

QCA and causal knowledge

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The debate on the import of QCA solutions within the configurational scholarship leaves the question open of how the technique can balance theory and its inferential rules to contribute to causal knowledge. The article finds a response in the wider methodological discourse that accounts for the relationship between a ‘treatment’ and an outcome by modeling the underlying causal situation. It shows the terms in QCA solutions can fit the causal graphs of the Structural Causal Models framework.

Introduction

Standard Qualitative Comparative Analysis (QCA: (Ragin, 2009), (Ragin, 2000), (Ragin, 2014); (Rihoux and Ragin, 2008)) is an eliminative strategy that employs Boolean algebra to identify the consistent bundles of factors before an outcome by dismissing irrelevant components. The causal standing of its findings, however, raises doubts within the same configurational scholarship. Despite the wide recognition that QCA solutions suit the rationale of *inut* causation (e.g., (Schneider and Wagemann, 2010), (Goertz and Mahoney, 2012), (Schneider and Rohlfing, 2014), (Thiem, 2019); (Duşa, 2019), (Thomann and Maggetti, 2017)), the configurational scholarship disagrees on two related points.

The first point concerns the causal import of QCA’s operations. Part of the scholarship maintains that they have none: the pruning algorithm simply merges empirical types with the

main purpose of fine-tuning theories to cases. Therefore, QCA's solutions are free from the ambitions of design-based techniques and remit conclusions to case studies, where actual causation takes place (e.g., (Schneider and Wagemann, 2012), (Schneider, 2018), (Duşa, 2019)). Just the opposite, the other part maintains the technique cannot help finding formal causes. Any factor, simple or compound, qualifies as such if it tells the instances with the outcome from those without, and none of its components has the same sorting power. In Standard QCA, the parsimonious minimizations identify at least some of these minimal implicants that, as "difference-makers," inevitably bear causal import (e.g., (Baumgartner and Thiem, 2015), (Baumgartner and Thiem, 2017), (Thiem, 2019)).

The second dimension, and related, pertains to the role of counterfactual reasoning in shaping results. Part of the scholarship maintains *in*us causation commits to a regularity mindset that dispenses with assumptions about unobserved circumstances: a *correct* difference-maker is any *minimal* bundle shared by at least some of the instances of the outcome and by them only. Therefore, Standard QCA's minimizations find some correct difference-makers as Type III errors when they employ unobserved configurations free from theoretical concerns for the plausibility of the counterfactual. Findings are incorrect and prone to confirmation bias, instead, when theoretical assumptions constrain the usage of remainders (e.g., (Baumgartner, 2009), (Baumgartner, 2014), (Thiem, 2019)). The opposite position contends causal analysis cannot help making assumptions about unrealized alternatives. A difference-making bundle entails the counterfactual claim that, had any of its components been in a different state, the outcome would have been different, too. Similarly, the claim that a factor is irrelevant to a bundle stands in light of the counterfactual consideration that, had the factor been in a different state, the bundle would still have obtained. Theoretical assumptions ensure the reasoning is plausible even when the counterfactual is unobserved in the population of reference (e.g., (Ragin and Sonnett, 2005), (Schneider and Wagemann, 2013)). Concerns of theoretical consistency necessarily override those of minimality (e.g., (Maggetti and Levi-Faur, 2013), (Schneider, 2018), (Duşa, 2019)).

The two positions, together, suggest the reasonable conclusion that neither when it preserves theory against evidence, nor when it assembles evidence before theory QCA can yield causally interpretable results. Nevertheless, the conclusion leaves the question open of how, if ever, the technique can contribute to causal knowledge.

This article advances its response in light of the wider discourse on causation (e.g., (Verba, 1967); (Mackie, 1980); (Pearl, 2009); (Morgan and Winship, 2014); (Andreas and Günther, 2018)). Thus, Section 1 summarizes the requirements developed in the philosophy of science and in methodology to license causal claims, that is, about designs and models; the Structural Causal Model framework is offered as an exemplary implementation of these requirements. Against this backdrop, supported with an original example, Section 2 argues that QCA can reconcile or overcome the dilemmas the configurational literature maintains would hinder the causal standing of its findings and borrows the criteria used by the Structural Causal Model to support its claim. Section 3 summarizes the key points.

As a matter of clarification, the following commits to the stipulations and conventions listed in the [Appendix §1](#).

1. Requirements for licensing causal claims

The recognized hallmark of causal phenomena lies in the asymmetry of the relationships between a “cause” and its “effect” (e.g., (Simon and Rescher, 1966): 323 ff, (Hausman, 1998)).

The asymmetry takes the shape of dependencies with a direction, as the effect depends on the cause, but the converse does not hold. The operational implications of asymmetry are two. First, the effect can be induced by inducing the cause but not *vice-versa*. Second, the suppression of the cause affects the effect, although the suffocation of the effect or an obstruction in the path of the dependence leaves the cause intact.

Research strategies rely on these criteria to establish which kind of evidence indicates a factor to be *causally relevant* to an outcome.

1.1. Designs

The methodological literature recognizes two fundamental and ideally alternative rationales for proving relevance that build on opposite designs (e.g., (Brady, 2009), (Goldthorpe, 2001), (Gerring, 2005), (Goertz, 2017); (Rohlfing and Zuber, 2019); (Salmon, 2017)).

1.1.1. Regularity

Regularity-based techniques understand dependence as sufficiency, often gauged as a conditional relationship observed on the instances of the outcome (e.g., (Hájek, 2011)) as in equation (1) below

$$\pi(Y|X) := \frac{X \cap Y}{(X \cap Y) \cup (X \cap \bar{Y})} = \frac{X \cap Y}{X} \approx 1.00 \quad (1)$$

where π is for the size of intersections and unions.

The regularity perspective considers causal events raise a robust pattern of sufficiency and make the ratio approximate the value of 1.00 across otherwise dissimilar instances of the same outcome. However, the inference may prove misleading of causal dependence from high ratio values in a design of greatest background diversity. In some subpopulations, the size of Y can still be due to an omitted shared background factor Z . Renowned “paradoxes of confirmation” recall the role of

- (i) atmospheric pressure (Z) in the relationship between the reading of a barometer (X_1) and the weather (Y),
- (ii) water (Z) in the dependence of dissolving salt (Y) from spells (X_2), or
- (iii) a functioning spinal cord (Z) in feeling burning pain (Y) after heating (X_3).

