Karlsruher Institut für Technologie

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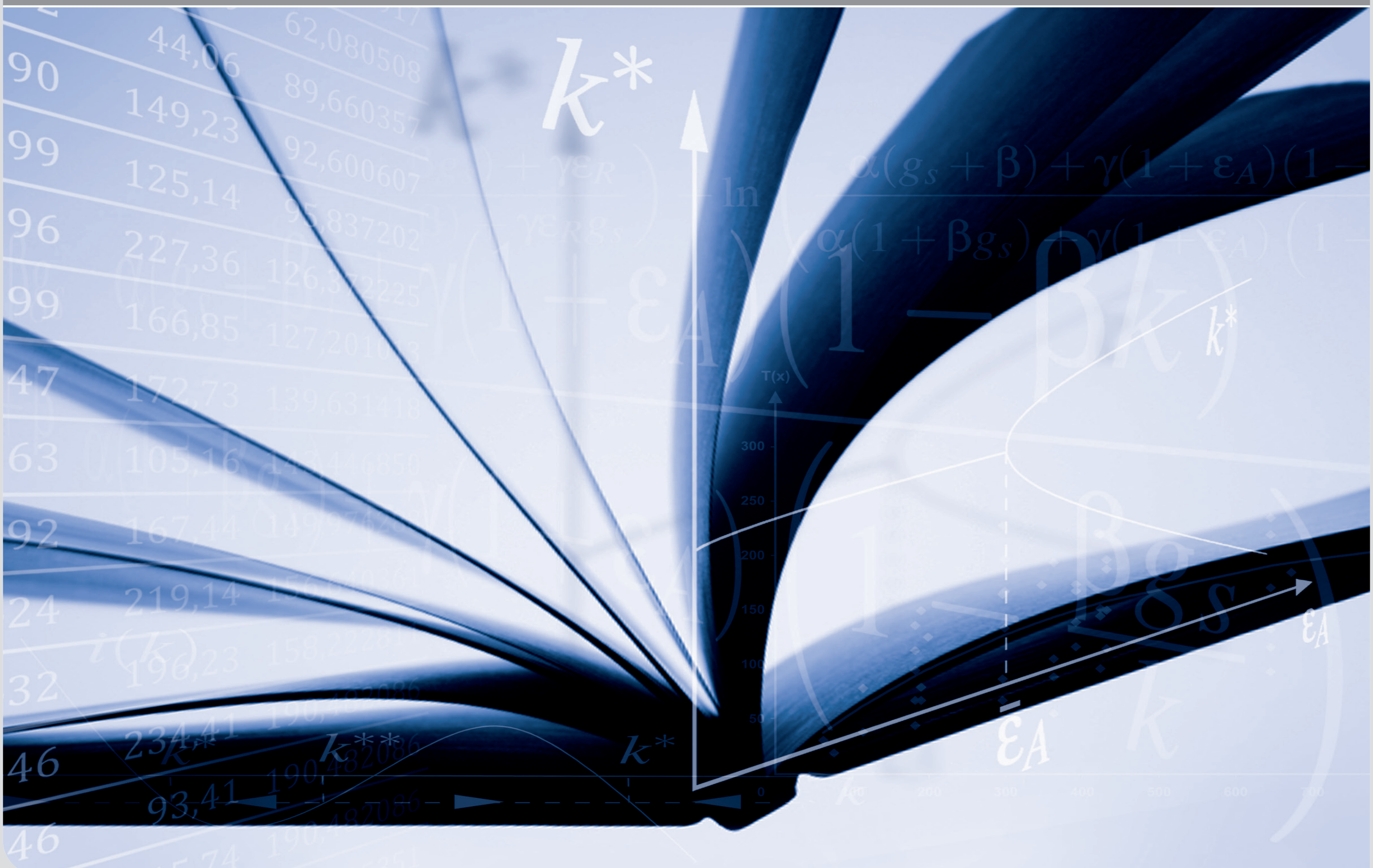
No. 113 | JANUARY 2018

WORKING PAPER SERIES IN ECONOMICS



Impressum

Karlsruher Institut für Technologie (KIT)
Fakultät für Wirtschaftswissenschaften
Institut für Volkswirtschaftslehre (ECON)

Kaiserstraße 12
76131 Karlsruhe

KIT – Die Forschungsuniversität in der Helmholtz-Gemeinschaft

Working Paper Series in Economics
No. 113, January 2018

ISSN 2190-9806

econpapers.wiwi.kit.edu

Dilution Effects, Population Growth and Economic Growth under Human Capital Accumulation and Endogenous Technological Change

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Abstract

This paper answers the following two questions: 1) In the data, can we find a dilution effect of population growth also on per-capita human capital investment? If yes, 2) how can we use this fact to explain theoretically the existence of a differential impact of population change on economic growth across countries? In the first part of the article we document empirically the considerable across-countries heterogeneity of a dilution effect of population growth also in regard to the process of per-capita human capital formation and observe that, at a country's level, population growth may be relevant (either positively or negatively) for economic growth depending on the specific way it affects the process of schooling-acquisition by agents. In the second part of the paper we use these results in order to build a multi-sector growth model which is capable of accounting (depending on the strength of the found dilution effect of population growth on per-capita human capital formation) for the non-monotonous correlation between demographic and economic growth rates in the long-run.

Keywords: Human Capital Investment, Economic Growth, Population Growth, Dilution Effects, Research & Development

JEL classification: J10, J24, O33, O41

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1. Introduction

Because the existence of a causal relationship and, eventually, the sign of such a relation are still controversial issues in the literature, it remains very important (not only for demographers and economists, but also for policy-makers) to investigate the impact that population growth may have on long-term economic growth (*i.e.*, the growth rate of real per-capita income). Many studies (Solow, 1956; Coale and Hoover, 1958; Ehrlich, 1968; Li and Zhang, 2007; Herzer et al., 2012, just to mention a few) find a negative influence of population growth on economic growth. In exogenous growth models, for example, this result is ultimately explained by the so-called physical capital dilution effect of population growth: an increase in population, by diluting the stock of physical capital held by each individual, lowers *ceteris paribus* the long-run (or steady-state) level and the short-run (or transitional) growth rate of physical capital per capita. Other studies (Kuznets, 1967; Boserup, 1981; Simon, 1981; Romer, 1987 and 1990; Kremer, 1993; Jones, 1995), instead, conclude that, once endogenous technological change is explicitly taken into account, the impact of population growth on economic growth is definitely positive as larger populations can stimulate the advancement of technical progress, are able to enjoy greater economies of scale, and are likely to have a greater number of geniuses. Finally, there are also strong arguments advocating that it is a better economic performance (*i.e.*, a higher economic growth rate) to cause an increased population growth rate.[1] Blanchet (1988) is among those who years ago have already used this claim to explain the presence of an insignificant correlation between economic and population growth rates.

In the light of all this, it seems that the main conclusion coming from the path-breaking paper by Kelley and Schmidt (1995, p. 554) still continues to hold today: "*...This analysis...provides a more balanced perspective on the consequences of demographic change because it rests on the proposition that population growth has both negative and positive effects... As this paper demonstrates, population growth is not all good or all bad for economic growth: it contains both elements, which can and...do change over time*".

The present paper is motivated by empirical and theoretical reasons. More concretely, the two questions that our work tries to answer are, respectively, the following: In the data, is the so-called dilution effect of population growth (that the literature often relates solely to the process of per-capita physical capital accumulation) capable of influencing also the evolution over time of per-capita human capital? If yes, how could we use this information to explain theoretically the existence of a differential cross-country impact of population change on economic growth?

In the first part of our paper we investigate and document empirically the existence (and the considerable across-countries heterogeneity) of a dilution effect of population growth also in regard to the process of per-capita human capital formation. More importantly, we also observe that, at a country's level, population growth may be relevant (either positively or negatively) for economic growth depending on the specific way it (population growth) affects the process of schooling-acquisition by agents.

As a consequence of these empirical results, in the second part of the paper we turn to the theory and propose a generalization of the very well-known Uzawa (1965)'s and Lucas (1988)'s models. Our contribution introduces two important differences with respect to Uzawa (1965) and Lucas (1988). First of all, we assume that firms

[1] See Chang et al. (2014) for a recent empirical analysis that confirms this result.

may undertake horizontal R&D activity. This means that disembodied technological progress (intended as the growth rate of the available varieties of intermediate inputs) is endogenous in our economy, as in the path-breaking papers by Romer (1990) and Jones (1995). Moreover, and inspired by our empirical findings, we also postulate that individual human capital investment is subject to some sort of dilution effect related to an increase in population. According to this effect, since newborns enter the world completely uneducated, they reduce the stock of human capital per-capita available in the population. Hence, population growth operates like a depreciation of human capital per capita and, thus, contributes to slow down human capital accumulation at an individual's level. In the original Lucas (1988, Eq. 13, p. 19)'s model such a dilution effect in per-capita human capital investment is totally missing, based on the belief that skill acquisition is a sort of "*social activity*" and, as such, it is completely different from physical capital investment.[2]

Our extended model shows that the magnitude of the dilution effect of population change on per-capita human capital investment can be used as an argument to explain the differential (*i.e.*, positive, negative, or neutral) impact that population growth may have on long-run economic growth across countries. In other words, our theoretical model can be employed to propose a new, alternative explanation as to the non-uniform effect of population growth on long-run per capita income growth. This new explanation is based on the recognition that population growth brings about two simultaneous, but different (in sign and size), effects on economic growth. The first is precisely the already-mentioned dilution effect. This effect is always negative because when newborns enter the world they reduce the existing per-capita stock of any reproducible factor-input (human capital in our case). So, in order to equip every single member of the growing population with an even amount of such input, some further resources need to be explicitly devoted to this aim (as opposed to other, alternative/more productive uses), which slows productivity growth down. The second effect, instead, describes the positive impact that population growth may have on the economy's growth rate of ideas,[3] and hence on per-capita income growth. What our theoretical model ultimately suggests is then a mechanism such that there exists a country-specific threshold level in the magnitude of the per-capita human capital accumulation dilution effect: for those countries in which the dilution effect of population growth is below (above) the threshold we should observe a positive (negative) impact of population change on long-run economic growth.

Although one contribution of the present paper is mainly empirical,[4] we believe that the theoretical model we present in the second part of this article has features that make

[2] "*...To obtain (13) for a family, one needs to assume both that each individual's capital follows this equation and that the initial level each new member begins with is proportional to (not equal to!) the level already attained by older members of the family. This is simply one instance of a general fact that I will emphasize again and again: that human capital accumulation is a social activity, involving groups of people in a way that has no counterpart in the accumulation of physical capital...*"(Lucas, 1988, p. 19).

[3] "*...More people means more Isaac Newtons and therefore more ideas*" (Jones, 2003, p. 505).

