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1

Computerized evaluation of the articular loads on the normal dynamic human hip: a study of biomechanics

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ABSTRACT: Since the beginning of this century hip biomechanics has been widely studied. This subject is of great interest to the orthopaedic surgeon, especially regarding dismorphic and degenerative pathology, and trauma. Biomechanical evaluation is one of the key points in the setting-up of various surgical methodologies (osteotomies, arthroplasties, osteosyntheses, etc). Calculations referring to this subject are complex and almost impossible to do manually, as so hip operations orthopaedics base themselves on theoretical notions, empirically applied to individual cases and to personal experience. To obviate such inconveniences a computerized procedure has been realized to effect a biomechanical evaluation of the hip for each single subject. The Authors describe their own computerized procedure for a biomechanical analysis of normal hip. (HIP International, 1991; 1: 45-58)

KEY WORDS: Hip biomechanics, Biomechanical analysis

INTRODUCTION

A personal computer Apple Macintosh II CX and a spreadsheet Excel of Microsoft C were employed. The choice of this software was based on both its power and compatibility in Macintosh and MS-DOS fields, which make this method applicable to most personal computers presently in use.

Simply collected data are requested and neither the aid of a peculiar technique, nor the presence of a specialized staff are necessary; it is sufficient to know the patients weight and to have a standard radiography of the pelvis.

This study regards all the 12 phases of the single-support period of gait (according to Braune and Fischer).

The computer output is expressed numerically and it is represented in each phase by:

- a) Force \bar{K} , its components its angles of inclination.
- b) Force \bar{M} , its components its angles of inclination.
- c) Force \bar{R} , its components its angles of inclination.
- d) Lever arm of \bar{M} .
- e) Per cent variation between load (\bar{K}) and its vertical component (\bar{kz}).
- f) Per cent variation between partial body weight (\bar{P}) and

total load (\bar{K}).

This programme may turn out to be useful to point out cases of still asymptomatic joint hypersolicitations, but the Authors think that such a procedure may represent a reference for analogous studies on the pathological and prosthesis-ized hip.

Essentials of normal hip biomechanics

In orthostatism, with the support of both feet, hips can bear the load of the head, trunk and upper limbs (62% of human body weight). The centre of gravity of the body lies on a coronal plane perpendicular to the middle, the point of the segment linking the two centres of the femur heads in upper position, in comparison with the segment itself.

If the fulcrum is symmetric, the load each hip has to bear corresponds to 31% of human body weight. When a subject stands on one leg, the loaded hip supports the head, trunk, upper limbs and the other leg (81% of human body

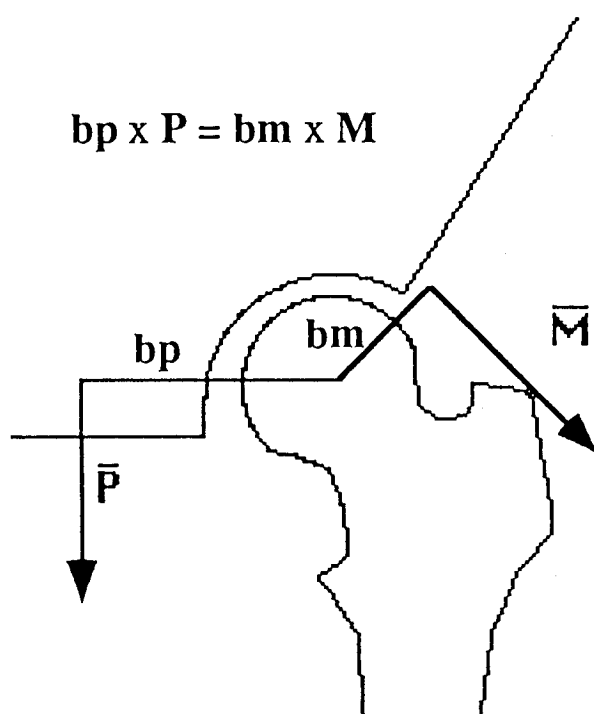


Fig. 1 - Balanced coxo-femoral system.

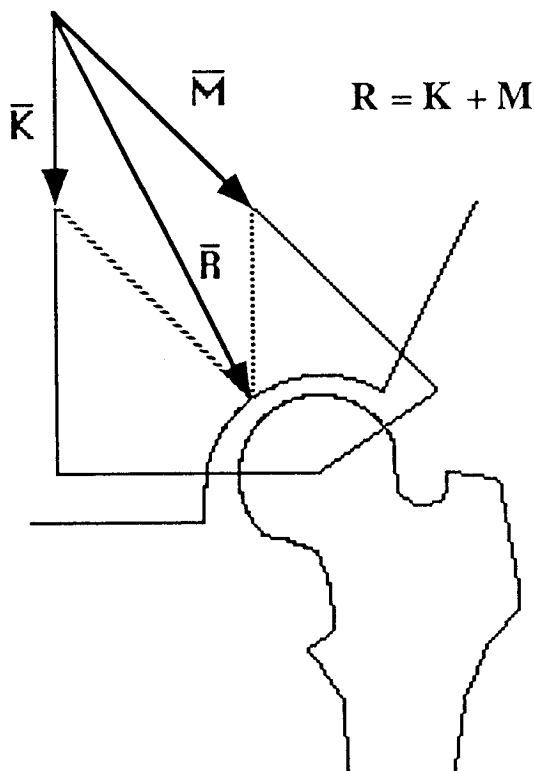


Fig. 2 - Resultant of the forces acting on head of femur.

weight) (T) (4).

The centre of gravity of the body lies on the perpendicular through the supporting foot on the ground, while the partial centre of gravity ($S5$), referring to the body with the exception of the lower limb in support, referring to the real load exerted on hip joint, departs towards the opposite side, going beyond the axis of pubis, so that P acts eccentrically on the hip and tends to fold up the pelvis, pushing it towards the femur.

This above described fall of the pelvis is counter-balanced by the action of the abductor muscles, belonging to the loaded hip engendering \bar{M} force.

A lever is moved, the fulcrum of which is the centre of the loaded femur head, \bar{P} is resistant force, and motive power is \bar{M} . In order to balance the system, it is necessary that the moment of the forces are equal, that is $bpP=bmM$ (Fig. 1). Under static conditions the resulting force (\bar{R}), acting on loaded hip corresponds to the vectorial addition of \bar{P} and \bar{M} .

According to Pawels (1), bp is about three times longer than bm , so in order to keep the system balanced, \bar{M} must be about three times greater than \bar{P} . During gait each hip has to bear, transitorily, \bar{P} ; but also \bar{D} , force of inertia, due the acceleration of $S5$.

