

# Three loop form factors of a massive spin-2 particle with nonuniversal coupling

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We investigate the interaction of spin-2 fields with those of the Standard Model in a model independent framework. We have considered interactions where the spin-2 fields couple to two sets of gauge invariant tensorial operators that are not conserved unlike the energy momentum tensor with different coupling strengths. Such interactions not only change the ultraviolet behavior of the couplings but also expand the scope of the searches of spin-2 particles at the colliders. We present all the relevant renormalization constants up to three loop level in QCD and also the form factors that contribute to potential observables. This sets the ground to investigate the phenomenological consequences of these interactions with spin-2 fields through more than one tensorial operator.

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## I. INTRODUCTION

The discovery of the Higgs boson by ATLAS [1] and CMS [2] Collaborations at the LHC has put the Standard Model (SM) on a strong footing. The ongoing precise measurements with the Higgs boson will shed light on the nature of its coupling to the particles of the SM. While the SM has been enormously successful, it is not a complete theory of particle physics. For example, it does not accommodate dark matter, nonzero neutrino mass, etc. These are some of the compelling reasons to go further to investigate the physics beyond the SM (BSM). In this context, supersymmetric extensions of the SM have been studied intensively both theoretically as well as experimentally. Similarly, models with extra dimensions, the strong contenders to supersymmetry (SUSY), have also been studied extensively. These models contain extra degrees of freedom through additional spin-2 bosons.

In the Arkani-Dimopoulos-Dvali (ADD) [3–5] and the Randall-Sundrum (RS) [6] models spin-2 particles couple to the SM particles through energy momentum (EM) tensor of the SM with a single coupling denoted by  $\kappa$ . The phenomenology with this universal coupling has been studied rigorously. There are also phenomenological investigations with spin-2 particles with nonuniversal coupling to the SM particles. While the later may not belong to any of the popular extra-dimensional models, they can provide an opportunity to study the distinct signatures at the colliders which is not

possible with theories with universal couplings. Independent of the nature of these couplings, these are effective theories and hence non-renormalizable in the conventional sense. In the ADD and the RS, thanks to conservation of EM tensor of the SM, the leading interaction term that describes the coupling of spin-2 with those of the SM does not require any additional renormalization. To this order in the coupling, which is good enough for all the phenomenological studies, the infrared (IR) structure of the SM is also not affected and hence factorization properties continue to hold. This allows us to compute successfully various observables beyond leading order in the SM coupling using perturbative methods. All the infrared singularities do cancel giving finite perturbative results that can be used to constrain the model parameters unambiguously. While this is true with theories with an interaction term containing a conserved EM tensor, it is not clear how the ultraviolet (UV) and IR structure would look like when spin-2 couples to particles of the SM with different (nonuniversal) couplings.

Soon after the discovery of the 125 GeV boson, these models with spin-2 nonuniversal coupling, have become important in the context of imposter to the Higgs boson [7–9]. This was necessary to extend the scope of spin-2 models as the experimental bounds on RS resonance (universal coupling) was much higher [10–17]. To next-to-leading order (NLO) in QCD the UV and IR behavior for the nonuniversal couplings for a spin-2 had been studied in the context of Higgs characterisation [18]. UV renormalization is needed as a result of the nonuniversal couplings and with regard to the IR structure, the double and single pole terms contained the appropriate universal IR coefficients that canceled against those coming from real emission processes and mass factorization counterterms. This was a demonstration of IR factorization to NLO for the nonuniversal couplings scenario [18]. Recently, in the context of the 750 GeV spin-2 resonance with nonuniversal couplings have been considered [19] to NLO + parton shower (PS) accuracy.

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The SM at the LHC is being scrutinized at an unprecedented level of precision. It is only natural to have the competing BSM scenarios match the same order of accuracy in QCD as the SM observables. At the hadron collider, the first step to such a phenomenological study would be to compute form factors to the production of a singlet on-shell state  $X$  via the quark  $q\bar{q} \rightarrow X$  or gluon  $gg \rightarrow X$  production channels, to the same order of accuracy. Presently, form factors are available to up to the three loop level in the SM [20–24] for some BSM spin-2 that couples to the EM tensor [25,26] and for the pseudoscalar Higgs boson [27]. NLO QCD corrections have been computed for the extra dimension models viz. ADD and RS for most of the di-final state process [28–35] and this has been extended to NLO + PS accuracy [36–38]. Recently, for the di-lepton production to next-to-next-to-leading order (NNLO) in QCD for the ADD model was computed [39].

In this article, we investigate the UV structure of the interaction term up to the three loop level in QCD. We restrict ourself to the QCD sector of the SM because the phenomenology with such operators has immediate application at the LHC where such interactions are probed by strong interaction. There are of course many ways spin-2 can couple to the SM. Here, we study the minimal version where spin-2 fields couple to QCD through two different operators with two different couplings, each operator is invariant under gauge group of QCD. Note that spin-2 is the gauge singlet. These operators are rank-2 but unfortunately not conserved unlike the EM tensor of QCD [40]. As a consequence of this, both the operators as well as the couplings get an additional UV renormalization order by order in perturbation theory.

In addition, we intend to compute the on-shell form factors of these operators between quark and gluon states that are important ingredients of any observable at the LHC to study such interactions. In this article we achieve this by computing the on-shell form factors of these operators. This is possible, thanks to the universal IR structure QCD amplitudes even in the case of a nonuniversal spin-2 coupling.

In Sec. II, we describe the theoretical framework, which includes the details of the interaction Lagrangian, UV and IR renormalization procedure. Computational details, the unrenormalized form factors and anomalous dimensions to the three loop level are given in Sec. III. Finally, we present our conclusions in Sec. IV.

## II. THEORETICAL FRAMEWORK

### A. The effective action

The minimal effective action that describes the coupling of spin-2 fields denoted by  $h_{\mu\nu}$  with those of QCD consists of two gauge invariant operators  $\hat{\mathcal{O}}^{G,\mu\nu}$  and  $\hat{\mathcal{O}}^{Q,\mu\nu}$ <sup>1</sup>:

$$S = \int d^4x \mathcal{L}_{\text{QCD}} - \frac{1}{2} \int d^4x h_{\mu\nu}(x) (\hat{\kappa}_G \hat{\mathcal{O}}^{G,\mu\nu}(x) + \hat{\kappa}_Q \hat{\mathcal{O}}^{Q,\mu\nu}(x)) \quad (2.1)$$

where  $\hat{\kappa}_I, I = G, Q$  are dimensionful couplings.  $G$  denotes the pure gauge sector, while  $Q$  denotes the fermionic sector and its gauge interaction. The gauge invariant operators  $\hat{\mathcal{O}}^{G,\mu\nu}$  and  $\hat{\mathcal{O}}^{Q,\mu\nu}$  are given by

$$\begin{aligned} \hat{\mathcal{O}}_{\mu\nu}^G = & \frac{1}{4} g_{\mu\nu} \hat{F}_{\alpha\beta}^a \hat{F}^{a\alpha\beta} - \hat{F}_{\mu\rho}^a \hat{F}_{\nu}^{a\rho} - \frac{1}{\xi} g_{\mu\nu} \partial^\rho (\hat{A}_\rho^a \partial^\sigma \hat{A}_\sigma^a) \\ & - \frac{1}{2\xi} g_{\mu\nu} \partial_\alpha \hat{A}^{a\alpha} \partial_\beta \hat{A}^{a\beta} + \frac{1}{\xi} (\hat{A}_\nu^a \partial_\mu (\partial^\sigma \hat{A}_\sigma^a) + \hat{A}_\mu^a \partial_\nu (\partial^\sigma \hat{A}_\sigma^a)) \\ & + \partial_\mu \hat{\omega}^a (\partial_\nu \hat{\omega}^a - \hat{g}_s f^{abc} \hat{A}_\nu^c \hat{\omega}^b) \\ & + \partial_\nu \hat{\omega}^a (\partial_\mu \hat{\omega}^a - \hat{g}_s f^{abc} \hat{A}_\mu^c \hat{\omega}^b) \\ & - g_{\mu\nu} \partial_\alpha \hat{\omega}^a (\partial^\alpha \hat{\omega}^a - \hat{g}_s f^{abc} \hat{A}^{ca} \hat{\omega}^b), \end{aligned} \quad (2.2)$$

$$\begin{aligned} \hat{\mathcal{O}}_{\mu\nu}^Q = & \frac{i}{4} [\bar{\psi} \gamma_\mu (\vec{\partial}_\nu - i\hat{g}_s T^a \hat{A}_\nu^a) \psi - \bar{\psi} (\vec{\partial}_\nu + i\hat{g}_s T^a \hat{A}_\nu^a) \gamma_\mu \psi \\ & + \bar{\psi} \gamma_\nu (\vec{\partial}_\mu - i\hat{g}_s T^a \hat{A}_\mu^a) \psi - \bar{\psi} (\vec{\partial}_\mu + i\hat{g}_s T^a \hat{A}_\mu^a) \gamma_\nu \psi] \\ & - i g_{\mu\nu} \bar{\psi} \gamma^\alpha (\vec{\partial}_\alpha - i\hat{g}_s T^a \hat{A}_\alpha^a) \psi \end{aligned} \quad (2.3)$$

where  $\hat{A}_\mu^a, \hat{\psi}, \hat{\omega}^a$  and  $h_{\mu\nu}$  are gauge, quark, ghost and spin-2 fields, respectively.  $\hat{g}_s$  is the strong coupling constant and  $\xi$  is the gauge fixing parameter. The hat on all the quantities indicate that they are bare/unrenormalized.  $T^a$  and  $f^{abc}$  are the Gell-Mann matrices and structure constants of SU(N) gauge theory, respectively. In the above, we have retained terms only up to order  $\hat{\kappa}$  and in the rest of the paper, we restrict ourselves to this approximation.

### B. Ultraviolet renormalization

Note that the sum  $\hat{\mathcal{O}}^{G,\mu\nu} + \hat{\mathcal{O}}^{Q,\mu\nu}$  is nothing but the EM tensor of QCD. Unlike the EM tensor, neither of these composite operators is individually conserved and hence are not protected by QCD radiative corrections. In other words, they develop additional UV divergences which need to be factored out in terms of UV renormalization constants and then removed by renormalization procedure. This is achieved by renormalizing bare coupling constants  $\hat{\kappa}_I, I = G, Q$  with the help of those renormalization constants. The resulting interaction terms expressed in terms of renormalized operators with appropriate renormalized couplings are guaranteed to predict UV finite correlation functions to all orders in the strong coupling constant. Note that, the operator  $\hat{\mathcal{O}}^{G,\mu\nu}$  is free from quark fields which means in the theory where spin-2 field couples exclusively to the pure Yang-Mills, the operator  $\hat{\mathcal{O}}^{G,\mu\nu}$  is conserved. However, in the presence of the quark fields in QCD, this property ceases to hold true beyond tree level.

<sup>1</sup>This is not the unique decomposition of an original EM tensor. One can adjust gauge invariant terms between these two.

