

Rayleigh loops in the random-field Ising model on the Bethe lattice

Francesca Colaiori, Andrea Gabrielli, and Stefano Zapperi

INFN unità di Roma I and SMC, Dipartimento di Fisica, Università "La Sapienza," Piazzale A. Moro 2, 00185 Roma, Italy

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We analyze the demagnetization properties of the random-field Ising model on the Bethe lattice focusing on the behavior near the disorder induced phase transition. We derive an exact recursion relation for the magnetization and integrate it numerically. Our analysis shows that demagnetization is possible only in the continuous high disorder phase, where at low field the loops are described by the Rayleigh law. In the low disorder phase, the saturation loop displays a discontinuity that is reflected by a nonvanishing magnetization m_∞ after a series of nested loops. In this case, at low fields the loops are not symmetric and the Rayleigh law does not hold.

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I. INTRODUCTION

A ferromagnetic material is characterized by a remanent magnetization even at zero field. In several instances, however, it is convenient to *demagnetize* the sample, bringing it to a state of zero magnetization at zero field. In practice, this can be done by applying a slowly varying ac field, decreasing its amplitude after each cycle. In this way, the system explores a complex energy landscape, due to the interplay between structural disorder and interactions, until it is trapped into a low-energy minimum. If the demagnetization process is performed adiabatically and thermal effects do not play an important role, the demagnetized state is reproducible for a given realization of the quenched disorder and can thus be used as a reference to define magnetic properties.

The hysteresis loops at low fields, starting from the demagnetized state, are usually described by the Rayleigh¹ law: when the field is cycled between $\pm H^*$, the magnetization m follows $m = (a + bH^*)H \pm b[(H^*)^2 - H^2]/2$, where the signs \pm distinguish the upper and lower branch of the loop. Consequently the area of the loop scales with the peak field H^* as $W = 4/3b(H^*)^3$ and the response to a small field change, starting from the demagnetized state is given by $M^* = a(H^*) \pm b(H^*)^2$.^{2,3} This law has been measured in a variety of materials, but a few papers have reported significant deviations from the simple quadratic law but no explanation has been provided.⁴

The current theoretical interpretation of this law is based on a 1942 paper by Néel,⁵ who derived the law formulating the magnetization process as the dynamics of a point (i.e., the position of a domain wall) in a random potential. In this framework, the initial susceptibility a is associated to reversible motions inside one of the many minima of the random potential, while the hysteretic coefficient b is due to irreversible jumps between different valleys. The main drawback of Néel theory relies in its purely phenomenological nature, being based on a *zero-dimensional* model that does not include collective effects considered very important for the magnetization process.⁶⁻⁸

In the past few years, the zero-temperature random-field Ising model (RFIM) has been used to describe the competition between quenched disorder and exchange interactions and their effect on the hysteresis loop.⁹ In three and higher

dimensions, the model shows a phase transition between a continuous cycle for strong disorder and a discontinuous loop, with a macroscopic jump, at low disorder. The two phases are separated by a second-order critical point, characterized by universal scaling laws.⁹⁻¹¹ A behavior of this kind is not restricted to the RFIM but has also been observed in other models, with random bonds,¹² random anisotropies,¹³ or vectorial spins.¹⁴ In addition, a similar disorder induced phase transition in the hysteresis loop has been experimentally reported for a Co-CoO bilayers.¹⁵

The RFIM is probably the simplest model including disorder and exchange interactions that can be treated analytically. The equilibrium properties of the RFIM on the Bethe lattice has been first studied in Refs. 16 and 17, that report exact results for various disorder distributions, mostly for the bimodal one. To describe hysteresis one should, however, focus on out of equilibrium properties. For this case, exact results were found in one dimension in Refs. 18 and 19 and on the Bethe lattice in Refs. 20–22, while mean-field theory⁹ and renormalization group¹⁰ have been used to analyze the transition. Recently the one-dimensional solution of the model, has been generalized to obtain the complete demagnetization process and to derive the Rayleigh loops.²³ The RFIM does not display a phase transition in one dimensions, while numerical simulations indicate that the transition has an important effect on the demagnetization process. In particular, in the low disorder phase the discontinuity in the saturation curve prevents the magnetization to reach a demagnetized state²³ but this behavior has not been understood theoretically. It has been shown exactly²⁰⁻²² that the RFIM displays a disorder induced phase transition on the Bethe lattice when the coordination number $z \geq 4$ and can thus be used to clarify the issue.

