A basic New Keynesian DSGE model with dispersed information: An agent-based approach*

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October 2017

Abstract

The aim of this paper is to bridge macro agent-based models with mainstream macroeconomic models by *agentifying* the baseline New Keynesian DSGE model. The model features multiple, boundedly rational, optimizing agents and is analyzed through numerical simulations. We exploit the flexibility of agent-based modeling to explore the effect of dispersed information on the learning process and on macroeconomic outcomes. We find that with dispersed information monetary and fiscal policy acquire the role of public signals.

Keywords: DSGE models, Agent-based models, heterogeneity, learning, dispersed information.

JEL classification: C63, D83, E52.

*This is a pre-copyedited, author-produced version of an article accepted for publication in Journal of Economic Behavior & Organization following peer review. The version of record (*Journal of Economic Behavior & Organization*, Volume 157, January 2019, Pages 101-116) is available online at: `https://doi.org/10.1016/j.jebo.2017.12.015`.

The authors would like to thank two anonymous referees, the participants in the WEHIA 2015 Conference, the workshop "Agent-Based and DSGE Macroeconomic Modelling: Bridging the Gap" organized by Paul Levine at University of Guildford and the JAFEE20 International Conference. The usual disclaimer applies. The research leading to these results has received funding from the European Union, Seventh Framework Programme FP7/2007-2013 Socio-economic Sciences and Humanities under Grant Agreement No. 612796 - MACFINROBODS.

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1 Introduction

The financial crisis of 2007 and the ensuing global recession questioned the ability of dynamic stochastic general equilibrium (DSGE) models to timely predict major episodes of distress and to detect the growing instability in economic systems. As a consequence, further attention has shifted on a new stream of macroeconomic computational models: agent-based (AB) models (Farmer and Foley, 2009; Fagiolo and Roventini, 2012, 2017; Battiston et al., 2016). These models incorporate elements such as bounded rationality, network structures, direct and indirect interactions among heterogeneous agents, bankruptcies, as these aspects are seen as essential features of an actual economy.

From a different perspective, AB modeling can also be considered as a computational methodology to tackle complex models in which an analytic solution is absent. Agents are simulated as autonomous individual entities and their decisions are aggregated numerically (Jennings, 2000; Tesfatsion, 2002; Richiardi, 2003; Pyka and Fagiolo, 2007). This feature of AB models resolves the theoretical issues related to an aggregate representation of the economic system (Kirman, 1992; Fagiolo and Roventini, 2017), and provides the flexibility to analyze the economy as an *evolving complex system* (Tesfatsion, 2006; Delli Gatti et al., 2010). Macroeconomic AB models are able to reproduce both micro stylized facts, such as the distributions of firm size and household wealth, and realistic macro behavior, such as aggregate fluctuations and endogenous crises (e.g., Delli Gatti et al., 2005; Russo et al., 2007; Delli Gatti et al., 2011; Dosi et al., 2015; Assenza et al., 2015; Giri et al., 2016).[1]

However, complexity bears a cost: AB models fail to keep the analytical tractability of DSGE models and lack well-established econometric approaches to estimation. On the contrary, DSGE models represent a multifaceted yet unitary theoretical framework that allows economists to assess the effect of different economic hypotheses and parametric choices. Despite their empirical and theoretical limitations, DSGE models still stand as the *de facto* paradigm for macroeconomic analysis.

The literature streams of AB and DSGE models have grown independently and it is very challenging to compare results and policy implications. The aim of this paper is to analyze and understand the differences and facilitate the communication between these two approaches. To bridge AB modeling with standard macroeconomic modeling, we consider the baseline New

[1]Recently, Beaudry et al. (2015) proposed a DSGE model that is capable of producing a limit cycle equilibrium.

Keynesian DSGE model described in Galí (2008) and depart from the original framework by assuming dispersed information and, following Woodford (2013), bounded rationality. In our view, the essential feature of AB modeling is that all agents are represented by their individual dynamic equations and aggregation is performed numerically. To *agentify* the model, therefore, we assume that the economy is populated by a large, yet *finite*, number of agents and we explicitly model and simulate the behavior of each agent. Households and firms are individual entities that solve an intertemporal optimization problem conditional on subjective expectations. Agents are boundedly rational in the sense that they are unaware of the structure of the economy. Instead, they use observed data to learn economic relations, form forecasts, and make decisions.[2]

We first analyze the model assuming that all agents are able to observe aggregate productivity. In this scenario, agents share the same information set and, as a consequence, beliefs are *homogeneous*. Second, we introduce dispersed information and assume that each household observes an individual noisy signal of aggregate productivity, while each firm observes only its own individual productivity. Dispersed information implies that agents have distinct information sets, so that beliefs are *heterogeneous*.[3] In this second scenario, the dynamic behavior of the model is altered. The response of output to a technology shock is smaller on impact, while the response of hours worked is more persistent. A monetary policy shock has greater impact on output and smaller impact on inflation, while tax shocks have significant short-run effects. We stress that these results originate from the fact that agents have different information sets in the second setting.[4] When agents do not observe aggregate productivity, they estimate forecast rules that deviate from the one obtained under homogeneous information. In turn, different beliefs will alter the aggregate behavior of the economy. In particular, under dispersed information agents try improve their forecasts of aggregate productivity by exploiting the correlation between the individual productivity process, interest rates, and government debt. In other words, monetary and fiscal policy variables are used as public signals to infer the state of aggregate productivity.

[2]We borrow the formalization of expectations from the adaptive learning literature (Marcet and Sargent, 1989; Evans and Honkapohja, 2001; Eusepi and Preston, 2011). However, once we assume that information is dispersed, each of the finitely many agents in the economy will have a specific law of motion.

[3]The presence of heterogeneous expectations is consistent with empirical evidence (e.g., Mankiw et al., 2004) and experimental evidence (see Hommes, 2011, for a survey).

[4]The macroeconomic effects of idiosyncratic productivity shocks in our model are purely due to their influence on the information sets, differently from Cooper and Schott (2013), in which idiosyncratic capital productivity shocks have macroeconomic effects through a non-linear production function and frictions.

The flexibility of the AB approach allows us to deal with the complexity of learning in a dispersed information framework. The learning mechanism coupled with dispersed information implies that in every period agents' expectations depend on their individual history of observed data, so that each agent learns an individual forecast rule. Therefore, the evolution of the macroeconomic system depends on the full distribution of the estimated rules. In this case, to avoid using an approximate solution method, we simulate the dynamic equations of each agent taking an AB approach.

The paper is organized as follows. Section 2 discusses the literature related to our work. Section 3 describes the model. Section 4 describes the expectation formation mechanism used by agents and the assumptions about their information sets. Section 5 shows the outcomes of the simulations under homogeneous and dispersed information and describes the mechanism leading to the results. Section 6 concludes.

2 Related Literature

The literature evaluating the differences between DSGE and AB models is scarce but growing. Fagiolo and Roventini (2012, 2017) compare methodologies and results in these models. Guerini et al. (2017) use impulse response functions to analyze the results of an AB model with both a centralized and a decentralized market structure. They show that the centralized market mechanism broadly responds to a negative supply shock as a DSGE does, while the response of the same model with decentralized markets displays persistent deviation from the steady state. Dilaver et al. (2016), to "bridge the gap" between DSGE and AB models, study a baseline DSGE model with boundedly rational agents. Other authors have attempted to build models with a New Keynesian skeleton and some AB features. Salle et al. (2013) and Salle (2015), for example, analyze the influence of transparency and credibility of inflation targeting on macroeconomic stability, when agents' behaviors depart from the optimizing framework taking decisions using boundedly rational heuristics. In our model, instead, agents form expectations using past data and optimize conditional on their expectations. The point we make by assuming the optimizing behavior typical of DSGE models is methodological: We aim at solving a DSGE model using an AB modeling approach. In other words, we introduce forward looking behavior in a simple AB model.[5]

[5]We do not necessarily believe that agents maximize utility functions, but we want to specify explicitly the incentive structures motivating agents' decisions. We choose to model these incentives as utility and profit functions

Our model is consistent with the institutional setting and the behavioral assumptions found in most of the DSGE literature while it shares with AB modeling the solution method and the flexibility. We depart from the usual assumptions in New Keynesian literature by assuming a finite number of boundedly rational agents; furthermore, we simulate autonomous and independent agents and we perform aggregation numerically.

Many authors exploit the assumption of heterogeneous expectations and bounded rationality in macroeconomics (e.g. Kurz et al., 2013; Massaro, 2013; Woodford, 2013) and in particular analyze DSGE models with heterogeneous expectations through numerical simulations. De Grauwe (2011, 2012) considers a model with aggregate supply, aggregate demand and a Taylor rule. Agents use heuristic switching à la Brock and Hommes (1997) to forecast inflation and the output gap. The aggregate forecast is a weighted average of agents' forecasts. Assenza et al. (2014) describe the results of a laboratory experiment studying subjects' behavior in a New Keynesian framework. They find that the expectation formation is well described by a set of simple heuristics. They simulate the model using heuristic switching and find that one-period-ahead simulations provide a satisfying representation of experimental data. Motolese et al. (2015) analyze monetary policy when agents have diverse beliefs, and show that diverse beliefs have relevant consequences on the trade-off between inflation and output volatility. Usually, in this literature, agents are not modeled explicitly, but aggregated to obtain a reduced-form representation. However, aggregation is challenging, and is regarded as one of the major theoretical issues in DSGE models (Fagiolo and Roventini, 2017). Kurz et al. (2013) analyze the aggregation problem in a New Keynesian log-linearized model assuming that agents have diverse beliefs and find that the solution depends on the belief structure. Branch and McGough (2009) use a baseline DSGE model to show the restrictive assumptions needed to aggregate heterogeneous boundedly rational agents. We depart from this strand of literature, as we analyze a New Keynesian model with boundedly rational heterogeneous agents employing an AB approach. We model autonomous agents, simulate their interactions and aggregate numerically from the bottom-up. We do not impose assumptions to obtain a reduced form representation and we avoid using any concept of representative agent.

Our model relates also to the literature studying environments with incomplete information and public signals.[6] Morris and Shin (2002) and Angeletos and Pavan (2004) analyze the

to follow the mainstream literature, however, the model can be easily extended to consider different specification of the incentive structure.