These paradoxes, in short, show how regularity may mistake a predictor or a false antecedent for a cause and mistake partial causes for irrelevant factors, which has long justified the move of the analysis onto the counterfactual ground.

1.1.2. Counterfactuals

Counterfactually, causal is the (regular) factor X that would have changed the state of the outcome, had it taken a different value “everything else being equal.” Thus, twin units to the instances of regularity but missing the causal factor should provide evidence of the counterfactual claim as in equation (2):

$$\pi(y|x) \approx 1.00 \quad (2)$$

The equation renders that the presence of the causal factor is necessary for the outcome to occur as the factor’s absence would have prevented it. Such a “difference-making” criterion allows establishing the relevance of the factor by telling between spurious and true causes. To witness, while the broken barometer does not affect the weather and the missing spell cannot impede the dissolution of salt in water, no heating prevents burning, and no pain can be felt if the spinal cord is damaged.

However, the criterion from equation (2), too, suffers from a major weakness. The counterfactual test returns false negatives under alternative (rival, equifinal) causation: for instance,

- (iv) chili pepper (Z_3) can raise a burning sensation (Y) under no heating (x_3) in people with a working spinal cord.

Under a strict application of the counterfactual test, an undetected rival can disprove the true cause as a relevant factor to the outcome.

The manipulation approach finds a solution to rivals in the researcher’s control on the units’ exposition to the factor of interest. Control allows construing twin groups except for units’

exposition so that the factor's causal effect lies in the arithmetic difference in each group's average response. Thus, the manipulation approach relies on regularity evidence from twin populations, each providing a counterfactual to the other by design. Nevertheless, the average treatment effect can still be biased. Especially in social domains, the perfect groups' similarity is hardly attainable; hence, the units' selection criteria cannot rule out that the causal effect is driven by relevant factors that are unevenly distributed across each group.

Both the regularity and the counterfactual strategies for establishing causation, in short, have limitations that cannot be overcome by solely relying on design criteria.

1.2. Models

Designs build either on the assumption of units' difference or similarity in respect of the causal phenomenon but leave the ground unclear to establish comparability. A fruitful solution makes comparability depend on whether units are instances of a certain mechanism to the outcome.

Data-generation mechanisms are hypotheses that localize causation in the special organization of entities and activities in a field (e.g., (Illari and Williamson, 2011)). Across traditions (e.g., (Cartwright, 1994), (Hausman, 2005), (Kim, 2005), (Zhang and Spirtes, 2008), (Scriven, 2009), (Woodward, 2011), (Schurz, 2017), (Kaiser and Krickel, 2017), (Craver and Kaplan, 2020), (Ross, 2021); (Walton, 1992), (Van Evera, 1997), (Bennett, 2009), (Blatter and Haverland, 2014), (Rohlfing and Schneider, 2016), (Fairfield and Charman, 2017), (Beach and Pedersen, 2019), (Dowding, 2020)), they vary at least in the grain of the factors that compose them and in their grasp on causation.

1.2.1. Factors' grain

Factors can be coarse or fine, and the difference entails the long-standing distinction between singular and general causation. Singular accounts make sense of one actual situation, often retrospectively. Their factors can be as fine-grained as needed, and their richness promises to preserve the faithfulness or plausibility of the account, although possibly at the cost of comparability. The other way round, general accounts are abstractions that accommodate facets

of singular accounts at different space-time points under coarser labels, which imposes information loss and lower faithfulness.

The wider discourse considers a balance possible between coarseness and faithfulness. The loss of information can be made purposeful and explicit when the move from the singular to a general account is governed by gauging rules that systematically establish special local details and their relationships as equivalent manifestations of a coarser factor. (Verba, 1967) illustrates it with Mount Vesuvius' eruption as a crucial circumstance in the disruption of the Pompeiian political system. Eruptions only invite reasoning about systems close to volcanoes unless they are reframed as instances of the wider category of disasters and gauged by some metric of destructive power. In this shape, the category can encompass a number of singular occurrences such as floods, wildfires, or earthquakes. Heterogeneity is dismissed as irrelevant or entailed in the metric that narrows on the trait of interest to account for the special outcome. As a further elaboration, the circumstance can be reduced to the units' response to the exogenous factor of interest and their (coarse) drivers – for instance, to the types of resources and capacities that can make the system resilient or fragile to a disaster.

Thus, the faithfulness of types can be tested by unboxing the coarse factors into their local drivers or components. Faithfulness is not violated when causation in the unboxed scheme at the local level is proven to flow in the same direction as in the boxed one at the general level.

1.2.2. Grasp on causation

The generation of the outcome is traditionally reduced to the unfolding of a capacity. Intuitively, a capacity is a property that some entities have and that natural language often acknowledges with attributes ending in “-able” or “-ible.” The capacity remains latent until, under the right conditions, it unfolds as a power. The power, in turn, yields the outcome unless it meets some obstruction.

Against this backdrop, there are two possible points of attack. First, causation can be addressed as an unimpeded *process* to the outcome. The corresponding model structures the hypothesis as a sequence of occurrences; the hypothesis, then, can be proven by finding in the field the traces or marks of those peculiar events. Alternatively, causation can be captured as the

local interaction of enabling, triggering, and shielding factors beneath the process. Such interaction renders the hypothesis of the *causal setting* where the unfolding takes place unimpeded and can account for both the identifying traces of the process and the occurrence of the outcome.

Processes and settings are often understood as mechanisms of a different kind. As the argument has long gone in the philosophy of science, processes offer an “etioloical” causal story in which the outcome follows from the transmission of some causal quantity, like in a pool game. Settings, instead, provide “constitutive” mechanisms that capture the “emergence” of qualitatively different phenomena from the interaction of special factors, as in a recipe or a chemical reaction. The etioloical approach builds models with an observational advantage: the effect of a transmission can be reduced to the algebraic sum of the previous vectors without any meaningful loss of information, and, the other way round, the elements of the mechanism can be reasonably backtracked given the outcome. The same does not apply to the constitutive understanding, instead, as the compound is the merger of its constituents. Although some facets of the resulting phenomenon can ultimately be reduced to the sum of its components, as is the weight of water molecules, these relationships only become clear after the components are identified. The knowledge problem consists of identifying the constitutive elements – which requires developing and testing hypotheses in successive approximations.