Thus, to keep balance, the action abductor muscles have to counter-balance the load represented by the addition (\bar{K}) of \bar{P} and \bar{D} ; in other words the moment of \bar{M} has to equal the moment of \bar{K} (Fig. 2).

With reference to Fick's data (1860, 1879), Pawels noticed that in normal hip, \bar{M} acts a lever arm with an angle of 21° , while \bar{R} forms an angle of 16° and, during gait it varies from 1,5 to about 5 times the body weight.

Lastly, we have to remember one element that plays an important role in the determination of the loads the joint has to bear, that is almost no friction (0.007 according to Barnett and Cobol (1962); 0.002 according to Radim et al (1970). The absence of friction presents, as an immediate consequence, the uninfluence of cephalic ray and of the entity and distribution of joint stresses.

MATERIAL AND METHODS

With our procedure it is possible to calculate the total load (\bar{R}) of normal hip in a subject deambulating harmonically.

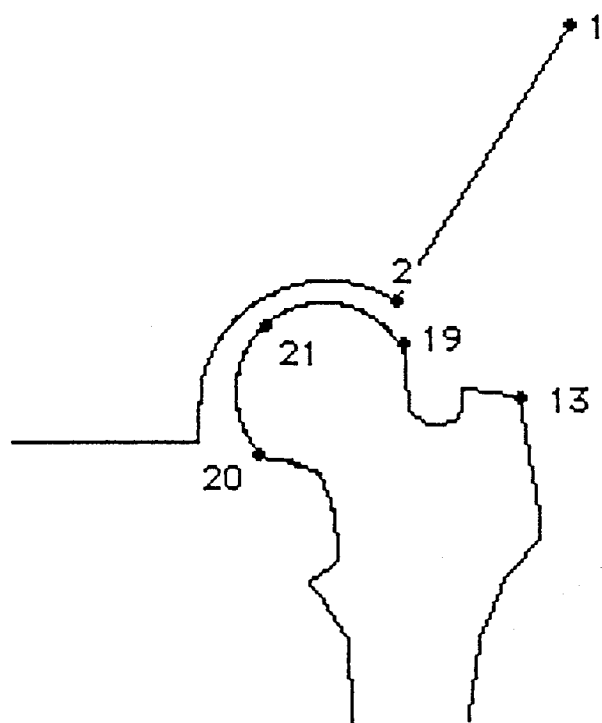


Fig. 3 - Points to be identified in pelvis radiogram:
 $p1$ = S.I.A.S.
 $p2$ = Cotyloid cilium
 $p13$ = Great trochanter apex
 $p19$ = Lateral limit of the head of femur
 $p20$ = Medial limit of the head of femur
 $p21$ = Any other point in the profile of the head of femur.

Radiological technique

A pelvis teleradiography in orthostatic position is performed, in order to reproduce the situation of the loaded hip as exactly as possible. It is preferable that the distance focus-film is the greatest possible (in any case almost 2 m).

The control x-ray has to be characterized by horizontal direction, *point of incidence* corresponding to the middle point of the line linking pubis with bisiliac line, *emergency point* in correspondence with the third sacral vertebra.

Data collecting

A bidimensional system of Cartesian axes, is inscribed on pelvis radiogram where the "X" axis corresponds to a line tangent the lower border of the tuberosities of the ischium,

and the "Y" axis crosses the pubis. With a common ruler, the co-ordinates of six points (in cm) are taken down.

Technical notes

Some Authors have tackled this problem basing themselves on the data of the subject studies by Braune and Fischer at the end of the past century and on a fixed length for the lever arm of \bar{M} force (4 cm). In contrast to realize a procedure that would give the evaluation of the joint stresses for the hip in every phase of gait, and which could be utilisable in every subject: there were however some approximations which were taken into account as follows:

- application point of \bar{M} on the hip bone it is a point with an abscissa belonging to cotyloid cilium and an ordinate belonging to SIAS;
- application point of \bar{M} on greater trochanter: the apex of greater trochanter;
- acceleration of $S5$: the one Braune and Fischer calculated (our subject is able to deambulate harmonically) (1,4).

As far as lever arm \bar{M} is concerned a very important element in the calculation of \bar{R} , its theoretical length has been calculated i.e., the one necessary for the balance of the system, supposing that this corresponds to the real one, according to the original hypothesis assuming that the subject deambulates without limping and that the system is therefore, balanced.

The study regarded all twelve phases of the right single-support period of gait (according to Braune and Fischer).

Method description

Radiographical enlargement

The co-ordinates of the points on the radiogram are subject to an increase, because of radiographical enlargement; so it is necessary to correct them: if not corrected, substantial errors in calculation will possibly occur.

In order to know the value of radiographical enlargement and the real dimensions of the radiographed object, it is necessary to know: *focus/film distance*, *object/film distance* and the *evident dimensions of the object* (Fig. 4).

Generally, among these three elements the only one

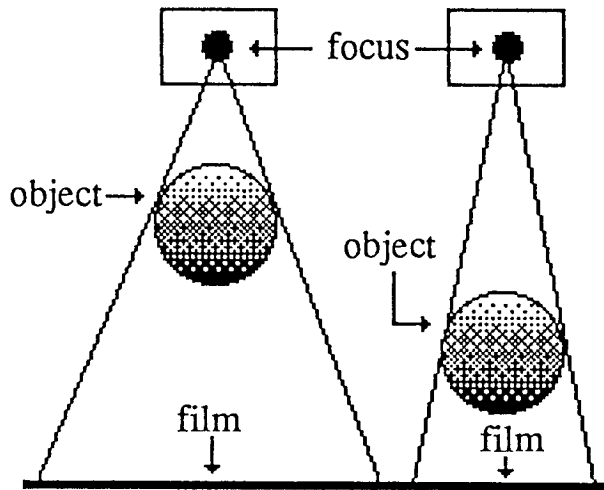


Fig. 4 - Radiological enlargement.

known is the evident dimension of the object. In fact, even if it is theoretically possible to calculate the focus/film distance, this measurement can, through the use of some radiographical tubes, either turn out to be inconvenient or only a rough estimate.

It is even more difficult, if not impossible, to make a precise measurement of the focus film distance: for two reasons:

- a) the object discussed (hip joint) is tridimensional;
- b) *in vivo* the interested anatomical structures are, obviously, unapproachable for the usual instruments of measurement.

The problem regarding radiographical enlargement has been solved by employing the following: during the radiography a reference radio-opaque element is located at a known distance from the film; the dimensions of this element are known; on the radiogram the evident dimension of the reference element is measured, and the theoretical distance of the focus from the film by:

$$\text{distF} = (\text{dma} \cdot \text{dsr}) / (\text{dma} - \text{dmr})$$

(**dma**=evident dimension; **dmr**=real dimension; **dsr**=film reference distance).

The greatest and shortest distance object film is approximately stated. This distance should comprehend the whole hip joint.