[6]See Angeletos and Lian (2016) for a comprehensive survey on models with incomplete information.

welfare effects of the precision of the public signal, and show that in some cases public information might lead to deviations of agents' decisions from the fundamental. Lorenzoni (2009) studies the role of public signals in driving the business cycle in a general equilibrium model. In particular, when agents can observe only their own idiosyncratic productivity and a public signal on aggregate productivity, shocks to the public signal affect output, employment and inflation in the short run. In our setting the public signal is provided by the policy actions. The effect of the informational role of economic policy in global games is examined in Angeletos et al. (2006) and Angeletos and Pavan (2013). Baeriswyl and Cornand (2010) study the effect of the signaling role of monetary policy on the optimal policy in a model with flexible prices. Recently, Melosi (2017) builds a DSGE model with dispersed information and studies the signaling effect of monetary policy on the monetary transmission. Melosi (2017) is closely related to this paper, with some important differences. First, we use an AB approach to analyze the model. Second, we introduce learning. Third, we add the study of the signaling effect of fiscal policy.

3 The model

We consider a model that is closely related to the baseline New Keynesian model with staggered price setting described, among others, in Clarida et al. (1999), Woodford (2003), and Galí (2008). This model represents the benchmark tool for analyzing monetary policy and aggregate fluctuations under rational expectations. We depart from the standard framework in two ways. First, we explicitly assume that expectations are boundedly rational and heterogeneous as each agent forms forecasts using only the information available to the agent. Conditional on their subjective beliefs, agents will solve for their optimal plan with an infinite horizon approach. Second, we adopt an AB approach and assume the existence of a *finite* number of agents. All households and firms act according to individual decision rules given by the first order condition of their specific optimization problem. These agent-specific choices are then aggregated numerically to obtain macroeconomic variables.

Our model closely follows Woodford (2013), who provides a structural environment that handles non-rational heterogeneous expectations. The next subsections will present the optimal consumption function for households (Section 3.1), the optimal price-setting function for firms (Section 3.2), the monetary and fiscal policy rules (Section 3.3), and the issue of aggregation (Section 3.4). Detailed derivations can be found in Appendix A.

3.1 Households

The economy is made up of H infinitely-lived households that face the same optimization problem. In period t, each household chooses how much to save and consume by maximizing its expected life-time utility subject to a budget constraint. Following Woodford (2013), we assume that nonfinancial income—total income other than interests earned from bond holdings—is identical for every household and depends only on the evolution of aggregate variables. Therefore, each household needs to forecast the future path of aggregate variables, which are all outside the household's control.

The log-linearized expression of the optimal consumption decision for household i is given by

$$\begin{aligned}\hat{c}_t^i = (1-\beta)\hat{b}_t^i + \hat{E}_{t-1}^i \sum_{k=0}^{\infty} \beta^k \big\{ &(1-\beta)\left(\hat{y}_{t+k} - \hat{\tau}_{t+k}\right) - \beta\sigma\left(\hat{\imath}_{t+k} - \pi_{t+k+1}\right) \\ &+ (1-\beta)s_b\left(\beta\hat{\imath}_{t+k} - \pi_{t+k}\right) - \beta\left(\hat{\xi}_{t+k+1} - \hat{\xi}_{t+k}\right)\big\},\end{aligned} \tag{1}$$

for $i = 1, \ldots, H$. In this equation, hatted variables indicate deviations from the non-stochastic steady state with zero inflation. The parameter β is the intertemporal discount factor, σ represents the intertemporal elasticity of substitution, while s_b is the debt-to-output ratio at steady state. The term $\hat{E}_{t-1}^i$ indicates that expectations are subjective and conditional on the information available prior to the consumption decision.[7] Real individual consumption depends on $\hat{b}_t^i$, the value of maturing bonds carried into period t, deflated at the price level in $t-1$, implying that $\hat{b}_t^i$ is predetermined and treated as given by the household in period t. $\hat{c}_t^i$ also depends on the expected present and future values of aggregate output net of taxes ($\hat{y}_t - \hat{\tau}_t$), nominal interest rates ($\hat{\imath}_t$), and inflation ($\pi_t = \log P_t - \log P_{t-1}$). Consumption further depends on the exogenous shock $\hat{\xi}_t$, which can be interpreted as a preference shock common to all households.

Equation (1) is derived under the assumption that the household explicitly satisfies its intertemporal life-time budget constraint when choosing consumption.[8] This means that the

[7]The assumption of lagged expectations is common in the literature on adaptive learning (Evans and Honkapohja, 2001). It allows us to separate the moment in which agents forecast a variable from the moment in which the actual value of the variable is determined.

[8]The formulation of the consumption rule follows the infinite-horizon approach advocated by Preston (2005), as opposed to the Euler-equation approach of Evans et al. (2013). See Branch and McGough (2016) for a comparison of the two schemes.

consumption decision is contingent on the subjective forecasts of the individual nonfinancial income into the infinite future, rather than on the one-step ahead forecast of individual consumption. And since every household receives the same income, we have a framework in which heterogeneity in the perception of future aggregate variables gives rise to heterogeneity in consumption behavior.

There are F available varieties of consumption goods in the economy, so that $\hat{c}_t^i$ is in fact a Dixit-Stiglitz index representing the household's total purchases of the F goods. After determining total expenditure for period t, households must allocate consumption across the F varieties. The solution of this problem yields the usual CES demand functions

$$C_t^{ij} = \left(\frac{P_t^j}{P_t} \right)^{-\epsilon} C_t^i, \tag{2}$$

for $j = 1, \ldots, F$, where C_t^{ij} is the quantity of good j consumed by household i in period t, P_t^j is the price of good j, while ϵ is the elasticity of substitution among goods.[9]

Finally, households buy government bonds in the exact amount that allows them to satisfy the budget constraint

$$\hat{b}_{t+1}^i = s_b \left(\hat{\imath}_t - \beta^{-1} \pi_t \right) + \beta^{-1} \left(\hat{b}_t^i + \hat{y}_t - \hat{\tau}_t - \hat{c}_t^i \right), \tag{3}$$

here written in log-linear form.

In our framework the labor choice is identical for every household, even if beliefs are heterogeneous. In fact, we follow Woodford (2013) and assume that households supply the hours of work that are demanded by firms at a wage bargained separately by a union. The union maximizes households' average welfare and allocates labor equally across households. The

[9]The Dixit-Stiglitz aggregator for F differentiated goods is given (in levels) by

$$C_t^i = \left[\sum_{j=1}^{F} (C_t^{ij})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

The problem of optimal allocation of consumption leads to the definition of total expenditure of household i as $P_t C_t^i = \sum_{j=1}^{F} P_t^j C_t^{ij}$, where P_t is the usual Dixit-Stiglitz price index

$$P_t = \left[\sum_{j=1}^{F} (P_t^j)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$

resulting labor supply equation is

$$\hat{w}_t = \phi \hat{n}_t + \sigma^{-1}(\hat{c}_t - \hat{\xi}_t), \tag{4}$$

where $\hat{w}_t$ is the real wage, ϕ is the marginal disutility of additional work, $\hat{n}_t$ are total hours worked, $\hat{c}_t$ is average consumption (i.e. $\hat{c}_t = \sum_{i=1}^{H} \hat{c}_t^i / H$), so that $\sigma^{-1}(\hat{c}_t - \hat{\xi}_t)$ is the average marginal utility of additional real income. The labor union has the important role of keeping wages and labor income homogeneous, thus simplifying the determination of the equilibrium in the labor market.

3.2 Firms

There are F firms operating in the economy. The generic firm j produces a single variety of consumption goods using a production function with constant returns to scale with labor as only input

$$\hat{y}_t^j = a_t^j + \hat{n}_t^j, \tag{5}$$

where a_t^j (expressed in logs) is the individual productivity process for the firm.

We introduce nominal rigidities through the staggered price-setting scheme à la Calvo: firms can reset prices with probability $1-\theta$ and therefore face an intertemporal maximization problem. If a firm is allowed to re-optimize in t, it will choose the price level P_t^{*j} that maximizes the expected present value of the profits generated while that price remains effective, subject to the demand schedule for good j. The log-linear approximation of the first order condition of the firm problem gives the individual optimal price-setting rule

$$p_t^{*j} = p_{t-1} + \frac{\log F}{\epsilon - 1} + (1 - \beta\theta)\hat{E}_{t-1}^j \sum_{k=0}^{\infty} (\beta\theta)^k \left\{ \hat{w}_{t+k} - a_t^j + p_{t+k} - p_{t-1} \right\}, \tag{6}$$

where p_t and p_t^{*j} are expressed in logs.[10] Note that individual optimal prices are heterogeneous in our framework, as each firm uses its subjective beliefs to forecast the real wage, inflation, and its own productivity process.

Productivity is made of a persistent component common to all firms and a temporary firm-

[10] The constant term of (6) is related to the fact that relative prices P_t^j / P_t are different from unity at the steady state, according to our definition of the Dixit-Stiglitz price index for F goods.

specific component. The former is a stationary AR(1) process

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \tag{7}$$

while the latter is an idiosyncratic shock ε_t^j whose variance is common across firms. We thus have that

$$a_t^j = a_t + \varepsilon_t^j. \tag{8}$$

3.3 Monetary policy and fiscal policy

The monetary authority follows a strict inflation targeting and sets the nominal interest rate only in response to realized inflation

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1 - \rho_i)\phi_\pi \pi_t + \varepsilon_{m,t}, \tag{9}$$

where ρ_i is the parameter governing monetary policy inertia, ϕ_π determines the response of the monetary authority to deviations of inflation from the target, and $\varepsilon_{m,t}$ denotes the monetary policy shocks. We assume that the central banks follows the so-called Taylor principle, i.e. $\phi_\pi > 1$.