1.3. An exemplary implementation: the SCM

The Structural Causal Model framework (SCM: *e.g.*, (Pearl, 1998); (Geiger et al., 1990), (Pearl and Verma, 1995), (Halpern and Pearl, 2005), (Halpern and Pearl, 2005), (Shpitser and Pearl, 2008), (Pearl and Bareinboim, 2014)) offers a recognized strategy to test and refine the shape of mechanistic hypotheses. Seemingly, the proposal dovetails with the etioloical understanding, as it renders data-generation mechanisms as graphs. However, the Bayesian feature of the graphs means that they do not render the world’s structures by default. Instead, they are devices for laying our assumptions open to scrutiny on which information is relevant to make correct inferences regarding some states of the world.

In better detail, graphs entail a universe in which units carry information on a selection of observed variables $\mathcal{V} = \{X, Y, Z, \dots\}$, background unobserved variables $\mathcal{U} = \{U_X, U_Y, U_Z, \dots\}$ that affect the explicit variables in the model, and functions $\mathcal{F} = \{f_X, f_Y, f_Z, \dots\}$ mapping the dependence of each observed variable's information on that from any remaining observed and unobserved ones.

Variables are literals and graph nodes, while the functions are edges that connect the nodes pairwise. The graph's structure is decided by independencies – the missing edges between two nodes – and the direction of each dependence – turning an edge into an arrow. A structure is analytically meaningful when it includes at least three nodes such that knowing two makes the knowledge of the third irrelevant. Three positions satisfy the requirement: the mediator in a linear chain of dependencies; the common source of two effects in a forked structure; and the common effect from two independent factors, dubbed “collider.”

1.3.1. Chains

In the linear chain graph \mathcal{G}_l

$$\mathcal{G}_l: \begin{array}{ccccc} X & \rightarrow & Z & \rightarrow & Y \\ \uparrow & & \uparrow & & \uparrow \\ U_X & & U_Z & & U_Y \end{array}$$

the factor Y follows from Z that follows from X , each with independent background conditions, captured by the nested functions in model \mathcal{M}_l

$$\mathcal{M}_l := \begin{cases} X = U_X \\ Z = \beta_{ZX}X + U_Z \\ Y = \beta_{YZ}Z + U_Y \end{cases}$$

and entailing the following in/equalities:

$$\mathcal{G}_\ell \models \begin{cases} P(Y|Z) \neq P(Y) \\ P(Z|X) \neq P(Z) \\ P(Y|X) \neq P(Y) \\ \mathbf{P}(Y|ZX) = \mathbf{P}(Y|X) \end{cases}$$

where P reads as the frequentist probability. These conditionals say that every consequent in the chain is possibly dependent on its antecedent; however, the middle node Z captures the whole of it. The structure is the archetype of a proper causal relation as it famously renders the chains from smoking to cancer through tar or from heating to pain through the spinal cord.

1.3.2. Forks

Similarly, in the fork graph \mathcal{G}_f

$$\mathcal{G}_f: \begin{array}{ccccc} X & \leftarrow & Z & \rightarrow & Y \\ & \uparrow & & \uparrow & \uparrow \\ & U_X & & U_Z & U_Y \end{array}$$

Z is the common antecedent of both X and Y , meaning that, by theoretical assumption,

$$\mathcal{M}_f := \begin{cases} Z = U_Z \\ X = \beta_{XZ}Z + U_X \\ Y = \beta_{YZ}Z + U_Y \end{cases}$$

and entailing the following relationships:

$$\mathcal{G}_f \models \begin{cases} P(Z|X) \neq P(Z) \\ P(Z|Y) \neq P(Z) \\ P(Y|X) \neq P(Y) \\ \mathbf{P}(Y|ZX) = \mathbf{P}(Y|Z) \end{cases}$$

The consequents seem to depend on each other due to the common antecedent Z . The dependence, however, vanishes when assessed within each subpopulation of units sharing the same value of Z . Controlling for the common antecedent ensures the independence holds for any consequent of X and Y , too.

The fork renders spurious causal situations such as the relationship between the reading of the barometer and the weather, but also the spell cast on salt and its dissolution in water – hence, it provides reasons for dismissing the false causal factor after the true one has been added to the model.

1.3.3. Colliders

Last, in the collision graph \mathcal{G}_c

$$\mathcal{G}_c: \begin{array}{ccccc} X & \rightarrow & Z & \leftarrow & Y \\ & \uparrow & & \uparrow & \uparrow \\ & U_X & & U_Z & U_Y \end{array}$$

Z follows from both X and Y , as in the theoretical model \mathcal{M}_c

$$\mathcal{M}_c := \begin{cases} X = U_X \\ Y = U_Y \\ Z = \beta_{ZX}X + \beta_{ZY}Y + U_Z \end{cases}$$

which entails the following relationships

$$\mathcal{G}_c \models \begin{cases} P(X|Z) \neq P(X) \\ P(Y|Z) \neq P(Y) \\ \mathbf{P(X|Y)} = \mathbf{P(X)} \\ P(X|ZY) \neq P(X|Z) \end{cases}$$

The common outcome is dependent on each alternative antecedents, which are independent of each other. However, their independence vanishes when the relationship is controlled for the common outcome Z , or any of its dependents.

Colliders capture those causal situations where the same outcome can be produced by partial equifinal causes as heating and capsaicin are of burning sensations. The alternatives, however, can again be interpreted as causal when the collider is a mediator to the outcome in a causal chain.

Knowledge of the structure allows for calculating the causal effect of an antecedent X on the outcome Y . The SCM framework provides rules for simulating the experimental intervention to the alleged causal factor – that is, removing key edges – indicated by the operator $do(\cdot)$. Subsetting units can pinpoint the units that are proper instances of the mechanism and hence provide regularity and counterfactual estimates of its effect on the outcome (see [Appendix §2](#)). Furthermore, proven structural models allow calculating counterfactuals for those single units for which the value of the background variables U are known, as well as the average treatment effect on classes of units.

2. QCA and causal claims

Against the backdrop of the general debates on causal analysis and the example of the SCM framework, considerations can be advanced about the causal standing of QCA’s operations and findings.

2.1. Beyond regularity vs. “counterfactual” designs: factual and counterfactual regularities

The definition of QCA as a regularity-based technique rests on its reliance on sufficiency (calculated as in equation (1) and dubbed “consistency of sufficiency”) in observational designs.

