

Interview Article

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My introduction to copulas

An interview with Roger Nelsen

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Roger Nelsen is Professor Emeritus of Mathematics at Lewis & Clark College in Portland, Oregon, USA. He studied mathematics at DePauw University (BA, 1964) and Duke University (PhD, 1969). Roger joined the faculty at Lewis & Clark in the fall of 1969, and retired in 2009. Prior to Lewis & Clark, Roger spent a year with the Biostatistics Unit of the Centre International de Recherche sur le Cancer in Lyon, France. He has had visiting appointments at the University of Massachusetts in Amherst and Mount Holyoke College in South Hadley, Massachusetts. In addition to his monograph *An Introduction to Copulas*, Roger has authored or co-authored eleven books published by the Mathematical Association of America. He has served on the editorial boards of two MAA journals and several of their book series.

The fifth interview of this series features a conversation with Roger Nelsen. His Springer book *An Introduction to Copulas* is eponymous for a mathematical precise and well written entrance into the fascinating field of copulas. Moreover, it serves as a classical reference book, a large number of citations bear witness to this. He collaborated with seminal researchers in our field, and kindly shares his view and memories with us. Roger's second strand of research – a combination of mathematical puzzles, art, and visualization – are “Proofs Without Words.” He published several books, containing elegant visual justifications of mathematical statements. In the following, our questions to Roger Nelsen are typeset in bold-face.

1 Biographical information

Was there any particular event or person during your childhood that influenced your choice to study mathematics?

As a child I always enjoyed puzzles, especially those using arithmetic and elementary geometry. My father was an engineer, so I had a great resource for answers to my questions. But my father rarely answered my questions; he would in turn ask me questions, to enable me to more or less solve the puzzle or problem on my own. In high school I had a truly inspiring teacher. My high school offered two levels of a one-semester calculus course (high school calculus was rare in Indiana in the 1950s), and I was in the upper level. With one semester of that high school course, I was placed into third semester calculus when I entered DePauw University. The credit belongs to that very special teacher. He passed away several years ago, and I wasn't surprised to learn that many of his students went on to earn PhDs in mathematics.

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Figure 1: Roger Nelsen in 1963 when he was an undergraduate at DePauw University.

You studied in the wild sixties. In retrospective, what do you think of these times and how did these influence your life?

I graduated from high school in June of 1960 and entered college that fall, and received my PhD in June of 1969. So I spent “the Sixties” in universities (but sociologists told me that “the Sixties” really date from the assassination of John Kennedy in 1963 to the resignation of Richard Nixon in 1974). The decade of the Sixties definitely influenced my later life because of where I was and what I was doing at the time. I was, however, less influenced by events associated with that decade. The myth of the Sixties is sex, drugs, and rock and roll, but no way (especially in rural Indiana and small town North Carolina) did any of that get in the way of the job at hand – making grades and staying in school.

Where and when did you decide to get a PhD?

At DePauw University during my senior year (1963–1964; see Figure 1). At one time I thought I would like to become an actuary, so I sat for and passed the first two exams for the Society of Actuaries in May of 1963, and had a trainee position in an insurance company in the summer of 1963. That ended my aspiration to be an actuary. In addition, most of the professors of mathematics at DePauw were more than excellent teachers, they were good mentors and advisors – and friends. I could go to the Math Department, have coffee with two or three of the faculty and talk mathematics, talk baseball, talk whatever. Being a college teacher at a small school like DePauw (or Lewis & Clark) seemed very appealing. Recall that in 1963 the U.S. was at war in Vietnam, and young men were subject to the draft for military service. Members of the U.S. Congress, aware and concerned that the U.S. was behind the Soviet Union in the “space race,” passed the National Defense Education Act (NDEA) in 1958, which in part provided funds for graduate fellowships for students in mathematics and the sciences – and, as a bonus, priority was given to students with a stated interest in becoming a professor. So, with the help of the math faculty at DePauw, I applied to several universities for admission to graduate studies in mathematics – to become a math professor – and was fortunate to receive an NDEA fellowship for study at Duke.

How did you then become a professor of mathematics? Ever regretted?

