

Threshold Region for Higgs Boson Production in Gluon Fusion

Marco Bonvini

Deutsches Elektronen-Synchrotron, DESY, Notkestraße 85, D-22603 Hamburg, Germany

Stefano Forte

Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

Giovanni Ridolfi

Dipartimento di Fisica, Università di Genova and INFN, Sezione di Genova, Via Dodecaneso 33, I-16146 Genova, Italy

(Received 26 April 2012; published 7 September 2012)

We provide a quantitative determination of the effective partonic kinematics for Higgs boson production in gluon fusion in terms of the collider energy at the LHC. We use the result to assess, as a function of the Higgs boson mass, whether the large m_t approximation is adequate and Sudakov resummation advantageous. We argue that our results hold to all perturbative orders. Based on our results, we conclude that the full inclusion of finite top mass corrections is likely to be important for accurate phenomenology for a light Higgs boson with $m_H \sim 125$ GeV at the LHC with $\sqrt{s} = 14$ TeV.

DOI: [10.1103/PhysRevLett.109.102002](https://doi.org/10.1103/PhysRevLett.109.102002)

PACS numbers: 12.38.Cy, 14.80.Bn

An accurate computation of the Higgs boson production cross section [1] is essential in the search for this particle, which might be on the verge of being discovered at the LHC [2]. If the Higgs boson is light, and in particular is in the region $m_H \sim 125$ GeV, perhaps favored by LHC data, the dominant Higgs production mechanism is gluon fusion, which starts at leading order $O(\alpha_s^2)$ through a (predominantly top) quark loop. Higher-order corrections, which turn out to be quite large, are accordingly difficult to compute, and the full next-to-next-to-leading order (NNLO) result is known either in the large m_t limit, at the fully differential level [3], or as an expansion in inverse powers of m_t for the fully inclusive cross section [4]. As $m_t \rightarrow \infty$, the quark loop shrinks to a point (pointlike approximation) and the leading order (LO) process becomes a tree-level process of an effective theory.

At next-to-leading order (NLO), where the exact result is known, the large m_t approximation turns out to work surprisingly well, even up to values of the Higgs boson mass at and above the top pair production threshold. This result can be understood at least in part on the basis of the observation that the partonic cross section is dominated by logarithmically enhanced contributions related to soft gluon radiation, which are independent of m_t , so that the pointlike approximation becomes exact, up to an overall factor that starts at NNLO [5]. This soft dominance should take place when m_H is raised at fixed s so that the energy \hat{s} of the partonic production subprocess approaches threshold $\hat{s} \sim m_H^2$.

Close enough to the threshold, it is advantageous to resum these logarithmically enhanced terms (threshold resummation), and it has indeed been observed that this resummation significantly corrects and stabilizes the perturbative result in regions in which the pointlike approximation

holds to satisfactory accuracy [6]. It is important to understand that this may happen even if the expansion in powers of the strong coupling α_s behaves in a perturbative way, i.e., if the size of the higher-order logarithmically enhanced contributions decreases with the perturbative order, so that an actual all-order resummation is not really necessary. For this, it is sufficient that these enhanced contributions approximate the missing higher orders well enough so that their inclusion actually improves the accuracy of the computation, and the desirability of resummation should be thus judged by the accuracy of the logarithmic approximation.

Even so, this can only be possible if the center-of-mass energy of the partonic collision \hat{s} is significantly lower than the hadronic one s , which at the LHC is very far from threshold. Since the gluon distribution is strongly peaked at small values of the momentum fraction of each hadron carried by the gluon itself, this is especially likely in a gluon fusion channel. A quantitative assessment of this effect is thus important in order to determine the accuracy of the pointlike limit, and also whether threshold resummation is advantageous.

