Financial Contagion and Economic Development: an Epidemiological Approach

Alberto Bucci∗ Davide La Torre† Danilo Liuzzi‡ Simone Marsiglio§

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We develop an epidemiological approach to analyze how financial contagion may affect and be affected by economic activities. We show that, according to specific parameter values, the economy may converge either to a non-speculative or a speculative equilibrium: in the former situation the level of per capita income is maximal, while in the latter it is reduced by financial contagion. The presence of economic and financial feedback effects may also give rise to macroeconomic fluctuations during the transitional path, clearly showing that such economic and financial links are an important driver of the short run macroeconomic performance. By extending the analysis to a spatial dimension, we also show that financial contagion in some specific region may propagate quickly also in regions far away from those in which the contagion initially occurs, highlighting the role of regional policy coordination to avoid interregional contagion.

Keywords: Diffusion, Economic Fluctuations, Economic Growth, Financial Contagion

JEL Classification: C60, E32, O40

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Abstract

We develop an epidemiological approach to analyze how financial contagion may affect and be affected by economic activities. We show that, according to specific parameter values, the economy may converge either to a non-speculative or a speculative equilibrium: in the former situation the level of per capita income is maximal, while in the latter it is reduced by financial contagion. The presence of economic and financial feedback effects may also give rise to macroeconomic fluctuations during the transitional path, clearly showing that such economic and financial links are an important driver of the short run macroeconomic performance. By extending the analysis to a spatial dimension, we also show that financial contagion in some specific region may propagate quickly also in regions far away from those in which the contagion initially occurs, highlighting the role of regional policy coordination to avoid interregional contagion.

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1 Introduction

Following the pioneering work by King and Levine (1993), a large literature aiming to assess the implications of financial activities on economic development has rapidly grown (see, among others, a survey by Levine, 2005). From a theoretical perspective, a limited number of works analyze the mechanisms through which financial intermediation impacts on economic growth through physical capital accumulation (Trew, 2014; Bucci et al., 2018), human capital formation (De Gregorio, 1996; De Gregorio and Kim, 2000; Bucci and Marsiglio, 2016) and technological progress (Pagano, 1993; Morales, 2003; Trew, 2008). From an empirical point of view, instead, an extensive body of studies discusses whether financial intermediation is beneficial or detrimental for growth, and the most recent view on this issue concludes that answering such a question is not simple at all, since the relation between economic growth and finance is non-monotonic, and most likely bell-shaped (Cecchetti and Kharrouri, 2012; Law and Singh, 2014; Arcand et al., 2015; Bucci et al., 2018). This clearly suggests that predicting how financial and economic activities interact is all but trivial. This type of conclusion has been reinforced after the recent global financial crisis, which has shown us that the financial and real sides of an economy are interconnected in a quite complex fashion which we still do not fully understand. Indeed, very little is known about the mechanisms through which a financial crisis can give rise to an economic crisis and how this in turn may feed the financial crisis again. Our paper wishes to

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shed some light on this by analyzing how financial contagion and economic development affect each other and how specific policies can be used to improve the overall economic and financial outcomes.

Several papers, especially following the great recession in 2008, have tried to analyze how a financial crisis can give birth to financial contagion. By relying on either an empirical (Mistrulli, 2011; Baur, 2012; Mondria and Quintana-Domeque, 2013) or a theoretical (Allen and Gale, 2000; Martinez–Jaramillo et al., 2010; Ait–Sahalia et al, 2015) approach, extant literature focuses mainly on risk transmission between financial intermediaries and within the financial system, eventually accounting for transmission across national borders (Reinhart and Rogoff, 2009; Mishkin, 2011; Campello et al., 2010). To the best of our knowledge, however, none of these works analyzes what contagion within the financial sector implies for the real side of an economy and how economic activities at macroeconomic level in turn determine financial contagion. This is exactly the goal of our paper which wishes to conceptualize the mutual links between an economy’s real and financial sides with particular emphasis on financial contagion. Specifically, we rely on an epidemiological approach to characterize how the exchange of assets between banks can determine the overall health status of the financial systems, which determines the level of productivity in the whole economy. Since macroeconomic activity affects the number of asset exchanged between banks, the financial and economic sides are mutually related. Our model allows for two different equilibria: in the non-speculative equilibrium the level of per capita income is maximal, while in the speculative equilibrium it is reduced by financial contagion (which depends on both economic and financial factors). We also show that the convergence to the speculative equilibrium may give rise to economic fluctuations even in absence of random shocks, and such fluctuations are simply driven by the interactions between the economic and financial sides of an economy. By allowing for a spatial dimension we also analyze how the presence of financial contagion in some specific region may generate dramatic effects even in regions far away from those in which the contagion initially occurs. This extension allows us to provide a simple and intuitive explanation of why the recent financial crisis has rapidly become a global phenomenon, but also to provide a straightforward explanation of the widely-spread argument claiming that policy coordination across regions is effectively needed.

The paper proceeds as follows. Section 2 presents our a-spatial model of financial contagion and economic development which is entirely summarized by a system of differential equations. We analyze first the case in which the financial system affects the real economy, and then the case in which financial and economic systems are mutually related. We show that according to which parametric configuration holds true, and specifically according to how the effective risk-transmission rate and the effective risk-decay rate compare, the economy may converge either to a non-speculative or a speculative equilibrium. Section 3 presents our spatial model in which financial activities and capital may diffuse across the entire spatial economy; the model is thus described by a system of partial differential equations. We show that the presence of financial contagion in some specific region may generate dramatic effects even in regions far away from those in which the contagion initially occurs. Section 4 as usual presents concluding remarks and highlights directions for future research. Appendix A presents a generalization of the explicit results discussed in the main text, while appendix B extends our analysis to the case in which the exchange of asset between banks is proportional to the level of per capita income rather than per capita capital showing that also in this setting our main qualitative results still apply.

The extent and speed of contagion is indeed the most striking novelty of the recent global financial crisis, in the sense that a rapid transmission occurred not only across different segments of the financial market but also across countries (see Beck et al., 2010).
2 The Baseline Model

We develop a simple model that analyzes how financial contagion arising from the financial system can affect the real economy. We focus on a setup with heterogeneous banks which through their normal business activities exchange assets leading to transmission of risk from speculative to non-speculative banks. We embed such a setting within a Solow-type (1956) neoclassical growth model in which non-speculative banks positively influence the total factor productivity, determining the level of income per capita. The transmission of risk from speculative to non-speculative banks affects capital accumulation through its impact on productivity. Our goal is to analyze how the degree of financial contagion can affect the real economy with a special emphasis on its level of economic development, proxied by the stock of per capita capital (we shall show in appendix B that the results do not change even if we use per capita income as a proxy of economic development).

