^4 \Big) H(1, 0, z)(s+u)^4 - 198(s+t)^4(t+u)^4 \left( s^4 + 2ts^3 + 3t^2s^2 + 2t^3s + (t^2 + ut + u^2 \right. \\
 & + u^2)^2 \Big) H(2, 0, y)(s+u)^4 + 198(s+t)^4(t+u)^4 \left( 2s^4 + 2(t+u)s^3 + 3(t^2 + u^2)s^2 + 2(t^3 + u^3)s + 2(t^2 + ut + u^2)^2 \right) H(3, 2, y)(s+u)^4 \\
 & + (s+t)(t+u) \left( 1939(t+u)^3 s^{10} + (9662t^4 + 37856ut^3 + 56586u^2t^2 + 37856u^3t + 9662u^4) s^9 + 3(7701t^5 + 35213ut^4 + 67475u^2t^3 \right. \\
 & + 67475u^3t^2 + 35213u^4t + 7701u^5) s^8 + 3(11524t^6 + 58638ut^5 + 131001u^2t^4 + 167972u^3t^3 + 131001u^4t^2 + 58638u^5t + 11524u^6) s^7 \\
 & + (34572t^7 + 204628ut^6 + 511062u^2t^5 + 754525u^3t^4 + 754525u^4t^3 + 511062u^5t^2 + 204628u^6t + 34572u^7) s^6 + 3(7701t^8 \\
 & + 58638ut^7 + 170354u^2t^6 + 278144u^3t^5 + 317652u^4t^4 + 278144u^5t^3 + 170354u^6t^2 + 58638u^7t + 7701u^8) s^5 + (9662t^9 + 105639ut^8 \\
 & + 393003u^2t^7 + 754525u^3t^6 + 952956u^4t^5 + 952956u^5t^4 + 754525u^6t^3 + 393003u^7t^2 + 105639u^8t + 9662u^9) s^4 + (1939t^{10} + 37856ut^9 \\
 & + 202425u^2t^8 + 503916u^3t^7 + 754525u^4t^6 + 834432u^5t^5 + 754525u^6t^4 + 503916u^7t^3 + 202425u^8t^2 + 37856u^9t + 1939u^{10}) s^3 \\
 & + 3tu(1939t^9 + 18862ut^8 + 67475u^2t^7 + 131001u^3t^6 + 170354u^4t^5 + 170354u^5t^4 + 131001u^6t^3 + 67475u^7t^2 + 18862u^8t + 1939u^9) s^2 \\
 & + t^2u^2(t+u)^2(5817t^6 + 26222ut^5 + 47378u^2t^4 + 54936u^3t^3 + 47378u^4t^2 + 26222u^5t + 5817u^6) s + t^3u^3(t+u)^3(1939t^4 + 3845u^2t^3 \\
 & + 5751u^2t^2 + 3845u^3t + 1939u^4) \Big) (s+u) + 33(s+t)^4(t+u)^4 \left( 11s^8 + 22(3t+u)s^7 + (33t^2 + 64ut + 187u^2)s^6 + (22t^3 + 78ut^2 + 60u^2t \right. \\
 & + 330u^3) s^5 + (11t^4 + 36ut^3 + 51u^2t^2 + 14u^3t + 396u^4) s^4 + 2u(10t^4 + 7ut^3 + 6u^2t^2 + 7u^3t + 165u^4) s^3 + u^2(24t^4 + 14ut^3 + 51u^2t^2 \\
 & + 60u^3t + 187u^4) s^2 + 2u^3(10t^4 + 18ut^3 + 39u^2t^2 + 32u^3t + 33u^4) s + 11u^4(t^2 + ut + u^2)^2 \Big) H(0, z) \Big\} / (9st(s+t)^4u(s+u)^4(t+u)^4)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{2;C_A^{n_f}}^{(2)} = & - \left\{ 3(t+u)^4 \left( 44s^8 + 88(3t+u)s^7 + 2(374t^2 + 119ut + 66u^2)s^6 + (1320t^3 + 186ut^2 + 321u^2t + 88u^3)s^5 + 2(792t^4 - 8ut^3 \right. \right. \\
 & + 183u^2t^2 + 36u^3t + 22u^4) s^4 + 2t(660t^4 - 8ut^3 + 177u^2t^2 - 8u^3t + 31u^4) s^3 + 2t^2(374t^4 + 93u^3t^3 + 183u^2t^2 - 8u^3t + 57u^4) s^2 \\
 & + t^3(264t^4 + 238ut^3 + 321u^2t^2 + 72u^3t + 62u^4) s + 44t^4(t^2 + ut + u^2)^2 \Big) H(0, y)(s+u)^4 + 9(s+t)^4(t+u)^4 \left( 4s^4 + 8(t+u)s^3 \right. \\
 & + 12(t^2 + u^2)s^2 + 8(t^3 + u^3)s + 4t^4 + 4u^4 - 11tu^3 - 11t^3u \Big) H(0, y)H(0, z)(s+u)^4 - 3(s+t)^4 \left( 2(22t^4 + 31u^3t^3 + 57u^2t^2 \right. \\
 & + 31u^3t + 22u^4) s^4 + 8(11t^5 + 9ut^4 - 2u^2t^3 - 2u^3t^2 + 9u^4t + 11u^5) s^3 + 3(t+u)^2(44t^4 + 19ut^3 + 40u^2t^2 + 19u^3t + 44u^4) s^2 \\
 & + 2(t+u)^3(44t^4 - 13ut^3 - 13u^3t + 44u^4) s + 44(t+u)^4(t^2 + ut + u^2)^2 \Big) H(1, z)(s+u)^4 - 9(s+t)^4(t+u)^4 \left( 4s^4 + (8t - 11u)s^3 \right. \\
 & + 12t^2s^2 + (8t^3 - 11u^3)s + 4(t^2 + ut + u^2)^2 \Big) H(0, y)H(1, z)(s+u)^4 - 3(s+t)^4 \left( 2(22t^4 + 31ut^3 + 57u^2t^2 + 31u^3t + 22u^4) s^4 \right. \\
 & + 8(11t^5 + 9ut^4 - 2u^2t^3 - 2u^3t^2 + 9u^4t + 11u^5) s^3 + 3(t+u)^2(44t^4 + 19ut^3 + 40u^2t^2 + 19u^3t + 44u^4) s^2 + 2(t+u)^3(44t^4 \\
 & - 13ut^3 - 13u^3t + 44u^4) s + 44(t+u)^4(t^2 + ut + u^2)^2 \Big) H(2, y)(s+u)^4 - 9(s+t)^4(t+u)^4 \left( 4s^4 + (8u - 11t)s^3 + 12u^2s^2 + (8u^3 \right. \\
 & - 11t^3)s + 4(t^2 + ut + u^2)^2 \Big) H(0, z)H(2, y)(s+u)^4 + 9(s+t)^4(t+u)^4 \left( 8s^4 - 3(t+u)s^3 + 12(t^2 + u^2)s^2 - 3(t^3 + u^3)s \right. \\
 & + 8(t^2 + ut + u^2)^2 \Big) H(1, z)H(3, y)(s+u)^4 + 9(s+t)^4(t+u)^4 \left( 4s^4 + (8t - 11u)s^3 + 12t^2s^2 + (8t^3 - 11u^3)s + 4(t^2 + ut + u^2 \right. \\
 & + u^2)^2 \Big) H(0, 1, z)(s+u)^4 - 9(s+t)^4(t+u)^4 \left( 4s^4 + (8t - 11u)s^3 + 12t^2s^2 + (8t^3 - 11u^3)s + 4(t^2 + ut + u^2)^2 \Big) H(0, 2, y)(s+u)^4 \\
 & + 9(s+t)^4(t+u)^4 \left( 8s^4 + (16t - 3u)s^3 + 12(2t^2 + u^2)s^2 + (16t^3 - 3u^3)s + 8t^4 + 8u^4 - 3tu^3 + 12t^2u^2 - 3t^3u \right) H(1, 0, y)(s+u)^4 \\
 & + 9(s+t)^4(t+u)^4 \left( 4s^4 + 8(t+u)s^3 + 12(t^2 + u^2)s^2 + 8(t^3 + u^3)s + 4t^4 + 4u^4 - 11tu^3 - 11t^3u \right) H(1, 0, z)(s+u)^4 \\
 & - 9(s+t)^4(t+u)^4 \left( 4s^4 + (8t - 11u)s^3 + 12t^2s^2 + (8t^3 - 11u^3)s + 4(t^2 + ut + u^2)^2 \right) H(2, 0, y)(s+u)^4 + 9(s+t)^4(t+u)^4 \left( 8s^4 \right. \\
 & - 3(t+u)s^3 + 12(t^2 + u^2)s^2 - 3(t^3 + u^3)s + 8(t^2 + ut + u^2)^2 \Big) H(3, 2, y)(s+u)^4 + (s+t)(t+u) \left( 499(t+u)^3 s^{10} + 2(1228t^4 + 4741ut^3 \right.
 \end{aligned}$$



$$\begin{aligned}
 & + 7143u^2t^2 + 4741u^3t + 1228u^4)s^9 + 3(1931t^5 + 8547ut^4 + 16389u^2t^3 + 16389u^3t^2 + 8547u^4t + 1931u^5)s^8 + 3(2864t^6 \\
 & + 13918ut^5 + 30487u^2t^4 + 39100u^3t^3 + 30487u^4t^2 + 13918u^5t + 2864u^6)s^7 + (8592t^7 + 48196ut^6 + 115584u^2t^5 + 168841u^3t^4 \\
 & + 168841u^4t^3 + 115584u^5t^2 + 48196u^6t + 8592u^7)s^6 + 3(1931t^8 + 13918ut^7 + 38528u^2t^6 + 61228u^3t^5 + 69608u^4t^4 \\
 & + 61228u^5t^3 + 38528u^6t^2 + 13918u^7t + 1931u^8)s^5 + (2456t^9 + 25641ut^8 + 91461u^2t^7 + 168841u^3t^6 + 208824u^4t^5 \\
 & + 208824u^5t^4 + 168841u^6t^3 + 91461u^7t^2 + 25641u^8t + 2456u^9)s^4 + (499t^{10} + 9482ut^9 + 49167u^2t^8 + 117300u^3t^7 + 168841u^4t^6 \\
 & + 183684u^5t^5 + 168841u^6t^4 + 117300u^7t^3 + 49167u^8t^2 + 9482u^9t + 499u^{10})s^3 + 3tu(499t^9 + 4762ut^8 + 16389u^2t^7 + 30487u^3t^6 \\
 & + 38528u^4t^5 + 38528u^5t^4 + 30487u^6t^3 + 16389u^7t^2 + 4762u^8t + 499u^9)s^2 + t^2u^2(t+u)^2(1497t^6 + 6488ut^5 + 11168u^2t^4 \\
 & + 12930u^3t^3 + 11168u^4t^2 + 6488u^5t + 1497u^6)s + t^3u^3(t+u)^3(499t^4 + 959ut^3 + 1419u^2t^2 + 959u^3t + 499u^4)(s+u) \\
 & + 3(s+t)^4(t+u)^4(44s^8 + 88(t+3u)s^7 + 2(66t^2 + 119ut + 374u^2)s^6 + (88t^3 + 321ut^2 + 186u^2t + 1320u^3)s^5 + 2(22t^4 \\
 & + 36ut^3 + 183u^2t^2 - 8u^3t + 792u^4)s^4 + 2u(31t^4 - 8ut^3 + 177u^2t^2 - 8u^3t + 660u^4)s^3 + 2u^2(57t^4 - 8u^3t^3 + 183u^2t^2 + 93u^3t \\
 & + 374u^4)s^2 + u^3(62t^4 + 72ut^3 + 321u^2t^2 + 238u^3t + 264u^4)s + 44u^4(t^2 + ut + u^2)^2)H(0, z)\} / (9st(s+t)^4u(s+u)^4(t+u)^4)
 \end{aligned}$$

$$\mathcal{A}_{2;C_F n_f}^{(2)} = -\left\{8(s^4 + 2s^3(t+u) + 3s^2(t^2 + u^2) + 2s(t^3 + u^3) + (t^2 + tu + u^2)^2)\right\} / (stu)$$

$$\begin{aligned}
 \mathcal{A}_{2;n_f}^{(2)} = & \left\{-18u(s+u)^4(t+u)^4(s^2 - ts + t^2 + u^2)H(1, 0, y)(s+t)^5 + 6(t+u)^4(2s^8 + 4(t+3u)s^7 + 2(3t^2 + 5ut + 17u^2)s^6 \right. \\
 & + (4t^3 + 15ut^2 + 6u^2t + 60u^3)s^5 + 2(t^4 + 12u^2t^2 - 2u^3t + 36u^4)s^4 + 2u(t^4 - 2ut^3 + 15u^2t^2 - 2u^3t + 30u^4)s^3 + 2u^2(3t^4 - 2ut^3 \\
 & + 12u^2t^2 + 3u^3t + 17u^4)s^2 + (12u^7 + 10tu^6 + 15t^2u^5 + 2t^4u^3)s + 2u^4(t^2 + u + t + u^2)^2)H(0, z)(s+t)^4 - 18tu(s+u)^4(t+u)^4(t^2 \\
 & + u^2)H(0, y)H(0, z)(s+t)^4 - 6(s+u)^4(2(t^4 + u^3t + 3u^2t^2 + u^3t + u^4)s^4 + 4(t^5 - u^2t^3 - u^3t^2 + u^5)s^3 + 3(t+u)^2(2t^4 + ut^3 \\
 & + 4u^2t^2 + u^3t + 2u^4)s^2 + 2(t+u)^3(2t^4 - ut^3 - u^3t + 2u^4)s + 2(t+u)^4(t^2 + u + t + u^2)^2)H(1, z)(s+t)^4 + 18su(s+u)^4(t+u)^4(s^2 \\
 & + u^2)H(0, y)H(1, z)(s+t)^4 - 6(s+u)^4(2(t^4 + u^3t + 3u^2t^2 + u^3t + u^4)s^4 + 4(t^5 - u^2t^3 - u^3t^2 + u^5)s^3 + 3(t+u)^2(2t^4 \\
 & + ut^3 + 4u^2t^2 + u^3t + 2u^4)s^2 + 2(t+u)^3(2t^4 - ut^3 - u^3t + 2u^4)s + 2(t+u)^4(t^2 + u + t + u^2)^2)H(2, y)(s+t)^4 \\
 & + 18st(s^2 + t^2)(s+u)^4(t+u)^4H(0, z)H(2, y)(s+t)^4 - 18s(s+u)^4(t+u)^4(s^2 + t^2 + u^2 - tu)H(1, z)H(3, y)(s+t)^4 \\
 & - 18su(s+u)^4(t+u)^4(s^2 + u^2)H(0, 1, z)(s+t)^4 + 18su(s+u)^4(t+u)^4(s^2 + u^2)H(0, 2, y)(s+t)^4 - 18tu(s+u)^4(t+u)^4(t^2 \\
 & + u^2)H(1, 0, z)(s+t)^4 + 18su(s+u)^4(t+u)^4(s^2 + u^2)H(2, 0, y)(s+t)^4 - 18s(s+u)^4(t+u)^5(s^2 + t^2 + u^2 - tu)H(3, 2, y)(s+t)^4 \\
 & + 6(s+u)(t+u)(5(t+u)^3s^{10} + 6(4t^4 + 15ut^3 + 23u^2t^2 + 15u^3t + 4u^4)s^9 + (55t^5 + 227ut^4 + 437u^2t^3 + 437u^3t^2 + 227u^4t + 55u^5)s^8 \\
 & + (80t^6 + 350ut^5 + 731u^2t^4 + 940u^3t^3 + 731u^4t^2 + 350u^5t + 80u^6)s^7 + (80t^7 + 396ut^6 + 852u^2t^5 + 1211u^3t^4 + 1211u^4t^3 + 852u^5t^2 \\
 & + 396u^6t + 80u^7)s^6 + (55t^8 + 350ut^7 + 852u^2t^6 + 1252u^3t^5 + 1408u^4t^4 + 1252u^5t^3 + 852u^6t^2 + 350u^7t + 55u^8)s^5 + (24t^9 + 227ut^8 \\
 & + 731u^2t^7 + 1211u^3t^6 + 1408u^4t^5 + 1408u^5t^4 + 1211u^6t^3 + 731u^7t^2 + 227u^8t + 24u^9)s^4 + (5t^{10} + 90ut^9 + 437u^2t^8 + 940u^3t^7 \\
 & + 1211u^4t^6 + 1252u^5t^5 + 1211u^6t^4 + 940u^7t^3 + 437u^8t^2 + 90u^9t + 5u^{10})s^3 + tu(15t^9 + 138ut^8 + 437u^2t^7 + 731u^3t^6 + 852u^4t^5 \\
 & + 852u^5t^4 + 731u^6t^3 + 437u^7t^2 + 138u^8t + 15u^9)s^2 + t^2u^2(t+u)^2(15t^6 + 60ut^5 + 92u^2t^4 + 106u^3t^3 + 92u^4t^2 + 60u^5t + 15u^6)s \\
 & + t^3u^3(t+u)^3(5t^4 + 9u^3t^3 + 13u^2t^2 + 9u^3t + 5u^4)(s+t) + 6(s+u)^4(t+u)^4(2s^8 + 4(3t+u)s^7 + 2(17t^2 + 5ut + 3u^2)s^6 + (60t^3 \\
 & + 6u^2t^2 + 15u^2t + 4u^3)s^5 + 2(36t^4 - 2ut^3 + 12u^2t^2 + u^4)s^4 + 2t(30t^4 - 2ut^3 + 15u^2t^2 - 2u^3t + u^4)s^3 + 2t^2(17t^4 + 3ut^3 \\
 & + 12u^2t^2 - 2u^3t + 3u^4)s^2 + (12t^7 + 10u^6t^6 + 15u^2t^5 + 2u^4t^3)s + 2t^4(t^2 + ut + u^2)^2)H(0, y)\} / (9st(s+t)^4u(s+u)^4(t+u)^4)
 \end{aligned}$$

$$\mathcal{A}_{3;C_A}^{(2)} = \left\{49s^4 + 98s^3(t+u) + 147s^2(t^2 + u^2) + 98s(t^3 + u^3) + 49t^4 - 450t^3u + 165t^2u^2 - 450tu^3 + 49u^4\right\} / (5stu)$$

$$\mathcal{A}_{3;C_A n_f}^{(2)} = \left\{353t^2 - 588tu + 353u^2\right\} / (5s)$$

$$\mathcal{A}_{3;C_F n_f}^{(2)} = -18\{t^2 + u^2\} / (s)$$

$$\mathcal{A}_{3;n_f}^{(2)} = 0$$

$$\begin{aligned}
 \mathcal{A}_{4;C_A^2}^{(2)} = & \left\{ 9s^2t^2u^2(231s^{16} + 1896(t+u)s^{15} + 6(1205t^2 + 2582u\,t + 1205u^2)s^{14} + 2(8485t^3 + 28497ut^2 + 28497u^2t + 8485\,u^3)s^{13} \right. \\
 & + (27247t^4 + 132002ut^3 + 187458u^2t^2 + 132002u^3\,t + 27247u^4)s^{12} + 12(2609t^5 + 18137ut^4 + 32471u^2t^3 + 32471\,u^3t^2 + 18137u^4t \\
 & + 2609u^5)s^{11} + (26071t^6 + 265278u\,t^5 + 584223u^2t^4 + 758272u^3t^3 + 584223u^4t^2 + 265278u^5t + 26071\,u^6)s^{10} + 2(7765t^7 + 118994ut^6 \\
 & + 329001u^2t^5 + 544812u^3t^4 + 544812u^4t^3 + 329001u^5t^2 + 118994u^6t + 7765u^7)s^9 + 3(2110t^8 + 50618ut^7 + 186907u^2t^6 + 405032u^3t^5 \\
 & + 468376u^4t^4 + 405032u^5t^3 + 186907u^6t^2 + 50618u^7t + 2110u^8)s^8 + 4(398t^9 + 16089ut^8 + 86178u^2t^7 + 272710u^3t^6 + 333261u^4t^5 \\
 & + 333261\,u^5t^4 + 272710u^6t^3 + 86178u^7t^2 + 16089u^8t + 398u^9)s^7 + (187t^{10} + 16046ut^9 + 137715u^2t^8 + 724384u^3t^7 + 1065452u^4t^6 \\
 & + 947616u^5t^5 + 1065452u^6t^4 + 724384u^7t^3 + 137715u^8t^2 + 16046u^9t + 187u^{10})s^6 + 6tu(299t^9 + 5160ut^8 + 51597u^2t^7 + 106788\,u^3t^6 \\
 & + 101934u^4t^5 + 101934u^5t^4 + 106788u^6t^3 + 51597u^7t^2 + 5160u^8t + 299u^9)s^5 + t^2u^2(2973t^8 + 74726ut^7 + 236013u^2t^6 + 329784u^3t^5 \\
 & + 314484u^4t^4 + 329784u^5t^3 + 236013u^6t^2 + 74726u^7t + 2973u^8)s^4 + 12t^3u^3(651t^7 + 3762ut^6 + 9502u^2t^5 + 11138\,u^3t^4 + 11138u^4t^3 \\
 & + 9502u^5t^2 + 3762u^6t + 651u^7)s^3 + 3t^4u^4(1027t^6 + 7446ut^5 + 12323u^2t^4 + 11392u^3t^3 + 12323u^4t^2 + 7446u^5t + 1027u^6)s^2 \\
 & + 6t^5u^5(315t^5 + 904ut^4 + 837u^2t^3 + 837u^3t^2 + 904u^4t + 315u^5)s + t^6u^6(215t^4 + 250ut^3 + 357u^2t^2 + 250u^3t + 215u^4)(t+u)^6 \\
 & - 108s^2t^2(s+t)^6u^2(s+u)^6(s^4 + 2(t+u)s^3 + 3(t^2+u^2)s^2 + 2(t^3+u^3)s + t^4+u^4 - 12tu^3 + 18t^2u^2 - 24t^3u)H(0,y)(t+u)^6 \\
 & - 108s^2t^2(s+t)^6u^2(s+u)^6(s^4 + 2(t+u)s^3 + 3(t^2+u^2)s^2 + 2(t^3+u^3)s + t^4+u^4 - 24tu^3 + 18t^2u^2 - 12t^3u)H(0,z)(t+u)^6 \\
 & + 216s^2t^2(s+t)^6u^3(s+u)^6(2s^3 - 6us^2 + 10u^2s + 3t(2t^2 - 3ut + 4u^2))H(1,y)(t+u)^6 + 108s^2t^2(s+t)^6u^2(s+u)^6(s^4 + 2(t-u)s^3 \\
 & + 3(t^2+5u^2)s^2 + 2(t^3-9u^3)s + t^4+u^4 + 18tu^3 - 9t^2u^2 + 30t^3u)H(1,z)(t+u)^6 + 108s^2t^2(s+t)^6u^2(s+u)^6(s^4 - 2(t+u)s^3 \\
 & + 15(t^2+u^2)s^2 - 18(t^3+u^3)s + t^4+u^4 + 6tu^3 + 9t^2u^2 + 6t^3u)H(2,y)(t+u)^6 \Big\} / (27s^3t^3u^3(s+t)^6(s+u)^6(t+u)^6)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{4;C_A^{nf}}^{(2)} = & \left\{ 9s^2t^2u^2(54s^{16} + 420(t+u)s^{15} + 24(64t^2 + 93ut + 64\,u^2)s^{14} + (3520t^3 + 6099ut^2 + 6099u^2t + 3520u^3)s^{13} \right. \\
 & + 2(2821t^4 + 4943ut^3 + 9330u^2t^2 + 4943u^3t + 2821u^4)s^{12} + 6(1106t^5 + 1697ut^4 + 6163u^2t^3 + 6163u^3t^2 + 1697u^4t + 1106\,u^5)s^{11} \\
 & + (5810t^6 + 7365ut^5 + 65076u^2t^4 + 38726u^3t^3 + 65076u^4t^2 + 7365u^5t + 5810u^6)s^{10} + (3736t^7 + 3503u\,t^6 + 105864u^2t^5 + 17977u^3t^4 \\
 & + 17977u^4t^3 + 105864u^5t^2 + 3503u^6t + 3736u^7)s^9 + 3(560t^8 + 208ut^7 + 38974u^2t^6 + 8059u^3t^5 - 15308u^4t^4 + 8059u^5t^3 + 38974u^6t^2 \\
 & + 208u^7t + 560u^8)s^8 + (472t^9 + 60ut^8 + 81318u^2t^7 - 8215u^3t^6 + 18453u^4t^5 + 18453\,u^5t^4 - 8215u^6t^3 + 81318u^7t^2 + 60u^8t \\
 & + 472u^9)s^7 + (62\,t^{10} + 229ut^9 + 37632u^2t^8 - 50045u^3t^7 + 25292u^4t^6 + 177060u^5t^5 + 25292u^6t^4 - 50045u^7t^3 + 37632u^8t^2 \\
 & + 229u^9t + 62u^{10})s^6 + 3tu(24t^9 + 3679ut^8 - 12820u^2t^7 + 195u^3t^6 + 50191u^4t^5 + 50191u^5t^4 + 195u^6t^3 - 12820u^7t^2 + 3679u^8t \\
 & + 24u^9)s^5 + 2\,t^2u^2(735t^8 - 6322ut^7 + 3033u^2t^6 + 30537u^3t^5 + 41982u^4t^4 + 30537u^5t^3 + 3033u^6t^2 - 6322u^7t + 735u^8)s^4 \\
 & + t^3u^3(-1700t^7 + 5703ut^6 + 17862u^2t^5 + 21167u^3t^4 + 21167u^4t^3 + 17862\,u^5t^2 + 5703u^6t - 1700u^7)s^3 + 12t^4u^4(109t^6 \\
 & + 190u\,t^5 + 297u^2t^4 + 423u^3t^3 + 297u^4t^2 + 190u^5t + 109u^6)s^2 - 6t^5u^5(12t^5 - 24ut^4 - 209u^2t^3 - 209u^3t^2 - 24u^4t + 12u^5)s \\
 & + t^6u^6(20t^4 + 131ut^3 + 174u^2t^2 + 131u^3t + 20u^4)(t+u)^6 - 162s^2t^3(s+t)^6u^3(s+u)^6(8t^2 - 21ut \\
 & + 17u^2)H(0,y)(t+u)^6 - 162s^2t^3(s+t)^6u^3(s+u)^6(17t^2 - 21ut + 8\,u^2)H(0,z)(t+u)^6 - 54s^2t^2(s+t)^6u^3(s+u)^6(41s^3 \\
 & - 42u\,s^2 + 5u^2s + t(50t^2 - 63ut + 23u^2))H(1,y)(t+u)^6 + 54s^2t^2(s+t)^6u^3(s+u)^6(41s^3 - 42us^2 \\
 & + 5u^2s + t(-25t^2 + 63u\,t - 52u^2))H(1,z)(t+u)^6 + 54s^2t^2(s+t)^6u^2(s+u)^6(41(t+u)s^3 - 42(t^2 \\
 & + u^2)s^2 + 5(t^3+u^3)s - 2t\,u(t^2+u^2))H(2,y)(t+u)^6 \Big\} / (27s^3t^3u^3(s+t)^6(s+u)^6(t+u)^6)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{4;C_F n_f}^{(2)} = & \left\{ 432s^3t^2(s+t)^6u^2(s+u)^6 \left( -3s^2+t^2+u^2-tu \right) H(2,y) (t+u)^7 - 18s^2t^2u^2 \left( 48s^{16} + 384(t+u)s^{15} + 12 \left( 120t^2 + 227ut \right. \right. \right. \\
 & + 120u^2 \left. \right) s^{14} + \left( 3360t^3 + 9087ut^2 + 9087u^2t + 3360u^3 \right) s^{13} + 3 \left( 1808t^4 + 6521ut^3 + 8604u^2t^2 + 6521u^3t + 1808u^4 \right) s^{12} \\
 & + \left( 6336t^5 + 30735ut^4 + 47598u^2t^3 + 47598u^3t^2 + 30735u^4t + 6336u^5 \right) s^{11} + \left( 5424t^6 + 36543ut^5 + 67614u^2t^4 + 73464u^3t^3 \right. \\
 & + 67614u^4t^2 + 36543u^5t + 5424u^6 \left. \right) s^{10} + \left( 3360t^7 + 32505ut^6 + 77922u^2t^5 + 91295u^3t^4 + 91295u^4t^3 + 77922u^5t^2 + 32505u^6t \right. \\
 & + 3360u^7 \left. \right) s^9 + 3 \left( 480t^8 + 6939ut^7 + 23404u^2t^6 + 34497u^3t^5 + 31012u^4t^4 + 34497u^5t^3 + 23404u^6t^2 + 6939u^7t + 480u^8 \right) s^8 \\
 & + 3 \left( 128t^9 + 2999ut^8 + 15226u^2t^7 + 33508u^3t^6 + 30137u^4t^5 + 30137u^5t^4 + 33508u^6t^3 + 15226u^7t^2 + 2999u^8t + 128u^9 \right) s^7 \\
 & + \left( 48t^{10} + 2337ut^9 + 19494u^2t^8 + 70630u^3t^7 + 86678u^4t^6 + 65250u^5t^5 + 86678u^6t^4 + 70630u^7t^3 + 19494u^8t^2 + 2337u^9t \right. \\
 & + 48u^{10} \left. \right) s^6 + 3tu \left( 92t^9 + 1601u^8t^8 + 10417u^7t^7 + 20343u^6t^6 + 17708u^5t^5 + 17708u^4t^4 + 20343u^3t^3 + 10417u^2t^2 + 1601u^8t \right. \\
 & + 92u^9 \left. \right) s^5 + 3t^2u^2 \left( 172t^8 + 2556ut^7 + 8524u^2t^6 + 11797u^3t^5 + 10692u^4t^4 + 11797u^5t^3 + 8524u^6t^2 + 2556u^7t + 172u^8 \right) s^4 \\
 & + t^3u^3 \left( 788t^7 + 5538ut^6 + 14103u^2t^5 + 16252u^3t^4 + 16252u^4t^3 + 14103u^5t^2 + 5538u^6t + 788u^7 \right) s^3 + 3t^4u^4 \left( 148t^6 \right. \\
 & + 959ut^5 + 1726u^2t^4 + 1748u^3t^3 + 1726u^4t^2 + 959u^5t + 148u^6 \left. \right) s^2 + 3t^5u^5 \left( 72t^5 + 265ut^4 + 347u^2t^3 + 347u^3t^2 + 265u^4t \right. \\
 & + 72u^5 \left. \right) s + 2t^6u^6 \left( 16t^4 + 39ut^3 + 54u^2t^2 + 39u^3t + 16u^4 \right) (t+u)^6 + 108s^2t^3(s+t)^6u^3(s+u)^6 \left( t^2 + 13u^2 \right) H(0,y)(t+u)^6 \\
 & + 108s^2t^3(s+t)^6u^3(s+u)^6 \left( 13t^2 + u^2 \right) H(0,z)(t+u)^6 + 108s^2t^2(s+t)^6u^3(s+u)^6 \left( 12s^3 - 4u^2s + 13t^3 + tu^2 \right) H(1,y)(t+u)^6 \\
 & - 108s^2t^2(s+t)^6u^3(s+u)^6 \left( 12s^3 - 4u^2s - t(t^2 + 13u^2) \right) H(1,z)(t+u)^6 \left. \right\} / \left( 27s^3t^3u^3(s+t)^6(s+u)^6(t+u)^6 \right)
 \end{aligned}$$

$$\mathcal{A}_{4;n_f^2}^{(2)} = 0$$

$$\begin{aligned}
 \mathcal{A}_{5;C_A^2}^{(2)} = & \left\{ 9s^2t^2(s+t)^2u^2 \left( 55s^{14} + (330t + 416u)s^{13} + (935t^2 + 1844ut + 1500u^2)s^{12} + 30(55t^3 + 135ut^2 + 167u^2t + 115u^3)s^{11} \right. \right. \\
 & + (1980t^4 + 5670ut^3 + 8826u^2t^2 + 8462u^3t + 5643u^4)s^{10} + 2(825t^5 + 2487ut^4 + 6495u^2t^3 + 3799u^3t^2 + 5061u^4t + 3426u^5)s^9 \\
 & + (935t^6 + 2568ut^5 + 15141u^2t^4 + 4070u^3t^3 - 1902u^4t^2 + 9678u^5t + 6231u^6)s^8 + 2(165t^7 + 443ut^6 + 6261u^2t^5 - 139u^3t^4 - 8575u^4t^3 \\
 & - 4065u^5t^2 + 3881u^6t + 2085u^7)s^7 + (55t^8 + 334ut^7 + 7728u^2t^6 - 6262u^3t^5 - 17396u^4t^4 - 22194u^5t^3 - 7172u^6t^2 + 5130u^7t \\
 & + 1950u^8)s^6 + 2u(45t^8 + 1695ut^7 - 3483u^2t^6 - 4305u^3t^5 - 4851u^4t^4 - 8249u^5t^3 - 1455u^6t^2 + 1362u^7t + 284u^8)s^5 + u^2(717t^8 \\
 & - 2782ut^7 - 1182u^2t^6 + 5322u^3t^5 - 4938u^4t^4 - 7502u^5t^3 + 711u^6t^2 + 998u^7t + 77u^8)s^4 - 2tu^3(158t^7 - 735ut^6 - 2541u^2t^5 \\
 & - 2255u^3t^4 + 577u^4t^3 + 150u^5t^2 - 512u^6t - 82u^7)s^3 + t^2u^4(717t^6 + 1098ut^5 + 2632u^2t^4 + 2622u^3t^3 + 1308u^4t^2 + 852u^5t + 234u^6)s^2 \\
 & + 2t^3u^5(45t^5 + 157ut^4 + 619u^2t^3 + 516u^3t^2 + 340u^4t + 102u^5)s + t^4u^6(55t^4 + 134ut^3 + 207u^2t^2 + 134u^3t + 105u^4) \left. \right) H(0,y)(t+u)^6 \\
 & + 9s^2t^2u^2(s+u)^2 \left( 55s^{14} + (416t + 330u)s^{13} + (1500t^2 + 1844ut + 935u^2)s^{12} + 30(115t^3 + 167ut^2 + 135u^2t + 55u^3)s^{11} + (5643t^4 \right. \\
 & + 8462ut^3 + 8826u^2t^2 + 5670u^3t + 1980u^4)s^{10} + 2(3426t^5 + 5061ut^4 + 3799u^2t^3 + 6495u^3t^2 + 2487u^4t + 825u^5)s^9 + (6231t^6 \\
 & + 9678ut^5 - 1902u^2t^4 + 4070u^3t^3 + 15141u^4t^2 + 2568u^5t + 935u^6)s^8 + 2(2085t^7 + 3881ut^6 - 4065u^2t^5 - 8575u^3t^4 - 139u^4t^3 \\
 & + 6261u^5t^2 + 443u^6t + 165u^7)s^7 + (1950t^8 + 5130ut^7 - 7172u^2t^6 - 22194u^3t^5 - 17396u^4t^4 - 6262u^5t^3 + 7728u^6t^2 + 334u^7t \\
 & + 55u^8)s^6 + 2t(284t^8 + 1362ut^7 - 1455u^2t^6 - 8249u^3t^5 - 4851u^4t^4 - 4305u^5t^3 - 3483u^6t^2 + 1695u^7t + 45u^8)s^5 + t^2(77t^8 \\
 & + 998ut^7 + 711u^2t^6 - 7502u^3t^5 - 4938u^4t^4 + 5322u^5t^3 - 1182u^6t^2 - 2782u^7t + 717u^8)s^4 + 2t^3u(82t^7 + 512ut^6 - 150u^2t^5 - 577u^3t^4 \\
 & + 2255u^4t^3 + 2541u^5t^2 + 735u^6t - 158u^7)s^3 + t^4u^2(234t^6 + 852ut^5 + 1308u^2t^4 + 2622u^3t^3 + 2632u^4t^2 + 1098u^5t + 717u^6)s^2 \\
 & + 2t^5u^3(102t^5 + 340ut^4 + 516u^2t^3 + 619u^3t^2 + 157u^4t + 45u^5)s + t^6u^4(105t^4 + 134ut^3 + 207u^2t^2 + 134u^3t + 55u^4) \left. \right) H(0,z)(t+u)^6 \\
 & + 108s^2t^2(s+t)^6u^2(s+u)^6 \left( 3s^4 + 6(t+u)s^3 + 9(t^2 + u^2)s^2 + 6(t^3 + u^3)s + 3t^4 + 3u^4 - 4tu^3 - 6t^2u^2 - 4t^3u \right) H(0,y)H(0,z)(t+u)^6 \\
 & - 18s^2t^2(s+t)^2u^2(s+u)^6 \left( 3s^8 + 2(51t + u)s^7 + (486t^2 + 200ut + 30u^2)s^6 + 6(179t^3 + 130ut^2 + 96u^2t + 7u^3)s^5 + 2(687t^4 + 677ut^3 \right. \\
 & + 945u^2t^2 + 265u^3t + 29u^4)s^4 + 2t(537t^4 + 677ut^3 + 1344u^2t^2 + 658u^3t + 170u^4)s^3 + 2t^2(243t^4 + 390ut^3 + 945u^2t^2 + 658u^3t \\
 & + 270u^4)s^2 + 2t^3(51t^4 + 100ut^3 + 288u^2t^2 + 265u^3t + 170u^4)s + t^4(3t^4 + 2ut^3 + 30u^2t^2 + 42u^3t + 58u^4) \left. \right) H(1,y)(t+u)^6
 \end{aligned}$$