Thus, its solutions are identified on types of instances of the outcome, but, like in experimental designs that aim to make the counterfactual “observable,” the proper analysis is required to draw its conclusions based on the antecedents to both states of the outcome.

The proper protocol, therefore, does apply the test displayed in equation (2) as a regularity test of the sufficiency of the missing factor on the instances of the missing outcome. The analysis on the absence of the outcome reads weird and indeed stemmed heated debates in the philosophy of science as, ideally, it commits to consider white shoes before deciding whether it is true that, if u_i is a raven, then it is black. The analysis becomes viable, however, if the universe of the discourse \mathcal{U} is defined at the outset as a meaningful population so that its units display as many observable realizations as possible of the factors before any outcome.

The universe of discourse, hence, provides the scope condition of the analysis and makes the boundaries clear of the validity of any findings while ensuring that units are proper instances of the explanatory model (Walker and Cohen 1985, Ragin 2014). To witness, speaking of ravens, the universe can be of birds; by extension, to establish whether administrative accountability designs explain the perception of corruption, one may limit the analysis to democracies in certain space-time regions to ensure interpretable concepts and, then, reliable measures.

2.2. Beyond singular vs. general causation: cases as instances of property-sets

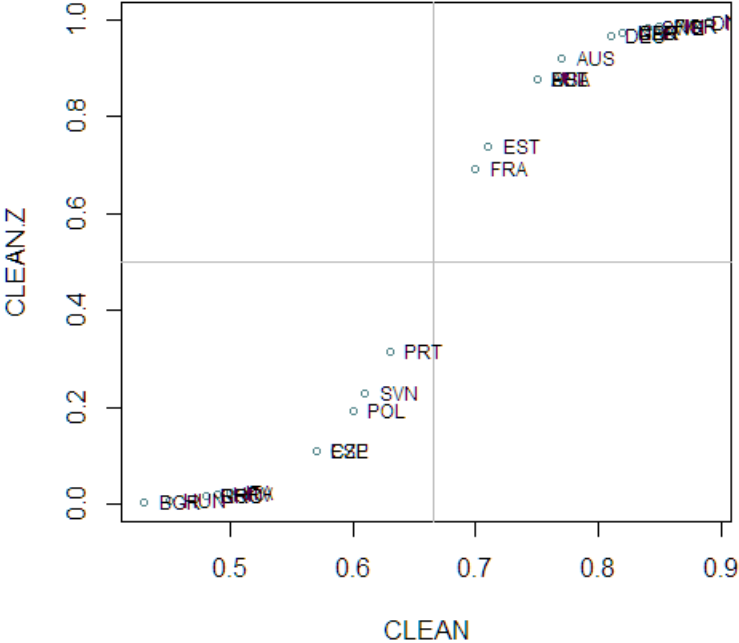
Given a universe, the relationship between the factors sufficient to the occurrence of the outcome and those sufficient to its non-occurrence rests on the gauging strategy. In Standard QCA, the strategy boils down to a reasoned classification of units as members of a set or its complement, *tertium non datur*. The classification establishes the qualitative similarity of units based on their manifestation of certain attributes or lack thereof – thus resolving the linkage of singular to general causation –, and comes in two versions.

The “crisp” one follows the original Boolean stipulations that only allow for two membership values, 1 for the inclusion of a unit in a set and 0 for its exclusion. The “fuzzy” version recognizes the possibility of ambiguity or errors in classification and allows units to take any value between 0 and 1 except 0.5. While the extreme values convey certainty in classification,

0.5 renders the highest ambiguity: thus, the corresponding units, neither be in nor out the corresponding property-set, would carry information to no use for the analysis. This critical point, however, has great analytical relevance. It provides the rationale for turning fuzzy scores into crisp: by definition, any fuzzy score corresponds to the crisp score of 0 if lower than 0.5 and of 1 if higher; thus, a unit is an instance of the set in which it scores higher than 0.5 and makes that set “observed” or “empirically true.” The critical point, besides, provides the central anchor or “crossover” in the transformation of continuous “raw” metrics into fuzzy membership scores (Ragin 2000: 156 ff, 2008:31 ff). The 0.5 indicates the change in the slope of the logistic filter function to which the raw values are conventionally pegged, whereas the inclusion and the exclusion points indicate the thresholds above and below which the difference in raw values is almost irrelevant to establish the membership.

As an example, the selection of the thresholds to calibrate the levels of perception of corruption in selected democracies in 2017 shapes the filter function as in Figure 1.

Figure 1. Raw (CLEAN) by fuzzy (CLEAN.Z) scores of selected units



Notes: exclusion = 0.535, crossover = 0.665, inclusion = 0.790. Reference year of the raw data: 2017. The shape assigns fuzzy scores close to 0 to Italy, Croatia, Romania, Greece, Hungary, and Bulgaria; between 0.1 and 0.4 to Czechia, Spain, Poland,

Slovenia, and Portugal; between 0.6 and 0.8 to France and Estonia, and scores increasingly close to 1 to Austria, Belgium, the U.S., Australia, Germany, Canada, the U.K., the Netherlands, Sweden, Finland, Norway, Denmark, and New Zealand.

Source: raw scores by Transparency International.

From this perspective, crisp scores are special, rougher cases of fuzzy scores, resulting from a logistic filter function with no classification errors – in which inclusion, exclusion, and highest ambiguity points coincide. The other way round, fuzzy scores afford a more fine-grained calculation of sufficiency that shows “rougher” crisp calculations are inflated. Indeed, fuzzy scores leave residuals in the intersection of a set with its complement and higher the closer they are to the point of ambiguity. To witness, a unit scoring $Y_i = 0.99$ would have a membership in the intersection of $Y_i \cap y_i = \min(0.99, 0.01) = 0.01$, whereas one scoring $Y_i = 0.55$ would have a membership in the same intersection of 0.45. Being all lower than 0.5, they would not violate stipulation (S2.b); nevertheless, high residuals complicate the analysis as they blur the ascription of a configuration to a state of the outcome, hence can undermine the credibility of the analysis.

2.3. Beyond configurations vs. mechanisms: configurations as mechanisms

QCA suits testing and refining *if-then* hypotheses where the antecedent is an explanatory type or otherwise renders a *constitutive* mechanism (e.g., (Amenta and Poulsen 1994), (Elman 2005), (Damonte 2021)). Its configurational mindset encourages building on theory and empirical literature *à la* Verba to conceive of the antecedent as the complete team or bundle of conditions under which the causal process has to unravel unimpeded regardless of the remaining background conditions.