In order to show more clearly our results and the different mechanisms that we wish to emphasize, we proceed in two steps. In the first we assume that the rate of risk-transmission is exogenous. In this case, the direction of the relation between the financial and the real sides of the economy goes from the former to the latter: the share of speculative banks in the economy affects negatively the dynamics of physical capital accumulation, whereas capital is unable to affect the evolution of the share of speculative banks.

As a second step, we extend such a basic setup to a more realistic framework where the risk-transmission rate is endogenous and dependent on the stock of per capita capital the economy is endowed with, hence on its level of economic development. In this more interesting case, the direction of the relation between the financial side and the real side of the economy is twofold: the share of speculative banks affects the dynamics of capital investment, with the capital stock being itself able, in turn, to affect the evolution of the share of speculative banks. Irrespective of the fact that the risk-transmission rate is exogenous or endogenous, in both cases we find the possibility of multiple equilibria, namely equilibria where the degree of financial contagion is either null or strictly positive. Our setting allows us to characterize the determinants of such equilibria and their dynamic properties, ultimately contributing to a better understanding not only of the complexity of the mutual relations between real and financial sides of the economy, but also of how such relations might be the source of endogenous fluctuations in per capita income, even in the absence of random shocks.

2.1 The Financial Side Affecting the Real Side

We consider a setting in which the interactions across private banks in the financial (or banking) sector are modeled as in a basic epidemiological framework (Kermack and McKendrick, 1927; Hethcote, 2000). The whole population of private banks (simply referred to as “banks” hereafter), $B_t$, is composed of two groups: non-speculative, $N_t$, and speculative, $S_t$ banks, such that at any time $B_t = N_t + S_t$. The strategies of these two groups of banks differ according to their chosen risk-profile. While speculative banks are not concerned at all about risk and thus opt for holding short-term very risky assets (e.g., asset-backed securities, or junk bonds, among others), non-speculative banks are instead concerned about risk and thus hold in their portfolio only long-term, non-risky assets (e.g., perpetual bonds, for example). The size and the composition of the financial sector changes over time according to business dynamics (some new banks are created and others close down) and to the interactions between the two groups of banks. At any moment in time new others close down) and to the interactions between the two groups of banks. At any moment in time new

In the remainder of the paper with the term “banks” we shall refer to the vast array of financial institutions intended in a broad sense.

For the sake of simplicity, we rely on one of the simplest epidemiological frameworks, namely the susceptible–infected–susceptible model. More sophisticated setups which been developed in the mathematical epidemiology literature (see, among others, Imran et al., 2014; Sharomi and Malik, 2015; Thompson et al., 2016) could be considered as well, but this would complicate our analysis without providing additional insights on the mutual relation between financial contagion and economic development.
banks are created at a given rate $0 < b < 1$, and we assume that newly created banks are all non-speculative at birth but may become speculative themselves by trading assets with speculative banks. Indeed, due to normal business interactions, speculative and non-speculative banks normally do trade assets with each other and this implies that high-risk assets held by speculative banks may spread across the entire financial system, such that financial contagion takes place. We postulate that risk is transmitted from speculative to non-speculative banks through the mutual exchange of assets at a rate $\theta > 0$, measuring the average number of asset transactions required for the risk-exposure of the portfolio of a non-speculative bank to increase enough to be considered, in turn, speculative itself. High-risk assets held by speculative banks have a finite duration $\eta > 0$, which determines the decay-rate of risk-exposure. We also assume that the life span of banks is potentially infinite, however holding high-risk assets might lead banks to bankruptcy which occurs with a certain probability $0 < d < 1$, meaning that due to default reasons speculative banks may cease to exist. Thus, while speculative banks leave the financial market with a positive probability, such a probability is null for non-speculative banks. In light of the above characterization of the financial sector, given the initial conditions, $N_0 \geq 0$, $S_0 \geq 0$, and $B_0 = N_0 + S_0 \geq 0$, the interactions between speculative and non-speculative banks along with assets and vital business characteristics determine the flow of banks between the two subgroups, meaning that the number of speculative, non-speculative and total banks evolve according to the following differential equations:

\[
\begin{align*}
\dot{N}_t &= bB_t + \eta I_t - \theta \frac{N_t S_t}{B_t} \\
\dot{S}_t &= \theta \frac{N_t S_t}{B_t} - \eta S_t - dS_t \\
\dot{B}_t &= bB_t - dS_t
\end{align*}
\]

The above equations state that the risk-transmission, that is the extent to which risk propagates across speculative and non-speculative banks through the mutual exchange of assets, is determined by $\theta \frac{N_t S_t}{B_t}$, where $\theta$ is the risk-transmission rate (measuring the average number of transactions required for transmission to occur), and $N_t \frac{S_t}{B_t}$ represents the number of random matchings per unit of time between the two groups of banks. Hence, for any $\theta > 0$ and $N_t > 0$, the risk-transmission ultimately depends on the share $\frac{S_t}{B_t}$ (rather than the absolute number, $S_t$) of speculative banks in the whole banks population, suggesting that banks do not modify the level of their normal business activities according to the spread of risk in the financial system (i.e., the level of transactions between banks does not change with risk).\footnote{By relying on epidemiology terminology, we would say that risk-transmission is frequency dependent, rather than density dependent (see Hethcote, 2000).} Also note that the rate at which speculative banks leave the market ($d$) is given by the probability of default induced by risk exposure: since non-speculative banks exchange assets with speculative banks, they might become speculative banks and forced to default as well. By defining the share of non-speculative and speculative banks as $n_t = \frac{N_t}{B_t}$ and $s_t = \frac{S_t}{B_t}$, respectively, the above system can be recast as follows:

\[
\begin{align*}
\dot{n}_t &= b(1 - n_t) + \eta s_t - (\theta - d)n_t s_t \\
\dot{s}_t &= \theta n_t s_t - (\eta + b)s_t - ds_t(1 - s_t)
\end{align*}
\]