$$\begin{aligned}
 & -216s^2t^2(s+t)^6u^3(s+u)^6(2s^3-3us^2+4u^2s+t(2t^2-3ut+4u^2))H(0,z)H(1,y)(t+u)^6-108s^2t^2(s+t)^6u^2(s+u)^6(3s^4 \\
 & + (6t-8u)s^3 + (9t^2+6u^2)s^2 + (6t^3-4u^3)s + 3t^4+3u^4+12tu^3+6t^2u^2+16t^3u)H(0,y)H(1,z)(t+u)^6 \\
 & -1296s^2t^2(s+t)^6u^3(s+u)^6(s^3-us^2+u^2s+t(t^2-ut+u^2))H(1,y)H(1,z)(t+u)^6-108s^2t^2(s+t)^6u^2(s+u)^6(3s^4+(2u-8t)s^3 \\
 & +3(2t^2+5u^2)s^2-2(2t^3+u^3)s+3t^4+3u^4+8tu^3+12t^2u^2+8t^3u)H(0,z)H(2,y)(t+u)^6-108s^2t^2(s+t)^6u^2(s+u)^6(8s^4 \\
 & +(4t-2u)s^3+9(2t^2+3u^2)s^2-2u^3s+8t^4+8u^4+18tu^3+27t^2u^2+18t^3u)H(1,z)H(2,y)(t+u)^6-108s^2t^2(s+t)^6u^2(s+u)^6(2s^4 \\
 & +10(t+u)s^3-3(t^2+u^2)s^2+6(t^3+u^3)s+2(t^2+ut+u^2)^2)H(1,z)H(3,y)(t+u)^6-108s^2t^2(s+t)^6u^3(s+u)^6(6t^2-9ut \\
 & +2u^2)H(0,0,y)(t+u)^6-108s^2t^2(s+t)^6u^3(s+u)^6(2t^2-9ut+6u^2)H(0,0,z)(t+u)^6+432s^2t^2(s+t)^6u^2(s+u)^6(2s^4 \\
 & +2(2t+u)s^3+3(2t^2+u^2)s^2+2(2t^3+u^3)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u)H(0,1,y)(t+u)^6+108s^2t^2(s+t)^6u^2(s+u)^6(3s^4 \\
 & +(6t-2u)s^3+3(3t^2+5u^2)s^2+2(3t^3+u^3)s+3t^4+3u^4-14tu^3+9t^2u^2-14t^3u)H(0,1,z)(t+u)^6-108s^2t^2(s+t)^6u^2(s+u)^6(3s^4 \\
 & +8(t-u)s^3+6(2t^2+u^2)s^2+(8t^3-4u^3)s+3(t^4+6ut^3-u^2t^2+6u^3t+u^4))H(0,2,y)(t+u)^6+108s^2t^2(s+t)^6u^2(s+u)^6(6s^4 \\
 & +2(6t-u)s^3+3(6t^2+5u^2)s^2+2(6t^3+u^3)s+6t^4+6u^4+6tu^3+9t^2u^2+6t^3u)H(1,0,y)(t+u)^6+108s^2t^2(s+t)^6u^2(s+u)^6(3s^4 \\
 & +2(3t+5u)s^3+3(3t^2+u^2)s^2+2(3t^3+7u^3)s+3t^4+3u^4-2tu^3-3t^2u^2-2t^3u)H(1,0,z)(t+u)^6+216s^2t^2(s+t)^6u^2(s+u)^6(4s^4 \\
 & +8(t+u)s^3+3(4t^2+u^2)s^2+(8t^3+6u^3)s+4t^4+4u^4+6tu^3+3t^2u^2+8t^3u)H(1,1,y)(t+u)^6+108s^2t^2(s+t)^6u^3(s+u)^6(18s^3 \\
 & -3us^2+18u^2s-t(18t^2+9ut+14u^2))H(1,1,z)(t+u)^6-1296s^2t^2(s+t)^6u^3(s+u)^6(s^3-us^2+u^2s+t(t^2-ut \\
 & +u^2))H(1,2,y)(t+u)^6-108s^2t^2(s+t)^6u^2(s+u)^6(3s^4+2(t-4u)s^3+3(5t^2+2u^2)s^2-2(t^3+2u^3)s+3t^4+3u^4+8tu^3 \\
 & +12t^2u^2+8t^3u)H(2,0,y)(t+u)^6-108s^2t^2(s+t)^6u^3(s+u)^6(6s^3-9us^2+2u^2s+t(2t^2+3ut+2u^2))H(2,1,y)(t+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(s+u)^6(8s^4-2(t+u)s^3+27(t^2+u^2)s^2-2(t^3+u^3)s+8(t^2+ut+u^2)^2)H(2,2,y)(t+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(s+u)^6(2s^4+10(t+u)s^3-3(t^2+u^2)s^2+6(t^3+u^3)s+2(t^2+ut+u^2)^2)H(3,2,y)(t+u)^6 \\
 & +9t^2(s+t)u^2(s+u)(366(t+u)^6s^{16}+12(213t^7+1475ut^6+4463u^2t^5+7365u^3t^4+7365u^4t^3+4463u^5t^2+1475u^6t+213u^7)s^{15} \\
 & +2(t+u)^2(4186t^6+23989ut^5+61240u^2t^4+77078u^3t^3+61240u^4t^2+23989u^5t+4186u^6)s^{14}+2(8526t^9+72111ut^8+283103u^2t^7 \\
 & +646677u^3t^6+948359u^4t^5+948359u^5t^4+646677u^6t^3+283103u^7t^2+72111u^8t+8526u^9)s^{13}+(t+u)^2(23886t^8+172264u^7t^7 \\
 & +582649u^2t^6+1131132u^3t^5+1344162u^4t^4+1131132u^5t^3+582649u^6t^2+172264u^7t+23886u^8)s^{12}+(23866t^{11}+243086u^10t^{10} \\
 & +1157425u^2t^9+3382695u^3t^8+6575795u^4t^7+8973837u^5t^6+8973837u^6t^5+6575795u^7t^4+3382695u^8t^3+1157425u^9t^2 \\
 & +243086u^{10}t+23866u^{11})s^{11}+(t+u)^2(17016t^{10}+164440u^9t^9+724400u^2t^8+1917401u^3t^7+3236864u^4t^6+3710558u^5t^5 \\
 & +3236864u^6t^4+1917401u^7t^3+724400u^8t^2+164440u^9t+17016u^{10})s^{10}+2(4176t^{13}+58901ut^{12}+382615u^2t^{11}+1458232u^3t^{10} \\
 & +3634444u^4t^9+6299163u^5t^8+8074581u^6t^7+8074581u^7t^6+6299163u^8t^5+3634444u^9t^4+1458232u^{10}t^3+382615u^{11}t^2 \\
 & +58901u^{12}t+4176u^{13})s^9+(t+u)^2(2552t^{12}+42616u^11t^{11}+321337u^2t^{10}+1192960u^3t^9+2636700u^4t^8+3851544u^5t^7 \\
 & +4187542u^6t^6+3851544u^7t^5+2636700u^8t^4+1192960u^9t^3+321337u^{10}t^2+42616u^{11}t+2552u^{12})s^8+(366t^{15}+11590u^14t^{14} \\
 & +150117u^2t^{13}+868535u^3t^{12}+2906129u^4t^{11}+6451075u^5t^{10}+10280514u^6t^9+12609962u^7t^8+12609962u^8t^7+10280514u^9t^6 \\
 & +6451075u^{10}t^5+2906129u^{11}t^4+868535u^{12}t^3+150117u^{13}t^2+11590u^{14}t+366u^{15})s^7+tu(t+u)^2(1254t^{12}+30458ut^{11} \\
 & +192025u^2t^{10}+634366u^3t^9+1369170u^4t^8+2072140u^5t^7+2324670u^6t^6+2072140u^7t^5+1369170u^8t^4+634366u^9t^3+192025u^{10}t^2 \\
 & +30458u^{11}t+1254u^{12})s^6+2t^2u^2(1605t^{13}+19222ut^{12}+110551u^2t^{11}+398318u^3t^{10}+989437u^4t^9+1764628u^5t^8+2329703u^6t^7 \\
 & +2329703u^7t^6+1764628u^8t^5+989437u^9t^4+398318u^{10}t^3+110551u^{11}t^2+19222u^{12}t+1605u^{13})s^5+t^3u^3(t+u)^2(1678t^{10} \\
 & +19862ut^9+93582u^2t^8+253073u^3t^7+439352u^4t^6+520162u^5t^5+439352u^6t^4+253073u^7t^3+93582u^8t^2+19862u^9t+1678u^{10})s^4 \\
 & +t^4u^4(t+u)^3(1260t^8+9096ut^7+30583u^2t^6+61244u^3t^5+75426u^4t^4+61244u^5t^3+30583u^6t^2+9096u^7t+1260u^8)s^3 \\
 & +t^5u^5(t+u)^4(324t^6+2168ut^5+6197u^2t^4+8622u^3t^3+6197u^4t^2+2168u^5t+324u^6)s^2+10t^6u^6(t+u)^7(14t^2+27ut+14u^2)s \\
 & +40t^7u^7(t+u)^6(t^2+ut+u^2)+9s^2t^2u^2(3(11t^6+130ut^5+141u^2t^4+604u^3t^3+141u^4t^2+130u^5t+11u^6)s^{16}+8(45t^7+521ut^6 \\
 & +1056u^2t^5+2598u^3t^4+2488u^4t^3+858u^5t^2+411u^6t+23u^7)s^{15}+6(285t^8+3120u^7t^7+8470u^2t^6+17852u^3t^5+25104u^4t^4 \\
 & +15024u^5t^3+5642u^6t^2+1908u^7t+83u^8)s^{14}+2(2375t^9+24561ut^8+80646u^2t^7+164370u^3t^6+268500u^4t^5+242676u^5t^4 \\
 & +112746u^6t^3+43806u^7t^2+11685u^8t+539u^9)s^{13}+(8669t^{10}+85044ut^9+320811u^2t^8+677420u^3t^7+1161930u^4t^6+1394268u^5t^5 \\
 & +895510u^6t^4+373044u^7t^3+149697u^8t^2+34384u^9t+2343u^{10})s^{12}+12(915t^{11}+8573u^10t^{10}+35593u^2t^9+81037u^3t^8+140831u^4t^7 \\
 & +205671u^5t^6+184091u^6t^5+94279u^7t^4+41772u^8t^3+16990u^9t^2+3718u^{10}t+370u^{11})s^{11}+(9845t^{12}+88872ut^{11}+394182u^2t^{10} \\
 & +982076u^3t^9+1680477u^4t^8+2723040u^5t^7+3377880u^6t^6+2469456u^7t^5+1323771u^8t^4+704312u^9t^3+264210u^{10}t^2+55188u^{11}t \\
 & +6179u^{12})s^{10}+2(3095t^{13}+27024ut^{12}+127632u^2t^{11}+352506u^3t^{10}+563960u^4t^9+824355u^5t^8+1409056u^6t^7+1665520u^7t^6 \\
 & +1351875u^8t^5+966790u^9t^4+514400u^{10}t^3+159702u^{11}t^2+28798u^{12}t+2919u^{13})s^9+3(870t^{14}+7314u^13t^{13}+37677u^2t^{12}
 \end{aligned}$$

$$\begin{aligned}
 & +112780u^3t^{11}+143003u^4t^{10}+53282u^5t^9+249844u^6t^8+845260u^7t^7+1239890u^8t^6+1252614u^9t^5+905087u^{10}t^4+401760u^{11}t^3 \\
 & +101733u^{12}t^2+14670u^{13}t+1176u^{14})s^8+4(166t^{15}+1359ut^{14}+8343u^2t^{13}+18170u^3t^{12}-29355u^4t^{11}-193428u^5t^{10} \\
 & -186082u^6t^9+373479u^7t^8+1074360u^8t^7+1349559u^9t^6+1156713u^{10}t^5+673434u^{11}t^4+244731u^{12}t^3+51600u^{13}t^2+5604u^{14}t \\
 & +307u^{15})s^7+(77t^{16}+712ut^{15}+7926u^2t^{14}-22420u^3t^{13}-310304u^4t^{12}-964008u^5t^{11}-1004190u^6t^{10}+1290836u^7t^9 \\
 & +5018184u^8t^8+6920472u^9t^7+6102774u^{10}t^6+3900588u^{11}t^5+1764592u^{12}t^4+528392u^{13}t^3+92178u^{14}t^2+6772u^{15}t \\
 & +187u^{16})s^6+6tu(5t^{15}+338ut^{14}-3532u^2t^{13}-33260u^3t^{12}-97374u^4t^{11}-95800u^5t^{10}+181294u^6t^9+739773u^7t^8 \\
 & +1163599u^8t^7+1123652u^9t^6+758126u^{10}t^5+368632u^{11}t^4+127306u^{12}t^3+29970u^{13}t^2+4032u^{14}t+151u^{15})s^5+t^2u^2(327t^{14} \\
 & -6804ut^{13}-59454u^2t^{12}-167472u^3t^{11}-99128u^4t^{10}+699492u^5t^9+2552097u^6t^8+4572060u^7t^7+5195055u^8t^6+4069272u^9t^5 \\
 & +2242188u^{10}t^4+847572u^{11}t^3+212250u^{12}t^2+33688u^{13}t+2793u^{14})s^4+4t^3u^3(-233t^{13}-1616u^{12}-3051u^{11}+10663u^{10} \\
 & +80192u^9t+249270u^8t^2+483611u^6t^7+633999u^7t^6+583800u^8t^5+382847u^9t^4+174178u^{10}t^3+51444u^{11}t^2+8727u^{12}t \\
 & +617u^{13})s^3+3t^4u^4(t+u)^2(109t^{10}+662ut^9+4205u^2t^8+17972u^3t^7+46152u^4t^6+74668u^5t^5+76440u^6t^4+53188u^7t^3 \\
 & +24579u^8t^2+6934u^9t+931u^{10})s^2+2t^5u^5(t+u)^3(15t^8+481ut^7+3156u^2t^6+8739u^3t^5+13780u^4t^4+12711u^5t^3 \\
 & +7500u^6t^2+2637u^7t+453u^8)s+ t^6u^6(t+u)^6(77t^4+270ut^3+477u^2t^2+350u^3t+187u^4)H(1,z)+9s^2t^2u^2(3(13t^6 \\
 & +142ut^5+171u^2t^4+644u^3t^3+171u^4t^2+142u^5t+13u^6)s^{16}+8(50t^7+578ut^6+1293u^2t^5+3103u^3t^4+3103u^4t^3 \\
 & +1293u^5t^2+578u^6t+50u^7)s^{15}+12(157t^8+1789ut^7+5420u^2t^6+12039u^3t^5+17342u^4t^4+12039u^5t^3+5420u^6t^2+1789u^7t \\
 & +157u^8)s^{14}+(5374t^9+59826ut^8+223092u^2t^7+513996u^3t^6+871536u^4t^5+871536u^5t^4+513996u^6t^3+223092u^7t^2 \\
 & +59826u^8t+5374u^9)s^{13}+(10359t^{10}+114484ut^9+503079u^2t^8+1279432u^3t^7+2390090u^4t^6+3044424u^5t^5+2390090u^6t^4 \\
 & +1279432u^7t^3+503079u^8t^2+114484u^9t+10359u^{10})s^{12}+12(1186t^{11}+13429u^{10}t+67778u^9t^2+197005u^8t^3+403359u^7t^4 \\
 & +605123u^6t^5+605123u^7t^6+403359u^8t^7+197005u^9t^8+13429u^{10}t+1186u^{11})s^{11}+(14195t^{12}+171720ut^{11} \\
 & +991722u^2t^{10}+3343932u^3t^9+7591317u^4t^8+12777132u^5t^7+15424700u^6t^6+12777132u^7t^5+7591317u^8t^4+3343932u^9t^3 \\
 & +991722u^{10}t^2+171720u^{11}t+14195u^{12})s^{10}+2(5067t^{13}+68848u^{12}t+464430u^{11}t^2+1834210u^{10}t^3+4687302u^9t^4 \\
 & +8645967u^8t^5+11943168u^7t^6+11943168u^8t^7+8645967u^9t^8+4687302u^{10}t^9+1834210u^{11}t^{10}+464430u^{12}t^{11}+68848u^{13}t^{12} \\
 & +5067u^{14})s^9+3(1638t^{14}+26822ut^{13}+219527u^2t^{12}+1022692u^3t^{11}+2994269u^4t^{10}+6115342u^5t^9+9403374u^6t^8 \\
 & +10916304u^7t^7+9403374u^8t^6+6115342u^9t^5+2994269u^{10}t^4+1022692u^{11}t^3+219527u^{12}t^2+26822u^{13}t+1638u^{14})s^8 \\
 & +4(361t^{15}+8109ut^{14}+85470u^2t^{13}+471328u^3t^{12}+1600674u^4t^{11}+3733632u^5t^{10}+6488383u^6t^9+8627643u^7t^8+8627643u^8t^7 \\
 & +6488383u^9t^6+3733632u^{10}t^5+1600674u^{11}t^4+471328u^{12}t^3+85470u^{13}t^2+8109u^{14}t+361u^{15})s^7+(193t^{16}+8108ut^{15} \\
 & +123366u^2t^{14}+816896u^3t^{13}+3259172u^4t^{12}+8952972u^5t^{11}+18149594u^6t^{10}+27988696u^7t^9+32478774u^8t^8+27988696u^9t^7 \\
 & +18149594u^{10}t^6+8952972u^{11}t^5+3259172u^{12}t^4+816896u^{13}t^3+123366u^{14}t^2+8108u^{15}t+193u^{16})s^6+6tu(157t^{15}+4612ut^{14} \\
 & +39024u^2t^{13}+191670u^3t^{12}+643658u^4t^{11}+1557030u^5t^{10}+2799334u^6t^9+3770803u^7t^8+3770803u^8t^7+2799334u^9t^6 \\
 & +1557030u^{10}t^5+643658u^{11}t^4+191670u^{12}t^3+39024u^{13}t^2+4612u^{14}t+157u^{15})s^5+t^2u^2(2883t^{14}+38608ut^{13}+269730u^2t^{12} \\
 & +1182108u^3t^{11}+3470348u^4t^{10}+7219608u^5t^9+11105895u^6t^8+12818744u^7t^7+11105895u^8t^6+7219608u^9t^5+3470348u^{10}t^4 \\
 & +1182108u^{11}t^3+269730u^{12}t^2+38608u^{13}t+2883u^{14})s^4+4t^3u^3(647t^{13}+9737ut^{12}+60783u^2t^{11}+220492u^3t^{10}+533350u^4t^9 \\
 & +931704u^5t^8+1224463u^6t^7+1224463u^7t^6+931704u^8t^5+533350u^9t^4+220492u^{10}t^3+60783u^{11}t^2+9737u^{12}t+647u^{13})s^3 \\
 & +3t^4u^4(t+u)^2(961t^{10}+7506ut^9+28145u^2t^8+66084u^3t^7+107838u^4t^6+127716u^5t^5+107838u^6t^4+66084u^7t^3+28145u^8t^2 \\
 & +7506u^9t+961u^{10})s^2+6t^5u^5(t+u)^3(157t^8+937ut^7+2766u^2t^6+5043u^3t^5+6226u^4t^4+5043u^5t^3+2766u^6t^2+937u^7t \\
 & +157u^8)s+t^6u^6(t+u)^6(193t^4+354ut^3+537u^2t^2+354u^3t+193u^4)H(2,y)\Big\}/\Big(27s^3t^3u^3(s+t)^6(s+u)^6(t+u)^6\Big)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{5;C_A^{n_f}}^{(2)} = & \left\{ 216s^2t^2u^3(s+u)^6(t+u)^6(s^2-ts+t^2+u^2)H(0,1,y)(s+t)^7+162s^2t^3u^3(s+u)^6(t+u)^6(t^2+u^2)H(0,y)H(0,z)(s+t)^6 \right. \\
 & +54s^2t^2u^3(s+u)^6(t+u)^6(16s^3-21us^2+7u^2s+t(16t^2-21ut+7u^2))H(0,z)H(1,y)(s+t)^6-54s^2t^2u^3(s+u)^6(t+u)^6(10s^3 \\
 & -21us^2+19u^2s+t(-8t^2+21ut-17u^2))H(0,y)H(1,z)(s+t)^6+1134s^2t^2u^3(s+u)^6(t+u)^6(s^3-2us^2 \\
 & +u^2s+t(t-u)^2)H(1,y)H(1,z)(s+t)^6+54s^2t^2u^2(s+u)^6(t+u)^6(-2(5t+8u)s^3+21(t^2+u^2)s^2-19t^3 \\
 & +7u^3)s+tu(t^2+u^2))H(0,z)H(2,y)(s+t)^6+54s^2t^2u^2(s+u)^6(t+u)^6(-4(5t+6u)s^3+21(t^2+2u^2)s^2 \\
 & -(11t^3+24u^3)s+tu(t^2+u^2))H(1,z)H(2,y)(s+t)^6+162s^3t^2u^2(s+u)^6(t+u)^6(2(t+u)s^2-7t^2 \\
 & +u^2)s+5(t^3+u^3))H(1,z)H(3,y)(s+t)^6+162s^2t^3u^3(s+u)^6(t+u)^6(2t^2-7ut+5u^2)H(0,0,y)(s+t)^6 \\
 & \left. \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + 162s^2t^3u^3(s+u)^6(t+u)^6(5t^2-7ut+2u^2)H(0,0,z)(s+t)^6 + 54s^2t^2u^3(s+u)^6(t+u)^6(7s^3-21us^2+16u^2s+t(25t^2-42ut \\
 & + 25u^2))H(0,1,z)(s+t)^6 + 54s^2t^2u^2(s+u)^6(t+u)^6((t-10u)s^3+21u^2s^2+(t^3-19u^3)s+21t(t-u)^2u)H(0,2,y)(s+t)^6 \\
 & + 54s^2t^2u^3(s+u)^6(t+u)^6(10s^3-21us^2+19u^2s+t^3+tu^2)H(1,0,y)(s+t)^6 - 54s^2t^2u^3(s+u)^6(t+u)^6(16s^3-21us^2+7u^2s \\
 & - 2t(t^2+u^2))H(1,0,z)(s+t)^6 - 54s^2t^2u^3(s+u)^6(t+u)^6(s^3-21us^2+10u^2s+t(t^2-21ut+10u^2))H(1,1,y)(s+t)^6 \\
 & - 162s^2t^2u^3(s+u)^6(t+u)^6(8s^3-14us^2+8u^2s+t(-4t^2+7ut-7u^2))H(1,1,z)(s+t)^6 + 1134s^2t^2u^3(s+u)^6(t+u)^6(s^3 \\
 & - 2us^2+u^2s+t(t-u)^2)H(1,2,y)(s+t)^6 + 54s^2t^2u^2(s+u)^6(t+u)^6(-2(8t+5u)s^3+21(t^2+u^2)s^2-(7t^3 \\
 & + 19u^3)s+tu(t^2+u^2))H(2,0,y)(s+t)^6 + 54s^2t^2u^3(s+u)^6(t+u)^6(4s^3-21us^2+13u^2s+t^3+tu^2)H(2,1,y)(s+t)^6 \\
 & - 324s^3t^2u^2(s+u)^6(t+u)^6(4(t+u)s^2-7(t^2+u^2)s+4(t^3+u^3))H(2,2,y)(s+t)^6 + 162s^3t^2u^2(s+u)^6(t+u)^6(2(t+u)s^2 \\
 & - 7(t^2+u^2)s+5(t^3+u^3))H(3,2,y)(s+t)^6 - 9s^2t^2u^2(t+u)^6(10s^{14}+(60t+74u)s^{13}+2(85t^2+94ut+129u^2)s^{12}+3(100t^3 \\
 & + 74ut^2+123u^2t+190u^3)s^{11}+(360t^4+199u^3t^3+954u^2t^2+467u^3t+900u^4)s^{10}+(300t^5-194ut^4+485u^2t^3+635u^3t^2+348u^4t \\
 & + 1074u^5)s^9+2(85t^6-429u^5t^5+4734u^2t^4+2414u^3t^3-498u^4t^2+135u^5t+492u^6)s^8+(60t^7-874ut^6+8919u^2t^5+5677u^3t^4 \\
 & + 1791u^4t^3-159u^5t^2+165u^6t+678u^7)s^7+(10t^8-337ut^7+4848u^2t^6-2634u^3t^5+4506u^4t^4+5835u^5t^3+288u^6t^2+39u^7t \\
 & + 330u^8)s^6+u(-36t^8+1746ut^7-7930u^2t^6+1614u^3t^5+14679u^4t^4+3583u^5t^3-339u^6t^2+150u^7t+100u^8)s^5+u^2(360t^8 \\
 & - 4417u^7t-1014u^2t^6+13797u^3t^5+8278u^4t^4-689u^5t^3+306u^6t^2+172u^7t+14u^8)s^4+tu^3(-708t^7-45ut^6+4974u^2t^5 \\
 & + 6885u^3t^4+1171u^4t^3-108u^5t^2+489u^6t+44u^7)s^3+3t^2u^4(120t^6+114ut^5+580u^2t^4+562u^3t^3+152u^4t^2+117u^5t+42u^6)s^2 \\
 & + t^3u^5(-36t^5-53ut^4+526u^2t^3+402u^3t^2+271u^4t+104u^5)s+t^4u^6(10t^4+40ut^3+72u^2t^2+49u^3t+56u^4)H(0,y)(s+t)^2 \\
 & + 9s^2t^2u^2(s+u)^6(t+u)^6(6s^8+(78t+22u)s^7+(324t^2+202ut+42u^2)s^6+3(230t^3+202ut^2+186u^2t+7u^3)s^5+(876t^4 \\
 & + 962ut^3+1710u^2t^2+103u^3t+58u^4)s^4+2t(345t^4+481ut^3+1194u^2t^2+38u^3t+122u^4)s^3+2t^2(162t^4+303u^3t+855u^2t^2 \\
 & + 38u^3t+162u^4)s^2+t^3(78t^4+202ut^3+558u^2t^2+103u^3t+244u^4)s+t^4(6t^4+22ut^3+42u^2t^2+21u^3t+58u^4)H(1,y)(s+t)^2 \\
 & - \frac{3}{2}t^2u^2(s+u)(462(t+u)^6s^{16}+6(533t^7+3647ut^6+11355u^2t^5+18041u^3t^4+18041u^4t^3+11355u^5t^2+3647u^6t+533u^7)s^{15} \\
 & + 6(t+u)^2(1725t^6+9572ut^5+26881u^2t^4+27580u^3t^3+26881u^4t^2+9572u^5t+1725u^6)s^{14}+(20814t^9+169590ut^8+698271u^2t^7 \\
 & + 1572623u^3t^6+2180718u^4t^5+2180718u^5t^4+1572623u^6t^3+698271u^7t^2+169590u^8t+20814u^9)s^{13}+(t+u)^2(28872t^8+187896ut^7 \\
 & + 702435u^2t^6+1379212u^3t^5+1402562u^4t^4+1379212u^5t^3+702435u^6t^2+187896u^7t+28872u^8)s^{12}+(28752t^{11}+247032u^2t^{10} \\
 & + 1200159u^2t^9+3905948u^3t^8+7929314u^4t^7+10661099u^5t^6+10661099u^6t^5+7929314u^7t^4+3905948u^8t^3+1200159u^9t^2 \\
 & + 247032u^{10}t+28752u^{11})s^{11}+(t+u)^2(20598t^{10}+137298u^2t^9+671079u^2t^8+2337710u^3t^7+4295427u^4t^6+4577312u^5t^5 \\
 & + 4295427u^6t^4+2337710u^7t^3+671079u^8t^2+137298u^9t+20598u^{10})s^{10}+(10230t^{13}+94398ut^{12}+670143u^2t^{11}+3176397u^3t^{10} \\
 & + 9253014u^4t^9+16795202u^5t^8+21138216u^6t^7+21138216u^7t^6+16795202u^8t^5+9253014u^9t^4+3176397u^{10}t^3+670143u^{11}t^2 \\
 & + 94398u^{12}t+10230u^{13})s^9+(t+u)^2(3174t^{12}+28308u^2t^{11}+353829u^2t^{10}+1598364u^3t^9+3798910u^4t^8+5149552u^5t^7 \\
 & + 4884942u^6t^6+5149552u^7t^5+3798910u^8t^4+1598364u^9t^3+353829u^{10}t^2+28308u^{11}t+3174u^{12})s^8+(462t^{15}+7482u^2t^{14} \\
 & + 190719u^2t^{13}+1263290u^3t^{12}+4354796u^4t^{11}+9652847u^5t^{10}+14817008u^6t^9+17262724u^7t^8+17262724u^8t^7+14817008u^9t^6 \\
 & + 9652847u^{10}t^5+4354796u^{11}t^4+1263290u^{12}t^3+190719u^{13}t^2+7482u^{14}t+462u^{15})s^7+tu(t+u)^2(690t^{12}+52407u^2t^{11} \\
 & + 300320u^2t^{10}+958425u^3t^9+2341562u^4t^8+3965828u^5t^7+4535936u^6t^6+3965828u^7t^5+2341562u^8t^4+958425u^9t^3+300320u^{10}t^2 \\
 & + 52407u^{11}t+690u^{12})s^6+t^2u^2(6786t^{13}+57592ut^{12}+322024u^2t^{11}+1440278u^3t^{10}+4550721u^4t^9+9574691u^5t^8+13690372u^6t^7 \\
 & + 13690372u^7t^6+9574691u^8t^5+4550721u^9t^4+1440278u^{10}t^3+322024u^{11}t^2+57592u^{12}t+6786u^{13})s^5+t^3u^3(t+u)^2(74t^{10} \\
 & + 34768u^9t^9+270574u^2t^8+947815u^3t^7+1872940u^4t^6+2304682u^5t^5+1872940u^6t^4+947815u^7t^3+270574u^8t^2+34768u^9t \\
 & + 74u^{10})s^4+t^4u^4(t+u)^3(4432t^8+44590ut^7+177657u^2t^6+382429u^3t^5+481348u^4t^4+382429u^5t^3+177657u^6t^2+44590u^7t \\
 & + 4432u^8)s^3+t^5u^5(t+u)^4(2948t^6+20395ut^5+53828u^2t^4+72306u^3t^3+53828u^4t^2+20395u^5t+2948u^6)s^2+720t^6u^6(t+u)^7(2t^2 \\
 & + 3ut+2u^2)s+288t^7u^7(t+u)^6(t^2+ut+u^2)(s+t)-9s^2t^2u^2(s+u)^2(t+u)^6(10s^{14}+(74t+60u)s^{13}+2(129t^2+94ut+85u^2)s^{12} \\
 & + 3(190t^3+123ut^2+74u^2t+100u^3)s^{11}+(900t^4+467ut^3+954u^2t^2+199u^3t+360u^4)s^{10}+(1074t^5+348ut^4+635u^2t^3 \\
 & + 4854u^3t^2-194u^4t+300u^5)s^9+2(492t^6+135ut^5-498u^2t^4+2414u^3t^3+4734u^4t^2-429u^5t+85u^6)s^8+(678t^7+165ut^6 \\
 & - 159u^2t^5+1791u^3t^4+5677u^4t^3+8919u^5t^2-874u^6t+60u^7)s^7+(330t^8+39ut^7+288u^2t^6+5835u^3t^5+4506u^4t^4-2634u^5t^3 \\
 & + 4848u^6t^2-337u^7t+10u^8)s^6+t(100t^8+150ut^7-339u^2t^6+3583u^3t^5+14679u^4t^4+1614u^5t^3-7930u^6t^2+1746u^7t-36u^8)s^5 \\
 & + t^2(14t^8+172ut^7+306u^2t^6-689u^3t^5+8278u^4t^4+13797u^5t^3-1014u^6t^2-4417u^7t+360u^8)s^4+t^3u(44t^7+489u^6t-108u^2t^5 \\
 & + 1171u^3t^4+6885u^4t^3+4974u^5t^2-45u^6t-708u^7)s^3+3t^4u^2(42t^6+117ut^5+152u^2t^4+562u^3t^3+580u^4t^2+114u^5t+120u^6)s^2
 \end{aligned}$$