The most commonly used method of obtaining the renormalization constants in quantum field theory is to compute off-shell amplitudes and extract the UV divergent contributions order by order in perturbation theory. For composite operators, there exists an alternative approach, namely, the method of the operator product expansion. We will not follow any of these approaches in this article. Instead, we apply the method discussed in [27] to obtain both UV renormalization constants as well as on-shell form factors of these operators. In [27] we have demonstrated that UV renormalization constants of composite operators can be extracted order by order in perturbation theory from their on-shell form factors exploiting their universal IR structure. Note that the renormalized on-shell form factors are important components of higher order radiative corrections to observables as they give contributions to the pure virtual part and hence will be useful for further studies.

We use dimensional regularization to regulate both UV and IR divergences. The space-time dimension is taken to be  $d = 4 + \epsilon$ . Both these divergences appear as poles in  $\epsilon$ . Introducing the scale  $\mu$  to keep the bare strong coupling constant  $\hat{a}_s \equiv \hat{g}_s^2/16\pi^2$  dimensionless, we relate bare strong coupling constant  $\hat{a}_s$  to the renormalized one  $a_s \equiv a_s(\mu_R^2)$ , at renormalization scale  $\mu_R$ , through

$$\hat{a}_s S_\epsilon = \left(\frac{\mu^2}{\mu_R^2}\right)^{\epsilon/2} Z_{a_s} a_s \quad (2.4)$$

with  $S_\epsilon = \exp[(\gamma_E - \ln 4\pi)\epsilon/2]$ , where  $\gamma_E$  is the Euler constant. The renormalization constant  $Z_{a_s}$  up to  $\mathcal{O}(a_s^3)$  is given by

$$Z_{a_s} = 1 + a_s \left[ \frac{2}{\epsilon} \beta_0 \right] + a_s^2 \left[ \frac{4}{\epsilon^2} \beta_0^2 + \frac{1}{\epsilon} \beta_1 \right] + a_s^3 \left[ \frac{8}{\epsilon^3} \beta_0^3 + \frac{14}{3\epsilon^2} \beta_0 \beta_1 + \frac{2}{3\epsilon} \beta_2 \right]. \quad (2.5)$$

$\beta_i$ 's are the coefficients of QCD  $\beta$  function [41].

According to the Joglekar and Lee theorem [42], the two operators  $\mathcal{O}^I$  are closed under renormalization which can be accomplished through the renormalization mixing matrix  $Z$ , as follows:

$$\begin{bmatrix} \mathcal{O}^G \\ \mathcal{O}^Q \end{bmatrix} = \begin{bmatrix} Z_{GG} & Z_{GQ} \\ Z_{QG} & Z_{QQ} \end{bmatrix} \begin{bmatrix} \hat{\mathcal{O}}^G \\ \hat{\mathcal{O}}^Q \end{bmatrix}. \quad (2.6)$$

The renormalization constants  $Z_{IJ}$  satisfy the following renormalization group equation (RGE):

$$\mu_R^2 \frac{d}{d\mu_R^2} Z_{IJ} \equiv \gamma_{IK} Z_{KJ} \quad \text{with} \quad I, J, K = G, Q \quad (2.7)$$

where  $\gamma_{IK}$ 's are the corresponding anomalous dimensions and the summation over the repeated index is understood. The general solution to the RGE up to  $a_s^3$  is obtained as

$$\begin{aligned} Z_{IJ} = & \delta_{IJ} + a_s \left[ \frac{2}{\epsilon} \gamma_{IJ}^{(1)} \right] \\ & + a_s^2 \left[ \frac{1}{\epsilon^2} \{ 2\beta_0 \gamma_{IJ}^{(1)} + 2\gamma_{IK}^{(1)} \gamma_{KJ}^{(1)} \} + \frac{1}{\epsilon} \{ \gamma_{IJ}^{(2)} \} \right] \\ & + a_s^3 \left[ \frac{1}{\epsilon^3} \left\{ \frac{8}{3} \beta_0^2 \gamma_{IJ}^{(1)} + 4\beta_0 \gamma_{IK}^{(1)} \gamma_{KJ}^{(1)} + \frac{4}{3} \gamma_{IK}^{(1)} \gamma_{KL}^{(1)} \gamma_{LJ}^{(1)} \right\} \right. \\ & + \frac{1}{\epsilon^2} \left\{ \frac{4}{3} \beta_1 \gamma_{IJ}^{(1)} + \frac{4}{3} \beta_0 \gamma_{IJ}^{(2)} + \frac{2}{3} \gamma_{IK}^{(1)} \gamma_{KJ}^{(2)} + \frac{4}{3} \gamma_{IK}^{(2)} \gamma_{KJ}^{(1)} \right\} \\ & \left. + \frac{1}{\epsilon} \left\{ \frac{2}{3} \gamma_{IJ}^{(3)} \right\} \right] \end{aligned} \quad (2.8)$$

where,  $\gamma_{IJ}$  is expanded in powers of  $a_s$  as

$$\gamma_{IJ} = \sum_{n=1}^{\infty} a_s^n \gamma_{IJ}^{(n)}. \quad (2.9)$$

The second term of the Lagrangian can be written in terms of renormalized quantities:

$$-\frac{1}{2} \int d^4 x h_{\mu\nu} (\kappa_G \mathcal{O}^{G,\mu\nu} + \kappa_Q \mathcal{O}^{Q,\mu\nu}) \quad (2.10)$$

where the  $\kappa_I$  are related to the bare ones by

$$\hat{\kappa}_I = Z_{IJ} \kappa_J. \quad (2.11)$$

### C. Infrared structure

In the color space, the matrix elements of unrenormalized composite operators  $\hat{\mathcal{O}}^I$ ,  $I = G, Q$  between a pair of on-shell partonic states  $i = q, g$  and the vacuum state are expanded in powers of the bare coupling constant  $\hat{a}_s$  as

$$|\mathcal{M}_i^I\rangle = \sum_{n=0}^{\infty} \hat{a}_s^n \left( \frac{Q^2}{\mu^2} \right)^{n\epsilon/2} S_\epsilon^n |\hat{\mathcal{M}}_i^{I,(n)}\rangle \quad (2.12)$$

where  $i = q, \bar{q}, g$ . In terms of these, we can define the on-shell form factor of  $\hat{\mathcal{O}}^I$ ,  $I = G, Q$  by taking the overlap of  $|\mathcal{M}_i^I\rangle$  with its leading order amplitude normalized with respect to the leading order contribution. Given these two operators, one finds four independent form factors:

$$\begin{aligned} \hat{\mathcal{F}}^{I,g,(n)} &= \frac{\langle \hat{\mathcal{M}}_g^{G,(0)} | \hat{\mathcal{M}}_g^{I,(n)} \rangle}{\langle \hat{\mathcal{M}}_g^{G,(0)} | \hat{\mathcal{M}}_g^{G,(0)} \rangle}, \\ \hat{\mathcal{F}}^{I,q,(n)} &= \frac{\langle \hat{\mathcal{M}}_q^{Q,(0)} | \hat{\mathcal{M}}_q^{I,(n)} \rangle}{\langle \hat{\mathcal{M}}_q^{Q,(0)} | \hat{\mathcal{M}}_q^{Q,(0)} \rangle} \quad I = G, Q. \end{aligned} \quad (2.13)$$

Note that, the nondiagonal amplitudes i.e.  $|\hat{\mathcal{M}}_g^{Q,(n)}\rangle$  and  $|\hat{\mathcal{M}}_q^{G,(n)}\rangle$ , start at one-loop level and hence, the corresponding form factors start at  $\mathcal{O}(\hat{a}_s)$ .

The form factors are often ill defined in 4 dimensions even after UV renormalization due to the presence of

infrared divergences when massless modes are present. The massless gluons and light quarks and antiquarks bring in these IR divergences beyond the leading order in perturbation theory. As we mentioned earlier, we regulate both UV and IR divergences using dimensional regularization without disturbing the gauge symmetry of the theory. The UV divergences are renormalized away by coupling constant as well as overall operator renormalizations. The resulting UV finite form factors will contain IR divergences which appear in terms of poles in  $\epsilon$ . Thanks to factorization properties and universality of these IR divergences, these on-shell form factors satisfy the Sudakov differential equation, famously known as the  $K$ - $G$  equation.<sup>2</sup> A generalization to multiparton amplitudes up to two loop level in QCD was proposed by Catani [43] using the universal IR dipole subtraction operators, see also [44]. The generalization of IR subtraction operators of Catani beyond two loops were proposed by Becher and Neubert [45] and by Gardi and Magnea [46]. Following closely the notation used in [47], we find that the UV finite form factors  $\mathcal{F}^{I,i}(\hat{a}_s, Q^2, \mu^2, \epsilon)$ , after performing strong coupling constant and operator renormalizations, satisfy the integro-differential  $K$ - $G$  equation [48–51] given by

$$Q^2 \frac{d}{dQ^2} \ln \mathcal{F}^{I,i}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \frac{1}{2} \left[ K^i \left( \hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon \right) + G^{I,i} \left( \hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon \right) \right] \quad (2.14)$$

where the  $Q^2 = -q^2 = -(p_1 + p_2)^2$  with  $p_i$  being the momenta of external on-shell states. The  $Q^2$  independent function  $K^i$  contains all the poles in the dimensional regulator  $\epsilon$  and the terms, finite in  $\epsilon \rightarrow 0$ , are encapsulated in  $G^{I,i}$ .

The solutions present a universal structure of the singularities, except the single pole in  $\epsilon$ . Single poles are controlled by the finite functions  $G^{I,i}$ . We find

$$G^{I,i} \left( \hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon \right) = G^{I,i} \left( a_s(\mu_R^2), \frac{Q^2}{\mu_R^2}, \epsilon \right) = G^{I,i}(a_s(Q^2), 1, \epsilon) + \int_{\frac{Q^2}{\mu_R^2}}^1 \frac{d\lambda^2}{\lambda^2} A^i(\lambda^2 \mu_R^2) \quad (2.15)$$

where  $A^i$  are cusp anomalous dimensions that do not depend on the type of operator  $I$ .