I have never regretted my career choice. I attended the joint meetings of the American Mathematical Society and the Mathematical Association of America in New Orleans in January 1969, went to the “Employment Register” (the job fair), interviewed with several small colleges and universities, and was fortunate to be hired by Lewis & Clark to begin teaching in the fall of 1969. In hindsight I was fortunate. Not everyone gets it right the first time, and I was able to retire from my one and only position forty years later.

Was there a person (or event) of particular importance for your professional career?

Yes: my sabbatical year 1983–1984, which I spent at the University of Massachusetts in Amherst. At that time I wasn’t doing any research to speak of (my thesis research in queuing theory had led nowhere). Earlier that year the math department at Lewis & Clark had received an announcement from UMass seeking applica-



Figure 2: From left to right: Roger Nelsen, Bert Schweizer, his wife Judie, and Claudi Alsina in 2009 during a visit to the construction of the Sagrada Família in Barcelona, Spain.

tions for their “sabbatical lectureship” program, a program for small college faculty wishing to spend a year in a research institution. I applied. I was accepted, and drove east. When I arrived in Amherst, I went to the office of the Director of the Sabbatical Lectureship Program to introduce myself and offer thanks. The Director? Professor Berthold Schweizer. We talked a bit, he asked about my interests, I mentioned probability. He took a book off his shelf, handed it to me, and said that I might enjoy reading Chapter 6. The book? *Probabilistic Metric Spaces* by B. Schweizer and A. Sklar, published earlier that year; see [34] (reprinted in [35]). The title of Chapter 6? *Copulas*. I did enjoy reading Chapter 6 (and more). It changed the direction of my career.

Perhaps, nowadays, not everyone in dependence modeling has heard about Probabilistic Metric Spaces. Could you briefly explain what they are and why copulas appear in this context?

Sure. Recall that (informally) a metric space consists of a set S and a metric d that measures “distances” between points, say p and q , in S . In a probabilistic metric space, we replace the distance $d(p, q)$ by a distribution function F_{pq} , whose value $F_{pq}(x)$ for any real x is the probability that the distance between p and q is less than x . The first difficulty in the construction of a probabilistic metric space arises when one tries to find a probabilistic analog of the triangle inequality $d(p, r) \leq d(p, q) + d(q, r)$, that is, what is the corresponding relationship among the distribution functions F_{pr} , F_{pq} , and F_{qr} for all p, q , and r in S ? Karl Menger (Bert Schweizer’s PhD advisor) proposed

$$F_{pr}(x + y) \geq T(F_{pq}(x), F_{qr}(y)),$$

where T is a triangular norm, or *t-norm* (see [24]). Like copulas, *t-norms* map $[0, 1]^2$ to $[0, 1]$ and some of them can be used to join distribution functions. In fact, a *t-norm* is a bivariate copula if and only if it is 2-increasing, and a bivariate copula is a *t-norm* if and only if it is commutative and associative (see, e.g., [1, 23]). So, in a sense, it was inevitable that copulas would arise in the study of probabilistic metric spaces.

Do you have a scientific role model?

Without a doubt the role model and mentor in my adult life was Bert Schweizer; see Figure 2. The sabbatical year at UMass was just the beginning of a long and productive relationship. I will forever be grateful for his wise counsel and guidance.

You have worked for a long time in a college and have been clearly very active in research. Are these two activities compatible? Or was your experience quite singular?

I think the two are quite compatible, for persons who enjoy both research and teaching. When I first started to teach at Lewis & Clark, research was encouraged but not required. The teaching load was 6 courses a year, which took up most of one’s time (there were no graduate students at the College, and still aren’t). But during the late 70s and 80s, that changed. The teaching load decreased to 5 courses, and the expectation shifted towards the idea that to be an effective and inspiring teacher, one needs to be active in research. I



Figure 3: The participants of the conference *Distributions with given marginals and statistical modelling*, held in Barcelona in 2000 (see the proceedings in [5]). Roger Nelsen is standing, fourth from the left.

agree; it makes one's teaching better. It is common now for the math faculty at Lewis & Clark to do publishable joint work with undergraduate students.