$$\begin{aligned}
 & + t^5 u^3 \left( 104t^5 + 271ut^4 + 402u^2 t^3 + 526u^3 t^2 - 53u^4 t - 36u^5 \right) s + t^6 u^4 \left( 56t^4 + 49ut^3 + 72 u^2 t^2 + 40u^3 t + 10u^4 \right) H(0, z) \\
 & - 9s^2 t^2 u^2 \left( 2 \left( 3 t^6 + 60ut^5 - 78u^2 t^4 + 490u^3 t^3 - 78u^4 t^2 + 60u^5 t + 3u^6 \right) s^{16} + \left( 72t^7 + 1289ut^6 + 414u^2 t^5 + 8005u^3 t^4 + 7845u^4 t^3 \right. \right. \\
 & + 126u^5 t^2 + 1129u^6 t + 40u^7 \Big) s^{15} + 6 \left( 60t^8 + 900ut^7 + 1076u^2 t^6 + 4412u^3 t^5 + 9687u^4 t^4 + 3894u^5 t^3 + 558u^6 t^2 + 678u^7 t + 23 u^8 \right) s^{14} \\
 & + \left( 1024t^9 + 12954ut^8 + 24576u^2 t^7 + 46761u^3 t^6 + 163314u^4 t^5 + 154677u^5 t^4 + 29274u^6 t^3 + 11781u^7 t^2 + 8316u^8 t + 331 u^9 \right) s^{13} \\
 & + \left( 1874t^{10} + 20222ut^9 + 53220u^2 t^8 + 61201u^3 t^7 + 246846u^4 t^6 + 451965u^5 t^5 + 210152u^6 t^4 + 17091u^7 t^3 + 26370u^8 t^2 + 11421u^9 t \right. \\
 & + 638u^{10} \Big) s^{12} + 3 \left( 780t^{11} + 7249u t^{10} + 25014u^2 t^9 + 30013u^3 t^8 + 86909u^4 t^7 + 276042u^5 t^6 + 269826u^6 t^5 + 71475u^7 t^4 + 13798u^8 t^3 \right. \\
 & + 15298u^9 t^2 + 4109u^{10} t + 359 u^{11} \Big) s^{11} + \left( 2042t^{12} + 16125ut^{11} + 76122u^2 t^{10} + 155760u^3 t^9 + 296460u^4 t^8 + 1068120u^5 t^7 \right. \\
 & + 1786074u^6 t^6 + 1133742 u^7 t^5 + 352110u^8 t^4 + 166875u^9 t^3 + 68808u^{10} t^2 + 12114u^{11} t + 1520 u^{12} \Big) s^{10} + \left( 1240t^{13} + 7152ut^{12} \right. \\
 & + 58566u^2 t^{11} + 232980 u^3 t^{10} + 464512u^4 t^9 + 1090308u^5 t^8 + 2341921u^6 t^7 + 2520886u^7 t^6 + 1455108u^8 t^5 + 740077u^9 t^4 + 345463u^{10} t^3 \\
 & + 84624u^{11} t^2 + 10790 u^{12} t + 1573u^{13} \Big) s^9 + 3 \left( 168t^{14} + 208ut^{13} + 10592u^2 t^{12} + 73981u^3 t^{11} + 192464u^4 t^{10} + 350678u^5 t^9 + 731518u^6 t^8 \right. \\
 & + 1095507u^7 t^7 + 938868u^8 t^6 + 596751u^9 t^5 + 345880u^{10} t^4 + 133260 u^{11} t^3 + 25570u^{12} t^2 + 2627u^{13} t + 352u^{14} \Big) s^8 + \left( 124 t^{15} \right. \\
 & - 1050ut^{14} + 11760u^2 t^{13} + 109681u^3 t^{12} + 348969u^4 t^{11} + 744780u^5 t^{10} + 1812415u^6 t^9 + 3571941u^7 t^8 + 4045692u^8 t^7 + 2838503u^9 t^6 \\
 & + 1635744u^{10} t^5 + 818670u^{11} t^4 + 274406u^{12} t^3 + 50556u^{13} t^2 + 4374u^{14} t + 403u^{15} \Big) s^7 + \left( 14t^{16} - 459 ut^{15} + 4194u^2 t^{14} + 16667u^3 t^{13} \right. \\
 & + 21936u^4 t^{12} + 169446u^5 t^{11} + 1113116u^6 t^{10} + 3298442u^7 t^9 + 4969926u^8 t^8 + 4145565u^9 t^7 + 2172862u^{10} t^6 + 899067u^{11} t^5 \\
 & + 350974u^{12} t^4 + 118004u^{13} t^3 + 24360u^{14} t^2 + 1596u^{15} t + 66u^{16} \Big) s^6 + 3tu \left( -20 t^{15} + 510ut^{14} - 2846u^2 t^{13} - 25726u^3 t^{12} \right. \\
 & - 36057u^4 t^{11} + 137862 u^5 t^{10} + 687022u^6 t^9 + 1363652u^7 t^8 + 1488125u^8 t^7 + 962136u^9 t^6 + 387285u^{10} t^5 + 104624u^{11} t^4 + 27898u^{12} t^3 \\
 & + 10766u^{13} t^2 + 2505u^{14} t + 88u^{15} \Big) s^5 + t^2 u^2 \left( 276t^{14} - 4915u t^{13} - 35832u^2 t^{12} - 60573u^3 t^{11} + 131822u^4 t^{10} + 839823u^5 t^9 \right. \\
 & + 1989780u^6 t^8 + 2753037u^7 t^7 + 2394378u^8 t^6 + 1355826u^9 t^5 + 497554 u^{10} t^4 + 108936u^{11} t^3 + 16362u^{12} t^2 + 4782u^{13} t + 1056 u^{14} \Big) s^4 \\
 & + t^3 u^3 \left( -820t^{13} - 4665ut^{12} - 3330u^2 t^{11} + 50014u^3 t^{10} + 234315u^4 t^9 + 583398u^5 t^8 + 954377u^6 t^7 + 1055745 u^7 t^6 + 809880u^8 t^5 \right. \\
 & + 443150u^9 t^4 + 168838u^{10} t^3 + 40614u^{11} t^2 + 5192u^{12} t + 196u^{13} \Big) s^3 + 3t^4 u^4 (t + u)^2 \left( 92 t^{10} + 476ut^9 + 2090u^2 t^8 + 7269u^3 t^7 \right. \\
 & + 17036u^4 t^6 + 25733u^5 t^5 + 22780 u^6 t^4 + 14183u^7 t^3 + 6446u^8 t^2 + 2047u^9 t + 352u^{10} \Big) s^2 - t^5 u^5 (t + u)^3 \left( 60t^8 + 29ut^7 - 1449u^2 t^6 \right. \\
 & - 4932u^3 t^5 - 8177u^4 t^4 - 6609 u^5 t^3 - 3504u^6 t^2 - 1214u^7 t - 264u^8 \Big) s + t^6 u^6 (t + u)^6 \left( 14 t^4 + 90ut^3 + 126u^2 t^2 + 89u^3 t + 66u^4 \right) H(1, z) \\
 & - 9s^2 t^2 u^2 \left( 2 \left( 6t^6 + 78ut^5 - 33u^2 t^4 + 550u^3 t^3 - 33u^4 t^2 + 78u^5 t + 6 u^6 \right) s^{16} + \left( 130t^7 + 1727ut^6 + 1824u^2 t^5 + 10515u^3 t^4 \right. \right. \\
 & + 10515 u^4 t^3 + 1824u^5 t^2 + 1727u^6 t + 130u^7 \Big) s^{15} + 6 \left( 104t^8 + 1295u t^7 + 2603u^2 t^6 + 7743u^3 t^5 + 14182u^4 t^4 + 7743u^5 t^3 \right. \\
 & + 2603u^6 t^2 + 1295 u^7 t + 104u^8 \Big) s^{14} + \left( 1747t^9 + 20760ut^8 + 60177u^2 t^7 + 138345 u^3 t^6 + 311235u^4 t^5 + 311235u^5 t^4 + 138345u^6 t^3 \right. \\
 & + 60177u^7 t^2 + 20760u^8 t + 1747u^9 \Big) s^{13} + \left( 3218t^{10} + 37773ut^9 + 146652u^2 t^8 + 339358 u^3 t^7 + 769808u^4 t^6 + 1107894u^5 t^5 \right. \\
 & + 769808u^6 t^4 + 339358u^7 t^3 + 146652 u^8 t^2 + 37773u^9 t + 3218u^{10} \Big) s^{12} + 3 \left( 1403t^{11} + 16697u t^{10} + 82856u^2 t^9 + 227499u^3 t^8 \right. \\
 & + 514057u^4 t^7 + 898468u^5 t^6 + 898468u^6 t^5 + 514057u^7 t^4 + 227499u^8 t^3 + 82856u^9 t^2 + 16697u^{10} t + 1403 u^{11} \Big) s^{11} + 2 \left( 2050t^{12} \right. \\
 & + 24939ut^{11} + 154530u^2 t^{10} + 531155u^3 t^9 + 1271385u^4 t^8 + 2420490u^5 t^7 + 3112438u^6 t^6 + 2420490u^7 t^5 + 1271385u^8 t^4 \\
 & + 531155u^9 t^3 + 154530u^{10} t^2 + 24939 u^{11} t + 2050u^{12} \Big) s^{10} + \left( 2989t^{13} + 37142ut^{12} + 287298 u^2 t^{11} + 1240898u^3 t^{10} + 3321201u^4 t^9 \right. \\
 & + 6609462u^5 t^8 + 9856850u^6 t^7 + 9856850u^7 t^6 + 6609462u^8 t^5 + 3321201u^9 t^4 + 1240898u^{10} t^3 + 287298u^{11} t^2 + 37142u^{12} t \\
 & + 2989u^{13} \Big) s^9 + 3 \left( 514 t^{14} + 6775ut^{13} + 65664u^2 t^{12} + 346961u^3 t^{11} + 1076100u^4 t^{10} + 2314869u^5 t^9 + 3821756u^6 t^8 + 4598458u^7 t^7 \right. \\
 & + 3821756u^8 t^6 + 2314869u^9 t^5 + 1076100u^{10} t^4 + 346961u^{11} t^3 + 65664u^{12} t^2 + 6775u^{13} t + 514u^{14} \Big) s^8 + \left( 493t^{15} + 8076ut^{14} \right. \\
 & + 98952 u^2 t^{13} + 596673u^3 t^{12} + 2146416u^4 t^{11} + 5342982u^5 t^{10} + 10174467 u^6 t^9 + 14554545u^7 t^8 + 14554545u^8 t^7 + 10174467u^9 t^6 \\
 & + 5342982u^{10} t^5 + 2146416u^{11} t^4 + 596673u^{12} t^3 + 98952u^{13} t^2 + 8076u^{14} t + 493 u^{15} \Big) s^7 + \left( 72t^{16} + 2194ut^{15} + 36630u^2 t^{14} \right. \\
 & + 227075u^3 t^{13} + 910630u^4 t^{12} + 2784993u^5 t^{11} + 6611664u^6 t^{10} + 11660494u^7 t^9 + 14240640u^8 t^8 + 11660494u^9 t^7 + 6611664u^{10} t^6 \\
 & + 2784993u^{11} t^5 + 910630u^{12} t^4 + 227075u^{13} t^3 + 36630u^{14} t^2 + 2194u^{15} t + 72 u^{16} \Big) s^6 + 3tu \left( 100t^{15} + 3071ut^{14} + 18464u^2 t^{13} \right. \\
 & + 80084u^3 t^{12} + 323267u^4 t^{11} + 1009711u^5 t^{10} + 2219756u^6 t^9 + 3327843u^7 t^8 + 3327843u^8 t^7 + 2219756u^9 t^6 + 1009711u^{10} t^5 \\
 & + 323267u^{11} t^4 + 80084u^{12} t^3 + 18464u^{13} t^2 + 3071u^{14} t + 100 u^{15} \Big) s^5 + t^2 u^2 \left( 1146t^{14} + 7452ut^{13} + 43332u^2 t^{12} + 256857u^3 t^{11} \right. \\
 & + 1020516u^4 t^{10} + 2637270u^5 t^9 + 4640688u^6 t^8 + 5609726u^7 t^7 + 4640688u^8 t^6 + 2637270u^9 t^5 + 1020516u^{10} t^4 + 256857u^{11} t^3 \\
 & + 43332u^{12} t^2 + 7452u^{13} t + 1146u^{14} \Big) s^4 + t^3 u^3 \left( 316t^{13} + 7702ut^{12} + 60600u^2 t^{11} + 260422u^3 t^{10} + 721307u^4 t^9 + 1402338u^5 t^8 \right. \\
 & + 1962295u^6 t^7 + 1962295u^7 t^6 + 1402338u^8 t^5 + 721307u^9 t^4 + 260422u^{10} t^3 + 60600u^{11} t^2 + 7702 u^{12} t + 316u^{13} \Big) s^3 \\
 & + 3t^4 u^4 (t + u)^2 \left( 382t^{10} + 2457u t^9 + 8650u^2 t^8 + 21232u^3 t^7 + 37622u^4 t^6 + 46842u^5 t^5 + 37622u^6 t^4 + 21232u^7 t^3 + 8650u^8 t^2 \right. \\
 & + 2457u^9 t + 382u^{10} \Big) s^2 + t^5 u^5 (t + u)^3 \left( 300t^8 + 1544ut^7 + 4776u^2 t^6 + 9573u^3 t^5 + 12690u^4 t^4 + 9573u^5 t^3 + 4776u^6 t^2 \right. \\
 & + 1544u^7 t + 300u^8 \Big) s + 3t^6 u^6 (t + u)^6 \left( 24t^4 + 37ut^3 + 56u^2 t^2 + 37u^3 t + 24u^4 \right) H(2, y) \Big\} / \left( 27s^3 t^3 u^3 (s + t)^6 (s + u)^6 (t + u)^6 \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{5;CFnf}^{(2)} = & \left\{ -432s^2t^2(s^2 - ts + t^2)u^3(s+u)^6(t+u)^6H(0,z)H(1,y)(s+t)^7 - 324s^2t^2u^3(s+u)^6(t+u)^6(s^2 - ts \right. \\
 & + t^2 + u^2)H(1,y)H(1,z)(s+t)^7 - 108s^2t^2u^3(s+u)^6(t+u)^6(s^2 - ts + t^2 - 3u^2)H(1,1,y)(s+t)^7 \\
 & - 324s^2t^2u^3(s+u)^6(t+u)^6(s^2 - ts + t^2 + u^2)H(1,2,y)(s+t)^7 + 108s^2t^3u^3(s+u)^6(t+u)^6(t^2 + u^2)H(0,y)H(0,z)(s+t)^6 \\
 & + 432s^2(s-t)t^2u^5(s+u)^6(t+u)^6H(0,y)H(1,z)(s+t)^6 + 432s^3t^2u^2(s+u)^6(t+u)^6(t^3 + s^2u)H(0,z)H(2,y)(s+t)^6 \\
 & + 108s^3t^2u^2(s+u)^6(t+u)^6(3u^3 + s^2(4t+3u))H(1,z)H(2,y)(s+t)^6 - 432s^3t^2u^2(s+u)^6(t+u)^7(t^2 - ut + u^2)H(1,z)H(3,y)(s+t)^6 \\
 & - 432s^2t^3u^5(s+u)^6(t+u)^6H(0,0,y)(s+t)^6 - 432s^2t^5u^3(s+u)^6(t+u)^6H(0,0,z)(s+t)^6 - 108s^2t^2u^3(s+u)^6(t+u)^6(3t^3 + 3u^2t \\
 & + 4su^2)H(0,1,z)(s+t)^6 + 108s^2t^2u^3(s+u)^6(t+u)^6(-3t^3 - 3u^2t + 4su^2)H(0,2,y)(s+t)^6 + 108s^2t^2u^3(s+u)^6(t+u)^6(t^3 \\
 & + u^2t - 4su^2)H(1,0,y)(s+t)^6 + 108s^2t^2u^3(s+u)^6(t+u)^6(4s^3 + t^3 + tu^2)H(1,0,z)(s+t)^6 + 108s^2t^2u^3(s+u)^6(t+u)^6(3s^3 \\
 & + 3u^2s - 4tu^2)H(1,1,z)(s+t)^6 + 432s^3t^2u^2(s+u)^6(t+u)^6(u^3 + s^2t)H(2,0,y)(s+t)^6 + 108s^3t^2u^3(s+u)^6(t+u)^6(s^2 \\
 & - 3u^2)H(2,1,y)(s+t)^6 + 324s^3t^2u^2(s+u)^6(t+u)^7(s^2 + t^2 + u^2 - tu)H(2,2,y)(s+t)^6 - 432s^3t^2u^2(s+u)^6(t+u)^7(t^2 - ut \\
 & + u^2)H(3,2,y)(s+t)^6 - 18s^2t^2u^3(s+u)^6(t+u)^6(9(t+2u)s^4 + 9(3t^2 + 6ut + 2u^2)s^3 + (27t^3 + 72ut^2 + 12u^2t + 16u^3)s^2 + t(9t^3 \\
 & + 54ut^2 + 12u^2t + 20u^3)s + 2t^2u(9t^2 + 9ut + 8u^2))H(1,y)(s+t)^4 - 18s^2t^3u^3(t+u)^6(18s^{11} + 51(t+2u)s^{10} + (33t^2 + 360ut \\
 & + 203u^2)s^9 + (-27t^3 + 498ut^2 + 705u^2t + 156u^3)s^8 + (-39t^4 + 420ut^3 + 776u^2t^2 + 546u^3t - 63u^4)s^7 - 3(4t^5 - 108ut^4 - 91u^2t^3 \\
 & - 106u^3t^2 + 21u^4t + 98u^5)s^6 - 3u(-64t^5 - 9ut^4 + 98u^2t^3 + 142u^3t^2 + 236u^4t + 121u^5)s^5 - u(-48t^6 - 44ut^5 + 174u^2t^4 + 711u^3t^3 \\
 & + 1102u^4t^2 + 897u^5t + 240u^6)s^4 - u^2(-16t^6 - 100ut^5 + 333u^2t^4 + 948u^3t^3 + 1084u^4t^2 + 558u^5t + 83u^6)s^3 - 3u^3(-16t^6 + 8ut^5 \\
 & + 124u^2t^4 + 199u^3t^3 + 182u^4t^2 + 60u^5t + 4u^6)s^2 - t u^5(52t^4 + 135ut^3 + 186u^2t^2 + 147u^3t + 24u^4)s - 2t^2u^6(4t^3 + 9ut^2 + 13u^2t \\
 & + 8u^3))H(0,y)(s+t)^3 - 12t^3u^3(s+u)(24tu(3t^3 + 4ut^2 + 4u^2t + 3u^3)s^{15} - 6(t+u)^2(19t^4 - 7ut^3 + 86u^2t^2 - 7u^3t + 19u^4)s^{14} \\
 & - (528t^7 + 2241ut^6 + 5905u^2t^5 + 10806u^3t^4 + 10806u^4t^3 + 5905u^5t^2 + 2241u^6t + 528u^7)s^{13} - (t+u)^2(999t^6 + 3708ut^5 + 7757u^2t^4 \\
 & + 13636u^3t^3 + 7757u^4t^2 + 3708u^5t + 999u^6)s^{12} - (945t^9 + 6444ut^8 + 18478u^2t^7 + 38524u^3t^6 + 60985u^4t^5 + 60985u^5t^4 + 38524u^6t^3 \\
 & + 18478u^7t^2 + 6444u^8t + 945u^9)s^{11} - (t+u)^2(408t^8 + 1419ut^7 - 737u^2t^6 - 1221u^3t^5 + 6934u^4t^4 - 1221u^5t^3 - 737u^6t^2 + 1419u^7t \\
 & + 408u^8)s^{10} + 2(-9t^{11} + 1365ut^{10} + 9543u^2t^9 + 28308u^3t^8 + 43963u^4t^7 + 44166u^5t^6 + 44166u^6t^5 + 43963u^7t^4 + 28308u^8t^3 \\
 & + 9543u^9t^2 + 1365u^{10}t - 9u^{11})s^9 + (t+u)^2(33t^{10} + 3726ut^9 + 16908u^2t^8 + 31720u^3t^7 + 24859u^4t^6 + 8028u^5t^5 + 24859u^6t^4 \\
 & + 31720u^7t^3 + 16908u^8t^2 + 3726u^9t + 33u^{10})s^8 + (3t^{13} + 2076ut^{12} + 13514u^2t^{11} + 34430u^3t^{10} + 33548u^4t^9 - 29287u^5t^8 \\
 & - 114020u^6t^7 - 114020u^7t^6 - 29287u^8t^5 + 33548u^9t^4 + 34430u^{10}t^3 + 13514u^{11}t^2 + 2076u^{12}t + 3u^{13})s^7 + tu(t+u)^2(615t^{10} \\
 & + 2129ut^9 - 2862u^2t^8 - 27958u^3t^7 - 70915u^4t^6 - 96406u^5t^5 - 70915u^6t^4 - 27958u^7t^3 - 2862u^8t^2 + 2129u^9t + 615u^{10})s^6 \\
 & + tu(87t^{13} + 151ut^{12} - 4613u^2t^{11} - 33895u^3t^{10} - 117246u^4t^9 - 252106u^5t^8 - 365426u^6t^7 - 365426u^7t^6 - 252106u^8t^5 - 117246u^9t^4 \\
 & - 33895u^{10}t^3 - 4613u^{11}t^2 + 151u^{12}t + 87u^{13})s^5 - t^2u^2(t+u)^2(61t^{10} + 1421ut^9 + 9539u^2t^8 + 31118u^3t^7 + 60284u^4t^6 + 74918u^5t^5 \\
 & + 60284u^6t^4 + 31118u^7t^3 + 9539u^8t^2 + 1421u^9t + 61u^{10})s^4 - t^3u^3(t+u)^3(119t^8 + 1631ut^7 + 6699u^2t^6 + 14006u^3t^5 + 17750u^4t^4 \\
 & + 14006u^5t^3 + 6699u^6t^2 + 1631u^7t + 119u^8)s^3 - t^4u^4(t+u)^4(133t^6 + 842ut^5 + 1972u^2t^4 + 2532u^3t^3 + 1972u^4t^2 + 842u^5t + 133u^6)s^2 \\
 & - 6t^5u^5(t+u)^7(8t^2 + 9ut + 8u^2)s - 6t^6u^6(t+u)^6(t^2 + ut + u^2)(s+t) - 18s^2t^3u^3(s+u)^3(t+u)^6(18s^{11} + 51(2t+u)s^{10} + (203t^2 \\
 & + 360ut + 33u^2)s^9 + 3(52t^3 + 235ut^2 + 166u^2t - 9u^3)s^8 + (-63t^4 + 546ut^3 + 776u^2t^2 + 420u^3t - 39u^4)s^7 - 3(98t^5 + 21ut^4 - 106u^2t^3 \\
 & - 91u^3t^2 - 108u^4t + 4u^5)s^6 - 3t(121t^5 + 236ut^4 + 142u^2t^3 + 98u^3t^2 - 9u^4t - 64u^5)s^5 - t(240t^6 + 897ut^5 + 1102u^2t^4 + 711u^3t^3 \\
 & + 174u^4t^2 - 44u^5t - 48u^6)s^4 - t^2(83t^6 + 558ut^5 + 1084u^2t^4 + 948u^3t^3 + 333u^4t^2 - 100u^5t - 16u^6)s^3 - 3t^3(4t^6 + 60ut^5 + 182u^2t^4 \\
 & + 199u^3t^3 + 124u^4t^2 + 8u^5t - 16u^6)s^2 - t^5u(24t^4 + 147u^3t^3 + 186u^2t^2 + 135u^3t + 52u^4)s - 2t^6u^2(8t^3 + 13ut^2 + 9u^2t + 4u^3))H(0,z) \\
 & + 18s^2t^2u^3(16t^2u(3t^2 + ut + 3u^2)s^{16} - 4t(t^5 - 96ut^4 - 125u^2t^3 - 125u^3t^2 - 96u^4t + u^5)s^{15} + 3(-21t^7 + 376ut^6 + 969u^2t^5 \\
 & + 1014u^3t^4 + 1053u^4t^3 + 460u^5t^2 + 15u^6t + 6u^7)s^{14} + 3(-91t^8 + 434ut^7 + 2445u^2t^6 + 3548u^3t^5 + 4121u^4t^4 + 3594u^5t^3 \\
 & + 1259u^6t^2 + 200u^7t + 42u^8)s^{13} + (-507t^9 - 228ut^8 + 8958u^2t^7 + 21034u^3t^6 + 29016u^4t^5 + 37554u^5t^4 + 27842u^6t^3 \\
 & + 10398u^7t^2 + 2643u^8t + 394u^9)s^{12} + 3(-139t^{10} - 658ut^9 + 1402u^2t^8 + 7968u^3t^7 + 14642u^4t^6 + 25576u^5t^5 + 31288u^6t^4 \\
 & + 20652u^7t^3 + 8197u^8t^2 + 2094u^9t + 242u^{10})s^{11} + (-33t^{11} - 1944ut^{10} - 2626u^2t^9 + 15924u^3t^8 + 52455u^4t^7 + 115368u^5t^6 \\
 & + 193671u^6t^5 + 189594u^7t^4 + 109994u^8t^3 + 41340u^9t^2 + 9291u^{10}t + 870u^{11})s^{10} + (229t^{12} - 1206ut^{11} - 7826u^2t^{10} + 2016u^3t^9 \\
 & + 60801u^4t^8 + 168742u^5t^7 + 312211u^6t^6 + 377892u^7t^5 + 285136u^8t^4 + 142036u^9t^3 + 47343u^{10}t^2 + 9112u^{11}t + 698u^{12})s^9 \\
 & + 3(69t^{13} - 396ut^{12} - 3542u^2t^{11} - 3876u^3t^{10} + 19403u^4t^9 + 74332u^5t^8 + 145298u^6t^7 + 193458u^7t^6 + 172121u^8t^5 + 103198u^9t^4 \\
 & + 43468u^{10}t^3 + 12394u^{11}t^2 + 2015u^{12}t + 122u^{13})s^8 + (81t^{14} - 1338ut^{13} - 9602u^2t^{12} - 15144u^3t^{11} + 41583u^4t^{10} + 217006u^5t^9 \\
 & + 472374u^6t^8 + 692016u^7t^7 + 711667u^8t^6 + 501642u^9t^5 + 246150u^{10}t^4 + 85492u^{11}t^3 + 19953u^{12}t^2 + 2646u^{13}t + 114u^{14})s^7
 \end{aligned}$$



$$\begin{aligned}
 & + \left( 12t^{15} - 936ut^{14} - 5785u^2t^{13} - 9734u^3t^{12} + 23766u^4t^{11} + 151768u^5t^{10} + 374411u^6t^9 + 611526u^7t^8 + 725603u^8t^7 + 609872u^9t^6 \right. \\
 & + 355422u^{10}t^5 + 143724u^{11}t^4 + 39635u^{12}t^3 + 7044u^{13}t^2 + 696u^{14}t + 16u^{15} \Big) s^6 + 3tu \left( -112t^{14} - 711ut^{13} - 1428u^2t^{12} + 2680u^3t^{11} \right. \\
 & + 24408u^4t^{10} + 73333u^5t^9 + 137580u^6t^8 + 184601u^7t^7 + 179614u^8t^6 + 124026u^9t^5 + 59992u^{10}t^4 + 19831u^{11}t^3 + 4174u^{12}t^2 + 496u^{13}t \\
 & + 28u^{14} \Big) s^5 + t^2u \left( -48t^{14} - 380ut^{13} - 1422u^2t^{12} - 453u^3t^{11} + 19778u^4t^{10} + 90336u^5t^9 + 213354u^6t^8 + 330336u^7t^7 + 360906u^8t^6 \right. \\
 & + 281670u^9t^5 + 156716u^{10}t^4 + 61605u^{11}t^3 + 16236u^{12}t^2 + 2470u^{13}t + 144u^{14} \Big) s^4 + t^3u^2 \left( -16t^{13} - 340ut^{12} - 1209u^2t^{11} + 1822u^3t^{10} \right. \\
 & + 23522u^4t^9 + 76164u^5t^8 + 142722u^6t^7 + 179964u^7t^6 + 159444u^8t^5 + 99732u^9t^4 + 43795u^{10}t^3 + 13194u^{11}t^2 + 2510u^{12}t + 232u^{13} \Big) s^3 \\
 & - 3t^4u^3(t+u)^2 \left( 16t^{10} + 56ut^9 - 124u^2t^8 - 1147u^3t^7 - 3208u^4t^6 - 5170u^5t^5 - 5528u^6t^4 - 3835u^7t^3 - 1676u^8t^2 - 424u^9t \right. \\
 & - 48u^{10} \Big) s^2 + t^5u^5(t+u)^3 \left( 52t^7 + 321ut^6 + 921u^2t^5 + 1569u^3t^4 + 1779u^4t^3 + 1230u^5t^2 + 484u^6t + 84u^7 \right) s + 2t^6u^6(t+u)^7 \left( 4t^2 \right. \\
 & + 5ut + 8u^2 \Big) \Big) H(1, z) + 18s^2t^2u^2 \left( 16t^2u^2(3t^2 + ut + 3u^2)s^{16} - 4tu(t^5 - 96ut^4 - 125u^2t^3 - 125u^3t^2 - 96u^4t + u^5)s^{15} + 6(3t^8 \right. \\
 & + 9ut^7 + 242u^2t^6 + 567u^3t^5 + 582u^4t^4 + 567u^5t^3 + 242u^6t^2 + 9u^7t + 3u^8) s^{14} + 3(42t^9 + 221u^8t^8 + 1445u^2t^7 + 4311u^3t^6 \\
 & + 5693u^4t^5 + 5693u^5t^4 + 4311u^6t^3 + 1445u^7t^2 + 221u^8t + 42u^9) s^{13} + 2(197t^{10} + 1416ut^9 + 6153u^2t^8 + 18148u^3t^7 + 29594u^4t^6 \\
 & + 32160u^5t^5 + 29594u^6t^4 + 18148u^7t^3 + 6153u^8t^2 + 1416u^9t + 197u^{10}) s^{12} + 3(242t^{11} + 2199ut^{10} + 9451u^2t^9 + 27134u^3t^8 \\
 & + 50612u^4t^7 + 62634u^5t^6 + 62634u^6t^5 + 50612u^7t^4 + 27134u^8t^3 + 9451u^9t^2 + 2199u^{10}t + 242u^{11}) s^{11} + 2(435t^{12} + 4803ut^{11} \\
 & + 23010u^2t^{10} + 69466u^3t^9 + 145491u^4t^8 + 210555u^5t^7 + 230976u^6t^6 + 210555u^7t^5 + 145491u^8t^4 + 69466u^9t^3 + 23010u^{10}t^2 \\
 & + 4803u^{11}t + 435u^{12}) s^{10} + (698t^{13} + 9301ut^{12} + 51105u^2t^{11} + 170974u^3t^{10} + 406704u^4t^9 + 698997u^5t^8 + 884941u^6t^7 + 884941u^7t^6 \\
 & + 698997u^8t^5 + 406704u^9t^4 + 170974u^{10}t^3 + 51105u^{11}t^2 + 9301u^{12}t + 698u^{13}) s^9 + 6(61t^{14} + 1018ut^{13} + 6515u^2t^{12} + 24975u^3t^{11} \\
 & + 68497u^4t^{10} + 139578u^5t^9 + 209143u^6t^8 + 237090u^7t^7 + 209143u^8t^6 + 139578u^9t^5 + 68497u^{10}t^4 + 24975u^{11}t^3 + 6515u^{12}t^2 \\
 & + 1018u^{13}t + 61u^{14}) s^8 + (114t^{15} + 2655ut^{14} + 20511u^2t^{13} + 93946u^3t^{12} + 304122u^4t^{11} + 729081u^5t^{10} + 1284397u^6t^9 + 1678662u^7t^8 \\
 & + 1678662u^8t^7 + 1284397u^9t^6 + 729081u^{10}t^5 + 304122u^{11}t^4 + 93946u^{12}t^3 + 20511u^{13}t^2 + 2655u^{14}t + 114u^{15}) s^7 + 2(8t^{16} + 348ut^{15} \\
 & + 3558u^2t^{14} + 20893u^3t^{13} + 82679u^4t^{12} + 233298u^5t^{11} + 478228u^6t^{10} + 720901u^7t^9 + 821694u^8t^8 + 720901u^9t^7 + 478228u^{10}t^6 \\
 & + 233298u^{11}t^5 + 82679u^{12}t^4 + 20893u^{13}t^3 + 3558u^{14}t^2 + 348u^{15}t + 8u^{16}) s^6 + 3tu(28t^{15} + 496ut^{14} + 4255u^2t^{13} + 21403u^3t^{12} \\
 & + 71760u^4t^{11} + 172018u^5t^{10} + 302499u^6t^9 + 397333u^7t^8 + 397333u^8t^7 + 302499u^9t^6 + 172018u^{10}t^5 + 71760u^{11}t^4 + 21403u^{12}t^3 \\
 & + 4255u^{13}t^2 + 496u^{14}t + 28u^{15}) s^5 + 2t^2u^2(72t^{14} + 1235u^3t^{13} + 8343u^2t^{12} + 34020u^3t^{11} + 97435u^4t^{10} + 204801u^5t^9 + 317982u^6t^8 \\
 & + 367512u^7t^7 + 317982u^8t^6 + 204801u^9t^5 + 97435u^{10}t^4 + 34020u^{11}t^3 + 8343u^{12}t^2 + 1235u^{13}t + 72u^{14}) s^4 + t^3u^3(232t^{13} + 2510ut^{12} \\
 & + 13689u^2t^{11} + 49393u^3t^{10} + 127070u^4t^9 + 236640u^5t^8 + 321522u^6t^7 + 321522u^7t^6 + 236640u^8t^5 + 127070u^9t^4 + 49393u^{10}t^3 \\
 & + 13689u^{11}t^2 + 2510u^{12}t + 232u^{13}) s^3 + 6t^4u^4(t+u)^2(24t^{10} + 212ut^9 + 892u^2t^8 + 2315u^3t^7 + 4004u^4t^6 + 4762u^5t^5 + 4004u^6t^4 \\
 & + 2315u^7t^3 + 892u^8t^2 + 212u^9t + 24u^{10}) s^2 + t^5u^5(t+u)^3(84t^8 + 484ut^7 + 1347u^2t^6 + 2364u^3t^5 + 2802u^4t^4 + 2364u^5t^3 + 1347u^6t^2 \\
 & + 484u^7t + 84u^8) s + 2t^6u^6(t+u)^6(8t^4 + 13ut^3 + 18u^2t^2 + 13u^3t + 8u^4) \Big) H(2, y) \Big\} / \left( 27s^3t^3u^3(s+t)^6(s+u)^6(t+u)^6 \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{5;n_f^2}^{(2)} = & 2 \left\{ t^3 u H(2, y) + t^3 u H(1, z) - tu(t^2 + u^2) H(0, y) - tu(t^2 + u^2) H(0, z) + tu^3 H(2, y) + t u^3 H(1, z) \right. \\
 & \left. + s^4 + 2s^3t + 2s^3u + 3s^2t^2 + 3s^2u^2 + 2st^3 + 2su^3 + t^4 + 2t^3u + 3t^2u^2 + 2tu^3 + u^4 \right\} / (3stu)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{6;C_A^2}^{(2)} = & \left\{ 3s^2t^2(s+t)^6u^2(t+u)^2 \left( (363t^4 + 492ut^3 + 504u^2t^2 + 492u^3t + 363u^4) s^4 + 2(363t^5 + 479ut^4 - 58u^2t^3 - 58u^3t^2 + 479u^4t \right. \right. \\
 & + 363u^5) s^3 + 9(t+u)^2(121t^4 - 10ut^3 - 134u^2t^2 - 10u^3t + 121u^4) s^2 + 6(t+u)^3(121t^4 + 35ut^3 + 12u^2t^2 + 35u^3t + 121u^4) s \\
 & + 363(t+u)^4(t^2 + ut + u^2)^2 \Big) H(1, z) H(2, y)(s+u)^6 + 3s^2t^2(s+t)^2u^2(t+u)^6(363s^8 + 726(3t+u)s^7 + 3(2057t^2 + 796ut + 363u^2) s^6 \\
 & + 6(1815t^3 + 480ut^2 + 348u^2t + 121u^3) s^5 + (13068t^4 + 1782ut^3 - 297u^2t^2 + 958u^3t + 363u^4) s^4 + 2t(5445t^4 + 891ut^3 - 1296u^2t^2 \\
 & - 58u^3t + 246u^4) s^3 + t^2(6171t^4 + 2880ut^3 - 297u^2t^2 - 116u^3t + 504u^4) s^2 + 2t^3(1089t^4 + 1194ut^3 + 1044u^2t^2 + 479u^3t + 246u^4) s \\
 & + 363t^4(t^2 + ut + u^2)^2 \Big) H(0, 0, y)(s+u)^6 + 36s^2t^2(s+t)^2u^2(t+u)^6(22s^8 + 44(3t+u)s^7 + 2(187t^2 + 70ut + 33u^2) s^6 + (660t^3 \\
 & + 147ut^2 + 246u^2t + 44u^3) s^5 + (792t^4 + 41ut^3 + 360u^2t^2 + 115u^3t + 22u^4) s^4 + t(660t^4 + 2ut^3 + 246u^2t^2 + 95u^3t + 64u^4) s^3 \\
 & + t^2(374t^4 + 54ut^3 + 84u^2t^2 + 9u^3t + 84u^4) s^2 + t^3(132t^4 + 63u^3t^3 + 36u^2t^2 - 11u^3t + 54u^4) s + t^4(22t^4 + 21ut^3 + 18u^2t^2 + 4u^3t \\
 & + 18u^4) \Big) H(0, z) H(0, 0, y)(s+u)^6 - 36s^2t^2(s+t)^2u^2(t+u)^6(22s^8 + 3(44t+7u)s^7 + (374t^2 + 63ut + 18u^2) s^6 + (660t^3 + 54ut^2 \\
 & + 36u^2t + 4u^3) s^5 + (792t^4 + 2u^3t^3 + 84u^2t^2 - 11u^3t + 18u^4) s^4 + t(660t^4 + 41ut^3 + 246u^2t^2 + 9u^3t + 54u^4) s^3 + t^2(374t^4 + 147ut^3 \\
 & + 360u^2t^2 + 95u^3t + 84u^4) s^2 + t^3(132t^4 + 140ut^3 + 246u^2t^2 + 115u^3t + 64u^4) s + 22t^4(t^2 + ut + u^2)^2 \Big) H(1, z) H(0, 0, y)(s+u)^6
 \end{aligned}$$