The highest, lowest and probable factors of reduction are calculated by:

$$\text{frm} = (\text{distFP} - \text{dmx}) / \text{distFP}; (\text{dmx} = \text{greatest distance});$$

$$\text{frm} = (\text{distFP} - \text{dmn}) / \text{distFP}; (\text{dmn} = \text{shortest distance});$$

$$\text{fr} = (\text{frm} + \text{frm}) / 2; (\text{fr} = \text{probable factor pf reduction}).$$

The maximum error range for such procedure is calculated:

$$\text{range\%} = ((\text{frm} - \text{fr}) / \text{fr}) \cdot 100$$

Per cent enlargement is:

$$\text{fi} = ((1 - \text{fr}) / \text{fr}) \cdot 100; (\text{fi} = \text{probable per cent enlargement}).$$

The co-ordinates pointed out in the radiography for the factor of reduction are multiplied. Some values, which do not quite correspond to reality assure a percent unsubstantial error or, anyway, a known one.

Cephalic centre co-ordinates calculation

The centre of the femur head represents the centre of rotation of the normal whole hip joint system and also the fulcrum of the lever keeping the balance of the pelvis during gait.

The data we possess are the co-ordinates of the three points of the femur head radiological profile (p19, p20, p21).

On a Cartesian plane, the equation referring to a circumference is

$$x^2 + y^2 + \alpha + \beta y + \gamma = 0$$

where the co-ordinates of the centre are: $C(a;b)$, with:

$$a = -(\alpha/2) \quad \text{and} \quad b = -(\beta/2)$$

$$\text{and the radius is: } r = \sqrt{a^2 + b^2 - \gamma}; \quad r = (1/2) \sqrt{\alpha^2 + \beta^2 + 4\gamma}$$

If a circumference links 3 points: $A(x_1; y_1)$, $B(x_2; y_2)$,

$C(x_3; y_3)$; the value of the coefficients α, β, γ , is obtained through the solving of the following system:

$$\begin{aligned} x_1^2 + y_1^2 + \alpha x_1 + \beta y_1 + \gamma &= 0 \\ x_2^2 + y_2^2 + \alpha x_2 + \beta y_2 + \gamma &= 0 \\ x_3^2 + y_3^2 + \alpha x_3 + \beta y_3 + \gamma &= 0 \end{aligned}$$

We identify the three points with p19, p20 and p21 (Fig.3) and proceed as follows:

$$pp = (((bb^2) + (tt^2)) - ((aa^2) + (dd^2)) + zz) ;$$

$$rr = bb - aa + zz ;$$

$$ss = ((cc^2) + (gg^2)) - ((aa^2) + (dd^2)) + zz ;$$

$$yy = ((pp \cdot cc) - (aa \cdot p) - (rr \cdot ss) + rr) / ((cc \cdot tt) + (cc \cdot dd) + (gg \cdot rr) + (aa \cdot tt) - (aa \cdot dd) - (dd \cdot rr)) ;$$

$$ss = ((cc^2) + (gg^2)) - ((aa^2) + (dd^2)) + zz ;$$

$$yy = ((pp \cdot cc) - (aa \cdot p) - (rr \cdot ss) + rr) / ((cc \cdot tt) + (cc \cdot dd) + (gg \cdot rr) + (aa \cdot tt) - (aa \cdot dd) - (dd \cdot rr))$$

$$[yy = \beta] ;$$

$$xx = ((yy \cdot (dd - tt)) - pp) / rr [xx = \alpha] ;$$

$$z = -((aa \cdot xx) - (dd \cdot yy) - ((aa^2) + (dd^2))) [z = \gamma] ;$$

$$tx = -(xx/2) ; \quad ty = -(yy/2) ; \quad ra = \sqrt{(tx^2 + ty^2 - z)}$$

where tx and ty are the co-ordinates of the cephalic centre and the cephalic radius (which, normal hip being frictionless, is not important in our calculations)⁽¹⁾.

Computation of \bar{M} angle of elevation on OYZ plane

The co-ordinates pointed out in the radiography refe to a bidimensional Cartesian system; in order to spread this procedure to a three-dimensional space, we identify X-axis with Y-axis (belonging to the new system, positive on the right) and Y-axis with Z-axis (positive, below): the new X-axis will mark gait.

Let us compute the angle of inclination, or elevation, of \bar{M} , or better of \bar{M} projection on OYZ plane.

Consider right-angled triangle ABC, where A(p2x; p1y) and B(p13x; p13y) each represent the points of application of \bar{M} on the wing of ilium and on the greater trochanter, while C is the apex of right angle (Fig. 5):

Then:

$$a = \text{ABS}(p13x - p2x + zz) ; (a = \text{BC cathetus length})$$

⁽¹⁾ - In the effectuation of the calculation of cephalic centre co-ordinate, we verified that on employing mm/measures, calculations were exact, while on using cm, a quite unacceptable error came up, which was apparently unjustified. The explanation of such a phenomenon is that the computer is able to represent only a finite quantity of numbers: if a real number x is comprised between two engine consecutive numbers, A and B it will be represented as A or as B.

The process of appoximation of a real number to the nearest engine number, is called "rounding" and the error so included is named "cutting-off error".

If the computer rounds x to engine number A, will have an "absolute error" of A - X ; a more significant valuation of this error is given by "relative error" (A-x)/x · 100 , that is, the rounding error expressed percentually.

A particular great numerical mistake appears when two near numbers, are subtracted, for example:

$$0.991012312 - 0.991009987 = 0.000002325;$$

If the computer is able to make a calculation with 6 figures, it will cut results up to 0.000002, with an absolute error of 0.000000325, but with a "relative error" of 14%:

$$0.000000325 / 0.000002325 \cdot 100 = 13.97849462.$$

Obviously, if calculations of this kind are to be repeated, the error can increase so much as to make the result completely insignificant. Such a kind of mistake appears when high numbers are summed up with law numbers, and this addition is likely to cause the loss of many significant figures.

In our case, though Excel precision is composed by 14 figures, the passing from one decimal to another (from cm to mm) is enough to cause the numerical error.

In order to solve this problem a device has been employed, i.e. the values of the co-ordinates (in cm) are multiplied by 1000, then the calculations are made and the results connected by dividing by 1000.