Being in academia comes with the benefit of traveling to exotic places and meeting interesting people. What journeys (e.g. to conferences) were particularly remarkable and what meetings with people do you remember with pleasure?

After the 1983-84 sabbatical year, I returned to Massachusetts in the fall of 1986 for a one semester visit to Mount Holyoke College in South Hadley. That semester began with the 24th International Symposium on Functional Equations (ISFE), held at Mount Holyoke. It was at that meeting where I met Claudi Alsina, Juan Quesada Molina, Carlo Sempì, Howard Sherwood, and Mike Taylor. Abe Sklar and Jerry Frank were also there; I had met them during the sabbatical year. I think it was later that fall that I met Christian Genest (see [10]) when he came to UMass to present a talk. Another memorable conference was the first of the symposia on distributions with given marginals, held in Rome in April 1990; see the proceedings in [7]. At that conference I met Giorgio Dall'Aglio (see [11]), Ingram Olkin, Al Marshall, Carles Cuadras, Sam Kotz, Ludger Rüschendorf (see [9]), Marco Scarsini, José Antonio Rodríguez Lallena, and many others.

Many times you have collaborated with colleagues in Spain. How did this collaboration start?

As I mentioned earlier, I had met Claudi Alsina, Juan Quesada Molina, and José Antonio Rodríguez Lallena at conferences in 1986 and 1990. In May 1995 the 33rd ISFE was held in Caldes de Malavella, Spain (near Barcelona), and after that conference I travelled to Granada to visit Juan. We had collaborated earlier with Bert Schweizer and Carlo Sempì on the paper [30] via email. José Antonio had completed his PhD a few years earlier under Juan's direction (see [33]), and had joined the faculty at the University of Almería. It may have been at that time (or perhaps a subsequent visit to Granada) that I met José Antonio's student Manuel (Manolo) Úbeda Flores, and accepted their invitation to be a co-advisor for Manolo's thesis [36]. I consider this a special honor, as I have never held a position in a PhD-granting institution. Suffice it to say that I think the four of us (Juan, José Antonio, Manolo, and me) have made a pretty good team.

Besides research, what are your recommendations for a trip to Spain?

My recommendations for visiting Spain? First of all, do visit! Spain is a country of contrasts, be sure to visit Andalusia and Catalonia. I have yet to visit the Basque country and Galicia, but they are on my bucket list. When you go, spend some time, at least two or three months. The art, architecture, music, food, wine, all are great, but be sure to get to know the people. At Lewis & Clark we encourage our students to spend at least one semester studying abroad, and about two thirds of them do so. Many of the College's overseas programs utilize a faculty member to lead the program, and so after the visit to Spain in 1995 I studied Spanish and had the good fortune to lead our program in Sevilla three times, in 1999, 2003, and 2009. Each time I stayed in Europe after the program ended, and visited colleagues in Spain and Italy.

Can you describe a recipe for becoming a great researcher: are the ingredients technical skills, dedication, and having ideas? In which proportion? To which extent does luck or perhaps even “relational skills” play a role?

No, not really. I don't consider myself a “great researcher.” But I'll give you my recipe anyway. It's simple: who you know is just as important as what you know. Attend conferences, meetings, workshops, etc. Meet people. Cultivate collaborators and coauthors. Luck does play a role – I've been blessed to be in the right place at the right time to meet my collaborators. I think of mathematical research as a team sport – and I've been fortunate to have teammates from Spain, Italy, Belgium, Mexico, Brazil, Argentina, Turkey, and of course the U.S. The work I feel the best about is the work I've done with others.

Where do you usually get your inspirations and what was the most improbable place to have a mathematical idea?

The inspirations most often come from discussions with colleagues. There is something to the adage that “the whole is greater than the sum of its parts” when it comes to mathematical research! The most improbable place for an inspiration was undoubtedly on a beach in Almería one beautiful afternoon. I was in Almería with Manolo, José Antonio, and Juan working on a paper on properties of quasi-copulas. It was our habit to work together in the morning, then go our separate ways in the afternoon. One afternoon Juan and I went to the beach. We had been searching for an example of (or a counterexample to) one suspected property of quasi-copulas that morning, and were still thinking about it that afternoon. Without paper or a black board, we had only the sand to draw support diagrams – and that's the way we found the example (or counterexample) we were searching for.