Here we generalize the analysis of Refs. 20–23 to obtain exact recursion relations for the demagnetization process on the Bethe lattice and show that demagnetization is only possible in the high disorder phase. In the low disorder phase the remanent magnetization after a series of nested loops of decreasing amplitude does not vanish but scales to zero as the transition is approached. Furthermore, in the low disorder phase the Rayleigh law is not obeyed and low-field loops are not symmetric. The Rayleigh law is instead recovered in the high disorder phase and the Rayleigh parameters a and b

behave qualitatively as in $d=1$, displaying a peak in the disorder. All the results derived in this paper hold, in general, for any analytic and symmetric random-field distribution with infinite support and finite variance. However, in the numerical implementation of the analytical results we have used a Gaussian distribution. In the case of a discrete distribution, as the one used in Refs. 16 and 17, the generalization of the results is straightforward but lengthy as discussed, for instance, in Ref. 21.

The paper is organized as follows: in Sec. II we describe the model. In Sec. III we recall the results obtained in Refs. 20–22 and generalize them for a series of nested loops. Section IV discusses the effect of the phase transition on the magnetization and Sec. V analyzes the Rayleigh law. A brief discussion of the perspectives is reported in Sec. VI. Finally, the complete derivation of the recursion relations is reported in the Appendix.

II. THE MODEL

In this section we recall briefly a model used to describe hysteresis loops in magnetic materials:⁹ the ferromagnetic RFIM. This model is characterized by the Hamiltonian

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} s_i s_j - H \sum_i s_i - \sum_i h_i s_i, \quad (1)$$

where $J > 0$, the $\Sigma_{\langle i,j \rangle}$ is restricted on the pairs of nearest neighbors on a lattice of coordination number z , s_i is the Ising spin on the site i , H is a homogeneous external field, and h_i represents a quenched random field on the spin s_i modeling the presence of lattice defects. The fields $\{h_i\}$ are independently drawn from a symmetric distribution $\rho(h_i)$. In the following the numerical results are referred to a Gaussian distribution with variance R^2 .

In this paper we study the case of a Bethe lattice with a generic coordination number z . In particular, we are interested in the case $z=4$ that is known to be the minimal case showing a disorder induced phase transition towards a discontinuous hysteresis loop.

In order to mimic the microscopic spin dynamics, we use the flipping rules used in Refs. 9–11 and 18–22 obtained from the Glauber dynamics at temperature T and with an external field of frequency ω taking the limit $T \rightarrow 0$ first and then $\omega \rightarrow 0$. The basic rule of this $T=0$ dynamics is that the spins align with the local field

$$s_i = \text{sgn}(h_{i,eff}), \quad (2)$$

where the effective local field felt by the spin i is

$$h_{i,eff} = -\frac{\partial \mathcal{H}}{\partial s_i} = J \sum_{j \in n(i)} s_j + H + h_i \quad (3)$$

and the sum runs over the z nearest neighbors of the site i .

Note that though the model is defined through three external parameters J, H, R , the dynamics is determined by the two reduced quantities H/R and J/R only. For the sake of simplicity, from now on we rename these two ratios H and J , respectively, and consider $R=1$. Given H and J , we can

write the probability $p_m(H)$ that a spin i , with m ($0 \leq m \leq z$) of its neighbors up , is also up . This is given by the probability that $h_{i,eff} > 0$:

$$p_m(H) = P(h_{i,eff} > 0) = \int_{(z-2m)J-H}^{\infty} dh' \rho(h'). \quad (4)$$

III. HYSTERESIS LOOPS

In general a change in the applied field H produces a rearrangement of the spins, so that each spin i is stable being aligned with its effective field $h_{i,eff}$. It is important to note that each spin flip modifies the effective field on the nearest neighbors and sometimes generates an avalanche of spin flips through the lattice. In the following we will consider the case of a slowly varying external field: its value is kept constant until the next metastable state is reached. Two important properties of the $T=0$ dynamics are (i) the Abelian property—the stable state after an avalanche does not depend on the order in which the spins flip—and (ii) the return point memory—when the field is changed adiabatically the stable state only depends on the point where the field was last reversed. These two properties can be used to obtain exactly the shape of the hysteresis loops. We first recall the derivation of the saturation curve and those of the first minor loops and then proceed with the general analysis of minor loops.