The above equations describe how the relative composition of the banking sector changes over time. In particular, the spread of risk through the exchange of assets implies that some non-speculative ($N_t$) banks may become speculative ($S_t$) and thus the share of speculative banks ($s_t$) may increase over time. Therefore, the prevalence of speculative banks in the financial system entirely captures the contagion effects in the financial system and thus in the following we shall refer to $s_t$ as the “degree of financial contagion”. In particular, we wish to understand what are the determinants of financial contagion and what are its potential implications for economic development.
From the economic side we consider a Solow-type setting in which agents save a constant share \(0 < v < 1\) of their income. Output can be either consumed or invested to accumulate more capital or to replace depreciated capital. Specifically, output, \(Y_t\), is produced according to a Cobb-Douglas production function by employing capital \(K_t\) and labor \(L_t\) as follows: \(Y_t = A_t K_t^\alpha L_t^{1-\alpha}\), where \(0 < \alpha < 1\) measures the capital share of income and \(A_t\) the total factors productivity (TFP). One of the most important roles of banks in modern economies consists of funneling savings to those firms that are able to employ (scarce) resources in a more efficient way, which in turn (at an aggregate level) results in a higher TFP (see, among others, Pagano, 1993; Morales, 2003; Trew, 2008; Bucci et al., 2018). In order to capture this effect, we assume that banks determine the level of TFP and in particular we postulate that this level increases with the share of non-speculative banks in the economy as follows: \(A_t = a n_t^\alpha\), where \(a > 0\) is a scale parameter and \(\beta > 0\) measures the efficiency of intermediation activity in affecting productivity.\(^5\) By denoting with \(n > 0\) the growth rate of the labor force (coinciding with the population size, since we abstract from unemployment), the evolution of per capita capital, \(k_t = \frac{K_t}{L_t}\), is given by:

\[
\dot{k}_t = v an_t^\alpha k_t^\alpha - (\delta + n)k_t, \tag{6}
\]

As usual we denote with lowercase letters per capita variables and with uppercase letters aggregate variables, thus \(y_t = \frac{Y_t}{L_t}\) denotes per capita output which is given by \(y_t = an_t^\beta k_t^\alpha\). Since our focus is on per capita variables, in what follows we shall refer to per capita capital as capital and per capita income as income as a matter of expositional simplicity.

Equations (4), (5) and (6) describe how the interactions between non-speculative and speculative banks determine the outcome in the financial side of the economy (namely, the composition of the banking system) which in turn, by affecting capital accumulation, ultimately determines the outcome in the real side of the same economy (i.e., the level of economic development). Since the shares of speculative and non-speculative banks sum to one at any moment in time, the system can be analyzed by focusing on only one of the two shares. In other words, we can simply concentrate our attention on how the evolution of the share of speculative banks, \(s_t\) (and, therefore, of the degree of financial contagion), affects capital accumulation. The dynamics of these two variables are given by the following equations:

\[
\begin{align*}
\dot{s}_t &= s_t[(\theta - b)(1 - s_t) - \eta - d] \\
\dot{k}_t &= va(1 - s_t)^\beta k_t^\alpha - (\delta + n)k_t. \tag{7, 8}
\end{align*}
\]

Note that in the case we are now focusing on (exogenous \(\theta\)), the degree of financial contagion \((s_t)\) affects the dynamics of capital, while the opposite is not true. This means that in this setting, it is the financial side to influence univocally the real side of the economy. The above system (7) and (8) admits two equilibria, \(E_1 = (\bar{k}_1, \bar{s}_1)\) and \(E_2 = (\bar{k}_2, \bar{s}_2)\), where:

\[
\begin{align*}
\bar{k}_1 &= \left(\frac{va}{\delta + n}\right)^{\frac{1}{1-\alpha}} \bar{s}_1 = 0 \\
\bar{k}_2 &= \left(\frac{va}{\delta + n}\right)^{\frac{1}{1-\alpha}} \left(\frac{\eta + b}{\theta - d}\right)^{\frac{\beta}{1-\alpha}} \bar{s}_2 = \theta - d - \eta - b \tag{9, 10}
\end{align*}
\]

It is possible to observe that while \(E_1\) always exists, \(E_2\) does exist only whenever \(\theta > d + \eta + b\) and such a parameter condition determines the stability properties of the two equilibria. Indeed, whenever a unique equilibrium exists (i.e., \(\theta \leq d + \eta + b\)) \(E_1\) is asymptotically stable, while when two equilibria exist (i.e., \(\theta > d + \eta + b\)) \(E_1\) turns out to be unstable while \(E_2\) asymptotically stable. Thus, which equilibrium the

\(^5\)Notice that in our framework when all banks are non-speculative (\(n_t = 1\)) the level of the TFP is maximal \((A_t = a)\), while the presence of speculative banks \((n_t < 1)\) reduces the TFP \((A_t < a)\). This can occur, as an example, simply because, unlike non-speculative banks, the speculative ones may have objectives different from channeling available funds towards the most productive uses in an economy.
economy converges to crucially depends on the number of equilibria, determined by how the risk-transmission rate ($\theta$) and what we can refer to as the “effective risk-exposure decay rate” ($d + \eta + b$) compare. Note that the effective risk-exposure decay rate depends on the pure risk-exposure decay rate ($\eta$) but also on the default probability ($d$) and the birth rate of new banks ($b$). This is due to the fact that the probability of default, by forcing some speculative banks out of the financial system, and the birth rate, by diluting the presence of speculative banks in the financial sector (since at birth every bank is a non-speculative bank), both effectively increase the risk-exposure decay rate.

In particular, $E_1$ represents what we may refer to as a “non-speculative” equilibrium where the degree of financial contagion is null ($\overline{s}_1 = 0$) and thus capital achieves its maximal level ($\overline{k}_1$) determined only by economic fundamentals (namely, $v$, $a$, $\delta$, $n$ and $\alpha$). $E_2$ represents what we may label as a “speculative” equilibrium where the degree of financial contagion is positive ($\overline{s}_2 > 0$) and entirely determined by financial factors (namely $\theta$, $\eta$, $b$ and $d$); this implies that capital does not achieve its maximal level ($\overline{k}_2 < \overline{k}_1$) since it is negatively affected by financial factors. Such factors, indeed, by increasing the share of speculative banks existing in the economy, decrease the TFP level, and therefore the economic incentives to invest in capital accumulation. Our above discussion suggests thus that the economy converges towards a non-speculative equilibrium whenever the effective risk-decay rate is faster than the risk-transmission rate ($\theta \leq d + \eta + b$), while it converges towards a speculative equilibrium whenever the effective decay rate is slower than the transmission rate ($\theta > d + \eta + b$). The dynamics of the degree of financial contagion and capital are shown in Figure 1, in the case in which the non-speculative equilibrium is stable (dashed curves) and in the case in which the speculative equilibrium is stable (solid curves). It is worth observing that the evolution of financial contagion is monotonic, and so is the evolution of capital.