The necessary formalism was presented in Ref. [7], and applied to Drell-Yan production. The cross section for Higgs boson production is a function of the scale m_H^2 and a scaling variable $\tau = m_H^2/s$, given by the convolution

$$\frac{\sigma(\tau, m_H^2)}{\tau} = \int_{\tau}^1 \frac{dz}{z} \mathcal{L}\left(\frac{\tau}{z}, m_H^2\right) \frac{\hat{\sigma}(z, \alpha_s(m_H^2))}{z} \quad (1)$$

of a partonic cross section $\hat{\sigma}(\frac{m_H^2}{s}, \alpha_s(m_H^2))$ and a parton luminosity $\mathcal{L}(\frac{\hat{s}}{s}, m_H^2)$; a sum over relevant partonic subprocesses is understood. In our case, at LO the only subprocess is $gg \rightarrow H$, so that

$$\mathcal{L}(z, \mu^2) = \int_z^1 \frac{dy}{y} g(y, \mu^2) g\left(\frac{z}{y}, \mu^2\right), \quad (2)$$

where $g(z, \mu^2)$ is the gluon parton distribution (PDF) in the proton. It is convenient to write the partonic cross section in terms of a dimensionless coefficient function $C(z, \alpha_s)$

$$\hat{\sigma}(z, \alpha_s) = \sigma_0 z C(z, \alpha_s), \quad (3)$$

where σ_0 is the LO partonic cross section, so that the gg coefficient function has an expansion in powers of α_s

$$C(z, \alpha_s) = \delta(1-z) + \alpha_s C^{(1)}(z) + \alpha_s^2 C^{(2)}(z) + \mathcal{O}(\alpha_s^3). \quad (4)$$

Upon Mellin transformation

$$\sigma(N, m_H^2) = \int_0^1 d\tau \tau^{N-1} \frac{\sigma(\tau, m_H^2)}{\tau}, \quad (5)$$

$$C(N, \alpha_s) = \int_0^1 dz z^{N-1} C(z, \alpha_s), \quad (6)$$

$$\mathcal{L}(N, \mu^2) = \int_0^1 dz z^{N-1} \mathcal{L}(z, \mu^2), \quad (7)$$

the convolution in Eq. (1) turns into an ordinary product:

$$\sigma(N, m_H^2) = \sigma_0 \mathcal{L}(N, m_H^2) C(N, \alpha_s). \quad (8)$$

The Mellin transformation maps the large $\tau \rightarrow 1$ region into the large $N \rightarrow \infty$ region, and the small $\tau \rightarrow 0$ region into the small $N \rightarrow N_s$ region, with N_s the rightmost singularity of $\sigma(N, m_H^2)$ (i.e., the convergence abscissa of the Mellin transform). For gluon-initiated processes, $N_s = 1$ to all perturbative orders.

The dominant partonic kinematic region can then be determined through a saddle point argument, by computing the value of N which provides the dominant contribution to the Mellin inversion integral:

$$\frac{\sigma(\tau, m_H^2)}{\tau} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \sigma(N, m_H^2) \quad (c > N_s). \quad (9)$$

That is, we define

$$E(N, \tau, m_H^2) \equiv N \ln \frac{1}{\tau} + \ln \sigma(N, m_H^2), \quad (10)$$

so that

$$\frac{\sigma(\tau, m_H^2)}{\tau} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{E(N, \tau, m_H^2)}. \quad (11)$$

In the saddle point approximation, the integral Eq. (11) is dominated by the value of the exponent $E(N, \tau, m_H^2)$ at its stationary point N_0

$$\left. \frac{\partial E(N, \tau, m_H^2)}{\partial N} \right|_{N=N_0} = 0, \quad (12)$$

and by the behavior in its vicinity. The position of the saddle point is a function $N_0 = N_0(\tau, m_H^2)$, solution of Eq. (12). Hence, for any value of the physical kinematics, the questions of whether the partonic cross section is well approximated by its pointlike limit or resummation is advantageous are answered by verifying whether this is the case for $C(N_0, \alpha_s)$.