A. Saturation loop

When the external field H is cycled from $-\infty$ to $+\infty$ and back the magnetization describes the saturation loop. The key quantity describing the lower half of the saturation loop is the conditional probability $U_0(H)$ defined²⁰ as the probability that a given spin flips before a fixed nearest neighbor, conditioned to this neighbor being down. The probability $U_0(H)$ satisfies the following equation:²⁰

$$U_0(H) = \sum_{m=0}^{z-1} \binom{z-1}{m} [U_0(H)]^m [1 - U_0(H)]^{z-1-m} p_m(H). \quad (5)$$

It can be shown that the probability that a spin is up at external field H is²⁰

$$p(H) = \sum_{m=0}^z \binom{z}{m} [U_0(H)]^m [1 - U_0(H)]^{z-m} p_m(H). \quad (6)$$

The related magnetization is given by $m_l(H) = 1 - 2p(H)$, which describes the lower half of the hysteresis loop. The upper half of the hysteresis loop can then be obtained by symmetry [i.e., $m_u(H) = m_l(-H)$].

B. First minor loops

If the external field H is raised from $-\infty$ to a finite value H_0 (i.e., we are on the lower half of the major loop) and then it is reversed, the magnetization describes the upper half of a minor hysteresis loop. When the field is reversed from H_0 to $H_1 < H_0$ we define the conditional probability $D_1(H_1)$ for a

spin to be *down* before a fixed nearest neighbor, conditioned to this neighbor being *up*. The probability $D_1(H_1)$ satisfies the following equation:²²

$$D_1(H_1) = \sum_{m=0}^{z-1} \binom{z-1}{m} [U_0(H_0)]^m \times \{ [1 - U_0(H_0)]^{z-1-m} [1 - p_{m+1}(H_0)] + [D_1(H_1)]^{z-1-m} [p_{m+1}(H_0) - p_{m+1}(H_1)] \}. \quad (7)$$

The related probability that a spin is *up* at H_1 is

$$p(H_1) = p(H_0) - \sum_{m=0}^z \binom{z}{m} [U_0(H_0)]^m \times [D_1(H_1)]^{z-m} [p_m(H_0) - p_m(H_1)]. \quad (8)$$

Equation (7) holds as long as H_1 is larger than $H_0 - 2J$.²² In one dimension it has been shown¹⁹ that at $H = H_0 - 2J$ the upper half of minor loop merges with the upper half of the major loop with the same local slope. This proof can be extended to the case of a Bethe lattice as long as the saturation loop is continuous.²² The case in which the saturation loop displays a discontinuity is discussed below for the case $z = 4$.

C. General formula for nested loops

The method used to find U_0 and D_1 can be generalized to obtain a complete characterization of all minor loops. In particular, we are interested in *nested* minor loops, since they are directly related to the demagnetization process of the disordered ferromagnet. Nested loops is defined as follows: after having reached H_1 , we reverse again the field increasing its value up to $H_2 \leq H_0$ (lower half of the first minor loop). This process is then iterated in a sequence of fields $H_{2n} \in [H_{2n-1}, H_{2n+1}]$ and $H_{2n+1} \in [H_{2n}, H_{2n+2}]$ with $n \geq 1$, where H_{2n} and H_{2n+1} refer to the final value of the field H in the lower half of the n th minor loop, and for the upper half of the $(n+1)$ th minor loop, respectively. The generalizations of U_0 to the minor loops is called $U_{2n}(H_{2n})$ while the generalization of $D_1(H_1)$ is called $D_{2n+1}(H_{2n+1})$. In what follows, we simply indicate $U_{2n}(H_{2n})$ with U_{2n} and $D_{2n+1}(H_{2n+1})$ with D_{2n+1} .

Since $H_{2n} < H_{2n-2}$, the set of spins contributing to U_{2n} will be a subset of those contributing to U_{2n-2} , so that we can write

$$U_{2n} = U_{2n-2} - \eta_{2n-1} + \eta_{2n}, \quad (9)$$

where η_{2n-1} represents the fraction of spins that were *up* at H_{2n-2} before their fixed nearest neighbor and *down* at H_{2n-1} , while η_{2n} is the fraction of the set contributing to η_{2n-1} that flip *up* again at H_{2n} . The explicit derivation of η_{2n-1} and η_{2n} is a little involved and it is thus discussed in the Appendix.