2 The seminal book *An Introduction to Copulas*

What inspired you to write *An Introduction to Copulas* ([28], second edition in [29])?

I was in Amherst in November 1995 to attend a symposium at UMass celebrating Bert Schweizer's retirement. At a coffee break after one session with several talks about copulas, someone asked several of us what one should read to learn more about them. We mentioned several research papers and conference proceedings. I think I then mentioned that perhaps the time had come for “someone” to write an introductory monograph on copulas. A colleague, I forget whom, responded “Good idea Roger, why don't you write it?” I let the comment lie dormant until the following September when I was in Prague for the third conference on distributions with given marginals; see [3]. I asked Abe Sklar, Giorgio Dall'Aglio, and Ingram Olkin if they thought there might be interest among statisticians for such a book. They responded positively, and knowing that I was eligible for a sabbatical in 1997-1998, I began to give the idea of writing the book some serious thought.

Your book contains a long, analytical proof of Sklar's theorem. Where does the proof come from? Was Abe involved in this part of the book?

The outline for the proof comes from the book *Probabilistic Metric Spaces* by Bert and Abe. There it is given for the n -dimensional case, but it is incomplete. I re-worked it for bivariate copulas, and Bert and Abe helped me fill in some of the gaps.

Which researchers inspired and supported you during this project?

I wrote most of the book during my sabbatical leave in 1997-1998. I am grateful to my colleagues at Mount Holyoke College for graciously inviting me to spend the year with them. One advantage to working at Mount Holyoke was that it is only about ten miles south of UMass – and Bert Schweizer.

Was it difficult to find a publishing house?

No. At the time I was working on the book Ingram Olkin was one of the editors of Springer's *Lecture Notes in Statistics* Series, and he encouraged me to submit the manuscript to Springer.

Many notations and names of copula families have been introduced in your book and became standard in the community. Why did you use the symbols W , Π , and M for the three basic copulas?

Most of the names come from the literature, either from the authors (e.g., Ali–Mikhail–Haq), or the name used by the authors for the functions (e.g., ordinal sums and shuffles of M). About the symbols M , W , and Π



Figure 4: From left to right: Carlo Sempi, Abe Sklar, Gianfausto Salvadori, Bert Schweizer, Fabrizio Durante, and Roger Nelsen in 2004 during the DeMoSTAFI conference held in Québec from 20-22 May. This international meeting was intended as a sequel to the series of conferences on copulas and their applications that were held in Rome (1990), Seattle (1993), Prague (1996), and Barcelona (2000).

for copulas: they are the symbols used in the book *Probabilistic Metric Spaces* (see [34]). The symbols Π and M obviously stand for *product* and *minimum*. I think W is an upside-down M , since as a bivariate copula the graph of W is a twisted reflection in Π of the graph of M (see eq. 5.6.5 in [34]).

In your book, you presented a long list of bivariate copula families, some of them named after one or more colleagues who have worked on these. What is your “favorite” family?

It’s probably the family known as *Shuffles of M* (see [25]). I’ve found it to be quite useful for finding counterexamples to certain implications concerning dependence properties, and to show that certain inequalities are best-possible. It’s also easy to draw diagrams of their support.

Would you share some anecdotes about your book?

I can’t tell you how many times I’ve been told that it’s not a statistics book (i.e., no data), in spite of its appearance in the *Springer Series in Statistics*!

How many copies of your book have you signed?

Not too many. Maybe a dozen – maybe two dozen, no more.

What was your worst experience with the review process of any of your papers? Did you experience that it became easier to get your papers published after your book on copulas became a standard reference?

At first it was difficult to find the right journal for early papers on copulas with Bert. When we submitted the paper to a statistics journal, the rejection letter often stated that the paper was “too mathematical”; when submitted to a mathematics journal, “too statistical.” This was not surprising, as the copula concept does not appear to have been well known in the statistics community at that time. The first appearance of “copula” as a keyword in the *Current Index to Statistics* was in 1981, and in the first 18 volumes of the *Index*, there were only 11 references (total, not per year) to papers about copulas. That has clearly changed, and the book may have played a role. According to the Google Scholar website, the book in its two editions has been cited 10,000 times (as of 2017). That’s hard to believe – makes me think that one should not trust data one finds on the Web.