[4] As far as we know, this is the first paper that attempts at estimating a dilution effect of population growth on per-capita human capital formation, irrespective of the sources of the demographic change. Indeed, Boikos et al. (2013) estimate the effect that a change in the birth rate (as opposed to the whole population growth rate) has on per-capita human capital investment. Moreover, in their model Boikos et al. (2013) consider an economy where people do not save (aggregate output equals aggregate consumption at all times), there is no R&D activity (and, hence, no endogenous disembodied technological change), and in which the aggregate production function is linear in the only input employed, *i.e.* human capital.

it new and original within the existing literature. Unlike the canonical Romer (1990)'s and Jones (1995)' settings, we consider the possibility that the intermediate sector uses human capital as an input. Hence, in our model human capital is employed to produce new human capital, final and intermediate goods, and to invent new ideas. Despite the fact that in Mierau and Turnovsky (2014) — who use a one-sector endogenous growth framework *à la* Romer (1986) — the link between the whole population growth rate and the economic growth rate is monotonic, their model is still able to account for a hump-shaped relation between the two variables since population growth can be explained by either an increase in fertility or a reduction in mortality. Thus, in Mierau and Turnovsky (2014) the way in which population growth occurs (*i.e.*, through an increase in the birth rate or a decrease in the mortality rate) is important in making the relationship between demographic change and economic growth ultimately non-uniform in sign. Our paper differs from Mierau and Turnovsky (2014) in that we consider a multi-sector growth model with R&D activity and human capital accumulation. Moreover, we do not split the population growth rate into its birth- and mortality-rate components because we are interested in uncovering another potential source of non-monotonicity in the overall relationship between demographic change and economic growth. Using an endogenous growth model with human capital investment and two types of R&D activities (horizontal and vertical, respectively), Strulik (2005) concludes that the sign of the correlation between population growth and the growth rate of per capita income crucially depends on the form of households' preferences ('Millian', 'Benthamite', or an intermediate type between the two, respectively). By considering an economy where R&D activity is purely horizontal, Bucci (2008), instead, postulates that the investment in skill-acquisition by agents is directly influenced by technological progress. In his framework, the sign of the long-run correlation between population growth and economic growth is ultimately found to depend not only on the form of households' preferences, but also on the nature of technical change (whether 'skill-biased', 'eroding', or 'neutral'). Unlike Bucci (2013), it is not an objective of this article to highlight the role of the so-called 'returns-to-specialization' in shaping the link between population growth and economic growth. Moreover, contrary to Bucci (2008, 2013) and Strulik (2005), we pay no attention here to how the type of households' preferences might contribute to affect the correlation between population and economic growth rates. Finally, differently from Bucci (2015), in the present article we are not interested in highlighting the possible tension between *productivity-gains* (due to specialization) and *productivity-losses* (due to production-complexity) that arises from an expansion of input-variety as a major determinant of the sign of the long-run relation between economic and population growth rates. Instead, as mentioned above, we are interested here in emphasizing a totally new mechanism (based on the role played by the dilution effect of population change on human capital formation by agents) through which the relation between population and economic growth rates might be non-uniform in sign in the long-term. In a recent paper, Prettner (2014) has proposed another way through which a higher population growth can have a non-monotonous effect on economic growth within a Romer (1990)'s—Jones (1995)'s economy augmented with an education sector: in his model, an increase in population growth, while positively influencing aggregate human capital accumulation, decreases simultaneously schooling intensity (defined as the productivity of teachers times the resources spent on educating each child). The fall of schooling intensity has, in turn, a negative impact on the future evolution of aggregate human capital. Depending on whether this negative effect of population growth dominates on the positive one or not, we can ultimately observe a non-uniform impact of population change on economic growth in the long-run. Prettner (2014), however, is not interested in emphasizing the role of the dilution effect in per capita human capital investment, which is instead the main focus of our article.

The remainder of the paper is organized as follows. Section 2 empirically estimates the effects of population growth on economic growth through individuals' human capital formation. Section 3 lays out the theoretical model, and Section 4 analyzes its predictions along a Balanced Growth Path (BGP henceforth) equilibrium. Section 5 focuses on the correlation between population growth and per-capita income growth in the long-run, and explains the possibly non-monotonous sign of this correlation in terms of the extent of a dilution effect of population growth on per-capita human capital accumulation. The last Section concludes and provides grounds for possible future extensions.

2. Population growth, human capital formation and economic growth: An empirical investigation

This section consists of two parts. The first part investigates the relationship between population growth and human capital formation. The second part considers the impact of both population growth and human capital on long-run economic growth. In order to capture all these dimensions, we use population data from the United Nations (2015), the schooling attainment data of Barro and Lee (2013), and GDP data from the Penn World Tables (Feenstra et al., 2013).

Our sample excludes countries from East Europe, Central Asia, and Germany because of their transition from non-market to market economies during the 1980s and 1990s, and we exclude countries for which we have missing information from 1960 onwards. Furthermore, five outliers in terms of schooling are excluded from the sample. These restrictions leave us with 92 countries across five world regions.[5]

Population growth in a given year is measured as the natural population growth rate, i.e. crude birth rate minus crude death rate in that year, not taking into account the migration rate. Migration effects on human capital and economic growth are thus ignored in the analysis. Following the literature, we measure human capital in terms of (logged) years of schooling.[6] Economic wealth is measured in terms of (logged) output-side real GDP at chained PPPs. In the schooling regression we normalize GDP by the size of the population aged above twenty in order to separate this measure from the population growth rate, while in the long-run economic growth regression we normalize by the total size of the population to arrive at GDP per capita. Long-run economic growth is the difference of log GDP per capita over the time period.

In a first step, we investigate how population growth relates to years of schooling. The prior literature generally suggests that the impact of population growth on schooling may greatly vary across economies and over time (Bloom et al., 2000; Bloom

[5] See appendix A1 for the list of countries and descriptive statistics by region and year.

[6] The advantage of years of schooling is that it is relatively easy to measure and it is a clear policy variable. On the other hand, schooling as a measure of human capital has been increasingly criticized as it is not a direct measure of human capital but rather one of several inputs to human capital formation, informing on quantity and not on quality of schooling. There have been various attempts to overcome these limitations, notably considering students' cognitive skills as evaluated in international tests (e.g. Hanushek and Woessmann, 2008). This approach however has its own drawbacks. It considers only children actually being in school, results in very broad skill distributions that are relatively unstable over time within countries, and are difficult to compare across countries.

et al., 2010; Kelley, 1994; Montgomery et al., 2000). More specifically, Boikos et al. (2013) investigate the effect of fertility rates on schooling and find an inverse-U-shaped relationship. Furthermore, population growth realized through higher life expectancy is likely to have a positive impact on educational attainment (see e.g. Cervellati and Sunde, 2013). In general, population growth can be expected to be more relevant in resource-constrained environments where the schooling decision depends stronger on the (changes of) resources available per child (Kelley, 1994). In addition, incentives for education depend on the economic structure of the economy. It has been argued for example that in South Korea, during the 1980s, population growth through larger cohorts of the younger generation resulted in an increase of secondary and tertiary education as a rational response to increased competition for jobs in the growing high-tech industry (Kim, 2001). On the other hand, population growth in China during the 1980s did not result in comparable schooling because of severe resource constraints of the households in combination with a strong expansion of the low-skilled manufacturing sector (Li et al., 2017; Liu et al., 2009). Finally, population growth effects probably depend on education policy. Consider for example Kelley (1994)'s argument that population growth may impact schooling attainment even positively in cases where it increases the pressure on policy to implement appropriate education reforms towards higher efficiency in the education sector.

The schooling regression follows a production logic, where the schooling of the younger cohort is a function of the schooling of the adults, the GDP available to the adults, and population growth. The dependent variable is measured at the year when the younger cohort reached age 25 to 29 years, when schooling is mostly over. All independent variables are measured however twenty years earlier, when the cohort of the young has been 5 to 9 years old — at a time or before when most schooling decisions are actually made. We estimate one regression over consecutive five-year periods starting from 1960 until 1990. This time period is due to the gap of 20 years between the time when the education decision was made, e.g. 1990, and the time where we observe the final outcome of that decision, e.g. 2010.

The structure of the model is as follows:

$$s_{i,t}^{y} = c_i + \beta_i \mathbf{x_{i,t}} + \tau_t + \epsilon_{i,t} \quad \text{with } c_i = \delta \mathbf{x_{i,0}} + u_i$$
$$\begin{pmatrix} u_i \\ \beta_i \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \bar{\beta} \end{pmatrix}, \mathbf{\Omega} \right)$$

In the first equation, schooling of the young, $s_{i,t}^{y}$ depends on an intercept, c_i, various time-varying and constant regressors $\mathbf{x_{i,t}}$, time dummies τ_t and a random error term $\epsilon_{i,t}$. The model allows for country specific intercept c_i and slope β_i.

We may expect that c_i correlates with $\mathbf{x_{i,t}}$. The second equation implements a correlated random coefficients approach as it explicitly models the correlation between c_i and the country's initial conditions $\mathbf{x_{i,0}}$. The time constant, country-specific error term u_i can then be assumed to be uncorrelated with the right-hand side variables of the first equation.[7] In the estimation, the second equation is simply plugged into the

[7] The more common approach of controlling through averages of the regressors (rather than initial values) in our case provides very similar results but is less appropriate. First, it is reasonable that regressors are weakly and not strongly independent and, second, schooling of the adults is a stock built through schooling of the young. Both issues introduce (some) correlation between regressor averages and the error term $\epsilon_{i,t}$. Estimation in first differences is an alternative that suffers from the same

first equation - yielding a simple linear regression with additional country averages as controls (Hsiao, 2003, pp.44).

Heterogeneous slopes β_i can not be identified for each country individually. However, a random effects approach allows to obtain an estimate of the distribution of β_i under the assumption that $\epsilon_{i,t}$, u_i, and β_i are independent from regressors and follow a Normal distribution. The corresponding mean, i.e. average fixed effects $\bar{\beta}$, and covariance matrix, $\mathbf{\Omega}$, are then freely estimated.[8]

As we measure independent variables before the outcome realizes reverse causality — better educated have less kids — is not an issue. One may argue however that forward-looking, rational parents decide simultaneously on the number of children and associated schooling investments (Becker and Lewis, 1973). Typically it is assumed that such a decision involves a trade-off leading to more (fewer) children with lower (higher) education. Our estimation approach results into stronger (negative) population growth effects in economies where these considerations are more relevant. We further note that estimates of population growth coefficients remain subject to an omitted variables bias: Population growth is one of several dimensions reflecting the socio-economic state of an economy and hence is susceptible of taking over explanatory power from related but different factors that are not included in the regression. Consequently, we rather speak of correlations than causal effects in the following.

Table 1 presents the results of our schooling regressions. Model (1) presents first-difference panel estimates of our main variables as a base line model. Model (2) provides estimates implementing the correlated random coefficients approach in combination with a linear mixed model. Model (1) and (2) are two alternative approaches of dealing with country-specific (fixed) effects. Only the intercept and R^2 change considerably as we go from the estimation in differences (Model 1) to estimation in levels (Model 2). Despite that coefficient estimates are very similar in both models. Note that all factors enter with a curvilinear slope. In particular population growth follows an inverse-U-shape with an optimal point slightly above two percent population growth, which is consistent with (Boikos et al., 2013).

Model (3) allows for random country-specific coefficients of schooling and GDP of adults. Moving from Model (2) to (3) improves model fit significantly according to the likelihood ratio test, and while the marginal R^2 (based on fixed effects only) decreases, the conditional R^2 (based on fixed and random effects) increases. Population growth coefficient estimates remain very stable but become somewhat more precise.

The final model, Model (4), includes country-specific population growth effects. We note that coefficients do not change much compared to their standard deviations. Yet, the likelihood ratio test strongly rejects equivalence of Model (4) and Model (3) in terms of data fit. Similarly, conditional R^2, taking into account estimated random intercept and slopes increases. These results strongly suggest considerable heterogeneity across countries in terms of the effect of population growth on schooling.

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issues. Furthermore, differencing reduces the signal-to-noise ratio considerably in case measurement errors are uncorrelated over time, which is likely to be the case in our data (Cohen and Soto, 2007). For comparison, we provide nevertheless first-difference results in the main regression table.

[8]We also estimated simple OLS regressions in which we allow the population growth effect to vary over world regions (instead of individual countries) and obtained results that are consistent with the ones presented here.

Table 1: Estimates of the impact of population growth on schooling of the young from 1960 to 1990.

	(1)	(2)	(3)	(4)
Common effects				
Intercept	0.072*** (0.007)	0.33* (0.133)	0.319* (0.155)	0.376* (0.167)
Schooling adults	1.71*** (0.405)	1.764*** (0.288)	2.168*** (0.329)	2.007*** (0.353)
(Schooling adults)^2	-0.729*** (0.172)	-0.661*** (0.123)	-0.849*** (0.139)	-0.776*** (0.149)
GDP p. adult	0.138** (0.052)	0.266*** (0.043)	0.304*** (0.052)	0.288*** (0.054)
(GDP p. adult)^2	-0.036** (0.011)	-0.058*** (0.01)	-0.059*** (0.011)	-0.054*** (0.011)
Population growth	0.093* (0.038)	0.094** (0.035)	0.107*** (0.029)	0.124*** (0.03)
(Population growth)^2	-0.017 (0.009)	-0.022** (0.007)	-0.024*** (0.007)	-0.028*** (0.008)
Correlated random intercept controls				
Initial schooling	—	0.328*** (0.057)	0.189*** (0.044)	0.202*** (0.042)
Initial GDP 1960	—	-0.002 (0.028)	0.03 (0.022)	0.023 (0.021)
Initial Population growth 1960	—	-0.021 (0.025)	-0.021 (0.018)	-0.027 (0.018)
Random coefficients				
$\mathbf{\Omega}$ (Intercept, Schooling adults, Log GDP p. adult, Population growth, (Population growth)^2)	—	$\begin{pmatrix} 0.12 \\ - & - \\ - & - & - \\ - & - & - & - \\ - & - & - & - & - \end{pmatrix}$	$\begin{pmatrix} 0.43 \\ -0.77 & 0.31 \\ 0.14 & -0.74 & 0.13 \\ - & - & - & - \\ - & - & - & - & - \end{pmatrix}$	$\begin{pmatrix} 0.56 \\ -0.76 & 0.32 \\ -0.08 & -0.58 & 0.13 \\ 0.03 & -0.15 & 0.23 & 0.06 \\ -0.38 & 0.29 & -0.02 & -0.88 & 0.03 \end{pmatrix}$
log Likelihood	—	458.78	573.31	606.17
LLR test			χ^2 =229.06, DF=5, p-val=< 0.01%	χ^2 =65.736, DF=9, p-val=< 0.01%
Marginal/Conditional R^2	0.158/—	0.852/0.936	0.777/0.964	0.764/0.973

Notes: All regressions are on 92 observations and include time and "world region" dummies. Model (1) is on first-differences, Models (2) to (4) on levels. Likelihood ratio (LLR) test statistics for equivalence of Model (2) and Model (3), and Model (3) and Model (4) respectively. One star denotes the 5%, two stars 1%, and three stars 0.1% confidence levels. Standard errors in brackets.