$$\begin{aligned}
 & + 108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4(t+u)s^3+6(t^2+u^2)s^2+4(t^3+u^3)s+2t^4+2u^4-2tu^3-3t^2u^2 \\
 & - 2t^3u)H(0,0,y)H(0,0,z)(s+u)^6+18s^2t^2(s+t)^6u^2(t+u)^2((44t^4+296ut^3+486u^2t^2+316u^3t+58u^4)s^4+2(15t^5+247ut^4 \\
 & + 634u^2t^3+646u^3t^2+271u^4t+27u^5)s^3+3(t+u)^2(5t^4+150ut^3+228u^2t^2+146u^3t+15u^4)s^2-2(t+u)^3(2t^4-91ut^3-90u^2t^2 \\
 & - 97u^3t-4u^4)s+(t+u)^4(11t^4+94ut^3+105u^2t^2+72u^3t+6u^4))H(3,y)H(0,1,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4 \\
 & + 2(2t+u)s^3+3(2t^2+u^2)s^2+2(2t^3+u^3)s+2(t^4-2ut^3+6u^2t^2+u^4))H(0,0,y)H(0,1,z)(s+u)^6 \\
 & + 36s^2t^2(s+t)^6u^2(t+u)^2(2(11t^4+32ut^3+42u^2t^2+27u^3t+9u^4)s^4+(44t^5+115ut^4+95u^2t^3+9u^3t^2-11u^4t \\
 & + 4u^5)s^3+6(t+u)^2(11t^4+19ut^3+11u^2t^2+3u^4)s^2+(t+u)^3(44t^4+8ut^3-9u^2t^2+21u^4)s+22(t+u)^4(t^2+ut+u^2) \\
 & + u^2))H(1,z)H(0,2,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+2u)s^3+3(t^2+2u^2)s^2+2(t^3+2u^3)s+2(t^4+6u^2t^2 \\
 & - 2u^3t+u^4))H(0,0,z)H(0,2,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(-2(t-2u)s^3-3(t^2-2u^2)s^2-2(t^3-2u^3)s+12tu(t^2-ut+u^2) \\
 & + u^2))H(0,1,z)H(0,2,y)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6(s^4+2us^3+3u^2s^2+2u^3s+(t^2+ut+u^2)s^2)H(0,1,z)H(0,3,y)(s+u)^6 \\
 & + 18s^2t^2(s+t)^3u^2(t+u)^6(3s^7+(6u-9t)s^6+(-48t^2+12ut+15u^2)s^5-(56t^3+6ut^2+63u^2t-12u^3)s^4-t^2(7t^2+36ut+222u^2)s^3 \\
 & + 3t(11t^4-14ut^3-70u^2t^2-8u^3t-8u^4)s^2+t^2(28t^4-24ut^3-81u^2t^2-24u^3t-30u^4)s+t^3(8t^4-6ut^3-15u^2t^2-12u^3t-14u^4)) \\
 & H(0,z)H(1,0,y)(s+u)^6-18s^2t^2(s+t)^3u^2(t+u)^6(8s^7+(28t-6u)s^6+3(11t^2-8ut-5u^2)s^5-(7t^3+42ut^2+81u^2t+12u^3)s^4 \\
 & - 2(28t^4+18ut^3+105u^2t^2+12u^3t+7u^4)s^3-6t(8t^4+ut^3+37u^2t^2+4u^3t+5u^4)s^2-3(t^6-4ut^5+21u^2t^4+8u^4t^2)s \\
 & + 3t^4(t^3+2ut^2+5u^2t+4u^3))H(1,z)H(1,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(2t+u)s^3+3(2t^2+u^2)s^2+2(2t^3+u^3)s \\
 & + 2t^4+2u^4-2tu^3+9t^2u^2-6t^3u)H(0,1,z)H(1,0,y)(s+u)^6-18s^2t^2(s+t)^6u^2(t+u)^3(2t(7t^2+15ut+12u^2)s^4+12(t^4+2ut^3 \\
 & + 2u^2t^2-u^4)s^3+3(t+u)^2(5t^3+17ut^2+31u^2t-5u^3)s^2+6(t+u)^3(t^3+ut^2+u^2t-u^3)s-(t+u)^3(8t^4+4ut^3-3u^2t^2-18u^3t+3u^4)) \\
 & H(3,y)H(1,0,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4(t-u)s^3+6(t^2+2u^2)s^2+4t^3s+2t^4+2u^4+2tu^3+3t^2u^2 \\
 & + 2t^3u)H(0,0,y)H(1,0,z)(s+u)^6-216s^3t^2(s+t)^6u^2(t+u)^6(2(t-2u)s^2+3(t^2+u^2)s+2(t^3-u^3))H(0,2,y)H(1,0,z)(s+u)^6 \\
 & + 108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+3u)s^3+3(t^2-u^2)s^2+2(t^3+5u^3)s+2(t^2+ut+u^2)s^2)H(0,3,y)H(1,0,z)(s+u)^6 \\
 & + 108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+(4t-6u)s^3+(6t^2+9u^2)s^2+(4t^3-2u^3)s+2t^4+2u^4+2tu^3+3t^2u^2 \\
 & + 2t^3u)H(1,0,y)H(1,0,z)(s+u)^6+3s^2t^2(s+t)^6u^2(t+u)^2((363t^4+492ut^3+504u^2t^2+492u^3t+363u^4)s^4+2(363t^5+479ut^4 \\
 & - 58u^2t^3-58u^3t^2+479u^4t+363u^5)s^3+9(t+u)^2(121t^4-10ut^3-134u^2t^2-10u^3t+121u^4)s^2+6(t+u)^3(121t^4+35ut^3+12u^2t^2 \\
 & + 35u^3t+121u^4)s+363(t+u)^4(t^2+ut+u^2)s^2)H(1,1,z)(s+u)^6+36s^2t^2(s+t)^6u^2(t+u)^2(2(11t^4+32ut^3+42u^2t^2+27u^3t+9u^4)s^4 \\
 & + (44t^5+115ut^4+95u^2t^3+9u^3t^2-11u^4t+4u^5)s^3+6(t+u)^2(11t^4+19ut^3+11u^2t^2+3u^4)s^2+(t+u)^3(44t^4+8ut^3-9u^2t^2+21u^4)s \\
 & + 22(t+u)^4(t^2+ut+u^2)s^2)H(0,y)H(1,1,z)(s+u)^6-36s^2t^2(s+t)^6u^2(t+u)^2(2(20t^4+59ut^3+84u^2t^2+59u^3t+20u^4)s^4 \\
 & + 8(6t^5+13ut^4+13u^2t^3+13u^3t^2+13u^4t+6u^5)s^3+6(t+u)^2(14t^4+19ut^3+22u^2t^2+19u^3t+14u^4)s^2+(t+u)^3(65t^4+8ut^3-18u^2t^2 \\
 & + 8u^3t-18u^2t^2+8u^3t+65u^4)s+44(t+u)^4(t^2+ut+u^2)s^2)H(3,y)H(1,1,z)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4 \\
 & + (4t-2u)s^3+(6t^2-3u^2)s^2+(4t^3-2u^3)s+2(t^2+ut+u^2)s^2)H(0,0,y)H(1,1,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(4s^4 \\
 & + (6t-2u)s^3+9(t^2+u^2)s^2+6(t^3-u^3)s+4(t^2+ut+u^2)s^2)H(0,3,y)H(1,1,z)(s+u)^6+216s^2t^2(s+t)^6u^3(t+u)^6(2s^3 \\
 & - 3us^2+4u^2s+6t(t^2-ut+u^2))H(0,1,z)H(1,2,y)(s+u)^6-216s^2t^2(s+t)^6u^3(t+u)^6(6s^3-6us^2+6u^2s+t(2t^2-3ut+4u^2)) \\
 & H(1,0,z)H(1,2,y)(s+u)^6+36s^2t^2(s+t)^6u^2(t+u)^2(2(11t^4+32ut^3+42u^2t^2+27u^3t+9u^4)s^4+(44t^5+115ut^4+95u^2t^3 \\
 & + 9u^3t^2-11u^4t+4u^5)s^3+6(t+u)^2(11t^4+19ut^3+11u^2t^2+3u^4)s^2+(t+u)^3(44t^4+8ut^3-9u^2t^2+21u^4)s+22(t+u)^4(t^2+ut+u^2) \\
 & + u^2))H(1,z)H(2,0,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4-4(t-u)s^3+6(2t^2+u^2)s^2+4u^3s+2t^4+2u^4+2tu^3+3t^2u^2 \\
 & + 2t^3u)H(0,0,z)H(2,0,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(4s^4+2(5t+2u)s^3+3(t^2+2u^2)s^2+2(7t^3+2u^3)s+4t^4+4u^4 \\
 & + 4tu^3+6t^2u^2+4t^3u)H(0,1,z)H(2,0,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(4s^4+6ts^3+9(t^2+2u^2)s^2+(6t^3+4u^3)s+4t^4 \\
 & + 4u^4+6tu^3+9t^2u^2+6t^3u)H(1,0,z)H(2,0,y)(s+u)^6+3s^2t^2(s+t)^6u^2(t+u)^2((363t^4+492ut^3+504u^2t^2+492u^3t+363u^4)s^4 \\
 & + 2(363t^5+479ut^4-58u^2t^3-58u^3t^2+479u^4t+363u^5)s^3+9(t+u)^2(121t^4-10ut^3-134u^2t^2-10u^3t+121u^4)s^2+6(t+u)^3(121t^4 \\
 & + 35ut^3+12u^2t^2+35u^3t+121u^4)s+363(t+u)^4(t^2+ut+u^2)s^2)H(2,2,y)(s+u)^6+36s^2t^2(s+t)^6u^2(t+u)^2(2(9t^4 \\
 & + 27u^3t^3+42u^2t^2+32u^3t+11u^4)s^4+(4t^5-11ut^4+9u^2t^3+95u^3t^2+115u^4t+44u^5)s^3+6(t+u)^2(3t^4+11u^2t^2 \\
 & + 19u^3t+11u^4)s^2+(t+u)^3(21t^4-9u^2t^2+8u^3t+44u^4)s+22(t+u)^4(t^2+ut+u^2)s^2)H(0,z)H(2,2,y)(s+u)^6 \\
 & + 108s^2t^2(s+t)^6u^2(t+u)^6(2s^4-2(t-2u)s^3-3(t^2-2u^2)s^2-2(t^3-2u^3)s+2(t^2+ut+u^2)s^2)H(0,0,z)H(2,2,y)(s+u)^6 \\
 & + 432s^2t^2(s+t)^6u^2(t+u)^6(s^4+3(t^2+u^2)s^2-(t^3+u^3)s+(t^2+ut+u^2)s^2)H(0,1,z)H(2,2,y)(s+u)^6 \\
 & - 108s^2t^2(s+t)^6u^2(t+u)^6(4s^4-2(t+2u)s^3+3(5t^2+8u^2)s^2+2(t^3-2u^3)s+4(t^2+ut+u^2)s^2)H(1,0,z)H(2,2,y)(s+u)^6 \\
 & + 324s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+u)s^3+3(t^2+u^2)s^2+2(t^3+u^3)s+2(t^2+ut+u^2)s^2)H(0,1,z)H(2,3,y)(s+u)^6
 \end{aligned}$$

$$\begin{aligned}
 & -108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+u)s^3+3(t^2+u^2)s^2+2(t^3+u^3)s+2(t^2+ut+u^2)^2)H(1,0,z)H(2,3,y)(s+u)^6 \\
 & -18s^2t^2(s+t)^6u^2(t+u)^3(2u(12t^2+15ut+7u^2)s^4-12(t^4-2u^2t^2-2u^3t-u^4)s^3-3(t+u)^2(5t^3-31ut^2-17u^2t \\
 & -5u^3)s^2-6(t+u)^3(t^3-ut^2-u^2t-u^3)s-(t+u)^3(3t^4-18ut^3-3u^2t^2+4u^3t+8u^4))H(1,z)H(3,0,y)(s+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+u)s^3+3(t^2+u^2)s^2+2(t^3+u^3)s+2(t^2+ut+u^2)^2)H(0,1,z)H(3,0,y)(s+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+10(t+u)s^3-3(t^2+u^2)s^2+6(t^3+u^3)s+2(t^2+ut+u^2)^2)H(1,0,z)H(3,0,y)(s+u)^6 \\
 & -216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2ts^3+9t^2s^2+2(t^2+ut+u^2)^2)H(1,1,z)H(3,0,y)(s+u)^6 \\
 & -18s^2t^2(s+t)^6u^2(t+u)^3(2t(7t^2+15ut+12u^2)s^4+12(t^4+2ut^3+2u^2t^2-u^4)s^3+3(t+u)^2(5t^3+17ut^2+31u^2t \\
 & -5u^3)s^2+6(t+u)^3(t^3+ut^2+u^2t-u^3)s-(t+u)^3(8t^4+4ut^3-3u^2t^2-18u^3t+3u^4))H(0,z)H(3,2,y)(s+u)^6 \\
 & -36s^2t^2(s+t)^6u^2(t+u)^2(2(20t^4+59ut^3+84u^2t^2+59u^3t+20u^4)s^4+8(6t^5+13ut^4+13u^2t^3+13u^3t^2+13u^4t+6u^5)s^3 \\
 & +6(t+u)^2(14t^4+19ut^3+22u^2t^2+19u^3t+14u^4)s^2+(t+u)^3(65t^4+8ut^3-18u^2t^2+8u^3t+65u^4)s+44(t+u)^4(t^2+ut \\
 & +u^2)^2)H(1,z)H(3,2,y)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2ts^3+9t^2s^2+2(t^2+ut+u^2)^2)H(0,1,z)H(3,2,y)(s+u)^6 \\
 & -216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4(t+2u)s^3-3(t^2-u^2)s^2+2(t^3+3u^3)s+2(t^2+ut+u^2)^2)H(1,0,z)H(3,2,y)(s+u)^6 \\
 & +18s^2t^2(s+t)^6u^2(t+u)^2((58t^4+340ut^3+540u^2t^2+340u^3t+58u^4)s^4+2(21t^5+265ut^4+658u^2t^3+658u^3t^2+265u^4t \\
 & +21u^5)s^3+6(t+u)^2(5t^4+86ut^3+138u^2t^2+86u^3t+5u^4)s^2+2(t+u)^3(t^4+97ut^3+96u^2t^2+97u^3t+u^4)s+3(t+u)^4(t^4 \\
 & +30ut^3+36u^2t^2+30u^3t+u^4))H(1,z)H(3,3,y)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6(4s^4+(6t+4u)s^3+3(t^2+2u^2)s^2+4(2t^3 \\
 & +u^3)s+4(t^2+ut+u^2)^2)H(0,1,z)H(3,3,y)(s+u)^6-216s^3t^2(s+t)^6u^3(t+u)^6(2s^2-3us+4u^2)H(1,0,z)H(3,3,y)(s+u)^6 \\
 & +216s^2t^2(s+t)^6u^2(t+u)^6(4s^4+2(t+u)s^3+9(t^2+u^2)s^2+4(t^2+ut+u^2)^2)H(1,1,z)H(3,3,y)(s+u)^6 \\
 & +216s^2t^2(s+t)^6u^2(t+u)^6(s^4+2(t+u)s^3+3(t^2+u^2)s^2+2(t^3+u^3)s+t^4+u^4+6t^2u^2-2t^3u)H(0,y)H(0,0,1,z)(s+u)^6 \\
 & +216s^2t^2(s+t)^6u^3(t+u)^6(2s^3-3us^2+4u^2s+6t(t^2-ut+u^2))H(1,y)H(0,0,1,z)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(6s^4 \\
 & +2(4t+u)s^3+15u^2s^2+2(6t^3-u^3)s+6(t^2+ut+u^2)^2)H(2,y)H(0,0,1,z)(s+u)^6+432s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+u)s^3 \\
 & +3(t^2+u^2)s^2+2(t^3+u^3)s+2(t^2+ut+u^2)^2)H(3,y)H(0,0,1,z)(s+u)^6-36s^2t^2(s+t)^2u^2(t+u)^6(22s^8+3(44t+7u)s^7 \\
 & +(374t^2+63ut+18u^2)s^6+(660t^3+54ut^2+36u^2t+4u^3)s^5+(792t^4+2ut^3+84u^2t^2-11u^3t+18u^4)s^4+t(660t^4+41ut^3 \\
 & +246u^2t^2+9u^3t+54u^4)s^3+t^2(374t^4+147ut^3+360u^2t^2+95u^3t+84u^4)s^2+t^3(132t^4+140ut^3+246u^2t^2+115u^3t+64u^4)s \\
 & +22t^4(t^2+ut+u^2)^2)H(0,0,2,y)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6(s^4+2ts^3+3(t^2+2u^2)s^2+2(t^3-u^3)s+t^4+u^4 \\
 & -2tu^3+6t^2u^2)H(0,z)H(0,0,2,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+(4t-2u)s^3+(6t^2-3u^2)s^2+(4t^3-2u^3)s \\
 & +2(t^2+ut+u^2)^2)H(1,z)H(0,0,2,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(4s^4+2(4t+u)s^3+3(4t^2+5u^2)s^2+(8t^3-2u^3)s \\
 & +4(t^4-u^3t^3+6u^2t^2-u^3t+u^4))H(1,z)H(0,0,3,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(2t+u)s^3+3(2t^2+u^2)s^2 \\
 & +2(2t^3+u^3)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u)H(0,z)H(0,1,0,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(2t+u)s^3 \\
 & +3(2t^2+u^2)s^2+2(2t^3+u^3)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u)H(1,z)H(0,1,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4 \\
 & +(4t-6u)s^3+(6t^2+9u^2)s^2+(4t^3-2u^3)s+2(t^4+u^4))H(0,y)H(0,1,0,z)(s+u)^6-216s^2t^2(s+t)^6u^3(t+u)^6(6s^3 \\
 & -6us^2+6u^2s+t(2t^2-3ut+4u^2))H(1,y)H(0,1,0,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4-2(2t+5u)s^3+3(4t^2 \\
 & +5u^2)s^2-10u^3s+2(t^2+ut+u^2)^2)H(2,y)H(0,1,0,z)(s+u)^6+216s^3t^2(s+t)^6u^2(t+u)^6(4(t+u)s^2-3(t^2+u^2)s+2(t^3 \\
 & +u^3))H(3,y)H(0,1,0,z)(s+u)^6-36s^2t^2(s+t)^6u^2(t+u)^2(2(11t^4+32ut^3+42u^2t^2+27u^3t+9u^4)s^4+(44t^5+115ut^4+95u^2t^3 \\
 & +9u^3t^2-11u^4t+4u^5)s^3+6(t+u)^2(11t^4+19ut^3+11u^2t^2+3u^4)s^2+(t+u)^3(44t^4+8ut^3-9u^2t^2+21u^4)s+22(t+u)^4(t^2 \\
 & +ut+u^2)^2)H(0,1,1,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+(4t-2u)s^3+(6t^2-3u^2)s^2+(4t^3-2u^3)s+2(t^2+ut \\
 & +u^2)^2)H(0,y)H(0,1,1,z)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2ts^3+9t^2s^2+2(t^2+ut+u^2)^2)H(3,y)H(0,1,1,z)(s+u)^6 \\
 & -36s^2t^2(s+t)^2u^2(t+u)^6(22s^8+3(44t+7u)s^7+(374t^2+63ut+18u^2)s^6+(660t^3+54ut^2+36u^2t+4u^3)s^5+(792t^4+2ut^3 \\
 & +84u^2t^2-11u^3t+18u^4)s^4+t(660t^4+41ut^3+246u^2t^2+9u^3t+54u^4)s^3+t^2(374t^4+147ut^3+360u^2t^2+95u^3t+84u^4)s^2 \\
 & +t^3(132t^4+140ut^3+246u^2t^2+115u^3t+64u^4)s+22t^4(t^2+ut+u^2)^2)H(0,2,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4 \\
 & +12us^3+8u^3s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u)H(0,z)H(0,2,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+(4t-2u)s^3 \\
 & +(6t^2-3u^2)s^2+(4t^3-2u^3)s+2(t^2+ut+u^2)^2)H(1,z)H(0,2,0,y)(s+u)^6+36s^2t^2(s+t)^6u^2(t+u)^2(2(11t^4 \\
 & +32ut^3+42u^2t^2+27u^3t+9u^4)s^4+(44t^5+115ut^4+95u^2t^3+9u^3t^2-11u^4t+4u^5)s^3+6(t+u)^2(11t^4+19ut^3 \\
 & +11u^2t^2+3u^4)s^2+(t+u)^3(44t^4+8ut^3-9u^2t^2+21u^4)s+22(t+u)^4(t^2+ut+u^2)^2)H(0,2,2,y)(s+u)^6 \\
 & +108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2ts^3+3(t^2+4u^2)s^2+2(t^3-2u^3)s+2(t^2+ut+u^2)^2)H(0,z)H(0,2,2,y)(s+u)^6
 \end{aligned}$$

$$\begin{aligned}
 & -216s^2t^2(s+t)^6u^2(t+u)^6(s^4+2(t-u)s^3+3(t^2+u^2)s^2+2(t^3-2u^3)s+t^4+u^4-4tu^3+9t^2u^2-4t^3u)H(1,z)H(0,2,3,y)(s+u)^6 \\
 & -216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+u)s^3+3(t^2+u^2)s^2+2(t^3+u^3)s+2(t^2+u^2+u^2)^2)H(1,z)H(0,3,0,y)(s+u)^6 \\
 & +108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+3u)s^3+3(t^2-u^2)s^2+2(t^3+5u^3)s+2(t^2+ut+u^2)^2)H(0,z)H(0,3,2,y)(s+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(t+u)^6(4s^4+(6t-2u)s^3+9(t^2+u^2)s^2+6(t^3-u^3)s+4(t^2+u^2+u^2)^2)H(1,z)H(0,3,2,y)(s+u)^6 \\
 & -108s^3t^2(s+t)^6u^2(t+u)^6(2(t+u)s^2+3(t^2-3u^2)s+2(t^3+3u^3))H(1,z)H(0,3,3,y)(s+u)^6+36s^2t^2(s+t)^2u^2(t+u)^6(44s^8 \\
 & +(264t+65u)s^7+(748t^2+203ut+84u^2)s^6+3(440t^3+67ut^2+94u^2t+16u^3)s^5+(1584t^4+43u^3t^3+444u^2t^2+104u^3t \\
 & +40u^4)s^4+t(1320t^4+43ut^3+492u^2t^2+104u^3t+118u^4)s^3+t^2(748t^4+201ut^3+444u^2t^2+104u^3t+168u^4)s^2 \\
 & +t^3(264t^4+203ut^3+282u^2t^2+104u^3t+118u^4)s+t^4(44t^4+65ut^3+84u^2t^2+48u^3t+40u^4))H(1,0,0,y)(s+u)^6 \\
 & +216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4ts^3+(6t^2+9u^2)s^2+2(2t^3+u^3)s+2(t^4+u^4))H(0,z)H(1,0,0,y)(s+u)^6 \\
 & -216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4ts^3+6t^2s^2+4t^3s+2t^4+2u^4+2tu^3+9t^2u^2)H(1,z)H(1,0,0,y)(s+u)^6 \\
 & +108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4(t+u)s^3+6(t^2+u^2)s^2+4(t^3+u^3)s+2t^4+2u^4-2tu^3-3t^2u^2 \\
 & -2t^3u)H(0,y)H(1,0,0,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4-4(t-u)s^3+6(2t^2+u^2)s^2+4u^3s+2t^4+2u^4 \\
 & +2tu^3+3t^2u^2+2t^3u)H(2,y)H(1,0,0,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4ts^3+6t^2s^2+4t^3s+2t^4+2u^4 \\
 & -2tu^3+9t^2u^2-6t^3u)H(0,y)H(1,0,1,z)(s+u)^6+216s^2t^2(s+t)^6u^3(t+u)^6(2s^3-3us^2+4u^2s+6t(t^2-u^2 \\
 & +u^2))H(1,y)H(1,0,1,z)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6(2(t+2u)s^3-3t^2s^2+(4t^3+6u^3)s-tu(2t^2+3ut \\
 & +2u^2))H(2,y)H(1,0,1,z)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2ts^3+9t^2s^2+2(t^2+u^2+u^2)^2)H(3,y)H(1,0,1,z)(s+u)^6 \\
 & -18s^2t^2(s+t)^3u^2(t+u)^6(8s^7+(28t-6u)s^6+3(11t^2-8ut-5u^2)s^5-(7t^3+42ut^2+81u^2t+12u^3)s^4-2(28t^4+18u^3t^3 \\
 & +105u^2t^2+12u^3t+7u^4)s^3-6t(8t^4+ut^3+37u^2t^2+4u^3t+5u^4)s^2-3(3t^6-4ut^5+21u^2t^4+8u^4t^2)s+3t^4(t^3+2ut^2 \\
 & +5u^2t+4u^3))H(1,0,2,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+(4t+6u)s^3+(6t^2-3u^2)s^2+2(2t^3+5u^3)s+2t^4 \\
 & +2u^4+10tu^3-3t^2u^2+6t^3u)H(0,z)H(1,0,2,y)(s+u)^6-216s^2t^2(s+t)^6u^3(t+u)^6(2s^3-3us^2+4u^2s+6t(t^2-ut \\
 & +u^2))H(1,z)H(1,0,3,y)(s+u)^6-18s^2t^2(s+t)^2u^2(t+u)^6(3s^8+2(51t+u)s^7+(486t^2+200ut+30u^2)s^6+6(179t^3+130ut^2 \\
 & +96u^2t+7u^3)s^5+2(687t^4+677ut^3+945u^2t^2+265u^3t+29u^4)s^4+2t(537t^4+677ut^3+1344u^2t^2+658u^3t+170u^4)s^3 \\
 & +2t^2(243t^4+390ut^3+945u^2t^2+658u^3t+270u^4)s^2+2t^3(51t^4+100ut^3+288u^2t^2+265u^3t+170u^4)s+t^4(3t^4+2ut^3 \\
 & +30u^2t^2+42u^3t+58u^4))H(1,1,0,y)(s+u)^6+216s^3t^2(s+t)^6u^3(t+u)^6(4s^2-3us+2u^2)H(0,z)H(1,1,0,y)(s+u)^6 \\
 & -216s^2t^3(s+t)^6u^3(t+u)^6(4t^2-3ut+2u^2)H(1,z)H(1,1,0,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6(s^4+2(t-u)s^3+3(t^2 \\
 & +2u^2)s^2+2t^3s+(t^2+ut+u^2)^2)H(0,y)H(1,1,0,z)(s+u)^6-216s^2t^2(s+t)^6u^3(t+u)^6(6s^3-6us^2+6u^2s+t(2t^2-3ut \\
 & +4u^2))H(1,y)H(1,1,0,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(8s^4+4(2t+u)s^3+12(t^2+3u^2)s^2+4(2t^3+u^3)s+8t^4+8u^4 \\
 & +14t^3u^3+21t^2u^2+14t^3u)H(2,y)H(1,1,0,z)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4(t+2u)s^3-3(t^2-u^2)s^2+2(t^3 \\
 & +3u^3)s+2(t^2+ut+u^2)^2)H(3,y)H(1,1,0,z)(s+u)^6-18s^2t^2(s+t)^3u^2(t+u)^6(8s^7+(28t-6u)s^6+3(11t^2-8ut-5u^2)s^5 \\
 & -(7t^3+42ut^2+81u^2t+12u^3)s^4-2(28t^4+18ut^3+105u^2t^2+12u^3t+7u^4)s^3-6t(8t^4+ut^3+37u^2t^2+4u^3t+5u^4)s^2-3(3t^6 \\
 & -4ut^5+21u^2t^4+8u^4t^2)s+3t^4(t^3+2ut^2+5u^2t+4u^3))H(1,2,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+(4t-6u)s^3 \\
 & +(6t^2+9u^2)s^2+(4t^3-2u^3)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u)H(0,z)H(1,2,0,y)(s+u)^6+216s^2t^2(s+t)^6u^3(t+u)^6(2s^3 \\
 & -3us^2+4u^2s+6t(t^2-ut+u^2))H(1,z)H(1,2,3,y)(s+u)^6-36s^2t^2(s+t)^2u^2(t+u)^6(22s^8+3(44t+7u)s^7+(374t^2+63ut \\
 & +18u^2)s^6+(660t^3+54ut^2+36u^2t+4u^3)s^5+(792t^4+2u^3t^3+84u^2t^2-11u^3t+18u^4)s^4+t(660t^4+41ut^3+246u^2t^2 \\
 & +9u^3t+54u^4)s^3+t^2(374t^4+147ut^3+360u^2t^2+95u^3t+84u^4)s^2+t^3(132t^4+140ut^3+246u^2t^2+115u^3t+64u^4)s \\
 & +22t^4(t^2+ut+u^2)^2)H(2,0,0,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+4(t-u)s^3+6(t^2+2u^2)s^2+4t^3s+2t^4 \\
 & +2u^4+2tu^3+3t^2u^2+2t^3u)H(0,z)H(2,0,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+(4t-2u)s^3+(6t^2-3u^2)s^2 \\
 & +(4t^3-2u^3)s+2(t^2+ut+u^2)^2)H(1,z)H(2,0,0,y)(s+u)^6+36s^2t^2(s+t)^6u^2(t+u)^2(2(11t^4+32ut^3+42u^2t^2+27u^3t \\
 & +9u^4)s^4+(44t^5+115ut^4+95u^2t^3+9u^3t^2-11u^4t+4u^5)s^3+6(t+u)^2(11t^4+19ut^3+11u^2t^2+3u^4)s^2+(t+u)^3(44t^4 \\
 & +8ut^3-9u^2t^2+21u^4)s+22(t+u)^4(t^2+ut+u^2)^2)H(2,0,2,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(t+4u)s^3 \\
 & +3t^2s^2+2(t^3+6u^3)s+2(t^4+u^4))H(0,z)H(2,0,2,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6(s^4+(4t-2u)s^3+3u^2s^2 \\
 & +(6t^3-4u^3)s+(t^2+ut+u^2)^2)H(1,z)H(2,0,3,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6(2s^4+2(2t+5u)s^3+(6t^2 \\
 & -3u^2)s^2+(4t^3+6u^3)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u)H(0,z)H(2,1,0,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6(2s^4 \\
 & +2(2t+u)s^3+3(2t^2+u^2)s^2+2(2t^3+u^3)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u)H(1,z)H(2,1,0,y)(s+u)^6
 \end{aligned}$$

$$\begin{aligned}
 & + 36s^2t^2(s+t)^6u^2(t+u)^2\left(2\left(11t^4+32ut^3+42u^2t^2+27u^3t+9u^4\right)s^4+\left(44t^5+115ut^4+95u^2t^3+9u^3t^2-11u^4t\right.\right. \\
 & \left.+4u^5\right)s^3+6(t+u)^2\left(11t^4+19ut^3+11u^2t^2+3u^4\right)s^2+(t+u)^3\left(44t^4+8ut^3-9u^2t^2+21u^4\right)s+22(t+u)^4\left(t^2+ut\right. \\
 & \left.+u^2\right)^2\Big)H(2,2,0,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(s^4-2(t+u)s^3+6\left(t^2+u^2\right)s^2+\left(t^2+ut+u^2\right)^2\right)H(0,z)H(2,2,0,y)(s+u)^6 \\
 & +432s^2t^2(s+t)^6u^2(t+u)^6\left(s^4+3\left(t^2+u^2\right)s^2-\left(t^3+u^3\right)s+\left(t^2+ut+u^2\right)^2\right)H(1,z)H(2,2,3,y)(s+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(t+u)s^3+3\left(t^2+u^2\right)s^2+2\left(t^3+u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(1,z)H(2,3,0,y)(s+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(t+u)s^3+3\left(t^2+u^2\right)s^2+2\left(t^3+u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(0,z)H(2,3,2,y)(s+u)^6 \\
 & +432s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(t+u)s^3+3\left(t^2+u^2\right)s^2+2\left(t^3+u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(1,z)H(2,3,3,y)(s+u)^6 \\
 & -18s^2t^2(s+t)^6u^2(t+u)^3\left(2u\left(12t^2+15ut+7u^2\right)s^4-12\left(t^4-2u^2t^2-2u^3t-u^4\right)s^3-3(t+u)^2\left(5t^3-31ut^2-17u^2t\right.\right. \\
 & \left.-5u^3\right)s^2-6(t+u)^3\left(t^3-ut^2-u^2t-u^3\right)s-(t+u)^3\left(3t^4-18ut^3-3u^2t^2+4u^3t+8u^4\right)\Big)H(3,0,2,y)(s+u)^6 \\
 & -108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(t-u)s^3+3\left(t^2+3u^2\right)s^2+2\left(t^3-3u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(0,z)H(3,0,2,y)(s+u)^6 \\
 & -216s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2ts^3+9t^2s^2+2\left(t^2+ut+u^2\right)^2\right)H(1,z)H(3,0,2,y)(s+u)^6-216s^3t^2(s+t)^6u^3(t+u)^6\left(2s^2\right. \\
 & \left.-3us+4u^2\right)H(1,z)H(3,0,3,y)(s+u)^6-18s^2t^2(s+t)^6u^2(t+u)^3\left(2u\left(12t^2+15ut+7u^2\right)s^4-12\left(t^4-2u^2t^2-2u^3t\right.\right. \\
 & \left.-u^4\right)s^3-3(t+u)^2\left(5t^3-31ut^2-17u^2t-5u^3\right)s^2-6(t+u)^3\left(t^3-ut^2-u^2t-u^3\right)s-(t+u)^3\left(3t^4-18ut^3-3u^2t^2\right. \\
 & \left.+4u^3t+8u^4\right)\Big)H(3,2,0,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+10(t+u)s^3-3\left(t^2+u^2\right)s^2+6\left(t^3+u^3\right)s+2\left(t^2+ut\right.\right. \\
 & \left.+u^2\right)^2\Big)H(0,z)H(3,2,0,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2ts^3+9t^2s^2+2\left(t^2+ut+u^2\right)^2\right)H(1,z)H(3,2,0,y)(s+u)^6 \\
 & -36s^2t^2(s+t)^6u^2(t+u)^2\left(2\left(20t^4+59ut^3+84u^2t^2+59u^3t+20u^4\right)s^4+8\left(6t^5+13ut^4+13u^2t^3+13u^3t^2+13u^4t+6u^5\right)s^3\right. \\
 & \left.+6(t+u)^2\left(14t^4+19ut^3+22u^2t^2+19u^3t+14u^4\right)s^2+(t+u)^3\left(65t^4+8ut^3-18u^2t^2+8u^3t+65u^4\right)s+44(t+u)^4\left(t^2+ut\right.\right. \\
 & \left.+u^2\right)^2\Big)H(3,2,2,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2us^3+9u^2s^2+2\left(t^2+ut+u^2\right)^2\right)H(0,z)H(3,2,2,y)(s+u)^6 \\
 & +216s^2t^2(s+t)^6u^2(t+u)^6\left(4s^4+2(t+u)s^3+9\left(t^2+u^2\right)s^2+4\left(t^2+ut+u^2\right)^2\right)H(1,z)H(3,2,3,y)(s+u)^6-216s^3t^3(s+t)^6\left(2s^2\right. \\
 & \left.-3ts+4t^2\right)u^2(t+u)^6H(1,z)H(3,3,0,y)(s+u)^6+18s^2t^2(s+t)^6u^2(t+u)^2\left(\left(58t^4+340ut^3+540u^2t^2+340u^3t+58u^4\right)s^4+2\left(21t^5\right.\right. \\
 & \left.+265ut^4+658u^2t^3+658u^3t^2+265u^4t+21u^5\right)s^3+6(t+u)^2\left(5t^4+86ut^3+138u^2t^2+86u^3t+5u^4\right)s^2+2(t+u)^3\left(t^4+97ut^3\right. \\
 & \left.+96u^2t^2+97u^3t+u^4\right)s+3(t+u)^4\left(t^4+30ut^3+36u^2t^2+30u^3t+u^4\right)\Big)H(3,3,2,y)(s+u)^6-216s^3t^2(s+t)^6u^3(t+u)^6\left(2s^2\right. \\
 & \left.-3us+4u^2\right)H(0,z)H(3,3,2,y)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6\left(4s^4+2(t+u)s^3+9\left(t^2+u^2\right)s^2+4\left(t^2+ut\right.\right. \\
 & \left.+u^2\right)^2\Big)H(1,z)H(3,3,2,y)(s+u)^6+216s^2t^2(s+t)^6u^2(t+u)^6\left(4s^4+6(t+u)s^3+3\left(t^2+u^2\right)s^2+8\left(t^3+u^3\right)s+4\left(t^2+ut\right.\right. \\
 & \left.+u^2\right)^2\Big)H(1,z)H(3,3,3,y)(s+u)^6-324s^2t^2(s+t)^6u^3(t+u)^6\left(2s^3+3us^2+2u^2s-4t\left(t^2-ut+u^2\right)\right)H(0,0,0,1,z)(s+u)^6 \\
 & +432s^2t^2(s+t)^6u^2(t+u)^6\left(s^4+(2t-u)s^3+3\left(t^2+u^2\right)s^2+2t^3s+t^4+u^4+3t^2u^2-t^3u\right)H(0,0,1,0,y)(s+u)^6 \\
 & +432s^2t^2(s+t)^6u^3(t+u)^6\left(3s^3+2u^2s-2t^3-3tu^2\right)H(0,0,1,0,z)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+(4t-2u)s^3+\left(6t^2\right.\right. \\
 & \left.-3u^2\right)s^2+\left(4t^3-2u^3\right)s+2\left(t^2+ut+u^2\right)^2\Big)H(0,0,1,1,z)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+(4t-2u)s^3+\left(6t^2-3u^2\right)s^2\right. \\
 & \left.+4\left(t^3-2u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(0,0,2,2,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6\left(4s^4+2(4t+u)s^3+3\left(4t^2+5u^2\right)s^2+\left(8t^3\right.\right. \\
 & \left.-2u^3\right)s+4\left(t^4-ut^3+6u^2t^2-u^3t+u^4\right)\Big)H(0,0,3,2,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+4(t-u)s^3+6\left(t^2-u^2\right)s^2+4\left(t^3\right.\right. \\
 & \left.-u^3\right)s+2t^4+2u^4+18t^3u-3t^2u^2+18t^3u\Big)H(0,1,0,1,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(2t+u)s^3+3\left(2t^2+u^2\right)s^2\right. \\
 & \left.+2\left(t^3+u^3\right)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u\right)H(0,1,0,2,y)(s+u)^6+432s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(2t+u)s^3+3\left(2t^2\right.\right. \\
 & \left.+u^2\right)s^2+2\left(2t^3+u^3\right)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u\Big)H(0,1,1,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+4(t-u)s^3\right. \\
 & \left.+6\left(t^2+2u^2\right)s^2+4t^3s+2t^4+2u^4-10tu^3+3t^2u^2-6t^3u\right)H(0,1,1,0,z)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(2t+u)s^3\right. \\
 & \left.+3\left(2t^2+u^2\right)s^2+2\left(2t^3+u^3\right)s+2t^4+2u^4+2tu^3+3t^2u^2+2t^3u\right)H(0,1,2,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4\right. \\
 & \left.+4(t-2u)s^3+\left(6t^2-3u^2\right)s^2+\left(4t^3-2u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(0,2,0,2,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(s^4\right. \\
 & \left.+2(t-2u)s^3+3\left(t^2+u^2\right)s^2+2\left(t^3-u^3\right)s+t^4+u^4+4tu^3+6t^3u\right)H(0,2,1,0,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4\right. \\
 & \left.+4(t-2u)s^3+\left(6t^2-3u^2\right)s^2+\left(4t^3-2u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(0,2,2,0,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(s^4\right. \\
 & \left.+2(t-u)s^3+3\left(t^2+u^2\right)s^2+2\left(t^3-2u^3\right)s+t^4+u^4-4tu^3+9t^2u^2-4t^3u\right)H(0,2,3,2,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4\right. \\
 & \left.+2(t+u)s^3+3\left(t^2+u^2\right)s^2+2\left(t^3+u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(0,3,0,2,y)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+2(t+u)s^3\right. \\
 & \left.+3\left(t^2+u^2\right)s^2+2\left(t^3+u^3\right)s+2\left(t^2+ut+u^2\right)^2\right)H(0,3,2,0,y)(s+u)^6-108s^2t^2(s+t)^6u^2(t+u)^6\left(4s^4+\left(6t-2u\right)s^3+9\left(t^2\right.\right. \\
 & \left.+u^2\right)s^2+6\left(t^3-u^3\right)s+4\left(t^2+ut+u^2\right)^2\Big)H(0,3,2,2,y)(s+u)^6-108s^3t^2(s+t)^6u^2(t+u)^6\left(2(t+u)s^2+3\left(t^2-3u^2\right)s+2\left(t^3\right.\right. \\
 & \left.+3u^3\right)\Big)H(0,3,3,2,y)(s+u)^6+108s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+\left(4t-6u\right)s^3+3\left(2t^2+u^2\right)s^2+2\left(2t^3-5u^3\right)s+2\left(t^4-2ut^3+6u^2t^2\right.\right. \\
 & \left.+u^4\right)\Big)H(1,0,0,1,z)(s+u)^6-216s^2t^2(s+t)^6u^2(t+u)^6\left(2s^4+4ts^3+6t^2s^2+4t^3s+2t^4+2u^4+2tu^3+9t^2u^2\right)H(1,0,0,2,y)(s+u)^6 \\
 & +216s^2t^2(s+t)^6u^2(t+u)^6\left(4s^4+8ts^3+3\left(4t^2+3u^2\right)s^2+2\left(4t^3+u^3\right)s+4t^4+4u^4+2tu^3+9t^2u^2\right)H(1,0,1,0,y)(s+u)^6
 \end{aligned}$$