3 Copulas

When did you realize that copulas have become so popular?

About the time when I began to see papers with applications in finance, actuarial science, hydrology, etc.; one of the early papers concerning applications is [4]. An even earlier paper is [22]. I discussed copulas with Bob Clemen when he was at the University of Oregon; Jouini was his PhD student at the time.

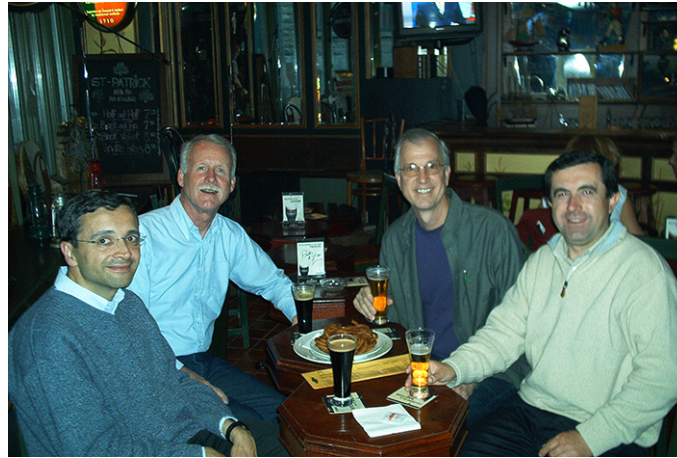


Figure 5: From left to right: José Antonio Rodríguez Lallena, Roger Nelsen, Greg Fredricks, José Juan Quesada Molina in 2004 during the DeMoSTAFI conference held in Québec from 20-22 May.

How was your collaboration with Abe Sklar and Bert Schweizer in the early age of copulas?

Most of the early collaboration was with Bert and those who had worked with Bert such as Jerry Frank and Claudi Alsina. Although Abe and I often discussed issues related to copulas, we don't have a joint paper, something I really regret.

Most young scientists have not had the chance to meet Abe Sklar. What kind of scientist is he? Is he surprised/happy that “his theorem” has become so famous?

Abe is a remarkable person. He is not only a fine mathematician, he is a warm, friendly, and kind person. He has a dry but keen sense of humor. His writing is precise and to the point. I suspect that he is pleased about the theorem – not so much that it is “famous,” but that it has found so many applications.

Is it correct to say that you started with an analytical view on copulas? During your career, has this ever changed to a probabilistic or statistical perspective?

The analytic view is no doubt due to the fact that my first exposure to copulas was via Bert and Abe's book *Probabilistic Metric Spaces*. As I've learned more about copulas, my view has become more multidisciplinary. I don't see the three perspectives you mention as being mutually exclusive.

Why did it take many years before copulas were appreciated in statistics and applications? What was the triggering event (if any) that has changed the development of this concept?

As I mentioned earlier, it was difficult at first to find journals interested in publishing papers about copulas. The copula concept was not new. It is present in the work of Maurice Fréchet [15], Giorgio Dall'Aglio [6], Robert Féron [12], Wassily Hoeffding [19, 20], and later Paul Deheuvels [8], Janos Galambos [17], and others. I suspect that the sequence of conferences on distributions with fixed or given marginal, beginning with the one in Rome in 1990, played a role in bringing together researchers from many countries.

It seems that you like to represent graphically mathematical concepts (see the PWW section below). In fact, some of your works are related to “geometric” constructions of copulas. When did you start considering such constructions? What was the main motivation behind?

You are correct; I like to illustrate mathematical concepts geometrically when it's possible. The geometric interpretation of positive quadrant dependence for two random variables – the graph of their copula lies on or above the graph of Π – led me ask if there are similar geometric interpretations of other dependence properties. The motivation for me was simple: if I can visualize something I'm more likely to understand it.

In one of your articles with Frank and Schweizer you presented a solution of the best-possible bounds for the sum of two random variables; see [14]. How did you arrive at the problem? Have you ever suspected that the problem would become so popular in risk management years later?