In [20,52], it was first observed that the coefficient  $G^{I,i}$  of the single pole in  $\epsilon$  manifests a universal structure, in terms of the anomalous dimensions. In [52], the factorization of

the single pole in quark and gluon form factors in terms of soft and collinear anomalous dimensions was first revealed up to two loop level whose validity at three loop was later established in the article [20]. That is, expanding  $G^{I,i}$  as

$$G^{I,i}(a_s(Q^2), 1, \epsilon) = \sum_{n=1}^{\infty} a_s^n(Q^2) G_n^{I,i}(\epsilon) \quad (2.16)$$

one finds

$$G_n^{I,i}(\epsilon) = 2B_n^i + f_n^i + C_n^{I,i} + \sum_{k=1}^{\infty} \epsilon^k g_n^{I,i,k}, \quad (2.17)$$

where, the constants  $C_n^{I,i}$  up to three loop are [53]

$$\begin{aligned} C_1^{I,i} &= 0, \\ C_2^{I,i} &= -2\beta_0 g_1^{I,i,1}, \\ C_3^{I,i} &= -2\beta_1 g_1^{I,i,1} - 2\beta_0 (g_2^{I,i,1} + 2\beta_0 g_1^{I,i,2}). \end{aligned} \quad (2.18)$$

In the above expressions,  $X_n^{I,i}$  with  $X = A, B, f$  are defined through

$$X^{I,i} \equiv \sum_{n=1}^{\infty} a_s^n X_n^{I,i}. \quad (2.19)$$

The constant  $G_n^{I,i}(\epsilon)$  in Eq. (2.17) depends not only on the universal collinear ( $B_n^i$ ) and soft ( $f_n^i$ ) anomalous dimensions, but also the operator as well as process dependent constants  $g_n^{I,i,k}$ . In other words, except  $g_n^{I,i,k}$ , the solution to the  $K$ - $G$  equation contains only universal quantities such as  $A^i$ ,  $B^i$  and  $f^i$ , in addition to  $\beta_i$ . Since,  $A^i$  [54–59],  $B^i$  [55] and  $f^i$  [20,52] are known up to three loop level, we can use the solution to the  $K$ - $G$  equation to determine the renormalization constants  $Z_{IJ}$ . Hence our next task is to compute the on-shell form factors order by order in perturbation theory and compare them against the predictions of the  $K$ - $G$  equation to determine the unknown renormalization constants  $\gamma_{IJ}$  in  $Z_{IJ}$ . Using these renormalization constants, we obtain UV finite on-shell form factors of  $\mathcal{O}^I$  up to three loop level.

### III. COMPUTATION AND RESULTS

In this section, after a brief discussion on how we have performed the computation, we present the unrenormalized form factors  $\hat{\mathcal{F}}^{I,i,(n)}$  and the anomalous dimensions  $\gamma_{IJ}$  up to three loop level. We closely follow the steps used in the derivation of three loop unrenormalized form factors of scalar and vector form factors [23,24], see also [26,27,60]. The relevant Feynman diagrams are generated using QGRAF [61]. The numbers of diagrams contributing to three loop amplitudes are 1586 for  $|\hat{\mathcal{M}}_g^{G,(3)}\rangle$ , 447 for  $|\hat{\mathcal{M}}_g^{Q,(3)}\rangle$ , 400 for  $|\hat{\mathcal{M}}_q^{G,(3)}\rangle$  and 244 for  $|\hat{\mathcal{M}}_q^{Q,(3)}\rangle$  where all the external particles are considered to be on shell. The QGRAF output is suitably converted to a format that can be

<sup>2</sup>The name is due to the presence of two functions in the Sudakov differential equation which are popularly denoted by letters  $K$  and  $G$ .



further used to perform the substitution of Feynman rules, contraction of Lorentz and color indices and simplification of Dirac and Gell-Mann matrices. We have used a set of in-house routines written in the symbolic manipulating program FORM [62] to achieve them. We have included ghost loops in the Feynman gauge. For the external on-shell gluons, we have kept only transversely polarization states of gluons in  $n$  dimensions. The resulting large number of integrals are further reduced to fewer scalar integrals, called master integrals (MIs), using integration by parts (IBP) [63,64] and Lorentz invariance (LI) [65] identities. While the LI identities are not linearly independent from the IBP identities [66], they however help to accelerate to solve the large number of equations resulting from IBP. Reduction to MIs is achieved using Laporta algorithm, [67] implemented

in various symbolic manipulation packages such as AIR [68], FIRE [69], Reduze2 [70,71] and LiteRed [72,73]. We first use Reduze2 [70,71] to shift loop momenta to get suitable integral classes and then make extensive use of LiteRed [72,73] to perform the reductions of all the integrals to MIs. We find that at three loop level, there are 22 topologically different MIs involving genuine three loop integrals with vertex functions ( $A_{t,i}$ ), three loop propagator integrals ( $B_{t,i}$ ) and products of one- and two loop integrals ( $C_{t,i}$ ). Each integral has been computed analytically as a Laurent series in  $\epsilon$  and they can be found in [74–78]. The entire set can also be found in the appendix of [23]. Substituting these MIs, we obtain the unrenormalized form factors which are listed below:

$$\hat{\mathcal{F}}^{G,g,(0)} = 1, \quad (3.1)$$

$$\begin{aligned} \hat{\mathcal{F}}^{G,g,(1)} = C_A \bigg\{ & \frac{1}{\epsilon^2}(-8) + \frac{1}{\epsilon} \left( \frac{22}{3} \right) + \left( -\frac{203}{18} + \zeta_2 \right) + \epsilon \left( +\frac{2879}{216} - \frac{7}{3}\zeta_3 - \frac{11}{12}\zeta_2 \right) + \epsilon^2 \left( -\frac{37307}{2592} + \frac{77}{36}\zeta_3 + \frac{203}{144}\zeta_2 + \frac{47}{80}\zeta_2^2 \right) \\ & + \epsilon^3 \left( \frac{465143}{31104} - \frac{31}{20}\zeta_5 - \frac{1421}{432}\zeta_3 - \frac{2879}{1728}\zeta_2 + \frac{7}{24}\zeta_2\zeta_3 - \frac{517}{960}\zeta_2^2 \right) \\ & + \epsilon^4 \left( -\frac{5695811}{373248} + \frac{341}{240}\zeta_5 + \frac{20153}{5184}\zeta_3 - \frac{49}{144}\zeta_3^2 + \frac{37307}{20736}\zeta_2 - \frac{77}{288}\zeta_2\zeta_3 + \frac{9541}{11520}\zeta_2^2 + \frac{949}{4480}\zeta_2^3 \right) \bigg\}, \quad (3.2) \end{aligned}$$

$$\begin{aligned} \hat{\mathcal{F}}^{G,g,(2)} = C_F n_f \bigg\{ & \frac{1}{\epsilon^2} \left( \frac{32}{9} \right) + \frac{1}{\epsilon} \left( -\frac{260}{27} \right) + \left( \frac{3037}{162} - \frac{8}{3}\zeta_2 \right) + \epsilon \left( -\frac{61807}{1944} + \frac{62}{27}\zeta_3 + \frac{65}{9}\zeta_2 \right) \\ & + \epsilon^2 \left( \frac{1158007}{23328} - \frac{461}{81}\zeta_3 - \frac{3185}{216}\zeta_2 - \frac{31}{45}\zeta_2^2 \right) \\ & + \epsilon^3 \left( -\frac{20551495}{279936} - \frac{28}{45}\zeta_5 + \frac{26131}{1944}\zeta_3 + \frac{22759}{864}\zeta_2 - \frac{17}{6}\zeta_2\zeta_3 + \frac{1721}{1080}\zeta_2^2 \right) \bigg\} \\ & + C_A n_f \bigg\{ \frac{1}{\epsilon^3} \left( -\frac{8}{3} \right) + \frac{1}{\epsilon^2} \left( \frac{64}{9} \right) + \frac{1}{\epsilon} \left( -\frac{499}{27} + 2\zeta_2 \right) + \left( \frac{6863}{162} - \frac{38}{9}\zeta_3 - \frac{16}{3}\zeta_2 \right) \\ & + \epsilon \left( -\frac{84433}{972} + \frac{277}{27}\zeta_3 + \frac{481}{36}\zeta_2 - \frac{73}{60}\zeta_2^2 \right) + \epsilon^2 \left( \frac{1913059}{11664} - \frac{151}{30}\zeta_5 - \frac{2269}{81}\zeta_3 - \frac{1009}{36}\zeta_2 + \frac{5}{2}\zeta_2\zeta_3 - \frac{131}{45}\zeta_2^2 \right) \\ & + \epsilon^3 \left( -\frac{40845067}{139968} + \frac{559}{45}\zeta_5 + \frac{251461}{3888}\zeta_3 - \frac{343}{108}\zeta_3^2 + \frac{68603}{1296}\zeta_2 - \frac{25}{4}\zeta_2\zeta_3 + \frac{6911}{864}\zeta_2^2 + \frac{781}{1680}\zeta_2^3 \right) \bigg\} \\ & + C_A^2 \bigg\{ \frac{1}{\epsilon^4} (32) + \frac{1}{\epsilon^3} (-44) + \frac{1}{\epsilon^2} \left( \frac{226}{3} - 4\zeta_2 \right) + \frac{1}{\epsilon} \left( -81 + \frac{50}{3}\zeta_3 + \frac{11}{3}\zeta_2 \right) + \left( \frac{5249}{108} - 11\zeta_3 - \frac{67}{18}\zeta_2 - \frac{21}{5}\zeta_2^2 \right) \\ & + \epsilon \left( \frac{59009}{1296} - \frac{71}{10}\zeta_5 + \frac{433}{18}\zeta_3 - \frac{337}{108}\zeta_2 - \frac{23}{6}\zeta_2\zeta_3 + \frac{99}{40}\zeta_2^2 \right) + \epsilon^2 \left( -\frac{1233397}{5184} + \frac{759}{20}\zeta_5 - \frac{8855}{216}\zeta_3 + \frac{901}{36}\zeta_3^2 \right. \\ & + \frac{12551}{648}\zeta_2 + \frac{77}{36}\zeta_2\zeta_3 - \frac{4843}{720}\zeta_2^2 + \frac{2313}{280}\zeta_2^3 \bigg) + \epsilon^3 \left( \frac{108841321}{186624} - \frac{3169}{28}\zeta_7 - \frac{4691}{60}\zeta_5 \right. \\ & + \frac{22231}{216}\zeta_3 - \frac{2365}{72}\zeta_3^2 - \frac{813499}{15552}\zeta_2 + \frac{313}{40}\zeta_2\zeta_5 - \frac{1609}{216}\zeta_2\zeta_3 + \frac{21901}{1440}\zeta_2^2 - \frac{1291}{80}\zeta_2^2\zeta_3 - \frac{65659}{3360}\zeta_2^3 \bigg) \bigg\}, \quad (3.3) \end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{F}}^{G,g,(3)} = & C_F n_f^2 \left\{ \frac{1}{\epsilon^3} \left( \frac{256}{81} \right) + \frac{1}{\epsilon^2} \left( -\frac{128}{9} \right) + \frac{1}{\epsilon} \left( \frac{30916}{729} - \frac{160}{27} \zeta_2 \right) + \left( -\frac{78268}{729} + \frac{208}{81} \zeta_3 + \frac{80}{3} \zeta_2 \right) \right\} \\
& + C_F^2 n_f \left\{ \frac{1}{\epsilon^3} \left( \frac{512}{81} \right) + \frac{1}{\epsilon^2} \left( -\frac{1600}{81} \right) + \frac{1}{\epsilon} \left( \frac{20180}{729} + \frac{320}{9} \zeta_3 - \frac{320}{27} \zeta_2 \right) \right. \\
& + \left. \left( \frac{35957}{2430} - \frac{45056}{405} \zeta_3 + \frac{1144}{27} \zeta_2 - \frac{32}{3} \zeta_2^2 \right) \right\} + C_A n_f^2 \left\{ \frac{1}{\epsilon^4} \left( -\frac{128}{81} \right) + \frac{1}{\epsilon^3} \left( \frac{1696}{243} \right) + \frac{1}{\epsilon^2} \left( -\frac{6328}{243} + \frac{80}{27} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{189167}{2187} - \frac{464}{81} \zeta_3 - \frac{1060}{81} \zeta_2 \right) + \left( -\frac{6734887}{26244} + \frac{5500}{243} \zeta_3 + \frac{3805}{81} \zeta_2 + \frac{293}{135} \zeta_2^2 \right) \Big\} \\
& + C_A C_F n_f \left\{ \frac{1}{\epsilon^4} \left( -\frac{256}{9} \right) + \frac{1}{\epsilon^3} \left( \frac{2032}{27} \right) + \frac{1}{\epsilon^2} \left( -\frac{10532}{81} - \frac{64}{9} \zeta_3 + \frac{224}{9} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{39715}{243} - \frac{944}{27} \zeta_3 - \frac{1490}{27} \zeta_2 + \frac{32}{15} \zeta_2^2 \right) \\
& + \left. \left( -\frac{1315651}{14580} - \frac{112}{9} \zeta_5 + \frac{29818}{405} \zeta_3 + \frac{11719}{162} \zeta_2 + \frac{40}{3} \zeta_2 \zeta_3 + \frac{50}{3} \zeta_2^2 \right) \right\} \\
& + C_A^2 n_f \left\{ \frac{1}{\epsilon^5} \left( \frac{64}{3} \right) + \frac{1}{\epsilon^4} \left( -\frac{4784}{81} \right) + \frac{1}{\epsilon^3} \left( \frac{35764}{243} - \frac{376}{27} \zeta_2 \right) + \frac{1}{\epsilon^2} \left( -\frac{7435}{27} + \frac{1208}{27} \zeta_3 + \frac{2458}{81} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{2991329}{8748} - \frac{6634}{81} \zeta_3 - \frac{27059}{486} \zeta_2 - \frac{1493}{90} \zeta_2^2 \right) \\
& + \left. \left( \frac{4440127}{524880} - \frac{3002}{45} \zeta_5 + \frac{219163}{810} \zeta_3 + \frac{229919}{5832} \zeta_2 - \frac{331}{9} \zeta_2 \zeta_3 + \frac{11461}{360} \zeta_2^2 \right) \right\} \\
& + C_A^3 \left\{ \frac{1}{\epsilon^6} \left( -\frac{256}{3} \right) + \frac{1}{\epsilon^5} \left( \frac{352}{3} \right) + \frac{1}{\epsilon^4} \left( -\frac{14744}{81} \right) + \frac{1}{\epsilon^3} \left( \frac{13126}{243} - \frac{176}{3} \zeta_3 + \frac{484}{27} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon^2} \left( \frac{149939}{486} - \frac{440}{27} \zeta_3 - \frac{4321}{81} \zeta_2 + \frac{494}{45} \zeta_2^2 \right) \\
& + \frac{1}{\epsilon} \left( -\frac{14639165}{17496} + \frac{1756}{15} \zeta_5 - \frac{634}{9} \zeta_3 + \frac{112633}{972} \zeta_2 + \frac{170}{9} \zeta_2 \zeta_3 + \frac{4213}{180} \zeta_2^2 \right) \\
& + \left. \left( \frac{1056263429}{1049760} + \frac{5014}{45} \zeta_5 + \frac{539}{2430} \zeta_3 - \frac{1766}{9} \zeta_2^2 - \frac{1988293}{11664} \zeta_2 - \frac{92}{9} \zeta_2 \zeta_3 - \frac{64997}{2160} \zeta_2^2 - \frac{22523}{270} \zeta_2^3 \right) \right\}, \quad (3.4)
\end{aligned}$$

