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Milano Retinex family

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Abstract. Several different implementations of the Retinex model have been derived from the original Land and McCann's paper. This paper aims at presenting the Milano-Retinex family, a collection of slightly different Retinex implementations, developed by the Department of Computer Science of Universitá degli Studi di Milano. One important difference is in their goals: while the original Retinex aims at modeling vision, the Milano-Retinex family is mainly applied as an image enhancer, mimicking some mechanisms of the human vision system. *© 2017 SPIE and IS&T* [DOI: 10.1117/1.JEI.26.3.031207]

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1 Introduction

Edwin Land's Retinex theory is a model of human color vision.[1] Retinex theory is reviewed in a paper in a "Retinex at 50" special section called "Retinex at 50: color theory and spatial algorithms."[2] This model uses three independent channels (each one related to a different type of cone) to calculate color sensations. Land and McCann[3] used a model of vision as the core of image processing algorithms that mimic human vision.

As an example, Land and McCann modeled lightness sensations in Land's "black and white Mondrian" experiment (Fig. 1).[3] Here, an achromatic Mondrian is placed under a gradient of illumination, darker at the top and more intense at the bottom. The gradient of illumination was carefully placed to control the light coming from the two different papers at the tips of the two arrows (Fig. 1). The upper paper patch had a high reflectance in dim illumination, while the dark reflectance paper in the bottom had higher illumination. The gradient in illumination was smooth and was difficult to notice. The change in illumination was as large as the change in reflectances. The top high-reflectance paper in dim light had the same luminance as the low-reflectance paper in bright light.

Land's experiment asked observers about the appearance of the two identical stimuli of Fig. 1. The observers reported the sensation near-white at the top and the sensation near-black at the bottom.

The challenge was to describe a computational algorithm that could predict the appearance of all areas in the black and white Mondrian, computing different observer responses to identical input stimuli.

Figure 2 is an example of how spatial content can influence the appearance. When the change in luminance is a gradient, then the change in appearance, associated with digits 160 and 200, is small (Fig. 2, top right), but just noticeable. When the change in luminance is an edge, then the same change luminance (from 160 to 200) is large (Fig. 2, bottom right).

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2 Retinex Model

Retinex has been designed as a model for predicting the above described differences in local sensation. It is a model of human vision: the idea was to make better reproductions by incorporating an algorithm that mimics vision. Land, McCann, and colleagues did extensive measurements of appearance in a wide variety of scenes where appearances did not correlate with luminances.[5]

The idea at the base of Retinex, a contraction of the words retina and cortex, is that these two parts of human body, which compose our vision system, realize a robust adjustment to compensate for the high photometric and colorimetric variability of the world around us. This is realized by spatial comparisons within the various areas of the visual input. Such comparisons are modeled as a series of ratios and multiplications among near and far areas. They write, "This multiplication can be done in a variety of ways. The simplest is to generate a series of paths [. . .] The operation along each path takes the ratio of two adjacent points and multiplies it by the ratio of the next pair of points along the path."[6] An example of Retinex computation relative to the target presented in the introduction is shown in Fig. 3. A Retinex elementary piece of computation is a ratio among two adjacent pixels, e.g., along the path shown in Fig. 3. This ratio propagates through the image with a subsequent series of multiplications. These operations are called "ratio chain." Even if this is not the case of the example shown in Fig. 3, according to the values found along the path, the ratio chain can overtake the unitary value. In this case, a reset to one is applied and the chain restarts from this value. This happens when an area lighter than the starting one is encountered. When we include this mechanism we refer it as "ratio-product-reset." Reset is a fundamental operation that characterizes Retinex.

In some Retinex works, a threshold mechanism along the ratio chain is discussed and used: if the ratio does not differ from the unit more than a certain threshold, the ratio is set equal to 1. After many tests, threshold has been proved not to be essential; thus, we only mention it and suggest that readers refer to Ref. 4 (p. 298–299).