The magnetization at $H = H_{2n}$ is as usual obtained as $m_{2n} \equiv m(H_{2n}) = 1 - 2p(H_{2n})$, where the probability $p(H_{2n})$

that a spin is *up* at H_{2n} is given by the probability $p(H_{2n-1})$ that it was already *up* at H_{2n-1} summed to the probability to flip *up* when the field goes from H_{2n-1} to H_{2n} :

$$p(H_{2n}) = p(H_{2n-1}) + \sum_{m=0}^z \binom{z}{m} [U_{2n}]^m [D_{2n-1}]^{z-m} \times [p_m(H_{2n}) - p_m(H_{2n-1})]. \quad (10)$$

The generalization of D_1 to nested minor loops is called D_{2n+1} and analogously to $U_{2n}(H_{2n})$ is given by

$$D_{2n+1} = D_{2n-1} - \zeta_{2n} + \zeta_{2n+1}, \quad (11)$$

where ζ_{2n} is the fraction of spins that were *down* at H_{2n-1} before their fixed nearest neighbor and *up* at H_{2n} and ζ_{2n+1} is the fraction of the set of spins contributing to ζ_{2n} that flip *down* again at H_{2n} . The exact expression for the fractions ζ_{2n} and ζ_{2n+1} are reported in the Appendix.

The magnetization at $H = H_{2n+1}$ is $m_{2n+1} \equiv m(H_{2n+1}) = 1 - 2p(H_{2n+1})$, where $p(H_{2n+1})$ is the probability for a spin to be *up* at H_{2n+1} . This probability can be written as the analogous probability at $H = H_{2n}$ minus the probability to flip *down* between H_{2n} and H_{2n+1} :

$$p(H_{2n+1}) = p(H_{2n}) - \sum_{m=0}^z \binom{z}{m} [U_{2n}]^m [D_{2n+1}]^{z-m} \times [p_m(H_{2n}) - p_m(H_{2n+1})]. \quad (12)$$

In principle, an arbitrary series of nested loop can be obtained solving the recursion relation for U_{2n} , D_{2n+1} , and using the result to obtain the magnetization. A similar procedure was used in $d = 1$ to obtain a closed expression for the magnetization along the demagnetization curve.²³ This is not possible for the Bethe lattice where an explicit solution for the problem is not available and one should resort to a numerical integration.

IV. DISORDER INDUCED PHASE TRANSITION AND DEMAGNETIZATION

Previous numerical studies of the zero-temperature dynamics of the RFIM on a regular lattice in finite dimension^{9,11} have shown that in $d \geq 3$ the system shows a phase transition from a strong disorder phase to a weak disorder phase, separated by a second-order critical point (in $d = 2$ the presence of a phase transition is still controversial). In the strong disorder phase the major hysteresis loop is continuous, whilst in the weak disorder phase the loop shows a macroscopic jump in the magnetization at a critical value of the field. Note that if we use the reduced parameters J/R and H/R , fixing $R = 1$, strong disorder corresponds to small values of J and the phase transition will be characterized by a critical value J_c of the exchange coupling J . On the Bethe lattice, a phase transition is observed for coordination numbers $z \geq 4$, while for $z \leq 3$ one has only the strong disorder phase.²⁰ The presence of the phase transition has strong implications on the possibility of demagnetizing the system. In particular, in the weak disorder phase it is not possible to

demagnetize the system through an oscillating external field with decreasing amplitude.

Before we specialize to the case $z=4$, let us note that for $H=J$ one has $p_{z-1-m}(J)=1-p_m(J)$. This allows, after a little algebra, to show that for $H=J$, $U_0(J)=1/2$ is a solution for any J , any z and any random-field distribution. For $z=4$ Eq. (5) is a cubic equation in U_0 , with coefficients depending on H and J through the p_i . In order to find the critical point it is enough to find the value of H and J corresponding to a triple solution of the equation. Implementing this requirement for any symmetric density function of the disorder, one finds $H_c=J_c$, where J_c satisfies the equation $p_0(J)+p_1(J)=1/3$. For a Gaussian distribution of the disorder, this translates in the following implicit equation for J_c :

$$\operatorname{erf}(J_c) + \operatorname{erf}(3J_c) = 1/3 \quad (13)$$

resulting in $J_c=0.561\,400\,995\,873\,19\dots$, in accordance with the result quoted Ref. 22.

Above the transition ($J>J_c$ or weak disorder phase) the hysteresis loop becomes discontinuous.²⁰ In fact at $J=J_c$ and $H=J_c$ the susceptibility $\partial m/\partial H$ diverges, and for $J>J_c$ one observes a discontinuity with a spinodal singularity. At this point one can measure a gap Δm in the magnetization. It is easy to show, through an expansion of Eq. (5) to the lowest order in $J-J_c$ around J_c , that for $J\rightarrow J_c^+$ it is

$$\Delta m \sim (J-J_c)^\beta \quad (14)$$

with $\beta=1/2$ as in the mean-field case. The analytical derivation of this result can be easily sketched as follows: first of all the three solutions of Eq. (5) at $H=J$ can be found explicitly. They are