$$\begin{aligned}
 & + 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2(2t+9u)s^3 + (6t^2-3u^2)s^2 + 2(2t^3+9u^3)s + 2(t^4-2ut^3-3u^2t^2-2u^3t \right. \\
 & \left. + u^4) \right) H(1,0,1,0,z)(s+u)^6 - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 4ts^3 + 6t^2s^2 + 4t^3s + 2t^4 + 2u^4 + 2tu^3 + 9t^2u^2 \right) H(1,0,2,0,y)(s+u)^6 \\
 & - 216s^2t^2(s+t)^6 u^3(t+u)^6 \left( 2s^3 - 3us^2 + 4u^2s + 6t(t^2-ut+u^2) \right) H(1,0,3,2,y)(s+u)^6 + 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 4s^4 + 8ts^3 \right. \\
 & \left. + 3(4t^2+3u^2)s^2 + 2(4t^3+u^3)s + 4t^4 + 4u^4 + 2tu^3 + 9t^2u^2 \right) H(1,1,0,0,y)(s+u)^6 + 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 4(t+u)s^3 \right. \\
 & \left. + 6(t^2+u^2)s^2 + 4(t^3+u^3)s + 2t^4 + 2u^4 - 2tu^3 - 3t^2u^2 - 2t^3u \right) H(1,1,0,0,z)(s+u)^6 - 432s^2t^2(s+t)^6 u^3(t+u)^6 \left( 2s^3 \right. \\
 & \left. + 3u^2s - 3t^3 - 2tu^2 \right) H(1,1,0,1,z)(s+u)^6 - 216s^2t^3(s+t)^6 u^3(t+u)^6 \left( 4t^2 - 3ut + 2u^2 \right) H(1,1,0,2,y)(s+u)^6 \\
 & + 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 4s^4 + 8(t+u)s^3 + 3(4t^2+u^2)s^2 + (8t^3+6u^3)s + 4t^4 + 4u^4 + 6tu^3 + 3t^2u^2 + 8t^3u \right) H(1,1,1,0,y)(s+u)^6 \\
 & + 324s^2t^2(s+t)^6 u^3(t+u)^6 \left( 4s^3 - 4us^2 + 4u^2s - t(2t^2+3ut+2u^2) \right) H(1,1,1,0,z)(s+u)^6 - 216s^2t^3(s+t)^6 u^3(t+u)^6 \left( 4t^2 - 3ut \right. \\
 & \left. + 2u^2 \right) H(1,1,2,0,y)(s+u)^6 - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 4ts^3 + 6t^2s^2 + 4t^3s + 2t^4 + 2u^4 + 2tu^3 + 9t^2u^2 \right) H(1,2,0,0,y)(s+u)^6 \\
 & - 216s^3t^2(s+t)^6 u^3(t+u)^6 \left( 4s^2 - 3us + 2u^2 \right) H(1,2,1,0,y)(s+u)^6 + 216s^2t^2(s+t)^6 u^3(t+u)^6 \left( 2s^3 - 3us^2 + 4u^2s + 6t(t^2 \right. \\
 & \left. - ut + u^2) \right) H(1,2,3,2,y)(s+u)^6 + 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + (4t-2u)s^3 + (6t^2-3u^2)s^2 + (4t^3-2u^3)s + 2(t^2 \right. \\
 & \left. + ut + u^2)^2 \right) H(2,0,0,2,y)(s+u)^6 - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( s^4 - 4(t+u)s^3 + 3(3t^2+u^2)s^2 - 2(2t^3+u^3)s + (t^2 \right. \\
 & \left. + ut + u^2)^2 \right) H(2,0,1,0,y)(s+u)^6 + 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + (4t-2u)s^3 + (6t^2-3u^2)s^2 + (4t^3-2u^3)s + 2(t^2 \right. \\
 & \left. + ut + u^2)^2 \right) H(2,0,2,0,y)(s+u)^6 - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( s^4 + (4t-2u)s^3 + 3u^2s^2 + (6t^3-4u^3)s + (t^2 + ut \right. \\
 & \left. + u^2)^2 \right) H(2,0,3,2,y)(s+u)^6 - 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 4s^4 + (8t-6u)s^3 + 3(4t^2+3u^2)s^2 + (8t^3-2u^3)s + 4t^4 + 4u^4 + 6tu^3 \right. \\
 & \left. + 9t^2u^2 + 6t^3u \right) H(2,1,0,0,y)(s+u)^6 - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2(2t+u)s^3 + 3(2t^2+u^2)s^2 + 2(2t^3+u^3)s + 2t^4 + 2u^4 + 2tu^3 \right. \\
 & \left. + 3t^2u^2 + 2t^3u \right) H(2,1,0,2,y)(s+u)^6 - 108s^2t^2(s+t)^6 u^3(t+u)^6 \left( 6s^3 - 9us^2 + 2u^2s + t(2t^2+3ut+2u^2) \right) H(2,1,1,0,y)(s+u)^6 \\
 & - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2(2t+u)s^3 + 3(2t^2+u^2)s^2 + 2(2t^3+u^3)s + 2t^4 + 2u^4 + 2tu^3 + 3t^2u^2 + 2t^3u \right) H(2,1,2,0,y)(s+u)^6 \\
 & + 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + (4t-2u)s^3 + (6t^2-3u^2)s^2 + (4t^3-2u^3)s + 2(t^2+ut+u^2)^2 \right) H(2,2,0,0,y)(s+u)^6 \\
 & - 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 4s^4 - 2(2t+u)s^3 + 3(8t^2+5u^2)s^2 + (2u^3-4t^3)s + 4(t^2+ut+u^2)^2 \right) H(2,2,1,0,y)(s+u)^6 \\
 & + 432s^2t^2(s+t)^6 u^2(t+u)^6 \left( s^4 + 3(t^2+u^2)s^2 - (t^3+u^3)s + (t^2+ut+u^2)^2 \right) H(2,2,3,2,y)(s+u)^6 - 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 \right. \\
 & \left. + 2(t+u)s^3 + 3(t^2+u^2)s^2 + 2(t^3+u^3)s + 2(t^2+ut+u^2)^2 \right) H(2,3,0,2,y)(s+u)^6 - 108s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2(t+u)s^3 \right. \\
 & \left. + 3(t^2+u^2)s^2 + 2(t^3+u^3)s + 2(t^2+ut+u^2)^2 \right) H(2,3,2,0,y)(s+u)^6 + 432s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2(t+u)s^3 + 3(t^2 \right. \\
 & \left. + u^2)s^2 + 2(t^3+u^3)s + 2(t^2+ut+u^2)^2 \right) H(2,3,3,2,y)(s+u)^6 + 216s^3t^2(s+t)^6 u^2(t+u)^6 \left( (6t+4u)s^2 - 3(2t^2+u^2)s \right. \\
 & \left. + 2(3t^3+u^3) \right) H(3,0,1,0,y)(s+u)^6 - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2ts^3 + 9t^2s^2 + 2(t^2+ut+u^2)^2 \right) H(3,0,2,2,y)(s+u)^6 \\
 & - 216s^3t^2(s+t)^6 u^3(t+u)^6 \left( 2s^2 - 3us + 4u^2 \right) H(3,0,3,2,y)(s+u)^6 - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2ts^3 + 9t^2s^2 + 2(t^2+ut \right. \\
 & \left. + u^2)^2 \right) H(3,2,0,2,y)(s+u)^6 - 216s^3t^2(s+t)^6 u^2(t+u)^6 \left( (6t+4u)s^2 - 3(2t^2+u^2)s + 2(3t^3+u^3) \right) H(3,2,1,0,y)(s+u)^6 \\
 & - 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 2s^4 + 2ts^3 + 9t^2s^2 + 2(t^2+ut+u^2)^2 \right) H(3,2,2,0,y)(s+u)^6 + 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 4s^4 + 2(t+u)s^3 \right. \\
 & \left. + 9(t^2+u^2)s^2 + 4(t^2+ut+u^2)^2 \right) H(3,2,3,2,y)(s+u)^6 - 216s^3t^3(s+t)^6 \left( 2s^2 - 3ts + 4t^2 \right) u^2(t+u)^6 H(3,3,0,2,y)(s+u)^6 \\
 & - 216s^3t^3(s+t)^6 \left( 2s^2 - 3ts + 4t^2 \right) u^2(t+u)^6 H(3,3,2,0,y)(s+u)^6 + 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 4s^4 + 2(t+u)s^3 + 9(t^2+u^2)s^2 \right. \\
 & \left. + 4(t^2+ut+u^2)^2 \right) H(3,3,2,2,y)(s+u)^6 + 216s^2t^2(s+t)^6 u^2(t+u)^6 \left( 4s^4 + 6(t+u)s^3 + 3(t^2+u^2)s^2 + 8(t^3+u^3)s + 4(t^2 \right. \\
 & \left. + ut + u^2)^2 \right) H(3,3,3,2,y)(s+u)^6 - s^2t^2(s+t)^3 u^2(t+u)^3 \left( 7948(t+u)^3 s^{10} + (39359t^4 + 155744ut^3 + 236082u^2t^2 + 155744u^3t \right. \\
 & \left. + 39359u^4) s^9 + (93651t^5 + 434331ut^4 + 850587u^2t^3 + 850587u^3t^2 + 434331u^4t + 93651u^5) s^8 + 3(46598t^6 + 240304ut^5 + 550627u^2t^4 \right. \\
 & \left. + 717154u^3t^3 + 550627u^4t^2 + 240304u^5t + 46598u^6) s^7 + (139794t^7 + 836962ut^6 + 2146599u^2t^5 + 3271156u^3t^4 + 3271156u^4t^3 \right. \\
 & \left. + 2146599u^5t^2 + 836962u^6t + 139794u^7) s^6 + 3(31217t^8 + 240304ut^7 + 715533u^2t^6 + 1216094u^3t^5 + 1422608u^4t^4 + 1216094u^5t^3 \right. \\
 & \left. + 715533u^6t^2 + 240304u^7t + 31217u^8) s^5 + (39359t^9 + 434331ut^8 + 1651881u^2t^7 + 3271156u^3t^6 + 4267824u^4t^5 + 4267824u^5t^4 \right. \\
 & \left. + 3271156u^6t^3 + 1651881u^7t^2 + 434331u^8t + 39359u^9) s^4 + (7948t^{10} + 155744ut^9 + 850587u^2t^8 + 2151462u^3t^7 + 3271156u^4t^6 \right. \\
 & \left. + 3648282u^5t^5 + 3271156u^6t^4 + 2151462u^7t^3 + 850587u^8t^2 + 155744u^9t + 7948u^{10}) s^3 + 3tu(7948t^9 + 78694ut^8 + 283529u^2t^7 \right. \\
 & \left. + 550627u^3t^6 + 715533u^4t^5 + 715533u^5t^4 + 550627u^6t^3 + 283529u^7t^2 + 78694u^8t + 7948u^9) s^2 + t^2u^2(t+u)^2 \left( 23844t^6 + 108056ut^5 \right. \\
 & \left. + 194375u^2t^4 + 224106u^3t^3 + 194375u^4t^2 + 108056u^5t + 23844u^6) s + t^3u^3(t+u)^3 \left( 7948t^4 + 15515ut^3 + 23262u^2t^2 + 15515u^3t \right. \right. \\
 & \left. \left. + 7948u^4 \right) \right) (s+u)^3 + 18s^2t^2u^2 \left( (11t^6 + 162ut^5 + 153u^2t^4 + 884u^3t^3 + 255u^4t^2 + 246u^5t + 33u^6) s^{13} + (62t^7 + 1339ut^6 \right. \\
 & \left. + 2160u^2t^5 + 8525u^3t^4 + 7390u^4t^3 + 3807u^5t^2 + 1728u^6t + 189u^7) s^{12} + 6(26t^8 + 774ut^7 + 1639u^2t^6 + 5548u^3t^5 + 7678u^4t^4 \right. \\
 & \left. + 4900u^5t^3 + 2621u^6t^2 + 826u^7t + 84u^8) s^{11} + 2(140t^9 + 4728ut^8 + 12480u^2t^7 + 37199u^3t^6 + 69840u^4t^5 + 60090u^5t^4 + 36224u^6t^3 \right. \\
 & \left. + 17091u^7t^2 + 4188u^8t + 404u^9) s^{10} + (534t^{10} + 13578ut^9 + 45012u^2t^8 + 119424u^3t^7 + 270061u^4t^6 + 311178u^5t^5 + 220301u^6t^4 \right. \\
 & \left. + 125964u^7t^3 + 47997u^8t^2 + 9560u^9t + 839u^{10}) s^9 + 3(340t^{11} + 5526ut^{10} + 22370u^2t^9 + 55138u^3t^8 + 129254u^4t^7 + 190253u^5t^6 \right.
 \end{aligned}$$

$$\begin{aligned}
 & + 166896u^6t^5 + 109793u^7t^4 + 53466u^8t^3 + 15573u^9t^2 + 2466u^{10}t + 189u^{11})s^8 + 2(736t^{12} + 9471u t^{11} + 44292u^2t^{10} + 110988u^3t^9 \\
 & + 235059u^4t^8 + 387831u^5t^7 + 413404 u^6t^6 + 317445u^7t^5 + 187035u^8t^4 + 72214u^9t^3 + 15774u^{10}t^2 + 1827 u^{11}t + 116u^{12})s^7 \\
 & + 2(708t^{13} + 9054ut^{12} + 47823u^2 t^{11} + 134712u^3t^{10} + 269098u^4t^9 + 435561u^5t^8 + 508839u^6t^7 + 430834 u^7t^6 + 288621u^8t^5 \\
 & + 141397u^9t^4 + 43186u^{10}t^3 + 7236u^{11}t^2 + 541 u^{12}t + 22u^{13})s^6 + 3t(285t^{13} + 4132ut^{12} + 24795u^2 t^{11} + 81510u^3t^{10} + 175113u^4t^9 \\
 & + 286188u^5t^8 + 351323u^6t^7 + 311558 u^7t^6 + 211822u^8t^5 + 112744u^9t^4 + 42788u^{10}t^3 + 10552u^{11}t^2 + 1454 u^{12}t + 56u^{13})s^5 \\
 & + t^2(294t^{13} + 5499ut^{12} + 38730u^2 t^{11} + 148723u^3t^{10} + 362736u^4t^9 + 642597u^5t^8 + 864762u^6t^7 + 853029 u^7t^6 + 614934u^8t^5 \\
 & + 328490u^9t^4 + 124788u^{10}t^3 + 32430u^{11} t^2 + 6072u^{12}t + 708u^{13})s^4 + 2t^3(22t^{13} + 707u t^{12} + 6399u^2t^{11} + 29412u^3t^{10} + 82842u^4t^9 \\
 & + 162087u^5t^8 + 239183 u^6t^7 + 267073u^7t^6 + 222465u^8t^5 + 137118u^9t^4 + 59592u^{10}t^3 + 16485 u^{11}t^2 + 2553u^{12}t + 174u^{13})s^3 \\
 & + 6t^4u(t+u)(26 t^{10} + 335ut^9 + 1643u^2t^8 + 4383u^3t^7 + 7465u^4t^6 + 9430u^5t^5 + 8350 u^6t^4 + 5555u^7t^3 + 2649u^8t^2 + 809u^9t + 127u^{10})s^2 \\
 & + t^5u^2(t+u)(162t^8 + 1164ut^7 + 3573u^2t^6 + 6255u^3t^5 + 7589u^4t^4 + 5865 u^5t^3 + 3192u^6t^2 + 1112u^7t + 216u^8)s + t^6u^3(t+u)(58 t^4 \\
 & + 100ut^3 + 153u^2t^2 + 100u^3t + 58u^4)H(2,y)H(1,0,z)(s+u)^3 + st(s+t)^2u^2(t+u)^2(1531t(t+u)^4s^{13} + 3(3029 t^6 + 15191ut^5 \\
 & + 28746u^2t^4 + 30440u^3t^3 + 14567u^4t^2 + 3351u^5t + 120 u^6)s^{12} + (25433t^7 + 151345ut^6 + 347338u^2t^5 + 471508u^3 t^4 + 364546u^4t^3 \\
 & + 144937u^5t^2 + 32273u^6t + 1800u^7) s^{11} + (44445t^8 + 300713ut^7 + 814816u^2t^6 + 1300511u^3t^5 + 1362455 u^4t^4 + 833113u^5t^3 \\
 & + 303956u^6t^2 + 65055u^7t + 3960u^8) s^{10} + (53136t^9 + 409309ut^8 + 1281415u^2t^7 + 2291071u^3t^6 + 2894942 u^4t^5 + 2471851u^5t^4 \\
 & + 1320295u^6t^3 + 459769u^7t^2 + 87252u^8t + 5040 u^9) s^9 + (44445t^{10} + 409309ut^9 + 1473770u^2t^8 + 2915618u^3 t^7 + 4102638u^4t^6 \\
 & + 4464012u^5t^5 + 3318386u^6t^4 + 1630106u^7t^3 + 508285 u^8t^2 + 77799u^9t + 3960u^{10}) s^8 + (25433t^{11} + 300713u t^{10} + 1281415u^2t^9 \\
 & + 2915618u^3t^8 + 4495118u^4t^7 + 5664674u^5 t^6 + 5440250u^6t^5 + 3552410u^7t^4 + 1539427u^8t^3 + 391013u^9t^2 + 44585 u^{10}t + 1800u^{11}) s^7 \\
 & + (9087t^{12} + 151345ut^{11} + 814816u^2 t^{10} + 2291071u^3t^9 + 4102638u^4t^8 + 5664674u^5t^7 + 6338504u^6 t^6 + 5147724u^7t^5 + 2865079u^8t^4 \\
 & + 1027309u^9t^3 + 196900u^{10}t^2 + 15021 u^{11}t + 360u^{12}) s^6 + t(1531t^{12} + 45573ut^{11} + 347338u^2 t^{10} + 1300511u^3t^9 + 2894942u^4t^8 \\
 & + 4464012u^5t^7 + 5440250u^6 t^6 + 5147724u^7t^5 + 3498698u^8t^4 + 1616591u^9t^3 + 448090u^{10}t^2 + 58725 u^{11}t + 2287u^{12}) s^5 + t^2u(6124t^{11} \\
 & + 86238u t^{10} + 471508u^2t^9 + 1362455u^3t^8 + 2471851u^4t^7 + 3318386u^5 t^6 + 3552410u^6t^5 + 2865079u^7t^4 + 1616591u^8t^3 + 581902u^9t^2 \\
 & + 110862 u^{10}t + 7726u^{11}) s^4 + t^3u^2(9186t^{10} + 91320ut^9 + 364546 u^2t^8 + 833113u^3t^7 + 1320295u^4t^6 + 1630106u^5t^5 + 1539427u^6 t^4 \\
 & + 1027309u^7t^3 + 448090u^8t^2 + 110862u^9t + 11166u^{10}) s^3 + t^4u^3(t+u)^2(6124t^7 + 31453ut^6 + 75907u^2t^5 + 120689u^3t^4 + 142484 u^4t^3 \\
 & + 102628u^5t^2 + 43273u^6t + 7726u^7) s^2 + t^5u^4(t+u)^4(1531t^4 + 3929ut^3 + 7371u^2t^2 + 5873u^3t + 2287u^4)s + 360t^6u^6(t+u)^4(t^2 \\
 & + ut + u^2)H(0,y)(s+u)^2 + st^2(s+t)^2u(t+u)^2(1531u(t+u)^4s^{13} + 3(120t^6 + 3351ut^5 + 14567u^2 t^4 + 30440u^3t^3 + 28746u^4t^2 \\
 & + 15191u^5t + 3029u^6) s^{12} + (1800t^7 + 32273ut^6 + 144937u^2t^5 + 364546u^3t^4 + 471508u^4 t^3 + 347338u^5t^2 + 151345u^6t + 25433u^7) s^{11} \\
 & + (3960t^8 + 65055 ut^7 + 303956u^2t^6 + 833113u^3t^5 + 1362455u^4t^4 + 1300511u^5t^3 + 814816 u^6t^2 + 300713u^7t + 44445u^8) s^{10} \\
 & + (5040t^9 + 87252u t^8 + 459769u^2t^7 + 1320295u^3t^6 + 2471851u^4t^5 + 2894942u^5t^4 + 2291071 u^6t^3 + 1281415u^7t^2 + 409309u^8t \\
 & + 53136u^9) s^9 + (3960 t^{10} + 77799ut^9 + 508285u^2t^8 + 1630106u^3t^7 + 3318386u^4t^6 + 4464012 u^5t^5 + 4102638u^6t^4 + 2915618u^7t^3 \\
 & + 1473770u^8t^2 + 409309u^9t + 44445 u^{10}) s^8 + (1800t^{11} + 44585ut^{10} + 391013u^2t^9 + 1539427 u^3t^8 + 3552410u^4t^7 + 5440250u^5t^6 \\
 & + 5664674u^6t^5 + 4495118u^7 t^4 + 2915618u^8t^3 + 1281415u^9t^2 + 300713u^{10}t + 25433u^{11}) s^7 + (360t^{12} + 15021ut^{11} + 196900u^2t^{10} \\
 & + 1027309u^3 t^9 + 2865079u^4t^8 + 5147724u^5t^7 + 6338504u^6t^6 + 5664674u^7t^5 + 4102638 u^8t^4 + 2291071u^9t^3 + 814816u^{10}t^2 \\
 & + 151345u^{11}t + 9087u^{12}) s^6 + u(2287t^{12} + 58725ut^{11} + 448090u^2t^{10} + 1616591u^3 t^9 + 3498698u^4t^8 + 5147724u^5t^7 + 5440250u^6t^6 \\
 & + 4464012u^7t^5 + 2894942 u^8t^4 + 1300511u^9t^3 + 347338u^{10}t^2 + 45573u^{11}t + 1531u^{12}) s^5 + tu^2(7726t^{11} + 110862ut^{10} + 581902u^2t^9 \\
 & + 1616591u^3 t^8 + 2865079u^4t^7 + 3552410u^5t^6 + 3318386u^6t^5 + 2471851u^7t^4 + 1362455 u^8t^3 + 471508u^9t^2 + 86238u^{10}t + 6124u^{11}) s^4 \\
 & + t^2u^3(11166t^{10} + 110862ut^9 + 448090u^2t^8 + 1027309u^3t^7 + 1539427u^4 t^6 + 1630106u^5t^5 + 1320295u^6t^4 + 833113u^7t^3 + 364546u^8t^2 \\
 & + 91320u^9 t + 9186u^{10}) s^3 + t^3u^4(t+u)^2(7726t^7 + 43273ut^6 + 102628 u^2t^5 + 142484u^3t^4 + 120689u^4t^3 + 75907u^5t^2 + 31453u^6t \\
 & + 6124 u^7) s^2 + t^4u^5(t+u)^4(2287t^4 + 5873ut^3 + 7371u^2t^2 + 3929 u^3t + 1531u^4)s + 360t^6u^6(t+u)^4(t^2 + ut + u^2)H(0,z)(s+u)^2 \\
 & + 3t^2(s+t)^2u^2(t+u)(259(t+u)^5s^{14} + 36(43 t^6 + 242ut^5 + 651u^2t^4 + 764u^3t^3 + 651u^4t^2 + 242u^5t + 43u^6) s^{13} + (4367t^7 + 26163ut^6 \\
 & + 84905u^2t^5 + 124733u^3t^4 + 124733u^4 t^3 + 84905u^5t^2 + 26163u^6t + 4367u^7) s^{12} + 4(1920t^8 + 12056 ut^7 + 43065u^2t^6 + 78684u^3t^5 \\
 & + 86470u^4t^4 + 78684u^5t^3 + 43065u^6 t^2 + 12056u^7t + 1920u^8) s^{11} + 6(1534t^9 + 10415u^8 + 38108 u^2t^7 + 83108u^3t^6 + 101419u^4t^5 \\
 & + 101419u^5t^4 + 83108u^6t^3 + 38108u^7 t^2 + 10415u^8t + 1534u^9) s^{10} + 12(640t^{10} + 4992ut^9 + 18355 u^2t^8 + 44658u^3t^7 + 62385u^4t^6 \\
 & + 60940u^5t^5 + 62385u^6t^4 + 44658u^7 t^3 + 18355u^8t^2 + 4992u^9t + 640u^{10}) s^9 + (4367t^{11} + 41875u t^{10} + 163877u^2t^9 + 417437u^3t^8 \\
 & + 677572u^4t^7 + 645896u^5t^6 + 645896 u^6t^5 + 677572u^7t^4 + 417437u^8t^3 + 163877u^9t^2 + 41875u^{10}t + 4367 u^{11}) s^8 + 2(774t^{12} \\
 & + 9924ut^{11} + 46320u^2t^{10} + 124333 u^3t^9 + 232260u^4t^8 + 249279u^5t^7 + 198284u^6t^6 + 249279u^7t^5 + 232260 u^8t^4 + 124333u^9t^3
 \end{aligned}$$

$$\begin{aligned}
 & +46320u^{10}t^2+9924u^{11}t+774u^{12})s^7+(259t^{13}+5481ut^{12}+35278u^2t^{11}+110676u^3t^{10}+240564u^4t^9+349066u^5t^8+300068u^6t^7 \\
 & +300068u^7t^6+349066u^8t^5+240564u^9t^4+110676u^{10}t^3+35278u^{11}t^2+5481u^{12}t+259u^{13})s^6+2tu(308t^{12}+3564ut^{11}+15365u^2t^{10} \\
 & +42591u^3t^9+94706u^4t^8+136033u^5t^7+141098u^6t^6+136033u^7t^5+94706u^8t^4+42591u^9t^3+15365u^{10}t^2+3564u^{11}t+308u^{12})s^5 \\
 & +t^2u^2(426t^{11}+3380ut^{10}+15835u^2t^9+65921u^3t^8+171209u^4t^7+259557u^5t^6+259557u^6t^5+171209u^7t^4+65921u^8t^3+15835u^9t^2 \\
 & +3380u^{10}t+426u^{11})s^4-2t^3u^3(t+u)^2(34t^8-527ut^7-6013u^2t^6-19815u^3t^5-26374u^4t^4-19815u^5t^3-6013u^6t^2-527u^7t \\
 & +34u^8)s^3+t^4u^4(t+u)^3(109t^6+2242ut^5+8867u^2t^4+13216u^3t^3+8867u^4t^2+2242u^5t+109u^6)s^2+30t^5u^5(t+u)^6(10t^2 \\
 & +23ut+10u^2)s+120t^6u^6(t+u)^5(t^2+u^2)H(0,y)H(0,z)(s+u)^2-s^2t(s+t)^2u(t+u)^2((360t^6+2287ut^5+7726u^2t^4 \\
 & +11166u^3t^3+7726u^4t^2+2287u^5t+360u^6)s^{12}+9(200t^7+1669ut^6+6525u^2t^5+12318u^3t^4+12318u^4t^3+6525u^5t^2+1669u^6t \\
 & +200u^7)s^{11}+(3960t^8+44585ut^7+196900u^2t^6+448090u^3t^5+581902u^4t^4+448090u^5t^3+196900u^6t^2+44585u^7t+3960u^8)s^{10} \\
 & +(5040t^9+77799ut^8+391013u^2t^7+1027309u^3t^6+1616591u^4t^5+1616591u^5t^4+1027309u^6t^3+391013u^7t^2+77799u^8t \\
 & +5040u^9)s^9+(3960t^{10}+87252ut^9+508285u^2t^8+1539427u^3t^7+2865079u^4t^6+3498698u^5t^5+2865079u^6t^4+1539427u^7t^3 \\
 & +508285u^8t^2+87252u^9t+3960u^{10})s^8+(1800t^{11}+65055u^{10}+459769u^2t^9+1630106u^3t^8+3552410u^4t^7+5147724u^5t^6 \\
 & +5147724u^6t^5+3552410u^7t^4+1630106u^8t^3+459769u^9t^2+65055u^{10}t+1800u^{11})s^7+(360t^{12}+32273ut^{11}+303956u^2t^{10} \\
 & +1320295u^3t^9+3318386u^4t^8+5440250u^5t^7+6338504u^6t^6+5440250u^7t^5+3318386u^8t^4+1320295u^9t^3+303956u^{10}t^2+32273u^{11}t \\
 & +360u^{12})s^6+tu(10053t^{11}+144937ut^{10}+833113u^2t^9+2471851u^3t^8+4464012u^4t^7+5664674u^5t^6+5664674u^6t^5+4464012u^7t^4 \\
 & +2471851u^8t^3+833113u^9t^2+144937u^{10}t+10053u^{11})s^5+tu(1531t^{12}+43701ut^{11}+364546u^2t^{10}+1362455u^3t^9+2894942u^4t^8 \\
 & +4102638u^5t^7+4495118u^6t^6+4102638u^7t^5+2894942u^8t^4+1362455u^9t^3+364546u^{10}t^2+43701u^{11}t+1531u^{12})s^4+t^2u^2(6124t^{11} \\
 & +91320u^{10}t+471508u^9t^2+1300511u^8t^3+2291071u^7t^4+2915618u^6t^5+2915618u^7t^4+1300511u^8t^3+471508u^9t^2 \\
 & +91320u^{10}t+6124u^{11})s^3+t^3u^3(t+u)^2(9186t^8+67866ut^7+202420u^2t^6+342110u^3t^5+394775u^4t^4+342110u^5t^3+202420u^6t^2 \\
 & +67866u^7t+9186u^8)s^2+t^4u^4(t+u)^3(6124t^6+27201u^5t^5+51370u^2t^4+58876u^3t^3+51370u^4t^2+27201u^5t+6124u^6)s \\
 & +t^5u^5(t+u)^4(1531t^4+2963ut^3+4395u^2t^2+2963u^3t+1531u^4)H(1,z)(s+u)^2-s^2t(s+t)^2u(t+u)^2((360t^6+2287ut^5 \\
 & +7726u^2t^4+11166u^3t^3+7726u^4t^2+2287u^5t+360u^6)s^{12}+9(200t^7+1669ut^6+6525u^2t^5+12318u^3t^4+12318u^4t^3+6525u^5t^2 \\
 & +1669u^6t+200u^7)s^{11}+(3960t^8+44585ut^7+196900u^2t^6+448090u^3t^5+581902u^4t^4+448090u^5t^3+196900u^6t^2+44585u^7t \\
 & +3960u^8)s^{10}+(5040t^9+77799ut^8+391013u^2t^7+1027309u^3t^6+1616591u^4t^5+1616591u^5t^4+1027309u^6t^3+391013u^7t^2 \\
 & +77799u^8t+5040u^9)s^9+(3960t^{10}+87252ut^9+508285u^2t^8+1539427u^3t^7+2865079u^4t^6+3498698u^5t^5+2865079u^6t^4 \\
 & +1539427u^7t^3+508285u^8t^2+87252u^9t+3960u^{10})s^8+(1800t^{11}+65055ut^{10}+459769u^2t^9+1630106u^3t^8+3552410u^4t^7 \\
 & +5147724u^5t^6+5147724u^6t^5+3552410u^7t^4+1630106u^8t^3+459769u^9t^2+65055u^{10}t+1800u^{11})s^7+(360t^{12}+32273ut^{11} \\
 & +303956u^2t^{10}+1320295u^3t^9+3318386u^4t^8+5440250u^5t^7+6338504u^6t^6+5440250u^7t^5+3318386u^8t^4+1320295u^9t^3 \\
 & +303956u^{10}t^2+32273u^{11}t+360u^{12})s^6+tu(10053t^{11}+144937ut^{10}+833113u^2t^9+2471851u^3t^8+4464012u^4t^7+5664674u^5t^6 \\
 & +5664674u^6t^5+4464012u^7t^4+2471851u^8t^3+833113u^9t^2+144937u^{10}t+10053u^{11})s^5+tu(1531t^{12}+43701ut^{11}+364546u^2t^{10} \\
 & +1362455u^3t^9+2894942u^4t^8+4102638u^5t^7+4495118u^6t^6+4102638u^7t^5+2894942u^8t^4+1362455u^9t^3+364546u^{10}t^2 \\
 & +43701u^{11}t+1531u^{12})s^4+t^2u^2(6124t^{11}+91320ut^{10}+471508u^2t^9+1300511u^3t^8+2291071u^4t^7+2915618u^5t^6+2915618u^6t^5 \\
 & +2291071u^7t^4+1300511u^8t^3+471508u^9t^2+91320u^{10}t+6124u^{11})s^3+t^3u^3(t+u)^2(9186t^8+67866ut^7+202420u^2t^6 \\
 & +342110u^3t^5+394775u^4t^4+342110u^5t^3+202420u^6t^2+67866u^7t+9186u^8)s^2+t^4u^4(t+u)^3(6124t^6+27201u^5t^5+51370u^2t^4 \\
 & +58876u^3t^3+51370u^4t^2+27201u^5t+6124u^6)s+t^5u^5(t+u)^4(1531t^4+2963ut^3+4395u^2t^2+2963u^3t+1531u^4)H(2,y)(s+u)^2 \\
 & -3s^2t^2(t+u)^2((120t^6+300ut^5+109u^2t^4-68u^3t^3+426u^4t^2+616u^5t+259u^6)s^{14}+2(420t^7+1395ut^6+1339u^2t^5+425u^3t^4 \\
 & +1903u^4t^3+3872u^5t^2+2870u^6t+774u^7)s^{13}+(2640t^8+11430ut^7+18489u^2t^6+14984u^3t^5+19215u^4t^4+37858u^5t^3+40759u^6t^2 \\
 & +21396u^7t+4367u^8)s^{12}+2(2460t^9+13545ut^8+31286u^2t^7+39401u^3t^6+40878u^4t^5+57956u^5t^4+72977u^6t^3+56244u^7t^2 \\
 & +23121u^8t+3840u^9)s^{11}+2(3000t^{10}+20475ut^9+62005u^2t^8+104385u^3t^7+118565u^4t^6+137297u^5t^5+175620u^6t^4+170653u^7t^3 \\
 & +102876u^8t^2+33792u^9t+4602u^{10})s^{10}+2(2460t^{11}+20475ut^{10}+77358u^2t^9+164395u^3t^8+215383u^4t^7+230739u^5t^6 \\
 & +294815u^6t^5+356593u^7t^4+290657u^8t^3+140082u^9t^2+35847u^{10}t+3840u^{11})s^9+(2640t^{12}+27090ut^{11}+124010u^2t^{10} \\
 & +328790u^3t^9+519114u^4t^8+554262u^5t^7+649134u^6t^6+963078u^7t^5+1095009u^8t^4+756156u^9t^3+291138u^{10}t^2+55904u^{11}t \\
 & +4367u^{12})s^8+2(420t^{13}+5715ut^{12}+31286u^2t^{11}+104385u^3t^{10}+215383u^4t^9+277131u^5t^8+300068u^6t^7+447563u^7t^6 \\
 & +661734u^8t^5+642258u^9t^4+363648u^{10}t^3+110242u^{11}t^2+15265u^{12}t+774u^{13})s^7+(120t^{14}+2790ut^{13}+18489u^2t^{12}+78802u^3t^{11} \\
 & +237130u^4t^{10}+461478u^5t^9+649134u^6t^8+895126u^7t^7+1291792u^8t^6+1479900u^9t^5+1107162u^{10}t^4+486996u^{11}t^3+111068u^{12}t^2
 \end{aligned}$$