$$\hat{\mathcal{F}}^{Q,g,(0)} = 0, \quad (3.5)$$

$$\begin{aligned}
\hat{\mathcal{F}}^{Q,g,(1)} = & n_f \left\{ \frac{1}{\epsilon} \left( -\frac{4}{3} \right) + \left( \frac{35}{18} \right) + \epsilon \left( -\frac{497}{216} + \frac{1}{6} \zeta_2 \right) + \epsilon^2 \left( \frac{6593}{2592} - \frac{7}{18} \zeta_3 - \frac{35}{144} \zeta_2 \right) \right. \\
& + \epsilon^3 \left( -\frac{84797}{31104} + \frac{245}{432} \zeta_3 + \frac{497}{1728} \zeta_2 + \frac{47}{480} \zeta_2^2 \right) + \epsilon^4 \left( \frac{1072433}{373248} - \frac{31}{120} \zeta_5 - \frac{3479}{5184} \zeta_3 \right. \\
& + \left. \left. -\frac{6593}{20736} \zeta_2 + \frac{7}{144} \zeta_2 \zeta_3 - \frac{329}{2304} \zeta_2^2 \right) \right\}, \quad (3.6)
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{F}}^{Q,g,(2)} = & C_F n_f \left\{ \frac{1}{\epsilon^2} \left( -\frac{32}{9} \right) + \frac{1}{\epsilon} \left( \frac{206}{27} \right) + \left( -\frac{695}{81} - 8\zeta_3 + \frac{8}{3}\zeta_2 \right) + \epsilon \left( \frac{149}{243} + \frac{469}{27}\zeta_3 - \frac{121}{18}\zeta_2 + \frac{12}{5}\zeta_2^2 \right) \right. \\
& + \epsilon^2 \left( \frac{143693}{5832} - 14\zeta_5 - \frac{2554}{81}\zeta_3 + \frac{1219}{108}\zeta_2 + 2\zeta_2\zeta_3 - \frac{95}{18}\zeta_2^2 \right) \\
& + \epsilon^3 \left( -\frac{1386569}{17496} + \frac{6037}{180}\zeta_5 + \frac{104639}{1944}\zeta_3 - \frac{23}{3}\zeta_3^2 - \frac{6581}{432}\zeta_2 - \frac{29}{12}\zeta_2\zeta_3 + \frac{20633}{2160}\zeta_2^2 + \frac{99}{35}\zeta_2^3 \right) \Big\} \\
& + C_A n_f \left\{ \frac{1}{\epsilon^3} \left( \frac{32}{3} \right) + \frac{1}{\epsilon^2} \left( -\frac{184}{9} \right) + \frac{1}{\epsilon} \left( \frac{868}{27} - \frac{8}{3}\zeta_2 \right) + \left( -\frac{15541}{324} + \frac{128}{9}\zeta_3 + \frac{53}{9}\zeta_2 \right) \right. \\
& + \epsilon \left( \frac{273061}{3888} - \frac{823}{27}\zeta_3 - \frac{649}{54}\zeta_2 - \frac{61}{15}\zeta_2^2 \right) + \epsilon^2 \left( -\frac{4764919}{46656} + \frac{182}{15}\zeta_5 + \frac{37373}{648}\zeta_3 + \frac{14545}{648}\zeta_2 - \frac{44}{9}\zeta_2\zeta_3 + \frac{541}{60}\zeta_2^2 \right) \\
& + \epsilon^3 \left( \frac{83029021}{559872} - \frac{8507}{360}\zeta_5 - \frac{219191}{1944}\zeta_3 + \frac{454}{27}\zeta_3^2 - \frac{604667}{15552}\zeta_2 + \frac{1307}{108}\zeta_2\zeta_3 - \frac{4783}{270}\zeta_2^2 + \frac{109}{420}\zeta_2^3 \right) \Big\}, \quad (3.7)
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{F}}^{Q,g,(3)} = & C_F n_f^2 \left\{ \frac{1}{\epsilon^3} \left( -\frac{256}{81} \right) + \frac{1}{\epsilon^2} \left( \frac{112}{9} \right) + \frac{1}{\epsilon} \left( -\frac{20440}{729} - \frac{32}{3}\zeta_3 + \frac{160}{27}\zeta_2 \right) + \left( \frac{27661}{729} + \frac{3500}{81}\zeta_3 - 26\zeta_2 + \frac{16}{5}\zeta_2^2 \right) \right\} \\
& + C_F^2 n_f \left\{ \frac{1}{\epsilon^3} \left( -\frac{512}{81} \right) + \frac{1}{\epsilon^2} \left( \frac{1600}{81} \right) + \frac{1}{\epsilon} \left( -\frac{19694}{729} - \frac{320}{9}\zeta_3 + \frac{320}{27}\zeta_2 \right) \right. \\
& + \left. \left( -\frac{34246}{1215} + 80\zeta_5 + \frac{25076}{405}\zeta_3 - \frac{1144}{27}\zeta_2 + \frac{32}{3}\zeta_2^2 \right) \right\} \\
& + C_A n_f^2 \left\{ \frac{1}{\epsilon^4} \left( \frac{32}{9} \right) + \frac{1}{\epsilon^3} \left( -\frac{1012}{81} \right) + \frac{1}{\epsilon^2} \left( \frac{8029}{243} - \frac{28}{9}\zeta_2 \right) + \frac{1}{\epsilon} \left( -\frac{237197}{2916} + \frac{52}{3}\zeta_3 + \frac{235}{18}\zeta_2 \right) \right. \\
& + \left. \left( \frac{34159189}{174960} - \frac{59047}{810}\zeta_3 - \frac{28457}{648}\zeta_2 - \frac{983}{180}\zeta_2^2 \right) \right\} + C_A C_F n_f \left\{ \frac{1}{\epsilon^4} \left( \frac{256}{9} \right) + \frac{1}{\epsilon^3} \left( -\frac{1648}{27} \right) \right. \\
& + \frac{1}{\epsilon^2} \left( \frac{5396}{81} + 64\zeta_3 - \frac{224}{9}\zeta_2 \right) + \frac{1}{\epsilon} \left( -\frac{4519}{243} - \frac{472}{9}\zeta_3 + \frac{1418}{27}\zeta_2 - \frac{96}{5}\zeta_2^2 \right) \\
& + \left. \left( -\frac{516221}{14580} + 152\zeta_5 - \frac{4508}{45}\zeta_3 - \frac{9883}{162}\zeta_2 - \frac{40}{3}\zeta_2\zeta_3 + \frac{146}{15}\zeta_2^2 \right) \right\} \\
& + C_A^2 n_f \left\{ \frac{1}{\epsilon^5} \left( -\frac{128}{3} \right) + \frac{1}{\epsilon^4} \left( \frac{736}{9} \right) + \frac{1}{\epsilon^3} \left( -\frac{9982}{81} + \frac{32}{3}\zeta_2 \right) + \frac{1}{\epsilon^2} \left( \frac{77047}{486} - \frac{296}{3}\zeta_3 - \frac{58}{3}\zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( -\frac{96755}{648} + \frac{1385}{9}\zeta_3 + \frac{115}{4}\zeta_2 + \frac{147}{5}\zeta_2^2 \right) \\
& + \left. \left( -\frac{1027661}{349920} - \frac{2842}{15}\zeta_5 - \frac{36668}{405}\zeta_3 + \frac{1109}{432}\zeta_2 + 37\zeta_2\zeta_3 - \frac{4019}{90}\zeta_2^2 \right) \right\}, \quad (3.8)
\end{aligned}$$