When I arrived in Amherst in the fall of 1983 Bert and Jerry had already started on that paper. After I learned a bit about copulas and triangle functions (Chapters 6 and 7 in *Probabilistic Metric Spaces*), they

invited me to join them on the paper, part of my training in working with copulas. At the time we had no idea that copulas would find all their current applications.

In one of your articles with Alsina and Schweizer, you introduced the concept of quasi-copulas. What was your original motivation?

The article to which you refer is the 1993 *Statistics & Probability Letters* paper characterizing a class of certain binary operations on distribution functions derivable from functions on random variables; see [2]. We discovered that the operation in the characterization was somewhat more general than a bivariate copula. The operation is “almost” a copula, hence our choice of the name *quasi-copula*. Our definition in that paper is cumbersome, and it was soon replaced by a much better one in a 1999 paper by Christian Genest, Juan Quesada Molina, José Antonio Rodríguez Lallena, and Carlo Sempi; see [18].

You contributed to popularizing Archimedean copulas. Why are they so popular in your opinion?

I think Archimedean copulas are “popular” for several reasons. First, they are rather easy to construct from univariate functions (the generators). Secondly, members of the Archimedean family exhibit a wide variety of dependence structures. Although the basic Archimedean copulas are bivariate with a single real parameter, many can be extended to yield multivariate copulas, and it is often possible to add parameters for more flexibility.

In your book you listed 22 families of Archimedean copulas. How did you select them? Why not 23 or 31?

When I was working on the first edition of the copulas book, Claudi Alsina, Jerry Frank, and Bert Schweizer were working on their book about associative functions on intervals (see [1]). They kindly shared a draft of that book with me, which included a table of 25 families of one parameter triangular norms. Many (but not all) triangular norms are copulas, and Claudi, Jerry, and Bert permitted me to adapt their table, using the families in their list that included copulas. I did not include two of their families in which there were no copulas and one family with very few copulas.

You “translated” many dependence concepts into the language of copulas. Now, while independence is clear, dependence has many facets. How would you explain “dependence”?

For me it’s like the “negative” definition of an irrational number – it’s a real number that is not rational. Similarly dependence has a “negative” definition – it is any relation between random variables other than independence. Since (in the continuous setting) each pair of independent random variables has the copula Π , dependent pairs are those with any copula other than Π .

Is there any problem related to dependence/association where copulas should not be used?

I must admit I’m not a fan of the normal copula. It seems to me that many of the important properties of the bivariate normal distribution are lost when the margins are replaced by non-normal distribution functions. But I’m not a statistician, and I’m sure many researchers would disagree with me.

What is the mathematical result/scientific paper you are most proud of?

As I recall, pride is one of the seven deadly sins, so I hesitate to answer this question! However, there are a couple of results in my papers that I rather like. One is in the 2005 *Comptes Rendus* paper [31] with Manolo Úbeda Flores, showing that the set of bivariate quasi-copulas is the lattice-theoretic completion of the set of bivariate copulas. This paper grew out of a speculation Bert Schweizer made at Manolo’s thesis defense in 2001. Another is the result in the 2007 *JSPI* paper [16] with Greg Fredricks, where we found sufficient conditions on the copula for the values of the population version of Spearman’s rho to be about 50% larger than those for Kendall’s tau for distributions near independence.

What is the copula or dependence result you would have expected to be very difficult but has been solved? What is the copula or dependence related result you would have expected to be very easy but still is open?

Excellent questions, but hard to answer. What you find to be easy, I probably find to be difficult. But rarely the reverse.