$$\begin{aligned}
& +10260u^{13}t+259u^{14})s^6+2tu(150t^{13}+1339ut^{12}+7492u^2t^{11}+40878u^3t^{10}+137297u^4t^9+294815u^5t^8+481539u^6t^7+661734u^7t^6 \\
& +739950u^8t^5+608514u^9t^4+330308u^{10}t^3+104819u^{11}t^2+16074u^{12}t+777u^{13})s^5+t^2u^2(109t^{12}+850u^{11}t+19215u^{10}t^2 \\
& +115912u^9t^3+351240u^8t^4+713186u^7t^5+1095009u^6t^6+1284516u^5t^7+1107162u^4t^8+660616u^3t^9+249466u^2t^{10}+50940u^{11}t \\
& +3885u^{12})s^4+2t^3u^3(-34t^{11}+1903u^{10}t+18929u^2t^9+72977u^3t^8+170653u^4t^7+290657u^5t^6+378078u^6t^5+363648u^7t^4 \\
& +243498u^8t^3+104819u^9t^2+25470u^{10}t+2590u^{11})s^3+t^4u^4(t+u)^2(426t^8+6892ut^7+26549u^2t^6+52498u^3t^5+74207u^4t^4 \\
& +79252u^5t^3+58427u^6t^2+24378u^7t+3885u^8)s^2+2t^5u^5(t+u)^3(308t^6+1946ut^5+3936u^2t^4+5167u^3t^3+4537u^4t^2+2799u^5t \\
& +777u^6)s+t^6u^6(t+u)^4(259t^4+512ut^3+765u^2t^2+512u^3t+259u^4))H(0,z)H(2,y)(s+u)^2+3s^2t^2(s+t)^6u^2(t+u)^6(363s^8 \\
& +726(t+3u)s^7+3(363t^2+796ut+2057u^2)s^6+6(121t^3+348ut^2+480u^2t+1815u^3)s^5+(363t^4+958ut^3-297u^2t^2+1782u^3t \\
& +13068u^4)s^4+2u(246t^4-58ut^3-1296u^2t^2+891u^3t+5445u^4)s^3+u^2(504t^4-116ut^3-297u^2t^2+2880u^3t+6171u^4)s^2 \\
& +2u^3(246t^4+479ut^3+1044u^2t^2+1194u^3t+1089u^4)s+363u^4(t^2+ut+u^2)^2)H(0,0,z)(s+u)^2+36s^2t^2(s+t)^6u^2(t+u)^6(22s^8 \\
& +44(t+3u)s^7+2(33t^2+70u^{11}t+187u^2)s^6+(44t^3+246ut^2+147u^2t+660u^3)s^5+(22t^4+115ut^3+360u^2t^2+41u^3t+792u^4)s^4 \\
& +u(64t^4+95ut^3+246u^2t^2+2u^3t+660u^4)s^3+u^2(84t^4+9ut^3+84u^2t^2+54u^3t+374u^4)s^2+u^3(54t^4-11u^3t+36u^2t^2+63u^3t \\
& +132u^4)s+u^4(18t^4+4ut^3+18u^2t^2+21u^3t+22u^4))H(0,y)H(0,0,z)(s+u)^2-36s^2t^2(s+t)^6u^2(t+u)^6(22s^8+3(7t+44u)s^7 \\
& +(18t^2+63ut+374u^2)s^6+(4t^3+36ut^2+54u^2t+660u^3)s^5+(18t^4-11u^3t+84u^2t^2+2u^3t+792u^4)s^4+u(54t^4+9ut^3 \\
& +246u^2t^2+41u^3t+660u^4)s^3+u^2(84t^4+95ut^3+360u^2t^2+147u^3t+374u^4)s^2+u^3(64t^4+115ut^3+246u^2t^2+140u^3t+132u^4)s \\
& +22u^4(t^2+ut+u^2)^2)H(2,y)H(0,0,z)(s+u)^2+18s^2t^2u^2(t+u)^2((25t^4+64ut^3+78u^2t^2+44u^3t+11u^4)s^{14}+2(92t^5+278ut^4 \\
& +393u^2t^3+307u^3t^2+133u^4t+27u^5)s^{13}+(633t^6+2332ut^5+3918u^2t^4+3896u^3t^3+2351u^4t^2+816u^5t+124u^6)s^{12}+2(684t^7 \\
& +2970u^6t^6+5715u^5t^5+6827u^4t^4+5495u^3t^3+2871u^2t^2+846u^6t+96u^7)s^{11}+(2094t^8+10548ut^7+21762u^2t^6+27160u^3t^5 \\
& +25897u^4t^4+19800u^5t^3+10356u^6t^2+2820u^7t+231u^8)s^{10}+2(1200t^9+6984ut^8+15768u^2t^7+19041u^3t^6+16319u^4t^5 \\
& +14613u^5t^4+13072u^6t^3+7593u^7t^2+2013u^8t+105u^9)s^9+(2094t^{10}+13968ut^9+35988u^2t^8+47158u^3t^7+37273u^4t^6 \\
& +27528u^5t^5+31140u^6t^4+31554u^7t^3+17931u^8t^2+4336u^9t+130u^{10})s^8+2(684t^{11}+5274ut^{10}+15768u^2t^9+23579u^3t^8 \\
& +22058u^4t^7+21537u^5t^6+26874u^6t^5+27399u^7t^4+18504u^8t^3+7571u^9t^2+1472u^{10}t+24u^{11})s^7+(633t^{12}+5940u^{11}t^{11} \\
& +21762u^2t^{10}+38082u^3t^9+37273u^4t^8+43074u^5t^7+83288u^6t^6+111834u^7t^5+83094u^8t^4+33834u^9t^3+7744u^{10}t^2+1056u^{11}t \\
& +8u^{12})s^6+2t(92t^{12}+1166ut^{11}+5715u^2t^{10}+13580u^3t^9+16319u^4t^8+13764u^5t^7+26874u^6t^6+55917u^7t^5+62742u^8t^4 \\
& +36199u^9t^3+9748u^{10}t^2+930u^{11}t+72u^{12})s^5+t^2(25t^{12}+556ut^{11}+3918u^2t^{10}+13654u^3t^9+25897u^4t^8+29226u^5t^7 \\
& +31140u^6t^6+54798u^7t^5+83094u^8t^4+72398u^9t^3+32492u^{10}t^2+6012u^{11}t+84u^{12})s^4+2t^3u(32t^{11}+393ut^{10}+1948u^2t^9 \\
& +5495u^3t^8+9900u^4t^7+13072u^5t^6+15777u^6t^5+18504u^7t^4+16917u^8t^3+9748u^9t^2+3006u^{10}t+368u^{11})s^3+t^4u^2(t+u)^2(78t^8 \\
& +458ut^7+1357u^2t^6+2570u^3t^5+3859u^4t^4+4898u^5t^3+4276u^6t^2+1692u^7t+84u^8)s^2+2t^5u^3(t+u)^3(22t^6+67ut^5+141u^2t^4 \\
& +200u^3t^3+320u^4t^2+312u^5t+72u^6)s+t^6u^4(t+u)^4(11t^4+10u^3t^3+18u^2t^2+16u^3t+8u^4))H(0,z)H(0,2,y)(s+u)^2 \\
& +3t^2(s+t)u^2((17t^6+1338ut^5+1209u^2t^4+3688u^3t^3+4269u^4t^2+762u^5t+293u^6)s^{15}+(t+u)^2(125t^5+9282ut^4-1574u^2t^3 \\
& +24402u^3t^2+2031u^4t+1740u^5)s^{14}+(421t^8+28668ut^7+79874u^2t^6+122578u^3t^5+210974u^4t^4+216624u^5t^3+90402u^6t^2 \\
& +18674u^7t+4873u^8)s^{13}+(t+u)^2(853t^7+47050u^6t^6+99979u^2t^5+64212u^3t^4+244839u^4t^3+99258u^5t^2+22313u^6t+8520u^7)s^{12} \\
& +4(288t^{10}+12551ut^9+71057u^2t^8+139212u^3t^7+179702u^4t^6+266064u^5t^5+276429u^6t^4+139626u^7t^3+41347u^8t^2+13917u^9t \\
& +2547u^{10})s^{11}+2(t+u)^2(546t^9+13510ut^8+92097u^2t^7+124154u^3t^6+79909u^4t^5+282798u^5t^4+136349u^6t^3+25806u^7t^2 \\
& +17949u^8t+4260u^9)s^{10}+(745t^{12}+5476u^{11}t+88790u^2t^{10}+380158u^3t^9+559919u^4t^8+624124u^5t^7+1280316u^6t^6+1630684u^7t^5 \\
& +908135u^8t^4+264280u^9t^3+94614u^{10}t^2+32670u^{11}t+4873u^{12})s^9+(t+u)^2(361t^{11}-5942ut^{10}-13109u^2t^9+45310u^3t^8 \\
& -83270u^4t^7-225424u^5t^6+359776u^6t^5+341850u^7t^4+131133u^8t^3+49098u^9t^2+8565u^{10}t+1740u^{11})s^8+(113t^{14}-5422ut^{13} \\
& -49927u^2t^{12}-155092u^3t^{11}-318038u^4t^{10}-778482u^5t^9-1327656u^6t^8-974758u^7t^7-19040u^8t^6+432306u^9t^5+380872u^{10}t^4 \\
& +185882u^{11}t^3+39939u^{12}t^2+2122u^{13}t+293u^{14})s^7+t(t+u)^2(17t^{12}-2466ut^{11}-25660u^2t^{10}-88158u^3t^9-136236u^4t^8 \\
& -237708u^5t^7-370210u^6t^6-209898u^7t^5-117978u^8t^4-9304u^9t^3+43600u^{10}t^2+14262u^{11}t+49u^{12})s^6-t^2u(424t^{13}+9046ut^{12} \\
& +64296u^2t^{11}+209881u^3t^{10}+368916u^4t^9+405336u^5t^8+306108u^6t^7+185900u^7t^6+201510u^8t^5+255266u^9t^4+158896u^{10}t^3 \\
& +31479u^{11}t^2-5494u^{12}t-2252u^{13})s^5-t^3u^2(t+u)^2(1038t^{10}+11826ut^9+41953u^2t^8+50370u^3t^7-10747u^4t^6-92766u^5t^5 \\
& -113853u^6t^4-35658u^7t^3+15137u^8t^2+11676u^9t+2344u^{10})s^4-t^4u^3(t+u)^3(1108t^8+5303ut^7+2515u^2t^6-23685u^3t^5 \\
& -48783u^4t^4-46581u^5t^3-17785u^6t^2-2753u^7t-67u^8)s^3-t^5u^4(t+u)^4(133t^6-1574ut^5-8533u^2t^4-14102u^3t^3-10237u^4t^2 \\
& -2876u^5t-167u^6)s^2+30t^6u^5(t+u)^6(10t^3+37ut^2+37u^2t+14u^3)s+120t^7u^6(t+u)^6(t^2+ut+u^2))H(1,0,z)(s+u)^2
\end{aligned}$$

$$\begin{aligned}
& + 18s^2t^2(s+t)^2u^2 \left( (11t^6 + 162ut^5 + 129u^2t^4 + 796u^3t^3 + 129u^4t^2 + 162u^5t + 11u^6)s^{12} + 6(11t^7 + 193ut^6 + 349u^2t^5 + 1047u^3t^4 \right. \\
& + 1027u^4t^3 + 313u^5t^2 + 173u^6t + 7u^7)s^{11} + (187t^8 + 3280ut^7 + 8506u^2t^6 + 20204u^3t^5 + 32372u^4t^4 + 18608u^5t^3 + 6910u^6t^2 \\
& + 2596u^7t + 73u^8)s^{10} + 2(165t^9 + 2522ut^8 + 8324u^2t^7 + 17309u^3t^6 + 35002u^4t^5 + 33469u^5t^4 + 14236u^6t^3 + 6119u^7t^2 + 1745u^8t \\
& + 53u^9)s^9 + (396t^{10} + 4920ut^9 + 19227u^2t^8 + 34792u^3t^7 + 71793u^4t^6 + 105726u^5t^5 + 63533u^6t^4 + 25164u^7t^3 + 13635u^8t^2 \\
& + 3190u^9t + 168u^{10})s^8 + 2(165t^{11} + 1725ut^{10} + 7356u^2t^9 + 10992u^3t^8 + 12447u^4t^7 + 31129u^5t^6 + 30585u^6t^5 + 10947u^7t^4 \\
& + 9212u^8t^3 + 6186u^9t^2 + 1323u^{10}t + 109u^{11})s^7 + (187t^{12} + 1872ut^{11} + 8220u^2t^{10} + 10632u^3t^9 - 15915u^4t^8 - 29212u^5t^7 - 9538u^6t^6 \\
& - 13152u^7t^5 + 1543u^8t^4 + 19604u^9t^3 + 10518u^{10}t^2 + 2096u^{11}t + 185u^{12})s^6 + 2(33t^{13} + 372ut^{12} + 1782u^2t^{11} + 2908u^3t^{10} \\
& - 8202u^4t^9 - 31332u^5t^8 - 35999u^6t^7 - 26337u^7t^6 - 11010u^8t^5 + 8183u^9t^4 + 10293u^{10}t^3 + 3681u^{11}t^2 + 623u^{12}t + 45u^{13})s^5 \\
& + (11t^{14} + 182ut^{13} + 1142u^2t^{12} + 3266u^3t^{11} - 2041u^4t^{10} - 30594u^5t^9 - 56568u^6t^8 - 48084u^7t^7 - 20480u^8t^6 + 13148u^9t^5 \\
& + 25835u^{10}t^4 + 13918u^{11}t^3 + 3530u^{12}t^2 + 452u^{13}t + 19u^{14})s^4 + 2tu(10t^{13} + 115ut^{12} + 604u^2t^{11} + 1366u^3t^{10} - 633u^4t^9 - 5650u^5t^8 \\
& - 5310u^6t^7 + 1442u^7t^6 + 9166u^8t^5 + 12427u^9t^4 + 8168u^{10}t^3 + 2766u^{11}t^2 + 499u^{12}t + 38u^{13})s^3 + t^2u^2(t+u)^2(24t^{10} + 190ut^9 \\
& + 841u^2t^8 + 1482u^3t^7 + 1601u^4t^6 + 4172u^5t^5 + 4881u^6t^4 + 5362u^7t^3 + 3151u^8t^2 + 930u^9t + 126u^{10})s^2 + 2t^3u^3(t+u)^3(10t^8 \\
& + 70ut^7 + 248u^2t^6 + 399u^3t^5 + 669u^4t^4 + 647u^5t^3 + 545u^6t^2 + 236u^7t + 48u^8)s + t^4u^4(t+u)^6(11t^4 + 22ut^3 + 39u^2t^2 + 28u^3t \\
& + 33u^4)H(0, y)H(1, 0, z)(s+u)^2 + 18s^2t^2(s+t)^2u^2(4(2t^6 + 36ut^5 + 21u^2t^4 + 184u^3t^3 + 21u^4t^2 + 36u^5t + 2u^6)s^{12} + 12(4t^7 \\
& + 88ut^6 + 155u^2t^5 + 501u^3t^4 + 501u^4t^3 + 155u^5t^2 + 88u^6t + 4u^7)s^{11} + 2(65t^8 + 1472ut^7 + 3872u^2t^6 + 9748u^3t^5 + 16246u^4t^4 \\
& + 9748u^5t^3 + 3872u^6t^2 + 1472u^7t + 65u^8)s^{10} + 2(105t^9 + 2168ut^8 + 7571u^2t^7 + 16917u^3t^6 + 36199u^4t^5 + 36199u^5t^4 + 16917u^6t^3 \\
& + 7571u^7t^2 + 2168u^8t + 105u^9)s^9 + 3(77t^{10} + 1342ut^9 + 5977u^2t^8 + 12336u^3t^7 + 27698u^4t^6 + 41828u^5t^5 + 27698u^6t^4 + 12336u^7t^3 \\
& + 5977u^8t^2 + 1342u^9t + 77u^{10})s^8 + 6(32t^{11} + 470ut^{10} + 2531u^2t^9 + 5259u^3t^8 + 9133u^4t^7 + 18639u^5t^6 + 18639u^6t^5 + 9133u^7t^4 \\
& + 5259u^8t^3 + 2531u^9t^2 + 470u^{10}t + 32u^{11})s^7 + 4(31t^{12} + 423ut^{11} + 2589u^2t^{10} + 6536u^3t^9 + 7785u^4t^8 + 13437u^5t^7 + 20822u^6t^6 \\
& + 13437u^7t^5 + 7785u^8t^4 + 6536u^9t^3 + 2589u^{10}t^2 + 423u^{11}t + 31u^{12})s^6 + 6(9t^{13} + 136ut^{12} + 957u^2t^{11} + 3300u^3t^{10} + 4871u^4t^9 \\
& + 4588u^5t^8 + 7179u^6t^7 + 7179u^7t^6 + 4588u^8t^5 + 4871u^9t^4 + 3300u^{10}t^3 + 957u^{11}t^2 + 136u^{12}t + 9u^{13})s^5 + (11t^{14} + 266ut^{13} \\
& + 2351u^2t^{12} + 10990u^3t^{11} + 25897u^4t^{10} + 32638u^5t^9 + 37273u^6t^8 + 44116u^7t^7 + 37273u^8t^6 + 32638u^9t^5 + 25897u^{10}t^4 + 10990u^{11}t^3 \\
& + 2351u^{12}t^2 + 266u^{13}t + 11u^{14})s^4 + 2tu(22t^{13} + 307ut^{12} + 1948u^2t^{11} + 6827u^3t^{10} + 13580u^4t^9 + 19041u^5t^8 + 23579u^6t^7 + 23579u^7t^6 \\
& + 19041u^8t^5 + 13580u^9t^4 + 6827u^{10}t^3 + 1948u^{11}t^2 + 307u^{12}t + 22u^{13})s^3 + 6t^2u^2(t+u)^2(13t^{10} + 105ut^9 + 430u^2t^8 + 940u^3t^7 \\
& + 1317u^4t^6 + 1682u^5t^5 + 1317u^6t^4 + 940u^7t^3 + 430u^8t^2 + 105u^9t + 13u^{10})s^2 + 4t^3u^3(t+u)^3(16t^8 + 91ut^7 + 262u^2t^6 + 410u^3t^5 \\
& + 530u^4t^4 + 410u^5t^3 + 262u^6t^2 + 91u^7t + 16u^8)s + t^4u^4(t+u)^6(25t^4 + 34ut^3 + 54u^2t^2 + 34u^3t + 25u^4)H(0, z)H(2, 0, y)(s+u)^2 \\
& + 18s^2t^2u^2(6(11t^6 + 46ut^5 + 156u^2t^4 + 102u^3t^3 + 156u^4t^2 + 46u^5t + 11u^6)s^{14} + 2(270t^7 + 1259ut^6 + 4389u^2t^5 + 4796u^3t^4 \\
& + 5120u^4t^3 + 3729u^5t^2 + 1049u^6t + 204u^7)s^{13} + (2055t^8 + 10770ut^7 + 38032u^2t^6 + 57306u^3t^5 + 58498u^4t^4 + 54214u^5t^3 + 27450u^6t^2 \\
& + 7450u^7t + 1185u^8)s^{12} + 2(2410t^9 + 14457ut^8 + 51810u^2t^7 + 97505u^3t^6 + 109719u^4t^5 + 108252u^5t^4 + 77806u^6t^3 + 32031u^7t^2 \\
& + 8199u^8t + 1059u^9)s^{11} + (7744t^{10} + 53588ut^9 + 198480u^2t^8 + 439066u^3t^7 + 579501u^4t^6 + 598332u^5t^5 + 517783u^6t^4 + 294682u^7t^3 \\
& + 105495u^8t^2 + 24196u^9t + 2541u^{10})s^{10} + 2(4458t^{11} + 35621ut^{10} + 138966u^2t^9 + 344982u^3t^8 + 538700u^4t^7 + 609874u^5t^6 \\
& + 590760u^6t^5 + 426236u^7t^4 + 200386u^8t^3 + 62613u^9t^2 + 12146u^{10}t + 1050u^{11})s^9 + (7450t^{12} + 68928ut^{11} + 290076u^2t^{10} \\
& + 782276u^3t^9 + 1406634u^4t^8 + 1787400u^5t^7 + 1901521u^6t^6 + 1654302u^7t^5 + 999621u^8t^4 + 402156u^9t^3 + 106839u^{10}t^2 + 16578u^{11}t \\
& + 1179u^{12})s^8 + 2(2230t^{13} + 24137ut^{12} + 113268u^2t^{11} + 330723u^3t^{10} + 666867u^4t^9 + 948111u^5t^8 + 1082558u^6t^7 + 1072690u^7t^6 \\
& + 809304u^8t^5 + 421377u^9t^4 + 148590u^{10}t^3 + 32739u^{11}t^2 + 3807u^{12}t + 207u^{13})s^7 + (1830t^{14} + 23864ut^{13} + 129744u^2t^{12} \\
& + 419622u^3t^{11} + 946629u^4t^{10} + 1526944u^5t^9 + 1879479u^6t^8 + 1986148u^7t^7 + 1729402u^8t^6 + 1108500u^9t^5 + 506934u^{10}t^4 \\
& + 157722u^{11}t^3 + 28413u^{12}t^2 + 2220u^{13}t + 69u^{14})s^6 + 2t(232t^{14} + 3972ut^{13} + 26409u^2t^{12} + 97770u^3t^{11} + 247866u^4t^{10} \\
& + 463269u^5t^9 + 652628u^6t^8 + 749862u^7t^7 + 712998u^8t^6 + 514895u^9t^5 + 272466u^{10}t^4 + 106176u^{11}t^3 + 28022u^{12}t^2 + 3996u^{13}t \\
& + 159u^{14})s^5 + t^2(55t^{14} + 1614ut^{13} + 14748u^2t^{12} + 66434u^3t^{11} + 190555u^4t^{10} + 406308u^5t^9 + 671989u^6t^8 + 896290u^7t^7 \\
& + 978039u^8t^6 + 811640u^9t^5 + 480198u^{10}t^4 + 201792u^{11}t^3 + 59561u^{12}t^2 + 11310u^{13}t + 1083u^{14})s^4 + 2t^3u(74t^{13} + 1262ut^{12} \\
& + 7944u^2t^{11} + 27682u^3t^{10} + 67049u^4t^9 + 125496u^5t^8 + 189647u^6t^7 + 237429u^7t^6 + 234672u^8t^5 + 169330u^9t^4 + 85057u^{10}t^3 \\
& + 28209u^{11}t^2 + 5389u^{12}t + 424u^{13})s^3 + t^4u^2(t+u)^2(192t^{10} + 1868ut^9 + 7244u^2t^8 + 17304u^3t^7 + 30738u^4t^6 + 40806u^5t^5 \\
& + 45573u^6t^4 + 36880u^7t^3 + 20764u^8t^2 + 7278u^9t + 1137u^{10})s^2 + 2t^5u^3(t+u)^3(64t^8 + 420u^7t + 1164u^2t^6 + 2189u^3t^5 + 2787u^4t^4 \\
& + 2772u^5t^3 + 1870u^6t^2 + 855u^7t + 183u^8)s + t^6u^4(t+u)^6(47t^4 + 64ut^3 + 117u^2t^2 + 92u^3t + 83u^4)H(0, 1, 0, z)(s+u)^2
\end{aligned}$$

$$\begin{aligned}
 & + 36s^2t^2(s+t)^6u^2(t+u)^6 \left( 22s^8 + 44(t+3u)s^7 + 2(33t^2 + 70ut + 187u^2) s^6 + (44t^3 + 246ut^2 + 147u^2t + 660u^3) s^5 + (22t^4 \right. \\
 & + 115u t^3 + 360u^2t^2 + 41u^3t + 792u^4) s^4 + u(64t^4 + 95ut^3 + 246u^2t^2 + 2u^3t + 660u^4) s^3 + u^2(84t^4 + 9ut^3 + 84u^2t^2 + 54u^3t \\
 & + 374u^4) s^2 + u^3(54t^4 - 11ut^3 + 36u^2t^2 + 63u^3t + 132u^4) s + u^4(18t^4 + 4ut^3 + 18u^2t^2 + 21u^3t + 22u^4) \Big) H(1, 0, 0, z)(s+u)^2 \\
 & + 36s^2t^2u^3 \left( (48t^5 - 6u t^4 + 332u^2t^3 + 45u^3t^2 + 90u^4t + 11u^5) s^{14} + (420t^6 + 282u t^5 + 2684u^2t^4 + 2292u^3t^3 + 810u^4t^2 \right. \\
 & + 542u^5t + 42u^6) s^{13} + (1386t^7 + 1149ut^6 + 7038u^2t^5 + 12044u^3t^4 + 4972u^4t^3 + 2592u^5t^2 + 1202u^6t + 73u^7) s^{12} \\
 & + (2181t^8 - 6ut^7 + 3u^2t^6 + 15414u^3t^5 + 7299u^4t^4 - 896u^5t^3 + 2931u^6t^2 + 1368u^7t + 106u^8) s^{11} + (985t^9 - 9867ut^8 \\
 & - 44480u^2t^7 - 48559u^3t^6 - 45186u^4t^5 - 50331u^5t^4 - 19580u^6t^3 + 585u^7t^2 + 1097u^8t + 168u^9) s^{10} - (2469t^{10} + 31352ut^9 \\
 & + 135204u^2t^8 + 241022u^3t^7 + 253354u^4t^6 + 222072u^5t^5 + 127806u^6t^4 + 30846u^7t^3 + 657u^8t^2 - 980u^9t - 218u^{10}) s^9 \\
 & - (5517t^{11} + 52233ut^{10} + 228083u^2t^9 + 506619u^3t^8 + 645063u^4t^7 + 575952u^5t^6 + 368997u^6t^5 + 131304u^7t^4 + 16832u^8t^3 \\
 & - 1527u^9t^2 - 936u^{10}t - 185u^{11}) s^8 - (5797t^{12} + 55314ut^{11} + 250929u^2t^{10} + 646444u^3t^9 + 990366u^4t^8 + 1001980u^5t^7 \\
 & + 717884u^6t^6 + 319380u^7t^5 + 58843u^8t^4 - 6082u^9t^3 - 4017u^{10}t^2 - 684u^{11}t - 90u^{12}) s^7 - (3849t^{13} + 40039ut^{12} + 191778u^2t^{11} \\
 & + 546879u^3t^{10} + 969791u^4t^9 + 1126182u^5t^8 + 922078u^6t^7 + 512534u^7t^6 + 148569u^8t^5 - 1403u^9t^4 - 14004u^{10}t^3 - 3726u^{11}t^2 \\
 & - 315u^{12}t - 19u^{13}) s^6 - t(1647t^{13} + 19818ut^{12} + 104248u^2t^{11} + 323862u^3t^{10} + 638973u^4t^9 + 819288u^5t^8 + 723606u^6t^7 + 455700u^7t^6 \\
 & + 180305u^8t^5 + 24144u^9t^4 - 13260u^{10}t^3 - 8070u^{11}t^2 - 1683u^{12}t - 66u^{13}) s^5 - t^2(413t^{13} + 6240ut^{12} + 38435u^2t^{11} + 133574u^3t^{10} \\
 & + 291906u^4t^9 + 405133u^5t^8 + 357472u^6t^7 + 200055u^7t^6 + 58195u^8t^5 - 7313u^9t^4 - 13779u^{10}t^3 - 6238u^{11}t^2 - 1920u^{12}t - 309u^{13}) s^4 \\
 & - t^3(48t^{13} + 1112ut^{12} + 8730u^2t^{11} + 35276u^3t^{10} + 86245u^4t^9 + 130602u^5t^8 + 113599u^6t^7 + 38248u^7t^6 - 30777u^8t^5 - 50450u^9t^4 \\
 & - 31999u^{10}t^3 - 10602u^{11}t^2 - 1774u^{12}t - 114u^{13}) s^3 - t^4u(t+u)^2(84t^{10} + 898ut^9 + 3400u^2t^8 + 6309u^3t^7 + 5217u^4t^6 - 2505u^5t^5 \\
 & - 8514u^6t^4 - 9343u^7t^3 - 5743u^8t^2 - 2007u^9t - 336u^{10}) s^2 - t^5u^2(t+u)^3(48t^8 + 200ut^7 + 135u^2t^6 - 571u^3t^5 - 1726u^4t^4 - 1854u^5t^3 \\
 & - 1219u^6t^2 - 459u^7t - 90u^8) s + t^6u^4(t+u)^6(23t^3 + 54ut^2 + 46u^2t + 26u^3) \Big) H(1, 1, 0, z)(s+u)^2 + 9(s+t)^2u^2(t+u) \Big( 2(92t^7 + 224ut^6 \\
 & + 807u^2t^5 + 41u^3t^4 + 227u^4t^3 + 71u^5t^2 + 70u^6t + 20u^7) s^{15} + 2(548t^8 + 1800ut^7 + 6507u^2t^6 + 5556u^3t^5 + 2001u^4t^4 + 2149u^5t^3 \\
 & + 864u^6t^2 + 535u^7t + 120u^8) s^{14} + (3080t^9 + 12672ut^8 + 43909u^2t^7 + 68677u^3t^6 + 39172u^4t^5 + 27176u^5t^4 + 19411u^6t^3 + 8427u^7t^2 \\
 & + 3620u^8t + 640u^9) s^{13} + (5400t^{10} + 26796ut^9 + 85362u^2t^8 + 182471u^3t^7 + 168303u^4t^6 + 105098u^5t^5 + 92562u^6t^4 + 53097u^7t^3 \\
 & + 21885u^8t^2 + 7050u^9t + 1000u^{10}) s^{12} + (6464t^{11} + 38196ut^{10} + 114563u^2t^9 + 283713u^3t^8 + 385450u^4t^7 + 275704u^5t^6 + 227431u^6t^5 \\
 & + 183473u^7t^4 + 90148u^8t^3 + 33770u^9t^2 + 8600u^{10}t + 1000u^{11}) s^{11} + (5400t^{12} + 38196ut^{11} + 123420u^2t^{10} + 320669u^3t^9 + 573417u^4t^8 \\
 & + 513000u^5t^7 + 343086u^6t^6 + 321375u^7t^5 + 218509u^8t^4 + 96742u^9t^3 + 32200u^{10}t^2 + 6690u^{11}t + 640u^{12}) s^{10} + (3080t^{13} + 26796ut^{12} \\
 & + 114563u^2t^{11} + 320669u^3t^{10} + 641380u^4t^9 + 698714u^5t^8 + 386310u^6t^7 + 282028u^7t^6 + 279284u^8t^5 + 162550u^9t^4 + 65139u^{10}t^3 \\
 & + 18779u^{11}t^2 + 3220u^{12}t + 240u^{13}) s^9 + (1096t^{14} + 12672ut^{13} + 85362u^2t^{12} + 283713u^3t^{11} + 573417u^4t^{10} + 698714u^5t^9 \\
 & + 392272u^6t^8 + 129314u^7t^7 + 178058u^8t^6 + 158492u^9t^5 + 72298u^{10}t^4 + 25025u^{11}t^3 + 6257u^{12}t^2 + 870u^{13}t + 40u^{14}) s^8 + t(184t^{14} \\
 & + 3600ut^{13} + 43909u^2t^{12} + 182471u^3t^{11} + 385450u^4t^{10} + 513000u^5t^9 + 386310u^6t^8 + 129314u^7t^7 + 92160u^8t^6 + 110380u^9t^5 \\
 & + 48211u^{10}t^4 + 11865u^{11}t^3 + 3700u^{12}t^2 + 994u^{13}t + 100u^{14}) s^7 + t^2u(448t^{13} + 13014ut^{12} + 68677u^2t^{11} + 168303u^3t^{10} \\
 & + 275704u^4t^9 + 343086u^5t^8 + 282028u^6t^7 + 178058u^7t^6 + 110380u^8t^5 + 33784u^9t^4 - 4267u^{10}t^3 - 3791u^{11}t^2 - 474u^{12}t + 42u^{13}) s^6 \\
 & + t^3u^2(1614t^{12} + 11112ut^{11} + 39172u^2t^{10} + 105098u^3t^9 + 227431u^4t^8 + 321375u^5t^7 + 279284u^6t^6 + 158492u^7t^5 + 48211u^8t^4 \\
 & - 4267u^9t^3 - 7188u^{10}t^2 - 1770u^{11}t - 164u^{12}) s^5 + t^4u^3(82t^{11} + 4002ut^{10} + 27176u^2t^9 + 92562u^3t^8 + 183473u^4t^7 + 218509u^5t^6 \\
 & + 162550u^6t^5 + 72298u^7t^4 + 11865u^8t^3 - 3791u^9t^2 - 1770u^{10}t - 204u^{11}) s^4 + t^5u^4(t+u)^2(454t^8 + 3390ut^7 + 12177u^2t^6 \\
 & + 25353u^3t^5 + 27265u^4t^4 + 16859u^5t^3 + 4156u^6t^2 - 146u^7t - 164u^8) s^3 + t^6u^5(t+u)^3(142t^6 + 1302ut^5 + 4095u^2t^4 + 5552u^3t^3 \\
 & + 3527u^4t^2 + 868u^5t + 42u^6) s^2 + 10t^7u^6(t+u)^5(14t^3 + 37ut^2 + 37u^2t + 10u^3) s + 40t^8u^7(t+u)^5(t^2 + ut + u^2) \Big) H(1, 0, y)(s+u) \\
 & - 3s^2(s+t)^2u^2(t+u)^2 \left( (259t^6 + 616ut^5 + 426u^2t^4 - 68u^3t^3 + 109u^4t^2 + 300u^5t + 120u^6) s^{14} + 2(774t^7 + 2870u t^6 + 3872u^2t^5 \right. \\
 & + 1903u^3t^4 + 425u^4t^3 + 1339u^5t^2 + 1395u^6t + 420u^7) s^{13} + (4367t^8 + 21396ut^7 + 40759u^2t^6 + 37858u^3t^5 + 19215u^4t^4 + 14984u^5t^3 \\
 & + 18489u^6t^2 + 11430u^7t + 2640u^8) s^{12} + 2(3840t^9 + 23121ut^8 + 56244u^2t^7 + 72977u^3t^6 + 57956u^4t^5 + 40878u^5t^4 + 39401u^6t^3 \\
 & + 31286u^7t^2 + 13545u^8t + 2460u^9) s^{11} + 2(4602t^{10} + 33792ut^9 + 102876u^2t^8 + 170653u^3t^7 + 175620u^4t^6 + 137297u^5t^5 + 118565u^6t^4 \\
 & + 104385u^7t^3 + 62005u^8t^2 + 20475u^9t + 3000u^{10}) s^{10} + 2(3840t^{11} + 35847ut^{10} + 140082u^2t^9 + 290657u^3t^8 + 356593u^4t^7 \\
 & + 294815u^5t^6 + 230739u^6t^5 + 215383u^7t^4 + 164395u^8t^3 + 77358u^9t^2 + 20475u^{10}t + 2460u^{11}) s^9 + (4367t^{12} + 55904ut^{11} \\
 & + 291138u^2t^{10} + 756156u^3t^9 + 1095009u^4t^8 + 963078u^5t^7 + 649134u^6t^6 + 554262u^7t^5 + 519114u^8t^4 + 328790u^9t^3 + 124010u^{10}t^2 \\
 & + 27090u^{11}t + 2640u^{12}) s^8 + 2(774t^{13} + 15265ut^{12} + 110242u^2t^{11} + 363648u^3t^{10} + 642258u^4t^9 + 661734u^5t^8 + 447563u^6t^7 \\
 & + 300068u^7t^6 + 277131u^8t^5 + 215383u^9t^4 + 104385u^{10}t^3 + 31286u^{11}t^2 + 5715u^{12}t + 420u^{13}) s^7 + (259t^{14} + 10260ut^{13} + 111068u^2t^{12}
 \end{aligned}$$