$$\hat{\mathcal{F}}^{G,q,(0)} = 0, \quad (3.9)$$

$$\begin{aligned}
\hat{\mathcal{F}}^{G,q,(1)} = & C_F \left\{ \frac{1}{\epsilon} \left( -\frac{16}{3} \right) + \left( \frac{34}{9} \right) + \epsilon \left( -\frac{79}{27} + \frac{2}{3}\zeta_2 \right) + \epsilon^2 \left( \frac{401}{162} - \frac{14}{9}\zeta_3 - \frac{17}{36}\zeta_2 \right) \right. \\
& + \epsilon^3 \left( -\frac{2179}{972} + \frac{119}{108}\zeta_3 + \frac{79}{216}\zeta_2 + \frac{47}{120}\zeta_2^2 \right) + \epsilon^4 \left( \frac{12377}{5832} - \frac{31}{30}\zeta_5 - \frac{553}{648}\zeta_3 - \frac{401}{1296}\zeta_2 \right. \\
& + \left. \frac{7}{36}\zeta_2\zeta_3 - \frac{799}{2880}\zeta_2^2 \right) \Big\}, \quad (3.10)
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{F}}^{G,q,(2)} = & C_F n_f \left\{ \frac{1}{\epsilon^2} \left( -\frac{64}{9} \right) + \frac{1}{\epsilon} \left( \frac{376}{27} \right) + \left( -\frac{1798}{81} + \frac{16}{9} \zeta_2 \right) + \epsilon \left( \frac{16259}{486} - \frac{256}{27} \zeta_3 - \frac{94}{27} \zeta_2 \right) \right. \\
& + \epsilon^2 \left( -\frac{289163}{5832} + \frac{1504}{81} \zeta_3 + \frac{899}{162} \zeta_2 + \frac{38}{15} \zeta_2^2 \right) \\
& + \epsilon^3 \left( \frac{5125571}{69984} - \frac{544}{45} \zeta_5 - \frac{7192}{243} \zeta_3 - \frac{16259}{1944} \zeta_2 + \frac{64}{27} \zeta_2 \zeta_3 - \frac{893}{180} \zeta_2^2 \right) \left. \right\} + C_F^2 \left\{ \frac{1}{\epsilon^3} \left( \frac{128}{3} \right) + \frac{1}{\epsilon^2} \left( -\frac{688}{9} \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{3340}{27} - \frac{32}{3} \zeta_2 \right) + \left( -\frac{14257}{81} + \frac{224}{9} \zeta_3 + \frac{236}{9} \zeta_2 \right) + \epsilon \left( \frac{229261}{972} - \frac{1012}{27} \zeta_3 - \frac{1211}{27} \zeta_2 - \frac{28}{5} \zeta_2^2 \right) \\
& + \epsilon^2 \left( -\frac{3597469}{11664} + \frac{248}{15} \zeta_5 + \frac{5437}{81} \zeta_3 + \frac{21233}{324} \zeta_2 - \frac{56}{9} \zeta_2 \zeta_3 + \frac{743}{90} \zeta_2^2 \right) \\
& + \epsilon^3 \left( \frac{56232181}{139968} - \frac{613}{45} \zeta_5 - \frac{51995}{486} \zeta_3 + \frac{196}{27} \zeta_2^2 - \frac{350153}{3888} \zeta_2 + \frac{461}{27} \zeta_2 \zeta_3 - \frac{3331}{216} \zeta_2^2 - \frac{31}{21} \zeta_2^3 \right) \left. \right\} \\
& + C_A C_F \left\{ \frac{1}{\epsilon^2} \left( \frac{176}{9} \right) + \frac{1}{\epsilon} \left( -\frac{1124}{27} \right) + \left( \frac{5651}{81} - \frac{16}{9} \zeta_2 \right) + \epsilon \left( -\frac{108275}{972} + \frac{356}{27} \zeta_3 - \frac{86}{27} \zeta_2 - \frac{16}{15} \zeta_2^2 \right) \right. \\
& + \epsilon^2 \left( \frac{2055287}{11664} - 24 \zeta_5 - \frac{961}{162} \zeta_3 + \frac{986}{81} \zeta_2 - \frac{16}{3} \zeta_2 \zeta_3 - \frac{5}{6} \zeta_2^2 \right) \\
& + \epsilon^3 \left( -\frac{38875571}{139968} + \frac{5377}{90} \zeta_5 - \frac{110159}{1944} \zeta_3 + \frac{88}{3} \zeta_2^2 - \frac{47947}{1944} \zeta_2 + \frac{287}{27} \zeta_2 \zeta_3 - \frac{5323}{1080} \zeta_2^2 + \frac{484}{35} \zeta_2^3 \right) \left. \right\}, \quad (3.11)
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{F}}^{G,q,(3)} = & C_F n_f^2 \left\{ \frac{1}{\epsilon^3} \left( -\frac{256}{27} \right) + \frac{1}{\epsilon^2} \left( \frac{2464}{81} \right) + \frac{1}{\epsilon} \left( -\frac{17216}{243} + \frac{32}{9} \zeta_2 \right) + \left( \frac{107816}{729} - \frac{800}{27} \zeta_3 - \frac{308}{27} \zeta_2 \right) \right. \\
& + C_F^2 n_f \left\{ \frac{1}{\epsilon^4} \left( \frac{640}{9} \right) + \frac{1}{\epsilon^3} \left( -\frac{18544}{81} \right) + \frac{1}{\epsilon^2} \left( \frac{130696}{243} - \frac{176}{9} \zeta_2 \right) + \frac{1}{\epsilon} \left( -\frac{776510}{729} + \frac{752}{9} \zeta_3 + \frac{706}{9} \zeta_2 \right) \right. \\
& + \left( \frac{8387353}{4374} - \frac{18250}{81} \zeta_3 - \frac{15461}{81} \zeta_2 - \frac{289}{15} \zeta_2^2 \right) \left. \right\} + C_F^3 \left\{ \frac{1}{\epsilon^5} \left( -\frac{512}{3} \right) + \frac{1}{\epsilon^4} \left( \frac{1472}{3} \right) + \frac{1}{\epsilon^3} \left( -\frac{89312}{81} + 64 \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon^2} \left( \frac{55964}{27} - \frac{832}{3} \zeta_3 - \frac{1592}{9} \zeta_2 \right) + \frac{1}{\epsilon} \left( -\frac{2565953}{729} + \frac{2296}{3} \zeta_3 + \frac{8644}{27} \zeta_2 + \frac{1148}{15} \zeta_2^2 \right) \\
& + \left( \frac{16239107}{2916} - \frac{656}{5} \zeta_5 - \frac{162008}{81} \zeta_3 - \frac{65755}{162} \zeta_2 + \frac{440}{3} \zeta_2 \zeta_3 - \frac{2451}{10} \zeta_2^2 \right) \left. \right\} \\
& + C_A C_F n_f \left\{ \frac{1}{\epsilon^3} \left( \frac{4928}{81} \right) + \frac{1}{\epsilon^2} \left( -\frac{51592}{243} \right) + \frac{1}{\epsilon} \left( \frac{127238}{243} + \frac{256}{9} \zeta_3 - \frac{280}{27} \zeta_2 \right) \right. \\
& + \left( -\frac{2526404}{2187} + \frac{5960}{81} \zeta_3 - \frac{13}{27} \zeta_2 - \frac{128}{9} \zeta_2^2 \right) \left. \right\} \\
& + C_A C_F^2 \left\{ \frac{1}{\epsilon^4} \left( -\frac{704}{3} \right) + \frac{1}{\epsilon^3} \left( \frac{21784}{27} - \frac{64}{3} \zeta_2 \right) + \frac{1}{\epsilon^2} \left( -\frac{487996}{243} + \frac{416}{3} \zeta_3 + \frac{352}{9} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{3102511}{729} - \frac{6092}{9} \zeta_3 - \frac{1321}{27} \zeta_2 - \frac{536}{15} \zeta_2^2 \right) \\
& + \left( -\frac{71606351}{8748} + \frac{1624}{3} \zeta_5 + \frac{13865}{9} \zeta_3 - \frac{7513}{162} \zeta_2 - \frac{52}{3} \zeta_2 \zeta_3 + \frac{14549}{90} \zeta_2^2 \right) \left. \right\} \\
& + C_A^2 C_F \left\{ \frac{1}{\epsilon^3} \left( -\frac{7744}{81} \right) + \frac{1}{\epsilon^2} \left( \frac{87352}{243} \right) + \frac{1}{\epsilon} \left( -\frac{704276}{729} - \frac{128}{9} \zeta_3 - \frac{88}{9} \zeta_2 \right) \right. \\
& + \left( \frac{5045099}{2187} - 240 \zeta_5 + \frac{6098}{81} \zeta_3 + 209 \zeta_2 - \frac{104}{3} \zeta_2 \zeta_3 + \frac{622}{15} \zeta_2^2 \right) \left. \right\}, \quad (3.12)
\end{aligned}$$