Retinex always underlined its experimental origin and its clear goal of modeling vision: "We are not implying that the visual system computes its lightness response in the manner of a digital computer, i.e., one ratio and product at a time. The model described in this paper is one of the many embodiments using the ratio-multiplication process. It is the description of a digital two-dimensional computation for testing the processing principles."[6] There are many variants derived from the Retinex model by Land and McCann. A general overview is given in the work by McCann,[7] at the Retinex at 50 workshop.

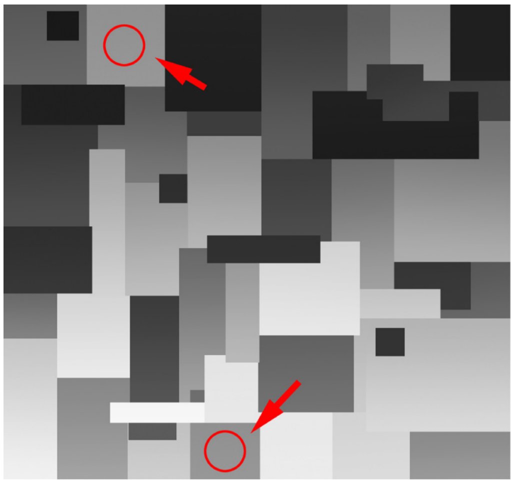

Fig. 1 Black and white Mondrian (Ref. 4 p. 71).

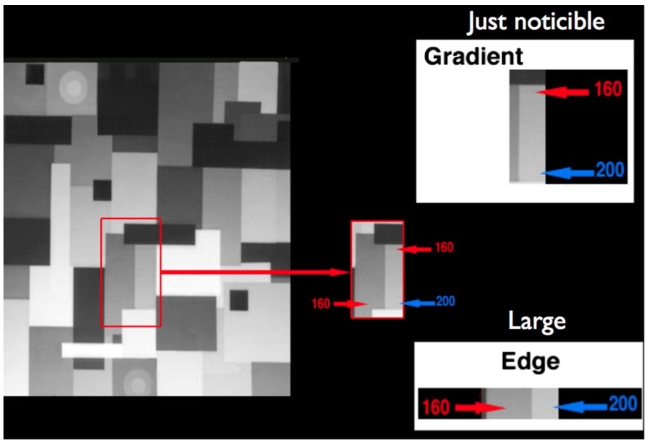

Fig. 2 Sensation difference between a gradient and an edge with the same values (Ref. 4, p. 298).

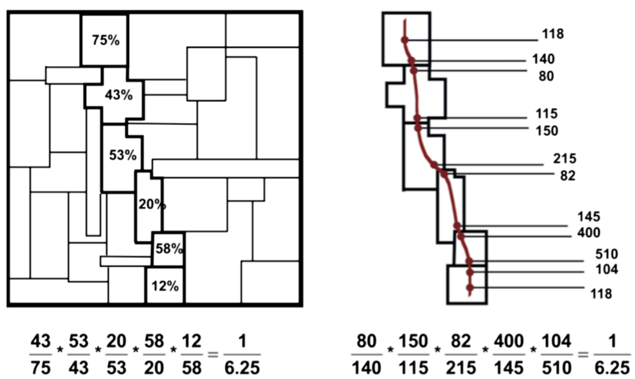

Fig. 3 Retinex processing over the black and white Mondrian. (Refs. 4, 5, p. 295).

Since the main goal of Retinex is to model human vision, they paid particular attention to the calibration of both the input and the output values. This is a mandatory procedure, described in a McCann et al. paper[6] and Ref. 4 (p. 70–71). However, since the Retinex model was the first attempt to model human vision computationally, it had a noticeable success; as a consequence of this success, its goal extended from modeling vision to enhancing images, such as the Milano-Retinex family, presented here [for a more complete description, see Ref. 4 (p. 324–328)]. Moving from modeling vision to image enhancement results in different needs about input and output calibration and different measures of success.[4,8]

3 Milano-Retinex

At the base of Retinex computation is the idea, presented above, of performing spatial comparisons among areas of the visual input (image) and computing a chain of ratio-product-reset. The ratio-product reset mechanism is also the common core of the Milano-Retinex family. Although the basic computation at each elementary iteration is the same, the way it is applied through the image generates differences.