A unique real saddle point is present due to the drop of the cross section $\sigma(N, m_H^2)$ as N grows. This drop is driven by the parton luminosity $\mathcal{L}(N, \mu^2)$, which thus controls the position of the saddle N_0 . The drop of the luminosity at large N (and its growth at small N) reflects in turn the drop of the PDFs and luminosity as $z \rightarrow 1$ (and their growth as $z \rightarrow 0$). However, for large N , $C(N, \alpha_s)$ actually grows with N . This growth, which is due to logarithmically enhanced contributions, is possible only because the partonic cross section is a distribution, rather than an ordinary function: the Mellin transform of an ordinary positive function is easily proven to be a decreasing function of N . Nevertheless, the parton luminosity always offsets this increase, because the physical cross section $\sigma(\tau, m_H^2)$ is an ordinary positive function and thus must decrease with N . The faster the growth of the cross section, the better the saddle point approximation, which is thus especially good for gluon-dominated processes, as the gluon is more peaked at small z than the quark, and thus the gluon-gluon luminosity drops faster than quark luminosity.

The stationary point is determined by the interplay in Eq. (10) of the rise of the first term and the drop of the hadronic cross section $\sigma(N, m_H^2)$, which in turn is the product of the coefficient function and the luminosity. The shape of the NLO and NNLO contributions to the coefficient function is shown in Figs. 1 and 2, for two values of the Higgs boson mass, which are allowed by present data [2]. Clearly, when the partonic cross section rises monotonically with N , it is the drop of the parton luminosity that determines the drop of the hadronic cross section and thus the position of the saddle, and even when it drops, the decrease of its hadronic counterpart is much stronger in the presence of a luminosity, so that the location of the saddle is substantially larger.

We have determined the position of the saddle at NLO and NNLO by using NNPDF2.1 parton distributions [8] at the corresponding order. The results are shown in Fig. 3, which is the main result of this Letter. The position of the saddle N_0 depends on two independent variables, which can be chosen from the three variables m_H , s , and $\tau = m_H^2/s$. If the results for N_0 are shown as a function of τ , the dependence on the other variable (be it m_H or s) becomes very slight, because it enters only through the scale dependence of α_s and the parton distributions: this is explicitly seen in the top plot of Fig. 3, where the results are shown for two different values of m_H ; the dependence would be similar if \sqrt{s} were varied instead by a comparable factor. The dependence of the position of the saddle on the

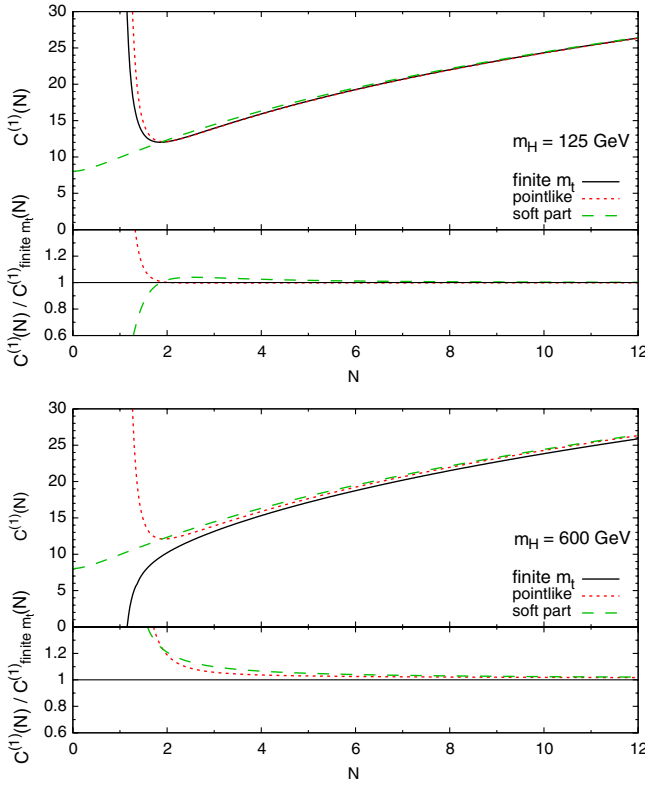


FIG. 1 (color online). The $O(\alpha_s)$ contribution to the coefficient function, Eq. (4), as a function of N , for $m_H = 125$ GeV (upper plot) and $m_H = 600$ GeV (lower plot). In each case we show the exact result and the pointlike and logarithmic approximations, as well as the ratio of the latter two to the exact result.

perturbative order is completely negligible (as also shown), and so is the dependence on whether the pointlike approximation is used or not. We have also checked that varying the renormalization and factorization scales by a factor of 2 changes the value of N_0 by 1% or less, even though more dramatic scheme changes that affect the infrared behavior of the PDF such as suggested in Ref. [9] might have somewhat larger effects. The impact of changing the PDF set or the value of α_s in a reasonable range is rather less than that.