To witness, following the literature, it is possible to claim that accountable administrative systems are those where independent technical bodies perform effective auditing and review of the government’s operations (ATE); independent media, civil society organizations, political parties, and individuals are free to report and comment on government policies without fear of retaliation (ASO); basic laws and information on legal rights are publicly available, presented in everyday language, and accessible, and administrative regulations, drafts of legislation, and high court decisions are promptly accessible to the public (APU); requests for relevant information from a government agency are timely granted, responses are pertinent and complete, and the cost

of access is reasonable and free from bribes (RTA); moreover, civil justice decisions – on which the proper enforcement of the freedoms and right above depend – are effective and timely (ENF). Thus, the constitutive mechanism before low perceived levels of corruption (CLEAN) reads as the antecedent in the Hp below:

$$\text{Hp: } ATE * ASO * APU * RTA * ENF \rightarrow CLEAN$$

where the star is for the intersection and the wedge. The hallmark of such antecedent is, the team is a compound causal factor that is expected to obtain when all its components are given in the right state and fail otherwise: hence, the components are necessary to an effective compound, but alone cannot account for the outcome. Otherwise said, the compound is the latent causal factor that arises when the right conditions are jointly present. This licenses expectations about empirical types as alternative realizations of the compound. Were the compound the true and only explanation, there would be only one realization with high sufficiency values to Y and, with K binary components, $2^K - 1$ realizations with high sufficiency values to y instead.

2.4. Beyond merging types vs. finding difference-makers: QCA as model specification to the universe

The analysis lays the expectation opens to inspection by listing all the possible realizations of the compound as a “truth table,” establishing which are observed, and then calculating the sufficiency to the positive, then to the negative outcome of each observed realization. To discount the inflation from fuzzy residuals, the sufficiency of equation (1) can be corrected as in equation (3.a) below, and the sufficiency of equation (2) rendered as equation (3.b) instead:

$$PRI_Y := \frac{(X \cap Y) - (X \cap Y \cap y)}{X - (X \cap Y \cap y)} \quad (3.a)$$

$$PRI_y := \frac{(x \cap y) - (x \cap Y \cap y)}{x - (x \cap Y \cap y)} \quad (3.b)$$

Dubbed “Proportional Reduction of Inconsistency” (PRI: (Ragin 2009), (Ragin 2015)), the parameter follows the rationale of the Proportional Reduction in Error in establishing the association of a factor (here, of a realization) and a state of the outcome and, as such, is more demanding than the original parameter of sufficiency.

To witness, after calibrating the indexes of the World Justice Project that operationalize the components of Hp.1, the corresponding truth table reports the information as in Table 1.

Table 1. Truth table: key information

Ω	ATE.Z	ASO.Z	APU.Z	RTA.Z	ENF.Z	n	PRI.Y	PRI.y	instances
26	1	1	0	0	1	2	1.000	0.000	AUT,BEL
30	1	1	1	0	1	2	1.000	0.000	AUS,CAN
24	1	0	1	1	1	1	1.000	0.000	FRA
32	1	1	1	1	1	10	0.999	0.000	NZL,DEU,DNK,EST,FIN,GBR,NLD,NOR,SWE,USA
25	1	1	0	0	0	1	0.204	0.665	PRT
17	1	0	0	0	0	1	0.381	0.943	ITA
1	0	0	0	0	0	9	0.148	0.989	BGR,CZE,ESP,GRC,HRV,HUN,POL,ROU,SVN
2	0	0	0	0	1	0	-	-	
3	0	0	0	1	0	0	-	-	
4	0	0	0	1	1	0	-	-	
5	0	0	1	0	0	0	-	-	
6	0	0	1	0	1	0	-	-	
7	0	0	1	1	0	0	-	-	
8	0	0	1	1	1	0	-	-	
9	0	1	0	0	0	0	-	-	
10	0	1	0	0	1	0	-	-	
11	0	1	0	1	0	0	-	-	
12	0	1	0	1	1	0	-	-	
13	0	1	1	0	0	0	-	-	
14	0	1	1	0	1	0	-	-	
15	0	1	1	1	0	0	-	-	
16	0	1	1	1	1	0	-	-	
18	1	0	0	0	1	0	-	-	
19	1	0	0	1	0	0	-	-	
20	1	0	0	1	1	0	-	-	
21	1	0	1	0	0	0	-	-	
22	1	0	1	0	1	0	-	-	
23	1	0	1	1	0	0	-	-	
27	1	1	0	1	0	0	-	-	
28	1	1	0	1	1	0	-	-	
29	1	1	1	0	0	0	-	-	
31	1	1	1	1	0	0	-	-	

The truth table contains 32 realizations, of which only seven are observed. Of the observed, the values of sufficiency to the outcome and its complement (respectively, “PRI.Y” and “PRI.y” in Table 1) of each are fairly symmetric, thus licensing the consideration that the first four realizations (namely, $\Omega_{26}, \Omega_{30}, \Omega_{24}, \Omega_{32}$) are conducive to Y , whereas the remaining three (that is, $\Omega_{25}, \Omega_{17}, \Omega_1$) to y . It is worth noting the relative exception of Ω_{25} , whose PRI is only slightly above the threshold of 0.5 conventionally established to assign a realization to an outcome with relative certainty.

The corresponding best instances of each realization also confirm that the starting hypothesis does contain proper explanatory factors, as each realization arises a “homogeneous” partition of instances of the same outcome, thus proving the starting hypothesis is not underspecified and contains all the factors to make sense of the diversity of the outcome (for instance, it includes the pressure to account for the weather). Furthermore, with ten instances, realization Ω_{32} – that, according to the hypothesis, was expected to produce the outcome – actually is the most populated partition. However, it is not the only one. The presence of further effective realizations, and of unobserved configurations, together suggest that the hypothesis is overspecified instead – by either irrelevant factors to some subpopulation (be it of the spurious kind, as is the barometer or the spell, or simply an alternative that is relevant to some subpopulation) or by chains within the compound (as is between the burning and the spinal cord).