3.1 *Milano-Retinex Origin and Its Differences*

The Milano-Retinex family started in 1993[9] with the publication of the graduation thesis of one of the authors. The main difference between this version and the original Retinex is presented in Fig. 4 on an example path: the

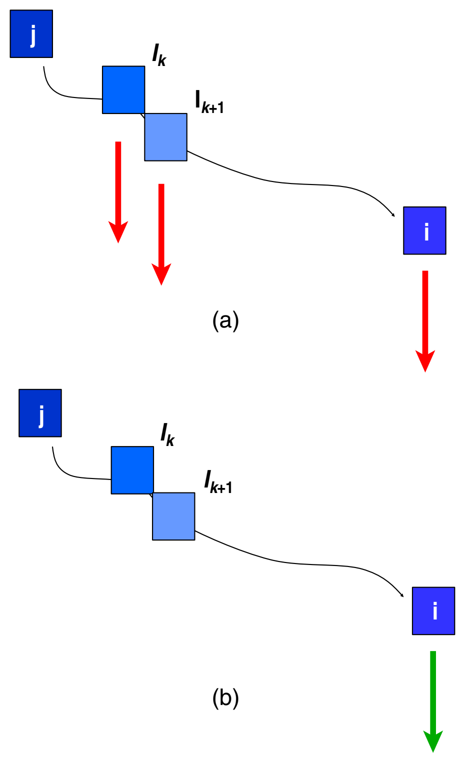

Fig. 4 The difference between (a) the original Retinex and (b) Milano Retinex.

original Retinex with the red arrows and Milano-Retinex with the green arrows.

Each arrow indicates a ratio-product-reset-average process. In the original Retinex, it takes place at each step along the path and it is used to change the output values relative to all the pixels traveled by the path. The calculated value of the average is stored at the traveled pixel, to be used the next time that a path crosses that pixel. The reset process occurs many times along the path, particularly at the beginning of the calculation. Every ratio-product-reset contributes to the output [Fig. 4(a)].

The Milano-Retinex does not change the pixels traveled by the path. It stores the ratio-product information along the entire path from a starting pixel to the pixel of interest at the end of the path and changes only this last pixel value with the result of the chain of ratios-product-reset. The final value of the target pixel is the average of all the ratio-chains computed along every other path ending in the pixel of interest.

The original Retinex and Milano-Retinex share the same core, but the different way of implementing it results in important differences: the sampling of the input values (image), the normalization due to reset operation, and the average to form the final output (Ref. 4, p. 293–337). The ratio-product-reset is performed at each step, as well as the original Retinex, but the fact that the computed value is used to change only the last pixel gives the paths a different meaning. Now the output [green arrow in Fig. 4(b)] is the ratio of the pixel of interest to the pixel with maximum value found along the path.

Milano-Retinex clearly shows its redundancy, making the entire path computation only for a single contribution to the final pixel. The advantage of this redundancy is that it is easier to formalize the implementation of the basic mechanism along the path.[10] On a more general level, path role can be now seen as the search for the local channel maxima, necessary for the normalization of the last pixel in the path. This search needs to be nonexhaustive, otherwise Milano-Retinex ends up as a standard global Von Kries channel normalization.

3.2 *Milano-Retinex Formalization*

Marini and Rizzi[11] described the Retinex model through the following equations

$$L(i) = \frac{1}{N}\sum_k l^{i,j_k}, \tag{1}$$

where

$$l^{i,j_k} = \sum_{x\in \text{path}} \delta \log\left|\frac{I_{x+1}}{I_x}\right|, \tag{2}$$

and

$$\delta = \begin{cases} 1 & \text{if } \log\left|\frac{I_{x+1}}{I_x}\right| > \text{Threshold} \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