Figure 1 plots observed population growth rates, g, against expected country-specific effects of population growth on schooling, i.e. expectations of $\left(\frac{\partial s^y}{\partial g}\right)$ conditional on the estimated distribution. The figure shows the additional insight gained from allowing heterogeneity across countries in the effect of population growth on schooling.

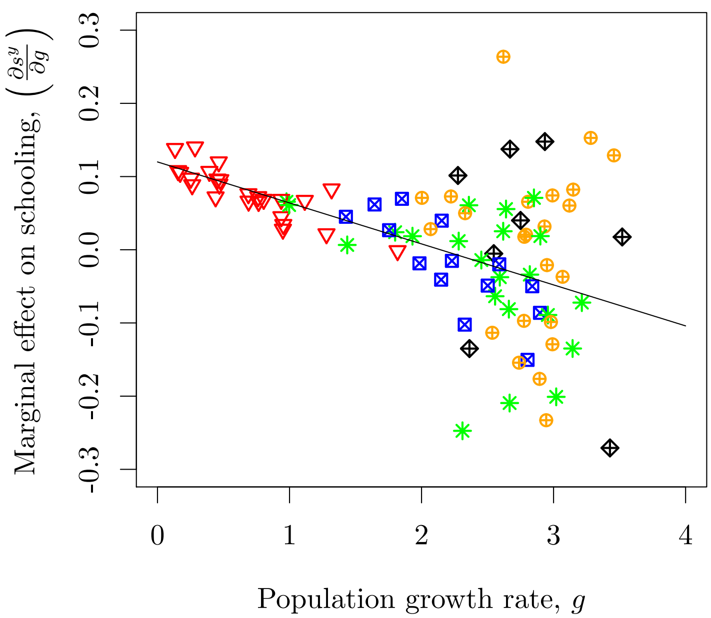

Figure 1: Population growth rate, g, against its marginal effect on schooling, $\frac{\partial s^y}{\partial g}$, taking into account fixed-common effects and random-individual effects (both averaged over years for each country). Advanced economies in red, Asia in blue, Latin America and Caribbean in green, Middle East and North Africa in black, Sub-Saharan Africa in orange. The black line marks the estimated marginal effect based on fixed, common effects only.

First, the black line provides estimated marginal effects without heterogeneity, i.e. when the effect is allowed to vary only with population growth. The estimated inverse-U-shaped relationship implies decreasing marginal effects as population growth increases. Noticing that countries with low population growth are often more advanced countries, this pattern is in line with the argument that population growth has a particularly negative effect on human capital formation in developing, resource-constrained economies. This finding is consistent with (Boikos et al., 2013).

Second, turning to country-specific marginal effects, we see that estimated marginal effects of advanced economies (red triangles) are very close to the common effects estimates (black line). However, for developing countries with higher population growth rate country-specific effects are widely dispersed around the common effects estimate.

Keeping population growth fixed, say at about 3 percent, some countries are expected to experience a negative impact of population growth on schooling (such as the African country in orange with $\frac{\partial s^y}{\partial g} = -0.2$) while others are at the other end of the spectrum (see the Middle East country in black at $\frac{\partial s^y}{\partial g} = 0.2$). Furthermore, the spread is considerable within 'world regions' for countries experiencing the same population growth rate. The observed heterogeneity among even otherwise 'similar' countries may well explain the ambiguity in the prior empirical literature regarding the dilution effect of population growth on human capital formation.

What is then the implication for economic growth? The second part of the analysis provides some intuition based on a regression of long-run economic growth on population growth and years of schooling. We consider the period 1980 to 2010 — a period of accelerated skill biased technological change (Acemoglu, 2002) — where we expect human capital to be a relevant mediator of the population-economic growth relationship. The idea is that population growth before the regression period influences schooling of the young, which in turn affects economic growth once they enter the labor force during the regression period. Following Benhabib and Spiegel (1994) we consider the average level of human capital as a driver of growth in a catching-up process. The regression is thus on initial GDP per capita (logged) at 1980, average years of schooling in logs over the regression period, and (natural) population growth rate in the prior period (1960 to 1980).

Table 2: Long-run economic growth regression (1980-2010).

	(1)	(2)	(3)
Intercept	1.448*** (0.285)	0.438 (0.336)	-0.163 (0.43)
GDP p.c. (1980)	-0.203** (0.085)	-0.407*** (0.083)	-0.41*** (0.082)
Avg. population growth	-0.208** (0.09)	-0.237*** (0.083)	-0.234*** (0.084)
Schooling (1980)		0.727*** (0.159)	
Avg. schooling			0.943*** (0.2)
Asia	0.333 (0.205)	0.434** (0.204)	0.426** (0.2)
Latin America	0.028 (0.184)	0.133 (0.182)	0.135 (0.183)
Middle East / N. Africa	0.454* (0.258)	0.792*** (0.241)	0.759*** (0.238)
S. Africa	-0.596** (0.247)	-0.324 (0.237)	-0.33 (0.236)
Observations	92	92	92
R^2	0.457	0.568	0.572

Notes: Avg. population growth is the average over the period 1960 to 1980. Schooling refers to average years of schooling of the population in logs. Standard errors in brackets, always robust to heteroskedasticity and clustering within "world regions". Model (1) and (2) estimated with OLS. Model (3) is a 2SLS regression with average schooling, i.e. log of schooling over the period 1980 to 2010, instrumented by schooling of the population in 1980, and schooling of the young (the dependent variable in the schooling regression) of the periods 1960 and 1965. F statistic of the first stage regression is 1108.93 and the test of weak Instruments rejects at a significance level below 0.01%. Sargan test p-value of 0.28, and Wu-Hausman test p-value of 0.42.

Table 2 provides results of three regressions. Model (1) is without any schooling variable. We obtain a negative population growth effect that is significant below a 5% level. Model (2) increases initial schooling which increases the negative effect of avg. population growth and the estimate becomes more precise, but not much compared to its standard error. GDP and regional dummy estimates tend to be more affected. Finally, Model (3) includes (instrumented) average years of schooling. Initial schooling in Model (2) is substantial and highly significant, and we obtain an elasticity of one for schooling in the final, our preferred, model. The coefficient of average population growth remains very stable over all three models.

After controlling for schooling, we consider the negative coefficient of population growth to be net of schooling, summarizing various other pathways. Note that the effect of introducing schooling in the regression on our population growth rate estimate is rather small. But this can be expected given the results of the first regression that, in average, population growth has a negligible impact on schooling of the young. Yet, the schooling regression lets us expect considerable heterogeneity of countries in their response on population growth.

In order to get some intuition, let us take the estimates at face value for a moment.[9] Consider again Figure 1. Pick the Latin American country in green with population growth rate of three percent per year and a marginal effect of minus 0.2.

Assuming the country has an average schooling of the young of Latin American countries in 1980, a one percent increase of that country's population growth would decrease years of schooling of the young from about six years to about five years. To simplify, assume that this effect occurs over all cohorts such that average years of schooling of the population in subsequent decades shifts accordingly.[10] With an elasticity of schooling of about one in the economic growth regression, the dilution effect in schooling simply adds to the negative impact of population growth on economic growth (i.e. $\partial\Delta GDP/\partial g \approx 1\ \partial s/\partial g - 0.23 = -0.2 - 0.23 = -0.43$). This implies a growth from about 5,000 US Dollars (PPP fixed at 2005) in 1980 to only 6,000 US Dollars in 2010 instead of the average 9,000 US Dollar observed. Other Latin American countries may be less affected by population growth. In Figure 1, for the Latin American country with the highest marginal effect, the net effect of population growth on economic growth would be rather small as the positive effect on schooling, about plus 0.1, improves on the negative population growth effect on economic growth, about minus 0.25. Thus, regressions suggest a threshold of the country-specific population growth effect that separates countries for which the net effect of population growth is negative from those experiencing a positive population growth effect on the economy.

The remainder of the paper provides a theoretical explanation for the emergence of such a threshold.

3. The model

In this section we present a multi-sector growth model based on expanding variety of intermediate inputs and human capital accumulation. In more detail we consider a closed economy where firms perform, among others, R&D activity aimed at discovering new ideas for new varieties of durables and individuals allocate their own time-endowment between working and education (human capital formation) activities. In this economy population grows at an exogenous rate, and there is no inbound or outbound migration of people.

Based on the empirical results found and discussed in the previous section, the key

[9] Our initial econometric study is of course not without limitations. On the one hand, the thin data basis and the complex economic development process makes it difficult to pin down causal effects. On the other hand, we would expect heterogeneity in all aspects of the schooling and economic growth process; notably the impact of education on schooling and the "remainder effect" of population growth on economic growth.

[10] The impact of the schooling of one cohort on subsequent schooling of the population is easily calculated in a mechanistic cohort model, but simplifying at this point does not invalidate the argument.

ingredient of our theoretical analysis resides in the presence of a dilution effect of the whole population growth rate on per-capita human capital investment. Such effect is not present in the original Uzawa (1965)'s and Lucas (1988)'s models of human capital accumulation.[11]

3.1 Production

Consider an environment in which three sectors of activity are vertically integrated. The research sector is characterized by free entry. Here, firms combine human capital and the existing stock of ideas to engage in innovative activity that results in the invention of new blueprints for firms operating in the intermediate sector. The intermediate sector is composed of monopolistic competitive firms. There is a distinct firm producing each single variety of intermediates/ durables and holding a perpetual monopoly power over its sale. In the competitive final output sector, atomistic firms produce a homogeneous consumption/ final good by employing human capital and all the available varieties of intermediate inputs. The representative firm producing final output has the following technology:[12]

$$Y_t = n_t^{\overline{\alpha}} H_{Y_t}^{1-Z} \int_0^{n_t} (x_{it})^Z di, \quad \overline{\alpha} \geq 0, \quad 0 < Z < 1 \tag{1}$$

In Eq. (1) Y denotes the total production of the homogeneous final good (the *numeraire* in the model), x_i and H_Y are the quantities of the i-th intermediate and human capital input employed in the sector, respectively. The number of ideas existing at a certain point in time (n_t) coincides with the number of intermediate-input varieties and represents the actual stock of non-rival knowledge capital available in the economy. We assume that having a larger number of available intermediate input varieties does not imply any detrimental effect on aggregate production.[13] As a whole, the aggregate production function (1) displays constant returns to scale to the two private and rival factor-inputs (H_Y and x_i), with $1 - Z$ and Z corresponding to their shares in GDP.[14] Since $Z \in (0; 1)$, final output production takes place by using simultaneously human capital and intermediates as inputs.

The inverse demand function for the i-th intermediate reads as:

$$p_{it} = Zn^{\overline{\alpha}} H_{Y_t}^{1-Z} (x_{it})^{Z-1} \tag{2}$$

The demand for the i-th durable has price elasticity (in absolute value) equal to $1/(1-Z) > 1$, which coincides with the elasticity of substitution between any two generic varieties of capital goods in the final output production.

In the intermediate sector, firms engage in monopolistic competition. Each of them produces one (and only one) horizontally differentiated durable and must purchase

[11] In the theoretical model we focus explicitly on the negative impact of population growth on human capital creation because our empirical results suggest that country specific dilution effects are particularly relevant for developing economies experiencing mostly a negative effect of population growth on schooling. We verified that in our model, the case of a positive effect of population growth on human capital always results in a positive effect on economic growth as well.

[12] We follow Ethier (1982) and Romer (1987, 1990).

[13] Notice that when $\overline{\alpha} > 0$, the higher n and the higher the productivity with which human capital and all the different varieties of intermediate inputs are combined in the manufacturing process.

[14] Since final output is produced competitively under constant returns to scale to rival inputs, at equilibrium H_Y and x_i are rewarded according to their own marginal products. Hence, $(1 - Z)$ is the share of Y going to human capital, and Z is that accruing to intermediate inputs.

a patented design before producing its own output. Thus, the price of the patent represents a fixed entry cost. Following Grossman and Helpman (1993, Chap. 3), we assume that local monopolists have access to the same one-to-one technology:

$$x_{it} = h_{it}, \quad \forall i \in [0; n_t], \quad n_t \in [0; \infty) \tag{3}$$

where h_i is the amount of skilled labor (human capital) required in the production of the i-th durable, whose output is x_i. For given n, Eq. (3) implies that the total amount of human capital used in the intermediate sector at time t (H_{It}) is:

$$\int_0^{n_t} (x_{it})di = \int_0^{n_t} (h_{it})di \equiv H_{It} \tag{4}$$

Making use of Eq. (2), maximization of the generic i-th firm's instantaneous flow of profits leads to the usual *constant markup* rule:

$$p_{it} = \frac{1}{Z} w_{It} = \frac{1}{Z} w_t = p_t, \quad \forall i \in [0; n_t], \quad n_t \in [0; \infty) \tag{5}$$

Eq. (5) says that the price is the same for all intermediate goods i and is equal to a constant markup $(1/Z) > 1$ over the marginal cost of production (w_I). In a moment it will be explained that in this economy the whole available stock of human capital (H) is employed and spread across production of final goods (H_Y), durables (H_I), and new ideas (H_n). Since it is assumed to be perfectly mobile across sectors, at equilibrium human capital will be rewarded according to the same wage rate $w_{Yt} = w_{It} = w_{nt} \equiv w_t$, with w_I denoting the wage paid to any generic unit of human capital employed in the intermediate sector. Under the hypothesis of symmetry, *i.e.*, p and x being equal across all the exisgting varieties of intermediates, it is immediate to conclude that:

$$x_{it} = H_{It}/n_t = x_t, \qquad \forall i \in [0; n_t] \tag{4'}$$

$$\pi_{it} = [Z(1-Z)H_{Yt}^{1-Z} H_{It}^{Z}] n_t^{\overline{\alpha}-Z} = \pi_t, \qquad \forall i \in [0; n_t] \tag{6}$$

Thus, each intermediate firm will decide at any time t to produce the same quantity of output (x) to sell it at the same price (p), accruing the same instantaneous profit (π). The symmetry across durables is a direct consequence of the fact that each intermediate firm uses the same production technology (3) and faces the same demand function (see 2 and 5). Notice that $Z \in (0; 1)$ in our framework and that π_t would have been equal to zero if Z had been equal to one (instantaneous profits are zero in a perfectly-competitive market). Under symmetry, Eq. (1) can be recast as:

$$Y_t = (H_{Yt}^{1-Z} H_{It}^{Z}) n_t^{R}, \quad R \equiv \overline{\alpha} + 1 - Z > 0 \tag{1'}$$

where R measures the degree of returns to specialization, that is "*the degree to which society benefits from 'specializing' production between a larger number of intermediates*" Benassy (1998, p. 63). In the present paper, it is immediate to verify that R is always positive. The hypothesis $R > 0$ implies that the impact on aggregate GDP (Y) of having a larger number of intermediate input varieties is always positive for any $H_I > 0$ and $H_Y > 0$. According to Eq. (1′), the aggregate production function exhibits constant returns to H_Y and H_I together, but either increasing ($R > 1$), or decreasing ($0 < R < 1$), or else constant ($R = 1$) returns to an expansion of variety, while holding the quantity employed of each other input fixed. With respect to other settings, this article introduces important novelties. Unlike Devereux et al. (1996a, 1996b, 2000) where —

if all intermediates are hired in the same quantity x — the returns to specialization are either unambiguously increasing[15] or at most constant,[16] we allow for the possibility that the returns to specialization might also be decreasing. Unlike Bucci (2013), we explicitly rule out the possibility that the returns to specialization R are negative.[17]

3.2 Research and development

There is a large number of small competitive firms undertaking R&D activity. These firms produce ideas indexed by zero through an upper bound $n \geq 0$. Ideas take the form of new varieties of intermediate inputs that are used in the production of final output. With access to the same stock of knowledge, n, a representative research firm uses only human capital to develop new ideas:

$$\dot{n}_t = \psi_t H_{nt}, \quad n(0) > 0 \tag{7}$$

In Eq. (7) H_n is the number of people attempting to discover new ideas, and ψ is the rate at which a single researcher can generate a new idea. Since the representative R&D firm is small enough with respect to the whole sector, it takes ψ as given. Hence, Eq. (7) suggests that R&D activity is conducted under constant returns to scale to the human capital input (H_n). We postulate that the arrival rate of a new idea ψ has the following specification:

$$\psi_t = \frac{1}{\chi}\frac{H_{nt}^{\mu-1}}{H_t^{\Phi}} n_t^{\eta}, \quad \chi > 0, \quad \mu > 0, \quad \Phi \gtreqless 0, \quad \eta < 1 \tag{7'}$$

Using together (7) and (7′), the R&D technology (the production-function of new ideas) finally reads as:

$$\dot{n}_t = \frac{1}{\chi}\frac{H_{nt}^{\mu}}{H_t^{\Phi}} n_t^{\eta}, \quad n(0) > 0, \quad \chi > 0, \quad \mu > 0, \quad \Phi \gtreqless 0, \quad \mu \neq \Phi, \quad \eta < 1 \tag{8}$$

In the equations above, χ is a strictly positive technological parameter and H is the aggregate amount of human capital available in the economy. The rate at which a researcher can generate a new idea (ψ) is related to three different effects. The parameter η measures the traditional *intertemporal spillover-effect* arising from the existing stock of knowledge, n: $\eta < 0$ reflects the case where the rate at which a new innovation arrives declines with the number of ideas already discovered ("*fishing-out effect*"); if $0 < \eta < 1$, previous discoveries raise the productivity of current research effort ("*standing-on-shoulders effect*"); $\eta = 0$ represents the situation in which the arrival rate of new ideas is independent of the available stock of knowledge.[18] The case $\eta = 1$ is ruled out from the analysis in order to avoid possible scale effects, whereby an increase in the level of available human capital may affect the rate at which new ideas are produced over

[15] In Devereux et al. (1996a, p. 236, Eq. 1; 2000, p. 549, Eq. 1) under symmetry ($x_i = x, \forall i$) the aggregate production function reads as: $Y = xN^{1/\rho}, \rho \in (0; 1)$. Therefore, the degree of returns to specialization equals $1/\rho > 1$. This is the "*increasing returns to specialization case*" in Devereux et al. (1996b, p. 633, Eq. 4b, with $\lambda = 0$).

[16] See Devereux et al. (1996b, p. 633, Eq. 4b, with $\lambda = 1 - 1/\rho$).

[17] A negative R means that an increase in n would lead to some sort of '*inefficiency*' in the economy since, following a rise in the number of intermediate-good varieties, aggregate GDP (Y) would *ceteris paribus* decline in this case.

[18] For a detailed discussion of the "fishing out" and "standing on shoulders" effects, see Jones (1995, 2005).

time. The parameter μ captures the effect on the arrival rate of a new innovation of the actual size of the R&D process (measured by the number of units of skilled labor-input actually devoted to it). A value $\mu = 0$ would imply that H_n is not an input to R&D-activity (Eq. 8). We rule out this unrealistic case by assuming that research human capital is indispensable to the discovery of new designs and that its contribution to the production of new ideas is always positive (*i.e.*, $\mu > 0$). If $\mu = 1$, doubling the number of researchers H_n would not affect the arrival rate of a new idea in Eq. (7′), so leading to exactly double the production of new innovations per unit of time (Eqs. 7 and 8); if $\mu \in (0; 1)$ due to the existence of congestion/duplication externality ("*stepping-on-toes*") effects, increasing the number of researchers leads to a reduction of the rate at which each of them can discover a new idea (Eq. 7′) and to an ultimate increase (but less than proportional) in the total number of new innovations produced in the unit of time (Eq. 8).[19] In accordance with Jones (2005, Eq. 16, p. 1074) we keep our analysis as general as possible and impose no upper-bound to μ. According to Eq. (8), inventing the latest idea requires an amount of skilled-labor input equal to $H_n = (\chi H^{\Phi}/n^{\eta})^{1/\mu}$, which can change over time either because of the growth of n (*intertemporal knowledge-spillover effect*), or because of the growth of H. In our model a rise of population leads *ceteris paribus* to a rise of H (see Section 3.3 for a formal definition of this variable in our setting) and, therefore, if Φ is positive, to a decrease in research human capital productivity (*i.e.*, to an increase in H_n). The hypothesis that the productivity of human capital employed in research may fall due to an increase in population can be justified by the fact that it becomes increasingly difficult to introduce successfully new varieties of (intermediate) goods in a more crowded market (*R&D-difficulty* grows also with the size of population, as suggested by Dinopoulos and Segerstrom (1999, p. 459)). In Eq. (8) Φ measures exactly the strength of this effect. Indeed, all the rest being equal, the larger $\Phi > 0$ and the larger the decline in the R&D human capital productivity following an increase in population size.[20] The Jones (2005)' formulation of the R&D process does not take this important feature of the inventive activity into account.[21]

The R&D sector is competitive and there is free entry. A representative R&D firm has instantaneous profits equal to:

$$\text{R\&D firm profits} = \underbrace{(\frac{1}{\chi}\frac{H_{nt}^{\mu}}{H_t^{\Phi}}n^{\eta})}_{\dot{n}} V_{nt} - w_{nt}H_{nt} \tag{9}$$

where:

$$V_{nt} = \int_t^{\infty} \pi_{i\tau} e^{-\int_t^{\tau} r(s)ds} d\tau, \quad \tau > t \tag{10}$$

[19] Likewise, if $\mu > 1$, increasing the number of researchers would imply an increase (more than proportional) in the total number of new innovations produced in the unit of time (Eq. 8).

[20] Formally, given the amount of R&D-human capital needed to invent the latest idea, *i.e.* $H_n = (\chi H^{\Phi}/n^{\eta})^{1/\mu}$, and with $H = h$ (per-capita human capital) $\times$ L (population size), it is immediate to show that $\frac{\partial H_n}{\partial L} = \frac{\Phi}{\mu}\frac{H_n}{L}$. For given $\mu > 0$, $H_n > 0$ and $L > 0$, the magnitude of $\frac{\partial H_n}{\partial L} > 0$ increases with $\Phi > 0$. Instead, with $\Phi < 0$ an increase in population, L, by reducing H_n, would *ceteris paribus* contribute to raise research human capital productivity. This could be explained, for example, by the fact that a growing population would lead, all the rest remaining equal, to an increase in the ease of exchanging/diffusing ideas across people, and/or creating research networks among researchers. In our model, both possibilities (positive or negative Φ) are left open.

[21] When $\Phi = 0$, Eq. (8) becomes: $\dot{n}_t = \frac{1}{\chi}H_{nt}^{\mu}n_t^{\eta}$, $\chi > 0$, $\mu > 0$, and $\eta < 1$. This specification coincides with Jones (2005, Eq. 16, p. 1074).

In the last two equations, V_n denotes the value of the generic *i-th* intermediate firm (the one that has got the exclusive right of producing the *i-th* variety of capital goods by employing the *i-th* blueprint); $\pi_{i\tau}$ is the flow of profits accruing to the same *i-th* intermediate firm at date τ; $\exp[-\int_t^\tau r(s)ds]$ is a present value factor which converts a unit of profit at time τ into an equivalent unit of profit at time t; r denotes the instantaneous interest rate (the real rate of return on households' asset holdings, to be introduced in a moment), and w_n is the wage rate going to one unit of research human capital. Eq. (9) says that profits of a representative R&D firm are equal to the difference between total R&D revenues (R&D output, $\dot{n}$, times the price of ideas, V_n) minus total R&D costs related to rival inputs (human capital employed in research, H_n, times the wage accruing to one unit of this input, w_n). Eq. (10), instead, reveals that the price of the generic *i-th* idea is equal to the present discounted value of the returns resulting from the production of the *i-th* variety of capital-goods by profit-making intermediate firm i.

Using Eq. (9), the zero-profit condition in the R&D sector implies:

$$w_{nt} = \frac{1}{\chi}\frac{H_{nt}^{\mu-1}}{H_t^{\Phi}}n^{\eta}V_{nt} = \psi_t V_{nt} \tag{9'}$$

3.3 Households

The economy is closed and consists of many structurally-identical households. Therefore, we focus on the choices of a single infinitely-lived family with perfect foresight whose size coincides with the size of the whole population (L) and that owns all the firms operating in the economy. Each member of the household can purposefully invest in human capital. Consequently, the aggregate stock of this factor-input ($H_t = h_t L_t$) can rise either because population grows at a constant and exogenously given rate $g_L > 0$, or because per capita human capital, h_t, endogenously increases over time. The household uses the income it does not consume to accumulate assets that take the form of ownership claims on firms. Thus:

$$\dot{A}_t = (r_t A_t + w_t H_{Et}) - C_t, \quad A(0) > 0 \tag{11}$$

where A and C denote, respectively, household's asset holdings and consumption and $H_E \equiv uH = H_Y + H_I + H_n$ is the fraction of the available human capital employed in production activities (namely, production of consumption goods and intermediate inputs, and discovery of new ideas).[22] Eq. (11) suggests that household's investment in assets (the left hand side) equals household's savings (the right hand side). Household's savings, in turn, are equal to the difference between household's total income - the sum of interest income, rA, and human capital income, wH_E - and household's consumption (C). Given Eq. (11), the law of motion of assets in per-capita terms ($a_t \equiv A_t/L_t$) reads as:

$$\dot{a}_t = (r_t - g_L)a_t + (u_t h_t)w_t - c_t, \quad a(0) > 0 \tag{11'}$$

where c_t and h_t denote consumption and human capital per capita, respectively. The term $-g_L$ in (11′) captures the negative *the dilution* effect occurring in per-capita asset holdings accumulation due to population growth, and reflects the '*cost*' of bringing the

[22]As already mentioned, at equilibrium all human capital employed in production activities (H_E) is rewarded according to the same wage, w.

amount of per-capita assets of the newcomers up to the average level of the existing population. This formulation implies that, *ceteris paribus*, population growth tends to slow down the investment in assets of the average individual in the population.

At each time $t \geq 0$, the household uses the remaining fraction $(1 - u_t)$ of H_t in educational assignments. Human capital per-capita accumulates as:

$$\dot{h}_t = [\sigma(1 - u_t) - \xi g_L]h_t, \quad \sigma > 0, \quad \xi \geq 0, \quad h(0) > 0 \tag{12}$$

where σ and ξ are parameters measuring the productivity of education and the strength, if any, of the negative effect of population growth on per-capita human capital investment respectively. When $\xi = 1$, Eq. (12) shows the existence of a dilution effect of population growth on per-capita human capital accumulation (analogous to that of Eq. (11′). A possible explanation of such effect would be that since newborns enter the world uneducated they naturally reduce, *ceteris paribus* and at a given point in time, the existing stock of human capital per-capita, hence population growth ultimately operates like a form of depreciation of individual skills, h. Indeed, this effect is not present in the original Lucas (1988, p. 19, Eq. 13)' formulation. Lucas' assumption (newborns enter the work-force endowed with a skill-level proportional to the level already attained by older members of the family, so population growth *per se* does not reduce the current skill level of the representative worker) is based on the social nature of human capital accumulation, which probably makes it different from the accumulation of physical capital and of any other tangible asset. Indeed when $\xi = 0$, Eq. (12) is able to recover also this idea. A value of $\xi \in (0;1)$ represents an intermediate case between the previous two, even though in our analysis we do not put any *a-priori* upper bound on the magnitude of parameter ξ.

With a *Constant Intertemporal Elasticity of Substitution (CIES)* instantaneous felicity function, the problem faced by a representative infinitely-lived family seeking to maximize the utility (attained from consumption) of its members is:

$$\underset{\{c_t, u_t, a_t, h_t\}_{t=0}^{\infty}}{Max} U \equiv \int_0^\infty (\frac{c_t^{1-\theta} - 1}{1 - \theta})e^{-(\rho - g_L)t}dt, \quad \rho > g_L > 0, \quad \theta > 0 \tag{13}$$

$$\begin{aligned} \text{s.t.} \quad & \dot{a}_t = (r_t - g_L)a_t + (u_t h_t)w_t - c_t, \quad u_t \in [0;1], \forall t \geq 0; \quad \dot{L}_t/L_t \equiv g_L > 0 \\ & \dot{h}_t = [\sigma(1 - u_t) - \xi g_L]h_t, \quad \sigma > 0, \quad \xi \geq 0 \\ & a(0) > 0, \quad h(0) > 0 \quad \text{given.} \end{aligned}$$

In Eq. (13) population at time 0, $L(0)$, has been normalized to one. The household chooses the optimal path of per-capita consumption (c) and the share of human capital to be devoted to production activities (u) . The other symbols have the following meaning: U and $(\frac{c_t^{1-\theta}-1}{1-\theta})$ are the household's intertemporal utility function and the instantaneous felicity function of each member of the dynasty. We indicate by $\rho > 0$ the pure rate of time-preference and by $1/\theta > 0$ the constant intertemporal elasticity of substitution in consumption. The hypothesis $\rho > g_L > 0$ ensures that U is bounded away from infinity if c remains constant over time.

4. General equilibrium and balanced growth path (BGP) analysis

Since human capital is fully employed, and there exists perfect mobility of this factor-input across sectors, the following equalities must hold at equilibrium:

$$H_E \equiv u_t H_t = H_{Yt} + H_{It} + H_{nt} \tag{14}$$

$$w_{It} = w_{nt} \tag{15}$$

$$w_{It} = w_{Yt} \tag{16}$$

Eq. (14) says that aggregate labor demand (the right hand side) should equal the fraction of the available human capital stock employed in production and R&D activities (the left hand side). Equations (15) and (16) together state that, for the previous equality to be checked, wages do adjust in such a way that the salary earned by one unit of skilled labor in the intermediate sector should be equal to the salary earned by the same unit of skilled labor if employed in research or in the production of final goods. Moreover, since household's asset holdings must equalize the aggregate value of firms, the following equation should also be met at equilibrium:

$$A_t = n_t V_{nt}, \tag{17}$$

where the market value (V_n) is given by Eq. (10) and satisfies the usual *no-arbitrage condition*:

$$\dot{V}_{nt} = r_t V_{nt} - \pi_t$$

In the model, the $i-th$ idea allows the $i-th$ intermediate firm to produce the $i-th$ variety of durables. This explains why in Eq. (17) total assets (A_t) equal the number of profit-making intermediate firms (n_t) times the market value (V_{nt}) of each of them (equal, in turn, to the price of the corresponding idea). On the other hand, the *no-arbitrage condition* suggests that the return on the value of the $i-th$ intermediate firm ($r_t V_{nt}$) must be equal to the sum of the instantaneous monopoly profit accruing to the $i-th$ intermediate input producer (π_t) and the capital gain/loss matured on V_{nt} during the time interval dt, $\dot{V}_{nt}$. We are now able to move to a formal definition and characterization of the model's BGP equilibrium.

Definition (BGP Equilibrium): A BGP Equilibrium in this economy is a long-run equilibrium path along which:

i All variables depending on time grow at constant (possibly positive) exponential rates;

ii The sectorial shares of human capital employment ($s_j = H_{jt}/H_t, j = Y, I, n$) are constant.

From this definition, Proposition 1 follows:

Proposition 1. Along a BGP equilibrium, the fraction of the aggregate stock of human capital employed in production activities is constant (that is, $u_t = u, \forall t \geq 0$).

Proof. Immediate from Eq. (12), and the fact that the growth rate of all time-dependent variables is constant along a BGP equilibrium. □

It is possible to show (mathematical derivation can be found in *Appendix A2*) that the following results do hold along a BGP equilibrium:

$$\frac{\dot{H}_{Yt}}{H_{Yt}} = \frac{\dot{H}_{It}}{H_{It}} = \frac{\dot{H}_{nt}}{H_{nt}} = \frac{\dot{H}_{t}}{H_{t}} \equiv \gamma_H = \frac{[(\sigma-\rho)-(\xi-1-\theta)g_L]}{\Upsilon R(\theta-1)+\theta} \tag{18}$$

$$\frac{\dot{n}_t}{n_t} \equiv \gamma_n = \frac{\Upsilon[(\sigma-\rho)-(\xi-1-\theta)g_L]}{\Upsilon R(\theta-1)+\theta} = \Upsilon\gamma_H \tag{19}$$

$$r = \frac{\sigma\theta+\Upsilon R(\sigma\theta-\rho)-\{\theta[\xi(1+\Upsilon R)-(1+2\Upsilon R)]\}g_L}{\Upsilon R(\theta-1)+\theta} \tag{20}$$

$$\gamma_a \equiv \frac{\dot{a}_t}{a_t} = \gamma_c \equiv \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r-\rho) \tag{21}$$

$$\gamma_y \equiv \frac{\dot{y}_t}{y_t} = \gamma_a = \gamma_c = \frac{(1+\Upsilon R)(\sigma-\rho)-[\Upsilon R(\xi-2)+(\xi-1)]g_L}{\Upsilon R(\theta-1)+\theta} \tag{22}$$

$$u = 1-\frac{\sigma-\rho-[\Upsilon R(1-\xi)(\theta-1)+\xi(1-\theta)-1]g_L}{\sigma[\Upsilon R(\theta-1)+\theta]} \tag{23}$$

$$s_n = \frac{Z(1-Z)\gamma_n}{[1-Z+Z^2][r+(1-R)\gamma_n-\gamma_H]+Z(1-Z)\gamma_n}u \tag{24}$$

$$s_I = [\frac{Z^2}{1-Z+Z^2}](u-s_n) \tag{25}$$

$$s_Y = [\frac{1-Z}{1-Z+Z^2}](u-s_n) \tag{26}$$

$$\frac{H_t^{\mu-\Phi}}{n_t^{1-\eta}} = \frac{\chi}{s_n^{\mu}}\gamma_n \tag{27}$$

$$R \equiv \overline{\alpha}+1-Z, \qquad \Upsilon \equiv \frac{\mu-\Phi}{1-\eta}$$

Eq. (18) gives the BGP equilibrium growth rate of the economy's human capital stock (H), and of the human capital employment in final output, intermediate and research sectors. Eq. (19) gives the BGP equilibrium growth rate of the economy's stock of knowledge (n). Eq. (20) provides the equilibrium real rate of return on asset holdings (r). According to equations (21) and (22) per-capita consumption (c), per-capita asset holdings (a) and per-capita real income (y) all grow at the same constant rate. Eq. (23) gives the allocation of the available stock of human capital between production and educational activities along the BGP. The equilibrium shares of the existing human capital stock devoted to production of ideas (s_n), production of intermediates (s_I) and production of consumption goods (s_Y) are reported in equations (24), (25) and (26), respectively. Finally, Eq. (27) expresses the ratio of (some function of) the two state-variables in terms of the growth rate of the number of ideas (γ_n), and the share of the available human capital stock devoted to R&D-activity (s_n). It is evident from this equation that the restriction $\mu \neq \Phi$ prevents, *ceteris paribus*, γ_n to be independent of H_t.

The following assumption introduces constraints on the (relationship among) the feasible values of the model's parameters.

Assumption: Assume that

(i) $\Upsilon > 0$

(ii) $\theta > Max\left\{\dfrac{\Upsilon R}{1+\Upsilon R}; \dfrac{\Upsilon R \rho}{\sigma(1+\Upsilon R) - [\xi(1+\Upsilon R) - (1+2\Upsilon R)]g_L}\right\}$

(iii) $(\sigma - \rho) > Max\left\{(\xi - 1 - \theta)g_L; \dfrac{[\Upsilon R(\xi - 2) + (\xi - 1)]g_L}{1+\Upsilon R}; [\Upsilon R(1-\xi)(\theta - 1) + \xi(1-\theta) - 1]g_L\right\}$

The assumption $\Upsilon > 0$ comes directly from the *semi-endogenous* growth literature that inspires the present work. In fact, simple comparison of Eq. (8) in this paper with Eq. (16) in Jones (2005, p. 1074, Chap. 16) suggests to set:

- $\mu > 0$;
- $\eta < 1$;
- $\Phi = 0$.

With this parameterization (which is consistent with ours) we clearly see that $\mu \neq \Phi$ and, more importantly, $\Upsilon \equiv \frac{\mu - \Phi}{1-\eta} = \frac{\mu}{1-\eta} > 0$. In other words, having $\Upsilon > 0$ is rather standard in *semi-endogenous* growth literature.

Proposition 2. If the previous assumptions are satisfied, then:

- γ_H and γ_n are positive;
- γ_y is positive;
- r is positive;
- $0 < u < 1$;
- $r > \gamma_H - (1 - R)\gamma_n$.*Ceteris paribus* this condition allows V_{nt} be positive at any time $t \geq 0$ along a BGP;
- The two transversality conditions:

$$\lim_{t \to \infty} \lambda_{at} a_t = 0 \quad \text{and} \quad \lim_{t \to \infty} \lambda_{ht} h_t = 0$$

  are simultaneously checked along a BGP.

Proof. When (i) and (ii) in *Assumption* are met, then the denominator of equations (18), (19), (20) and (22) is positive, *i.e.* $[\Upsilon R(\theta - 1) + \theta] > 0$. Given this, and the fact that in the model g_L and σ are positive. We conclude that: (i) and (ii) ensure $r > \gamma_H - (1 - R)\gamma_n$, $r > 0$, $u > 0$ and the respect of the two transversality conditions; (i)-(ii)-(iii) ensure $\gamma_y > 0$, $\gamma_H > 0$ and $\gamma_n > 0$. Finally, all ensures $u < 1$. □

With $r > \gamma_H - (1-R)\gamma_n$ and $\gamma_n > 0$ it is immediate to show that, for any $0 < Z < 1$,

$$0 < \Lambda \equiv \frac{Z(1-Z)\gamma_n}{[1 - Z + Z^2][r + (1-R)\gamma_n - \gamma_H] + Z(1-Z)\gamma_n} < 1.$$

In turn, with $u \in (0;1)$, this implies:

$$0 < s_n = \Lambda u < 1,$$

and

$$0 < (u - s_n) < 1.$$

Using the last result, along a BGP we also simultaneously observe: $s_I \in (0;1)$ and $s_Y \in (0;1)$.

5. Population growth and economic growth

The following proposition analyzes the interaction between population and economic growth rates in this economy.

Proposition 3. Assume that all parameter restrictions of *Assumption* are checked for ξ. Then;

- When the dilution effect of population growth on individual human capital investment is sufficiently large (*i.e.*, $\xi > 1$), then we observe an ambiguous correlation between population and economic growth rates, *i.e.*: $\frac{\partial \gamma_y}{\partial g_L} > 0$, or $\frac{\partial \gamma_y}{\partial g_L} < 0$, or else $\frac{\partial \gamma_y}{\partial g_L} = 0$;
- When the dilution effect of population growth on individual human capital investment is sufficiently small (*i.e.*, $0 \le \xi \le 1$), then we observe an unambiguously positive correlation between population and economic growth rates, *i.e.*: $\frac{\partial \gamma_y}{\partial g_L} > 0$.

Results are summarized in Table 3 (Supporting information can be found in Appendix A3).

The intuition behind the results of Proposition 3 (and Table 3) goes as follows. By using again the BGP equilibrium relation:

$$\gamma_y = \gamma_H + R\gamma_n - g_L \tag{28}$$

$0 \leq \xi \leq 1$	$\dfrac{\partial \gamma_y}{\partial g_L} > 0$
$1 < \xi < \frac{1+2\Upsilon R}{1+\Upsilon R}$	$\dfrac{\partial \gamma_y}{\partial g_L} > 0$
$\xi = \frac{1+2\Upsilon R}{1+\Upsilon R}$	$\dfrac{\partial \gamma_y}{\partial g_L} = 0$
$\xi > \frac{1+2\Upsilon R}{1+\Upsilon R}$	$\dfrac{\partial \gamma_y}{\partial g_L} < 0$

Table 3: The relationship between the magnitude of ξ and the sign of the effect of an increase in the population growth rate (g_L) on economic growth (γ_y).

One immediately observes that:

$$\begin{aligned}\frac{\partial \gamma_y}{\partial g_L} &= \left(\frac{\partial \gamma_H}{\partial g_L} + R\frac{\partial \gamma_n}{\partial g_L}\right) - 1 \\ &= \left(\frac{1}{\Upsilon}\frac{\partial \gamma_n}{\partial g_L} + R\frac{\partial \gamma_n}{\partial g_L}\right) - 1 \\ \frac{\partial \gamma_y}{\partial g_L} &= \underbrace{\left(\frac{1+\Upsilon R}{\Upsilon}\right)}_{\substack{>0 \\ \text{under our} \\ \text{assumptions}}} \underbrace{\frac{\partial \gamma_n}{\partial g_L}}_{\text{'ideas' effect}} - \underbrace{1}_{\text{'dilution' effect}}\end{aligned} \tag{29}$$

According to (29), the whole impact of population growth on real per-capita income growth depends on the relative contribution of two distinct effects:

- The direct '*dilution*' *effect*. This effect is always negative for the reasons that have already been explained.
- The indirect '*ideas*' *effect*. This effect describes the impact that at a certain point in time an exogenous change of population size (due to a change of g_L) may have on the economy's growth rate of ideas, γ_n (see Jones, 2003). Unlike the previous one, this effect is always positive, as long as $1 + \theta - \xi > 0$.[23]

So, all the rest remaining equal (in particular for given $\theta > 0$), the smaller ξ is, the more likely it is for the indirect '*ideas*' effect to be positive and for the first term of (29) to ultimately outweigh the negative '*dilution*' effect, so leading to an unambiguous $\frac{\partial \gamma_y}{\partial g_L} > 0$. This is what we observe in Table 3: when $\xi \leq 1$, the ideas-effect is clearly positive and the long-run correlation between population and economic growth rates is positive, as well. However, when ξ starts becoming large (*i.e.*, $\xi > 1$) a threshold level of ξ (namely, $\xi = (1+2\Upsilon R)/(1+\Upsilon R)$ in Table 3) does emerge such that the correlation between population growth and economic growth is no longer monotonous in sign along a BGP equilibrium: it continues to be positive below the ξ-threshold, but becomes negative above the ξ-threshold (the correlation between γ_y and g_L vanishes when ξ equals exactly the threshold). Note that if ξ is high enough the ideas-effect is negative (as $1 + \theta - \xi < 0$). In this case the whole impact of population growth on economic growth is undoubtedly negative, too (see Table 3 when $\xi > (1 + 2\Upsilon R)/(1 + \Upsilon R)$).

[23] The '*ideas*' effect is given by $\frac{\partial \gamma_n}{\partial g_L}$. From Eq. (19), $\frac{\partial \gamma_n}{\partial g_L} = \frac{\Upsilon(1+\theta-\xi)}{\Upsilon R(\theta-1)+\theta}$. Under requirements (*i*) and (*ii*) of our *Assumption*, $\frac{\partial \gamma_n}{\partial g_L} > 0$ as long as $1 + \theta - \xi > 0$.

6. Conclusion

Three concomitant facts have motivated the present paper: 1) The inconclusiveness of the existing debate concerning the ultimate long-run effects of demographic change (population growth) on per-capita income growth); 2) The need of clarifying whether the existence of a dilution effect of population growth can be documented empirically also for per-capita human capital investment (and not only for per-capita physical capital investment); 3) The possibility of introducing in the ongoing discussion a new channel through which one can explain theoretically why the long-run correlation between population and economic growth rates may be non-monotonous in sign, regardless of the possible sources of population growth (*i.e.*, fertility, mortality, migration, or ageing). In a word, this paper has tried to link the magnitude of a dilution-effect of population growth on per-capita human capital formation with the theoretical explanation of the sign that the correlation between population and economic growth rates can take along a BGP equilibrium.

In the first part of the paper we have had a thorough look at the data with the objective of providing evidence of the presence of a dilution effect of population growth also on per-capita human capital formation. Our empirical analysis confirms the idea that countries experiencing high population growth tend to be more negatively effected by population growth in terms of schooling than countries with low population growth. Furthermore, the analysis finds considerable heterogeneity of the dilution effect of population growth on schooling across countries experiencing the same level of population growth; in particular for developing economies. Such heterogeneity is likely to result from the combination of various factors including different drivers of population growth (birthrates versus life expectancy), different socio-economic characteristics affecting supply and demand of skilled labor, as well as differences in education policy. While future research may perform contingency analyses in order to single out the moderating effects of the various factors causing heterogeneity in the dilution effect of population growth on human capital formation, we proceeded with investigating its consequences on long-run economic growth. Our economic growth regression suggests that schooling is influential and positive for long-run economic growth, and that population growth — net of schooling — is in average negatively associated with economic growth. Hence, in total, population growth may affect economic growth either positively or negatively, depending on the country-specific dilution effect on schooling.

In the second part of the paper, instead, we have taken stock of the found empirical results and have proposed a theoretical framework (where firms can endogenously invest in technological progress — taking the form of the discovery of new ideas for new varieties of intermediate inputs —, and agents can endogenously invest in human capital formation) with the objective of explaining why it could realistic to come to a correlation between population growth and economic growth in the long-run which is undetermined in sign. The explanation provided by our theoretical model is based on the role played exactly by the size of the dilution effect in per-capita human capital accumulation. More precisely, we show that along a BGP equilibrium the whole impact of population growth on real per-capita income growth depends on the contrast of two opposing effects. The first is the already mentioned *dilution-effect* (of population growth on per-capita skill formation). This effect is always negative. The second is, instead, the *ideas-effect*. By ideas-effect we mean the effect that population growth may have on the economy's innovation rate. This effect is positive provided that the dilution-effect is not very large. So, depending on the size of the dilution effect, we may ultimately observe a different correlation between population and economic growth rates. When the dilution effect is sufficiently low, the ideas-effect is definitely positive and prevails over the negative dilution-effect, so determining a positive long-run correlation between population and economic growth rates. However, when the dilution-effect starts becoming large enough a threshold-level of this effect arises such that the correlation between population growth and economic growth is no longer monotonous in sign along a BGP equilibrium: it continues to be positive below the dilution-effect threshold, but becomes negative above it. When the dilution-effect gets extremely large, then the ideas-effect becomes negative itself, and the whole impact of population growth on economic growth is undoubtedly negative, too. We believe that these theoretical results are consistent with the empirical evidence discussed in the first part of the paper, and that suggests the existence of considerable heterogeneity across countries as far as a dilution effect of population growth on schooling is concerned. Our theory builds, hence, a bridge between such cross-country heterogeneity in the magnitude of a per-capita human capital dilution-effect of population growth and the sign of the population-economic growth rates correlation along a BGP equilibrium.

We think that our paper leaves, in particular, an important question still open. In our model the dilution-effect has been introduced as a parameter (greater or, at most, equal to zero) that multiplies the growth rate of population in the law of motion of per-capita human capital. For further research, it would be interesting to look into those factors that can make this parameter endogenous. Indeed, if countries have different threshold levels of the dilution-effect of population growth on the accumulation of per-capita human capital, then finding the determinants of the country-specific threshold becomes of fundamental importance, and may also lead us to make more accurate policy recommendations following the possibly different impacts of demographic change on economic prosperity across the various regions/countries of world.

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A Appendix

A1 Data

This subsection provides averages of the main variables used in the empirical section by region: i) 'Avg. years of schooling' (i.e. average years of schooling in the population of the 15 to 100 years old), ii) 'Years of schooling young cohort' (i.e. schooling of the then 5 to 9 years old cohort, attained until age 25 to 29 years), iii) 'GDP p. adult' (i.e. output-side real GDP at chained PPPs (in thousand 2005 US$) per person above 20 years, iv) 'GDP p. capita' (i.e. output-side real GDP at chained PPPs (in thousand 2005 US$) per person), v) 'Population growth rate' (i.e. natural population growth rate, crude birth rate minus crude death rate, per 100 population). All variables in logs. Five African countries (Botswana, Mali, Mozambique, Niger, Senegal) are outliers in terms of schooling advancement, and hence removed from the working sample.

Table 4: Advanced Economies[1]

Year	Avg. years of schooling in logs	Years of schooling young cohort in logs	GDP p. adult in logs	GDP p. capita in logs	Population growth rate
1960	1.97 (0.28)	2.41 (0.16)	2.59 (0.62)	2.14 (0.67)	1.09 (0.45)
1965	2.02 (0.27)	2.47 (0.13)	2.84 (0.49)	2.38 (0.53)	0.89 (0.4)
1970	2.1 (0.25)	2.51 (0.11)	3.03 (0.41)	2.6 (0.43)	0.74 (0.49)
1975	2.18 (0.22)	2.53 (0.1)	3.15 (0.4)	2.74 (0.41)	0.6 (0.5)
1980	2.25 (0.2)	2.55 (0.07)	3.32 (0.29)	2.93 (0.31)	0.52 (0.46)
1985	2.29 (0.17)	2.58 (0.06)	3.3 (0.3)	2.95 (0.32)	0.47 (0.42)
1990	2.33 (0.15)	2.6 (0.07)	3.49 (0.26)	3.16 (0.28)	0.45 (0.39)
1995	2.37 (0.12)	—	3.59 (0.25)	3.28 (0.27)	0.37 (0.36)
2000	2.41 (0.1)	—	3.79 (0.26)	3.49 (0.26)	0.34 (0.36)
2005	2.45 (0.11)	—	3.85 (0.3)	3.57 (0.29)	0.37 (0.37)
2010	2.49 (0.11)	—	3.9 (0.29)	3.63 (0.28)	0.31 (0.4)

[1] 25 countries: Australia, Austria, Belgium, Canada, Cyprus, Denmark, Finland, France, Greece, Iceland, Ireland, Israel, Italy, Japan, Luxembourg, Malta, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, USA.

Table 5: Asia[1]

Year	Avg. years of schooling in logs	Years of schooling young cohort in logs	GDP p. adult in logs	GDP p. capita in logs	Population growth rate
1960	1.21 (0.48)	1.8 (0.49)	1.23 (0.55)	0.51 (0.53)	2.65 (0.47)
1965	1.32 (0.49)	1.95 (0.45)	1.37 (0.59)	0.62 (0.57)	2.5 (0.44)
1970	1.46 (0.5)	2.02 (0.43)	1.58 (0.68)	0.83 (0.69)	2.35 (0.4)
1975	1.56 (0.49)	2.14 (0.39)	1.73 (0.79)	1.01 (0.82)	2.18 (0.59)
1980	1.67 (0.47)	2.2 (0.4)	1.84 (0.89)	1.17 (0.96)	2.11 (0.64)
1985	1.78 (0.42)	2.31 (0.32)	1.88 (0.84)	1.25 (0.94)	1.98 (0.69)
1990	1.88 (0.39)	2.38 (0.29)	2.07 (0.91)	1.48 (1.02)	1.8 (0.67)
1995	1.98 (0.38)	—	2.23 (0.88)	1.68 (1)	1.55 (0.75)
2000	2.05 (0.35)	—	2.26 (0.96)	1.74 (1.08)	1.36 (0.7)
2005	2.14 (0.3)	—	2.4 (1.03)	1.93 (1.16)	1.21 (0.63)
2010	2.21 (0.28)	—	2.66 (0.89)	2.22 (1.02)	1.09 (0.58)

[1] 14 countries: Bangladesh, China, China, Hong Kong Special Administrative Region, Fiji, India, Indonesia, Malaysia, Nepal, Pakistan, Philippines, Republic of Korea, Singapore, Sri Lanka, Thailand.

Table 6: Latin America and the Caribbean[1]

Year	Avg. years of schooling in logs	Years of schooling young cohort in logs	GDP p. adult in logs	GDP p. capita in logs	Population growth rate
1960	1.41 (0.34)	1.91 (0.29)	1.91 (0.63)	1.17 (0.65)	2.81 (0.67)
1965	1.5 (0.34)	2.04 (0.27)	2.05 (0.66)	1.28 (0.67)	2.65 (0.69)
1970	1.6 (0.34)	2.1 (0.24)	2.19 (0.71)	1.42 (0.72)	2.5 (0.68)
1975	1.69 (0.32)	2.19 (0.19)	2.3 (0.68)	1.55 (0.7)	2.43 (0.67)
1980	1.8 (0.28)	2.23 (0.19)	2.34 (0.66)	1.63 (0.69)	2.35 (0.68)
1985	1.9 (0.25)	2.29 (0.19)	2.22 (0.61)	1.54 (0.66)	2.19 (0.65)
1990	1.97 (0.23)	2.33 (0.19)	2.23 (0.61)	1.58 (0.68)	2 (0.64)
1995	2.04 (0.22)	—	2.36 (0.58)	1.75 (0.67)	1.79 (0.63)
2000	2.1 (0.2)	—	2.43 (0.54)	1.85 (0.64)	1.58 (0.57)
2005	2.16 (0.2)	—	2.55 (0.53)	2.02 (0.62)	1.41 (0.52)
2010	2.2 (0.18)	—	2.73 (0.57)	2.24 (0.64)	1.26 (0.48)

[1] 22 countries: Argentina, Barbados, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Rep., Ecuador, El Salvador, Guatemala, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Trinidad and Tobago, Uruguay, Venezuela.

Table 7: Middle East and North Africa[1]

Year	Avg. years of schooling in logs	Years of schooling young cohort in logs	GDP p. adult in logs	GDP p. capita in logs	Population growth rate
1960	0.75 (0.26)	1.63 (0.22)	1.69 (0.84)	0.94 (0.82)	2.95 (0.51)
1965	0.89 (0.26)	1.81 (0.21)	1.87 (0.67)	1.08 (0.66)	2.95 (0.54)
1970	1.05 (0.25)	1.96 (0.21)	2.04 (0.68)	1.22 (0.67)	2.95 (0.57)
1975	1.23 (0.23)	2.02 (0.17)	2.22 (0.74)	1.41 (0.72)	2.98 (0.49)
1980	1.41 (0.21)	2.11 (0.19)	2.13 (0.68)	1.33 (0.66)	2.94 (0.45)
1985	1.6 (0.2)	2.2 (0.18)	2.19 (0.66)	1.41 (0.67)	2.66 (0.47)
1990	1.73 (0.19)	2.27 (0.19)	2.15 (0.63)	1.39 (0.68)	2.24 (0.47)
1995	1.83 (0.2)	—	2.22 (0.64)	1.52 (0.71)	1.91 (0.57)
2000	1.92 (0.21)	—	2.34 (0.55)	1.71 (0.62)	1.71 (0.57)
2005	2.01 (0.19)	—	2.6 (0.38)	2.04 (0.43)	1.7 (0.5)
2010	2.1 (0.17)	—	2.84 (0.34)	2.34 (0.39)	1.72 (0.48)

[1] 8 countries: Algeria, Egypt, Iran, Jordan, Morocco, Syrian Arab Republic, Tunisia, Turkey.

Table 8: Sub-Saharan Africa[1]

Year	Avg. years of schooling in logs	Years of schooling young cohort in logs	GDP p. adult in logs	GDP p. capita in logs	Population growth rate
1960	0.85 (0.37)	1.47 (0.42)	1.21 (0.57)	0.46 (0.59)	2.57 (0.59)
1965	0.95 (0.37)	1.64 (0.37)	1.35 (0.61)	0.58 (0.64)	2.7 (0.47)
1970	1.07 (0.38)	1.76 (0.37)	1.49 (0.59)	0.7 (0.63)	2.84 (0.46)
1975	1.21 (0.36)	1.84 (0.35)	1.57 (0.63)	0.77 (0.67)	2.98 (0.43)
1980	1.35 (0.34)	1.89 (0.29)	1.58 (0.67)	0.78 (0.71)	3 (0.48)
1985	1.46 (0.33)	1.93 (0.27)	1.62 (0.64)	0.81 (0.68)	2.92 (0.44)
1990	1.58 (0.33)	1.99 (0.26)	1.52 (0.71)	0.71 (0.77)	2.59 (0.7)
1995	1.68 (0.31)	—	1.43 (0.78)	0.63 (0.85)	2.49 (0.53)
2000	1.74 (0.28)	—	1.44 (0.78)	0.66 (0.87)	2.38 (0.72)
2005	1.83 (0.26)	—	1.53 (0.8)	0.77 (0.89)	2.47 (0.8)
2010	1.89 (0.25)	—	1.63 (0.8)	0.88 (0.88)	2.5 (0.76)

[1] 23 countries: Benin, Burundi, Cameroon, Central African Republic, Congo, Cote d'Ivoire, Democratic Republic of the Congo, Gabon, Gambia, Ghana, Kenya, Lesotho, Malawi, Mauritania, Mauritius, Namibia, Rwanda, South Africa, Togo, Uganda, United Republic of Tanzania, Zambia, Zimbabwe.

A2 Eqs. (18) and (27)

The *Hamiltonian function* (J_t) related to the intertemporal problem (13) in the main-text is:

$$J_t = (\frac{c_t^{1-\theta}-1}{1-\theta})e^{-(\rho-g_L)t} + \lambda_{at}[(r_t - g_L)a_t + (u_t h_t)w_t - c_t] + \lambda_{ht}[\sigma(1-u_t) - \xi g_L]h_t$$

where λ_{at} and λ_{ht} are the *co-state variables* associated, respectively, to the *state variables* a_t and h_t.
The necessary first order conditions *FOC*s are:

$$\frac{\partial J_t}{\partial c_t} = 0 \Longleftrightarrow \frac{e^{-(\rho-g_L)t}}{c_t^{\theta}} = \lambda_{at} \tag{A1}$$

$$\frac{\partial J_t}{\partial u_t} = 0 \Longleftrightarrow \lambda_{at} = \frac{\sigma}{w_t}\lambda_{ht} \tag{A2}$$

$$\frac{\partial J_t}{\partial a_t} = -\dot{\lambda}_{at} \Longleftrightarrow \lambda_{at}(r_t - g_L) = -\dot{\lambda}_{at} \tag{A3}$$

$$\frac{\partial J_t}{\partial h_t} = -\dot{\lambda}_{ht} \Longleftrightarrow \lambda_{at}u_t w_t + \lambda_{ht}[\sigma(1-u_t) - \xi g_L] = -\dot{\lambda}_{ht} \tag{A4}$$

along with the two transversality conditions:

$$\lim_{t\to\infty} \lambda_{at}a_t = 0, \quad \lim_{t\to\infty} \lambda_{ht}h_t = 0$$

and the initial conditions:

$$a(0) > 0, \quad h(0) > 0$$

Combining ($A2$) and ($A4$) yields:

$$\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = -(\sigma - \xi g_L) \tag{A5}$$

Eqs. ($A3$) and ($A2$) imply, respectively:

$$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -(r_t - g_L) \tag{A6}$$

$$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = \frac{\dot{\lambda}_{ht}}{\lambda_{ht}} - \frac{\dot{w}_t}{w_t} \tag{A7}$$

Combination of ($A5$), ($A6$) and ($A7$) leads to:

$$r_t = (1-\xi)g_L + \sigma + \frac{\dot{w}_t}{w_t} \tag{A8}$$

Since human capital is perfectly mobile across sectors, at equilibrium it will be rewarded according to the same wage: $w_t \equiv w_{Yt} = w_{It} = w_{nt}$. Moreover, along the BGP this common wage would grow at a constant exponential rate, implying that γ_w is constant. Accordingly, in the BGP equilibrium the real rate of return on asset holdings, r, will be constant (Eq. $A8$). With r constant, and making use of Eqs. (6) and (10) in the main text, we find that along the BGP:

$$V_{nt} = Z(1-Z)(\frac{H_{Yt}}{n_t})^{1-Z}(\frac{H_{It}}{n_t})^{Z}\frac{n_t^R}{[r + (1-R)\gamma_n - \gamma_H]}, \tag{A9}$$

$$R \equiv \overline{\alpha} + 1 - Z, \qquad \frac{\dot{n}_t}{n_t} \equiv \gamma_n, \qquad \frac{\dot{H}_t}{H_t} \equiv \gamma_H$$

For any $0 < Z < 1$, $H_Y > 0$, $H_I > 0$, $n > 0$ and $R > 0$, V_{nt} is positive provided that:

$$r > \gamma_H - (1-R)\gamma_n \tag{A9'}$$

Given V_{nt} , from Eq. (9′) in the main text:

$$w_{nt} = \frac{Z}{\chi}(1-Z)s_n^{\mu-1}H_t^{\mu-1-\Phi}n_t^{\eta}(\frac{H_{Yt}}{n_t})^{1-Z}(\frac{H_{It}}{n_t})^{Z}\frac{n_t^R}{[r+(1-R)\gamma_n-\gamma_H]} \tag{A10}$$

where $s_n \equiv H_{nt}/H_t$ is constant along the BGP. We can now use Eqs. (5), (2) and (4′) in the main text, obtaining:

$$w_{It} = Z^2(\frac{H_{Yt}}{n_t})^{1-Z}(\frac{H_{It}}{n_t})^{Z-1}n_t^R \tag{A11}$$

From Eq. (15) in the main text, by equalizing ($A11$) and ($A10$) in this appendix, one gets:

$$s_I \equiv \frac{H_{It}}{H_t} = \frac{Z\chi}{(1-Z)}\frac{[r+(1-R)\gamma_n-\gamma_H]}{s_n^{\mu-1}}\frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}} \tag{A12}$$

Combining Eqs. (1) and (4′) in the text:

$$w_{Yt} \equiv \frac{\partial Y_t}{\partial H_{Yt}} = (1-Z)(\frac{H_{Yt}}{n_t})^{-Z}(\frac{H_{It}}{n_t})^{Z}n_t^R \tag{A13}$$

From (16) in the main text and ($A12$) above, equalization of Eqs. ($A11$) and ($A13$) in this appendix delivers:

$$s_Y \equiv \frac{H_{Yt}}{H_t} = (\frac{1-Z}{Z^2})s_I = \frac{\chi}{Z}\frac{[r+(1-R)\gamma_n-\gamma_H]}{s_n^{\mu-1}}\frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}} \tag{A14}$$

Along the BGP all variables depending on time grow at constant rates and the sector shares of human capital employment are also constant. Therefore, from Eq. (8) in the main text:

$$\frac{\dot{n}_t}{n_t} \equiv \gamma_n = (\frac{\mu-\Phi}{1-\eta})\gamma_H, \quad \gamma_H \equiv \frac{\dot{H}_t}{H_t} \tag{A15}$$

If $\mu - \Phi = 1 - \eta$ we have a very special case of the model in which human and technological capital grow at the same rate $\gamma_n = \gamma_H \equiv \gamma$ along a BGP. We rule out this "extreme" case and analyze the most general possible situation in which: $\mu \neq \Phi \neq \Phi + 1 - \eta$.

Using Eqs. ($A10$), ($A11$), ($A13$) and ($A15$) we see that along the BGP wages grow at a common and constant rate:

$$\frac{\dot{w_{nt}}}{w_{nt}} = \frac{\dot{w_{It}}}{w_{It}} = \frac{\dot{w_{Yt}}}{w_{Yt}} \equiv \frac{\dot{w}_t}{w_t} = R\gamma_n \tag{A15'}$$

Combining Eqs. ($A1$) and ($A6$), the usual Euler equation follows:

$$\frac{\dot{c}_t}{c_t} \equiv \frac{1}{\theta}(r-\rho), \quad c \equiv \frac{C}{L} \tag{A16'}$$

From (17) in the text and ($A9$) in this appendix we conclude that along a BGP:

$$\frac{\dot{a}_t}{a_t} \equiv \gamma_a = \gamma_H + R\gamma_n - g_L, \quad a_t \equiv \frac{A_t}{L_t}, \quad g_L \equiv \frac{\dot{L}_t}{L_t} \tag{A17}$$

Merging ($11'$) in the main text and ($A6$) in this appendix yields:

$$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -\gamma_a + u_t \frac{h_t w_t}{a_t} - \frac{c_t}{a_t}, \tag{$A18$}$$

where $u_t = u, \forall t \geq 0$ along the BGP. Instead, from the combination of (12) in the text and ($A5$) in this appendix we get:

$$\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = -\lambda_h - \sigma\mu, \quad h_t \equiv \frac{H_t}{L_t}, \quad \gamma_h \equiv \frac{\dot{h}_t}{h_t} \tag{$A19$}$$

Eqs. ($A7$), ($A15'$), ($A17$), ($A18$) and ($A19$) together lead to:

$$\frac{c_t}{a_t} = u[\frac{h_t w_t}{a_t} + \sigma], \tag{$A20$}$$

where $\gamma_h = \gamma_H - g_L$ has been used.

Using (12) in the main text, Eqs. ($A15'$), ($A17$) and ($A20$) and the fact that $\gamma_H = \gamma_h + g_L = \sigma(1-u) + (1-\xi)g_L$ one obtains:

$$\frac{c_t}{a_t} = u[\frac{h(0)w(0)}{a(0)} + \sigma] \tag{$A20'$}$$

where $h(0), w(0)$ and $a(0)$ are the initial values (*i.e.*, at $t = 0$) of h_t, w_t and a_t, respectively. With u constant, the just-mentioned initial values given, and $\sigma > 0$ the last equation implies:

$$\gamma_c = \gamma_a \tag{$A21$}$$

This means that along the BGP a_t and c_t grow at the same rate. Using ($A21$) and equating ($A16'$) and ($A17$) it is possible to get:

$$r = \rho + \theta(\gamma_H + R\gamma_n - g_L) \tag{$A22$}$$

Next, by equalizing ($A22$) to ($A8$), and using ($A15'$):

$$\gamma_n \equiv \frac{\dot{n}_t}{n_t} = \frac{[(\sigma-\rho) - (\xi - 1 - \theta)g_L - \theta\gamma_H]}{R(\theta-1)} \tag{$A23$}$$

Equating ($A23$) to ($A15$), and solving for γ_H, we finally obtain:

$$\gamma_H \equiv \frac{\dot{H}_t}{H_t} = \frac{[(\sigma-\rho) - (\xi-1-\theta)g_L]}{\Upsilon R(\theta-1)+\theta}, \quad \Upsilon \equiv [\frac{\mu-\Phi}{1-\eta}] \tag{$A23'$}$$

Given γ_H, it is possible to re-cast γ_n as:

$$\gamma_n \equiv \frac{\dot{n}_t}{n_t} = \frac{\Upsilon[(\sigma-\rho) - (\xi-1-\theta)g_L]}{\Upsilon R(\theta-1)+\theta} = \Upsilon\gamma_H \tag{$A23''$}$$

Eqs. ($A23'$) and ($A23''$) confirm that

$$\gamma_H = \gamma_n$$

in the special case in which $\Upsilon \equiv \frac{\mu-\Phi}{1-\eta} = 1$. The BGP equilibrium - value of r is obtained by combining $(A22)$, $(A23')$ and $(A23'')$:

$$r = \frac{\sigma\theta + \Upsilon R(\sigma\theta - \rho) - \{\theta[\xi(1+\Upsilon R) - (1+2\Upsilon R)]\}g_L}{\Upsilon R(\theta - 1) + \theta} \qquad (A22')$$

Eqs. $(A21)$ and $(A16')$ together imply:

$$\gamma_a \equiv \frac{\dot{a}_t}{a_t} = \gamma_c \equiv \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r-\rho) \quad \text{where } r \text{ is given by Eq. } (A22') \qquad (A21')$$

After using Eq. (12) in the main text, the definition of $h \equiv H/L$ and the fact that $\dot{L}/L \equiv g_L$, we conclude:

$$\gamma_H \equiv \frac{\dot{H}}{H} = \sigma(1-u) + (1-\xi)g_L \qquad (A24)$$

Equalization of $(A23')$ and $(A24)$ allows obtaining the BGP equilibrium value of u:

$$u = 1 - \frac{(\sigma-\rho) - [\Upsilon R(1-\xi)(\theta-1) + \xi(1-\theta) - 1]g_L}{\sigma[\Upsilon R(\theta-1)+\theta]} \qquad (A25)$$

From (1) in the text, $(A22')$, $(A23')$ and $(A23'')$ in this appendix, the hypothesis of symmetry (Eq. $4'$ in the text), the definitions of $y \equiv Y/L$, $R \equiv \overline{\alpha} + 1 - Z$ and $\dot{L}/L \equiv g_L$, we obtain the growth rate of real per-capita output along a BGP:

$$\gamma_y \equiv \frac{\dot{y}_t}{y_t} = \frac{(1+\Upsilon R)(\sigma-\rho) - [\Upsilon R(\xi-2) + (\xi-1)]g_L}{\Upsilon R(\theta-1)+\theta} = \gamma_a = \gamma_c = \frac{1}{\theta}(r-\rho) \quad (A26)$$

We now compute the BGP equilibrium values of s_n, s_I and s_Y. Eq. (14) in the main text suggests: $u = s_Y + s_I + s_n$ From (A14) in this appendix we use $s_y = (\frac{1-Z}{Z^2})s_I$ into the expression above and obtain:

$$s_I = [\frac{Z^2}{1-Z+Z^2}](u-s_n) \quad \text{where } u \text{ is given by } A25. \qquad (A27)$$

Hence:

$$s_Y = [\frac{1-Z}{1-Z+Z^2}](u-s_n) \qquad (A28)$$

According to $(A14)$, however, it is also true that:

$$s_Y \equiv \frac{H_{Yt}}{H_t} = \frac{\chi}{Z}\frac{[r + (1-R)\gamma_n - \gamma_H]}{s_n^{\mu-1}}\frac{n_t^{1-\eta}}{H_t^{\mu-\Phi}}$$

Equating this expression to $(A28)$ yields:

$$\frac{H_t^{\mu-\Phi}}{n_t^{1-\eta}} = \frac{\chi[1-Z+Z^2]}{Z(1-Z)}\frac{[r+(1-R)\gamma_n - \gamma_H]}{s_n^{\mu-1}(u-s_n)} \qquad (A29)$$

From Eq. (8) in the body-text:

$$\frac{H_t^{\mu-\Phi}}{n^{1-\eta}} = \frac{\chi}{s_n^{\mu}}\gamma_n \qquad (A30)$$

Equalization of $(A29)$ and $(A30)$ leads to:

$$s_n = \frac{Z(1-Z)\gamma_n}{[1-Z+Z^2][r+(1-R)\gamma_n-\gamma_H]+Z(1-Z)\gamma_n}u \qquad (A31)$$

Given Eqs. $(A22')$, $(A23')$, $(A23'')$, $(A25)$ and $(A31)$, it is possible to compute the BGP ratio $\frac{H_t^{\mu-\Phi}}{n^{1-\eta}}$ (by using either Eq. 29 or Eq. $A30$), along with s_I and s_Y (Eqs. $A27$ and $A28$).

Finally, by employing Eqs. $(A6)$, $(A7)$, $(A15')$, $(A17)$ and the definition of h, it can be showed that along a BGP the two transversality conditions $\lim_{t\to\infty}\lambda_{at}a_t = 0$ and $\lim_{t\to\infty}\lambda_{ht}h_t = 0$ are simultaneously checked when: $r > \gamma_H + R\gamma_n$. In turn, when the two transversality conditions are met, then the requirement (Eq. $A9'$): $r > \gamma_H - (1-R)\gamma_n$ is also met.

A3 Table 3

$$\frac{\partial\gamma_y}{\partial g_L} = \frac{-[\Upsilon R(\xi-2)+(\xi-1)]}{\Upsilon R(\theta-1)+\theta}$$

When assumptions (i) and (ii) in the main text are met, $[\Upsilon R(\theta-1)+\theta]$ is always positive. With $\Upsilon > 0$, $R > 0$, and $\xi \geq 0$ we conclude:

- $\frac{\partial\gamma_y}{\partial g_L} > 0 \Rightarrow$

$$\begin{aligned} -\Upsilon R(\xi-2)-(\xi-1) &> 0 \\ -\Upsilon R(\xi-2) &> (\xi-1) \\ \Upsilon R\xi+\xi &< 1+2\Upsilon R \\ \xi(1+\Upsilon R) &< 1+2\Upsilon R \\ 0<\xi &< \frac{1+2\Upsilon R}{1+\Upsilon R} \end{aligned}$$

- $\frac{\partial\gamma_y}{\partial\gamma_L} < 0 \Rightarrow -\Upsilon R(\xi-2)-(\xi-1) < 0 \quad \Rightarrow \xi > \frac{1+2\Upsilon R}{1+\Upsilon R}$
- $\frac{\partial\gamma_y}{\partial g_L} = 0 \Rightarrow -\Upsilon R(\xi-2)-(\xi-1) = 0 \quad \Rightarrow \xi = \frac{1+2\Upsilon R}{1+\Upsilon R}$
- if $\xi = 1 \Rightarrow \frac{\partial\gamma_y}{\partial g_L} = \frac{-[\Upsilon R(-1)]}{\Upsilon R(\theta-1)+\theta} = \frac{\Upsilon R}{\Upsilon R(\theta-1)+\theta} \quad \Rightarrow \frac{\partial\gamma_y}{\partial g_L} > 0$
- if $\xi = 0 \Rightarrow \frac{\partial\gamma_y}{\partial g_L} = \frac{-[\Upsilon R(-2)+(-1)]}{\Upsilon R(\theta-1)+\theta)} = \frac{2\Upsilon R+1}{\Upsilon R(\theta-1)+\theta} \Rightarrow \frac{\partial\gamma_y}{\partial g_L} > 0$

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