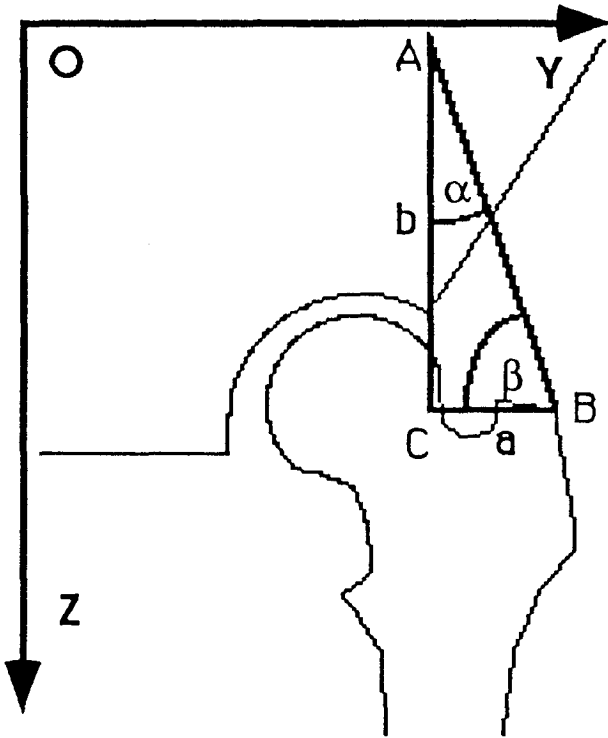


Fig. 5 - Angle of inclination of \bar{M} on plane OYZ (α) with reference to Z axis.

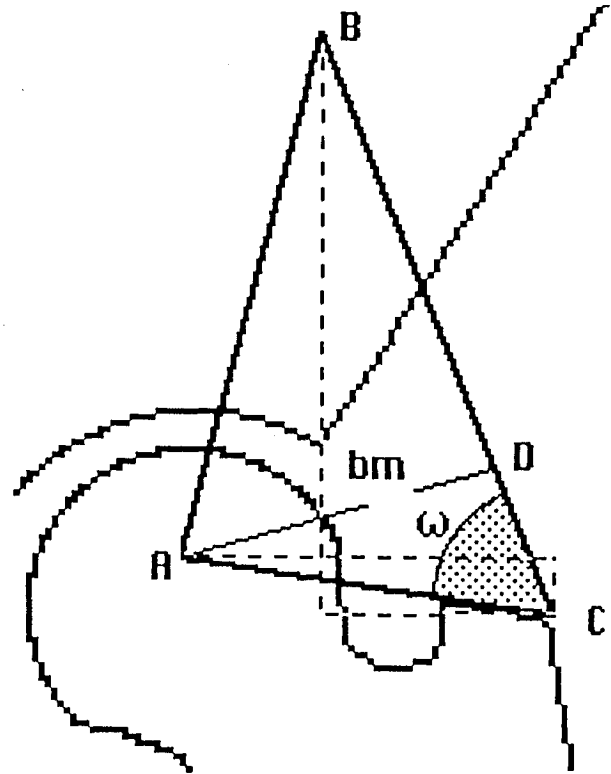


Fig. 6 - Calculation of bm .

$$b = \text{ABS}(p1y - p13y + zz) ; (b = AC \text{ cathetus length})$$

$$\beta = \text{ATAN}(b/a) ; (\beta = \text{angle opposite to } b \text{ in radians})$$

$$\alpha_{\text{rad}} = (\pi/2) - \beta ; (\alpha_{\text{rad}} = \text{angle opposite to } a, \text{ or } \bar{M} \text{ angle of elevation in radians})$$

$$\alpha = \alpha_{\text{rad}} (180/\pi) ; (\alpha = \text{angle opposite to } a, \text{ or } \bar{M} \text{ angle of elevation in degrees}).$$

Computation of the length of \bar{M} lever arm projection on OYZ plane

Analyze ABC triangle, where (Fig. 6):

$A(xA; yA)$ = centre of the femur head;
 $B(xB; yB)$ = \bar{M} point of application on the wing of ilium;
 $C(xC; yC)$ = greater trochanter apex.

The result is: $xA=tX, yA=ty ; xB=x, yB=y ;$

$$xC=p13X, yC=p13y .$$

The 3 sides of ABC are:

$$AB = \sqrt{(xA-xB)^2 + (yA-yB)^2} ;$$

$$BC = \sqrt{(xB-xC)^2 + (yB-yC)^2} ;$$

$$AC = \sqrt{(xA-xC)^2 + (yA-yC)^2} ;$$

We employ Briggs's formulas which allow us to calculate the amplitude of the angles of a triangle when we know

$$\text{its sides: } p = (AB+BC+AC)/2 ;$$

(p = ABC halfperimeter)

$$\omega = \arctan \sqrt{(p-BC)(p-AC)/2p(p-AB)} ;$$

(ω =angle apposite to **AB** side).

We draw **ABC** triangle height, with reference to **BC** side and verify that (**AD**) corresponds to **bM** or, better, to the projection of **bM** on OYZ plane, in the right-angle rectangle **ACD**, **bM** represents the cathetus apposite acute angle, to:

$$bM = AC \sin \omega .$$

Computation of (\bar{D}) force of inertia and of ms components

A very important factor contributing to the determination of balance of the system given by the joint of stressed hip, is \bar{D} force of inertia, due to the translation of the partial centre of gravity of the body (**S5**).

$D = ma$, where: **m** is body mass and **a** acceleration.
We obtain:

$$fp = 0.8136286201 \times \text{weight} ; (\text{fp} = \text{body weight, not}$$

included stressed limb)

$$\text{mass} = (fp/gi) ; (gi = 9,80665 \text{ m / sec}^2 = \text{gravity}$$

acceleration).

In order to calculate \bar{D} , it is necessary to know the acceleration of **S5** during the various phases of gait. As anticipated above, in our calculation we have supposed that the acceleration of the partial baricentre the subject studied is subjected to, coincide with those studied by Braune and Fischer (Tab. I).

As acceleration can also be considered a vector quantity (it has in fact one point of application (**S5**) one direction and one intensity or modulus), it can be divided into three components, according to X, Y, Z axes.

In the course of this study we could verify that the components of inertia, according to gait direction, and to orthogonal gait direction (X and Y axes) are not very meaningful, while vertical inertia has an important value.

TABLE I - ACCELERATION COMPONENTS IN THE PHASES OF RIGHT HEMIPACE (m/SEC²), ACCORDING TO BRAUNE AND FISCHER

	ACCX	ACCY	ACCZ
12	- 0,03	-1,08	-5,08
13	- 0,82	-0,88	-3,99
14	- 1,91	-0,71	-1,88
15	- 1,12	-0,54	0,8
16	- 0,42	-0,4	3,44
17	- 0,3	-0,28	5,04
18	- 0,24	-0,18	4,08
19	- 0,13	-0,23	2,5
20	0,06	-0,5	0,56
21	0,29	-0,89	-1,53
22	0,46	-1,4	-2,07
23	0,5	-1,97	-0,13

It is the last that will be balanced by abductor force.