$$\hat{\mathcal{F}}^{Q,q,(0)} = 1, \quad (3.13)$$



$$\begin{aligned}
\hat{\mathcal{F}}^{\mathcal{Q},q,(1)} = C_F & \left\{ \frac{1}{\epsilon^2}(-8) + \frac{1}{\epsilon} \left( \frac{34}{3} \right) + \left( -\frac{124}{9} + \zeta_2 \right) + \epsilon \left( \frac{403}{27} - \frac{7}{3} \zeta_3 - \frac{17}{12} \zeta_2 \right) \right. \\
& + \epsilon^2 \left( -\frac{2507}{162} + \frac{119}{36} \zeta_3 + \frac{31}{18} \zeta_2 + \frac{47}{80} \zeta_2^2 \right) + \epsilon^3 \left( \frac{15301}{972} - \frac{31}{20} \zeta_5 - \frac{217}{54} \zeta_3 - \frac{403}{216} \zeta_2 + \frac{7}{24} \zeta_2 \zeta_3 - \frac{799}{960} \zeta_2^2 \right) \\
& \left. + \epsilon^4 \left( -\frac{92567}{5832} + \frac{527}{240} \zeta_5 + \frac{2821}{648} \zeta_3 - \frac{49}{144} \zeta_3^2 + \frac{2507}{1296} \zeta_2 - \frac{119}{288} \zeta_2 \zeta_3 + \frac{1457}{1440} \zeta_2^2 + \frac{949}{4480} \zeta_2^3 \right) \right\}, \quad (3.14)
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{F}}^{\mathcal{Q},q,(2)} = C_F n_f & \left\{ \frac{1}{\epsilon^3} \left( -\frac{8}{3} \right) + \frac{1}{\epsilon^2} \left( \frac{40}{3} \right) + \frac{1}{\epsilon} \left( -\frac{89}{3} - \frac{2}{3} \zeta_2 \right) + \left( \frac{1909}{36} - \frac{26}{9} \zeta_3 + \frac{22}{9} \zeta_2 \right) \right. \\
& + \epsilon \left( -\frac{36925}{432} + \frac{86}{9} \zeta_3 - \frac{613}{108} \zeta_2 + \frac{41}{60} \zeta_2^2 \right) + \epsilon^2 \left( \frac{677941}{5184} - \frac{121}{30} \zeta_5 - \frac{2317}{108} \zeta_3 \right. \\
& + \frac{15745}{1296} \zeta_2 - \frac{13}{18} \zeta_2 \zeta_3 - \frac{359}{180} \zeta_2^2 \left. \right) + \epsilon^3 \left( -\frac{12131053}{62208} + \frac{67}{6} \zeta_5 + \frac{52237}{1296} \zeta_3 - \frac{169}{108} \zeta_3^2 \right. \\
& - \frac{364273}{15552} \zeta_2 + \frac{209}{54} \zeta_2 \zeta_3 + \frac{19369}{4320} \zeta_2^2 + \frac{127}{112} \zeta_2^3 \left. \right) \left. \right\} \\
& + C_F^2 \left\{ \frac{1}{\epsilon^4} (32) + \frac{1}{\epsilon^3} \left( -\frac{272}{3} \right) + \frac{1}{\epsilon^2} \left( \frac{1570}{9} - 8 \zeta_2 \right) + \frac{1}{\epsilon} \left( -\frac{15023}{54} + \frac{128}{3} \zeta_3 + \frac{32}{3} \zeta_2 \right) \right. \\
& + \left( \frac{257615}{648} - \frac{1034}{9} \zeta_3 - \frac{103}{18} \zeta_2 - 13 \zeta_2^2 \right) \\
& + \epsilon \left( -\frac{4112375}{7776} + \frac{92}{5} \zeta_5 + \frac{13967}{54} \zeta_3 - \frac{3767}{216} \zeta_2 - \frac{56}{3} \zeta_2 \zeta_3 + \frac{1033}{30} \zeta_2^2 \right) \\
& + \epsilon^2 \left( \frac{62375663}{93312} - \frac{1429}{30} \zeta_5 - \frac{356111}{648} \zeta_3 + \frac{652}{9} \zeta_3^2 + \frac{177023}{2592} \zeta_2 + \frac{691}{18} \zeta_2 \zeta_3 - \frac{56369}{720} \zeta_2^2 + \frac{223}{20} \zeta_2^3 \right) \\
& + \epsilon^3 \left( -\frac{911224295}{1119744} - \frac{4471}{28} \zeta_7 + \frac{9439}{72} \zeta_5 + \frac{8942747}{7776} \zeta_3 - \frac{21385}{108} \zeta_3^2 - \frac{5072471}{31104} \zeta_2 - \frac{23}{5} \zeta_2 \zeta_5 - \frac{16141}{216} \zeta_2 \zeta_3 \right. \\
& + \frac{488237}{2880} \zeta_2^2 - \frac{686}{15} \zeta_2^2 \zeta_3 - \frac{3001}{105} \zeta_2^3 \left. \right) \left. \right\} \\
& + C_A C_F \left\{ \frac{1}{\epsilon^3} \left( \frac{44}{3} \right) + \frac{1}{\epsilon^2} \left( -\frac{508}{9} + 4 \zeta_2 \right) + \frac{1}{\epsilon} \left( \frac{7169}{54} - 26 \zeta_3 + \frac{11}{3} \zeta_2 \right) \right. \\
& + \left( -\frac{165413}{648} + \frac{755}{9} \zeta_3 - \frac{235}{9} \zeta_2 + \frac{44}{5} \zeta_2^2 \right) \\
& + \epsilon \left( \frac{3429125}{7776} - \frac{51}{2} \zeta_5 - \frac{5629}{27} \zeta_3 + \frac{15449}{216} \zeta_2 + \frac{89}{6} \zeta_2 \zeta_3 - \frac{1057}{40} \zeta_2^2 \right) \\
& + \epsilon^2 \left( -\frac{66913709}{93312} + \frac{5411}{60} \zeta_5 + \frac{286661}{648} \zeta_3 - \frac{569}{12} \zeta_3^2 - \frac{383285}{2592} \zeta_2 - \frac{877}{36} \zeta_2 \zeta_3 + \frac{2527}{40} \zeta_2^2 - \frac{809}{280} \zeta_2^3 \right) \\
& + \epsilon^3 \left( \frac{1260896789}{1119744} + \frac{93}{2} \zeta_7 - \frac{42157}{180} \zeta_5 - \frac{6822089}{7776} \zeta_3 + \frac{29399}{216} \zeta_3^2 + \frac{8369333}{31104} \zeta_2 + \frac{497}{40} \zeta_2 \zeta_5 + \frac{3683}{108} \zeta_2 \zeta_3 \right. \\
& \left. - \frac{1142729}{8640} \zeta_2^2 + \frac{7103}{240} \zeta_2^2 \zeta_3 - \frac{143}{160} \zeta_2^3 \right) \left. \right\}, \quad (3.15)
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{F}}^{Q,q,(3)} = & C_F n_f^2 \left\{ \frac{1}{\epsilon^4} \left( -\frac{128}{81} \right) + \frac{1}{\epsilon^3} \left( \frac{3808}{243} \right) + \frac{1}{\epsilon^2} \left( -\frac{4240}{81} - \frac{16}{9} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{283256}{2187} - \frac{272}{81} \zeta_3 + \frac{284}{27} \zeta_2 \right) + \left( -\frac{1827880}{6561} + \frac{4348}{243} \zeta_3 - \frac{314}{9} \zeta_2 - \frac{83}{135} \zeta_2^2 \right) \Big\} \\
& + C_F^2 n_f \left\{ \frac{1}{\epsilon^5} \left( \frac{64}{3} \right) + \frac{1}{\epsilon^4} \left( -\frac{1232}{9} \right) + \frac{1}{\epsilon^3} \left( \frac{33784}{81} + \frac{8}{3} \zeta_2 \right) + \frac{1}{\epsilon^2} \left( -\frac{232876}{243} + \frac{584}{9} \zeta_3 - \frac{94}{3} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{1359371}{729} - \frac{8234}{27} \zeta_3 + \frac{3533}{27} \zeta_2 - \frac{337}{18} \zeta_2^2 \right) \\
& + \left( -\frac{28437107}{8748} + \frac{278}{45} \zeta_5 + \frac{3287}{3} \zeta_3 - \frac{849}{2} \zeta_2 - \frac{343}{9} \zeta_2 \zeta_3 + \frac{69809}{1080} \zeta_2^2 \right) \Big\} \\
& + C_F^3 \left\{ \frac{1}{\epsilon^6} \left( -\frac{256}{3} \right) + \frac{1}{\epsilon^5} \left( \frac{1088}{3} \right) + \frac{1}{\epsilon^4} \left( -\frac{2864}{3} + 32 \zeta_2 \right) + \frac{1}{\epsilon^3} \left( \frac{161240}{81} - \frac{800}{3} \zeta_3 - 40 \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon^2} \left( -\frac{97202}{27} + \frac{3256}{3} \zeta_3 - \frac{730}{9} \zeta_2 + \frac{426}{5} \zeta_2^2 \right) + \frac{1}{\epsilon} \left( \frac{8625031}{1458} - \frac{1288}{5} \zeta_5 \right. \\
& - 3050 \zeta_3 + \frac{15017}{27} \zeta_2 + \frac{428}{3} \zeta_2 \zeta_3 - \frac{633}{2} \zeta_2^2 \Big) + \left( -\frac{53150197}{5832} + \frac{14042}{15} \zeta_5 + \frac{590021}{81} \zeta_3 \right. \\
& - \frac{1826}{3} \zeta_3^2 - \frac{576475}{324} \zeta_2 - 267 \zeta_2 \zeta_3 + \frac{289927}{360} \zeta_2^2 - \frac{9095}{252} \zeta_2^3 \Big) \Big\} \\
& + C_A C_F n_f \left\{ \frac{1}{\epsilon^4} \left( \frac{1408}{81} \right) + \frac{1}{\epsilon^3} \left( -\frac{32816}{243} + \frac{128}{27} \zeta_2 \right) + \frac{1}{\epsilon^2} \left( \frac{12868}{27} - \frac{1024}{27} \zeta_3 + \frac{1264}{81} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon} \left( -\frac{2758264}{2187} + \frac{17480}{81} \zeta_3 - \frac{38542}{243} \zeta_2 + \frac{88}{5} \zeta_2^2 \right) + \left( \frac{18919184}{6561} - \frac{128}{3} \zeta_5 - \frac{70690}{81} \zeta_3 \right. \\
& + \frac{916919}{1458} \zeta_2 + \frac{392}{9} \zeta_2 \zeta_3 - \frac{1777}{27} \zeta_2^2 \Big) \Big\} \\
& + C_A C_F^2 \left\{ \frac{1}{\epsilon^5} \left( -\frac{352}{3} \right) + \frac{1}{\epsilon^4} \left( \frac{5560}{9} - 32 \zeta_2 \right) + \frac{1}{\epsilon^3} \left( -\frac{51404}{27} + 208 \zeta_3 + \frac{92}{3} \zeta_2 \right) \right. \\
& + \frac{1}{\epsilon^2} \left( \frac{1110322}{243} - \frac{3704}{3} \zeta_3 + \frac{2119}{9} \zeta_2 - \frac{332}{5} \zeta_2^2 \right) \\
& + \frac{1}{\epsilon} \left( -\frac{13792217}{1458} + 284 \zeta_5 + \frac{37901}{9} \zeta_3 - \frac{68459}{54} \zeta_2 - \frac{430}{3} \zeta_2 \zeta_3 + \frac{72523}{180} \zeta_2^2 \right) \\
& + \left( \frac{311359573}{17496} - \frac{42634}{45} \zeta_5 - \frac{23739}{2} \zeta_3 + \frac{1616}{3} \zeta_3^2 + \frac{1339027}{324} \zeta_2 + \frac{2026}{9} \zeta_2 \zeta_3 - \frac{2603779}{2160} \zeta_2^2 - \frac{18619}{1260} \zeta_2^3 \right) \Big\} \\
& + C_A^2 C_F \left\{ \frac{1}{\epsilon^4} \left( -\frac{3872}{81} \right) + \frac{1}{\epsilon^3} \left( \frac{75400}{243} - \frac{704}{27} \zeta_2 \right) + \frac{1}{\epsilon^2} \left( -\frac{10172}{9} + \frac{6688}{27} \zeta_3 - \frac{2212}{81} \zeta_2 - \frac{352}{45} \zeta_2^2 \right) \right. \\
& + \frac{1}{\epsilon} \left( \frac{6969164}{2187} + \frac{272}{3} \zeta_5 - \frac{36500}{27} \zeta_3 + \frac{123145}{243} \zeta_2 + \frac{176}{9} \zeta_2 \zeta_3 - \frac{1604}{15} \zeta_2^2 \right) \\
& + \left( -\frac{102217595}{13122} - \frac{428}{9} \zeta_5 + \frac{2427625}{486} \zeta_3 - \frac{1136}{9} \zeta_3^2 - \frac{1632292}{729} \zeta_2 - \frac{614}{9} \zeta_2 \zeta_3 + \frac{247963}{540} \zeta_2^2 - \frac{6152}{189} \zeta_2^3 \right) \Big\},
\end{aligned} \tag{3.16}$$

where  $C_A = N$  and  $C_F = (N^2 - 1)/2N$  are the quadratic Casimir of the SU(N) group.  $T_F = 1/2$  and  $n_f$  is the number of light active quark flavours.  $\zeta_i$  is the Riemann Zeta function.