$$U_0^{(a)} = 1/2 \quad \text{and}$$

$$U_0^{(b),(c)} = 1/2(1 \pm \sqrt{[1-3(p_0+p_1)]/[1-3p_1+p_0]}),$$

where, for $H=J$, $[1-3(p_0+p_1)]>0$ only for $J>J_c$ and $=0$ at $J=J_c$. The gap can be measured by $\Delta U \equiv |U_0^{(b)} - U_0^{(c)}|$, and it is simple to show that $\Delta U \sim (J-J_c)^{1/2}$, which gives Eq. (14) considering that to the lowest order $\Delta m \sim \Delta U$. Actually, the position of the gap in the magnetization of the major loop of the hysteresis cycle is located at $H_g > J$, which could give corrections to the previous result. However, one can show that $(H_g - J) \sim (J - J_c)$, implying corrections to the previous value of the gap of the same order in $(J - J_c)$. Then this correction does not alter the found scaling behavior.

We recall that usually it is possible to demagnetize a material by applying a slowly oscillating external field appropriately chosen. In practice, this corresponds to a series of nested loops, starting from the completely magnetized situation (e.g., $m = -1$ and $h \rightarrow -\infty$) and then applying in succession the fields $H_0=J, H_1=-H_0(1-\varepsilon), \dots, H_{2n+1} = -H_{2n}(1-\varepsilon)$ in the limits $\varepsilon \rightarrow 0^+$ and $n \rightarrow +\infty$.²³ This sequence, for $J < J_c$, leads to a completely demagnetized state. This is due to two fundamental properties of the strong disordered phase: (i) the ‘‘return point memory’’ property that has been defined in the preceding paragraph; (ii) the fact that,

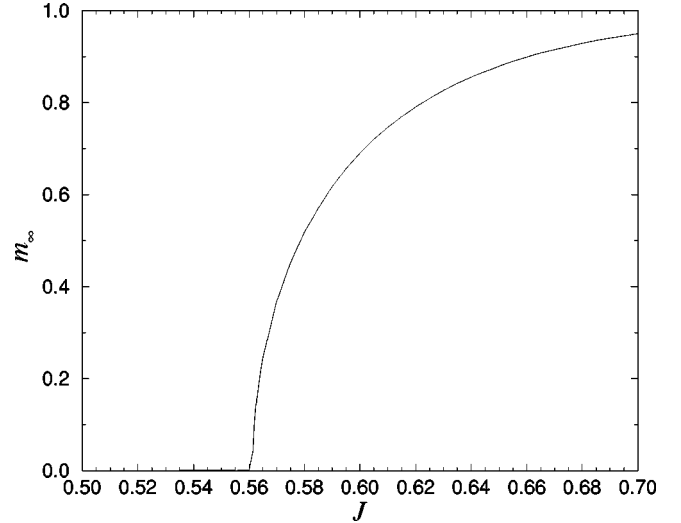


FIG. 1. Final magnetization m_∞ as a function of the exchange coefficient J .

if we are on a certain point of the saturation loop (e.g., at $H=J_0$ on the lower half) and invert the field to H_1 the system meets the other half on the saturation loop if $H_1=H_0 - 2J$. This implies that in order that the first minor loop is symmetric with respect to the origin $H=0$ and $m=0$ without touching the saturation curve we have to start from $H_0=J$ on the lower half of the saturation loop (or equivalently from $H_0=J$ on the upper half).

At $J>J_c$ the demagnetization process is no more possible, because the discontinuity prevents minor loops to be symmetric with respect to the origin of the axes. In fact there is now an inaccessible region of the plane (H, m) around the origin.²³ However, the field succession described above still provides a well-defined procedure (apart the broken symmetry $m \rightarrow -m$ and $H \rightarrow -H$) to minimal possible residual magnetization, that we denote m_∞ , at $H=0$. Following this procedure, by numerically integrating Eqs. (9) and (11) with $\varepsilon = 10^{-3}$, we find that m_∞ displays the same scaling behavior of the gap in the saturation loop as J approaches J_c from above: $m_\infty \propto (J - J_c)^{1/2}$ (see Fig. 6).

V. RAYLEIGH LAW

The Rayleigh law describes the hysteresis behavior at low field in a vast class of materials. Exact values for the Rayleigh parameters have been obtained for the one-dimensional RFIM,²³ where the initial susceptibility a and the hysteretic coefficient b both display a peak in the disorder R . A similar behavior is observed in simulation for $d=2,3$ but only in the high disorder phase, while in the low disorder phase a and b are not defined (Figs. 1 and 2).