The force of inertia is the reaction of acceleration and is therefore oriented in the opposite direction: if acceleration is oriented upwards, the force of inertia will be oriented downwards, inertia will tend upward and will evade the stress.

$$dx = \text{massa} \times \text{accx} ;$$

$$dy = \text{massa} \times \text{accy} ;$$

$$dz = \text{massa} \times \text{accz} ;$$

where **dx**, **dy**, and **dz** are the components of the force of inertia and **accx**, **accy** and **accz** are the components of acceleration.

Total load calculation (\bar{K}) and its components

The total load (\bar{K}) should be balanced by abductor force. It is a vector which can be divided into 3 components, according to the co-ordinate axes.

$$Kx=dx, Ky=dy, Kz=fp+dz .$$

Knowing the 3 components of a vector (\bar{V}) it is possible to calculate their module through:

$$|V| = \sqrt{vx^2 + vy^2 + vz^2}, \text{ which applied to the particular}$$

case, is

$$|K| = \sqrt{kx^2 + ky^2 + kz^2}.$$

The per-cent variation between \bar{K} and \bar{KZ} , has also been calculated verifying that this (**ratio $\bar{k}kz$**) is slight (1%) so, in a calculation where great accuracy is not requested, the 2 vector can be assimilated.

On the contrary, the per-cent variation between \bar{K} and \bar{P} (**ratio $\bar{k}pk$**) can be substantial, overpassing some phases of the pace (as the twelfth) 50%. This is a confirmation of the fact that in our calculation of hip joint stress, a statical valuation not considering the existence of the force of inertia, may lead to intolerable errors.

Lever arm calculation of \bar{K} and its components

In a first kind lever, in order to maintain the balance, the moment of power and the moment of resistance must clash.

The moment (**m**) of a force (**F**) is given by the product of **F** multiplied by its lever arm (**b**):

$$m = bF \sin \Theta, \text{ where } \Theta \text{ is the angle formed by the}$$

direction of the force and the force of the lever arm:

$$PbP = RbR.$$

In our case:

- the fulcrum is represented by the centre of the head of femur;
- resistance corresponds to the weight, or, better, to \bar{K} ;
- power is \bar{M} ,
- **bR** is the distance between fulcrum and baricentre,
- **bP** is the distance between point of application of \bar{M} and fulcrum.

We calculated the two lever arms and their moments, considering that **bP** and **bR** are not scalar quantities, but vectorial ones (in fact they present a point of application, a direction, a module, represented by their length), and that, therefore, the moments of the load and of the muscular force are vectorial products:

$$QP = bP \wedge P; \quad QM = bM \wedge M.$$

TABLE II - COORDINATES (CM) OF S5 (XS, YS, ZS) AND OF THE CENTRE OF HEAD OF FEMUR (XH, YH, ZH) ACCORDING TO BRAUNE AND FISCHER

Phases	XS	YS	ZS	XH	YH	ZH
12	102,4	-1,47	98,28	107,7	8,35	82,27
13	109,7	-1,12	98,67	113,9	9,19	82,83
14	116,4	-0,89	99,58	119,9	9,78	83,97
15	123,4	-0,75	101	125,4	10,16	85,28
16	129,4	-0,71	102,1	130,4	10,28	85,95
17	136,2	-0,72	102,8	135,4	10,2	86,32
18	142,3	-0,82	102,8	140,4	10,01	86,16
19	148,9	-0,88	102,2	145,9	9,92	85,42
20	155,2	-1	101,2	151,3	9,78	84,31
21	161,5	-1,17	100,1	156,9	9,63	83,3
22	167,8	-1,5	99,25	162,9	9,25	82,78
23	174,3	-2	98,64	169,4	8,7	82,56

As far as \bar{K} lever arm calculation is concerned, we should know **S5** and the centre of the head of femur movements.

Through the data in our possession such movements could not be calculated, so we were forced to operate some approximations.

Braune and Fischer calculated, for the subject they analyzed, the co-ordinate of **S5** and of the centre of the head of femur during different pace phases (Tab. II).

It was supposed that in the subject studied through such a procedure, the co-ordinates were proportionally correspondent to those calculated by Braune and Fischer:

a) A translation of the cartesian axes on a new reference system is effected with parallel axes to the previous ones and origin in correspondence to centre of femoral head.

The co-ordinates of **S5** in the new system will be:

$$XS_n = XS - HY; \quad YS_n = YS - YH;$$

$$ZS_n = -(ZS - ZY).$$

Co-ordinates on **Z** axis present inverted sign; this is due to the fact that in the new system the positive side of **Z** axis is directed below while **ZS** and **ZY** co-ordinates partake of a system in which **Z** axis (as usual) positive upwards;

b) The existence of a directed proportionality between half the distance between the centres of the heads of femur and \bar{K} Y co-ordinate: knowing that in Braune and Fischer's subject intercephalic distance is about 170 cm we calculate the coefficient of proportionality (**percy**) (Tab. III):

$$XSn = XS - HY ; YSn = YS - YH ;$$

$$ZSn = -(ZS - ZY) .$$

We notice that hemicephalic distance corresponds to the abscissa of the centre of femur head in our first bidimensional Cartesian system and that such distance is **tx**, so that the previous formula can be written as follows:

$$\text{percy} = (yH/tx) .$$

c) So: $\text{percx} = xH/yH$, where **percy** corresponds to the tangent of the angle formed by direction and by a straight line linking **S5** and the centre of the head of femur.

d) Consequently $\text{percz} = zH/yH$, where **percz** is the tangent of the angle formed by \bar{K} direction and a straight line linking **S5** and the centre of the head of femur.

Now, supposing that our subject is moving as harmonically as Braune and Fischer's, we calculate the components of \bar{K} lever arm.

$$bky = tx \cdot \text{percy} ; [\text{Y component}];$$

$$bkx = bky \cdot \text{percx} [\text{X component}];$$

$$bkz = bky \cdot \text{percz} [\text{Z component}],$$

$$\text{so } \bar{K} \text{ lever arm is: } bk = \sqrt{bkx^2 + bky^2 + bkz^2} .$$

\bar{K} and its components moment calculation

The moment of the stress or load, is the vectorial product of \bar{K} and its lever arm (**bk**) therefore in a space with three

TABLE III - S5 COORDINATES IN A CARTESIAN SYSTEM WITH ITS ORIGIN IN CORRESPONDENCE TO THE CENTRE OF THE HEAD OF FEMUR IN BRAUNE AND FISHER'S SUBJECT, AND FACTORS USED FOR THE CALCULATION OF SUCH COORDINATES IN ANY SUBJECT

Phases	XSn	YSn	ZSn	PERCX	PERCY	PERCZ
12	-5,29	-9,82	-16	0,539	-1,16	1,63
13	-4,23	-10,3	-15,8	0,41	-1,21	1,536
14	-3,49	-10,7	-15,6	0,327	-1,26	1,463
15	-2,03	-10,9	-15,8	0,186	-1,28	1,444
16	-0,97	-11	-16,1	0,088	-1,29	1,469
17	0,78	-10,9	-16,5	-0,07	-1,28	1,51
18	1,93	-10,8	-16,6	-0,18	-1,27	1,533
19	3,02	-10,8	-16,8	-0,28	-1,27	1,552
20	3,89	-10,8	-16,9	-0,36	-1,27	1,566
21	4,55	-10,8	-16,8	-0,42	-1,27	1,555
22	4,92	-10,8	-16,5	-0,46	-1,26	1,532
23	4,86	-10,7	-16,1	-0,45	-1,26	1,503

dimensions it can be identified by its components on X, Y, Z axes, that is: **qx, qy, qz**.