Having computed the unrenormalized form factors, our next task is to determine the operator renormalization constants  $Z_{IJ}$ . As we explained in the previous section, we can determine them by exploiting the universal IR structure of the form factors. We determine these constants by comparing order by order the results of renormalized form factors expressed in terms of unknown  $\gamma_{IJ}$  against the predictions of the  $K$ - $G$  equation expressed in terms of  $A^i$ ,  $B^i$  and  $f^i$  anomalous dimensions that are known to three loop level. The  $\gamma_{IJ}$  thus extracted are listed below:

$$\gamma_{GG}^{(1)} = -\frac{2}{3}n_f \quad (3.17)$$

$$\gamma_{GG}^{(2)} = -\frac{35}{27}C_A n_f - \frac{74}{27}C_F n_f \quad (3.18)$$

$$\begin{aligned} \gamma_{GG}^{(3)} = & C_A^2 n_f \left( -\frac{3589}{162} + 24\zeta_3 \right) + C_A C_F n_f \left( \frac{139}{9} - \frac{104}{3}\zeta_3 \right) \\ & + C_F^2 n_f \left( -\frac{2155}{243} + \frac{32}{3}\zeta_3 \right) \\ & + C_A n_f^2 \left( \frac{1058}{243} \right) - C_F n_f^2 \left( \frac{173}{243} \right) \end{aligned} \quad (3.19)$$

$$\gamma_{GQ}^{(1)} = C_F \left( \frac{8}{3} \right) \quad (3.20)$$

$$\gamma_{GQ}^{(2)} = C_A C_F \left( \frac{376}{27} \right) - C_F^2 \left( \frac{112}{27} \right) - C_F n_f \left( \frac{104}{27} \right) \quad (3.21)$$

$$\begin{aligned} \gamma_{GQ}^{(3)} = & C_A^2 C_F \left( \frac{20920}{243} + \frac{64}{3}\zeta_3 \right) + C_A C_F^2 \left( -\frac{8528}{243} - 64\zeta_3 \right) \\ & + C_F^3 \left( -\frac{560}{243} + \frac{128}{3}\zeta_3 \right) + C_A C_F n_f \left( -\frac{22}{9} - \frac{128}{3}\zeta_3 \right) \\ & + C_F^2 n_f \left( -\frac{7094}{243} + \frac{128}{3}\zeta_3 \right) - C_F n_f^2 \left( \frac{284}{81} \right). \end{aligned} \quad (3.22)$$

The remaining entries are  $\gamma_{QG}^{(n)} = -\gamma_{GG}^{(n)}$  and  $\gamma_{QQ}^{(n)} = -\gamma_{GQ}^{(n)}$  where  $n = 1, 2, 3$ . This is indeed a consequence of the fact that the sum of these operators is conserved. This provides a crucial check on the correctness of our computation. Note that, all the  $\gamma_{GG}^{(n)}$  are proportional to  $n_f$  which is consistent with the expectation that the conservation property of the operator  $\hat{\mathcal{O}}^{G,\mu\nu}$  breaks down beyond tree level due to the presence of quark loops.

The renormalized form factors can be obtained using Eqs. (2.4), (2.5), (2.6), (2.8). Setting  $\mu_R^2 = Q^2$ , expanding in terms of  $a_s(Q^2)$  as

$$F^{I,i}(Q^2) = \sum_{n=0}^{\infty} a_s^n(Q^2) \mathcal{F}^{I,i,(n)}, \quad I = G, \quad Qi = g, q, \quad (3.23)$$