In the case of the Bethe lattice, we could not obtain an explicit expression of the Rayleigh parameters even in the high disorder phase. We thus resort to numerical integration and analyze the demagnetization curves close to $H=0$. We estimate the susceptibility a and the hysteretic coefficient b using a linear fit of m_{2n}/H_{2n} vs H_{2n} . According to the Rayleigh law for $n \rightarrow \infty$ we have $m_{2n}/H_{2n} = a + bH_{2n}$, and simi-

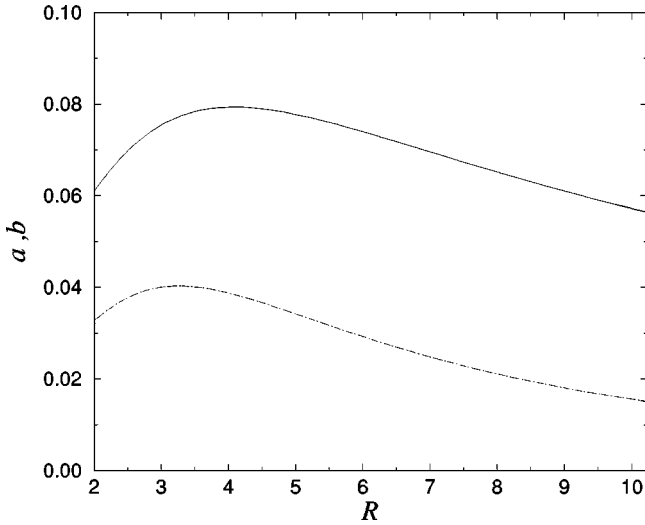


FIG. 2. The Rayleigh parameters (solid line is a , dashed line is b) as a function of the disorder width R for $R > R_c = 1/J_c$.

larly for negative fields $m_{2n+1}/H_{2n+1} = a - bH_{2n+1}$. The values of a and b as a function of the exchange coupling J are shown in Figs. 3 and 4. When plotted as a function of the disorder R , a , and b show a peak in the high disorder phase, in agreement with the results on Euclidean lattices.

In the low disorder phase the demagnetization curve is not symmetric with respect to $H=0$ and consequently the Rayleigh law does not hold. In particular, we can define two values for the coefficient b :

$$\begin{aligned} m_{2n}/H_{2n} &= a + b^+ H_{2n}, \\ m_{2n+1}/H_{2n+1} &= a - b^- H_{2n+1}. \end{aligned} \quad (15)$$

The values of a , b^+ , and b^- as a function of the exchange coefficient J are shown in Figs. 3 and 4; also the difference $\Delta b = b^+ - b^-$ is shown in Fig. 5. Again, as $J \rightarrow J_c^+$, Δb approaches 0 as $(J - J_c)^{1/2}$ (see Fig. 6).

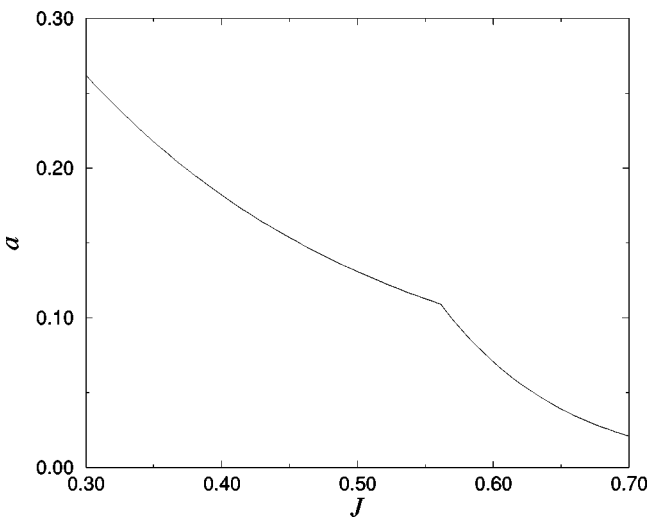


FIG. 3. Susceptibility a as a function of the exchange coefficient J .