A vectorial product can be expressed through the components of the factors, as follows: $c = a \wedge b$.

$$cx = aybz - azby ; cy = azbx - axbz ;$$

$$cz = axby - aybx ;$$

which in our case becomes:

$$qx = bky \cdot kz - bkz \cdot ky ; qy = bkz \cdot kx - bkx \cdot kz ;$$

$$qz = bkx \cdot ky - bky \cdot kx ;$$

$$\text{so moment (q) of the load is: } Q = \sqrt{qx^2 + qy^2 + qz^2} .$$

Directional cosines of moment of \bar{M} calculation

In tridimensional space, direction of a vector is individualized by the angles that the vector makes with the three

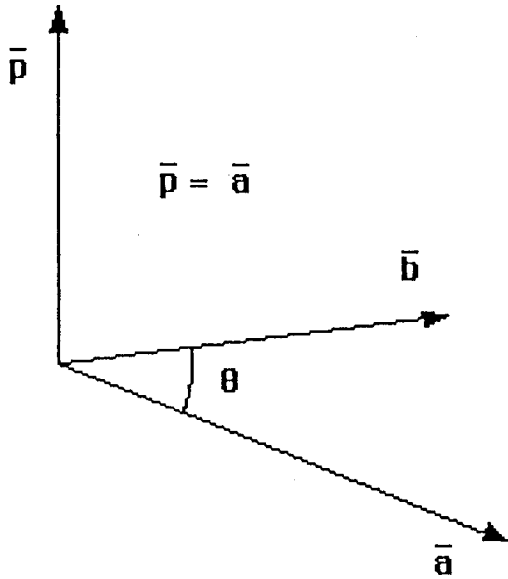


Fig. 7 - Graphic representation of vectorial product.

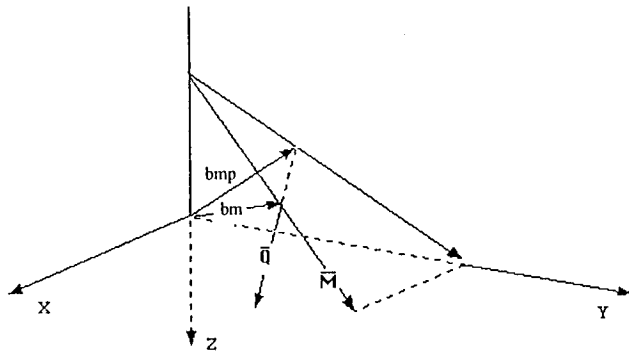


Fig. 8 - Graphic representation of \mathbf{M} and \mathbf{Q} .

Cartesian axis; cosines of these angles are said directional cosines of the vector.

\mathbf{P} = vertex of the vector;

m, n, p = co-ordinates of \mathbf{P} , proportional to directional cosines, and that is:

$$m = a \cos \alpha, n = a \cos \beta, p = a \cos \gamma; \text{ where } \alpha, \beta \text{ e } \gamma \text{ are the}$$

angles that the vector individualizes with X, Y, Z axes and a is the modulus of the vector:

$$a = \sqrt{m^2 + n^2 + p^2}.$$

In our case, the directional cosines of moment of \mathbf{K} are:

$$p_x = q_x/q, p_y = q_y/q, p_z = q_z/q.$$

Directional cosines of \mathbf{M} force calculation

Here we recall some principles of vectorial geometry:

1) The vectorial product between vector \mathbf{a} and vector \mathbf{b} is vector \mathbf{c} , forming with them a dextrorse tern $\mathbf{p} = \mathbf{a} \times \mathbf{b}$,

\mathbf{b} ving as a modulus: $|c| = |a| \cdot |b| \cdot \sin \Theta$, where Θ

is the angle, not wider than 180° , formed by \mathbf{a} and \mathbf{b} .

The vectorial product is a vector presenting direction orthogonal to the phase where we find vectors \mathbf{a} and \mathbf{b} , and verso observer standing in its direction, can see \mathbf{a} , belonging to Θ angle wheeling counter clockwise in order to superimpose itself to \mathbf{b} (Fig.7).

2) We name **versor** (\mathbf{v}) of a vector (\mathbf{V}) a vector having the same direction and versor of \mathbf{V} , as the unitary modulus.

Directional cosines are, practically, the components of its versor.

With such preliminary statements, we analyse our peculiar case (Fig. 8).

For definition \mathbf{M} and its projection on OYZ plane are complanar, so are \mathbf{bM} and its projection on OYZ (\mathbf{bMp}) plane (Fig. 9).

As \mathbf{bm} orthogonal to \mathbf{M} as they belong also to the same plane, \mathbf{bmp} is orthogonal to \mathbf{M} . Furthermore, also \mathbf{Q} is orthogonal to \mathbf{M} , and at the same time placed on the same plane as \mathbf{bm} so, \mathbf{Q} lies on the same plane as \mathbf{bm} and \mathbf{bmp} (Fig.10).

This means that the vectorial product of versor of \mathbf{Q} by \mathbf{bmp} is a vector having the same direction of \mathbf{M} , and its directional cosines clash with directional cosines of \mathbf{M} (Fig.10).

The components of a vectorial product can be calculated beginning with the components of the two vectors, as fol-

$$\text{lows: } c = a \wedge b.$$

$$c_x = a_y b_z - a_z b_y; \quad c_y = a_z b_x - a_x b_z;$$

$$c_z = a_x b_y - a_y b_x;$$

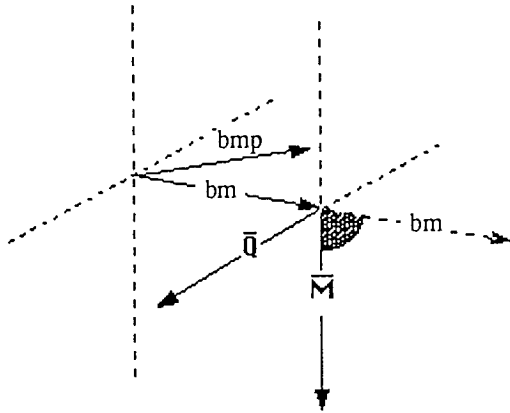


Fig. 9 - Graphic representation of \bar{M} and \bar{Q} .