![Figure 1: Evolution of the degree of financial contagion (left panel) and per capita capital (right panel) over time whenever $\theta \leq d + \eta + b$ (dashed curves) and $\theta > d + \eta + b$, starting from the same initial conditions, $i_0 = 0.2$ and $k_0 = 1$. Parameter values: $\alpha = 0.33$, $a = 1$, $v = 0.2$, $\delta = 0.05$, $n = 0.02$, $\theta = 0.2$, $\eta = 0.1$, $\beta = 1$, $b = 0.03$, with $d = 0.08$ (dashed curves) or $d = 0.02$ (solid curves).](image)

It is interesting at this stage to emphasize the twofold role of the parameter $d$ in our setting. As a matter of fact, the banks’ default probability affects not only the capital level in a speculative equilibrium, but also the effective risk-decay rate (that determines which equilibrium the economy converges to). As long as two equilibria do exist, that is the effective decay rate is sufficiently low, an increase in the probability of banks’ default increases the equilibrium capital level, as well; however, as soon as the effective decay rate becomes large enough, such that only one equilibrium does exist, then capital achieves its maximal level, which turns out to be independent of the probability of banks’ default (since entirely determined by economic fundamentals). This suggests that, despite the common wisdom that policymakers should do whatever is in their power in order to rescue banks from bankruptcy, in order to favor economic development it might be ultimately most convenient to allow them to default and exit the financial market. In the end,
the increased probability of banks’ default would result in a higher equilibrium capital and income levels. We can summarize the results as follows.

Proposition 1. In a Solow-type growth model, characterized by equations (7)-(8), in which the risk-transmission rate $\theta$ is exogenous, the economy converges to the non-speculative equilibrium whenever $\theta \leq d + \eta + b$, while it converges to the speculative equilibrium whenever $\theta > d + \eta + b$. Increases in the probability of banks’ default ($d$) may either increase the speculative equilibrium capital level ($K_2$) or allow the economy to achieve its maximal capital level ($K_1$).

In words, Proposition 1 states that an approach to economic policy aimed at rescuing banks no matter what might be detrimental to macroeconomic performance. Policymakers should, instead, allow banks eventually to fail, since this would ultimately result in higher levels of capital and income. The intuition behind this result is that a higher probability of default would lead speculative banks out of the financial market sooner; this will reduce the degree of financial contagion favoring a healthier (non-speculative) banking system which is conductive to faster capital accumulation and higher income levels. This type of conclusion is consistent with the result of some recent works by Hart and Zingales (2014) and Heitfield et al. (2010).

2.2 Mutual Interdependence between the Financial and the Real Sides

We now extend our baseline model by endogeneizing $\theta$ in order to account for the role of the real economy in determining the outcome in the financial system as well. Specifically, we now postulate that the average number of asset-transactions across banks required for risk to be transmitted is time-varying and increases with per capita capital as follows: $\theta_t = \theta k_t^\epsilon$, where $\epsilon \geq 0$ quantifies the elasticity of the transmission rate to the capital stock. By assuming that the risk-transmission rate increases with capital we explicitly take into account the fact that, everything else equal, in more developed economies on average a larger number of transactions between speculative and non-speculative banks is needed before transferring risk. This might be due, for example, to the role of financial regulation which is generally more stringent in advanced economies, where central banks’ efforts to promote financial stability are more effective. Under our new assumption on $\theta_t$ and its link with capital, the dynamic equations for per capita capital and the degree of financial contagion are mutually related as follows:

$$
\dot{s}_t = s_t[(\theta k_t^\epsilon - b)(1 - s_t) - \eta - d]
$$
(11)

$$
\dot{k}_t = va(1 - s_t)k_t^\alpha - (\delta + n)k_t.
$$
(12)

Note that in the special case in which $\epsilon = 0$ the model boils down to our baseline setup in which the transmission rate is an exogenous constant. Simple comparison of the systems (7)-(8) and (11)-(12) reveals that when the risk-transmission rate is endogenous and positively dependent on capital, the relation between the financial and the real sides of the economy is no longer unidirectional: while the composition of the banking sector continues to affect capital accumulation and therefore economic development (see (8) and

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6This is consistent with the view that any banking regulation needs a resolution scheme that governs bank failures in order to solve the trade-off between imposing market discipline and minimizing the effects of such failures on the rest of the banking system (Beck, 2011).

7By considering a simple model that abstracts from issues related to the degree of interconnectedness of financial institutions (which, instead, is the main focus of our paper), and where security markets are complete but consumers cannot pledge future income or wealth, Hart and Zingales (2014) show that a shock to banks disproportionately affects the agents who need liquidity the most, reducing thus aggregate demand and the level of economic activity. The optimal fiscal response to such a shock, however, is to help people, not banks. Therefore, even the special role played by banks: “[…] it does not necessarily justify banks’ bailouts.” (Hart and Zingales, 2014).

8Heitfield et al. (2010) strikingly conclude: “[…] pumping funds into the financial system may sustain zombie banks and amplify problem”
economic development (i.e., capital) now affects the banking sector (see (7) and (11)). As we shall see in a while this bidirectional relation between the financial and the real sides of the economy gives rise to important macroeconomic consequences.

Exactly as in our baseline model, the above system (11)-(12) admits two equilibria: $E_1 = (\bar{k}_1, \bar{s}_1)$ and $E_2 = (\bar{k}_2, \bar{s}_2)$. $E_1$ is the same non-speculative equilibrium as in the previous section with null degree of financial contagion and maximal capital level, as in equation (9). $E_2$ is again a speculative equilibrium characterized by a positive degree of financial contagion and capital less than its maximal level; clearly, with endogenous $\theta_t$ the equilibrium values of capital and share of speculative banks associated with the speculative equilibrium are different from those in (10) with an exogenous $\theta$. In fact, in order to derive the equilibrium values of capital and financial contagion in the speculative equilibrium, we need to solve the following equation for $s$:

$$\begin{align*}
\theta \left( \frac{va}{\delta + n} \right)^{1/\alpha} (1 - s)^{\frac{\alpha}{1-\alpha}} (1 - s) &= \eta + d
\end{align*}$$

Such an equation cannot be solved explicitly in general, nonetheless it is possible to show that a unique solution always exists (see appendix A). Moreover, from (13) it is also clear that the equilibrium value of the share of speculative banks now depends (unlike the case of exogenous $\theta$) on economic fundamentals ($v, a, \delta, n$, and $\alpha$), as well. In order to look at the nature of the relation between the real and financial system in the simplest possible way, we focus on the case in which an explicit solution for (13) does exist, but our main results apply even in a more general case (see appendix A). This occurs whenever $\epsilon = \frac{1-\alpha}{\beta}$, which we assume to hold true in what follows. Note that such a parametric restriction is very mild since it allows us to consider situations in which $\epsilon$ is smaller or larger than or even equal to unity, according to whether $\beta$ is larger or smaller than or equal to $1 - \alpha$, respectively. If such a condition is met then the speculative equilibrium $E_2$ is characterized by the following equilibrium values of capital and degree of financial contagion:

$$\bar{k}_2 = \left( \frac{va}{\delta + n} \right)^{\frac{1}{\alpha}} (1 - \bar{s}_2)^{\frac{\alpha}{1-\alpha}} 1 - \bar{s}_2 = \frac{b + \sqrt{b^2 + 4\theta(\frac{va}{\delta + n})^{\frac{1}{\alpha}}(\eta + d)}}{2\theta(\frac{va}{\delta + n})^{\frac{1}{\alpha}}}(14)$$