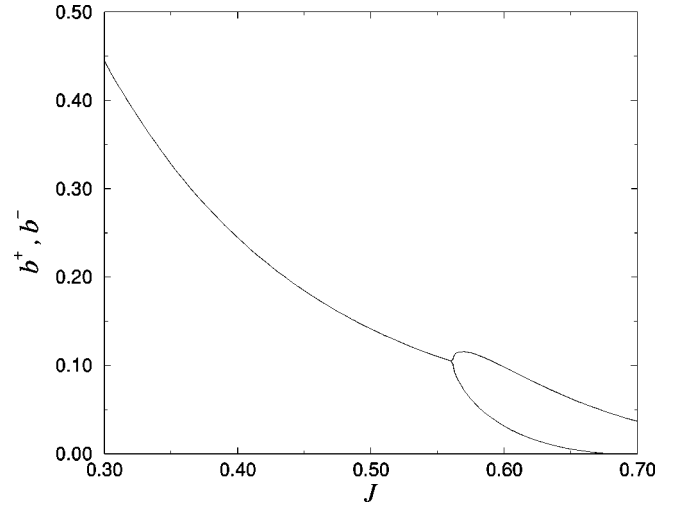


FIG. 4. Hysteretic coefficient b as a function of the exchange coefficient J .

VI. DISCUSSION

In conclusion, the present analysis allows to clarify the role of a disorder induced phase transition on the demagnetization properties of a ferromagnet. In particular, we find that in the low disorder phase the jump in the saturation curve gives rise to an inaccessible region in the (m, H) plane close to $H=0$ and $m=0$. Even after an infinitesimally fine series of nested loop the final magnetization m_∞ does not vanish. Approaching the transition, however, m_∞ scales to zero with an exponent $1/2$, which is the same as the one controlling the size jump in the saturation curve. While this value could be an artifact of the Bethe approximation, the impossibility to demagnetize the system in the low disordered phase has already been observed in three-dimensional numerical simulations.^{23,24}

The results discussed in this paper are derived for the case of Gaussian random-field distribution, but most of the derivation holds as well for the case of a generic distribution

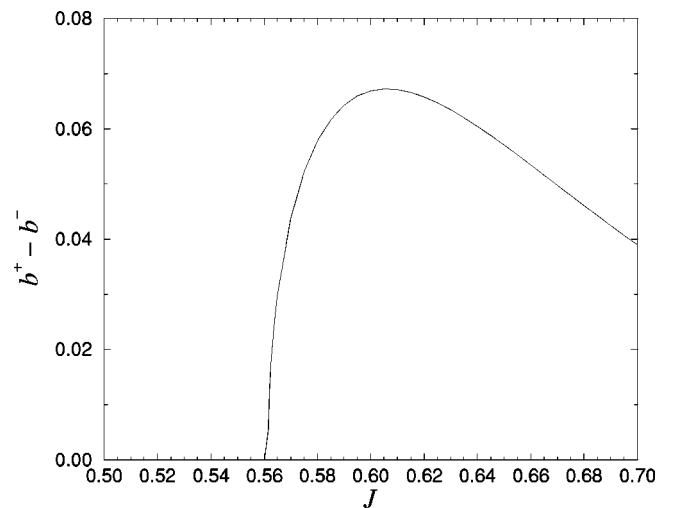


FIG. 5. The deviations from the Rayleigh law are measured by $\Delta b = b^+ - b^-$.

$\rho(x)$. The particular form of the distribution enters only when we try to obtain quantitative results about the transition point or the Rayleigh parameter, but the phenomenology should be the same independently on the distribution. A word of caution should, however, be spent when considering nonanalytic distributions, such as the uniform²¹ or the bimodal one.^{16,17} In this case the derivation follows the same steps, but typically some of the p_i integrals are zero for a nonvanishing interval of fields which makes the analysis cumbersome (see, for instance, Ref. 21).

Finally, it is interesting to compare the nonequilibrium behavior of the hysteresis loop with the corresponding equilibrium state. The equilibrium or, at $T=0$, the ground-state properties have been evaluated exactly on the Bethe lattice only for a bimodal distribution of random fields (i.e., $\rho(x) = [\delta(x-h) + \delta(x+h)]/2$).^{16,17} In this case, the system exhibits at $T=0$ a disorder induced ferromagnetic transition at $h=J$. In the corresponding hysteresis loop, this point also marks a transition, albeit somewhat trivial: for $h < J$ the cycle is perfectly squared and the first spin to flip [at $H = \pm(2J-h)$] leads to an avalanche that reverses all the other spins. The case of the Gaussian distribution is probably less trivial and thus more interesting to compare, but unfortunately no analytic solution for the ground state is available in the literature.