The minimizations in Standard QCA, performed with the Quine-McCluskey algorithm, can get rid of the irrelevant factors by applying a rationale close to the counterfactual analysis in manipulation designs but to declare the irrelevance of a factor. The minimizations compare realizations pairwise that lead to the same outcome and drop the “single different” component in otherwise identical teams. For instance, in Table 1, Ω_{26} and Ω_{30} only differ by $APU.Z$, which can be dismissed, revealing the more general implicant $ATE.Z * ASO.Z * rta.z * ENF.Z$. As the implicant covers Austria, Belgium, Australia, and Canada, it also accounts for each of their (positive) outcomes, and more credibly so. The process applies to implicants with the same number of literals and only stops when no single differences are available.

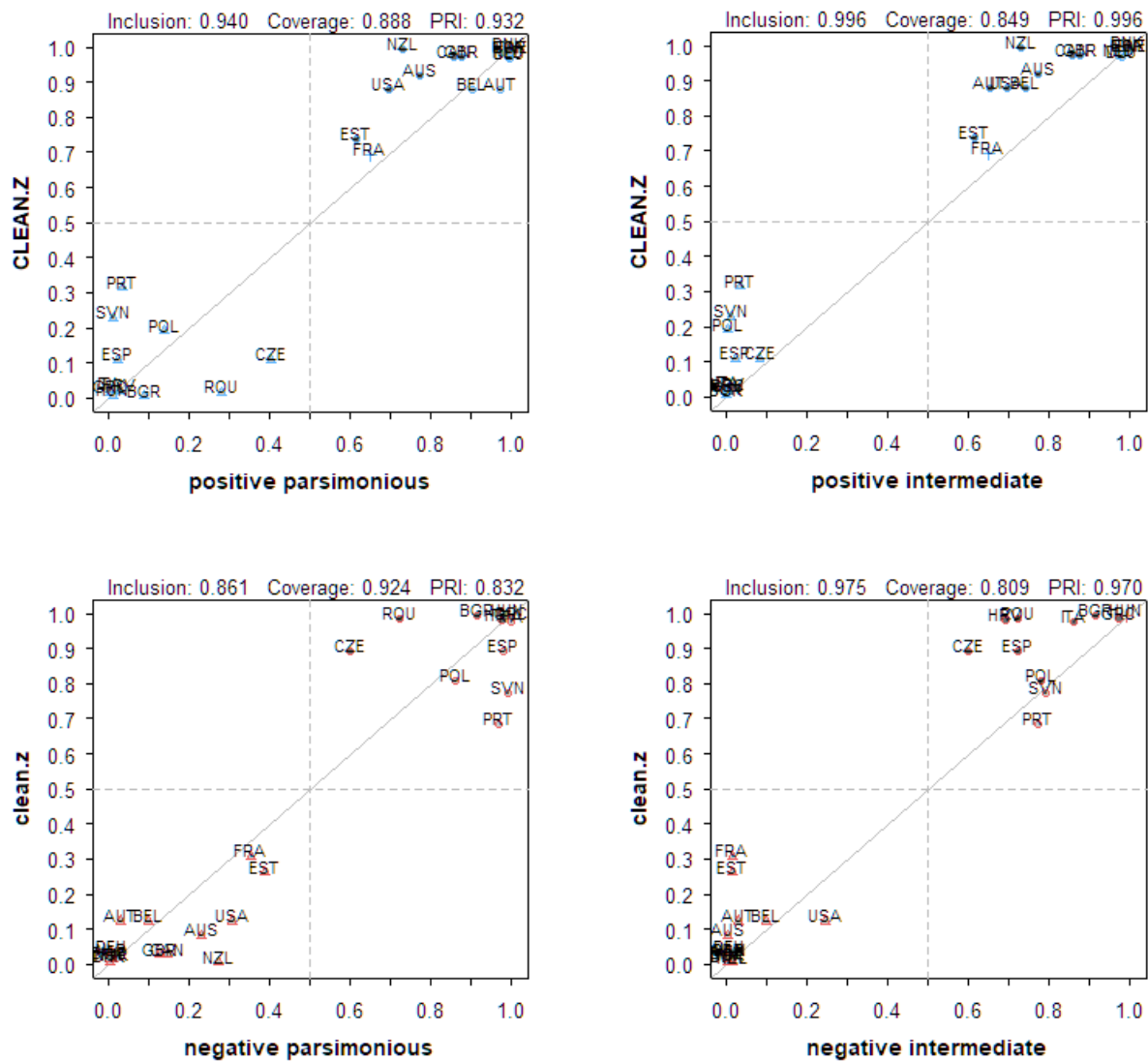
The truth table makes clear that minimizations always entail counterfactual assumptions as tenets about unobserved realizations. When they are performed on observed realizations only, the assumption is that the unobserved would never obtain, which returns the complex implicants. But they can be performed on observed realizations and any unobserved that satisfy the single-difference rule, which returns more parsimonious implicants – the essential difference-makers, without which the truth table would become contradictory. The matching can be a “hard counterfactual” when the unobserved configuration carries the *minimizand* in a state that theory would expect contributing to the opposite outcome. Minimizations that only rely on “easy counterfactuals” return the plausible implicants, usually of intermediate complexity.

To continue the accountability example, the parsimonious minimizations find a single implicant, $ENF.Z$, accounts for all the instances of the positive outcome, while its negation, $enf.z$, as the single implicant to the negative outcome. This is a (rare) example of a single essential factor: it is easy to prove that, were it dropped from the hypothesis, the new truth table would become contradictory. The interpretation is, an effective civil justice is sufficient to perceive low levels of administrative corruption, while an ineffective one triggers the opposite perceptions. The plausible minimizations add further components. The additions prove the source of unexpected positive realizations in the truth table is a hidden equifinality. The solution of the positive outcome reads $ATE.Z * ENF.Z * (ASO.Z + APU.Z * RTA.Z)$, where + is for the union and the vee. It means the plausible team before low perceived corruption always combines an effective civil justice and the review from technical bodies, reinforced, in a subpopulation, by societal oversight, and in a partially different subpopulation, by generalized rights to access information combined with effective administrative obligations to circulate proper information. The cases listed in both subpopulations are those that display both the societal and the legal drivers, which can be considered functionally equivalent. The plausible solution of the negative outcome, instead, reads $apu.z * rta.z * enf.z$, which is not symmetric to the positive one. The interpretation is, societal oversight and technical reviews become irrelevant when the legal provisions about administrative information are poor and civil justice ineffective.