The fiscal authority sets primary surpluses in reaction to past deviations of government debt from its steady state value

$$\hat{s}_t = \phi_b \hat{b}_{t-1} + \varepsilon_{s,t}, \tag{10}$$

where $1 - \beta < \phi_b < 1$ and $\varepsilon_{s,t}$ is an exogenous fiscal innovation. Aggregate government spending also follows the exogenous disturbance $\hat{g}_t = \varepsilon_{g,t}$, so that taxes are determined as the residual $\hat{\tau}_t = \hat{s}_t + \hat{g}_t$.[11] Finally, the law of motion of government debt is given by the government budget constraint

$$\hat{b}_{t+1} = \beta^{-1}(\hat{b}_t - s_b \pi_t - \hat{s}_t) + s_b \hat{\imath}_t. \tag{11}$$

[11] Given amount of total purchases G_t, we assume that the government demands good j according to the relation

$$G_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\epsilon} G_t.$$

3.4 Aggregation and market clearing

Once individual consumption decisions are made, conditional on the prevailing prices $\{p_t^j\}_{j=1}^F$ and on the full set of subjective beliefs, aggregate demand can be computed by aggregating across households and and adding government purchases. Market clearing then requires total supply by firms, bundled according to the Dixit-Stiglitz index, to match aggregate demand, so that we have the log-linear relation

$$\hat{y}_t = \frac{1}{H}\sum_{i=1}^{H}\hat{c}_t^i + \hat{g}_t. \tag{12}$$

In accordance, total production of firm j is given by

$$\hat{y}_t^j = -\epsilon\left(p_t^j - p_t - \frac{\log F}{\epsilon - 1}\right) + \hat{y}_t. \tag{13}$$

Market clearing in the labor market requires that total labor supplied by households equals aggregate labor demanded by firms, or

$$\hat{n}_t = \hat{y}_t - \frac{1}{F}\sum_{j=1}^{F} a_t^j, \tag{14}$$

which can be plugged into the labor supply (4) to determine the real wage.

4 Expectations formation

It is clear from the analysis of Section 3 that individual households and firms make economic decisions conditional on the expected path of future variables. We depart from the conventional assumption of rational expectations, which would imply that agents are aware of the exact structural relations governing the economy and are able to form predictions consistent with these relations. Instead, we follow the adaptive learning literature and assume that agents behave as econometricians and try to extrapolate forecasts from past observations. Agents form expectations according to a perceived law of motion (PLM), whose structural form is similar to the minimal state variable solution of the model under rational expectations. More formally, agent l (either a household or a firm) considers the PLM

$$\boldsymbol{Y}_t^l = \boldsymbol{\phi}_{0,t-1}^l + \boldsymbol{\phi}_{1,t-1}^l \boldsymbol{Z}_{t-1}^l + \boldsymbol{e}_t, \tag{15}$$

where $\boldsymbol{Y}_t^l$ is the vector that includes all the variables to be forecast, while $\boldsymbol{Z}_{t-1}^l$ is the vector that includes the variables that the agent observes and wants to use to form forecasts. At the beginning of period t, the agent generates subjective expectations conditional on the information set up to $t-1$:

$$\hat{E}_{t-1}^l \boldsymbol{Y}_t^l = \boldsymbol{\phi}_{0,t-1}^l + \boldsymbol{\phi}_{1,t-1}^l \boldsymbol{Z}_{t-1}^l. \tag{16}$$

Expectations for variables dated at $t+1$ and farther in the future are constructed by rolling forward equation (16) and applying the law of iterated expectations. Each agent updates recursively the parameters of the individual PLM ($\boldsymbol{\phi}_0^l$ and $\boldsymbol{\phi}_1^l$) once aggregate variables are determined and new data is thus available, using the following recursive least squares (RLS) algorithm with constant gain

$$\boldsymbol{\phi}_t^l = \boldsymbol{\phi}_{t-1}^l + \gamma (\boldsymbol{R}_t^l)^{-1} \boldsymbol{X}_{t-1}^l \left(\boldsymbol{Y}_t^l - (\boldsymbol{\phi}_{t-1}^l)' \boldsymbol{X}_{t-1}^l \right)', \tag{17}$$

$$\boldsymbol{R}_t^l = \boldsymbol{R}_{t-1}^l + \gamma \left(\boldsymbol{X}_{t-1}^l (\boldsymbol{X}_{t-1}^l)' - \boldsymbol{R}_{t-1}^l \right), \tag{18}$$

where $\boldsymbol{\phi}_t^l = [\boldsymbol{\phi}_{0,t}^l, \boldsymbol{\phi}_{1,t}^l]'$ and $\boldsymbol{R}_t^l$ is the estimated matrix of second moments of the stacked regressors $\boldsymbol{X}_t^l = [1, (\boldsymbol{Z}_t^l)']'$.[12] The key parameter of the RLS algorithm is the gain coefficient γ, which controls the rate at which agents weight past information in order to generate forecasts about the future. The assumption that the gain coefficient is constant (i.e. non-decreasing) rules out the possibility that the estimates of the PLM parameters converge to a fixed value. Still, constant gain learning is more appropriate if we assume that agents are unaware that they live in a stationary environment or if we seek to analyze the effects of a structural break, such as a change of policy regime.

4.1 Information

Remember from equation (1) that households must forecast aggregate output, aggregate taxes, interest rate, and inflation in order to take their consumption decision. Therefore we have $\boldsymbol{Y}_t^i = [\hat{y}_t, \hat{\tau}_t, \hat{\imath}_t, \pi_t]'$ for household i. Likewise, equation (6) shows that resetting firms choose optimal prices using the forecasts of the real wage, individual productivity, and inflation. In this case we have $\boldsymbol{Y}_t^j = [\hat{w}_t, a_t^j, \pi_t]'$ for firm j.

As for the right-hand side variables of equation (15), we consider two scenarios that correspond to two different assumptions about the information set available to agents. In the

[12] Refer to Evans and Honkapohja (2001) for a discussion of the RLS algorithm.

first case information sets and expectations are homogeneous across agents. In the second, we assume dispersed information, hence information sets are agent-specific and expectations, in turn, are heterogeneous.

Homogeneous information. In this scenario every agent is aware that the minimal state variable solution of the model (when H and F tend to infinity) contains the lagged values of productivity, interest rate and inflation. Agents lack knowledge of the realizations of the shocks but are assumed to observe (with a lag) the permanent component of productivity, along with the other aggregate variables. Therefore, the right-hand side of the PLM contains a constant[13] and the vector of independent variables $\boldsymbol{Z}_t^l = [a_t, \hat{b}_t, \hat{\imath}_t]'$. We emphasize that this information set is common to all agents and, as such, the estimated parameters will be identical across agents: $\boldsymbol{\phi}_{0,t}^l = \boldsymbol{\phi}_{0,t}$ and $\boldsymbol{\phi}_{1,t}^l = \boldsymbol{\phi}_{1,t}$. As a consequence, subjective beliefs about the evolution of the macro variables will be homogeneous.

It is important to note that firms, in principle, will try to forecast their individual productivity but this prediction coincides with the expected value of permanent component of productivity. From equation (8), $\hat{E}_{t-1}^j a_t^j = \hat{E}_{t-1}^j a_t$. In fact, even if firms observe the transitory component of productivity only after they form expectations, they do observe the permanent component and are able to make a correct expectation of individual productivity.

Dispersed information. In the second scenario we exploit the flexibility of AB modeling and introduce dispersed information to modify the assumptions about the information set available to agents. We assume that agents are unable to observe the aggregate productivity component a_t. Each firm j observes only its own idiosyncratic productivity, a_t^j, and will consider the vector $\boldsymbol{Z}_t^j = [a_t^j, \hat{b}_t, \hat{\imath}_t]'$ in the PLM. Each household i receives a noisy private signal on aggregate productivity

$$a_t^i = a_t + \varepsilon_t^i, \tag{19}$$

and in accordance will use the vector $\boldsymbol{Z}_t^i = [a_t^i, \hat{b}_t, \hat{\imath}_t]'$. As information sets are heterogeneous, agents will have heterogeneous beliefs about the evolution of the relevant variables.

[13] As the model is written in deviations from the steady state, the solution under rational expectations does not contain a constant. Nonetheless, we assume that agents ignore this information and try to learn about the intercept too. This is a common assumption in the learning literature (for an example see Milani, 2011).

5 Simulations

In this section we use numerical simulations to assess how the behavior of the model differs under the assumption of homogeneous versus dispersed information. The section is arranged as follows: Section 5.1 describes the sequence of events that we follow to simulate the model; Section 5.2 reports the calibration; Section 5.3 shows the impulse response function; Section 5.4 investigates the role of dispersed information and, finally, Section 5.5 discusses the effect of different values of the gain parameter.

5.1 Sequence of events

At the beginning of a generic period t, agents use past data to revise their PLMs according to equations (17) and (18). The new PLMs allow agents to update their expectations about current and future variables following equation (16). A subset of firms, drawn according to the Calvo noise process, is then selected. These firms reset their prices using the optimal individual rule (6), while the remaining firms simply keep their prices unchanged. The inflation rate for period t can be computed using the Dixit-Stiglitz price index. Each household chooses how much to consume according to equation (1). Government spending is set and aggregate demand can be computed as the sum of households' consumption and government spending, following equation (12). Since we know aggregate demand and individual prices, we use equation (13) to compute the production level for each firm. The common and idiosyncratic components of productivity are updated according to equations (7) and (8), so we obtain individual and aggregate labor demand. In the heterogeneous information case, households' noisy private signals on productivity are updated according to (19). Real wage is determined using equation (4). The monetary and fiscal authorities set the interest rate and the fiscal surplus according to equation (9) and (10), respectively. It is important to stress that agents form expectations using information available up to the previous period. This implies that current monetary and fiscal shocks do not immediately affect the decisions of households and firms. Given surplus and government spending, taxes are determined as a residual. Finally, households' after-tax income is computed and individual bond holdings are updated, following equation (3).[14]

[14] In appendix B we have listed the sequence of events in each simulated time period.

5.2 Calibration

Structural parameters are calibrated according to Table 1. We use a standard quarterly calibration, following the recent DSGE literature (e.g. Galí, 2008). The elasticity of substitution between goods is equal to 6. The Calvo probability θ of keeping prices fixed is set to $2/3$, implying an average duration of prices of three quarters. The intertemporal elasticity of substitution, the labor disutility parameter, and the debt-to-output ratio are all set to 1. Following Lubik and Schorfheide (2004), we set the interest rate inertia parameter ρ_i to 0.85 and the inflation feedback parameter ϕ_π to 2.5, a rather hawkish value but in line with the estimates for the U.S. in the post-1982 period.[15] The fiscal rule parameter ϕ_b is equal to 0.2, meaning that the fiscal authority would be "passive" in the sense of Leeper (1991) in a rational expectations framework. We set to 0.002 the gain parameter γ used by agents to update the estimates of the PLM coefficients, following Eusepi and Preston (2011).[16] Relatively to the standard literature, we must also set the number of agents. We assume that the economic system is populated by 100 firms and 100 households. The number of agents must be large enough to wash out any idiosyncratic noise. However, agents cannot be too many as we are constrained by the computational time of each simulation.[17]

Table 1: Parameters calibration

H	F	ϵ	β	σ	ϕ	θ	s_b	ρ_a	ρ_i	ϕ_π	ϕ_b	γ
100	100	6	0.99	1	1	2/3	1	0.9	0.85	2.5	0.2	0.002

Table 2: Standard deviations

ξ	ε_g	ε_m	ε_s	ε_a	ε^j	ε^i
0.0010	0.0014	0.0014	0.0014	0.0077	0.0154	0.0154

Table 2 reports the calibrated standard deviations of the exogenous shocks of the model. We set the standard deviation of the monetary policy shock according to Maćkowiak and Wiederholt (2015), while for the aggregate productivity shock we follow Lorenzoni (2009). We assume that the standard deviation of the fiscal policy shocks and of the consumption shock are similar in size to the standard deviation of the monetary policy shock. The shocks to idiosyncratic

[15] We ran a sensitivity analysis changing the value of ϕ_π. We found that the numerical convergence of the learning algorithm is in line the theoretical results by Bullard and Mitra (2002), namely numerical convergence is almost always obtained with $\phi_\pi > 1$. Moreover, we reproduced impulse responses corresponding to the case $\phi_\pi = 1.5$. Results are qualitatively the same and are available from the authors upon request.