As already discussed in the previous section $E_2$ exists only if a certain condition is met, and in this case the condition reads as $\theta(\frac{va}{\delta + n})^{\frac{1}{\alpha}} > d + \eta + b$, requiring what we can refer to as the “effective risk-transmission rate” ($\theta(\frac{va}{\delta + n})^{\frac{1}{\alpha}}$) to be larger than the effective risk-decay rate. Exactly as before, whenever a unique equilibrium exists (i.e., $\theta(\frac{va}{\delta + n})^{\frac{1}{\alpha}} \leq d + \eta + b$) $E_1$ is asymptotically stable, while when two equilibria exist (i.e., $\theta(\frac{va}{\delta + n})^{\frac{1}{\alpha}} > d + \eta + b$) $E_1$ turns out to be saddle point stable while $E_2$ asymptotically stable. This suggests that economic fundamentals ($v, a, \delta, n$, and $\alpha$) by determining the effective risk-transmission rate play an essential role in determining which equilibrium the economy converges to. Specifically, we observe that, ceteris paribus, the effective risk-transmission rate increases with the saving rate but decreases with the depreciation rate and the labor force growth rate. In other words, in more developed economies (i.e., those in which the saving rate is higher, and/or the labor force growth rate and the capital depreciation rate are lower) the condition for a speculative equilibrium to be reached is, all the rest being equal, more easily satisfied: in such economies the degree of financial contagion is likely to be higher (see equation (14)).

Regarding the determinants of the equilibrium outcomes in the economic and financial systems, the same results discussed in the previous section apply apart from the fact that now economic fundamentals affect the outcome in the financial sector as well. The dynamics of the degree of financial contagion and capital are shown in Figure 2 in the case in which the non-speculative equilibrium is stable (dashed curves) and in the case in which the speculative equilibrium is stable (solid curves). We now observe that, while in the case...
of convergence to the non-speculative equilibrium dynamics are monotonic as in the previous section, in the case of convergence to the speculative equilibrium the evolution of financial contagion and capital are non-monotonic, suggesting that the mutual relation between the economic and the financial systems might give rise to macroeconomic fluctuations during which phases of economic expansion and contraction alternate each other. Note that this occurs independently of the fact that our model is purely deterministic, thus the result is not driven by the presence of any exogenous source of random shocks, as in most theories of economic fluctuations (see, among many others, the path-breaking works by King et al., 1988a, 1988b).

Instead, in our setting this result is due to a totally different mechanism based on the persistent and reciprocal interaction between the composition of the banking sector and the economic incentives to invest in physical capital. Specifically, such a mechanism can be qualitatively explained as follows. Assume that \( \theta_t = \theta k_t^2 \) raises (due, for example to an exogenous increase in \( \theta \)): this will lead, ceteris paribus, to a rise in the number of speculative banks (\( S_t \)) and hence in the degree of financial contagion (\( s_t \)), as well; consequently, the share of non-speculative banks (\( n_t = 1 - s_t \)) and the productivity (\( A_t = \alpha n_t^2 \)) would both decrease; with a lower TFP, per capita income (\( y_t \)) and per capita capital (\( k_t \)) would ultimately decrease too and this, in turn, leads to a lower level of \( \theta_t \). With a lower \( \theta_t \) we would assist, respectively, to a decrease in \( S_t \) and \( s_t \), and to a rise of \( n_t, A_t, y_t, k_t, \) and \( \theta_t \). This process continues over time (with phases of declining capital that precede/follow phases of growing capital) until when a long run equilibrium is reached in which \( s_t, n_t, A_t, y_t, k_t, \) and \( \theta_t \) remain all constant.

![Figure 2](image_url)

**Figure 2**: Evolution of the degree of financial contagion (left panel) and per capita capital (right panel) over time whenever \( \theta(\frac{\alpha n_t^2}{\delta + n_t})^\beta > d + \eta + b \), starting from the same initial conditions, \( i_0 = 0.2 \) and \( k_0 = 1 \).

Parameter values: \( \alpha = 0.33, a = 1, v = 0.2, \delta = 0.05, n = 0.02, \theta = 0.2, \eta = 0.1, \beta = 1, b = 0.03, \epsilon = \frac{1-\alpha}{\beta} \), with \( d = 0.08 \) (dashed curves) or \( d = 0.02 \) (solid curves).

As a final remark, note that exactly as in the previous section banks’ default probability \( d \) plays a significant role not only on the speculative equilibrium capital level, but also on the level of the effective risk-decay rate, which determines what equilibrium the economy converges to. As long as the effective decay rate is sufficiently low and a speculative equilibrium exists, an increase in the probability of banks’ default will increase the capital level, \( \bar{k}_t \); however, if, following the same increase in banks’ default probability, the effective decay rate becomes too high (compared to the effective risk-transmission rate) then the economy

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10 Other theories of economic fluctuations based on deterministic settings are related to the existence of equilibrium indeterminacy (see for example, Benhabib and Farmer, 1994; Benhabib and Perli, 1994; Boldrin and Rustichini, 1994). Note that, differently from this branch of literature, our setup gives rise to fluctuations even if the equilibrium is determinate.

11 See Marsiglio and Tolotti (2016) for a model where social interactions within the research industry might give rise to income fluctuations.
converges to the non-speculative equilibrium and capital ($\bar{k}_1$) achieves its maximal level, which is independent of the probability of default and entirely determined by economic fundamentals. We can summarize the results as follows.

**Proposition 2.** In a Solow-type growth model, characterized by equations (14)-(16), in which the risk-transmission rate $\theta_t = \theta k^p_t$ is endogenous and $\epsilon = \frac{1-a}{\beta}$, the economy converges to the non-speculative equilibrium whenever $\theta_t (\frac{\epsilon}{\theta + \eta})^{1/\gamma} \leq d + \eta + b$, while it converges to the speculative equilibrium whenever $\theta_t (\frac{\epsilon}{\theta + \eta})^{1/\gamma} > d + \eta + b$. Increases in the probability of banks’ default ($d$) may either increase the speculative equilibrium capital level ($\bar{k}_2$) or allow the economy to achieve its maximal capital level ($\bar{k}_1$).

Proposition 2 is qualitative identical to Proposition 1 suggesting thus that rather than trying to rescue speculative banks at any cost policymakers might support economic development by allowing banks eventually to default. However, with respect to a situation in which the risk-transmission rate is fully exogenous, when it is endogenous the presence of reciprocal interactions between the economic and financial sides of the economy complicates policy decisions. Indeed, since (according to our model) in richer economies the effective transmission rate is larger, the probability of default needs to be substantially high in order to effectively achieve a non-speculative equilibrium characterized by no financial contagion and maximal level of capital and income.