2.5. Testing the credibility of parsimonious and plausible implicants

Both the plausible and the essential implicants support interpretations that make theoretical sense. The open question in the debate about the causal standing of QCA solutions, however, remains, asking whether the parsimonious solution should be granted higher credibility than the plausible.

Panel 1. XY-plots



A response can be provided from within the technique: beyond any value that the parameters can take, the ultimate information is visual. And indeed, the XY plots of the parsimonious and intermediate solutions in Panel 1 show the added explanatory value of the theoretical assumptions about the *minimizands*. Despite the fact that two inconsistent cases remain in the explanation of the negative outcome, the comparison of the left and the right distributions shows that added terms in the plausible solutions do improve the fit to the model.

The technique, however, cannot dispel the doubt that these solutions have little to share with proper mechanisms. On this point, the equalities and inequalities between conditionals established by the SCM framework to identify causal structures offer the blueprint for an external response. The application is straightforward when the fuzzy scores are turned into crisp.

chain		fork		collider
$X \rightarrow Z \rightarrow Y$	$W \rightarrow Z \rightarrow Y$	$X \leftarrow Z \rightarrow Y$	$W \leftarrow Z \rightarrow Y$	$X \rightarrow Z \leftarrow W$
$\pi(Y ZX) = \pi(Y X)$	$\pi(Y ZW) = \pi(Y W)$	$\pi(Y ZX) = \pi(Y Z)$	$\pi(Y ZW) = \pi(Y Z)$	$\pi(X W) = \pi(X)$
1.000 = 1.000	1.000 = 1.000	1.000 != 0.933	1.000 != 0.933	0.909 != 0.577
TRUE	TRUE	false	false	false
$V \rightarrow z \rightarrow y$		$V \leftarrow z \rightarrow y$		
$\pi(y zV) = \pi(y V)$		$\pi(y zV) = \pi(y z)$		
1.000 = 0.846		1.000 = 1.000		
false		TRUE		

Keys: $Z = ENF.Z$ $X = ATE.Z * ASO.Z,$ $W = ATE.Z * APU.Z * RTA.Z,$
 $z = enf.z$ $V = rta.z * apu.z$
N=26

The conditionals say the parsimonious term is the mediator in the chain from plausible components to the outcome, and provides the confounder in the explanation of the missing outcome – or the antecedent to $V = rta.z * apu.z$ that so becomes the actual mediator in the linear chain.

3. Some last remarks

The configurational literature has long questioned the causal standing of Standard QCA based on two keys dilemmas: counterfactual considerations would prove unsuitable to regularity techniques; actual mechanistic understanding would prove beyond the reach of general causation. This work contended that the dilemmas stand in light of the wider debate on causal analysis and exemplary implementations. Against this backdrop, it showed QCA can be geared toward rendering, testing, and refining models of constitutive mechanisms, and that its findings can afford a different structural rendering. The technique can return more sound and causally interpretable results than the scholarship often recognizes.

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Appendix

§1. Stipulations and conventions

The article commits to the tenets that the literals and connectives in QCA’s Boolean algebra afford a twofold reading – logical and set-theoretical (Stone, 1936).

Literals

Logically, the literals render the intensional dimension of predicates. Predicates are for the qualitative state that any unit can be said to take. Uppercase literals (such as A) indicate the set of the units for which the predicate “is A ” is empirically true. Lowercase literals (such as, a) indicate the set for which the negated predicate is true instead. Each predicate can be either “true” or “false” of a unit depending on classification rules established before the analysis.

Set-theoretically, the literal renders the extensional dimension of the predicate as a class or set. A class corresponds to a partition of the universe \mathcal{U} that clusters together those units $u \in \mathcal{U}$ with the same membership score. As classes or sets correspond to predicates, a unit’s logical truth-value and its set-membership score are one and the same.

When subscripted, the literal refers to an actual unit: a_i reads “unit i is not A ,” and the truth-value corresponds to its membership score in the set. By definition, in a universe of $0 < i \leq N$ units, the size of a set is the sum of the units’ membership scores in that set:

$$A = |A_i| = \sum_{i=1}^N A_i \quad (\text{S1.a})$$

$$a = |a_i| = \sum_{i=1}^N a_i \quad (\text{S1.b})$$

Relationships

By stipulation, A and a are *complementary* and *mutually exclusive*.

Extensionally, these relationships are rendered by set union (\cup) and intersection (\cap). Complementarity means that the union of A and a returns the universe, while their mutual exclusivity entails that their intersection returns an empty set:

$$A \cup a = \mathcal{U} \quad (\text{S2.a})$$

$$A \cap a = \emptyset \quad (\text{S2.b})$$

Union and intersection are the set-theoretical renderings of the logical connectives for, respectively, the disjunction “or” (\vee) and the conjunction “and” (\wedge). Intensionally, the disjunction of a state and its complement raises a logical tautology, which is always true, while their conjunction raises a contradiction, which is always false:

$$A_i \vee a_i = 1 \quad (\text{S3.a})$$

$$A_i \wedge a_i = 0 \quad (\text{S3.b})$$

More generally, a disjunction is true in a unit when at least one disjunct is true. It follows that a unit’s membership score in a union is given by the highest score in its sets. Hence, given a train of $0 < j \leq K$ literals $A_1, \dots, A_j, \dots, A_K$,

$$\bigvee A_{ji} = \bigcup A_{ji} = \max(A_{ji}) \quad (\text{S4})$$

Similarly, a conjunction is true in a unit where every conjunct is true, meaning that the units’ membership score in an intersection is given by the lowest scores in the sets that constitute it. Thus, given the same train of K literals,

$$\bigwedge A_{ji} = \bigcap A_{ji} = \min(A_{ji}) \quad (\text{S5})$$

Last, A is *sufficient* to Y when the statement “if A , then Y ” holds empirically. Intensionally, it means that the truth-value of statement A in the i -th unit is contained in the truth-value that statement Y takes in the same unit; at the level of the universe, instead, it means that the instances of A are all instances of Y so that the former is a subset of the latter:

$$A_i \leq Y_i \quad (\text{S6.a})$$

$$A \subseteq Y \quad (\text{S6.b})$$

Conversely, A is *necessary* to Y when the statement “only if A , then Y ” holds as the truth of the former contains that of the latter so that the set of the former’s instances encloses the set of the latter’s’:

$$Y_i \leq A_i \tag{S7.a}$$

$$Y \subseteq A \tag{S7.b}$$

In short, ideally, sufficient is the factor whose presence makes the outcome certainly occur, whereas necessary is the factor whose absence makes the outcome certainly void.