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APPENDIX

Here we derive the expressions for η_{2n-1} , η_{2n} , ζ_{2n} , and ζ_{2n+1} appearing in Eqs. (9) and (11). We first note that these quantities can be defined in a recursive way. In particular, η_{2n-1} represents the fraction of the set of spins which, at H_{2n-2} , contribute to U_{2n-2} , but are *down* at H_{2n-1} . To obtain the weight associated to this fraction, consider a spin i with a given neighbor j kept *down* (i.e., the spin j is conditioning the probabilities). When the spin i flips *down* at H_{2n-1} , apart from j all the other neighbors can be either *up* or *down*. Consider for instance the case in which m of these neighbors are *up* and $z-1-m$ *down*. Under these conditions, the spin i flips *down* if its effective field is positive at H_{2n-2} and negative at $H_{2n-1} < H_{2n-2}$. The associated contribution to η_{2n-1} is then given by

$$[U_{2n-2}]^m [D_{2n-1}]^{z-1-m} [p_m(H_{2n-2}) - p_m(H_{2n-1})], \quad (\text{A1})$$

where $[U_{2n-2}]^m [D_{2n-1}]^{z-1-m}$ is the probability that, at the moment at which the spin i flips *down*, m given neighbors are *up* and $z-1-m$ (other than j) are *down*, and $[p_m(H_{2n-2}) - p_m(H_{2n-1})]$ is the probability that the spin i , having m *up* neighbors, is *up* at H_{2n-2} but not at H_{2n-1} . To obtain η_{2n-1} , we have first to multiply Eq. (A1) by a com-

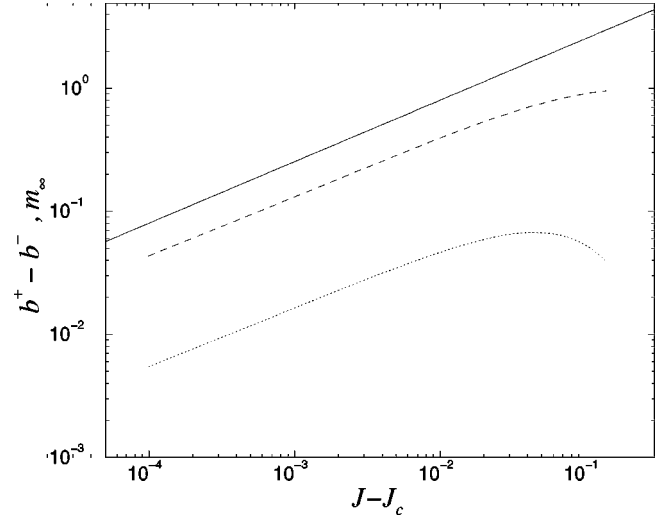


FIG. 6. Final magnetization m_∞ and Δb as a function of $J - J_c$. The solid line has a slope 1/2.

binatorial factor $\binom{z-1}{m}$, taking into account all the equivalent choices of the site j , and then sum over m from 0 to $z-1$. The result reads:

$$\eta_{2n-1} = \sum_{m=0}^{z-1} \binom{z-1}{m} [U_{2n-2}]^m [D_{2n-1}]^{z-1-m} \times [p_m(H_{2n-2}) - p_m(H_{2n-1})]. \quad (\text{A2})$$

An analogous procedure can be implemented to derive η_{2n} , the fraction of the set of spins contributing to η_{2n-1} which flip back *up* at H_{2n} . Using a derivation similar to the one discussed above, we obtain

$$\eta_{2n} = \sum_{m=0}^{z-1} \binom{z-1}{m} [U_{2n}]^m [D_{2n-1}]^{z-1-m} \times [p_m(H_{2n}) - p_m(H_{2n-1})]. \quad (\text{A3})$$

The quantities ζ_{2n} and ζ_{2n+1} can be obtained proceeding as in the evaluation of η_{2n-1} and η_{2n} , noticing that in this case the fixed neighbor j (conditioning the probabilities) has to be kept *up*. Moreover the spin i must flip from *down* to *up* at H_{2n} , in order to contribute to ζ_{2n} , and then flip back *down* at H_{2n+1} in order to contribute also to ζ_{2n+1} . The final results reads

$$\begin{aligned} \zeta_{2n} &= \sum_{m=0}^{z-1} \binom{z-1}{m} [U_{2n}]^m [D_{2n-1}]^{z-1-m} \\ &\quad \times [p_{m+1}(H_{2n}) - p_{m+1}(H_{2n-1})] \\ \zeta_{2n+1} &= \sum_{m=0}^{z-1} \binom{z-1}{m} [U_{2n}]^m [D_{2n+1}]^{z-1-m} [p_{m+1}(H_{2n}) \\ &\quad - p_{m+1}(H_{2n+1})]. \end{aligned} \quad (\text{A4})$$

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