$L(i)$ is the computed lightness of a pixel i in a given chromatic channel. i is the end-point of a set paths, while j_k is the starting point of a generic k'th path. I_x is the input image value at the pixel x, while I_{x+1} is at the subsequent pixel $x+1$ along the path. δ represents the threshold mechanism that can be skipped in the implementation (see above comments). In this early Milano Retinex paper,[11] the reset operation is not included in the equation but is only presented in the implementation description. Provenzi et al.[10] presented a mathematical description of Milano Retinex, now with the reset mechanism included in the equations.

$$L(i) = \frac{1}{N}\sum_{k=1}^{N}\prod_{t_k=1}^{n_k-1}\delta_k(R_{t_k}). \tag{4}$$

$\delta_k : \mathbb{R}^+ \to \mathbb{R}^+$, $k = 1, \ldots, N$ are functions defined by

$$\delta_k(R_0) = 1 \tag{5}$$

and for $t_k = 1, \ldots, n_k - 1$

$$\delta_k(R_{t_k}) = \begin{cases} R_{t_k} & \text{if } 0 < R_{t_k} \le 1-\epsilon \\ 1 & \text{if } 1-\epsilon < R_{t_k} < 1+\epsilon \\ R_{t_k} & \text{if } 1+\epsilon \le R_{t_k} \le \frac{1+\epsilon}{\prod_{m_k=0}^{t_k-1}\delta_k(R_{m_k})} \\ \frac{1}{\prod_{m_k=0}^{t_k-1}\delta_k(R_{m_k})} & \text{if } R_{t_k} > \frac{1+\epsilon}{\prod_{m_k=0}^{t_k-1}\delta_k(R_{m_k})} \end{cases}. \tag{6}$$

This formalization describes the Milano-Retinex ratio-product-reset chain computation but not the way that it is spatially performed using path or alternative methods.

3.3 *Milano-Retinex Family*

Milano Retinex evolved mainly with changes in the image sampling method. This originated a family of algorithms grouped together, as shown in Fig. 5. Here we want to present a descriptive overview of its members with their main differences to have a bird's-eye view of the genesis and approaches beyond each one of them. For more technical details, we ask readers to refer to the cited works.

For the sake of completeness, Fig. 5 also contains ACE for automatic color equalization,[12–15] an algorithm that does not derive directly from Retinex but is an indirect product of the research on it. ACE substituted the random path search with a complete scan of all the pixels in the image, realizing a local behavior with a distance function and the dependency from image content with a contrast amplification function. The main problem of considering all the pixels in the computation of each single pixel is the overall computational cost. To speed-up the computation time, some accelerated implementations have been devised,[16–18] some together with a variational version.[19,20]

In the following, a brief description of the members of the Milano-Retinex family is presented.

To have a visual idea of Milano Retinex filtering, a couple of examples are shown in Fig. 6.

3.4 *Brownian Path*

Since the very beginning, the study of Retinex at the University of Milano has been centered on the characterization of the image exploration, initially studying pathwise methods. Initial tests using very simple linear paths showed

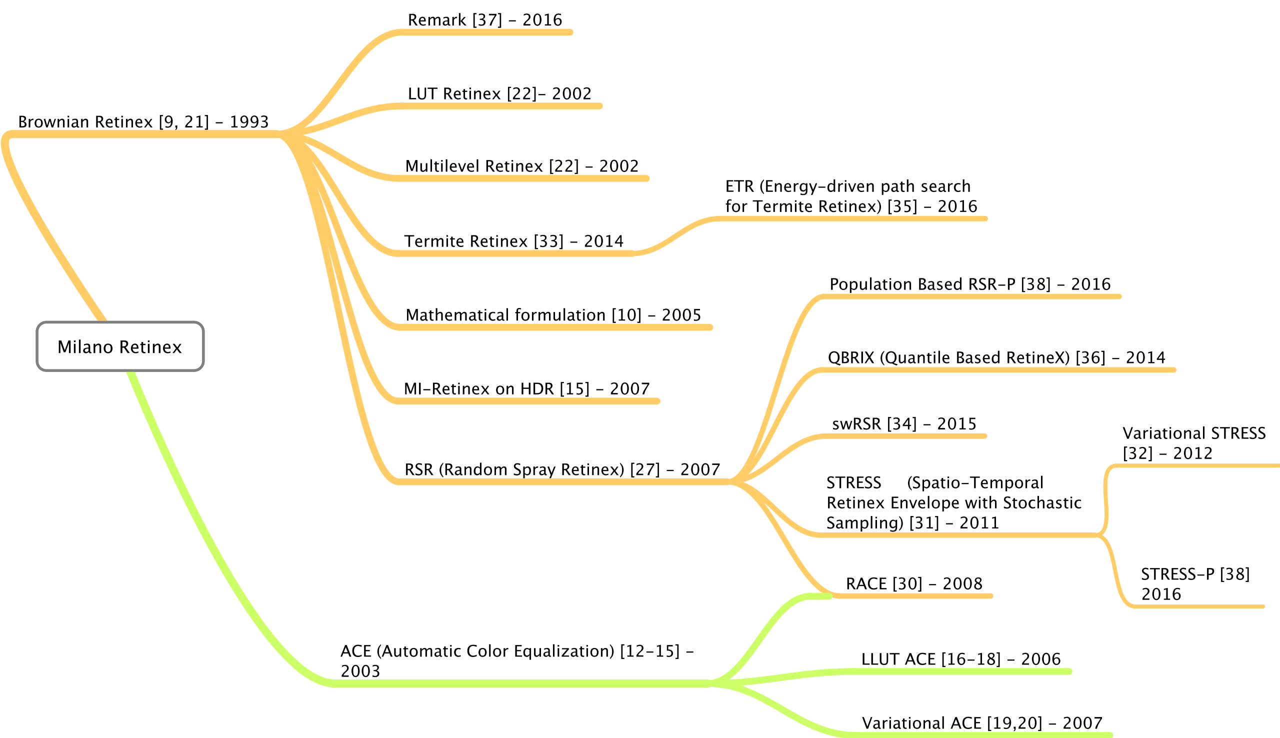

Fig. 5 The Milano Retinex family.

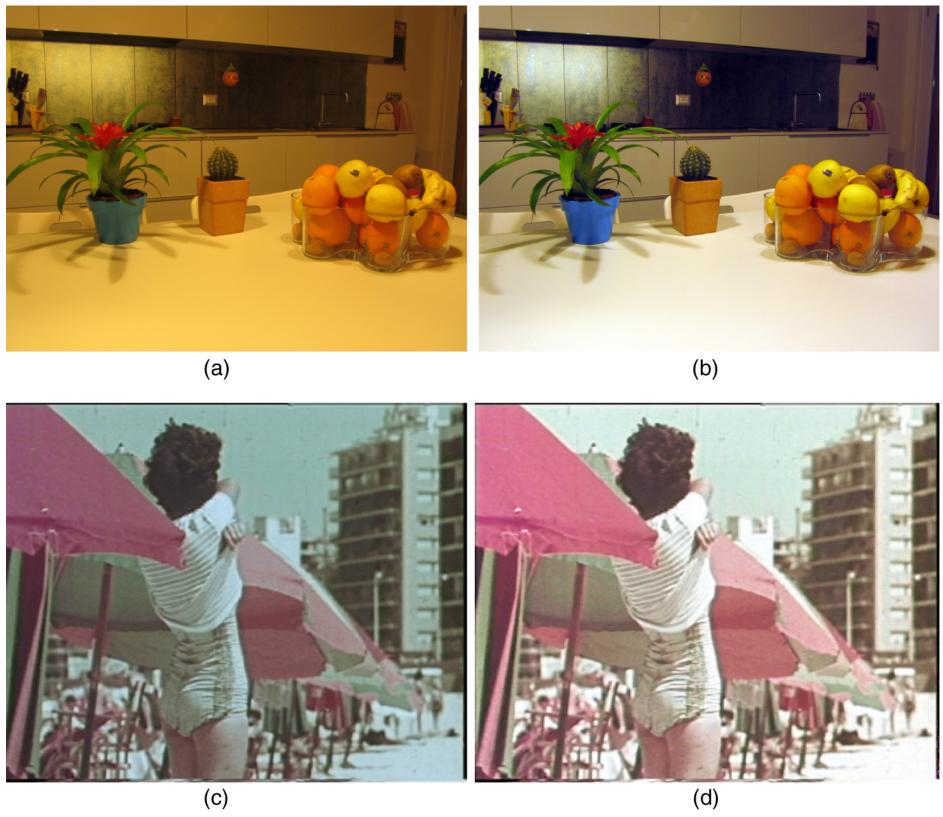

Fig. 6 Two examples of Milano-Retinex filtering. (a) original image, (b) the result of the filtering with STRESS, (c) original image, and (d) result of filtering with RSR.

that the characteristics of the path sampling could heavily influence Retinex output.

These preliminary tests lead to the first algorithm of the family, named Brownian Retinex,[21] where Brownian paths are generated with the midpoint displacement technique. The choice of Brownian paths was made to avoid directional bias that generates unwanted shadows and, at the same time, to realize an easy way to sample over a wide area.

To save on computational time, an LUT and a multilevel version were devised in 2002.[22] Initial interest was on color constancy[23,24] and color normalization.[25]

Since this is a Milano Retinex, which updates only the final pixel of the path, Brownian paths do not have to sample the whole image, thus avoiding a global normalization. On the contrary, Montagna and Finlayson realized a way to generate Brownian paths with the constraint of exploring all the pixels in the image, in this case, applied on the original Retinex.[26]

3.5 *Spray Sampling*

A noticeable quantum leap in computational costs arrived in 2007 with random spray Retinex (RSR).[27] Since in Milano Retinex only the pixel with maximum value found along the path counts, RSR substituted the Brownian paths with a random sampling made with a spray of points around the pixel to be computed. This simplification clearly shows the role of sampling and originated other probabilistic formulations (see Sec. 3.7). The random path exploration aims at finding the pixel with maximum value for every path that is independent from the path itself. This is substituted with the maximum value of pixels in the spray that is used as a local white to rescale the original pixel value.

Spray points generation uses polar coordinates; since the angular distribution of points is isotropic, when the radius increases, the density of pixels decreases.

To lower the noise, RSR uses N sprays; therefore, a final average between the spray contributions is computed.

We would like to note here that the noise problem has been addressed in the works of Banić et al.,[28,29] who developed light random spray Retinex (LRSR) in 2013 and smart light random memory spray Retinex (SLRMSR) in 2015. LRSR is able to remove the noise from the resulting image, reducing the high computation cost of RSR. SLRMSR is a further improvement that uses memory sprays to decrease the number of per-pixel operations of LRSR to a constant; then the algorithm reduces the halo effect present in LRSR and implements a different remapping of image intensities, for a better result.

From the mix of ACE and RSR originated RACE,[30] where the two approaches are mixed due to a spray-based implementation of ACE.

A variant of RSR is STRESS, spatio-temporal Retinex-inspired envelope with stochastic sampling.[31] The idea of STRESS is to calculate the local maxima (reference white, like RSR) and the local minima (reference black) for each pixel for each chromatic channel and then stretch the pixel values accordingly.

STRESS is implemented in GIMP (GNU Image Manipulation Program) from the version 2.8. A variational version is presented in Ref. 32.

3.6 *Image Driven Search*

The idea of paths is preserved in termite Retinex (TR).[33] In this implementation, the creation of paths is not completely random, they are realized by a swarm of agents, called "termites," that consider image contrast as a positive variable for influencing the path generation. Moreover, termites traveling along a path leave a trace called "poison" to avoid the use of previsited points in the generation of the next path.

Thus, an artificial termite at the pixel chooses to move to an adjacent pixel that does not currently belong to its working memory with a probabilistic method that considers the amount of poison of the pixels and the bilateral distance between them, with two parameters weighting the importance of the poison versus the closeness, which is directly related to the brightness of the pixel. If all the surrounding pixels have the same probability, one pixel is drawn randomly with uniform probability. In TR, the memory is the taboo list of the k'th termite, which contains the coordinates of the pixels that the termite has already visited. The poison is implemented as the inverse of pheromone; once a termite has transited on a pixel, the quantity of the relative poison is updated. This is the first work that implements a path generation dependent on the image content. The idea is that some areas are more relevant than others in the formation of the final visual sensation. The same approach is exploited by swRSR and ETR. In swRSR (spatially weighted RSR),[34] the spray sampling is weighted by a figure of distance. In ETR (energy-driven path search for TR),[35] the paths are computed as local minima of an energy functional, depending on the image gradient. In this way, ETR defines the paths through global mathematical conditions, not by step-by-step procedures as implemented, for instance, in TR.

3.7 *Statistical Models*

Approximate probabilistic models of spray sampling based on quantiles are the base of QBRIX,[36] with its global and local versions, and probabilistic models of sampling realized with random walks represented as reward Markov process are the base of remark.[37]

The key idea underlying QBRIX is that the sampling procedure used by RSR can be replaced by an equivalent one. The value computed by the RSR procedure corresponds, in statistical terms, to an estimator for the mean of the sampling maximum distribution (SMD); QBRIX consists of an alternative procedure that directly computes the SMD mean, based on the high percentile values of the pixels population. From a series of tests on a wider set of images, the authors of QBRIX found that a quantile between 95 and 99 leads to the most satisfactory results.

To implement a local behavior, we devised an alternative algorithm, named local QBRIX. Local QBRIX aims at using the same quantile approach as QBRIX, computing the locality by means of a distance-based weighting approach. The proposed strategy consists of computing a pixel-based reference white that changes according to the relative positions and intensities of all the pixels in the image, in relation to the target pixel.

Remark is the formalization of Retinex with absorbing Markov chains.[37] It passes from the path-sampling basic algorithm to the probabilistic representation of the corresponding diffusion process, solving the problem of sampling noise. Remark starts from the corresponding Retinex analytic model, accounting for the combined effects of path-function, path sampling process, and starting-point sampling process. Then, implementing a numerical solution, it computes the output brightness of a pixel based on the solution of a simple sparse linear system. The output of the random walk sampling algorithm and the Markov chain-based algorithm can be controlled by few model parameters.

Finally, RSR-P and STRESS-P[38] are two probabilistic formulation of RSR and STRESS, respectively. In RSR-P, the contribution of a spray to a target is computed by independently sampling *n* points and recording the reciprocal of the maximum. The quantity thus defined changes from spray to spray. Averaging *N* of such reciprocals is equivalent to computing an estimate of the sampling mean of such a random variable. In STRESS-P, a similar consideration holds. Both RSR-P and STRESS-P use samples from the population of sprays to estimate an average quantity; however, the whole population of sprays is available since it can be derived from the sampling function and can be computed directly, in a deterministic way. In the case of RSR-P, the sampling distribution of the maxima can be obtained by computing the probability that a given pixel becomes the maximum of a spray; in the case of STRESS-P, the relevant sampling distribution can be obtained by computing the probability that a pair of pixels form a minimum-maximum-pair of a spray and then summing over all the pairs yielding the same contribution.

4 Conclusions

This paper presents a brief overview about the Milano Retinex family. they all belong to the spatial color algorithms family[8] and go from the initial Brownian version to the more recent probabilistic formulations. Here, we introduce the differences in comparison to the original Retinex formulation and the rationale between the many Milano-Retinex variants. The main difference regards their goals: while the original Retinex model aims at predicting color sensation, the Milano-Retinex family is mainly applied to image enhancement, mimicking some mechanisms of the human vision system. They share the same basic mechanisms, but their different implementations lead to important differences: the way they sample the image, the effects of the reset mechanism, and the final averaging process for calculating the new pixel value.

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