### 3 The Spatial Model

Financial contagion can be the result of two different mechanisms: the financial systems affects the real side of the economy (which might feed back on the financial system as well), or an economy affects another giving rise to geographical patterns of contagion. Thus far, we have focused on the first mechanism showing the extent to which the financial and real side may be mutually related. We now focus on the second mechanisms trying to understand how financial contagion occurring in a single economy may spread and affect other economies as well. In order to look at this, we analyze the model’s implications in a spatial framework in which economic and financial activities diffuse across space. We assume a continuous space structure to represent that the spatial economy develops along a linear city, as in Hotelling (1929). A similar approach has been recently used to characterize the spatial implications of economic growth and environmental degradation (Boucekkine et al., 2009; Brock and Xepapadeas, 2010; Capasso et al., 2010; La Torre et al., 2015). We denote with $n_{x,t}$, $s_{x,t}$ and $k_{x,t}$, respectively the share of non-speculative and speculative banks and per capita capital level in the position $x$ at date $t$, in a compact interval $[x_a, x_b] \subset \mathbb{R}$, and we assume that there are no diffusion flows through the borders of $[x_a, x_b]$, that is the directional derivatives are null, $\frac{\partial n_{x,t}}{\partial x} = \frac{\partial s_{x,t}}{\partial x} = \frac{\partial k_{x,t}}{\partial x} = 0$, at $x = x_a$ and $x = x_b$. In this framework, any position $x$ may be interpreted as a specific location while a set of adjacent locations as a region in the spatial economy; such a possibility to distinguish between different regions allows us to account for the existence of regional heterogeneity and to understand what this might imply for the entire spatial economy. Differently from what discussed earlier for the a-spatial model, the outcome in the financial system cannot be fully characterized by focusing only on the evolution of the degree of financial contagion, $i_{x,t}$ since the population of banks is spatially distributed, $B_t = \int_{x_a}^{x_b} [N_{x,t} + S_{x,t}] dx$, and thus it is not necessarily true that the shares of speculative and non-speculative banks sum to one in each location $x$ (they do sum to one over the whole spatial domain).

We need thus to analyze the evolution of the share of speculative banks and the share of non-speculative banks over time and across space, along with their implications on per capita capital, and the spatial model.
can be represented through a system of partial differential equation as follows:

\begin{align}
\frac{\partial n_{x,t}}{\partial t} &= \lambda \frac{\partial^2 n_{x,t}}{\partial x^2} + (b + \eta) s_{x,t} - (\theta k_{x,t}^e - d_x) n_{x,t}, \quad (15) \\
\frac{\partial s_{x,t}}{\partial t} &= \lambda \frac{\partial^2 s_{x,t}}{\partial x^2} + (\theta k_{x,t}^e - d_x) s_{x,t} n_{x,t} - (\eta + b) s_{x,t}, \quad (16) \\
\frac{\partial k_{x,t}}{\partial t} &= \lambda \frac{\partial^2 k_{x,t}}{\partial x^2} + v a_n^\beta x k_{x,t}^\alpha - (\delta + n) k_{x,t}. \quad (17)
\end{align}

In the above equations the only difference with the respect to the a-spatial model earlier described is related to the introduction of a spatial characterization. In each location $x$ economic and financial activities evolve over time exactly as we discussed before. However, such activities evolve also across space and $\lambda \geq 0$ represents the diffusion parameter which measures the speed at which per capita capital, and the shares of speculative and non-speculative banks spread across space, which without loss of generality is assumed to be same for all variables. The spatial diffusion of capital has been recently discussed in a number of macroeconomic geography papers, where capital diffusion represents the effect of trade between adjacent locations (Boucekkine et al., 2009). The diffusion of speculative and non-speculative banks can be similarly interpreted as the effects of business transactions (i.e., trade of assets) between banks operating in adjacent locations. All parameters in the above equations could be space-dependent but for the sake of simplicity we assume that only the probability of default is, $d_x$, meaning that different locations may be characterized by a different banks’ default reflecting differences in local economic policy or legislation.

Explicitly analyzing systems of partial differential equations is cumbersome and goes well beyond the scope of this paper. Indeed, our goal is simply to understand what are the possible implications of introducing a spatial structure in our model of financial contagion, and the simplest way to do so consists of relying upon some numerical simulations which allow to visually represent the possible outcomes. Specifically, we wish to understand whether trade across adjacent locations, what we will refer to as a “spatial externality”, might be beneficial for long run economic development. In order to look at this let us consider a framework in which economies in different locations across the whole spatial domain are structurally identical (also in terms of initial conditions) apart from the probability of default, which is spatially heterogeneous. In particular, let us focus on a situation in which the probability of default is high (such that $\theta(\frac{v_n}{\delta + n})^\frac{1}{2} \leq d + \eta + b$ holds) in the lateral regions while it is low (such that $\theta(\frac{v_n}{\delta + n})^\frac{1}{2} > d + \eta + b$ holds) in the central region, thus from our previous analysis we expect that the lateral regions will achieve a non-speculative equilibrium while the central region a speculative equilibrium. We expect thus the presence of heterogeneity in the banks’ probability of default to give rise to heterogeneous outcomes in the spatial economy. We wish to understand whether the presence of a spatial externality, captured by the diffusion parameter $\lambda$, is likely to affect this type of conclusions.

The implications of spatial externalities can be seen from Figure 3. The figure represents the spatio-temporal dynamics of the degree of financial contagion (top panels) and per capita capital (bottom panels) in the cases with no (left panels) and with (right panels) spatial externalities. The left panels represent the outcome in a situation in which $\lambda = 0$, meaning that the economy is spatially structured but any economy located in a specific venue is completely independent from economies located in other venues. It is straightforward to note that the predictions of our a-spatial model are confirmed even in such a spatial framework: in the long run the lateral regions converge to a non-speculative equilibrium while the central region to a speculative equilibrium. Why this should be the case is obvious: the introduction of an exogenous spatial structure does not affect in any way the development experience of single economies since the spatial externality does not play any role and thus each region is completely independent from the others. The right panels represent instead the outcome in a situation in which $\lambda > 0$, meaning that the presence of diffusion due to trade implies that any economy located in a specific venue is affected by adjacent economies and thus each economy is no longer completely independent from all others. In this case we can note that the
predictions of our a-spatial model are no longer confirmed, and also lateral regions converge to a speculative
equilibrium. Since banks trade with each others independently of their specific location, contagion tends
to spread across the entire spatial economy affecting the evolution of per capita capital. In particular the
presence of diffusion tends to homogenize the long run spatial outcome, both in terms of financial and
economic activities. With respect to the no diffusion case (left panels), the degree of contagion in the
central region decreases and thus capital increases while contagion increases in the lateral regions and thus
capital decreases. Overall, while the central region benefits from the spatial externality, the lateral regions
suffer from trade spillovers. We can also note that with diffusion the dynamics of per capita capital and
contagion become non-monotonic suggesting that trade can magnify the economic fluctuations induced by
the interactions between the economic and financial sectors.