§2. Causal effect calculation with SCMs

When the antecedent has no parents, as is the vertex Z of a fork, the relationship with the outcome of interest Y is unconditional, and the intervention coincides with the conditional probability of the outcome given the vertex weighted by the probability of the vertex: $P(Y|do(Z)) = P(Y|Z)$.

When Z is a partial fork, the causal effect can be identified by operating a “back-door adjustment” to simulate the insulation of the antecedent of interest from any other factor on which it may be dependent. Thus, with binary variables, Z ’s unconditional realizations have the probabilities as in Table A1; X ’s conditional realizations have the probabilities as in Table A2; Y ’s conditional realizations have probability as in Table A3, and the probabilities of Y ’s realizations under the simulated intervention $do(X)$ read as in Table A4.

Similarly, when Z is a mediator, the causal effect is given by the chain of the partial effects with the “front-door adjustment” – of a selected state of the antecedent on the mediator, and of the state of the antecedent on the dependent in the subpopulation with the mediator in the right state.

These rules compose the building blocks of the “do-calculus” and allow simulating experimental interventions by narrowing on those instances that display a certain value of X (or, before, a certain propensity to display it) within identified causal structures.

Table A1. Unconditional probability of Z 's realizations when $X \rightarrow Y$ is partially confounded by forking Z

Ω	Z	$P(Z)$
ω_1	1	α
ω_2	0	$1-\alpha$

Table A2. Probability of X 's realizations given Z 's when $X \rightarrow Y$ is partially confounded by forking Z

Ω	Z	X	$P(X Z)$
ω_{1a}	1	1	β_1
ω_{1b}	1	0	$1-\beta_1$
ω_{2a}	0	1	β_0
ω_{2b}	0	0	$1-\beta_0$

Table A3. Probability of Y 's realizations given X 's when $X \rightarrow Y$ is partially confounded by forking Z

Ω	Z	X	Y	$P(Y X,Z)$
$\omega_{1a'}$	1	1	1	γ_{11}
$\omega_{1a''}$	1	1	0	$1-\gamma_{11}$
$\omega_{1b'}$	1	0	1	γ_{01}
$\omega_{1b''}$	1	0	0	$1-\gamma_{01}$
$\omega_{2a'}$	0	1	1	γ_{10}
$\omega_{2a''}$	0	1	0	$1-\gamma_{10}$
$\omega_{2b'}$	0	0	1	γ_{00}
$\omega_{2b''}$	0	0	0	$1-\gamma_{00}$

Table A4. Probability of Y 's realizations under a simulated intervention on X

Ω	$do(X)$	Y	$P(Y do(X))$
ω'_a	1	1	$\alpha\gamma_{11}+(1-\alpha)\gamma_{10}$
ω''_a	1	0	$\alpha(1-\gamma_{11})+(1-\alpha)(1-\gamma_{10})$
ω'_b	0	1	$\alpha\gamma_{01}+(1-\alpha)\gamma_{00}$
ω''_b	0	0	$\alpha(1-\gamma_{01})+(1-\alpha)(1-\gamma_{00})$

§3. Data, fuzzy (.Z) and crisp (.S)

ID	CLEAN.Z	ATE.Z	ASO.Z	APU.Z	RTA.Z	ENF.Z	CLEAN.S	ATE.S	ASO.S	APU.S	RTA.S	ENF.S
AUS	0.91847	0.96176	0.91988	0.99766	0.16552	0.76992	1	1	1	1	0	1
NZL	0.99446	0.98562	0.93076	0.98493	0.86001	0.7279	1	1	1	1	1	1
AUT	0.87658	0.65075	0.96939	0.34963	0.0742	0.97179	1	1	1	0	0	1
BEL	0.87658	0.98863	0.74216	0.17456	0.44125	0.90155	1	1	1	0	0	1
BGR	0.00542	0	0.00145	0.00671	0	0.08525	0	0	0	0	0	0
CAN	0.97275	0.92755	0.96297	0.99233	0.40659	0.8543	1	1	1	1	0	1
CZE	0.10844	0.29754	0.08109	0.10705	0.06879	0.40051	0	0	0	0	0	0
DEU	0.96592	0.99379	0.98006	0.65661	0.88143	0.99399	1	1	1	1	1	1
DNK	0.99303	0.99955	1	0.97705	0.96424	0.99608	1	1	1	1	1	1
ESP	0.10844	0.19585	0.16653	0.27971	0.04108	0.01964	0	0	0	0	0	0
EST	0.73844	0.99399	0.68979	0.98763	0.78699	0.61425	1	1	1	1	1	1
FIN	0.98617	0.99986	0.99759	0.99662	0.97902	0.99113	1	1	1	1	1	1
FRA	0.69152	0.96346	0.27975	0.98469	0.92084	0.64874	1	1	0	1	1	1
GBR	0.97275	0.97005	0.967	0.99703	0.54325	0.87393	1	1	1	1	1	1
GRC	0.01626	0.38271	0.02002	0.00449	0.02792	0.00132	0	0	0	0	0	0
HRV	0.02021	0.00083	0.01917	0.0033	0.31139	0.02627	0	0	0	0	0	0
HUN	0.00843	0	0	0.00624	0	0.00985	0	0	0	0	0	0
ITA	0.02511	0.69378	0.02129	0.01737	0.13963	0.00048	0	1	0	0	0	0
NLD	0.97275	0.99668	0.97295	0.85772	0.97286	0.99207	1	1	1	1	1	1
NOR	0.98617	0.99984	0.99907	0.99269	0.99753	0.99578	1	1	1	1	1	1
POL	0.19131	0.00301	0.00215	0.02515	0.22143	0.13857	0	0	0	0	0	0
PRT	0.31514	0.89354	0.86241	0.00227	0.2302	0.03233	0	1	1	0	0	0
ROU	0.01626	0.00064	0.34248	0.01983	0.01425	0.27924	0	0	0	0	0	0
SVN	0.22798	0.01294	0.03509	0.21067	0.10628	0.00874	0	0	0	0	0	0
SWE	0.98264	0.99986	0.99862	0.88074	0.9997	0.99714	1	1	1	1	1	1
USA	0.87658	0.83372	0.80737	0.75412	0.6419	0.69379	1	1	1	1	1	1