[16] The effects of a change of the gain parameter is described in Section 5.5.

[17] All simulations have been carried out using a Dell Precision R7910 with two 2.5 GHz Intel Xeon CPU E5-2680 v3 processors (each with 12 cores and 24 threads) and 128 GB of RAM, available at the Complexity Lab in Economics at the Catholic University of Milan.

productivity have two main effects. The first one is to increase the overall volatility of productivity. However, this effect tends rapidly to zero as the number of firms increases, and is negligible with 100 firms. The second effect is to reduce the precision of the information available to agents under the assumption of dispersed information. Higher standard deviation of the idiosyncratic productivity shock implies stronger differences between the homogeneous and dispersed information scenarios. We assume that the standard deviations of idiosyncratic productivity shock are twice as big as the standard deviation of the aggregate productivity shocks.

5.3 Impulse responses

To assess the role of learning and dispersed information in business cycle fluctuations, we initially consider the impulse response functions of the model. To construct impulse responses, we first simulate the model for 200000 periods using RLS with decreasing gain. The resulting estimated PLMs constitute the initial conditions for the simulations with constant gain. We simulate the model for another 2100 periods with constant gain and generate the baseline time series. We then re-simulate the model using the same exogenous shocks but adding one standard deviation at period 2000 to the shock under investigation. Impulse responses are obtained as the difference between these simulated series and the baseline ones. The whole process is repeated 500 times. Figure 1 shows the median responses to a positive shock to the common component of technology (ε_a) under the assumption of homogeneous (left column) and dispersed (right column) information. Similarly, Figure 2 shows the response to a contractionary monetary policy shock (ε_m) and Figure 3 shows the response to a contractionary fiscal policy shock (ε_s).

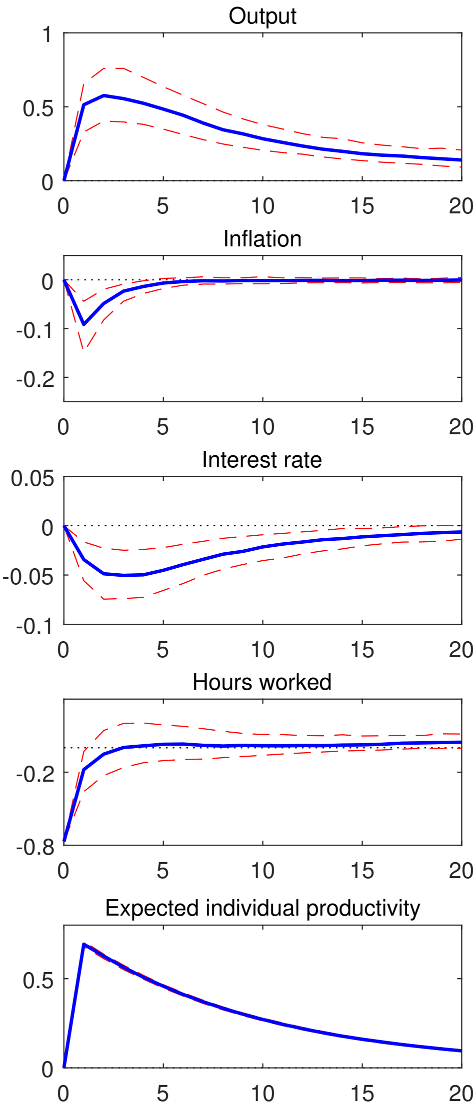
As apparent from Figure 1, a shock to the common component of productivity has different effects in the homogeneous and in the dispersed information scenarios. In the former case, agents include a_t in their information set and are able to identify the correct origin of the increase in productivity. Therefore, their forecasts of future productivity (bottom-left graph of Figure 1) are correct, in the sense that they exactly track the path followed by the persistent component of productivity. The consequences of the shock are an increase in output and a decrease in inflation and hours worked, in line with the behavior of New Keynesian models.

In the dispersed information case, agents observe only their individual productivity process and are unable to distinguish between its temporary and persistent components.

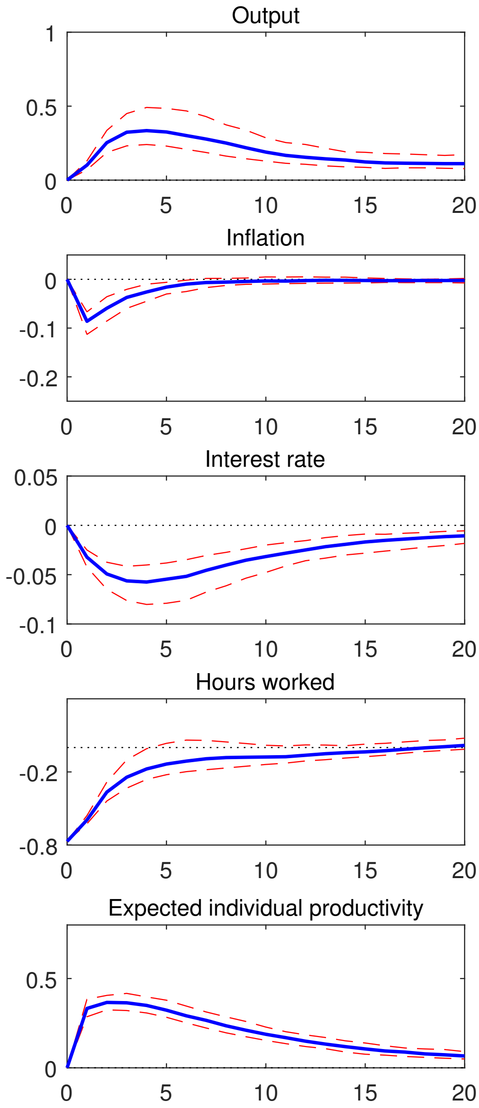
When they observe an increase in productivity, they revise upwards only partially their forecasts of future productivity (bottom-right graph of Figure 1), as they cannot say whether the improvement will be persistent or short-lived. In other words, under dispersed information, individual expectations under-react relative to the actual change in productivity. In later periods agents realize that the shock is more persistent than expected, and revise their expectations to catch up with the actual change in productivity. The initial subdued reaction of expectations and the following upward revision contribute to increase the persistence of the impulse responses, which is particularly evident for hours worked. This result is similar to what is found in Lorenzoni (2009) under dispersed information with public signals on aggregate productivity.

Let us now consider the impulse response functions to a contractionary monetary policy shock, shown in Figure 2. Under the assumption of homogeneous information, an unexpected increase in interest rates leads to a fall in output, inflation, and hours worked as predicted by New Keynesian models. In the meantime, expectations about future individual productivity remain unaffected. The dynamic behavior of the model changes once we assume dispersed information. On impact, the response of output and hours worked is increased and the response of inflation is less pronounced. The intuition is that under dispersed information monetary policy has two different transmission channels. The first channel is the traditional effect on aggregate demand given by higher nominal and real interest rates. A contractionary monetary shock will reduce output and inflation, exactly as in the homogeneous information case. The second channel is introduced by the assumption of dispersed information. In fact, the actions of the monetary authority are interpreted by agents as signals on aggregate productivity, so that expected future productivity drops after the increase of the nominal interest rate (bottom-right panel of Figure 2). We leave to Section 5.4 a detailed discussion of the mechanism leading to this result. This second channel reduces the impact of monetary policy on inflation and increases the impact of monetary policy on output. In fact, the response of output and inflation to a change in expected productivity is similar to the response to a technology shock. In particular, a decrease in expected productivity increases inflation and reduces output.[18] Since actual productivity does not change, the response of hours worked is completely determined by the reaction of output. The effect of dispersed information on the impulse response function van-

[18]In other words, under dispersed information a monetary policy shock induces also a shock on expectations on future productivity. Therefore, the overall response of the economy to a monetary shock is the combination between the traditional effect of higher interest rate—lower production and inflation—and the effect of lower expected productivity—lower production and higher inflation.



(a) Homogeneous information

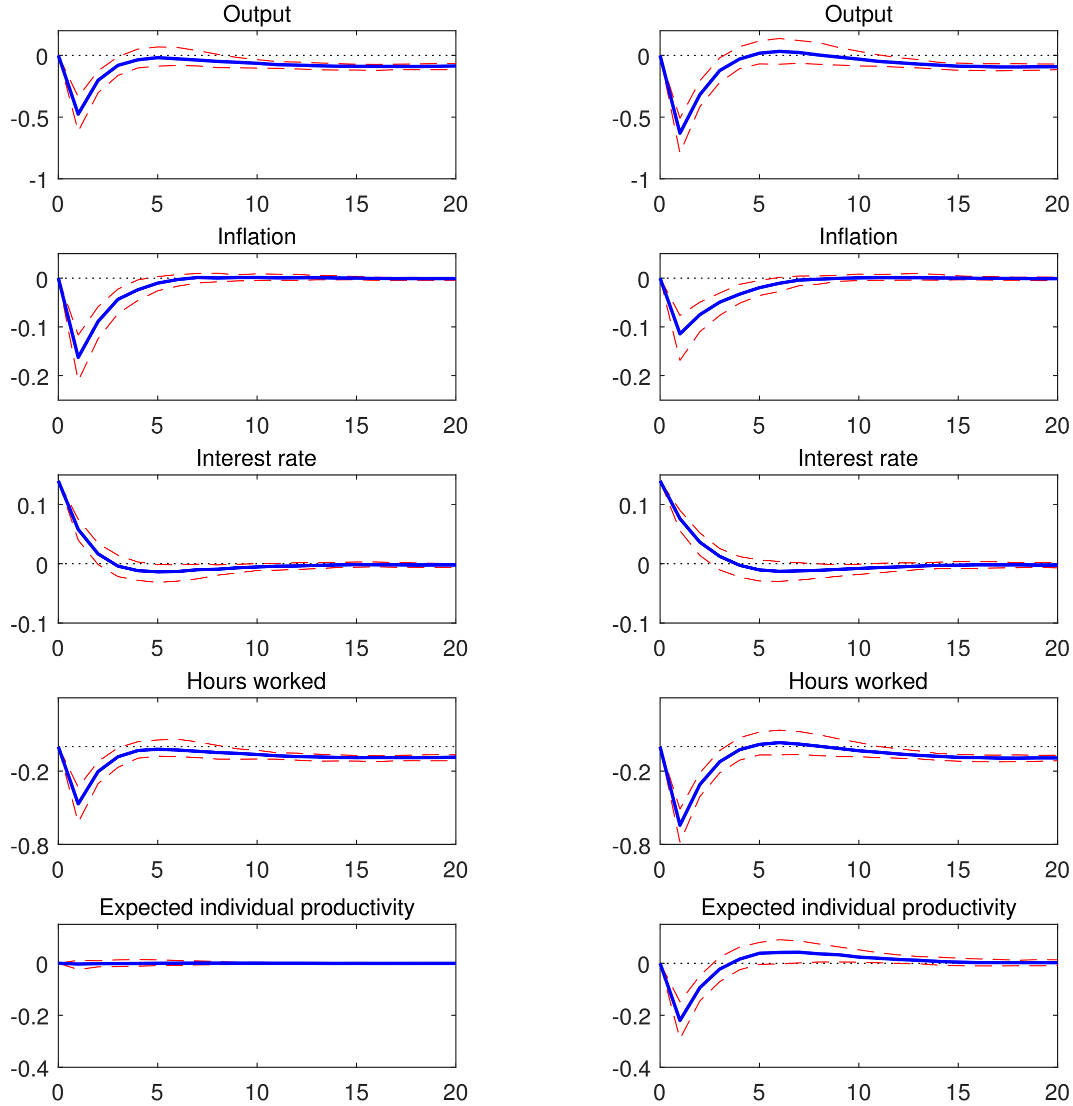


(b) Dispersed information

Figure 1: Impulse response function to a positive shock to aggregate productivity.
Notes: Blue lines: Median responses; Red dashed lines: 10-90 percentile bands.

ishes in the long-run, since agents realize that they have underestimated aggregate productivity and revise upwards their expectations.

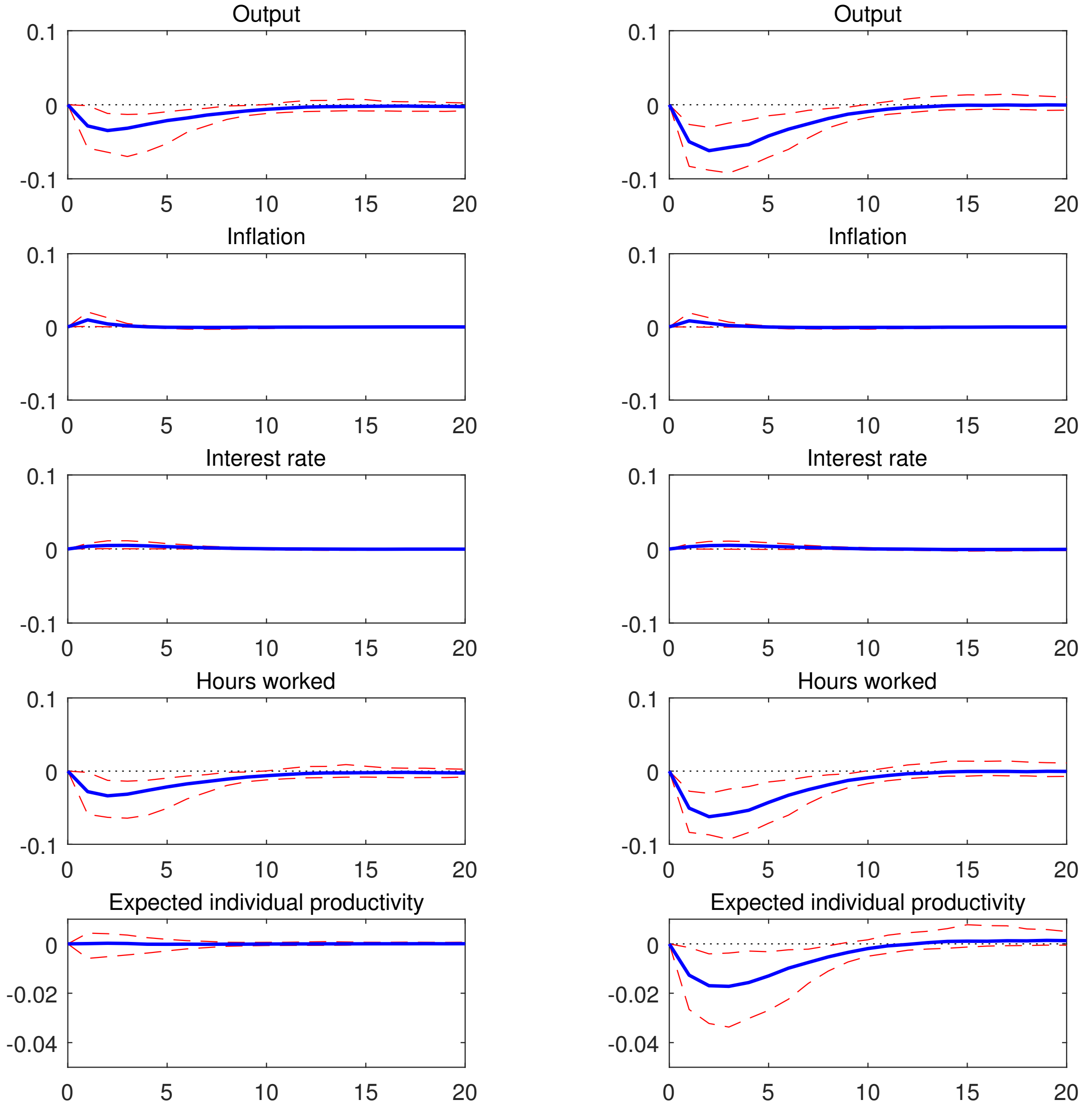
The reaction to an unexpected increase in aggregate taxes is shown in Figure 3. In a fully rational expectation environment, a shock to aggregate taxes would leave output and inflation unaffected. The behavior of the model changes slightly when we assume constant gain learning and homogeneous information. In fact, the left column in Figure 3 shows a small and mostly insignificant decrease in output and in hours worked. The effect of the fiscal shock is stronger under dispersed information. The intuition is the following. When agents deal with incomplete



(a) Homogeneous information (b) Dispersed information

Figure 2: Impulse response function to a contractionary monetary policy shock.
Notes: Blue lines: Median responses; Red dashed lines: 10-90 percentile bands.

information on productivity they use the behavior of the fiscal authority as a signal on the persistent component of productivity. Agents react to an increase of the fiscal surplus by reducing the expectations on future productivity. The consequence is a slight reduction of output and hours worked. Despite the signaling role of fiscal policy being much weaker than the monetary policy one, we believe that it contributes to the intuition of the main result. Once dispersed information on productivity is assumed, policy actions acquire the role of public signals. The interest rate and the fiscal surplus affect agents' beliefs about macroeconomic dynamics. The next subsection is devoted to further investigate the mechanism that leads to these results.



(a) Homogeneous information

(b) Dispersed information

Figure 3: Impulse response function to a contractionary shock to aggregate taxes.
Notes: Blue lines: Median responses; Red dashed lines: 10-90 percentile bands.

5.4 Inspecting the mechanism

The different dynamic behavior of the model under the assumption of homogeneous versus dispersed information can be understood by looking at how agents form their forecasts in the two settings. Let us consider, in particular, the forecasts about productivity. In the homogeneous information case, agents observe the persistent component of productivity (a_t), which is the only relevant information required to form correct predictions about future productivity. This implies that the other variables in the agents' information set ($\hat{b}_t$ and $\hat{\imath}_t$) are not used to forecast productivity.

In the dispersed information case, a generic agent l observes only a_t^l – the sum of the persistent component and the transitory (individual-specific) component of productivity. Therefore, to predict future productivity agents will try to exploit also the information content of b_t and i_t, as these variables are correlated with a_t, and a_t^l represents only a noisy measure of a_t. Importantly, the common component a_t is an exogenous stochastic process, so the correlation between productivity, the interest rate, and debt is generated solely by the innovations in $\varepsilon_{a,t}$. Therefore, this correlation can be seen by looking at the impulse responses to a technology shock, shown in Figure 1. A positive technology shock leads, obviously, to an increase in productivity, to a fall in inflation, and, through the monetary policy response, to a fall in nominal interest rates. Hence, the correlation between a_t and $\hat{\imath}_t$ is negative. Moreover, the inertial response of monetary policy generates an initial increase in real interest rates, and, as a consequence, an increase in real debt. The correlation between a_t and $\hat{b}_t$ is positive. Agents will therefore interpret the changes in interest rates and debt as a signal about the evolution of productivity. This explains the pattern of the impulse response functions following a monetary or fiscal shock under dispersed information (Figure 2 and 3, right columns). For example, a positive monetary shock leads agents to believe that the central bank may be reacting to an increase in inflation due to a negative technology shock, so they will forecast a lower productivity in the future. The same argument applies to the explanation of the signaling role of the fiscal surplus.

The mechanism outlined so far is reflected in the coefficients of the PLMs estimated by agents. To abstract from the time-variation in the estimates introduced by the constant gain assumption, we report in Table 3 the PLM estimates obtained after simulating the model for 200000 periods with *decreasing gain* learning.[19,20]

The most important differences between the homogeneous and dispersed information cases can be seen by comparing the estimates in the first column. In the homogeneous information case, agents correctly estimate the autocorrelation of aggregate productivity to be 0.90, and the dependence of aggregate productivity from past debt and interest rates to be (almost) zero. In the dispersed information case, agents correctly estimate a lower autocorrelation for the indi-

[19] Under decreasing gain, the parameter γ of the RLS algorithm in equations (17) and (18) is replaced by $1/t$. Marcet and Sargent (1989) and Evans and Honkapohja (2001) discuss the analytical conditions ensuring convergence of the learning mechanism. In our framework with a finite number of agents, we do not have analytic results for convergence but our simulations indicate that convergence is obtained.

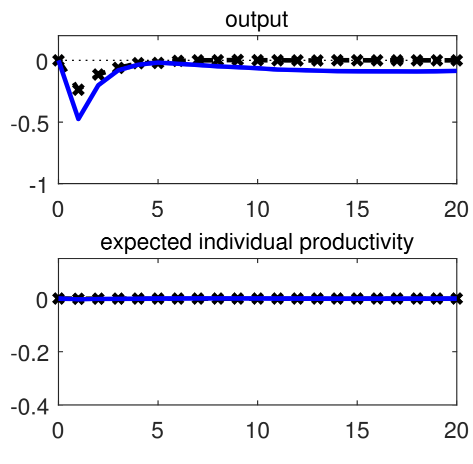
[20] The focus of Table 3 is on firms' PLM, but results for households are qualitatively identical.

Table 3: Firms' estimated PLM coefficients with decreasing gain.

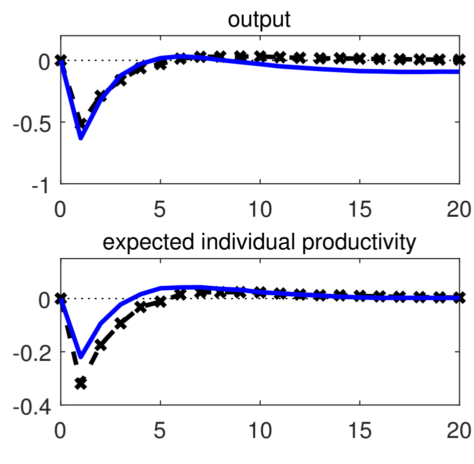
Homogeneous information case:	a_t	$\hat{b}_t$	$\hat{\imath}_t$	$\hat{w}_t$	π_t
a_{t-1}	0.9000	0.0874	-0.0523	0.4892	-0.1390
$\hat{b}_{t-1}$	-0.0004	0.8084	-0.0002	-0.0007	-0.0004
$\hat{\imath}_{t-1}$	0.0063	1.5074	0.4535	-3.4376	-1.0439
Dispersed information case:	a_t^j	$\hat{b}_t$	$\hat{\imath}_t$	$\hat{w}_t$	π_t
a_{t-1}^j	0.3924	0.0323	-0.0194	-0.2678	-0.0512
$\hat{b}_{t-1}$	0.1923	0.8245	-0.0097	0.3855	-0.0255
$\hat{\imath}_{t-1}$	-2.4970	0.9936	0.7633	-5.9707	-0.2308

Notes: Constants are here omitted. For the dispersed information case we report average estimates across firms.

vidual productivity process,[21] and they obtain estimates of the coefficients of past debt and interest rates which are significantly different from zero. The estimates in Table 3 confirm that under dispersed information agents exploit the correlation between a_t, b_t and i_t to improve their forecasts on future productivity. In other words, policy actions are used as signals on the productivity process.



(a) Homogeneous information



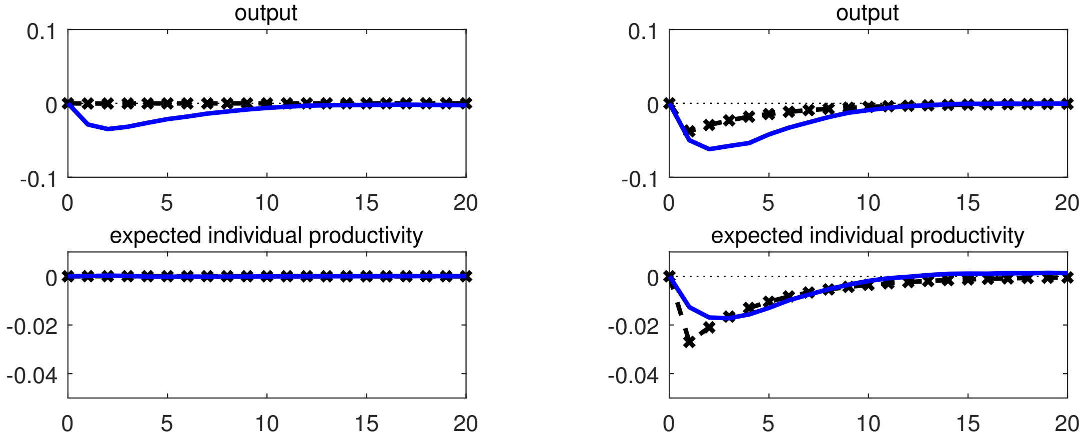
(b) Dispersed information

Figure 4: Impulse response function to a contractionary monetary shock under decreasing gain and constant gain. *Notes:* Blue lines: Median responses with constant gain; Black starred lines: responses with decreasing gain.

Next, it is interesting to compare the results under decreasing and constant gain. In Figure

[21] It is easy to show that the autocorrelation of the individual productivity process for a generic firm j is equal to

$$\frac{E[a_t^j a_{t-1}^j]}{E[(a_t^j)^2]} = \frac{\rho_a \sigma_{\varepsilon_a}^2}{\sigma_{\varepsilon_a}^2 + (1-\rho_a^2)\sigma_{\varepsilon^j}^2} < \rho_a.$$



(a) Homogeneous information

(b) Dispersed information

Figure 5: Impulse response function to a contractionary shock to aggregate taxes under decreasing gain and constant gain.
Notes: Blue lines: Median responses with constant gain; Black starred lines: responses with decreasing gain.

4 we overlap the impulse response functions to a contractionary monetary shock with constant gain (identical to the impulses shown in Figure 2) and the corresponding responses computed with decreasing gain. The differences between constant and decreasing gain seem negligible, apart from a slightly stronger reaction of expected productivity under decreasing gain learning. In Figure 5 we report the same comparison, with focus on the fiscal shock. With homogeneous information set and decreasing gain, output and expected forecasts do not react to fiscal shocks, so that the Ricardian equivalence holds. The same result would hold in a baseline New Keynesian model. The constant gain responses under homogeneous information show instead a small but evident reaction of output to the fiscal shock. This effect is solely due to the constant gain mechanism. With dispersed information, impulse responses are similar under constant and decreasing gain. In both cases the Ricardian equivalence does not hold.

Table 4: Firms' estimated PLM coefficients with constant gain.

Homogeneous information case:			*Dispersed information case:*		
	Dependent variable: a_t			Dependent variable: a_t^j	
	Mean	10-90 percent interval		Mean	10-90 percent interval
constant	0.0000	[-0.0004, 0.0003]	constant	0.0000	[-0.0017, 0.0016]
a_{t-1}	0.8943	[0.8765, 0.9121]	a_{t-1}^j	0.4290	[0.3916, 0.4695]
$\hat{b}_{t-1}$	0.0002	[-0.0382, 0.0407]	$\hat{b}_{t-1}$	0.0949	[0.0005, 0.1927]
$\hat{\imath}_{t-1}$	-0.0115	[-0.1557, 0.1438]	$\hat{\imath}_{t-1}$	-1.6825	[-2.0838, -1.2307]

Notes: For the dispersed information case we report average estimates across firms. We report only the estimates for the first column of the PLM.

The difference in the impulse response functions with constant gain and decreasing gain are reflected also in the PLM estimated coefficients. Since constant gain learning does not converge to a fixed value, we simulate the model 500 times and compute the average estimated coefficients and the 10th and 90th percentiles. The results are reported in Table 4. We report only the first column of the PLMs for both the homogeneous and dispersed information case. The estimates with constant gain are close to the estimate with decreasing gain. The differences are explained by the bias in constant gain learning reported in Eusepi and Preston (2011). The results on the signaling role of monetary and fiscal policy under dispersed information are confirmed also by the coefficients estimated with constant gain.

5.5 The role of the gain parameter

The constant gain parameter, γ, controls the rate at which agents update their PLM: larger values of the gain mean that agents give greater weight to new information. To investigate the implications of changing the gain parameter, we reproduce an experiment in line with Eusepi and Preston (2011) and report in Table 5 a set of summary statistics obtained after simulating the model using different values of γ. As a benchmark, we report the same statistics computed by simulating the model using the PLMs obtained after convergence with decreasing gain. An increase of γ raises the volatility of output and inflation, and the root-mean-square-error of expected inflation. With constant gain learning, agents update continuously their PLMs and their expectations. As γ increases, the effect of a shocks on expectations is greater and volatility is larger. This mechanism is at work both in the homogeneous case and in the dispersed information case, and is confirmed by the results with decreasing gain reported in the first column of the table.

Table 5 shows that the model with dispersed information displays lower volatility of output and inflation. To explain this result we must look again at the impulse response functions in Section 5.3 and compare the reaction of the model under homogeneous and dispersed information to the different shocks. Under dispersed information, output reacts less to a productivity shock and more to a monetary shock and a fiscal shock. Since the standard deviation of the persistent component of productivity is 5.5 times larger than the standard deviations of fiscal shocks and monetary shocks, the overall effect is a reduction of output volatility. Under dispersed information, inflation reacts less to a monetary shock and react similarly to a technology shock and a fiscal shock. The overall effect is a reduction of inflation volatility.

Table 5: The effect of different values of the gain parameter.

	Gain					
	$\simeq 0$	0.0010	0.0020	0.0030	0.0040	0.0050
Homogeneous information case:						
$\sigma_{\hat{y}}$	0.0130	0.0134	0.0147	0.0166	0.0180	0.0209
σ_{π}	0.0021	0.0022	0.0023	0.0025	0.0026	0.0029
RMSE π	0.0004	0.0014	0.0017	0.0020	0.0021	0.0025
cross-sectional std $\hat{b}^i$	-	-	-	-	-	-
cross-sectional std p^j	0.0047	0.0050	0.0053	0.0058	0.0059	0.0066
Dispersed information case:						
$\sigma_{\hat{y}}$	0.0098	0.0102	0.0116	0.0144	0.0168	0.0191
σ_{π}	0.0017	0.0018	0.0020	0.0023	0.0024	0.0027
RMSE π	0.0012	0.0016	0.0018	0.0021	0.0022	0.0025
cross-sectional std $\hat{b}^i$	0.0145	0.0580	0.0605	0.0716	0.0860	0.1023
cross-sectional std p^j	0.0063	0.0068	0.0071	0.0074	0.0075	0.0081

Notes: Statistics computed on simulated data for different values of the gain parameter. The first column ($\simeq 0$) corresponds to simulations computed after convergence under decreasing gain. We report output volatility ($\sigma_{\hat{y}}$), inflation volatility (σ_{π}), average root-mean-square-error of expectations on inflation (RMSE π), average cross-sectional standard deviation of households' individual wealth ($\hat{b}^i$), and average cross-sectional standard deviation of firms' individual prices (p^j).

Table 5 reports also the average cross-section standard deviation of individual wealth (std $\hat{b}^i$) and individual prices (std p_j). These statistics measure average heterogeneity between agents. By construction, in the homogeneous information case all agents have the same information set and the same expectations. This implies that households make the same consumption decisions and have the wealth. Therefore, the cross-sectional variance of $\hat{b}^i$ is zero. On the contrary, when we assume dispersed information, we are introducing heterogeneous information sets, heterogeneous expectations and, consequently, heterogeneous decisions. With dispersed information agents are heterogeneous in their wealth, and the size of the heterogeneity depends on the gain parameter.

In the homogeneous information case, the cross-sectional standard deviation in individual prices is positive. This is because, despite homogeneous expectations and homogeneous optimal prices, the Calvo noise process allows only a subset of firms to change their prices in each period. In this regard, it is interesting to stress that thanks to the AB approach we are explicitly modeling price dispersion. Table 5 shows that price dispersion is increasing with the gain parameter. Moreover, the assumption of dispersed information increases price dispersion. The reason is that firms have heterogeneous expectations on future variables, leading them to set heterogeneous optimal prices. Under this scenario, price dispersion is due both to the Calvo mechanism and to heterogeneous expectations.

6 Conclusions

In this paper we consider a framework that greatly differs from typical AB models. The institutional setting and the behavioral assumptions are consistent with the DSGE literature. Households maximize their utility and firms optimize their profits conditional on their expectations about current and future variables. Our aim, however, is to provide a first bridge between the macroeconomic AB literature and the mainstream macroeconomic literature. To do so, we simulate the model using an AB approach, i.e., agents are considered as autonomous individual entities and their decisions are aggregated numerically. The AB approach allows us to easily deal with the introduction of bounded rationality and dispersed information in the model.

We analyze two different scenarios. In the first, we assume that agents are able to observe aggregate productivity. Agents have therefore homogeneous information sets and homogeneous expectations. In the second, we assume that firms only observe their own idiosyncratic productivity, while households observe a noisy signal of aggregate productivity. This assumption has several implications. First, heterogeneous information sets imply heterogeneous expectations. Second, agents are not able to distinguish the persistent component (aggregate) from the temporary component (idiosyncratic) of the shock. The learning mechanism leads agents to use the information content of the other two variables in their information set, i.e., government debt and interest rates. In turn, policy actions acquire a signaling role. Monetary and fiscal policies are transmitted to the economy not only through the usual channels, but also by affecting agents forecasts on future productivity. This implies that monetary policy shocks have a greater effect on output and a smaller effect on inflation relative to the benchmark case. Similarly, a shock to taxes affects output in the short run.

The flexibility of the AB approach allows us to avoid the issues associated with an aggregate representation of the model. We have exploited this flexibility to study the effect of heterogeneous information sets in a learning environment. However, this flexibility can also be used to introduce different assumptions on the objective functions, e.g., by introducing behavioral biases, or different expectation formation mechanism, by assuming different PLMs (e.g., Hommes and Zhu, 2014) or heuristic switching behavior (e.g., Dilaver et al., 2016). Future research should introduce more heterogeneity, e.g, income inequality, input-output networks and financial systems. We believe that using an AB approach could allow to build more realistic models preserving, at the same time, the comparability with the mainstream literature.

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A Model

We consider a baseline New Keynesian model but we depart from the standard framework in two ways. First, we explicitly assume that expectations are non-rational and heterogeneous as each agent forms forecasts using only the information available to the agent. Conditional on their subjective beliefs, agents will solve for their optimal plan with an infinite horizon approach. Second, we adopt an agent-based approach and assume that the economy is populated by a large but finite number of agents. Each household and firm acts according to an individual decision rule given by the first order condition of its specific optimization problem. These agent-specific choices are then aggregated numerically to obtain macroeconomic variables.

The description of the model closely follows Woodford (2013), who provides a structural environment that handles non-rational heterogeneous expectations. We differ from Woodford by assuming lagged expectations (i.e., dated at $t-1$) and a finite number of of agents.

A.1 Households

At time t each of the H households of the economy chooses how much to save and consume by maximizing the expected utility

$$\hat{E}^i_{t-1} \sum_{k=0}^{\infty} \beta^k \left[u(C^i_{t+k}; \xi_{t+k}) - v(N^i_{t+k}) \right], \tag{20}$$

where $\hat{E}^i_{t-1}$ denotes that expectations are agent-specific and possibly non-rational. C^i_t is a Dixit-Stiglitz (CES) index of the household's purchases of the F available varieties of consumption goods

$$C^i_t = \left[\sum_{j=1}^{F} (C^{ij}_t)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}},$$

with C^{ij}_t denoting the quantity of good j consumed by household i in period t and ϵ the elasticity of substitution among goods. N^i_t is the number of hours worked by household i, β is the intertemporal discount factor, and ξ_t is an exogenous shock to consumption (say a preference shock) common to all households.

The budget constraint faced by the household is given by

$$B^i_{t+1} = (1+i_t) \left[B^i_t + W_t N^i_t + \frac{1}{H} \sum_{j=1}^{F} \left(P^j_t Y^j_t - W_t N^j_t \right) - \sum_{i=1}^{F} P^j_t C^{ij}_t - \frac{T_t}{H} \right],$$

where B_t^i is the nominal value at maturity of the government bonds carried into period t, i_t is the risk-free interest rate, the nominal wage W_t is common across workers, P_t^j is the price of good j, $P_t^j Y_t^j - W_t N_t^j$ are the profits of firm j (nominal revenues net of labor costs), and T_t are the aggregate nominal tax collection. The budget constraints assumes that households own the same share of each firm, so that aggregate dividends are distributed equally across households. Similarly, each household pays the same amount of taxes, i.e. $T_t^i = T_t/H$. Following Woodford (2013), we also assume that households supply the hours of work that are demanded by firms at a wage bargained separately by a union, which allocates labor equally to each household. This assumption ensures that

$$N_t^i = \frac{1}{H}\sum_{j=1}^{F} N_t^j = \frac{N_t}{H},$$

where N_t are total hours worked in the economy. The problem of optimal allocation of expenditure across different goods is standard and leads to the definition of total expenditure of household i as $P_t C_t^i = \sum_{j=1}^{F} P_t^j C_t^{ij}$, where

$$P_t = \left[\sum_{j=1}^{F} (P_t^j)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

is the usual Dixit-Stiglitz price index, while the demand of household i for good j is given by

$$C_t^{ij} = \left(\frac{P_t^j}{P_t}\right)^{-\epsilon} C_t^i.$$

As this allocation is valid for all households and for the government, total revenues in the economy can also be expressed as $P_t Y_t = \sum_{j=1}^{F} P_t^j Y_t^j$. In accordance, the budget constraint can be simplified as

$$B_{t+1}^i = (1+i_t)\left(B_t^i + \frac{P_t Y_t}{H} - P_t C_t^i - \frac{T_t}{H}\right).$$

It is now apparent that each households receives the same nonfinancial income $(P_t Y_t - T_t)/H$.

The first order conditions with respect to consumption and bond holdings give the Euler equation for household i

$$u_c\left(C_t^i;\xi_t\right) = \beta \hat{E}_{t-1}^i \left((1+i_t)\frac{P_t}{P_{t+1}} u_c\left(C_{t+1}^i;\xi_{t+1}\right)\right).$$

The non-linear Euler equation and the budget constraint are log-linearized around the non-stochastic steady state with zero inflation. To do so, we first define real debt and real tax collection as $b_t^i = B_t^i / P_{t-1}$ and $\tau_t = T_t/P_t$, respectively. Then, we define the following log-deviations from steady state as

$$\hat{y}_t = \frac{Y_t - \bar{Y}}{\bar{Y}}, \quad \hat{b}_t^i = \frac{b_t^i - \bar{b}^i}{\bar{Y}/H}, \quad \hat{c}_t^i = \frac{C_t^i - \bar{C}^i}{\bar{Y}/H}, \quad \hat{\tau}_t = \frac{\tau_t - \bar{\tau}}{\bar{Y}}, \quad \hat{\imath}_t = \log(1+i_t) - \log(1+\bar{\imath}),$$

where upper-bar variables indicate steady state values. The log-linear version of budget constraint is therefore

$$\hat{b}_{t+1}^i = s_b \left(\hat{\imath}_t - \beta^{-1}\pi_t\right) + \beta^{-1}\left(\hat{b}_t^i + \hat{y}_t - \hat{c}_t^i - \hat{\tau}_t\right),$$

where we used the definition of inflation $\pi_t = \log P_t - \log P_{t-1}$ and of the debt-to-GDP ratio $s_b = \frac{\bar{b}^i}{\bar{Y}/H}$. To log-linearize the Euler equation, we approximate the marginal utility of expenditure as

$$\ln u_c\left(C_t^i; \xi_t\right) = \ln u_c\left(\bar{C}^i; \bar{\xi}\right) - \sigma^{-1}\left(\hat{c}_t^i - \hat{\xi}_t\right)$$

with $\hat{\xi}_t$ being an exogenous shift in marginal utility. Then we take logs and get

$$\hat{c}_t^i - \hat{\xi}_t = -\sigma \hat{E}_{t-1}^i \left(\hat{\imath}_t - \pi_{t+1}\right) + \hat{E}_{t-1}^i \left(\hat{c}_{t+1}^i - \hat{\xi}_{t+1}\right).$$