$$\begin{aligned}
 & +486996u^3t^{11}+1107162u^4t^{10}+1479900u^5t^9+1291792u^6t^8+895126u^7t^7+649134u^8t^6+461478u^9t^5+237130u^{10}t^4+78802u^{11}t^3 \\
 & +18489u^{12}t^2+2790u^{13}t+120u^{14})s^6+2tu(777t^{13}+16074ut^{12}+104819u^2t^{11}+330308u^3t^{10}+608514u^4t^9+739950u^5t^8 \\
 & +661734u^6t^7+481539u^7t^6+294815u^8t^5+137297u^9t^4+40878u^{10}t^3+7492u^{11}t^2+1339u^{12}t+150u^{13})s^5+t^2u^2(3885t^{12} \\
 & +50940ut^{11}+249466u^2t^{10}+660616u^3t^9+1107162u^4t^8+1284516u^5t^7+1095009u^6t^6+713186u^7t^5+351240u^8t^4+115912u^9t^3 \\
 & +19215u^{10}t^2+850u^{11}t+109u^{12})s^4+2t^3u^3(2590t^{11}+25470ut^{10}+104819u^2t^9+243498u^3t^8+363648u^4t^7+378078u^5t^6 \\
 & +290657u^6t^5+170653u^7t^4+72977u^8t^3+18929u^9t^2+1903u^{10}t-34u^{11})s^3+t^4u^4(t+u)^2(3885t^8+24378u^7t^7+58427u^2t^6 \\
 & +79252u^3t^5+74207u^4t^4+52498u^5t^3+26549u^6t^2+6892u^7t+426u^8)s^2+2t^5u^5(t+u)^3(777t^6+2799u^5t^5+4537u^2t^4+5167u^3t^3 \\
 & +3936u^4t^2+1946u^5t+308u^6)s+t^6u^6(t+u)^4(259t^4+512ut^3+765u^2t^2+512u^3t+259u^4)H(0,y)H(1,z)+9s^2(t+u)^2(2(20t^8 \\
 & +50u^7t+21u^2t^6-82u^3t^5-102u^4t^4-82u^5t^3+21u^6t^2+50u^7t+20u^8)s^{16}+2(140t^9+505ut^8+568u^2t^7-298u^3t^6-1069u^4t^5 \\
 & -1069u^5t^4-298u^6t^3+568u^7t^2+505u^8t+140u^9)s^{15}+(880t^{10}+4370ut^9+8221u^2t^8+4262u^3t^7-6199u^4t^6-10932u^5t^5 \\
 & -6199u^6t^4+4262u^7t^3+8221u^8t^2+4370u^9t+880u^{10})s^{14}+2(820t^{11}+5395ut^{10}+14563u^2t^9+17988u^3t^8+5650u^4t^7-8508u^5t^6 \\
 & -8508u^6t^5+5650u^7t^4+17988u^8t^3+14563u^9t^2+5395u^{10}t+820u^{11})s^{13}+(2000t^{12}+16930ut^{11}+60889u^2t^{10}+115200u^3t^9 \\
 & +112888u^4t^8+52018u^5t^7+18062u^6t^6+52018u^7t^5+112888u^8t^4+115200u^9t^3+60889u^{10}t^2+16930u^{11}t+2000u^{12})s^{12}+2(820t^{13} \\
 & +8825ut^{12}+40630u^2t^{11}+106430u^3t^{10}+162506u^4t^9+145433u^5t^8+94054u^6t^7+94054u^7t^6+145433u^8t^5+162506u^9t^4 \\
 & +106430u^{10}t^3+40630u^{11}t^2+8825u^{12}t+820u^{13})s^{11}+(880t^{14}+12310ut^{13}+71305u^2t^{12}+252860u^3t^{11}+542940u^4t^{10} \\
 & +672624u^5t^9+495141u^6t^8+346704u^7t^7+495141u^8t^6+672624u^9t^5+542940u^{10}t^4+252860u^{11}t^3+71305u^{12}t^2+12310u^{13}t \\
 & +880u^{14})s^{10}+2(140t^{15}+2785ut^{14}+20491u^2t^{13}+99450u^3t^{12}+294436u^4t^{11}+490859u^5t^{10}+448931u^6t^9+254956u^7t^8 \\
 & +254956u^8t^7+448931u^9t^6+490859u^{10}t^5+294436u^{11}t^4+99450u^{12}t^3+20491u^{13}t^2+2785u^{14}t+140u^{15})s^9+(40t^{16}+1490ut^{15} \\
 & +14845u^2t^{14}+102820u^3t^{13}+419280u^4t^{12}+950788u^5t^{11}+1225773u^6t^{10}+975710u^7t^9+743060u^8t^8+975710u^9t^7+1225773u^{10}t^6 \\
 & +950788u^{11}t^5+419280u^{12}t^4+102820u^{13}t^3+14845u^{14}t^2+1490u^{15}t+40u^{16})s^8+2tu(90t^{15}+1540ut^{14}+16932u^2t^{13}+96123u^3t^{12} \\
 & +304282u^4t^{11}+583798u^5t^{10}+762212u^6t^9+803305u^7t^8+803305u^8t^7+762212u^9t^6+583798u^{10}t^5+304282u^{11}t^4+96123u^{12}t^3 \\
 & +16932u^{13}t^2+1540u^{14}t+90u^{15})s^7+t^2u^2(282t^{14}+6622ut^{13}+54887u^2t^{12}+264008u^3t^{11}+776536u^4t^{10}+1517240u^5t^9 \\
 & +2171441u^6t^8+2431080u^7t^7+2171441u^8t^6+1517240u^9t^5+776536u^{10}t^4+264008u^{11}t^3+54887u^{12}t^2+6622u^{13}t+282u^{14})s^6 \\
 & +2t^3u^3(298t^{13}+4418u^2t^{12}+40731u^2t^{11}+190625u^3t^{10}+505964u^4t^9+877790u^5t^8+1117090u^6t^7+1117090u^7t^6+877790u^8t^5 \\
 & +505964u^9t^4+190625u^{10}t^3+40731u^{11}t^2+4418u^{12}t+298u^{13})s^5+t^4u^4(536t^{12}+16810ut^{11}+131975u^2t^{10}+463360u^3t^9 \\
 & +936996u^4t^8+1292362u^5t^7+1406138u^6t^6+1292362u^7t^5+936996u^8t^4+463360u^9t^3+131975u^{10}t^2+16810u^{11}t+536u^{12})s^4 \\
 & +2t^5u^5(848t^{11}+13094ut^{10}+64600u^2t^9+162207u^3t^8+255217u^4t^7+298424u^5t^6+298424u^6t^5+255217u^7t^4+162207u^8t^3 \\
 & +64600u^9t^2+13094u^{10}t+848u^{11})s^3+t^6u^6(t+u)^2(2062t^8+13122ut^7+32971u^2t^6+48846u^3t^5+54292u^4t^4+48846u^5t^3 \\
 & +32971u^6t^2+13122u^7t+2062u^8)s^2+4t^7u^7(t+u)^3(158t^6+746ut^5+1500u^2t^4+1923u^3t^3+1500u^4t^2+746u^5t+158u^6)s \\
 & +8t^8u^8(t+u)^4(23t^4+45ut^3+67u^2t^2+45u^3t+23u^4)H(1,z)H(3,y)+3s^2(s+t)u^2(t+u)^2((17t^6-424ut^5-1038u^2t^4-1108u^3t^3 \\
 & -133u^4t^2+300u^5t+120u^6)s^{15}+(113t^7-2432ut^6-9046u^2t^5-13902u^3t^4-8627u^4t^3+1042u^5t^2+2910u^6t+840u^7)s^{14}+(361t^8 \\
 & -5422ut^7-30575u^2t^6-64296u^3t^5-66643u^4t^4-21748u^5t^3+14031u^6t^2+12270u^7t+2640u^8)s^{13}+(745t^9-5220ut^8-49927u^2t^7 \\
 & -141944u^3t^6-209881u^4t^5-146102u^5t^4-877u^6t^3+57146u^7t^2+29730u^8t+4920u^9)s^{12}+2(546t^{10}+2738ut^9-12316u^2t^8 \\
 & -77546u^3t^7-169106u^4t^6-184458u^5t^5-65973u^6t^4+53495u^7t^3+62003u^8t^2+22935u^9t+3000u^{10})s^{11}+2(576t^{11}+14602u^2t^{10} \\
 & +44395u^2t^9+6575u^3t^8-159019u^4t^7-299169u^5t^6-202668u^6t^5+31945u^7t^4+130735u^8t^3+82071u^9t^2+23475u^{10}t+2460u^{11})s^{10} \\
 & +(853t^{12}+50204ut^{11}+239326u^2t^{10}+380158u^3t^9-5759u^4t^8-778482u^5t^7-981862u^6t^6-306108u^7t^5+310132u^8t^4 \\
 & +327562u^9t^3+138034u^{10}t^2+32010u^{11}t+2640u^{12})s^9+(421t^{13}+48756ut^{12}+284228u^2t^{11}+643716u^3t^{10}+559919u^4t^9 \\
 & -346654u^5t^8-1327656u^6t^7-1188026u^7t^6-185900u^8t^5+356130u^9t^4+244634u^{10}t^3+72974u^{11}t^2+14070u^{12}t+840u^{13})s^8 \\
 & +(125t^{14}+28668ut^{13}+194932u^2t^{12}+556848u^3t^{11}+840628u^4t^{10}+624124u^5t^9-174342u^6t^8-974758u^7t^7-907984u^8t^6 \\
 & -201510u^9t^5+170032u^{10}t^4+108262u^{11}t^3+22743u^{12}t^2+3630u^{13}t+120u^{14})s^7+t(17t^{14}+9532ut^{13}+79874u^2t^{12}+311220u^3t^{11} \\
 & +718808u^4t^{10}+1133540u^5t^9+1280316u^6t^8+835978u^7t^7-19040u^8t^6-455158u^9t^5-255266u^{10}t^4-6292u^{11}t^3+26245u^{12}t^2 \\
 & +3544u^{13}t+420u^{14})s^6+t^2u(1338t^{13}+17115ut^{12}+122578u^2t^{11}+473242u^3t^{10}+1064256u^4t^9+1563708u^5t^8+1630684u^6t^7 \\
 & +1174609u^7t^6+432306u^8t^5-92986u^9t^4-158896u^{10}t^3-40833u^{11}t^2+2954u^{12}t+167u^{13})s^5+t^3u^2(1209t^{12}+30536ut^{11} \\
 & +210974u^2t^{10}+653148u^3t^9+1105716u^4t^8+1162604u^5t^7+908135u^6t^6+653214u^7t^5+380872u^8t^4+92158u^9t^3-31479u^{10}t^2 \\
 & -16364u^{11}t+67u^{12})s^4+t^4u^3(3688t^{11}+49261ut^{10}+216624u^2t^9+465668u^3t^8+558504u^4t^7+411820u^5t^6+264280u^6t^5
 \end{aligned}$$

$$\begin{aligned}
 & + 237894u^7t^4 + 185882u^8t^3 + 72173u^9t^2 + 5494u^{10}t - 2344u^{11})s^3 + t^5u^4(t+u)^2(4269t^8 + 21666ut^7 + 42801u^2t^6 + 45136u^3t^5 \\
 & + 32315u^4t^4 + 22162u^5t^3 + 17975u^6t^2 + 9856u^7t + 2252u^8)s^2 + t^6u^5(t+u)^3(762t^6 + 3225u^5t^5 + 6713u^2t^4 + 8777u^3t^3 + 5973u^4t^2 \\
 & + 1975u^5t + 49u^6)s + t^7u^6(t+u)^4(293t^4 + 568ut^3 + 843u^2t^2 + 568u^3t + 293u^4)H(0,1,z) + 18s^2t^2(s+t)^2u^2(t+u)^2((11t^4 + 20ut^3 \\
 & + 24u^2t^2 + 20u^3t + 11u^4)s^{14} + 2(33t^5 + 91ut^4 + 115u^2t^3 + 119u^3t^2 + 100u^4t + 44u^5)s^{13} + (187t^6 + 744ut^5 + 1142u^2t^4 + 1208u^3t^3 \\
 & + 1245u^4t^2 + 976u^5t + 336u^6)s^{12} + 2(165t^7 + 936ut^6 + 1782u^2t^5 + 1633u^3t^4 + 1366u^4t^3 + 1677u^5t^2 + 1363u^6t + 406u^7)s^{11} \\
 & + (396t^8 + 3450ut^7 + 8220u^2t^6 + 5816u^3t^5 - 2041u^4t^4 - 1266u^5t^3 + 5406u^6t^2 + 5360u^7t + 1391u^8)s^{10} + 2(165t^9 + 2460ut^8 \\
 & + 7356u^2t^7 + 5316u^3t^6 - 8202u^4t^5 - 15297u^5t^4 - 5650u^6t^3 + 4428u^7t^2 + 4099u^8t + 897u^9)s^9 + (187t^{10} + 5044ut^9 + 19227u^2t^8 \\
 & + 21984u^3t^7 - 15915u^4t^6 - 62664u^5t^5 - 56568u^6t^4 - 10620u^7t^3 + 14826u^8t^2 + 9784u^9t + 1783u^{10})s^8 + 2(33t^{11} + 1640ut^{10} \\
 & + 8324u^2t^9 + 17396u^3t^8 + 12447u^4t^7 - 14606u^5t^6 - 35999u^6t^5 - 24042u^7t^4 + 1442u^8t^3 + 9648u^9t^2 + 4481u^{10}t + 668u^{11})s^7 \\
 & + (11t^{12} + 1158ut^{11} + 8506u^2t^{10} + 34618u^3t^9 + 71793u^4t^8 + 62258u^5t^7 - 9538u^6t^6 - 52674u^7t^5 - 20480u^8t^4 + 18332u^9t^3 \\
 & + 18756u^{10}t^2 + 6076u^{11}t + 702u^{12})s^6 + 2u(81t^{12} + 1047ut^{11} + 10102u^2t^{10} + 35002u^3t^9 + 52863u^4t^8 + 30585u^5t^7 - 6576u^6t^6 \\
 & - 11010u^7t^5 + 6574u^8t^4 + 12427u^9t^3 + 6297u^{10}t^2 + 1397u^{11}t + 113u^{12})s^5 + u^2(129t^{12} + 6282ut^{11} + 32372u^2t^{10} + 66938u^3t^9 \\
 & + 63533u^4t^8 + 21894u^5t^7 + 1543u^6t^6 + 16366u^7t^5 + 25835u^8t^4 + 16336u^9t^3 + 5137u^{10}t^2 + 760u^{11}t + 33u^{12})s^4 + 2tu^3(398t^{11} \\
 & + 3081u^{10} + 9304u^2t^9 + 14236u^3t^8 + 12582u^4t^7 + 9212u^5t^6 + 9802u^6t^5 + 10293u^7t^4 + 6959u^8t^3 + 2766u^9t^2 + 591u^{10}t + 48u^{11})s^3 \\
 & + t^2u^4(t+u)^2(129t^8 + 1620ut^7 + 3541u^2t^6 + 3536u^3t^5 + 3022u^4t^4 + 2792u^5t^3 + 1912u^6t^2 + 746u^7t + 126u^8)s^2 + 2t^3u^5(t+u)^3(81t^6 \\
 & + 276ut^5 + 227u^2t^4 + 155u^3t^3 + 173u^4t^2 + 112u^5t + 38u^6)s + t^4u^6(t+u)^4(11t^4 - 2ut^3 + 15u^2t^2 + 14u^3t + 19u^4)H(0,y)H(0,1,z) \\
 & - 18s^2t^2u^2(t+u)^2((47t^4 + 116ut^3 + 144u^2t^2 + 96u^3t + 39u^4)s^{16} + 2(211t^5 + 697ut^4 + 1032u^2t^3 + 886u^3t^2 + 487u^4t + 147u^5)s^{15} \\
 & + 3(587t^6 + 2466ut^5 + 4371u^2t^4 + 4476u^3t^3 + 3059u^4t^2 + 1466u^5t + 355u^6)s^{14} + 2(2246t^7 + 11795ut^6 + 25344u^2t^5 + 30555u^3t^4 \\
 & + 24384u^4t^3 + 14649u^5t^2 + 6280u^6t + 1255u^7)s^{13} + (7761t^8 + 49924ut^7 + 132409u^2t^6 + 195056u^3t^5 + 184034u^4t^4 + 125820u^5t^3 \\
 & + 68009u^6t^2 + 25468u^7t + 4325u^8)s^{12} + 6(1587t^9 + 12289ut^8 + 40330u^2t^7 + 74208u^3t^6 + 86445u^4t^5 + 69435u^5t^4 + 42172u^6t^3 \\
 & + 20076u^7t^2 + 6418u^8t + 944u^9)s^{11} + (8447t^{10} + 78224ut^9 + 319917u^2t^8 + 731786u^3t^7 + 1046888u^4t^6 + 1017372u^5t^5 + 727368u^6t^4 \\
 & + 404718u^7t^3 + 167685u^8t^2 + 44164u^9t + 5599u^{10})s^{10} + 2(2688t^{11} + 30091ut^{10} + 156379u^2t^9 + 443281u^3t^8 + 763361u^4t^7 \\
 & + 878974u^5t^6 + 745371u^6t^5 + 500624u^7t^4 + 259751u^8t^3 + 91571u^9t^2 + 19006u^{10}t + 2015u^{11})s^9 + 3(784t^{12} + 11080ut^{11} + 75305u^2t^{10} \\
 & + 263492u^3t^9 + 545731u^4t^8 + 746656u^5t^7 + 740518u^6t^6 + 577912u^7t^5 + 364335u^8t^4 + 170508u^9t^3 + 50345u^{10}t^2 + 7940u^{11}t \\
 & + 660u^{12})s^8 + 2(318t^{13} + 6360ut^{12} + 59340u^2t^{11} + 252696u^3t^{10} + 631038u^4t^9 + 1061526u^5t^8 + 1285210u^6t^7 + 1158562u^7t^6 \\
 & + 806991u^8t^5 + 444563u^9t^4 + 181058u^{10}t^3 + 45120u^{11}t^2 + 5167u^{12}t + 295u^{13})s^7 + (80t^{14} + 3066ut^{13} + 43714u^2t^{12} + 225298u^3t^{11} \\
 & + 680647u^4t^{10} + 1441224u^5t^9 + 2222801u^6t^8 + 2456276u^7t^7 + 1932681u^8t^6 + 1122364u^9t^5 + 510799u^{10}t^4 + 177102u^{11}t^3 \\
 & + 37210u^{12}t^2 + 2790u^{13}t + 80u^{14})s^6 + 6tu(58t^{13} + 1689u^{12} + 11116u^2t^{11} + 42847u^3t^{10} + 116103u^4t^9 + 225254u^5t^8 + 308029u^6t^7 \\
 & + 293932u^7t^6 + 196649u^8t^5 + 94795u^9t^4 + 34537u^{10}t^3 + 9491u^{11}t^2 + 1574u^{12}t + 58u^{13})s^5 + t^2u^2(1092t^{12} + 11428ut^{11} + 67756u^2t^{10} \\
 & + 245052u^3t^9 + 575825u^4t^8 + 929600u^5t^7 + 1064520u^6t^6 + 875584u^7t^5 + 515905u^8t^4 + 213096u^9t^3 + 59096u^{10}t^2 + 10508u^{11}t \\
 & + 1092u^{12})s^4 + 2t^3u^3(394t^{11} + 5820ut^{10} + 29696u^2t^9 + 83756u^3t^8 + 156154u^4t^7 + 208169u^5t^6 + 204213u^6t^5 + 148024u^7t^4 \\
 & + 77656u^8t^3 + 27425u^9t^2 + 5475u^{10}t + 394u^{11})s^3 + 6t^4u^4(t+u)^2(173t^8 + 987ut^7 + 2643u^2t^6 + 4684u^3t^5 + 5690u^4t^4 + 4568u^5t^3 \\
 & + 2519u^6t^2 + 941u^7t + 173u^8)s^2 + 2t^5u^5(t+u)^3(150t^6 + 629ut^5 + 1361u^2t^4 + 1851u^3t^3 + 1353u^4t^2 + 606u^5t + 150u^6)s \\
 & + 66t^6u^6(t+u)^4(t^2 + ut + u^2)^2H(2,y)H(0,1,z) - 3s^2(s+t)^2u^2(t+u)^2((259t^6 + 616ut^5 + 426u^2t^4 - 68u^3t^3 + 109u^4t^2 + 300u^5t \\
 & + 120u^6)s^{14} + 2(774t^7 + 2870ut^6 + 3872u^2t^5 + 1903u^3t^4 + 425u^4t^3 + 1339u^5t^2 + 1395u^6t + 420u^7)s^{13} + (4367t^8 + 21396ut^7 \\
 & + 40759u^2t^6 + 37858u^3t^5 + 19215u^4t^4 + 14984u^5t^3 + 18489u^6t^2 + 11430u^7t + 2640u^8)s^{12} + 2(3840t^9 + 23121ut^8 + 56244u^2t^7 \\
 & + 72977u^3t^6 + 57956u^4t^5 + 40878u^5t^4 + 39401u^6t^3 + 31286u^7t^2 + 13545u^8t + 2460u^9)s^{11} + 2(4602t^{10} + 33792ut^9 + 102876u^2t^8 \\
 & + 170653u^3t^7 + 175620u^4t^6 + 137297u^5t^5 + 118565u^6t^4 + 104385u^7t^3 + 62005u^8t^2 + 20475u^9t + 3000u^{10})s^{10} + 2(3840t^{11} \\
 & + 35847ut^{10} + 140082u^2t^9 + 290657u^3t^8 + 356593u^4t^7 + 294815u^5t^6 + 230739u^6t^5 + 215383u^7t^4 + 164395u^8t^3 + 77358u^9t^2 \\
 & + 20475u^{10}t + 2460u^{11})s^9 + (4367t^{12} + 55904ut^{11} + 291138u^2t^{10} + 756156u^3t^9 + 1095009u^4t^8 + 963078u^5t^7 + 649134u^6t^6 \\
 & + 554262u^7t^5 + 519114u^8t^4 + 328790u^9t^3 + 124010u^{10}t^2 + 27090u^{11}t + 2640u^{12})s^8 + 2(774t^{13} + 15265ut^{12} + 110242u^2t^{11} \\
 & + 363648u^3t^{10} + 642258u^4t^9 + 661734u^5t^8 + 447563u^6t^7 + 300068u^7t^6 + 277131u^8t^5 + 215383u^9t^4 + 104385u^{10}t^3 + 31286u^{11}t^2 \\
 & + 5715u^{12}t + 420u^{13})s^7 + (259t^{14} + 10260ut^{13} + 111068u^2t^{12} + 486996u^3t^{11} + 1107162u^4t^{10} + 1479900u^5t^9 + 1291792u^6t^8 \\
 & + 895126u^7t^7 + 649134u^8t^6 + 461478u^9t^5 + 237130u^{10}t^4 + 78802u^{11}t^3 + 18489u^{12}t^2 + 2790u^{13}t + 120u^{14})s^6 + 2tu(777t^{13} \\
 & + 16074ut^{12} + 104819u^2t^{11} + 330308u^3t^{10} + 608514u^4t^9 + 739950u^5t^8 + 661734u^6t^7 + 481539u^7t^6 + 294815u^8t^5 + 137297u^9t^4
 \end{aligned}$$

$$\begin{aligned}
 & + 40878u^{10}t^3 + 7492u^{11}t^2 + 1339u^{12}t + 150u^{13})s^5 + t^2u^2(3885t^{12} + 50940ut^{11} + 249466u^2t^{10} + 660616u^3t^9 + 1107162u^4t^8 \\
 & + 1284516u^5t^7 + 1095009u^6t^6 + 713186u^7t^5 + 351240u^8t^4 + 115912u^9t^3 + 19215u^{10}t^2 + 850u^{11}t + 109u^{12})s^4 + 2t^3u^3(2590t^{11} \\
 & + 25470ut^{10} + 104819u^2t^9 + 243498u^3t^8 + 363648u^4t^7 + 378078u^5t^6 + 290657u^6t^5 + 170653u^7t^4 + 72977u^8t^3 + 18929u^9t^2 + 1903u^{10}t \\
 & - 34u^{11})s^3 + t^4u^4(t+u)^2(3885t^8 + 24378u^7t + 58427u^2t^6 + 79252u^3t^5 + 74207u^4t^4 + 52498u^5t^3 + 26549u^6t^2 + 6892u^7t + 426u^8)s^2 \\
 & + 2t^5u^5(t+u)^3(777t^6 + 2799u^5t^5 + 4537u^2t^4 + 5167u^3t^3 + 3936u^4t^2 + 1946u^5t + 308u^6)s + t^6u^6(t+u)^4(259t^4 + 512ut^3 + 765u^2t^2 \\
 & + 512u^3t + 259u^4)H(0, 2, y) - 18s^2t^2u^2(t+u)^3(2t(7t^2 + 15u^2t + 12u^2)s^{16} + 12(8t^4 + 24ut^3 + 29u^2t^2 + 12u^3t - u^4)s^{15} + 3(101t^5 \\
 & + 423ut^4 + 720u^2t^3 + 556u^3t^2 + 111u^4t - 31u^5)s^{14} + (598t^6 + 3066ut^5 + 7134u^2t^4 + 8992u^3t^3 + 5154u^4t^2 + 510u^5t - 318u^6)s^{13} \\
 & + (851t^7 + 4495ut^6 + 13290u^2t^5 + 25506u^3t^4 + 26949u^4t^3 + 12309u^5t^2 + 688u^6t - 652u^7)s^{12} + 6(158t^8 + 586ut^7 + 2628u^2t^6 \\
 & + 7708u^3t^5 + 11649u^4t^4 + 9092u^5t^3 + 3324u^6t^2 + 218u^7t - 155u^8)s^{11} + (851t^9 - 417ut^8 + 9564u^2t^7 + 62150u^3t^6 + 123813u^4t^5 \\
 & + 121269u^5t^4 + 67982u^6t^3 + 23208u^7t^2 + 2838u^8t - 1002u^9)s^{10} + 2(299t^{10} - 2075ut^9 - 1896u^2t^8 + 23751u^3t^7 + 72903u^4t^6 \\
 & + 97515u^5t^5 + 71423u^6t^4 + 31963u^7t^3 + 10800u^8t^2 + 1966u^9t - 409u^{10})s^9 + 3(101t^{11} - 1463ut^{10} - 3896u^2t^9 + 208u^3t^8 + 20429u^4t^7 \\
 & + 58669u^5t^6 + 80268u^6t^5 + 55172u^7t^4 + 19400u^8t^3 + 4832u^9t^2 + 1000u^{10}t - 156u^{11})s^8 + 2(48t^{12} - 1128ut^{11} - 4410u^2t^{10} \\
 & - 17090u^3t^9 - 31983u^4t^8 + 6084u^5t^7 + 100874u^6t^6 + 143160u^7t^5 + 88338u^8t^4 + 23800u^9t^3 + 2622u^{10}t^2 + 582u^{11}t - 81u^{12})s^7 \\
 & + (14t^{13} - 572ut^{12} - 3036u^2t^{11} - 35818u^3t^{10} - 106376u^4t^9 - 101688u^5t^8 + 48822u^6t^7 + 213630u^7t^6 + 239397u^8t^5 + 134809u^9t^4 \\
 & + 31274u^{10}t^3 - 36u^{11}t^2 + 157u^{12}t - 25u^{13})s^6 - 6tu(8t^{12} + 37ut^{11} + 3213u^2t^{10} + 11984u^3t^9 + 14592u^4t^8 + 1987u^5t^7 - 12914u^6t^6 \\
 & - 20563u^7t^5 - 20499u^8t^4 - 11612u^9t^3 - 2423u^{10}t^2 + 147u^{11}t + 3u^{12})s^5 + t^2u^2(102t^{11} - 5234ut^{10} - 25530u^2t^9 - 39126u^3t^8 \\
 & - 12408u^4t^7 + 24648u^5t^6 + 35505u^6t^5 + 40505u^7t^4 + 40476u^8t^3 + 21240u^9t^2 + 3629u^{10}t - 267u^{11})s^4 + 2t^3u^3(-266t^{10} - 1804u^9t^9 \\
 & - 4752u^2t^8 - 4830u^3t^7 + 378u^4t^6 + 5301u^5t^5 + 6806u^6t^4 + 6178u^7t^3 + 3642u^8t^2 + 1119u^9t + 156u^{10})s^3 + 3t^4u^4(t+u)^2(16t^7 \\
 & - 382ut^6 - 532u^2t^5 + 8u^3t^4 + 467u^4t^3 + 637u^5t^2 + 265u^6t - 71u^7)s^2 - 2t^5u^5(t+u)^3(48t^5 + 151ut^4 - 84u^2t^3 + 18u^3t^2 - 160u^4t \\
 & - 15u^5)s + t^6u^6(t+u)^3(12t^3 + 3ut^2 + 2u^2t - 11u^3)H(1, z)H(0, 3, y) - 3s^2(s+t)^2u^2(t+u)^2((259t^6 + 616ut^5 + 426u^2t^4 - 68u^3t^3 \\
 & + 109u^4t^2 + 300u^5t + 120u^6)s^{14} + 2(774t^7 + 2870ut^6 + 3872u^2t^5 + 1903u^3t^4 + 425u^4t^3 + 1339u^5t^2 + 1395u^6t + 420u^7)s^{13} + (4367t^8 \\
 & + 21396ut^7 + 40759u^2t^6 + 37858u^3t^5 + 19215u^4t^4 + 14984u^5t^3 + 18489u^6t^2 + 11430u^7t + 2640u^8)s^{12} + 2(3840t^9 + 23121ut^8 \\
 & + 56244u^2t^7 + 72977u^3t^6 + 57956u^4t^5 + 40878u^5t^4 + 39401u^6t^3 + 31286u^7t^2 + 13545u^8t + 2460u^9)s^{11} + 2(4602t^{10} + 33792ut^9 \\
 & + 102876u^2t^8 + 170653u^3t^7 + 175620u^4t^6 + 137297u^5t^5 + 118565u^6t^4 + 104385u^7t^3 + 62005u^8t^2 + 20475u^9t + 3000u^{10})s^{10} \\
 & + 2(3840t^{11} + 35847ut^{10} + 140082u^2t^9 + 290657u^3t^8 + 356593u^4t^7 + 294815u^5t^6 + 230739u^6t^5 + 215383u^7t^4 + 164395u^8t^3 \\
 & + 77358u^9t^2 + 20475u^{10}t + 2460u^{11})s^9 + (4367t^{12} + 55904ut^{11} + 291138u^2t^{10} + 756156u^3t^9 + 1095009u^4t^8 + 963078u^5t^7 \\
 & + 649134u^6t^6 + 554262u^7t^5 + 519114u^8t^4 + 328790u^9t^3 + 124010u^{10}t^2 + 27090u^{11}t + 2640u^{12})s^8 + 2(774t^{13} + 15265ut^{12} \\
 & + 110242u^2t^{11} + 363648u^3t^{10} + 642258u^4t^9 + 661734u^5t^8 + 447563u^6t^7 + 300068u^7t^6 + 277131u^8t^5 + 215383u^9t^4 + 104385u^{10}t^3 \\
 & + 31286u^{11}t^2 + 5715u^{12}t + 420u^{13})s^7 + (259t^{14} + 10260ut^{13} + 111068u^2t^{12} + 486996u^3t^{11} + 1107162u^4t^{10} + 1479900u^5t^9 \\
 & + 1291792u^6t^8 + 895126u^7t^7 + 649134u^8t^6 + 461478u^9t^5 + 237130u^{10}t^4 + 78802u^{11}t^3 + 18489u^{12}t^2 + 2790u^{13}t + 120u^{14})s^6 \\
 & + 2tu(777t^{13} + 16074ut^{12} + 104819u^2t^{11} + 330308u^3t^{10} + 608514u^4t^9 + 739950u^5t^8 + 661734u^6t^7 + 481539u^7t^6 + 294815u^8t^5 \\
 & + 137297u^9t^4 + 40878u^{10}t^3 + 7492u^{11}t^2 + 1339u^{12}t + 150u^{13})s^5 + t^2u^2(3885t^{12} + 50940ut^{11} + 249466u^2t^{10} + 660616u^3t^9 \\
 & + 1107162u^4t^8 + 1284516u^5t^7 + 1095009u^6t^6 + 713186u^7t^5 + 351240u^8t^4 + 115912u^9t^3 + 19215u^{10}t^2 + 850u^{11}t + 109u^{12})s^4 \\
 & + 2t^3u^3(2590t^{11} + 25470ut^{10} + 104819u^2t^9 + 243498u^3t^8 + 363648u^4t^7 + 378078u^5t^6 + 290657u^6t^5 + 170653u^7t^4 + 72977u^8t^3 \\
 & + 18929u^9t^2 + 1903u^{10}t - 34u^{11})s^3 + t^4u^4(t+u)^2(3885t^8 + 24378u^7t + 58427u^2t^6 + 79252u^3t^5 + 74207u^4t^4 + 52498u^5t^3 \\
 & + 26549u^6t^2 + 6892u^7t + 426u^8)s^2 + 2t^5u^5(t+u)^3(777t^6 + 2799u^5t^5 + 4537u^2t^4 + 5167u^3t^3 + 3936u^4t^2 + 1946u^5t + 308u^6)s \\
 & + t^6u^6(t+u)^4(259t^4 + 512ut^3 + 765u^2t^2 + 512u^3t + 259u^4)H(2, 0, y) - 18s^2t^2u^2(t+u)^2((83t^4 + 224u^3t^3 + 312u^2t^2 + 224u^3t \\
 & + 83u^4)s^{16} + (646t^5 + 2236ut^4 + 3738u^2t^3 + 3738u^3t^2 + 2236u^4t + 646u^5)s^{15} + 3(795t^6 + 3434u^5t^5 + 6735u^2t^4 + 8252u^3t^3 + 6735u^4t^2 \\
 & + 3434u^5t + 795u^6)s^{14} + (5590t^7 + 29722ut^6 + 68154u^2t^5 + 96362u^3t^4 + 96362u^4t^3 + 68154u^5t^2 + 29722u^6t + 5590u^7)s^{13} + (9297t^8 \\
 & + 59792u^7t + 162893u^2t^6 + 265856u^3t^5 + 307366u^4t^4 + 265856u^5t^3 + 162893u^6t^2 + 59792u^7t + 9297u^8)s^{12} + 6(1912t^9 + 14697ut^8 \\
 & + 48146u^2t^7 + 92627u^3t^6 + 123322u^4t^5 + 123322u^5t^4 + 92627u^6t^3 + 48146u^7t^2 + 14697u^8t + 1912u^9)s^{11} + (10571t^{10} + 97460ut^9 \\
 & + 390525u^2t^8 + 895950u^3t^7 + 1372200u^4t^6 + 1560372u^5t^5 + 1372200u^6t^4 + 895950u^7t^3 + 390525u^8t^2 + 97460u^9t + 10571u^{10})s^{10} \\
 & + 2(3555t^{11} + 40253ut^{10} + 202415u^2t^9 + 561210u^3t^8 + 989305u^4t^7 + 1257298u^5t^6 + 1257298u^6t^5 + 989305u^7t^4 + 561210u^8t^3 \\
 & + 202415u^9t^2 + 40253u^{10}t + 3555u^{11})s^9 + 3(1100t^{12} + 16144u^{11}t + 105097u^2t^{10} + 356156u^3t^9 + 736699u^4t^8 + 1061804u^5t^7 \\
 & + 1182566u^6t^6 + 1061804u^7t^5 + 736699u^8t^4 + 356156u^9t^3 + 105097u^{10}t^2 + 16144u^{11}t + 1100u^{12})s^8 + 2(471t^{13} + 10059u^{12}t^2
 \end{aligned}$$