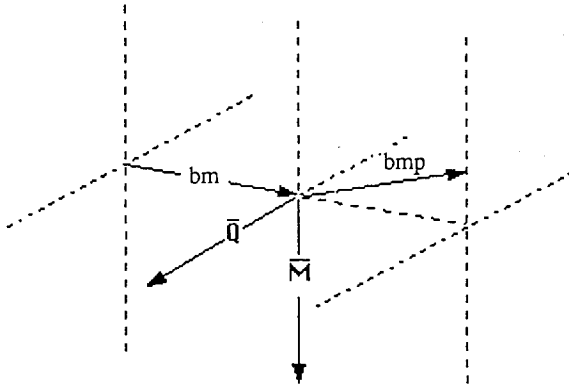


Fig. 10 - Graphic representation of \bar{M} and \bar{Q} .

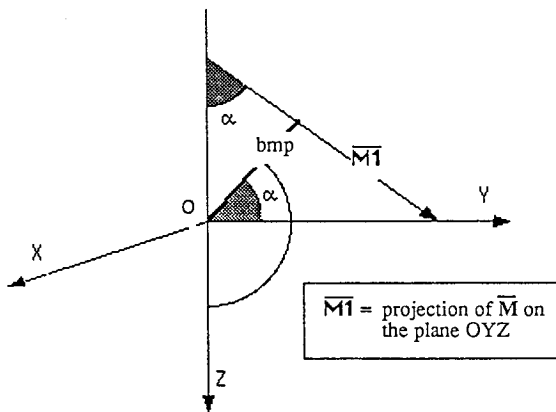


Fig. 11 - Graphic representation of the angle formed by \bar{bmp} and coordinates axes.

The components of versor of \bar{Q} are: $\bar{p}_x, \bar{p}_y, \bar{p}_z$.

The components of versor of \bar{bmp} are the cosines of the angles \bar{bmp} forms with co-ordinates axes (Fig.11), that is:

- for X axis: $\cos 0^\circ = 0$
- for Y axis: $\cos \alpha$
- for Z axis: $\cos 90^\circ + \alpha = \sin \alpha$.

Applying the above mentioned formula, we calculate the components of a vector presenting direction equal to \bar{M} :

$$m_x = -p_y \cdot \sin(\alpha \text{ rad}) - p_z \cdot \cos(\alpha \text{ rad}) ;$$

$$m_y = p_x \cdot \sin(\alpha \text{ rad}) ; \quad m_z = p_x \cdot \cos(\alpha \text{ rad}) .$$

The modulus of such vector is:

$$m_t = \sqrt{m_x^2 + m_y^2 + m_z^2} ;$$

And respective directional cosines:

$$m_{mx} = m_x / m_t ; \quad m_{my} = m_y / m_t ;$$

$$m_{mz} = m_z / m_t ; \quad \text{these are in fact, directional cosines of } \bar{M}.$$

\bar{M} lever arm real length calculation

We analyze the right-angled triangle \bar{AOC} , where \bar{A} is the point of intersection of \bar{M} direction and Y axis, \bar{O} is the centre of the head of femur and \bar{OC} corresponds to \bar{bm} , that is, to the projection of abductor force lever arm on \bar{OYZ} plane (Fig.12).

$$\bar{OA} \text{ side: named } \bar{fmctf}, \text{ is: } \bar{fmctf} = \bar{bm} / \tan \alpha \text{ rad.}$$

We analyze a second right-angled triangle, \bar{AOD} where \bar{OD} cathetus, is \bar{M} (\bar{bmc}) lever arm (Fig.12).

$$\text{It appears that: } \bar{bmc} = \bar{fmctf} \cdot \tan(\text{ACOS}(m_{mz})) ;$$

where $\text{ACOS}(m_{mz})$ is the angle formed by \bar{M} and Z axis;

The angle of elevation of muscular force, considering the three co-ordinates axes, is:

$$\bar{aifmx} = \text{ACOS}(m_{mx}) \cdot 180 / \pi ;$$

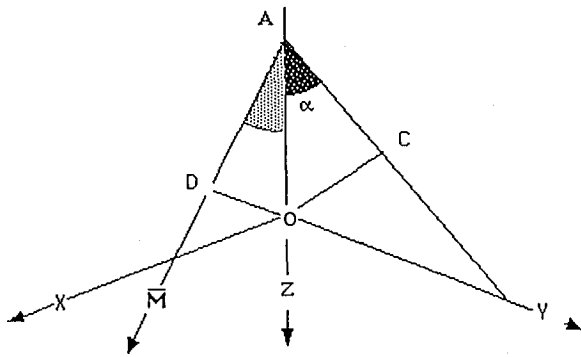


Fig. 12 - Graphic representation of \bar{M} (OD) .

$$\text{aifmy} = \text{ACOS}(\text{mmy}) \cdot 180/\pi ;$$

$$\text{aifmz} = \text{ACOS}(\text{mmz}) \cdot 180/\pi .$$

\bar{M} and its component calculation

As the system is, by assumption, balanced, the moment of the load and that of muscular force are equivalent.

Therefore: $\bar{Q} = \bar{M} \cdot \text{bme}$.

abductor force \bar{M} , named **f_{musc}** in the programme is therefore:

$$\bar{M} = \bar{Q} / \text{bme} ; \text{ and each respective component:}$$

$$\text{fmuscx} = \text{fmusc} \cdot \text{mmx} ; \quad \text{fmuscy} = \text{fmusc} \cdot \text{mmy} ;$$

$$\text{fmuscz} = \text{fmusc} \cdot \text{mmz} .$$

Resultant load \bar{R} and its components calculation

Total load exerted on the head of femur, \bar{R} (which in the programme is named **result**, is the vectorial addition of \bar{K} and \bar{M} .

\bar{R} components are, therefore, represented by the addition of \bar{K} and \bar{M} components, according to the co-ordinate axes.

So: $\text{rx} = \text{kx} + \text{fmuscx} ; \quad \text{ry} = \text{ky} + \text{fmuscy} ;$

$$\text{rz} = \text{kz} + \text{fmuscz} ;$$

$$\text{result} = \sqrt{\text{rx}^2 + \text{ry}^2 + \text{rz}^2}$$

Also \bar{R} projection on OYZ plane has been calculated (**risoyz**):

$$\text{risoyz} = \sqrt{\text{ry}^2 + \text{rz}^2} \text{ as the per cent variation between}$$

$$\bar{R} \text{ and } \text{risoyz} (\text{rtioris}): (\text{rtioris} = 100 - (\text{risoyz} / \text{result}) \cdot 100) .$$

A substantial difference between \bar{R} and its projection on OYZ plan does not exist, as per cent relative variation is about 1%.