The size of the region around the value of N_0 that dominates the integral can be estimated by computing the second derivative of $\ln\sigma(N, m_H^2)$, which gives the width of the Gaussian integral that approximates the Mellin inversion in the complex N plane. We find that a 1σ region corresponds to a variation of N_0 by about 25% about its central value. So, within the accuracy of our saddle point approximation, the distinction between different curves at fixed τ is of little import. For clarity and completeness, we show (see Fig. 3) the position of the saddle as a function of the Higgs boson mass for the three values of s relevant for the LHC, and as a function of the center-of-mass energy for two values of the Higgs boson mass.

We can now assess both the adequacy of the pointlike approximation and the desirability of resummation for given values of s and m_H . First, from Fig. 3, the given

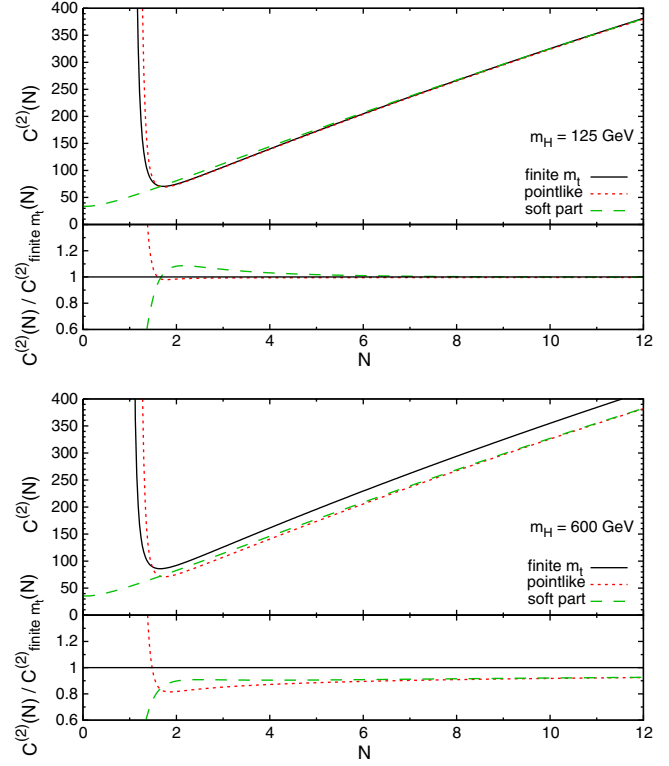


FIG. 2 (color online). Same as Fig. 1, but to $O(\alpha_s^2)$.

hadronic kinematics can be translated into a value of N_0 . Then, for this value of N_0 , we check whether the pointlike approximation is accurate and threshold resummation advisable. To this purpose, in Figs. 1 and 2 we compare the exact NLO and NNLO contributions to the coefficient function to their various approximations. At NLO, we use the implementation of Ref. [10] of the original exact result of Refs. [11,12]; at NNLO, a full exact computation is not available, and so for the light Higgs boson we use the expansion of Ref. [4] matched to the exact small z limit of Ref. [13], while for the heavy Higgs boson, where the expansion of Ref. [4] is unstable, we give the “finite m_t ” curve of Fig. 2 by correcting the pointlike result through the inclusion of the first-order correction in $\frac{m_t}{m_H}$ from Ref. [4] (it thus only indicates the location of the region where the finite m_t corrections are likely relevant).

The pointlike results at NLO [14,15] and NNLO [16] have been known for long. As a soft part, we show all contributions that survive the $N \rightarrow \infty$ limit, defined as the exact Mellin transform of all contributions of the form $z[\ln^k\{(1-z)/\sqrt{z}\}/(1-z)]_+$ and all contributions proportional to $\delta(1-z)$ to the NLO and NNLO coefficient functions. While the coefficients of these terms are fixed by soft resummation, there is a certain latitude in defining which subleading terms to include. Our choice reflects the exact soft kinematics [7,17], thereby optimizing the agreement with the exact result to all orders. The region in which the soft limit defined in this way is close to the full result is the region in which one expects soft resummation to

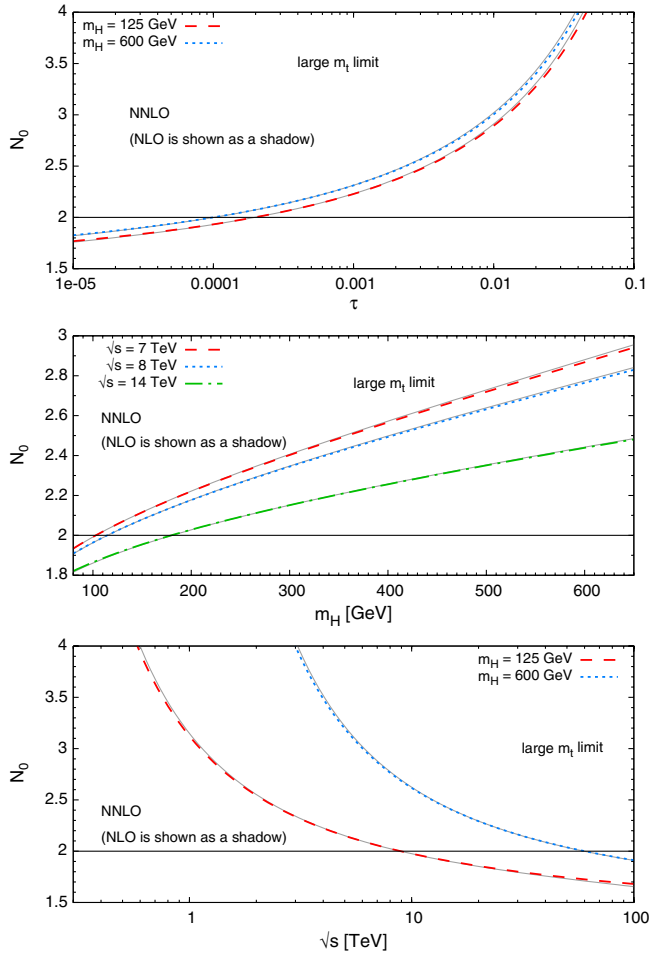


FIG. 3 (color online). The position of the saddle point N_0 for the Mellin inversion integral, Eq. (9), as a function of $\tau = m_H^2/s$ (top), m_H for three different values of the collider energy s (middle), s for two different values of m_H (bottom). The curves in the top plot depend very weakly on either m_H (shown) or s . Both the NLO and NNLO are shown, computed using the appropriate NNPDF2.1 PDF set [8] in each case.

improve the accuracy of the computation, even when all-order resummation is not mandatory.

An analysis of Figs. 1 and 2 shows that the region in which the pointlike approximation is adequate is very close to the region in which the soft approximation is good, and so the success of the former might indeed be explained by the accuracy of the latter. Both at NLO and NNLO, the relevant region is roughly $N \gtrsim 2$. This is the region where the partonic cross section starts to rise with N , driven by logarithmically enhanced contributions. The apparent failure of both approximations for the heavy Higgs boson at NNLO is likely due to the fact that in this case the “finite m_t ” computation is only approximate.

The fact that the same behavior is observed at NLO and NNLO is not accidental. On the one hand, the position of the saddle is largely determined by the parton luminosity, which is perturbatively very stable at NLO and beyond [8].

On the other hand, the shape of the coefficient functions and the dominance of soft terms are mostly controlled by the location of the leading small- and large- N singularities, which can be checked to be stable even upon all-order resummation [7]. This supports the expectation that the desirability of resummation and the all-order reliability of the pointlike approximation can be assessed on the basis of the known low orders.

At the LHC, τ is quite small: if $m_H \sim 125$ GeV, $\tau \sim 10^{-4}$, and if $m_H \sim 600$ GeV, $\tau \sim 10^{-3}-10^{-2}$, thereby leading to values of N_0 , close to the transition value $N \sim 2$, for which resummation is at best desirable, but certainly not mandatory, as $\alpha_s \ln^2 N_0 \ll 1$. Nevertheless, Fig. 3, in which the $N = 2$ line has been drawn for ease of reference, shows that for a heavy Higgs boson with $m_H \sim 600$ GeV, the pointlike approximation is adequate, and Sudakov resummation clearly advantageous, for any collider with center-of-mass energy up to $\sqrt{s} \lesssim 30$ TeV, and thus certainly at the LHC.

Even for a light Higgs boson with $m_H \sim 125$ GeV, the pointlike approximation is fine for a collider with energy of 7 TeV but may start failing as the energy is raised, and becomes inadequate at the LHC with $\sqrt{s} = 14$ TeV, where the saddle drops at $N_0 \approx 1.9$ so that the finite-mass corrections to $C^{(1)}$ in Fig. 1 reach the percent level, though care should be taken because, close to the region where the approximation breaks down, the conclusion may depend on various details and approximations. However, because the breakdown of the pointlike approximation is in significant part due to the presence of spurious high-energy double logs [13], it is expected to be more noticeable in less inclusive observables. Correspondingly, with this higher center-of-mass energy, Sudakov resummation is likely to cease to be advantageous for a light Higgs boson.

In summary, we have provided a way of assessing whether the pointlike approximation is adequate and Sudakov resummation desirable on the basis of the behavior of the known first few orders of the partonic cross section. Based on this, we conclude that for a light Higgs boson with $m_H \sim 125$ GeV, the full inclusion of finite top mass corrections to its cross section is likely to be important for accurate phenomenology at the LHC with $\sqrt{s} = 14$ TeV.

We thank F. Tackmann for a critical reading of the manuscript, and R. Harlander and A. Vicini for providing computing codes.

-
- [1] S. Dittmaier *et al.* (LHC Higgs Cross Section Working Group Collaboration), [arXiv:1101.0593](https://arxiv.org/abs/1101.0593).
 - [2] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **710**, 49 (2012); S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **710**, 26 (2012).
 - [3] C. Anastasiou, K. Melnikov, and F. Petriello, *Nucl. Phys. B* **724**, 197 (2005).

- [4] R. V. Harlander and K. J. Ozeren, *J. High Energy Phys.* **11** (2009) 088.
- [5] M. Kramer, E. Laenen, and M. Spira, *Nucl. Phys.* **B511**, 523 (1998).
- [6] S. Catani, D. de Florian, M. Grazzini, and P. Nason, *J. High Energy Phys.* **07** (2003) 028.
- [7] M. Bonvini, S. Forte, and G. Ridolfi, *Nucl. Phys.* **B847**, 93 (2011).
- [8] R. D. Ball, V. Bertone, F. Cerutti, L. Del Debbio, S. Forte, A. Guffanti, J. I. Latorre, J. Rojo, and M. Ubiali, (The NNPDF Collaboration), *Nucl. Phys.* **B849**, 296 (2011); *Nucl. Phys.* **B855**, 153 (2012).
- [9] E. G. de Oliveira, A. D. Martin, and M. G. Ryskin, [arXiv:1206.2223](https://arxiv.org/abs/1206.2223).
- [10] R. Bonciani, G. Degrossi, and A. Vicini, *J. High Energy Phys.* **11** (2007) 095.
- [11] D. Graudenz, M. Spira, and P. M. Zerwas, *Phys. Rev. Lett.* **70**, 1372 (1993).
- [12] M. Spira, A. Djouadi, D. Graudenz, and R. M. Zerwas, *Nucl. Phys.* **B453**, 17 (1995).
- [13] S. Marzani, R. D. Ball, V. Del Duca, S. Forte, and A. Vicini, *Nucl. Phys.* **B800**, 127 (2008).
- [14] S. Dawson, *Nucl. Phys.* **B359**, 283 (1991).
- [15] A. Djouadi, M. Spira, and P. M. Zerwas, *Phys. Lett. B* **264**, 440 (1991).
- [16] C. Anastasiou and K. Melnikov, *Nucl. Phys.* **B646**, 220 (2002).
- [17] S. Forte and G. Ridolfi, *Nucl. Phys.* **B650**, 229 (2003).