$$\begin{aligned}
 &+ 89007u^2t^{11} + 370856u^3t^{10} + 918809u^4t^9 + 1568172u^5t^8 + 2010976u^6t^7 + 2010976u^7t^6 + 1568172u^8t^5 + 918809u^9t^4 + 370856u^{10}t^3 \\
 &+ 89007u^{11}t^2 + 10059u^{12}t + 471u^{13})s^7 + (124t^{14} + 5166ut^{13} + 68942u^2t^{12} + 360694u^3t^{11} + 1093775u^4t^{10} + 2281660u^5t^9 \\
 &+ 3515021u^6t^8 + 4060864u^7t^7 + 3515021u^8t^6 + 2281660u^9t^5 + 1093775u^{10}t^4 + 360694u^{11}t^3 + 68942u^{12}t^2 + 5166u^{13}t + 124u^{14})s^6 \\
 &+ 6tu(102t^{13} + 2718u^{12} + 19299u^{11}t + 75952u^{10}t^2 + 198691u^9t^3 + 370135u^8t^4 + 503605u^7t^5 + 503605u^6t^6 + 370135u^5t^7 \\
 &+ 198691u^4t^8 + 75952u^3t^9 + 19299u^2t^{10} + 2718u^{12}t + 102u^{13})s^5 + t^2u^2(1752t^{12} + 21508ut^{11} + 127396u^2t^{10} + 438996u^3t^9 \\
 &+ 986209u^4t^8 + 1559032u^5t^7 + 1808388u^6t^6 + 1559032u^7t^5 + 986209u^8t^4 + 438996u^9t^3 + 127396u^{10}t^2 + 21508u^{11}t + 1752u^{12})s^4 \\
 &+ 2t^3u^3(834t^{11} + 10755ut^{10} + 52845u^2t^9 + 146657u^3t^8 + 271322u^4t^7 + 363015u^5t^6 + 363015u^6t^5 + 271322u^7t^4 + 146657u^8t^3 \\
 &+ 52845u^9t^2 + 10755u^{10}t + 834u^{11})s^3 + 6t^4u^4(t+u)^2(283t^8 + 1733ut^7 + 4779u^2t^6 + 8360u^3t^5 + 10092u^4t^4 + 8360u^5t^3 + 4779u^6t^2 \\
 &+ 1733u^7t + 283u^8)s^2 + 2t^5u^5(t+u)^3(282t^6 + 1178u^5t^5 + 2417u^2t^4 + 3162u^3t^3 + 2417u^4t^2 + 1178u^5t + 282u^6)s + 110t^6u^6(t+u)^4(t^2 \\
 &+ ut + u^2)^2H(1, z)H(2, 3, y) + 9s^2(t+u)^2(2(20t^8 + 50ut^7 + 21u^2t^6 - 82u^3t^5 - 102u^4t^4 - 82u^5t^3 + 21u^6t^2 + 50u^7t + 20u^8)s^{16} \\
 &+ 2(140t^9 + 505u^8t^8 + 568u^2t^7 - 298u^3t^6 - 1069u^4t^5 - 1069u^5t^4 - 298u^6t^3 + 568u^7t^2 + 505u^8t + 140u^9)s^{15} + (880t^{10} + 4370ut^9 \\
 &+ 8221u^2t^8 + 4262u^3t^7 - 6199u^4t^6 - 10932u^5t^5 - 6199u^6t^4 + 4262u^7t^3 + 8221u^8t^2 + 4370u^9t + 880u^{10})s^{14} + 2(820t^{11} + 5395u^2t^{10} \\
 &+ 14563u^3t^9 + 17988u^3t^8 + 5650u^4t^7 - 8508u^5t^6 - 8508u^6t^5 + 5650u^7t^4 + 17988u^8t^3 + 14563u^9t^2 + 5395u^{10}t + 820u^{11})s^{13} \\
 &+ (2000t^{12} + 16930ut^{11} + 60889u^2t^{10} + 115200u^3t^9 + 112888u^4t^8 + 52018u^5t^7 + 18062u^6t^6 + 52018u^7t^5 + 112888u^8t^4 + 115200u^9t^3 \\
 &+ 60889u^{10}t^2 + 16930u^{11}t + 2000u^{12})s^{12} + 2(820t^{13} + 8825ut^{12} + 40630u^2t^{11} + 106430u^3t^{10} + 162506u^4t^9 + 145433u^5t^8 \\
 &+ 94054u^6t^7 + 94054u^7t^6 + 145433u^8t^5 + 162506u^9t^4 + 106430u^{10}t^3 + 40630u^{11}t^2 + 8825u^{12}t + 820u^{13})s^{11} + (880t^{14} + 12310ut^{13} \\
 &+ 71305u^2t^{12} + 252860u^3t^{11} + 542940u^4t^{10} + 672624u^5t^9 + 495141u^6t^8 + 346704u^7t^7 + 495141u^8t^6 + 672624u^9t^5 + 542940u^{10}t^4 \\
 &+ 252860u^{11}t^3 + 71305u^{12}t^2 + 12310u^{13}t + 880u^{14})s^{10} + 2(140t^{15} + 2785u^2t^{14} + 20491u^2t^{13} + 99450u^3t^{12} + 294436u^4t^{11} \\
 &+ 490859u^5t^{10} + 448931u^6t^9 + 254956u^7t^8 + 254956u^8t^7 + 448931u^9t^6 + 490859u^{10}t^5 + 294436u^{11}t^4 + 99450u^{12}t^3 + 20491u^{13}t^2 \\
 &+ 2785u^{14}t + 140u^{15})s^9 + (40t^{16} + 1490ut^{15} + 14845u^2t^{14} + 102820u^3t^{13} + 419280u^4t^{12} + 950788u^5t^{11} + 1225773u^6t^{10} \\
 &+ 975710u^7t^9 + 743060u^8t^8 + 975710u^9t^7 + 1225773u^{10}t^6 + 950788u^{11}t^5 + 419280u^{12}t^4 + 102820u^{13}t^3 + 14845u^{14}t^2 + 1490u^{15}t \\
 &+ 40u^{16})s^8 + 2tu(90t^{15} + 1540u^2t^{14} + 16932u^2t^{13} + 96123u^3t^{12} + 304282u^4t^{11} + 583798u^5t^{10} + 762212u^6t^9 + 803305u^7t^8 \\
 &+ 803305u^8t^7 + 762212u^9t^6 + 583798u^{10}t^5 + 304282u^{11}t^4 + 96123u^{12}t^3 + 16932u^{13}t^2 + 1540u^{14}t + 90u^{15})s^7 + t^2u^2(282t^{14} \\
 &+ 6622ut^{13} + 54887u^2t^{12} + 264008u^3t^{11} + 776536u^4t^{10} + 1517240u^5t^9 + 2171441u^6t^8 + 2431080u^7t^7 + 2171441u^8t^6 + 1517240u^9t^5 \\
 &+ 776536u^{10}t^4 + 264008u^{11}t^3 + 54887u^{12}t^2 + 6622u^{13}t + 282u^{14})s^6 + 2t^3u^3(298t^{13} + 4418ut^{12} + 40731u^2t^{11} + 190625u^3t^{10} \\
 &+ 505964u^4t^9 + 877790u^5t^8 + 1117090u^6t^7 + 1117090u^7t^6 + 877790u^8t^5 + 505964u^9t^4 + 190625u^{10}t^3 + 40731u^{11}t^2 + 4418u^{12}t \\
 &+ 298u^{13})s^5 + t^4u^4(536t^{12} + 16810ut^{11} + 131975u^2t^{10} + 463360u^3t^9 + 936996u^4t^8 + 1292362u^5t^7 + 1406138u^6t^6 + 1292362u^7t^5 \\
 &+ 936996u^8t^4 + 463360u^9t^3 + 131975u^{10}t^2 + 16810u^{11}t + 536u^{12})s^4 + 2t^5u^5(848t^{11} + 13094ut^{10} + 64600u^2t^9 + 162207u^3t^8 \\
 &+ 255217u^4t^7 + 298424u^5t^6 + 298424u^6t^5 + 255217u^7t^4 + 162207u^8t^3 + 64600u^9t^2 + 13094u^{10}t + 848u^{11})s^3 + t^6u^6(t+u)^2(2062t^8 \\
 &+ 13122ut^7 + 32971u^2t^6 + 48846u^3t^5 + 54292u^4t^4 + 48846u^5t^3 + 32971u^6t^2 + 13122u^7t + 2062u^8)s^2 + 4t^7u^7(t+u)^3(158t^6 \\
 &+ 746ut^5 + 1500u^2t^4 + 1923u^3t^3 + 1500u^4t^2 + 746u^5t + 158u^6)s + 8t^8u^8(t+u)^4(23t^4 + 45ut^3 + 67u^2t^2 + 45u^3t + 23u^4)H(3, 2, y) \\
 &- 36s^2t^2u^3(t+u)^2(-12t(4t^2 + 7ut + 4u^2)s^{16} - (413t^4 + 1112ut^3 + 1066u^2t^2 + 344u^3t - 23u^4)s^{15} - 3(549t^5 + 2080ut^4 + 2910u^2t^3 \\
 &+ 1760u^3t^2 + 293u^4t - 64u^5)s^{14} - (3849t^6 + 19818ut^5 + 38435u^2t^4 + 35276u^3t^3 + 14007u^4t^2 + 482u^5t - 715u^6)s^{13} - (5797t^7 \\
 &+ 40039u^6t^6 + 104248u^2t^5 + 133574u^3t^4 + 86245u^4t^3 + 21235u^5t^2 - 2834u^6t - 1572u^7)s^{12} - 3(1839t^8 + 18438ut^7 + 63926u^2t^6 \\
 &+ 107954u^3t^5 + 97302u^4t^4 + 43534u^5t^3 + 4746u^6t^2 - 2870u^7t - 757u^8)s^{11} - (2469t^9 + 52233ut^8 + 250929u^2t^7 + 546879u^3t^6 \\
 &+ 638973u^4t^5 + 405133u^5t^4 + 113599u^6t^3 - 8307u^7t^2 - 12530u^8t - 2258u^9)s^{10} + (985t^{10} - 31352ut^9 - 228083u^2t^8 - 646444u^3t^7 \\
 &- 969791u^4t^6 - 819288u^5t^5 - 357472u^6t^4 - 38248u^7t^3 + 28876u^8t^2 + 11404u^9t + 1557u^{10})s^9 + 3(727t^{11} - 3289ut^{10} - 45068u^2t^9 \\
 &- 168873u^3t^8 - 330122u^4t^7 - 375394u^5t^6 - 241202u^6t^5 - 66685u^7t^4 + 10259u^8t^3 + 10981u^9t^2 + 2326u^{10}t + 240u^{11})s^8 + (1386t^{12} \\
 &- 6ut^{11} - 44480u^2t^{10} - 241022u^3t^9 - 645063u^4t^8 - 1001980u^5t^7 - 922078u^6t^6 - 455700u^7t^5 - 58195u^8t^4 + 50450u^9t^3 + 22836u^{10}t^2 \\
 &+ 2866u^{11}t + 202u^{12})s^7 + (420t^{13} + 1149ut^{12} + 3u^2t^{11} - 48559u^3t^{10} - 253354u^4t^9 - 575952u^5t^8 - 717884u^6t^7 - 512534u^7t^6 \\
 &- 180305u^8t^5 + 7313u^9t^4 + 31999u^{10}t^3 + 10093u^{11}t^2 + 729u^{12}t + 26u^{13})s^6 + 3t(16t^{13} + 94ut^{12} + 2346u^2t^{11} + 5138u^3t^{10} - 15062u^4t^9 \\
 &- 74024u^5t^8 - 122999u^6t^7 - 106460u^7t^6 - 49523u^8t^5 - 8048u^9t^4 + 4593u^{10}t^3 + 3534u^{11}t^2 + 893u^{12}t + 30u^{13})s^5 + t^2u^2(-6t^{12} \\
 &+ 2684ut^{11} + 12044u^2t^{10} + 7299u^3t^9 - 50331u^4t^8 - 127806u^5t^7 - 131304u^6t^6 - 58843u^7t^5 + 1403u^8t^4 + 13260u^9t^3 + 6238u^{10}t^2 \\
 &+ 1774u^{11}t + 336u^{12})s^4 + 2t^3u^2(166t^{11} + 1146u^2t^{10} + 2486u^2t^9 - 448u^3t^8 - 9790u^4t^7 - 15423u^5t^6 - 8416u^6t^5 + 3041u^7t^4
 \end{aligned}$$

$$\begin{aligned}
& + 7002u^8t^3 + 4035u^9t^2 + 960u^{10}t + 57u^{11})s^3 + 3t^4u^3(t+u)^2(15t^8 + 240ut^7 + 369u^2t^6 - u^3t^5 - 172u^4t^4 + 126u^5t^3 + 429u^6t^2 \\
& + 355u^7t + 103u^8)s^2 + t^5u^4(t+u)^3(90t^6 + 272ut^5 + 116u^2t^4 + 114u^3t^3 + 135u^4t^2 + 117u^5t + 66u^6)s + t^6u^5(t+u)^4(11t^4 - 2ut^3 \\
& + 15u^2t^2 + 14u^3t + 19u^4)H(0, 0, 1, z) + 18s^2t^2(s+t)^2u^2(2(55t^6 + 282ut^5 + 849u^2t^4 + 834u^3t^3 + 876u^4t^2 + 306u^5t + 62u^6)s^{14} \\
& + 2(330t^7 + 2024ut^6 + 6897u^2t^5 + 10755u^3t^4 + 10754u^4t^3 + 8154u^5t^2 + 2583u^6t + 471u^7)s^{13} + 2(935t^8 + 6797ut^7 + 25584u^2t^6 \\
& + 52845u^3t^5 + 63698u^4t^4 + 57897u^5t^3 + 34471u^6t^2 + 10059u^7t + 1650u^8)s^{12} + 2(1650t^9 + 14229ut^8 + 58953u^2t^7 + 146657u^3t^6 \\
& + 219498u^4t^5 + 227856u^5t^4 + 180347u^6t^3 + 89007u^7t^2 + 24216u^8t + 3555u^9)s^{11} + (3960t^{10} + 40664ut^9 + 189546u^2t^8 + 542644u^3t^7 \\
& + 986209u^4t^6 + 1192146u^5t^5 + 1093775u^6t^4 + 741712u^7t^3 + 315291u^8t^2 + 80506u^9t + 10571u^{10})s^{10} + 2(1650t^{11} + 20332ut^{10} \\
& + 110712u^2t^9 + 363015u^3t^8 + 779516u^4t^7 + 1110405u^5t^6 + 1140830u^6t^5 + 918809u^7t^4 + 534234u^8t^3 + 202415u^9t^2 + 48730u^{10}t \\
& + 5736u^{11})s^9 + (1870t^{12} + 28458ut^{11} + 189546u^2t^{10} + 726030u^3t^9 + 1808388u^4t^8 + 3021630u^5t^7 + 3515021u^6t^6 + 3136344u^7t^5 \\
& + 2210097u^8t^4 + 1122420u^9t^3 + 390525u^{10}t^2 + 88182u^{11}t + 9297u^{12})s^8 + 2(330t^{13} + 6797ut^{12} + 58953u^2t^{11} + 271322u^3t^{10} \\
& + 779516u^4t^9 + 1510815u^5t^8 + 2030432u^6t^7 + 2010976u^7t^6 + 1592706u^8t^5 + 989305u^9t^4 + 447975u^{10}t^3 + 144438u^{11}t^2 + 29896u^{12}t \\
& + 2795u^{13})s^7 + (110t^{14} + 4048ut^{13} + 51168u^2t^{12} + 293314u^3t^{11} + 986209u^4t^{10} + 2220810u^5t^9 + 3515021u^6t^8 + 4021952u^7t^7 \\
& + 3547698u^8t^6 + 2514596u^9t^5 + 1372200u^{10}t^4 + 555762u^{11}t^3 + 162893u^{12}t^2 + 29722u^{13}t + 2385u^{14})s^6 + 2u(282t^{14} + 6897ut^{13} \\
& + 52845u^2t^{12} + 219498u^3t^{11} + 596073u^4t^{10} + 1140830u^5t^9 + 1568172u^6t^8 + 1592706u^7t^7 + 1257298u^8t^6 + 780186u^9t^5 + 369966u^{10}t^4 \\
& + 132928u^{11}t^3 + 34077u^{12}t^2 + 5151u^{13}t + 323u^{14})s^5 + u^2(1698t^{14} + 21510u^2t^{13} + 127396u^2t^{12} + 455712u^3t^{11} + 1093775u^4t^{10} \\
& + 1837618u^5t^9 + 2210097u^6t^8 + 1978610u^7t^7 + 1372200u^8t^6 + 739932u^9t^5 + 307366u^{10}t^4 + 96362u^{11}t^3 + 20205u^{12}t^2 + 2236u^{13}t \\
& + 83u^{14})s^4 + 2tu^3(834t^{13} + 10754ut^{12} + 57897u^2t^{11} + 180347u^3t^{10} + 370856u^4t^9 + 534234u^5t^8 + 561210u^6t^7 + 447975u^7t^6 \\
& + 277881u^8t^5 + 132928u^9t^4 + 48181u^{10}t^3 + 12378u^{11}t^2 + 1869u^{12}t + 112u^{13})s^3 + t^2u^4(t+u)^2(1752t^{10} + 12804ut^9 + 41582u^2t^8 \\
& + 82046u^3t^7 + 109617u^4t^6 + 103550u^5t^5 + 73808u^6t^4 + 37710u^7t^3 + 13665u^8t^2 + 3114u^9t + 312u^{10})s^2 + 2t^3u^5(t+u)^3(306t^8 \\
& + 1665ut^7 + 4146u^2t^6 + 6477u^3t^5 + 6719u^4t^4 + 4996u^5t^3 + 2469u^6t^2 + 782u^7t + 112u^8)s + t^4u^6(t+u)^6(124t^4 + 198ut^3 + 252u^2t^2 \\
& + 148u^3t + 83u^4)H(0, 1, 0, y) - 18s^2t^2u^2(t+u)^3(2t(7t^2 + 15ut + 12u^2)s^{16} + 12(8t^4 + 24ut^3 + 29u^2t^2 + 12u^3t - u^4)s^{15} \\
& + 3(101t^5 + 423ut^4 + 720u^2t^3 + 556u^3t^2 + 111u^4t - 31u^5)s^{14} + (598t^6 + 3066ut^5 + 7134u^2t^4 + 8992u^3t^3 + 5154u^4t^2 + 510u^5t \\
& - 318u^6)s^{13} + (851t^7 + 4495ut^6 + 13290u^2t^5 + 25506u^3t^4 + 26949u^4t^3 + 12309u^5t^2 + 688u^6t - 652u^7)s^{12} + 6(158t^8 + 586u^7t^7 \\
& + 2628u^2t^6 + 7708u^3t^5 + 11649u^4t^4 + 9092u^5t^3 + 3324u^6t^2 + 218u^7t - 155u^8)s^{11} + (851t^9 - 417ut^8 + 9564u^2t^7 + 62150u^3t^6 \\
& + 123813u^4t^5 + 121269u^5t^4 + 67982u^6t^3 + 23208u^7t^2 + 2838u^8t - 1002u^9)s^{10} + 2(299t^{10} - 2075ut^9 - 1896u^2t^8 + 23751u^3t^7 \\
& + 72903u^4t^6 + 97515u^5t^5 + 71423u^6t^4 + 31963u^7t^3 + 10800u^8t^2 + 1966u^9t - 409u^{10})s^9 + 3(101t^{11} - 1463ut^{10} - 3896u^2t^9 \\
& + 208u^3t^8 + 20429u^4t^7 + 58669u^5t^6 + 80268u^6t^5 + 55172u^7t^4 + 19400u^8t^3 + 4832u^9t^2 + 1000u^{10}t - 156u^{11})s^8 + 2(48t^{12} - 1128ut^{11} \\
& - 4410u^2t^{10} - 17090u^3t^9 - 31983u^4t^8 + 6084u^5t^7 + 100874u^6t^6 + 143160u^7t^5 + 88338u^8t^4 + 23800u^9t^3 + 2622u^{10}t^2 + 582u^{11}t \\
& - 81u^{12})s^7 + (14t^{13} - 572u^2t^{12} - 3036u^2t^{11} - 35818u^3t^{10} - 106376u^4t^9 - 101688u^5t^8 + 48822u^6t^7 + 213630u^7t^6 + 239397u^8t^5 \\
& + 134809u^9t^4 + 31274u^{10}t^3 - 36u^{11}t^2 + 157u^{12}t - 25u^{13})s^6 - 6tu(8t^{12} + 37u^2t^{11} + 3213u^2t^{10} + 11984u^3t^9 + 14592u^4t^8 + 1987u^5t^7 \\
& - 12914u^6t^6 - 20563u^7t^5 - 20499u^8t^4 - 11612u^9t^3 - 2423u^{10}t^2 + 147u^{11}t + 3u^{12})s^5 + t^2u^2(102t^{11} - 5234ut^{10} - 25530u^2t^9 \\
& - 39126u^3t^8 - 12408u^4t^7 + 24648u^5t^6 + 35505u^6t^5 + 40505u^7t^4 + 40476u^8t^3 + 21240u^9t^2 + 3629u^{10}t - 267u^{11})s^4 + 2t^3u^3(- \\
& - 266t^{10} - 1804ut^9 - 4752u^2t^8 - 4830u^3t^7 + 378u^4t^6 + 5301u^5t^5 + 6806u^6t^4 + 6178u^7t^3 + 3642u^8t^2 + 1119u^9t + 156u^{10})s^3 \\
& + 3t^4u^4(t+u)^2(16t^7 - 382ut^6 - 532u^2t^5 + 8u^3t^4 + 467u^4t^3 + 637u^5t^2 + 265u^6t - 71u^7)s^2 - 2t^5u^5(t+u)^3(48t^5 + 151ut^4 \\
& - 84u^2t^3 + 18u^3t^2 - 160u^4t - 15u^5)s + t^6u^7(t+u)^3(12t^3 + 3ut^2 + 2u^2t - 11u^3)H(0, 3, 2, y) - 18s^2t^2u^2(t+u)^2((55t^4 \\
& + 148ut^3 + 192u^2t^2 + 128u^3t + 47u^4)s^{16} + 2(232t^5 + 807ut^4 + 1262u^2t^3 + 1126u^3t^2 + 612u^4t + 173u^5)s^{15} + 6(305t^6 + 1324ut^5 \\
& + 2458u^2t^4 + 2648u^3t^3 + 1862u^4t^2 + 872u^5t + 201u^6)s^{14} + 2(2230t^7 + 11932ut^6 + 26409u^2t^5 + 33217u^3t^4 + 27682u^4t^3 \\
& + 16830u^5t^2 + 7005u^6t + 1347u^7)s^{13} + (7450t^8 + 48274ut^7 + 129744u^2t^6 + 195540u^3t^5 + 190555u^4t^4 + 134098u^5t^3 + 72590u^6t^2 \\
& + 26532u^7t + 4375u^8)s^{12} + 6(1486t^9 + 11488ut^8 + 37756u^2t^7 + 69937u^3t^6 + 82622u^4t^5 + 67718u^5t^4 + 41832u^6t^3 + 19931u^7t^2 \\
& + 6288u^8t + 910u^9)s^{11} + (7744t^{10} + 71242ut^9 + 290076u^2t^8 + 661446u^3t^7 + 946629u^4t^6 + 926538u^5t^5 + 671989u^6t^4 + 379294u^7t^3 \\
& + 157923u^8t^2 + 41472u^9t + 5271u^{10})s^{10} + 2(2410t^{11} + 26794ut^{10} + 138966u^2t^9 + 391138u^3t^8 + 666867u^4t^7 + 763472u^5t^6 \\
& + 652628u^6t^5 + 448145u^7t^4 + 237429u^8t^3 + 84416u^9t^2 + 17568u^{10}t + 1903u^{11})s^9 + 3(685t^{12} + 9638ut^{11} + 66160u^2t^{10} + 229988u^3t^9 \\
& + 468878u^4t^8 + 632074u^5t^7 + 626493u^6t^6 + 499908u^7t^5 + 326013u^8t^4 + 156448u^9t^3 + 46699u^{10}t^2 + 7420u^{11}t + 638u^{12})s^8 \\
& + 2(270t^{13} + 5385ut^{12} + 51810u^2t^{11} + 219533u^3t^{10} + 538700u^4t^9 + 893700u^5t^8 + 1082558u^6t^7 + 993074u^7t^6 + 712998u^8t^5 \\
& + 405820u^9t^4 + 169330u^{10}t^3 + 42843u^{11}t^2 + 4984u^{12}t + 295u^{13})s^7 + (66t^{14} + 2518ut^{13} + 38032u^2t^{12} + 195010u^3t^{11} + 579501u^4t^{10}
\end{aligned}$$



$$\begin{aligned}
& +1219748u^5t^9 + 1901521u^6t^8 + 2145380u^7t^7 + 1729402u^8t^6 + 1029790u^9t^5 + 480198u^{10}t^4 + 170114u^{11}t^3 + 36457u^{12}t^2 + 2808u^{13}t \\
& + 83u^{14})s^6 + 6tu(46t^{13} + 1463u^2t^{12} + 9551u^3t^{11} + 36573u^4t^{10} + 99722u^5t^9 + 196920u^6t^8 + 275717u^7t^7 + 269768u^8t^6 + 184750u^9t^5 \\
& + 90822u^{10}t^4 + 33632u^{11}t^3 + 9403u^{12}t^2 + 1592u^{13}t + 61u^{14})s^5 + t^2u^2(936t^{12} + 9592ut^{11} + 58498u^2t^{10} + 216504u^3t^9 + 517783u^4t^8 \\
& + 852472u^5t^7 + 999621u^6t^6 + 842754u^7t^5 + 506934u^8t^4 + 212352u^9t^3 + 59561u^{10}t^2 + 10778u^{11}t + 1137u^{12})s^4 + 2t^3u^3(306t^{11} \\
& + 5120u^{10} + 27107u^9t + 77806u^8t^2 + 147341u^7t^3 + 200386u^6t^4 + 201078u^5t^5 + 148590u^4t^6 + 78861u^3t^7 + 28022u^2t^8 + 5655u^1t^9 \\
& + 424u^{11})s^3 + 3t^4u^4(t+u)^2(312t^8 + 1862ut^7 + 5114u^2t^6 + 9264u^3t^5 + 11523u^4t^4 + 9432u^5t^3 + 5226u^6t^2 + 1942u^7t + 361u^8)s^2 \\
& + 2t^5u^5(t+u)^3(138t^6 + 635ut^5 + 1406u^2t^4 + 1938u^3t^3 + 1431u^4t^2 + 633u^5t + 159u^6)s + 3t^6u^6(t+u)^4(22t^4 + 48ut^3 + 71u^2t^2 \\
& + 46u^3t + 23u^4)H(1, 0, 1, z) - 18s^2t^2(s+t)^3u^2((11t^6 - 30ut^5 + 213u^2t^4 - 312u^3t^3 + 267u^4t^2 + 18u^5t + 25u^6)s^{13} + (31t^7 - 410ut^6 \\
& - 369u^2t^5 - 2238u^3t^4 - 3629u^4t^3 + 882u^5t^2 - 157u^6t + 162u^7)s^{12} + 6(4t^8 - 169ut^7 - 548u^2t^6 - 1214u^3t^5 - 3540u^4t^4 - 2423u^5t^3 \\
& + 6u^6t^2 - 194u^7t + 78u^8)s^{11} - 2(8t^9 + 525ut^8 + 3009u^2t^7 + 6178u^3t^6 + 20238u^4t^5 + 34836u^5t^4 + 15637u^6t^3 + 2622u^7t^2 + 1500u^8t \\
& - 409u^9)s^{10} - (47t^{10} + 414ut^9 + 4737u^2t^8 + 13612u^3t^7 + 40505u^4t^6 + 122994u^5t^5 + 134809u^6t^4 + 47600u^7t^3 + 14496u^8t^2 + 3932u^9t \\
& - 1002u^{10})s^9 - 3(13t^{11} - 178ut^{10} - 49u^2t^9 + 3534u^3t^8 + 11835u^4t^7 + 41126u^5t^6 + 79799u^6t^5 + 58892u^7t^4 + 19400u^8t^3 + 7200u^9t^2 \\
& + 946u^{10}t - 310u^{11})s^8 - 2(6t^{12} - 513ut^{11} - 2157u^2t^{10} + 378u^3t^9 + 12324u^4t^8 + 38742u^5t^7 + 106815u^6t^6 + 143160u^7t^5 \\
& + 82758u^8t^4 + 31963u^9t^3 + 11604u^{10}t^2 + 654u^{11}t - 326u^{12})s^7 + 2u(295t^{12} + 1920u^2t^{10} + 4830u^2t^{10} + 6204u^3t^9 + 5961u^4t^8 \\
& - 2441u^5t^7 - 100874u^6t^6 - 120402u^7t^5 - 71423u^8t^4 - 33991u^9t^3 - 9972u^{10}t^2 - 344u^{11}t + 159u^{12})s^6 + 3u(32t^{13} + 350ut^{12} \\
& + 3168u^2t^{11} + 13042u^3t^{10} + 29184u^4t^9 + 33896u^5t^8 - 4056u^6t^7 - 58669u^7t^6 - 65010u^8t^5 - 40423u^9t^4 - 18184u^{10}t^3 \\
& - 4103u^{11}t^2 - 170u^{12}t + 31u^{13})s^5 + u^2(-48t^{13} + 3608ut^{12} + 25530u^2t^{11} + 71904u^3t^{10} + 106376u^4t^9 + 63966u^5t^8 - 61287u^6t^7 \\
& - 145806u^7t^6 - 123813u^8t^5 - 69894u^9t^4 - 26949u^{10}t^3 - 5154u^{11}t^2 - 333u^{12}t + 12u^{13})s^4 + 2tu^3(266t^{12} + 2617ut^{11} + 9639u^2t^{10} \\
& + 17909u^3t^9 + 17090u^4t^8 - 312u^5t^7 - 23751u^6t^6 - 31075u^7t^5 - 23124u^8t^4 - 12753u^9t^3 - 4496u^{10}t^2 - 834u^{11}t - 72u^{12})s^3 \\
& - 6tu^4(t+u)^2(17t^{10} - 71ut^9 - 381u^2t^8 - 637u^3t^7 - 293u^4t^6 + 591u^5t^5 + 705u^6t^4 + 627u^7t^3 + 256u^8t^2 + 50u^9t + 4u^{10})s^2 \\
& + t^2u^5(t+u)^3(48t^8 + 428ut^7 + 828u^2t^6 + 573u^3t^5 - 481u^4t^4 - 687u^5t^3 - 585u^6t^2 - 198u^7t - 30u^8)s - t^3u^6(t+u)^6(14t^4 \\
& + 12ut^3 + 21u^2t^2 + 12u^3t + 14u^4)H(2, 1, 0, y) - 18s^2t^2u^2(t+u)^2((83t^4 + 224ut^3 + 312u^2t^2 + 224u^3t + 83u^4)s^{16} + (646t^5 \\
& + 2236ut^4 + 3738u^2t^3 + 3738u^3t^2 + 2236u^4t + 646u^5)s^{15} + 3(795t^6 + 3434u^5t^5 + 6735u^2t^4 + 8252u^3t^3 + 6735u^4t^2 + 3434u^5t \\
& + 795u^6)s^{14} + (5590t^7 + 29722ut^6 + 68154u^2t^5 + 96362u^3t^4 + 96362u^4t^3 + 68154u^5t^2 + 29722u^6t + 5590u^7)s^{13} + (9297t^8 \\
& + 59792u^7t^7 + 162893u^2t^6 + 265856u^3t^5 + 307366u^4t^4 + 265856u^5t^3 + 162893u^6t^2 + 59792u^7t + 9297u^8)s^{12} + 6(1912t^9 \\
& + 14697ut^8 + 48146u^2t^7 + 92627u^3t^6 + 123322u^4t^5 + 123322u^5t^4 + 92627u^6t^3 + 48146u^7t^2 + 14697u^8t + 1912u^9)s^{11} \\
& + (10571t^{10} + 97460ut^9 + 390525u^2t^8 + 895950u^3t^7 + 1372200u^4t^6 + 1560372u^5t^5 + 1372200u^6t^4 + 895950u^7t^3 + 390525u^8t^2 \\
& + 97460u^9t + 10571u^{10})s^{10} + 2(3555t^{11} + 40253ut^{10} + 202415u^2t^9 + 561210u^3t^8 + 989305u^4t^7 + 1257298u^5t^6 + 1257298u^6t^5 \\
& + 989305u^7t^4 + 561210u^8t^3 + 202415u^9t^2 + 40253u^{10}t + 3555u^{11})s^9 + 3(1100t^{12} + 16144u^7t^{11} + 105097u^2t^{10} + 356156u^3t^9 \\
& + 736699u^4t^8 + 1061804u^5t^7 + 1182566u^6t^6 + 1061804u^7t^5 + 736699u^8t^4 + 356156u^9t^3 + 105097u^{10}t^2 + 16144u^{11}t + 1100u^{12})s^8 \\
& + 2(471t^{13} + 10059u^2t^{12} + 89007u^2t^{11} + 370856u^3t^{10} + 918809u^4t^9 + 1568172u^5t^8 + 2010976u^6t^7 + 2010976u^7t^6 + 1568172u^8t^5 \\
& + 918809u^9t^4 + 370856u^{10}t^3 + 89007u^{11}t^2 + 10059u^{12}t + 471u^{13})s^7 + (124t^{14} + 5166ut^{13} + 68942u^2t^{12} + 360694u^3t^{11} \\
& + 1093775u^4t^{10} + 2281660u^5t^9 + 3515021u^6t^8 + 4060864u^7t^7 + 3515021u^8t^6 + 2281660u^9t^5 + 1093775u^{10}t^4 + 360694u^{11}t^3 \\
& + 68942u^{12}t^2 + 5166u^{13}t + 124u^{14})s^6 + 6tu(102t^{13} + 2718u^2t^{12} + 19299u^2t^{11} + 75952u^3t^{10} + 198691u^4t^9 + 370135u^5t^8 \\
& + 503605u^6t^7 + 503605u^7t^6 + 370135u^8t^5 + 198691u^9t^4 + 75952u^{10}t^3 + 19299u^{11}t^2 + 2718u^{12}t + 102u^{13})s^5 + t^2u^2(1752t^{12} \\
& + 21508ut^{11} + 127396u^2t^{10} + 438996u^3t^9 + 986209u^4t^8 + 1559032u^5t^7 + 1808388u^6t^6 + 1559032u^7t^5 + 986209u^8t^4 + 438996u^9t^3 \\
& + 127396u^{10}t^2 + 21508u^{11}t + 1752u^{12})s^4 + 2t^3u^3(834t^{11} + 10755ut^{10} + 52845u^2t^9 + 146657u^3t^8 + 271322u^4t^7 + 363015u^5t^6 \\
& + 363015u^6t^5 + 271322u^7t^4 + 146657u^8t^3 + 52845u^9t^2 + 10755u^{10}t + 834u^{11})s^3 + 6t^4u^4(t+u)^2(283t^8 + 1733ut^7 + 4779u^2t^6 \\
& + 8360u^3t^5 + 10092u^4t^4 + 8360u^5t^3 + 4779u^6t^2 + 1733u^7t + 283u^8)s^2 + 2t^5u^5(t+u)^3(282t^6 + 1178u^5t^5 + 2417u^2t^4 + 3162u^3t^3 \\
& + 2417u^4t^2 + 1178u^5t + 282u^6)s + 110t^6u^6(t+u)^4(t^2 + ut + u^2)^2H(2, 3, 2, y)\Big\} / (27s^3t^3u^3(s+t)^6(s+u)^6(t+u)^6)
\end{aligned}$$

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