where  $F^{I,i,(n)}$  up to the three loop level are given by

$$\begin{aligned} \mathcal{F}^{G,g,(1)} &= \frac{2}{\epsilon} \gamma_{GG}^{(1)} + \hat{\mathcal{F}}^{G,g,(1)} \\ \mathcal{F}^{G,g,(2)} &= \frac{2}{\epsilon^2} \{ \beta_0 \gamma_{GG}^{(1)} + (\gamma_{GG}^{(1)})^2 + \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} \} + \frac{1}{\epsilon} \{ 2\hat{\mathcal{F}}^{G,g,(1)} (\beta_0 + \gamma_{GG}^{(1)}) 2\hat{\mathcal{F}}^{Q,g,(1)} \gamma_{GQ}^{(1)} + \gamma_{GG}^{(2)} \} + \hat{\mathcal{F}}^{G,g,(2)} \\ \mathcal{F}^{G,g,(3)} &= \frac{1}{\epsilon^3} \left\{ \frac{8}{3} \beta_0^2 \gamma_{GG}^{(1)} + 4\beta_0 (\gamma_{GG}^{(1)})^2 + \frac{4}{3} (\gamma_{GG}^{(1)})^3 + 4\beta_0 \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{8}{3} \gamma_{GG}^{(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{4}{3} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} \gamma_{QG}^{(1)} \right\} \\ &+ \frac{1}{\epsilon^2} \left\{ 4\beta_0^2 \hat{\mathcal{F}}^{G,g,(1)} + \frac{4}{3} \beta_1 \gamma_{GG}^{(1)} + 6\beta_0 \hat{\mathcal{F}}^{G,g,(1)} \gamma_{GG}^{(1)} + 2\hat{\mathcal{F}}^{G,g,(1)} (\gamma_{GG}^{(1)})^2 + 6\beta_0 \hat{\mathcal{F}}^{Q,g,(1)} \gamma_{GQ}^{(1)} \right. \\ &+ 2\hat{\mathcal{F}}^{Q,g,(1)} \gamma_{GG}^{(1)} \gamma_{GQ}^{(1)} + \hat{\mathcal{F}}^{G,g,(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \hat{\mathcal{F}}^{Q,g,(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{4}{3} \beta_0 \gamma_{GG}^{(2)} + 2\gamma_{GG}^{(1)} \gamma_{GG}^{(2)} + \frac{4}{3} \gamma_{GQ}^{(2)} \gamma_{QG}^{(1)} + \frac{2}{3} \gamma_{GQ}^{(1)} \gamma_{QG}^{(2)} \} \\ &+ \frac{1}{\epsilon} \left\{ \beta_1 \hat{\mathcal{F}}^{G,g,(1)} + 4\beta_0 \hat{\mathcal{F}}^{G,g,(2)} + 2\hat{\mathcal{F}}^{G,g,(2)} \gamma_{GG}^{(1)} + 2\hat{\mathcal{F}}^{Q,g,(2)} \gamma_{GQ}^{(1)} + \hat{\mathcal{F}}^{G,g,(1)} \gamma_{GG}^{(2)} + \hat{\mathcal{F}}^{Q,g,(1)} \gamma_{GQ}^{(2)} + \frac{2}{3} \gamma_{GG}^{(3)} \right\} + \hat{\mathcal{F}}^{G,g,(3)} \\ \mathcal{F}^{G,q,(1)} &= \frac{2}{\epsilon} \gamma_{GQ}^{(1)} + \hat{\mathcal{F}}^{G,q,(1)} \\ \mathcal{F}^{G,q,(2)} &= \frac{2}{\epsilon^2} \{ \beta_0 \gamma_{GQ}^{(1)} + \gamma_{GG}^{(1)} \gamma_{GQ}^{(1)} + \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} \} + \frac{1}{\epsilon} \{ 2\beta_0 \hat{\mathcal{F}}^{G,q,(1)} + 2\hat{\mathcal{F}}^{G,q,(1)} \gamma_{GG}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(1)} \gamma_{GQ}^{(1)} + \gamma_{GQ}^{(2)} \} + \hat{\mathcal{F}}^{G,q,(2)} \\ \mathcal{F}^{G,q,(3)} &= \frac{1}{\epsilon^3} \left\{ \frac{8}{3} \beta_0^2 \gamma_{GQ}^{(1)} + 4\beta_0 \gamma_{GG}^{(1)} \gamma_{GQ}^{(1)} + \frac{4}{3} (\gamma_{GG}^{(1)})^2 \gamma_{GQ}^{(1)} + \frac{4}{3} (\gamma_{GQ}^{(1)})^2 \gamma_{QG}^{(1)} + 4\beta_0 \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{4}{3} \gamma_{GG}^{(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{4}{3} \gamma_{GQ}^{(1)} (\gamma_{QG}^{(1)})^2 \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\epsilon^2} \left\{ 4\beta_0^2 \hat{\mathcal{F}}^{G,q,(1)} + 6\beta_0 \hat{\mathcal{F}}^{G,q,(1)} \gamma_{GG}^{(1)} + 2\hat{\mathcal{F}}^{G,q,(1)} (\gamma_{GG}^{(1)})^2 + \frac{4}{3} \beta_1 \gamma_{GQ}^{(1)} + 6\beta_0 \hat{\mathcal{F}}^{Q,q,(1)} \gamma_{GQ}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(1)} \gamma_{GG}^{(1)} \gamma_{GQ}^{(1)} \right. \\
& + 2\hat{\mathcal{F}}^{G,q,(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(1)} \gamma_{GQ}^{(1)} \gamma_{QQ}^{(1)} + \frac{4}{3} \gamma_{GQ}^{(1)} \gamma_{GG}^{(2)} + \frac{4}{3} \beta_0 \gamma_{GQ}^{(2)} + \frac{2}{3} \gamma_{GG}^{(1)} \gamma_{GQ}^{(2)} + \frac{4}{3} \gamma_{QQ}^{(1)} \gamma_{GQ}^{(2)} + \frac{2}{3} \gamma_{GQ}^{(1)} \gamma_{QQ}^{(2)} \left. \right\} \\
& + \frac{1}{\epsilon} \left\{ \beta_1 \hat{\mathcal{F}}^{G,q,(1)} + 4\beta_0 \hat{\mathcal{F}}^{G,q,(2)} + 2\hat{\mathcal{F}}^{G,q,(2)} \gamma_{GG}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(2)} \gamma_{GQ}^{(1)} + \hat{\mathcal{F}}^{G,q,(1)} \gamma_{GG}^{(2)} + \hat{\mathcal{F}}^{Q,q,(1)} \gamma_{GQ}^{(2)} + \frac{2}{3} \gamma_{GQ}^{(3)} \right\} + \hat{\mathcal{F}}^{G,q,(3)} \\
\mathcal{F}^{Q,g,(1)} &= \frac{2}{\epsilon} \gamma_{QG}^{(1)} + \hat{\mathcal{F}}^{Q,g,(1)} \\
\mathcal{F}^{Q,g,(2)} &= \frac{2}{\epsilon^2} \{ \beta_0 \gamma_{QG}^{(1)} + \gamma_{GG}^{(1)} \gamma_{QG}^{(1)} + \gamma_{QG}^{(1)} \gamma_{QQ}^{(1)} \} + \frac{1}{\epsilon} \{ 2\beta_0 \hat{\mathcal{F}}^{Q,g,(1)} + 2\hat{\mathcal{F}}^{G,g,(1)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{Q,g,(1)} \gamma_{QQ}^{(1)} + \gamma_{QG}^{(2)} \} + \hat{\mathcal{F}}^{Q,g,(2)} \\
\mathcal{F}^{Q,g,(3)} &= \frac{1}{\epsilon^3} \left\{ \frac{8}{3} \beta_0^2 \gamma_{QG}^{(1)} + 4\beta_0 \gamma_{GG}^{(1)} \gamma_{QG}^{(1)} + \frac{4}{3} (\gamma_{GG}^{(1)})^2 \gamma_{QG}^{(1)} + \frac{4}{3} \gamma_{GQ}^{(1)} (\gamma_{QG}^{(1)})^2 + 4\beta_0 \gamma_{QG}^{(1)} \gamma_{QQ}^{(1)} + \frac{4}{3} \gamma_{GG}^{(1)} \gamma_{QG}^{(1)} \gamma_{QQ}^{(1)} + \frac{4}{3} \gamma_{QG}^{(1)} (\gamma_{QQ}^{(1)})^2 \right\} \\
& + \frac{1}{\epsilon^2} \left\{ 4\beta_0^2 \hat{\mathcal{F}}^{Q,g,(1)} + \frac{4}{3} \beta_1 \gamma_{QG}^{(1)} + 6\beta_0 \hat{\mathcal{F}}^{G,g,(1)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{G,g,(1)} \gamma_{GG}^{(1)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{Q,g,(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + 6\beta_0 \hat{\mathcal{F}}^{Q,g,1} \gamma_{QQ}^{(1)} \right. \\
& + 2\hat{\mathcal{F}}^{G,g,(1)} \gamma_{QG}^{(1)} \gamma_{QQ}^{(1)} + 2\hat{\mathcal{F}}^{Q,g,(1)} (\gamma_{QQ}^{(1)})^2 + \frac{2}{3} \gamma_{QG}^{(1)} \gamma_{GG}^{(2)} + \frac{4}{3} \beta_0 \gamma_{QG}^{(2)} + \frac{4}{3} \gamma_{GG}^{(1)} \gamma_{QG}^{(2)} + \frac{2}{3} \gamma_{QG}^{(1)} \gamma_{QQ}^{(2)} + \frac{4}{3} \gamma_{QG}^{(1)} \gamma_{QQ}^{(2)} \left. \right\} \\
& + \frac{1}{\epsilon} \left\{ \beta_1 \hat{\mathcal{F}}^{Q,g,(1)} + 4\beta_0 \hat{\mathcal{F}}^{Q,g,(2)} + 2\hat{\mathcal{F}}^{G,g,(2)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{Q,g,(2)} \gamma_{QQ}^{(1)} + \hat{\mathcal{F}}^{G,g,(1)} \gamma_{QG}^{(2)} + \hat{\mathcal{F}}^{Q,g,(1)} \gamma_{QQ}^{(2)} + \frac{2}{3} \gamma_{QG}^{(3)} \right\} + \hat{\mathcal{F}}^{Q,g,(3)} \\
\mathcal{F}^{Q,q,(1)} &= \frac{2}{\epsilon} \gamma_{QQ}^{(1)} + \hat{\mathcal{F}}^{Q,q,(1)} \\
\mathcal{F}^{Q,q,(2)} &= \frac{2}{\epsilon^2} \{ \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \beta_0 \gamma_{QQ}^{(1)} + (\gamma_{QQ}^{(1)})^2 \} + \frac{1}{\epsilon} \{ 2\beta_0 \hat{\mathcal{F}}^{Q,q,(1)} + 2\hat{\mathcal{F}}^{G,q,(1)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(1)} \gamma_{QQ}^{(1)} + \gamma_{QQ}^{(2)} \} + \hat{\mathcal{F}}^{Q,q,(2)} \\
\mathcal{F}^{Q,q,(3)} &= \frac{1}{\epsilon^3} \left\{ 4\beta_0 \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{4}{3} \gamma_{GG}^{(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{8}{3} \{ \beta_0^2 \gamma_{QQ}^{(1)} + \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} \gamma_{QQ}^{(1)} \} + 4\beta_0 (\gamma_{QQ}^{(1)})^2 + \frac{4}{3} (\gamma_{QQ}^{(1)})^3 \right\} \\
& + \frac{1}{\epsilon^2} \left\{ 4\beta_0^2 \hat{\mathcal{F}}^{Q,q,(1)} + 6\beta_0 \hat{\mathcal{F}}^{G,q,(1)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{G,q,(1)} \gamma_{GG}^{(1)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(1)} \gamma_{GQ}^{(1)} \gamma_{QG}^{(1)} + \frac{4}{3} \beta_1 \gamma_{QQ}^{(1)} + 6\beta_0 \hat{\mathcal{F}}^{Q,q,(1)} \gamma_{QQ}^{(1)} \right. \\
& + 2\hat{\mathcal{F}}^{G,q,(1)} \gamma_{QG}^{(1)} \gamma_{QQ}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(1)} (\gamma_{QQ}^{(1)})^2 + \frac{2}{3} \gamma_{QG}^{(1)} \gamma_{GG}^{(2)} + \frac{4}{3} \gamma_{GQ}^{(1)} \gamma_{QG}^{(2)} + \frac{4}{3} \beta_0 \gamma_{QQ}^{(2)} + 2\gamma_{QG}^{(1)} \gamma_{QQ}^{(2)} \left. \right\} \\
& + \frac{1}{\epsilon} \left\{ \beta_1 \hat{\mathcal{F}}^{Q,q,(1)} + 4\beta_0 \hat{\mathcal{F}}^{Q,q,(2)} + 2\hat{\mathcal{F}}^{G,q,(2)} \gamma_{QG}^{(1)} + 2\hat{\mathcal{F}}^{Q,q,(2)} \gamma_{QQ}^{(1)} + \hat{\mathcal{F}}^{G,q,(1)} \gamma_{QG}^{(2)} + \hat{\mathcal{F}}^{Q,q,(1)} \gamma_{QQ}^{(2)} + \frac{2}{3} \gamma_{QQ}^{(3)} \right\} + \hat{\mathcal{F}}^{Q,q,(3)}.
\end{aligned} \tag{3.24}$$

The explicit results of the above renormalized form factors can be obtained from the authors on request.

### A. Leading transcendentality principle

Recently, on-shell form factors in supersymmetric Yang-Mills theory have attracted a lot of attention to understand their field theoretic structure. There are already several results [79–81] in  $\mathcal{N} = 4$  super-Yang-Mills (SYM) with gauge group  $SU(N)$ .  $\mathcal{N} = 4$  SYM is UV finite in  $d = 4$  dimensions and also dual to type IIB string theory on  $AdS_5 \times S^5$  with self dual RR field strength. This implies that one can relate quantities computed in  $\mathcal{N} = 4$  SYM in the strong coupling limit with those obtained in the weak coupling limit of the gravity theory. There have been efforts

to compute on-shell amplitudes, correlation functions and form factors in SYM using perturbative approach to very good accuracy to make nonperturbative predictions through systematic resummation procedures. The important advances in this direction include resummation of perturbative contributions [82,83] to MHV amplitudes to all orders in 't Hooft coupling. The developments in this direction have not only improved our understanding of the quantum field theories in general but also provided very sophisticated analytical tools to compute multiloop multi-leg processes that are essential in the present collider phenomenology. Thanks to maximal supersymmetry in  $\mathcal{N} = 4$ , large cancellations between various contributions result in elegant and simple looking predictions that have

lot of resemblance with those in QCD. For example, the leading transcendentality principle [84–86] relates anomalous dimensions of the twist two operators in  $\mathcal{N} = 4$  SYM to the leading transcendental (LT) terms of such operators computed in QCD.

Due to the presence of massless modes both in QCD and SYM, the IR divergences show up when loop corrections are involved. The IR regulated results for the scattering amplitudes and form factors can be expressed as a linear combinations of polylogarithmic functions whose maximum degree of transcendentality depends on the order of perturbation theory. Unlike QCD which receives contributions from all degrees of transcendentality up to  $2l$ , where “ $l$ ” denotes the order in perturbative expansion, certain scattering amplitudes and FFs in  $\mathcal{N} = 4$  SYM exhibit uniform transcendentality at each order.

An interesting relation between QCD quark and gluon form factors [23] and a scalar form factor in SYM has been observed up to the three loop. If we replace [85] the color factors  $C_A = C_F = N$  and  $n_f = N$  in the quark and gluon form factors, then their LT parts not only coincide with each other but also become identical, to the form factors of half-BPS scalar operator in  $\mathcal{N} = 4$  SYM [81]. Similar behavior was observed for the diagonal pseudoscalar form factors  $\mathcal{F}^{G,g}$  and  $\mathcal{F}^{J,q}$  in [27]. A similar relation for three point form factors at the two loop level between LT terms of  $H \rightarrow ggg$  in QCD [87] and those of a half-BPS operator in  $\mathcal{N} = 4$  SYM were found in [88].

In the present context, we have found the LT terms of the diagonal form factors,  $\hat{\mathcal{F}}^{G,g}$ ,  $\hat{\mathcal{F}}^{Q,q}$  with the above prescribed color replacement, are not only identical to each other but also coincide with the LT terms of the scalar form

factors in  $\mathcal{N} = 4$  SYM [81]. This is true for terms proportional to positive powers of  $\epsilon$  available up to transcendentality 8 [89]. On the other hand, the LT terms of the off-diagonal ones namely,  $\hat{\mathcal{F}}^{G,q}$ ,  $\hat{\mathcal{F}}^{Q,g}$ , while identical to each other after the replacement of color factors, do not coincide with those of the diagonal ones.

#### IV. CONCLUSIONS

In this article, we have studied in detail the theoretical issues with the interactions of spin-2 fields with those of the SM. We have considered a set of gauge invariant tensorial operators constructed out of fields of the SM that couple to spin-2 fields. These operators are in general not conserved like the usual EM tensor. Hence they require additional renormalization. To compute these additional renormalization constants, we have exploited the universal infrared structure of on-shell amplitudes with composite operators. Computing these form factors order by order in perturbation theory and using the  $K$ - $G$  equation we obtain the UV anomalous dimensions and the renormalization constants up to the three loop level. The renormalization constants and the on-shell FFs are important components of observables that can probe the physics of spin-2 fields. We have reserved the detailed phenomenological study with these two operators at the LHC for future publication [90].

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