So we have exactly the same formulation as in Woodford (2013) for the log-linearized Euler equation and budget constraint. We can then do the same forward iterations and substitutions to obtain the optimal consumption rule, reported in the text as equation (1),

$$\begin{aligned}\hat{c}_t^i = (1-\beta)\hat{b}_t^i + \hat{E}_{t-1}^i \sum_{k=0}^{\infty} \beta^k \Big[&(1-\beta)\left(\hat{y}_{t+k} - \hat{\tau}_{t+k}\right) - \beta\sigma\left(\hat{\imath}_{t+k} - \pi_{t+k+1}\right) \\ &+ (1-\beta)s_b\left(\beta\hat{\imath}_{t+k} - \pi_{t+k}\right) - \beta\left(\hat{\xi}_{t+k+1} - \hat{\xi}_{t+k}\right)\Big].\end{aligned}$$

To determine labor supply and wages, imagine that a union chooses the total amount of labor N_t supplied in equal parts by all households in order to maximize the average value of

$$\frac{1}{H}\sum_{i=1}^{H} V_t^i\left(b_t^i \frac{P_{t-1}}{P_t} + \frac{w_t N_t}{H}\right) - v\left(\frac{N_t}{H}\right),$$

for a given real wage $w_t = W_t/P_t$, where V_t^i is the household's evaluation in period t of the maximum continuation utility. The first order condition combined with the envelope theorem yields

$$w_t \frac{1}{H} \sum_{i=1}^{H} u_c \left(C_t^i; \xi_t \right) = v' \left(N_t \right),$$

that can be log-linearized, after introducing the notation

$$\hat{w}_t = \log w_t - \log \bar{w}, \quad \hat{c}_t = \frac{1}{H} \sum_{i=1}^{H} \hat{c}_t^i, \quad \log v' \left(N_t \right) = \log v' \left(\bar{N} \right) + \phi \hat{n}_t, \quad \hat{n}_t = \frac{N_t - \bar{N}}{\bar{N}},$$

to get the labor supply equation (4) in the paper

$$\hat{w}_t = \sigma^{-1} \left(\hat{c}_t - \hat{\xi}_t \right) + \phi \hat{n}_t. \tag{21}$$

A.2 Firms

Each of the F varieties of consumption goods is produced by one of the F firms operating in the economy. Firm j uses a production function with constant returns to scale with labor as only input

$$Y_t^j = A_t^j N_t^j, \tag{22}$$

where A_t^j is the individual productivity process for the firm. Nominal rigidities are introduced through the staggered price-setting scheme à la Calvo: in each period a firm can reset its price with probability $1 - \theta$ and has to keep its price unchanged with probability θ. A firm that is allowed to re-optimize in t will therefore choose the price level P_t^{*j} that maximizes the discounted sum of present and future profits given its subjective expectations

$$\max_{P_t^{*j}} \hat{E}_{t-1}^j \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(P_t^{*j} Y_{t+k|t}^j - \frac{W_{t+j} Y_{t+k|t}^j}{A_{t+k}^j} \right), \tag{23}$$

subject to the demand schedules

$$Y_{t+k|t}^j = \left(\frac{P_t^{*j}}{P_{t+k}} \right)^{-\epsilon} \left(\sum_{i=1}^{H} C_{t+k}^i + G_{t+k} \right). \tag{24}$$

In the above relations, $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\frac{1}{\sigma}} (P_t/P_{t+k})$ is the stochastic discount factor, while $Y_{t+k|t}^j$ denotes output in period $t+k$ for a firm that last reset its price in period t. Note that we have inverted the production function (22) to express nominal total costs in terms of output rather than labor. The first order condition with respect to P_t^{*j} gives

$$\hat{E}_{t-1}^j \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t}^j \left(P_t^{*j} - \frac{\epsilon}{\epsilon - 1} \frac{W_{t+j}}{A_{t+k}^j} \right) = 0,$$

which can be approximated around the zero-inflation steady state to obtain

$$p_t^{*j} = p_{t-1} + \frac{\log F}{\epsilon - 1} + (1 - \beta\theta) \hat{E}_{t-1}^j \sum_{k=0}^{\infty} (\beta\theta)^k \left\{ \hat{w}_{t+k} - a_t^j + p_{t+k} - p_{t-1} \right\},$$

where p_t and p_t^{*j} are expressed in logs. This expression represents the optimal price-setting rule for the individual firm j, and corresponds to equation (6) in main text. The derivation of the log-linearized price-setting rule is standard (see Galí, 2008, ch. 3), with the difference that relative prices P_t^j / P_t are equal to $F^{\frac{1}{\epsilon-1}}$ at the steady state. This explains the additional normalization constant that appears in the last equation.

The individual productivity process is made of a persistent component common to all firms and a temporary firm-specific component. The former is a stationary zero-mean AR(1) process

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t},$$

while the latter is an idiosyncratic shock ε_t^j whose variance is common across firms. Thus, we have that

$$a_t^j = \log A_t^j = a_t + \varepsilon_t^j.$$

A.3 Monetary and fiscal policy

The monetary authority follows a strict inflation targeting and sets the nominal interest rate in response to realized inflation

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1 - \rho_i) \phi_\pi \pi_t + \varepsilon_{m,t},$$

where ρ_i is the inertia in monetary policy, ϕ_π determines the response of the monetary authority to deviations of inflation from the target, and $\varepsilon_{m,t}$ denotes the monetary policy shocks.

The fiscal authority sets primary surpluses in reaction to past deviations of government debt from its steady state value

$$\hat{s}_t = \phi_b \hat{b}_{t-1} + \varepsilon_{s,t},$$

where $1-\beta < \phi_b < 1$, and $\varepsilon_{s,t}$ is an exogenous fiscal innovation. Total government expenditure also follows the exogenous disturbance $\hat{g}_t = \varepsilon_{g,t}$, so that tax revenues are determined as the residual $\hat{\tau}_t = \hat{s}_t + \hat{g}_t$. Note that we expressed the deviations from steady state of real government debt, primary surplus, expenditure, and tax revenues as percentage of steady state output (i.e., $\hat{b}_t = (b_t - \bar{b})/\bar{Y}$ and so forth). As for households, we assume that the government demands good j according to the relation

$$G_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\epsilon} G_t,$$

for a given amount of total purchases G_t. Finally, the law of motion of government debt is given by the real flow budget constraint

$$b_{t+1} = (1+i_t)\left(b_t \frac{P_{t-1}}{P_t} + G_t - \tau_t\right),$$

which can be log-linearized around the steady state to obtain

$$\hat{b}_{t+1} = \beta^{-1}\left(\hat{b}_t - s_b \pi_t - \hat{s}_t\right) + s_b \hat{\imath}_t, \tag{25}$$

which corresponds to equation (11) in the main text.

A.4 Aggregation and market clearing

Market clearing in the goods market requires each firm j to produce the exact amount that matches demand, so that

$$Y_t^j = \sum_{i=1}^{H} C_t^{ij} + G_t^j,$$

where the right-hand side is total demand for variety j, obtained after aggregating across households and adding government expenditures. Aggregate output can be defined as the Dixit-Stiglitz aggregate of individual productions, i.e.

$$Y_t = \left[\sum_{j=1}^{F} (Y_t^j)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{26}$$

Combining the previous two relations, it is possible to write

$$Y_t^{\frac{\epsilon-1}{\epsilon}} = \sum_{j=1}^{F}\left(\sum_{i=1}^{H} C_t^{ij} + G_t^j\right)^{\frac{\epsilon-1}{\epsilon}},$$

so that, exploiting the CES form of the demands for good j, we can obtain

$$Y_t = \sum_{i=1}^{H} C_t^i + G_t. \tag{27}$$

This relation can be plugged into equation (24) (evaluated for $k = 0$) to express the individual production of firm j as a function of aggregate output

$$Y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\epsilon} Y_t. \tag{28}$$

Market clearing in the labor market requires that aggregate labor demanded by firms equals total labor supplied by households, or

$$\sum_{j=1}^{F} N_t^j = \sum_{i=1}^{H} N_t^i = N_t,$$

where the last equality exploits the fact that total labor is split equally across households ($N_t^i = N_t/H$). Using the production function (22), total hours worked can be rewritten as

$$N_t = \sum_{j=1}^{F} \frac{Y_t^j}{A_t^j}.$$

Note that in the paper we consider the log-linear version of the decision rules of households, firms, and policymakers. Aggregation, therefore, must be performed using log-linear relations. The definition of aggregate output (26) can be approximated as

$$\hat{y}_t = \frac{1}{F} \sum_{j=1}^{F} \hat{y}_t^j,$$

where $\hat{y}_t^j = (Y_t^j - \bar{Y}^j)/\bar{Y}^j$ and $\hat{y}_t = (Y_t - \bar{Y})/\bar{Y}$, while equation (27) yields

$$\hat{y}_t = \frac{1}{H} \sum_{i=1}^{H} \hat{c}_t^i + \hat{g}_t,$$

which corresponds to equation (12) in the main text. The approximation to total demand for good j, equation (28), is given by

$$\hat{y}_t^j = -\epsilon \left(p_t^j - p_t - \frac{\log F}{\epsilon - 1} \right) + \hat{y}_t.$$

Finally, total hours can be re-expressed as

$$\hat{n}_t = \frac{1}{F}\sum_{j=1}^{F}(\hat{y}_t^j - a_t^j) = \hat{y}_t - \frac{1}{F}\sum_{j=1}^{F} a_t^j,$$

corresponding to equation (14) in the main text. This relation can be plugged into the labor supply (21) to determine the real wage as

$$\hat{w}_t = \sigma^{-1}\left(\hat{c}_t - \hat{\xi}_t\right) + \varphi\hat{y}_t - \varphi\frac{1}{F}\sum_{j=1}^{F} a_t^j.$$

B Sequence of events

This appendix describes the sequence of events in an arbitrary time period $t > 1$.

1. Agents update their perceived law of motion using recursive least square with constant gain using past data, according to equations (17) and (18).
2. Agents forecast current and future variables, according to equation (16).
3. A subset of firms is drawn according to the Calvo parameter. These firms optimize their prices according to equation (6). Inflation is computed.
4. Households choose their consumption according to equation (1).
5. Government spending is set and aggregate demand is determined as the sum of households' consumption and government spending, according to equation (12).
6. Aggregate and idiosyncratic productivity is updated according to equations (7) and (8). In the heterogeneous information case, households' noisy private signals on productivity are updated according to (19).
7. From aggregate demand we compute individual production, according to equation (13), individual labor demand and aggregate labor, according to (14).
8. Real wage is determined according to equation (4).
9. The monetary authority sets the interest rate according to equation (9).
10. The fiscal authority sets the fiscal surplus according to equation (10). Given the surplus and government spending, taxes are determined as a residual.

11. Households' after-tax income is computed and individual savings are updated, according to equation (3).