The above analysis suggests that because of the spatial externality, lateral regions suffer a reduction in
income levels due to the fall in capital and rise in financial contagion. This occurs because the central region,
due to its local economic policy and legislation, is not able to achieve a non-speculative equilibrium even in
absence of trade spillovers. In such a case, by taking into account the presence of this detrimental effect due
to trade with the central region, lateral regions can improve their long run outcome by relying on specific
local policies targeting the effective risk-transmission rate or the effective risk-decay rate. For example, they
could intervene in order to increase the probability of banks’ default. If the probability of default in the
lateral regions increases enough to more than compensate for the negative effect induced by trade with the
central region, then the entire spatial economy will monotonically converge to a non-speculative equilibrium
(see Figure 4). This means that not only lateral regions will achieve a non-speculative equilibrium but also
the central region will: an adequate change in economic policy in some (lateral) regions will benefit other

Figure 3: Evolution of the degree of financial contagion (top) and per capita capital (bottom), with no
diffusion (left) and with diffusion (right). Probability of default relatively low in the lateral regions.
Figure 4: Evolution of the degree of financial contagion (top) and per capita capital (bottom), with no diffusion (left) and with diffusion (right). Probability of default relatively high in the lateral regions.

(central) regions as well, improving the economic and financial outcomes in the entire spatial economy; this will also dampen the fluctuations in economic activity induced by the interactions between the real and the financial systems, promoting a smooth process of economic development. These results suggest that policy coordination between different regions is essential in order to deal with financial contagion and the implications of the spatial externality.

Our analysis from Figures 3 and 4 allows us to state some interesting conclusions. Spatial externalities in the form of trade of assets between banks operating in different regions might be a source of economic instability: in case of localized shocks, contagion effects may propagate quickly across the entire spatial economy potentially giving rise to a global crisis, and policy decisions in a single economy are completely irrelevant (see Figure 3). In order to avoid a global spread of financial contagion, it is imperative that single economies cooperate with each other: if they do not, a too low probability of default in a single region may negatively affect other regions as well, meaning that economic and financial policies need to be mutually determined by taking into account the effects induced by trade spillovers (see Figures 4). We believe that these very simple and intuitive results can explain quite well the interactions between the economic and financial sectors at world level during the recent global financial crisis.

4 Conclusion

The fact that the real and the financial systems are mutually related has been known for a long time, but how macroeconomic and financial activities affect each other is still an open question. This paper develops a simple framework to shed some light on such mutual feedback and in particular it tries to characterize
the extent to which financial contagion arising from the financial sector propagates to and is determined by the real economy. Specifically, we rely on an epidemiological approach to describe the transmission of risk within the banking sector which ultimately determines the degree of financial contagion; by allowing contagion to affect the TFP, introducing such epidemic dynamics in a Solow-type model of economic growth is straightforward; then, allowing economic development, measured by the level of per capita capital, to impact on the risk-transmission between banks permits us to close the loop. Such a simplistic framework allows us to derive some interesting conclusions: (i) the existence of multiple equilibria implies that economic policy can be effectively used to address the economy towards the speculation-free equilibrium, and this might simply require to promote banking efficiency by avoiding to rescue banks in distress; (ii) the complicated interactions between economic and financial systems imply that convergence to the speculative equilibrium might be characterized by cyclical fluctuations even in absence of random shocks, and such fluctuations naturally disappear if the economy is addressed to the speculation-free equilibrium. These results suggest that economic policy may play a fundamental role not only in improving long run (increasing the level of income per capita) but also short run (dampening the size of business fluctuations) macroeconomic outcomes. By extending the analysis to a spatial framework to account for spatial externalities we show that such conclusions are even more relevant in presence of spatial heterogeneity, since regional policy if effective enough could improve the macroeconomic outcomes in the entire spatial economy.

To the best of our knowledge this is the first attempt to formally characterize the mutual links between financial contagion and macroeconomic activities in such a neat framework. Therefore, we have tried to maintain the analysis as simple as possible in order to clarify the various mechanisms in place. In order to do so, the analysis has been carried out in a purely dynamic setting without considering the optimal determination of economic and financial policy. Extending the analysis in order to characterize the associated optimal control problem may provide some further insight on how economic and financial systems are mutually related, both in an a-spatial and in a spatial framework; moreover, it may permit to quantify the welfare effects associated with economic and financial policy, allowing to assess thus the effective desirability of certain policies. These additional tasks are left for future research.

### A Generalization

In this appendix we show that the results discussed in the main text, based on some specific parametric restriction \( \epsilon = \frac{1-a}{b} \), hold true also if we remove such a restriction. The cost of relaxing this condition is that we lose the explicit expression for the speculative equilibrium values of the degree of financial contagion and per capita capital. The equilibrium degree of financial contagion in the speculative equilibrium requires that \( \epsilon = \frac{1-a}{b} \), and therefore, it is clearly strictly monotone decreasing, with \( f(\epsilon) = \frac{\theta(\frac{a}{a+n})}{\eta+\beta} \). This implies that to have a unique speculative equilibrium \( s^* \in [0, 1] \) if and only if the equation (13) is verified. The following proposition states an existence and uniqueness result for the solution of this equation.

**Proposition 3.** The equation (13) has a unique solution \( s^* \in [0, 1] \) if and only if \( \theta(\frac{v_a}{\theta+n}) \geq d + \eta + b \).

**Proof.** Equation (13) can be rewritten as

\[
 f_1(s) = \theta \left( \frac{va}{\theta+n} \right)^{\frac{1}{1-\alpha}} (1-s)^{\frac{\alpha}{1-s}} = \frac{\eta + d}{1-s} + b = f_2(s) \tag{18}
\]

The function \( f_1 \) is clearly strictly monotone decreasing, with \( f_1(0) = \theta(\frac{v_a}{\theta+n}) \) and \( f_1(1) = 0 \). The function \( f_2 \) is monotone strictly increasing instead, with \( f_2(0) = d + \eta + b \) and \( f_2(1) = +\infty \). This implies that to have a unique solution to (13) is necessary and sufficient that \( f_1(0) \geq f_2(0) \) which implies \( \theta(\frac{v_a}{\theta+n}) \geq d + \eta + b \). ■

Whenever the condition in Proposition 3 is met, then a unique speculative equilibrium \( E_2 = (\kappa_2, \bar{s}_2) \).
exists. Specifically, the equilibrium value of capital is given by:

$$\bar{k}_2 = \left( \frac{va}{\delta + n} \right)^{\frac{1}{1-\alpha}} (1 - \bar{\pi}_2)^{\frac{\beta}{1-\alpha}} = \frac{1}{\theta} \left[ \frac{\eta + d}{1 - \bar{\pi}_2 + b} \right]$$

while the equilibrium value of the degree of contagion is the implicit solution to the following nonlinear equation:

$$\left[ \theta \left( \frac{va}{\delta + n} \right)^{\frac{1}{1-\alpha}} (1 - \bar{\pi}_2)^{\frac{\beta}{1-\alpha}} - b \right] (1 - \bar{\pi}_2) = \eta + d$$

In order to analyze the stability properties of this equilibrium, we proceed via linearization. The Jacobian of the system of differential equations \([11] - [12]\) evaluated at \(E_2\) is given by the following expression:

$$J = \begin{bmatrix} (\theta \bar{k}_2 - b)(1 - 2\bar{\pi}_2) - \eta - d & \epsilon \bar{\pi}_2 \bar{k}_2^{-1} (1 - \bar{\pi}_2) \\ -va\beta(1 - \bar{\pi}_2)^{\beta - 1}\bar{\pi}_2^{\alpha} & va\alpha \bar{k}_2^{-1} (1 - \bar{\pi}_2)^{\beta} - \delta - n \end{bmatrix}$$

It is straightforward to show that \(J_{21} < 0\) and \(J_{12} > 0\). The following calculations allow to determine the sign of \(J_{22}\) and \(J_{11}\):

$$J_{22} = va\alpha \left( \frac{1 - \bar{\pi}_2}{\bar{k}_2^{-1} - 1} - \delta - n \right) = va\alpha \left( \frac{\delta + n}{va} \right) - \delta - n = (\delta + n)(\alpha - 1) < 0$$

$$J_{11} = -\bar{\pi}_2 (\theta \bar{k}_2 - b) = -\bar{\pi}_2 \left( \frac{\delta + n}{1 - \bar{\pi}_2} \right) < 0$$

These conditions imply that the trace of \(J\) is negative and the determinant of \(J\) is positive which leads to the conclusion that both eigenvalues of \(J\) are strictly negative, suggesting that \(E_2\) is asymptotically stable. As discussed in the main text under the parametric restriction \(\epsilon = \frac{1-\alpha}{\beta}\), also whenever this condition is not met a unique speculative equilibrium exists whenever a certain parametric restriction holds true, and whenever such an equilibrium exists this will be asymptotically stable. Not surprisingly, note that the condition in Proposition 3 ensuring existence and uniqueness of the speculative equilibrium is a generalization of the condition discussed in the main text, which indeed can be restored by setting \(\epsilon = \frac{1-\alpha}{\beta}\). This discussion suggests that the results discussed in the main text apply even in more general situations in which the condition \(\epsilon = \frac{1-\alpha}{\beta}\) is not met. The evolution of per capita capital and financial contagion in the case in which \(\theta \left( \frac{va}{\delta + n} \right)^{\frac{1}{1-\alpha}} \geq d + \eta + b\) are illustrated in Figure 3\), which shows that the qualitative behavior of the variables is identical to what discussed in the body of the paper; the only noticeable difference is related to the fact that the size of fluctuations in capital and contagion are much larger, suggesting that whenever \(\epsilon\) is not restricted to take some specific value the macroeconomic effects of financial contagion can be substantially large.

B A Different Formulation

We now present a different formulation of the relation between the financial system and the real economy to show that even in more general setting our main results still hold true. We now assume that the average number of asset-transactions across banks required for risk to be transmitted increases no longer with per capita capital but with per capita income as follows: \(\theta_t = \theta y_t\). The relation between financial contagion and capital accumulation in this case is summarized by the following equations:

$$\dot{s}_t = s_t \{ [\theta a^\epsilon (1 - s_t)^{\beta_k} k_t^{\alpha} - b](1 - s_t) - \eta - d \} \quad (19)$$

$$\dot{k}_t = va(1 - s_t)^{\beta_k} k_t^{\alpha} - (\delta + n)k_t. \quad (20)$$
Figure 5: Evolution of the degree of financial contagion (left panel) and per capita capital (right panel) over time whenever $\theta(\frac{\nu \alpha}{\delta + n})^\frac{\alpha}{\beta} > d + \eta + b$, starting from the same initial conditions, $i_0 = 0.2$ and $k_0 = 1$. Parameter values: $\alpha = 0.33$, $a = 1$, $v = 0.2$, $\delta = 0.05$, $n = 0.02$, $\theta = 0.2$, $\eta = 0.1$, $\beta = 1$, $b = 0.03$, $\epsilon = 50$, with $d = 0.02$.

As in the body of the paper, if $\epsilon = \frac{1-\alpha}{\beta}$, it is possible to explicitly find the equilibria. The speculative equilibrium $E_2$ is characterized by the following equilibrium values of capital and degree of financial contagion:

$$k_2 = \left( \frac{\nu a}{\delta + n} \right)^{\frac{1-\alpha}{1-\alpha}} (1-\sigma_2)^{\frac{\alpha}{1-\alpha}} - \sigma_2 = \frac{b + \sqrt{b^2 + 4\theta(\frac{\nu a}{\delta + n})^\frac{\alpha}{\beta} (\eta + d)}}{2\theta(\frac{\nu a}{\delta + n})^\frac{\alpha}{\beta}}$$

In this case, the economy converges to the non-speculative equilibrium whenever $\theta(\frac{\nu a}{\delta + n})^\frac{\alpha}{\beta} \leq d + \eta + b$, while it converges to the speculative equilibrium whenever $\theta(\frac{\nu a}{\delta + n})^\frac{\alpha}{\beta} > d + \eta + b$. Also in this case increases in the probability of banks’ default ($d$) may either increase the speculative equilibrium capital level ($k_2$) or allow the economy to achieve its maximal capital level ($k_1$). The evolution of per capita capital and financial contagion are illustrated in Figure 5. We can observe that apart from some quantitative differences in the level of capital and contagion both during the transition and at equilibrium, from a qualitative point of view capital and contagion behave exactly as in our earlier formulation. This suggests that our main results hold true even in more general settings, and no matter the proxy of financial development (capital vs income) our model is able to effectively characterize the mutual links between the financial and the real sides of the economy.

References

Parameter values: $\alpha = 0.33$, $a = 1$, $v = 0.2$, $\delta = 0.05$, $n = 0.02$, $\theta = 0.2$, $\eta = 0.1$, $\beta = 1$, $b = 0.03$, $\epsilon = \frac{1-\alpha}{\beta}$, with $d = 0.08$ (dashed curves) or $d = 0.02$ (solid curves).