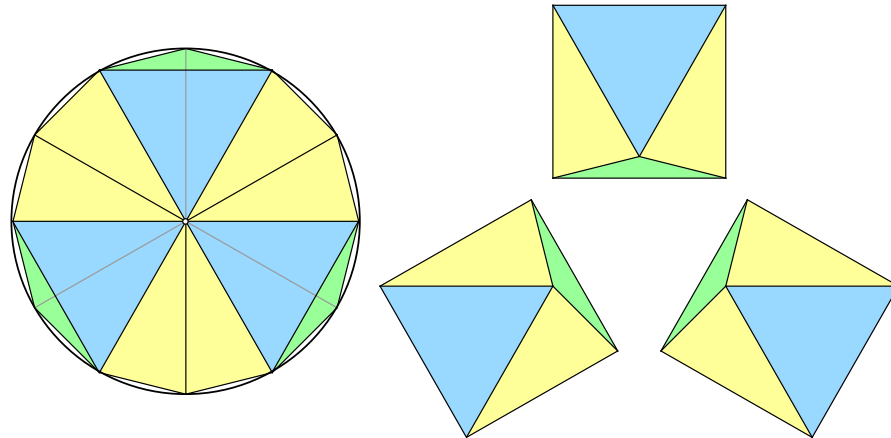


Figure 6: Proof without words: the area of a regular dodecagon inscribed in a unit circle is 3. © Mathematical Association of America, 2015. All rights reserved.

4 Proofs Without Words

One of your favorite activities is to create proofs that explain mathematical statements without words. In short: “Proofs Without Words” (PWWs). How did this interest arise?

In the fall of 1975 the first “proofs without words” appeared in *Mathematics Magazine*, see [21], and soon PWWs were appearing regularly as end-of-article filler. Like lots of readers, I enjoyed the mental challenge of figuring out how a PWW illustrates the truth of a statement and hints at its more formal proof. I thought that perhaps I could create PWWs, and after a number of disappointing efforts, I succeeded in having my first PWW appear in the *Magazine* in 1987; see [26] and [27]. Collecting and creating PWWs continues to be an enjoyable hobby.

Is creating a PWW more art or science?

Probably a combination of the two. For me it is like solving a puzzle. Here’s an example: Theorem: The area of a regular dodecagon inscribed in a unit circle is 3. The puzzle? Dissect the dodecagon into a finite number of pieces and reassemble them to form three unit squares. And do it with not too many pieces (that’s the science). Some symmetry in the solution would look nice, too (that’s the art).

What is your favorite PWW?

Currently it is the one I’ve just described. My favorites are those that are totally visual, with no labels or equations. Here’s the PWW (see Figure 6) for the puzzle in the answer to the preceding question (which appeared in the January 2015 issue of the *College Mathematics Journal*). I hope readers noticed that the reassembly of the pieces into squares requires only translations; no rotations or reflections are needed.

What do you think about the argument that Archimedes – on purpose! – did not draw geometric figures in the correct scale and reduced his diagrams to their topological features, in order to avoid drawing the wrong conclusions from a specific figure (See Ch. 4 in [32])?

I think George Pólya would agree, when he wrote “Geometry is the art of correct reasoning on incorrect figures.” And maybe so did Jean Dieudonné when he wrote “I have decided to introduce not a single figure in the text.” But yet Pólya’s advice in his classic book *How to Prove It* is “Draw a figure.” As I wrote in the introduction to the first collection of PWW, if PWW are not proofs, what are they? They are pictures or diagrams that help one see why a particular statement is true, and how one might go about proving it is true.

Is there any PWW in the field of copulas/dependence modeling?

I haven’t seen one yet. But I haven’t given up looking for one, either.

What activities do you enjoy after work?

Being retired I don’t consider anything I currently do “work.” I enjoy travel, especially visiting my sister in Colorado and her sons and grandchildren, and visiting my brother and his wife and their sons and grand-

children in Texas. I really enjoy going to conferences overseas, not just for the mathematics, but also to visit good friends, and make new ones. Here in Portland, I enjoy outdoor activities, visits to the Oregon Coast, the Columbia Gorge, and to the Cascades. For many years I had a small sailboat (named the *Aftermath*) on the Willamette River, and enjoyed sailing with friends until the boat disintegrated recently. Maybe I should get a new one.

A final question: Will you miss academia and what do you wish the copula community for the (near and far) future?

I am very grateful to Lewis & Clark for letting me continue to be a member of the math department with an office, library privileges, internet, etc. So in a sense I don't miss academia—not yet. My wish for the copula community is simple: I wish each member will enjoy the collegiality of this community and its many friendships as I have, and have as much pleasure working with others in the community as I have enjoyed the past 30-plus years. It is a remarkable community, and I am honoured to be part of it.

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