So, the two forces can be assimilated in not particularly complex calculations.

Directional cosines calculation

A vector directional cosine, according to one of co-ordinate axes is the ratio between the components modulus of the vector itself. Thus:

$$\text{crx} = \text{rx} / \text{result} ; \quad \text{cry} = \text{ry} / \text{result} ;$$

$$\text{crz} = \text{rz} / \text{result} .$$

Knowing directional cosines it is possible to calculate the angles of elevation of \bar{R} , with reference to the three Cartesian axes:

$$\text{gradorx} = \text{ACOS}(\text{crx}) \cdot 180/\pi ,$$

$$\text{gradory} = \text{ACOS}(\text{cry}) \cdot 180/\pi ,$$

$$\text{gradorz} = \text{ACOS}(\text{crz}) \cdot 180/\pi .$$

The angle of the elevation of \bar{R} projection on OYZ plane (**risoyz**), with reference to (**gradoyz**) is:

$$\text{gradoyz} = \text{ACOS}(\text{rz} / \text{risoyz}) \cdot 180/\pi .$$

This angle is about 16°, according to Pawels.

Every calculation has been applied to each phase of right single-support period of gait, according to Braune and Fischer.

INPUT DATA			
dmr=—>	10	reference object real dimensions in cm	
dma=—>	10,4	reference object apparent dimensions in cm	
dsr=—>	8	reference distance from film in cm	
dmx=—>	15	minimum distance object/film	
dmn=—>	5	maximum distance object film	
	input	rectific.	
cxp1=—>	14,4	13707,6923	SIAS abscissa in cm
cyp1=—>	16,22	15440,1923	SIAS ordinate in cm
cyp2=—>	9,89	9414,51923	cotyloid cilium abscissa
cyp2=—>	9,02	8586,34615	cotyloid cilium ordinate in cm
cxp13=—>	13,44	12793,8462	greater trochanter apex abscissa in cm
cyp13=—>	7	6663,46154	greater trochanter apex ordinate in cm
cxp19 (cc)-->	10,13	9642,98077	abscissa head of femur side limit
cyp19 (gg)-->	6,8	6473,07692	ordinate head of femur side limit
cxp20 (aa)-->	8,4	7996,15385	abscissa head of femur medial limit
cyp20 (dd)-->	5,2	4950	ordinate head of femur medial limit
cxp21 (bb)-->	7,2	6853,84615	every point of femur profile abscissa in cm
cyp21 (tt)-->	8	7615,38462	every point of femur profile ordinate in cm
weight=—>	70	subject's body weight in kg	
ca=—>	0,08	friction «coefficient»	
ff=—>	0,07982999	friction angle in radians	
	4,57392126	friction angle in degrees	
zz=—>	0	rectification factor avoiding errors of division for 0	
gi=—>	9,80665	m/sec^2 acceleration of gravity	

OUTPUT NORMAL HIP							
FASES	load	fmusc	result	bme	bk	aifmz	gradorz
12	86,684478	166,011212	249,379941	4,99660748	18,4689433	23,2746958	16,6224749
13	80,4306331	152,657571	230,42751	4,90347361	18,3315902	22,8859181	16,649251
14	68,8964599	126,715971	193,570026	5,01215011	18,1993177	23,3393577	17,8810384
15	52,8039452	105,595851	156,761852	4,70844825	18,2364561	22,0645691	16,6746076
16	37,1286586	77,2146761	113,18571	4,53188555	18,5049016	21,3126421	15,7995139
17	27,7856354	59,4181871	86,2710625	4,47274848	18,7347559	21,059049	15,501257
18	33,3042253	74,3784347	106,302454	4,50743096	18,8491489	21,2078808	15,1669035
19	42,4625045	94,0732666	134,725004	4,61066738	19,0872954	21,6491196	15,4249527
20	53,7812782	113,490003	165,243317	4,7329967	19,3116707	22,1684898	16,0499313
21	66,0638268	133,175481	197,088254	4,83798438	19,3804791	22,6111978	16,6967937
22	69,5048551	130,980584	198,76868	4,88259622	19,1896207	22,7984597	17,3513778
23	58,9038316	97,153679	155,371209	4,80439029	18,8515775	22,4698463	18,3311336

load = total load in kg
bme = real lever arm of muscular force in cm
aifmz = angle of elevation of muscular force with reference to Z axis
fmusc = muscular force
result = resultant force in kg
gradorz = angle of elevation of the resultant, with reference to Z axis

CONCLUSION

The present study focuses on normal hip.

All formulas described above have been elaborated by Excel and, in synthesis, the output from the computer, numerically expressed, is represented for each single phase of pace, by:

- a) \bar{K} force, its components and respective angles of elevation.
- b) \bar{M} force, its components and respective angles of elevation.
- c) \bar{R} force, its components and respective angles of elevation.
- d) \bar{M} lever arm.
- e) Per cent variation between partial weight of (\bar{P}) body and total load (\bar{K}) (see enclosure 1: computer output synthesis).

Particular relevance has been attributed to the determination of the total load exerted on normal hip in gait, as this is considered to be an element of some importance in the evaluation of hip joint geometrical and functional situation. Also the data collected through the use of this method have pointed out that the main stress on hip joint is exerted during the monopodal phase in gait.

This is gradually reduced, down to its minimum, during

the phase of intermediate oscillation.

Then it rises again during contralateral limb thrust phase according to what has been described in scientific literature.

The advantage of this procedure is that it is possible to apply these calculations to any subject, simply and without the support of a sophisticated apparatus and of a specialized staff.

In fact, after the input of a few requested data for the computer, all calculations are made automatically, without the intervention of the operator.

In clinical practice this programme can turn out to be useful in the individuation of still asymptomatic hypersolicitation-situations, making it possible to elaborate a therapeutic plan, studied in order to avoid such over loads evident in arthritis, and in order to limit possible damage.

We judge, however, that, apart from this clinical application, our procedure can be useful as a reference basis for analogous studies on pathological and prosthesis hip, studies already under discussion in our department.

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REFERENCES

- 1. Braune W, Fischer O. Der gang des Menschen. Leipzig - bei S. Hirzel, 1894.
- 2. Pawels F. Biomechanics of the normal and diseased hip. Berlin Heidelberg New York: Springer Verlag, 1976.
- 3. Pawel F. Biomechanics of the locomotor apparatus. Heidelberg New York: Springer Verlag, 1980.
- 4. Maquet PGJ. Biomechanics of the hip as applied to osteoarthritis and related conditions. