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Rotating BPS black holes in matter-coupled AdS_4 supergravity

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Abstract: Using the general recipe given in arXiv:0804.0009, where all timelike supersymmetric solutions of $\mathcal{N}=2$, $D=4$ gauged supergravity coupled to abelian vector multiplets were classified, we construct genuine rotating supersymmetric black holes in AdS_4 with nonconstant scalar fields. This is done for the $\mathrm{SU}(1,1)/\mathrm{U}(1)$ model with prepotential $F=-iX^0X^1$. In the static case, the black holes are uplifted to eleven dimensions, and generalize the solution found in hep-th/0105250 corresponding to membranes wrapping holomorphic curves in a Calabi-Yau five-fold. The constructed rotating black holes preserve one quarter of the supersymmetry, whereas their near-horizon geometry is one half BPS. Moreover, for constant scalars, we generalize (a supersymmetric subclass of) the Plebanski-Demianski solution of cosmological Einstein-Maxwell theory to an arbitrary number of vector multiplets. Remarkably, the latter turns out to be related to the dimensionally reduced gravitational Chern-Simons action.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence, Superstring Vacua

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Contents

1 Introduction

Black holes in anti-de Sitter (AdS) spaces provide an important testground to address fundamental questions of quantum gravity like holography. These ideas originally emerged from string theory, but became then interesting in their own right, for instance in recent applications to condensed matter physics (cf. [1] for a review), where black holes are again instrumental, since they provide the dual description of certain condensed matter systems at finite temperature, like e.g. holographic superconductors [2].

On the other hand, among the extremal black holes (which have zero Hawking temperature), those preserving a sufficient amount of supersymmetry are of particular interest, as this allows (owing to non-renormalization theorems) to extrapolate an entropy computation at weak string coupling (when the system is generically described by a bound state of strings and branes) to the strong-coupling regime, where a description in terms of a black hole is valid [3]. However, this picture, which has been essential for our current understanding of black hole microstates, might be modified in gauged supergravity (arising from flux compactifications) due to the presence of a potential for the moduli, generated by the fluxes. This could even lead to a stabilization of the dilaton, so that one cannot extrapolate between weak and strong coupling anymore. Obviously, the explicit knowledge of supersymmetric black hole solutions in AdS is a necessary ingredient if one wants to study this new scenario.

A first step in this direction was made in [4], where the first examples of extremal static BPS black holes in AdS_4 with nontrivial scalar field profiles were constructed. This analysis was facilitated by the results of [5], where all timelike supersymmetric backgrounds of $\mathcal{N} = 2$, $D = 4$ gauged supergravity coupled to abelian vector multiplets were classified. This provides a systematic method to obtain BPS solutions, without the necessity to guess some suitable ansätze. The upshot of [4] was the construction of a genuine static supersymmetric black hole with spherical horizon. This came as a surprise, since up to now the common folklore was that static spherical AdS black holes develop naked singularities in the BPS limit [6]. This is indeed true in minimal gauged supergravity, but the no-go theorems of [6] were circumvented in [4] by admitting nonconstant moduli. The spherical solutions of [4] were then further studied and generalized in [7, 8].

In this paper, we shall go one step further with respect to [4], and include also rotation. Apart from the supersymmetric Kerr-Newman-AdS family [9, 10] and its cousins with noncompact horizons [10], there are not many known solutions of this type. One of the most notable exceptions is perhaps the rotating two-charge black hole in SO(4)-gauged $\mathcal{N}=4$, $D=4$ supergravity [11], whose BPS limit was studied in [12]. Notice that the black holes constructed below are qualitatively different from the ones in [11], since they are solitonic objects that admit no smooth limit when the gauging is turned off.

In addition to the motivation given above, a further reason for considering supersymmetric rotating black holes is the attractor mechanism [13–17], which states that the scalar fields on the horizon and the entropy are independent of the asymptotic values of the moduli. (The scalars are attracted towards their purely charge-dependent horizon values). However, in gauged supergravity, the moduli fields have a potential, and typically approach the critical points of this potential asymptotically, where the solution approaches AdS. Thus, unless there are flat directions in the scalar potential, the values of the moduli at infinity are completely fixed (in terms of the gauge coupling constants), and therefore a more suitable formulation of the attractor mechanism in AdS would be to say that the black hole entropy is determined entirely by the charges, and is independent of the values of the moduli on the horizon that are not fixed by the charges. First steps towards a systematic analysis of the attractor flow in AdS were made in [18, 19] for the non-BPS and in [4, 8] for the BPS case, but it would be very interesting to generalize in particular the results of [4] to include also rotation.

The remainder of this paper is organized as follows: In the next section, we briefly review $\mathcal{N}=2$, $D=4$ gauged supergravity coupled to abelian vector multiplets (presence of U(1) Fayet-Iliopoulos terms) and give the general recipe to construct supersymmetric solutions found in [5]. In 3, the equations of [5] are solved for constant scalars. This leads to a generalization of the Plebanski-Demianski solution of cosmological Einstein-Maxwell theory to an arbitrary number of vector multiplets. We also find a remarkable relationship of the latter with the dimensionally reduced gravitational Chern-Simons action. In section 4, the case of nonconstant scalars is considered, using the $\mathrm{SU}(1,1)/\mathrm{U}(1)$ model with prepotential $F=-iX^0X^1$. First, we present in 4.1 a class of one half BPS near-horizon geometries, where the moduli field still has a nontrivial dependence on one of the horizon coordinates. Then, in section 4.2, a two-parameter family of rotating black holes is constructed. These solutions preserve one quarter of the supersymmetries, and approach the geometries of section 4.1 near the horizon. (4.3) contains an uplifting of the obtained black holes to M-theory, together with some comments on their higher-dimensional interpretation. We conclude in 5 with some final remarks.

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The reader who wants to skip the technical details can, instead of reading sections 3 and 4.2, immediately jump to eqs. (3.18) ff. and (4.31) ff. respectively for a summary of the results.

2 $\mathcal{N}=2$, $D=4$ gauged supergravity and its BPS geometries

We consider $\mathcal{N}=2$, $D=4$ gauged supergravity coupled to n_V abelian vector multiplets [20].[1] Apart from the vierbein e_μ^a, the bosonic field content includes the vectors A_μ^I enumerated by $I=0,\dots,n_V$, and the complex scalars z^α where $\alpha=1,\dots,n_V$. These scalars parametrize a special Kähler manifold, i. e. , an n_V-dimensional Hodge-Kähler manifold that is the base of a symplectic bundle, with the covariantly holomorphic sections

$$\mathcal{V}=\begin{pmatrix} X^I \\ F_I \end{pmatrix}, \qquad \mathcal{D}_{\bar\alpha}\mathcal{V}=\partial_{\bar\alpha}\mathcal{V}-\frac{1}{2}(\partial_{\bar\alpha}\mathcal{K})\mathcal{V}=0\,, \tag{2.1}$$

where $\mathcal{K}$ is the Kähler potential and $\mathcal{D}$ denotes the Kähler-covariant derivative. $\mathcal{V}$ obeys the symplectic constraint

$$\langle\mathcal{V}\,,\bar{\mathcal{V}}\rangle = X^I\bar F_I - F_I\bar X^I = i\,. \tag{2.2}$$

To solve this condition, one defines

$$\mathcal{V}=e^{\mathcal{K}(z,\bar z)/2}v(z)\,, \tag{2.3}$$

where $v(z)$ is a holomorphic symplectic vector,

$$v(z)=\begin{pmatrix} Z^I(z) \\ \frac{\partial}{\partial Z^I}F(Z) \end{pmatrix}. \tag{2.4}$$

F is a homogeneous function of degree two, called the prepotential, whose existence is assumed to obtain the last expression. The Kähler potential is then

$$e^{-\mathcal{K}(z,\bar z)}=-i\langle v\,,\bar v\rangle\,. \tag{2.5}$$

The matrix $\mathcal{N}_{IJ}$ determining the coupling between the scalars z^α and the vectors A_μ^I is defined by the relations

$$F_I=\mathcal{N}_{IJ}X^J\,, \qquad \mathcal{D}_{\bar\alpha}\bar F_I=\mathcal{N}_{IJ}\mathcal{D}_{\bar\alpha}\bar X^J\,. \tag{2.6}$$

The bosonic action reads

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos}} = {}& \frac{1}{2}R+\frac{1}{4}(\operatorname{Im}\mathcal{N})_{IJ}F^I_{\mu\nu}F^{J\mu\nu}-\frac{1}{8}(\operatorname{Re}\mathcal{N})_{IJ}\,e^{-1}\epsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma} \\ & -g_{\alpha\bar\beta}\partial_\mu z^\alpha\partial^\mu\bar z^{\bar\beta}-V\,, \end{aligned} \tag{2.7}$$

with the scalar potential

$$V=-2g^2\xi_I\xi_J[(\operatorname{Im}\mathcal{N})^{-1|IJ}+8\bar X^I X^J]\,, \tag{2.8}$$

[1] Throughout this paper, we use the notations and conventions of [21].

that results from U(1) Fayet-Iliopoulos gauging. Here, g denotes the gauge coupling and the ξ_I are constants. In what follows, we define $g_I = g\xi_I$.

The most general timelike supersymmetric background of the theory described above was constructed in [5], and is given by

$$ds^2 = -4|b|^2(dt+\sigma)^2 + |b|^{-2}(dz^2 + e^{2\Phi}dwd\bar{w})\,, \tag{2.9}$$

where the complex function $b(z,w,\bar{w})$, the real function $\Phi(z,w,\bar{w})$ and the one-form $\sigma = \sigma_w dw + \sigma_{\bar{w}}d\bar{w}$, together with the symplectic section (2.1)[2] are determined by the equations

$$\partial_z\Phi = 2ig_I\left(\frac{\bar{X}^I}{b} - \frac{X^I}{\bar{b}}\right)\,, \tag{2.10}$$

$$\begin{aligned}&4\partial\bar{\partial}\left(\frac{X^I}{\bar{b}} - \frac{\bar{X}^I}{b}\right) + \partial_z\left[e^{2\Phi}\partial_z\left(\frac{X^I}{\bar{b}} - \frac{\bar{X}^I}{b}\right)\right]\\ &-2ig_J\partial_z\left\{e^{2\Phi}\left[|b|^{-2}(\mathrm{Im}\,\mathcal{N})^{-1|IJ} + 2\left(\frac{X^I}{\bar{b}} + \frac{\bar{X}^I}{b}\right)\left(\frac{X^J}{\bar{b}} + \frac{\bar{X}^J}{b}\right)\right]\right\} = 0\,,\end{aligned} \tag{2.11}$$

$$\begin{aligned}&4\partial\bar{\partial}\left(\frac{F_I}{\bar{b}} - \frac{\bar{F}_I}{b}\right) + \partial_z\left[e^{2\Phi}\partial_z\left(\frac{F_I}{\bar{b}} - \frac{\bar{F}_I}{b}\right)\right]\\ &-2ig_J\partial_z\left\{e^{2\Phi}\left[|b|^{-2}\mathrm{Re}\,\mathcal{N}_{IL}(\mathrm{Im}\,\mathcal{N})^{-1|JL} + 2\left(\frac{F_I}{\bar{b}} + \frac{\bar{F}_I}{b}\right)\left(\frac{X^J}{\bar{b}} + \frac{\bar{X}^J}{b}\right)\right]\right\}\\ &-8ig_Ie^{2\Phi}\left[\langle\mathcal{I}\,,\partial_z\mathcal{I}\rangle - \frac{g_J}{|b|^2}\left(\frac{X^J}{\bar{b}} + \frac{\bar{X}^J}{b}\right)\right] = 0\,,\end{aligned} \tag{2.12}$$

$$2\partial\bar{\partial}\Phi = e^{2\Phi}\left[ig_I\partial_z\left(\frac{X^I}{\bar{b}} - \frac{\bar{X}^I}{b}\right) + \frac{2}{|b|^2}g_Ig_J(\mathrm{Im}\,\mathcal{N})^{-1|IJ} + 4\left(\frac{g_IX^I}{\bar{b}} + \frac{g_I\bar{X}^I}{b}\right)^2\right]\,, \tag{2.13}$$

$$d\sigma + 2\,\star^{(3)}\langle\mathcal{I}\,,d\mathcal{I}\rangle - \frac{i}{|b|^2}g_I\left(\frac{\bar{X}^I}{b} + \frac{X^I}{\bar{b}}\right)e^{2\Phi}dw\wedge d\bar{w} = 0\,. \tag{2.14}$$

Here $\star^{(3)}$ is the Hodge star on the three-dimensional base with metric[3]

$$ds_3^2 = dz^2 + e^{2\Phi}dwd\bar{w}\,, \tag{2.15}$$

and we defined $\partial = \partial_w$, $\bar{\partial} = \partial_{\bar{w}}$, as well as

$$\mathcal{I} = \mathrm{Im}\left(\mathcal{V}/\bar{b}\right)\,. \tag{2.16}$$

Given b, Φ, σ and $\mathcal{V}$, the fluxes read

$$\begin{aligned}F^I =\;& 2(dt+\sigma)\wedge d\left[bX^I + \bar{b}\bar{X}^I\right] +\\ &|b|^{-2}dz\wedge d\bar{w}\left[\bar{X}^I(\bar{\partial}\bar{b} + iA_{\bar{w}}\bar{b}) + (\mathcal{D}_\alpha X^I)b\bar{\partial}z^\alpha - X^I(\bar{\partial}b - iA_{\bar{w}}b) - (\mathcal{D}_{\bar{\alpha}}\bar{X}^I)\bar{b}\bar{\partial}\bar{z}^{\bar{\alpha}}\right] -\\ &|b|^{-2}dz\wedge dw\left[\bar{X}^I(\partial\bar{b} + iA_w\bar{b}) + (\mathcal{D}_\alpha X^I)b\partial z^\alpha - X^I(\partial b - iA_wb) - (\mathcal{D}_{\bar{\alpha}}\bar{X}^I)\bar{b}\partial\bar{z}^{\bar{\alpha}}\right] -\\ &\frac{1}{2}|b|^{-2}e^{2\Phi}dw\wedge d\bar{w}\Big[\bar{X}^I(\partial_z\bar{b} + iA_z\bar{b}) + (\mathcal{D}_\alpha X^I)b\partial_zz^\alpha - X^I(\partial_zb - iA_zb) -\\ &\qquad(\mathcal{D}_{\bar{\alpha}}\bar{X}^I)\bar{b}\partial_z\bar{z}^{\bar{\alpha}} - 2ig_J(\mathrm{Im}\,\mathcal{N})^{-1|IJ}\Big]\,.\end{aligned} \tag{2.17}$$

[2] Note that also σ and $\mathcal{V}$ are independent of t.

[3] Whereas in the ungauged case, this base space is flat and thus has trivial holonomy, here we have U(1) holonomy with torsion [5].

In (2.17), A_μ is the gauge field of the Kähler U(1),

$$A_\mu = -\frac{i}{2}(\partial_\alpha \mathcal{K}\partial_\mu z^\alpha - \partial_{\bar\alpha}\mathcal{K}\partial_\mu \bar z^{\bar\alpha})\,. \tag{2.18}$$

3 Constant scalars

Let us first assume $g_I \mathcal{D}_\alpha X^I = 0$, which implies that the scalars are constant.[4]

In order to solve the system (2.10)–(2.14), inspired by the analysis in pure gauged supergravity [22], we make the ansatz

$$\frac{\bar X}{b} = \frac{f(z)+p(w,\bar w)}{g(z)}\,, \qquad e^{2\Phi} = h(z)\ell(w,\bar w)\,, \tag{3.1}$$

where we defined $X \equiv g_I X^I$. Here, $f(z)$, $g(z)$ and $p(w,\bar w)$ are complex functions, while $h(z)$ and $\ell(w,\bar w)$ are real. (2.10) implies then that $\bar g p - g\bar p$ is independent of $w, \bar w$. This in turn leads to

$$p = (1+i\lambda_1)\mathrm{Re}\,p + i\lambda_2\,,$$

where $\lambda_1, \lambda_2 \in \mathbb{R}$ are constants. From the ansatz (3.1) it is clear that the prefactor $1+i\lambda_1$ as well as $i\lambda_2$ can be absorbed into $f(z)$ and $g(z)$, so that we can choose p real without loss of generality. But then $g(z)$ is also real, if we want p to have a nontrivial dependence on $w, \bar w$. Thus, equ. (2.10) boils down to

$$\partial_z \ln h = -\frac{8\mathrm{Im}\,f}{g}\,, \tag{3.2}$$

while (2.13) gives

$$\frac{\partial\bar\partial \ln \ell}{\ell} = h\left[-\frac{1}{4}\partial_z^2 \ln h + \frac{2}{g^2|X|^2}|f+p|^2 g_I g_J (\mathrm{Im}\,\mathcal{N})^{-1|IJ} + \frac{4}{g^2}(f+\bar f+2p)^2\right]\,. \tag{3.3}$$

This is of the type

$$A(w,\bar w) = -B(z) + C(z)p(w,\bar w) + D(z)p^2(w,\bar w)\,, \tag{3.4}$$

for some functions A, B, C, D. Applying the operator $\partial\partial_z$ to (3.4) yields $p =$ const. or C, D constant. The former case is trivial, so we shall consider the latter in what follows. (3.4) implies then that B is constant as well. Explicitely we have

$$D = \frac{h}{g^2}\left[\frac{2}{|X|^2}g_I g_J (\mathrm{Im}\,\mathcal{N})^{-1|IJ} + 16\right]\,, \qquad C = (f+\bar f)D\,, \tag{3.5}$$

and hence the real part of $f(z)$ is independent of z, and can be absorbed into p. One can thus choose f imaginary and $C = 0$ without loss of generality. Using the special geometry relation

$$g^{\alpha\bar\beta}\mathcal{D}_\alpha X^I \mathcal{D}_{\bar\beta}\bar X^J = -\frac{1}{2}(\mathrm{Im}\,\mathcal{N})^{-1|IJ} - \bar X^I X^J\,, \tag{3.6}$$

[4]This is true if the scalar potential has no flat directions.

we get $D = 12h/g^2$. Taking into account (3.2), the expression for B becomes

$$B = \frac{h}{4}(\ln h)'' + \frac{h}{16}(\ln h)'^2 = \text{constant} \;. \tag{3.7}$$

This is a differential equation for h, with solution

$$h = \begin{cases} \left(\frac{B}{u_0} + u_0 z^2\right)^2 & , \quad u_0 \neq 0\,, \\ -4Bz^2 & , \quad u_0 = 0\,, \end{cases} \tag{3.8}$$

where u_0 denotes a real integration constant.[5] In the following, we are interested in the case $u_0 \neq 0$ only. For the functions g and f one has

$$g = \pm 2\sqrt{\frac{3h}{D}}\,, \qquad f = \mp\frac{i}{2}\left(\sqrt{\frac{3h}{D}}\right)' \;. \tag{3.9}$$

By rescaling $p \to \pm p\sqrt{3/D}/2$ in the ansatz (3.1) we can choose the upper sign and set $\sqrt{3/D}/2 = 1$, i.e., $D = 3/4$ without loss of generality. Then (3.3) reduces to

$$\partial\bar{\partial}\ln\ell = \ell\left[-B + \frac{3}{4}p^2\right] \;. \tag{3.10}$$

The Bianchi identities (2.11) are automatically satisfied, while the Maxwell equations (2.12) imply

$$\partial\bar{\partial}p = \ell\left[\frac{1}{4}p^3 - Bp\right] \;. \tag{3.11}$$

As was noticed in [22], (3.10) and (3.11) follow from the dimensionally reduced gravitational Chern-Simons action [23]

$$S = \int d^2x\sqrt{g}\left[pR + p^3\right] \,, \tag{3.12}$$

if we choose the conformal gauge $g_{ij}dx^i dx^j = \ell dw d\bar{w}$. Note that in (3.12), p is not a fundamental field, rather it is the curl of a vector potential, $\sqrt{g}\epsilon_{ij}p = \partial_i A_j - \partial_j A_i$. Actually, the equations of motion following from the action (3.12) are slightly stronger than our system (3.10), (3.11), which does not include the traceless part of the constraints $\delta S/\delta g^{ij} = 0$. Grumiller and Kummer were able to write down the most general solution of (3.12), using the fact that the dimensionally reduced Chern-Simons theory can be written as a Poisson-sigma model with four-dimensional target space and degenerate Poisson tensor of rank two [24]. This solution is given by [24]

$$\begin{aligned} \ell &= \frac{1+\delta}{\cosh^4(\sqrt{B}\tilde{x})}\,, \qquad p = 2\sqrt{B}\tanh(\sqrt{B}\tilde{x})\,, \\ \delta &= \left(\frac{\mathcal{C}}{2B^2} - 1\right)\cosh^4(\sqrt{B}\tilde{x})\,, \end{aligned} \tag{3.13}$$

[5] A further integration constant can be eliminated by shifting z.

where the coordinate $\tilde{x}$ is related to $x = (w+\bar{w})/2$ by

$$\frac{dx}{d\tilde{x}} = \frac{\cosh^2(\sqrt{B}\tilde{x})}{1+\delta}\,, \tag{3.14}$$

and $\mathcal{C}$ denotes an integration constant.[6] Note that the solution for negative B can be obtained by a simple analytical continuation from (3.13) [22].

The shift vector σ can now be determined from (2.14), with the result

$$\sigma = \sigma_y dy\,, \qquad \sigma_y = \frac{1}{4g|X|^2}\left(\frac{p^4}{8B} - p^2 + \frac{\mathcal{C}}{B}\right) - \frac{u_0 p^2}{32B|X|^2}\,, \qquad y = \frac{w-\bar{w}}{2i}\,. \tag{3.15}$$

Putting all together and using p as a new coordinate in place of x, the metric (2.9) becomes[7]

$$\begin{aligned} ds^2 = &-\frac{64|X|^2(B/u_0+u_0z^2)^2}{4u_0^2z^2+p^2}(dt+\sigma_y dy)^2 + \frac{4u_0^2z^2+p^2}{16|X|^2(B/u_0+u_0z^2)^2}dz^2 \\ &+\frac{4u_0^2z^2+p^2}{32B|X|^2\left(\frac{p^4}{8B}-p^2+\frac{\mathcal{C}}{B}\right)}dp^2 + \frac{4u_0^2z^2+p^2}{32B|X|^2}\left(\frac{p^4}{8B}-p^2+\frac{\mathcal{C}}{B}\right)dy^2\,. \end{aligned} \tag{3.16}$$

This resembles the Plebanski-Demianski (PD) solution [25] of cosmological Einstein-Maxwell theory, which is also specified by quartic structure functions. Indeed, in the case $1-\mathcal{C}/(2B^2) > 0$, consider the coordinate transformation

$$\begin{pmatrix}\tau\\ \varsigma\end{pmatrix} = \left(1-\frac{\mathcal{C}}{2B^2}\right)^{-1/2}\begin{pmatrix}\frac{1}{u_0} & -\frac{\mathcal{C}}{16B^2|X|^2}\\ \frac{1}{4Bu_0} & -\frac{1}{32B|X|^2}\end{pmatrix}\begin{pmatrix}t\\ y\end{pmatrix}\,, \qquad q = 2u_0 z\,, \tag{3.17}$$

which casts the line element (3.16) into the PD form

$$ds^2 = \frac{p^2+q^2}{\mathcal{P}}dp^2 + \frac{\mathcal{P}}{p^2+q^2}\left(d\tau+q^2d\varsigma\right)^2 + \frac{p^2+q^2}{\mathcal{Q}}dq^2 - \frac{\mathcal{Q}}{p^2+q^2}\left(d\tau-p^2d\varsigma\right)^2\,, \tag{3.18}$$

with the structure functions

$$\mathcal{P} = \gamma - \mathsf{E}p^2 + \frac{p^4}{l^2}\,, \qquad \mathcal{Q} = \hat{\gamma} + \mathsf{E}q^2 + \frac{q^4}{l^2}\,, \tag{3.19}$$

where

$$\gamma = 32\mathcal{C}|X|^2\,, \qquad \hat{\gamma} = 64B^2|X|^2\,, \qquad \mathsf{E} = 32B|X|^2\,, \qquad l^2 = \frac{1}{4|X|^2}\,. \tag{3.20}$$

l is related to the effective cosmological constant by $\Lambda = -3/l^2$. In these coordinates, the fluxes (2.17) read[8]

$$F^I = \frac{2X^I\bar{X}}{|X|(p^2+q^2)^2}(\hat{\gamma}-\gamma)^{1/2}\left[(p^2-q^2)(d\tau+q^2d\varsigma)\wedge dp + 2pq(d\tau-p^2d\varsigma)\wedge dq\right]\,. \tag{3.21}$$

[6] More precisely, $\mathcal{C}$ and B are the Casimir functions of the Poisson sigma model that can be interpreted respectively as energy and charge [24].

[7] Notice that the analogue of (3.16) in minimal gauged supergravity was found in [22].

[8] Observe that contraction of (3.6) with g_I and taking into account $g_I\mathcal{D}_\alpha X^I = 0$ yields $\bar{X}X^J = -\frac{1}{2}(\mathrm{Im}\,\mathcal{N})^{-1|IJ}g_I$, and thus $\bar{X}X^J$ is real.

Comparing the functions (3.19) and the field strengths (3.21) with the general expressions given in [25], we see that in our case the mass, nut and electric charge parameters M, N and Q vanish. It would be very interesting to see how one has to generalize the ansatz (3.1) in order to get solutions with nonzero M, N and Q. That there must be supersymmetric solutions of this type is clear from the analysis for minimal gauged supergravity in [26]. Notice also that for $\mathsf{M} = \mathsf{N} = \mathsf{Q} = 0$, the BPS conditions obtained in [26] boil down to

$$\mathsf{E}^2 = \frac{4}{l^2}\hat{\gamma}\,, \tag{3.22}$$

which is exactly what follows from (3.20). As a by-product, we have thus shown that the PD solution in minimal gauged supergravity with $\mathsf{M} = \mathsf{N} = \mathsf{Q} = 0$ satisfying (3.22) does really admit a Killing spinor. This was not obvious, since [26] analyzes only the first integrability conditions, which are in general necessary but not sufficient for the existence of Killing spinors.

4 Nonconstant scalar fields

In this section we shall obtain supersymmetric rotating black holes as well as their near-horizon geometries, which both have nontrivial moduli turned on. This is done for the SU(1,1)/U(1) model with prepotential $F = -iX^0X^1$, that has $n_V = 1$ (one vector multiplet), and thus just one complex scalar τ. Choosing $Z^0 = 1$, $Z^1 = \tau$, the symplectic vector v becomes

$$v = \begin{pmatrix} 1 \\ \tau \\ -i\tau \\ -i \end{pmatrix}. \tag{4.1}$$

The Kähler potential, metric and kinetic matrix for the vectors are given respectively by

$$e^{-\mathcal{K}} = 2(\tau + \bar{\tau})\,, \qquad g_{\tau\bar{\tau}} = \partial_\tau \partial_{\bar{\tau}} \mathcal{K} = (\tau + \bar{\tau})^{-2}\,, \tag{4.2}$$

$$\mathcal{N} = \begin{pmatrix} -i\tau & 0 \\ 0 & -\frac{i}{\tau} \end{pmatrix}. \tag{4.3}$$

Note that positivity of the kinetic terms in the action requires $\mathrm{Re}\tau > 0$. For the scalar potential one obtains

$$V = -\frac{4}{\tau + \bar{\tau}}(g_0^2 + 2g_0g_1\tau + 2g_0g_1\bar{\tau} + g_1^2\tau\bar{\tau})\,, \tag{4.4}$$

which has an extremum at $\tau = \bar{\tau} = |g_0/g_1|$. In what follows we assume $g_I > 0$.

4.1 1/2 BPS near-horizon geometries

An interesting class of half-supersymmetric backgrounds was obtained in [27]. It includes the near-horizon geometry of extremal rotating black holes. The metric and the fluxes read respectively

$$
\begin{aligned}
ds^2 &= -z^2 e^{\xi}\left[dt + 4(e^{-2\xi} - L)\frac{dx}{z}\right]^2 + 4e^{-\xi}\frac{dz^2}{z^2} \\
&\quad +16e^{-\xi}(e^{-2\xi} - L)dx^2 + \frac{4e^{-2\xi}d\xi^2}{Y^2(e^{-\xi} - Le^{\xi})}\,,
\end{aligned} \tag{4.5}
$$

$$
\begin{aligned}
F^I &= 8i\left(\frac{\bar{X}X^I}{1-iY} - \frac{X\bar{X}^I}{1+iY}\right)dt\wedge dz \\
&\quad +\frac{4}{Y}\left[\frac{2\bar{X}X^I}{1-iY} + \frac{2X\bar{X}^I}{1+iY} + (\mathrm{Im}\,\mathcal{N})^{-1|IJ}g_J\right](zdt - 4Ldx)\wedge d\xi\,,
\end{aligned} \tag{4.6}
$$

where L is a real integration constant and Y is defined by

$$
Y^2 = 64e^{-\xi}|X|^2 - 1\;. \tag{4.7}
$$

The moduli fields z^α depend on the coordinate ξ only, and obey the flow equation

$$
\frac{dz^\alpha}{d\xi} = \frac{i}{2\bar{X}Y}(1-iY)g^{\alpha\bar{\beta}}\mathcal{D}_{\bar{\beta}}\bar{X}\;. \tag{4.8}
$$

For $L > 0$, the line element (4.5) can be cast into the simple form

$$
\begin{aligned}
ds^2 &= 4e^{-\xi}\left(-z^2d\hat{t}^2 + \frac{dz^2}{z^2}\right) + 16L(e^{-\xi} - Le^{\xi})\left(dx - \frac{z}{2\sqrt{L}}d\hat{t}\right)^2 \\
&\quad +\frac{4e^{-2\xi}d\xi^2}{Y^2(e^{-\xi} - Le^{\xi})}\,,
\end{aligned} \tag{4.9}
$$

where $\hat{t} \equiv t/(2\sqrt{L})$. (4.9) is of the form (3.3) of [28], and describes the near-horizon geometry of extremal rotating black holes,[9] with isometry group $\mathrm{SL}(2,\mathbb{R})\times\mathrm{U}(1)$. From (4.8) it is clear that the scalar fields have a nontrivial dependence on the horizon coordinate ξ unless $g_I\mathcal{D}_\alpha X^I = 0$. As was shown in [27], the solution with constant scalars is the near-horizon limit of the supersymmetric rotating hyperbolic black holes in minimal gauged supergravity [10]. We shall now give an explicit example of a near-horizon geometry with varying scalars, taking the simple model introduced above, with prepotential $F = -iX^0X^1$. In this case the flow equation (4.8) becomes

$$
\frac{d\tau}{d\xi} = \frac{i}{2Y}(1-iY)\frac{-g_0 + g_1\tau}{g_0 + g_1\bar{\tau}}(\tau + \bar{\tau})\;. \tag{4.10}
$$

Using Y in place of ξ as a new variable, this boils down to

$$
\frac{d\tau}{dY} = -\frac{g_1^2\tau^2 - g_0^2}{2g_0g_1(Y-i)}\,, \tag{4.11}
$$

which is solved by

$$
\tau = \frac{g_0}{g_1}\frac{Y - i + C}{Y - i - C}\,, \tag{4.12}
$$

[9] Metrics of the type (4.9) were discussed for the first time in [29] in the context of the extremal Kerr throat geometry.

with $C \in \mathbb{C}$ an integration constant. This allows to compute $|X|^2$ as a function of Y,

$$|X|^2 = g_0 g_1 \frac{Y^2+1}{Y^2+1-|C|^2} \,. \tag{4.13}$$

Plugging this into (4.7) yields an expression for ξ in terms of Y,

$$e^{-\xi} = \frac{Y^2+1-|C|^2}{64 g_0 g_1} \,. \tag{4.14}$$

4.2 Supersymmetric rotating black holes

We now want to obtain stationary BPS black holes with nonconstant moduli, that approach the geometries of the previous subsection in the near-horizon limit. To this end, we use the ansatz

$$\frac{\bar{X}^I}{b} = \frac{f^I(z)+\eta^I(w,\bar{w})}{g(z)} \,, \qquad e^{2\Phi} = h(z)\ell(w,\bar{w}) \,, \tag{4.15}$$

where $f^I(z)$ is an imaginary function, while $g(z)$, $\eta^I(w,\bar{w})$, $h(z)$ and $\ell(w,\bar{w})$ are real. Then, (2.10) reduces again to (3.2), where f is defined by $f \equiv f^I g_I$. (2.13) becomes

$$\frac{\partial\bar{\partial}\ln\ell}{\ell} = h\left[-\frac{1}{4}\partial_z^2 \ln h - \frac{8}{g^2}\sum_I g_I^2({\eta^I}^2 - {f^I}^2) + \frac{16}{g^2}\eta^2\right] \,, \tag{4.16}$$

with $\eta \equiv \eta^I g_I$. Guided by the constant scalar case (cf. section 3) we take $h/g^2 = \text{const.} \equiv c_1 > 0$ and[10]

$$-\frac{h}{4}\partial_z^2 \ln h + \frac{8h}{g^2}\sum_I g_I^2 {f^I}^2 = \text{const.} \equiv c_2 c_1 \,. \tag{4.17}$$

With these assumptions, (4.16) gives

$$\frac{\partial\bar{\partial}\ln\ell}{\ell} = c_1 c_2 - 8c_1 \sum_I g_I^2 {\eta^I}^2 + 16 c_1 \eta^2 \,. \tag{4.18}$$

In order to solve (4.17), we make the ansatz

$$g = c + az^2 \,, \qquad h = c_1(c+az^2)^2 \,, \qquad f^I = i(\alpha^I z + \beta^I) \,, \tag{4.19}$$

for some real constants a, c, α^I, β^I. One finds that (4.17) is satisfied if the following constraints hold:

$$a^2 = 8\sum_I g_I^2 {\alpha^I}^2 \,, \qquad \sum_I g_I^2 \alpha^I \beta^I = 0 \,, \qquad -ac - 8\sum_I g_I^2 {\beta^I}^2 = c_2 \,. \tag{4.20}$$

Moreover, (3.2) yields

$$g_I \alpha^I = -\frac{a}{2} \,, \qquad g_I \beta^I = 0 \,. \tag{4.21}$$

[10] It is easy to show that for constant scalars one must have $f^I = \gamma^I f$, $\eta^I = \gamma^I \eta$, where the constants γ^I satisfy $\gamma^I g_I = 1$. Using this together with (3.2), equ. (4.17) reduces to (3.7).

The Bianchi identities (2.11) lead to

$$\alpha^I = -\frac{a}{4g_I}\,, \tag{4.22}$$

which implies the first equation of (4.21). Finally, the Maxwell equations (2.12) hold provided that

$$\alpha_{IJ}\partial\bar{\partial}\eta^J + 4g_I c_1 \ell\eta\alpha_{LJ}\left(\frac{c}{a}\alpha^L\alpha^J - \beta^L\beta^J - \eta^L\eta^J\right) = 0\,, \tag{4.23}$$

where

$$(\alpha_{IJ}) \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,. \tag{4.24}$$

It would be interesting to see if (4.18) and (4.23), similar to (3.10) and (3.11), follow from an action principle of the type (3.12).

We shall now solve the eqs. (4.18), (4.23) under the additional assumption $\ell = \ell(x)$, $\eta^I = \eta^I(x)$, using an ansatz analogous to (3.13):

$$\begin{aligned} \ell &= \frac{1+\delta}{\cosh^4(\kappa\tilde{x})}\,, & \eta^I &= \hat{\eta}^I\tanh(\kappa\tilde{x})\,, \\ \delta &= A\cosh^4(\kappa\tilde{x})\,, & \frac{dx}{d\tilde{x}} &= \frac{\cosh^2(\kappa\tilde{x})}{1+\delta}\,, \end{aligned} \tag{4.25}$$

where κ, $\hat{\eta}^I$ and A are constants. Plugging this into (4.18) and (4.23) gives

$$g_0\hat{\eta}^0 = g_1\hat{\eta}^1\,, \qquad (4g_0\hat{\eta}^0)^2 c_1 = \kappa^2 = -c_1 c_2\,. \tag{4.26}$$

At the end, the shift vector σ is determined by (2.14), which yields

$$\sigma = \sigma_y dy\,, \qquad \sigma_y = \frac{\hat{\eta}^0\kappa(\cosh^{-4}(\kappa\tilde{x}) + A)}{2g_1(c + az^2)} + \frac{c_1 a\hat{\eta}^0}{2g_1\kappa}\tanh^2(\kappa\tilde{x})\,. \tag{4.27}$$

Similar to the case of constant scalars, for $A < 0$ the solution can be cast into a Plebanski-Demianski-type form by the coordinate transformation

$$\begin{pmatrix} t \\ y \end{pmatrix} \mapsto \frac{lg_1\sqrt{\mathsf{E}}}{\hat{\eta}^0\sqrt{-2A}}\begin{pmatrix} \frac{al^2}{2} & -\frac{(1+A)al^4\mathsf{E}}{4} \\ -\frac{1}{\sqrt{c_1}} & \frac{l^2\mathsf{E}}{2\sqrt{c_1}} \end{pmatrix}\begin{pmatrix} t \\ y \end{pmatrix}\,, \tag{4.28}$$

$$p = l\sqrt{\frac{\mathsf{E}}{2}}\tanh(\kappa\tilde{x})\,, \qquad q = \frac{al\sqrt{\mathsf{E}}}{4\sqrt{2}g_0\hat{\eta}^0}z\,, \tag{4.29}$$

where

$$l^2 \equiv \frac{1}{4g_0 g_1}\,. \tag{4.30}$$

The metric becomes then

$$\begin{aligned} ds^2 = &\frac{p^2+q^2-\Delta^2}{\mathcal{P}}dp^2 + \frac{\mathcal{P}}{p^2+q^2-\Delta^2}\left(dt + (q^2-\Delta^2)dy\right)^2 \\ &+\frac{p^2+q^2-\Delta^2}{\mathcal{Q}}dq^2 - \frac{\mathcal{Q}}{p^2+q^2-\Delta^2}\left(dt - p^2 dy\right)^2\,, \end{aligned} \tag{4.31}$$

with the structure functions

$$\mathcal{P} = (1+A)\frac{\mathsf{E}^2 l^2}{4} - \mathsf{E}p^2 + \frac{p^4}{l^2}\,, \qquad \mathcal{Q} = \frac{1}{l^2}\left(q^2 + \frac{\mathsf{E}l^2}{2} - \Delta^2\right)^2\,, \tag{4.32}$$

and the parameter Δ is defined by

$$\Delta \equiv \frac{\beta^0 l\sqrt{\mathsf{E}}}{\sqrt{2}\hat{\eta}^0}\,. \tag{4.33}$$

Notice that, although we must have obviously $\mathsf{E} > 0$ in the above coordinate transformation, the final solution in the PD form can be safely continued to $\mathsf{E} \leq 0$.[11]

In the new coordinates, the complex scalar field τ reads

$$\tau = \frac{g_0}{g_1}\frac{p^2 + q^2 - \Delta^2 + 2ip\Delta}{p^2 + (q-\Delta)^2}\,. \tag{4.34}$$

For $\Delta = 0$, τ is thus constant, and assumes the value $\tau = g_0/g_1$, for which the potential (4.4) is extremized. Note also that for p fixed and $q \to \infty$ or viceversa, τ tends to g_0/g_1 as well. The positivity domain $\mathrm{Re}\tau > 0$ is determined by $p^2+q^2-\Delta^2 > 0$. Finally, the fluxes (2.17) are given by $F^I = dA^I$, where

$$A^I = -\frac{\mathsf{E}p\sqrt{-A}}{4g_I(p^2+q^2-\Delta^2)}\left(dt + (q^2-\Delta^2)dy\right)\,. \tag{4.35}$$

The solution is thus specified by three free parameters A, E, Δ. A particular case is obtained by choosing

$$\sqrt{-A} = \frac{l^2+j^2}{l^2-j^2}\,, \qquad \mathsf{E} = \frac{j^2}{l^2} - 1\,, \tag{4.36}$$

$$p = j\cosh\theta\,, \qquad y = -\frac{\phi}{j\Xi}\,, \qquad t = \frac{T - j\phi}{\Xi}\,, \qquad \Xi \equiv 1 + \frac{j^2}{l^2}\,. \tag{4.37}$$

Defining also

$$\rho^2 = q^2 + j^2\cosh^2\theta\,, \qquad \Delta_q = \frac{1}{l^2}\left(q^2 + \frac{j^2 - l^2}{2} - \Delta^2\right)^2\,, \qquad \Delta_\theta = 1 + \frac{j^2}{l^2}\cosh^2\theta\,,$$

the metric (4.31), scalar field (4.34) and U(1) gauge potentials (4.35) become

$$\begin{aligned} ds^2 &= \frac{\rho^2-\Delta^2}{\Delta_q}dq^2 + \frac{\rho^2-\Delta^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta\sinh^2\theta}{(\rho^2-\Delta^2)\Xi^2}\left(jdT - (q^2+j^2-\Delta^2)d\phi\right)^2 \\ &\quad - \frac{\Delta_q}{(\rho^2-\Delta^2)\Xi^2}\left(dT + j\sinh^2\theta d\phi\right)^2\,, \end{aligned} \tag{4.38}$$

$$\tau = \frac{g_0}{g_1}\frac{j^2\cosh^2\theta + q^2 - \Delta^2 + 2ij\Delta\cosh\theta}{j^2\cosh^2\theta + (q-\Delta)^2}\,, \tag{4.39}$$

$$A^I = \frac{\cosh\theta}{4g_I(\rho^2-\Delta^2)}\left(jdT - (q^2+j^2-\Delta^2)d\phi\right)\,. \tag{4.40}$$

[11]In fact, it is easy to see that the case of negative E corresponds to the analytical continuation $\kappa = ik$, $\hat{\eta}^I = i\hat{n}^I$, where $k, \hat{n}^I \in \mathbb{R}$. The hyperbolic functions in (4.25) become then trigonometric.

This solution contains two arbitrary constants j and Δ. The former can be interpreted as rotation parameter, since for $j = 0$ the geometry is static. Moreover, there is an event horizon determined by $\Delta_q = 0$, i.e., for

$$q^2 = q_{\rm h}^2 = \Delta^2 + \frac{1}{2}(l^2 - j^2) \ . \tag{4.41}$$

From (4.40) it is also clear that these rotating black holes carry two magnetic charges that are inversely proportional to the coupling constants g_I. Notice that the positivity domain of the scalar is $q^2 + j^2\cosh^2\theta > \Delta^2$, but since for $q \geq q_{\rm h}$ we have $q^2 + j^2\cosh^2\theta \geq q_{\rm h}^2 + j^2 = \Delta^2 + (l^2 + j^2)/2 > \Delta^2$, there are no ghosts outside the horizon.[12] For $\Delta = 0$, the scalar is constant, and we recover the supersymmetric rotating black hole with hyperbolic horizon in minimal gauged supergravity found in [10].[13] For $j = 0$ and $\Delta \neq 0$, (4.38)–(4.40) boil down to

$$ds^2 = -l^2N^2dT^2 + \frac{dq^2}{l^2N^2} + \left(q^2 - \frac{l^2}{2}\sinh^2\nu\right)(d\theta^2 + \sinh^2\theta d\phi^2)\,, \tag{4.42}$$

$$\tau = \frac{g_0}{g_1}\frac{q - \frac{l}{\sqrt{2}}\sinh\nu}{q + \frac{l}{\sqrt{2}}\sinh\nu}\,, \qquad A^I = -\frac{\cosh\theta}{4g_I}d\phi\,, \tag{4.43}$$

where

$$l^2N^2 = \frac{\left(\frac{q^2}{l^2} - \frac{1}{2}\cosh^2\nu\right)^2}{\frac{q^2}{l^2} - \frac{1}{2}\sinh^2\nu}\,, \qquad \Delta \equiv -\frac{l}{\sqrt{2}}\sinh\nu\ . \tag{4.44}$$

This is exactly the BPS black hole found in section 3.1 of [4], with hyperbolic horizon and nontrivial profile for the scalar field. It is interesting to note that for zero rotation parameter, τ becomes real, whereas for the rotating solution it is complex, i.e., one has a nonzero axion. A similar scenario was encountered in [11].

Let us now take a closer look at the near-horizon geometry of (4.38), which is obtained by introducing new coordinates $z, \hat{t}, \hat{\phi}$ according to

$$q = q_{\rm h} + \epsilon q_0 z\,, \qquad T = \frac{\hat{t}q_0}{\epsilon}\,, \qquad \phi = \hat{\phi} + \Omega\frac{\hat{t}q_0}{\epsilon}\,, \tag{4.45}$$

and then taking the limit $\epsilon \to 0$. Here, $\Omega = j/(q_{\rm h}^2 + j^2 - \Delta^2)$ is the angular velocity of the horizon, and q_0 is defined by

$$q_0^2 = \frac{l^4\Xi^2}{8q_{\rm h}^2}\ .$$

In this way one gets

$$\begin{aligned} ds^2 = &\ \frac{\rho_{\rm h}^2 - \Delta^2}{4q_{\rm h}^2z^2}l^2dz^2 + \frac{\rho_{\rm h}^2 - \Delta^2}{\Delta_\theta}d\theta^2 + \frac{l^4\Delta_\theta\sinh^2\theta}{4(\rho_{\rm h}^2 - \Delta^2)}\left(d\hat{\phi} + \frac{j}{q_{\rm h}}zd\hat{t}\right)^2 \\ &- \frac{\rho_{\rm h}^2 - \Delta^2}{4q_{\rm h}^2}l^2z^2d\hat{t}^2\,, \end{aligned} \tag{4.46}$$

[12] Presumably there is a curvature singularity at $q^2 + j^2\cosh^2\theta = \Delta^2$, although we did not check this explicitely.

[13] Actually, even for $\Delta = 0$, the above solution slightly generalizes the one of [10], in that it carries two charges instead of one.

where

$$\rho_{\rm h}^2 = q_{\rm h}^2 + j^2 \cosh^2\theta \ .$$

The final coordinate transformation

$$e^{-\xi} = \frac{q_{\rm h}^2 + j^2\cosh^2\theta - \Delta^2}{16 q_{\rm h}^2} l^2 \,, \qquad x = -\frac{16 q_{\rm h}^3}{j l^4 \Xi}\hat{\phi}\,, \tag{4.47}$$

casts the metric (4.46) into the form (4.9), with the constant L in (4.9) given by

$$L = \frac{l^8 \Xi^2}{1024 q_{\rm h}^4}\,,$$

and we used also (4.14). The parameter C appearing in (4.14) turns out to be related to Δ by $\Delta^2 = q_{\rm h}^2 |C|^2$. The phase of C is fixed by requiring that the scalar field (4.39) coincides (after taking the limit $\epsilon \to 0$) with the expression (4.12), which leads to

$$C = -i\frac{\Delta}{q_{\rm h}} \ .$$

Note that there is a simple relationship between θ and the coordinate Y used in section 4.1, namely

$$Y = -\frac{j}{q_{\rm h}}\cosh\theta\,,$$

and hence Y is up to a prefactor identical to the coordinate p that appears in the PD form of the metric.

In conclusion, we have found a two-parameter family (4.38)–(4.40) of extremal rotating black holes preserving one quarter of the supersymmetries, i.e., two real supercharges. The solutions interpolate between AdS_4 at infinity and the geometry (4.9) near the horizon, which is 1/2 BPS. Notice also that there is a nontrivial scalar field profile (4.39), and for $q \to q_{\rm h}$, τ does not become constant, but still depends on the horizon coordinate θ.

4.3 Lifting to M-theory

We now want to uplift some of the black hole solutions obtained above to M-theory, and comment on their higher-dimensional interpretation. To this end, let us be slightly more general, and consider the stu model of $\mathcal{N} = 2$, $D = 4$ gauged supergravity (which, as we shall see below, contains the $F = -iX^0X^1$ model used in this section as a truncation). In the zero-axion case, i.e., for real scalars, this can be embedded into $D = 11$ supergravity using the reduction ansatz presented in [30], that we briefly review in what follows. The eleven-dimensional metric reads

$$ds_{11}^2 = \tilde{\Delta}^{2/3} ds_4^2 + g^{-2}\tilde{\Delta}^{-1/3}\sum_{I=0}^{3} X^{I^{-1}}\left(d\mu_I^2 + \mu_I^2(d\phi_I + gA^I)^2\right)\,, \tag{4.48}$$

where $\tilde{\Delta} = \sum_{I=0}^3 X^I \mu_I^2$. The four quantities μ_I satisfy $\sum_I \mu_I^2 = 1$, and can be parametrized in terms of angles on S^3 as

$$\mu_0 = \sin\vartheta\,, \qquad \mu_1 = \cos\vartheta\sin\chi\,, \qquad \mu_2 = \cos\vartheta\cos\chi\sin\psi\,, \qquad \mu_3 = \cos\vartheta\cos\chi\cos\psi \ .$$

The X^I are given by

$$X^I = e^{-\frac{1}{2}\vec{a}_I\cdot\vec{\varphi}}\,, \qquad \vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3)\,, \tag{4.49}$$

with

$$\vec{a}_0 = (1,1,1)\,, \qquad \vec{a}_1 = (1,-1,-1)\,, \qquad \vec{a}_2 = (-1,1,-1)\,, \qquad \vec{a}_3 = (-1,-1,1)\,,$$

and satisfy $X^0X^1X^2X^3 = 1$.

The reduction ansatz for the four-form field strength is

$$\begin{aligned} F_{(4)} &= 2g\sum_I \left({X^I}^2\mu_I^2 - \tilde{\Delta}X^I\right)\epsilon_{(4)} + \frac{1}{2g}\sum_I {X^I}^{-1}\bar{*}dX^I \wedge d(\mu_I^2) \\ &\quad -\frac{1}{2g^2}\sum_I {X^I}^{-2} d(\mu_I^2)\wedge(d\phi_I + gA^I)\wedge\bar{*}F^I\,, \end{aligned} \tag{4.50}$$

where $F^I = dA^I$, $\bar{*}$ denotes the Hodge dual operator of ds_4^2 and $\epsilon_{(4)}$ is the corresponding volume form.

This leads to the four-dimensional stu model with bosonic action

$$e^{-1}\mathcal{L}_4 = \frac{1}{2}\left[R - \frac{1}{2}(\partial\vec{\varphi})^2 - \frac{1}{4}\sum_I e^{\vec{a}_I\cdot\vec{\varphi}}{F^I}^2 + 8g^2(\cosh\varphi_1 + \cosh\varphi_2 + \cosh\varphi_3)\right]\,, \tag{4.51}$$

which can also be obtained from the general theory (2.7) by choosing a prepotential proportional to $(X^0X^1X^2X^3)^{1/2}$, taking all g_I equal, and subsequently setting the axions to zero, cf. [4] for details.[14] In order to obtain from (4.51) the model with $F = -iX^0X^1$ considered in this section, one has to further truncate according to

$$\varphi_1 = \varphi_3 = 0\,, \qquad e^{\varphi_2} = \tau\,, \qquad F^2 = F^0\,, \qquad F^3 = F^1\,, \tag{4.52}$$

such that $X^2 = X^0$ and $X^3 = X^1$. This yields exactly the model introduced in eqs. (4.1) ff., with the additional restriction that τ must be real[15] and $g_0 = g_1 = g$.

Due to the zero-axion condition $\tau = \bar{\tau}$, we cannot uplift the rotating black holes (4.38)–(4.40), since these have a complex scalar unless $\Delta = 0$.[16] For the static solution (4.42), (4.43), τ is real, and the above reduction ansatz leads to an eleven-dimensional metric

$$\begin{aligned} ds_{11}^2 &= \tilde{\Delta}^{2/3}ds_4^2 + g^{-2}\tilde{\Delta}^{-1/3}\left\{\tau^{1/2}\left[d\mu_0^2 + \mu_0^2(d\phi_0 - \frac{1}{4}\cosh\theta d\phi)^2 + d\mu_2^2\right.\right. \\ &\quad \left. + \mu_2^2(d\phi_2 - \frac{1}{4}\cosh\theta d\phi)^2\right] + \tau^{-1/2}\left[d\mu_1^2 + \mu_1^2(d\phi_1 - \frac{1}{4}\cosh\theta d\phi)^2\right. \\ &\quad \left.\left. + d\mu_3^2 + \mu_3^2(d\phi_3 - \frac{1}{4}\cosh\theta d\phi)^2\right]\right\}\,, \end{aligned} \tag{4.53}$$

---

[14] The X^I used here are related to the X^I in section 3.2 of [4] by $X^I_{\text{here}} = 2\sqrt{2}X^I_{\text{there}}$. Moreover, one has to identify $g_I = g/\sqrt{2}$, $F^I_{\text{here}} = \sqrt{2}F^I_{\text{there}}$, and $e^{\varphi_\alpha} = \tau^\alpha$, $\alpha = 1,2,3$.

[15] Notice that X^0, X^1 computed from (4.1) differ from X^0 and X^1 defined in (4.49) by a factor one half.

[16] Actually, the $F = -iX^0X^1$ model can be embedded into $\mathcal{N} = 4$, $D = 4$ SO(4) gauged supergravity as well. To see this, set $\tau = e^{-\varphi} + i\chi$, which casts the action (2.7) into an abelian truncation of the bosonic $\mathcal{N} = 4$ SO(4) gauged supergravity action (17) of [31]. Solutions of the latter can (even for $\chi \neq 0$) in principle be lifted to eleven dimensions along the lines of [31], but we shall leave this for future work.

where

$$\tilde{\Delta} = \tau^{-1/2}(\mu_0^2 + \mu_2^2) + \tau^{1/2}(\mu_1^2 + \mu_3^2)\,,$$

while ds_4^2 and τ are given by (4.42) and (4.43) (with $g_0 = g_1 = g$) respectively. Finally, the four-form field strength can be easily computed from (4.50).

In the case $\nu = 0$, the scalar τ becomes constant ($\tau = 1$), and the solution (4.53) can be interpreted as the gravity dual corresponding to membranes wrapping holomorphic curves in a Calabi-Yau five-fold [32]. Moreover, for $\nu = 0$ (i.e., $\Delta = 0$), τ in (4.39) is real (and constant), which allows to uplift also the rotating black holes, which in eleven dimensions correspond to supersymmetric waves on wrapped membranes [32]. It would be interesting to see whether the general solution (4.53) (for $\nu \neq 0$) has a similar interpretation. This might allow for a microscopic entropy computation of the four-dimensional black hole (4.42), which can then be compared with the macroscopic Bekenstein-Hawking result

$$S_{\rm BH} = \frac{A_{\rm hor}}{4G_4} = \frac{\pi V}{4g^2}\,, \qquad V \equiv \int \sinh\theta d\theta d\phi\,, \tag{4.54}$$

where we used that $8\pi G = 1$ in our conventions. Notice that, for a noncompact horizon H^2, only the entropy density $s = S/V$ is finite. If instead the hyperbolic space is compactified to a Riemann surface of genus h, we can use Gauss-Bonnet to get $V = 4\pi(h-1)$, and thus

$$S_{\rm BH} = \frac{\pi^2}{g^2}(h-1)\ . \tag{4.55}$$

5 Final remarks

In this paper we have constructed new magnetically charged rotating BPS black holes in $\mathcal{N} = 2$, $D = 4$ gauged supergravity coupled to abelian vector multiplets. One of our results is a two-parameter family of solutions with noncompact horizon that preserve two real supercharges and have a nontrivial scalar field profile. In the near-horizon limit, there is a supersymmetry enhancement to 1/2 BPS. We limited the calculations of section 4 to the prepotential $F = -iX^0X^1$, but it would be very interesting to generalize them to the stu or at least to the so-called t^3 model, since this admits spherically symmetric static BPS black holes [4], that can in principle be given rotation.

A further point to explore would be how the attractor equations in gauged supergravity [4, 8] get modified if one includes rotation. This involves solving eqs. (2.10)–(2.14) for the most general stationary near-horizon geometry.

Finally, another possible generalization of our work is the inclusion of hypermultiplets. Black holes in $\mathcal{N} = 2$, $D = 4$ gauged supergravity with charged hypers were constructed and analyzed in [33]. When the hypermultiplet scalars are charged, black holes of this type might have applications in the emerging field of holographic superconductivity, where usually no analytical solution is known, and one has to resort to numerical techniques.

Work along these directions is in progress [34].

Acknowledgments

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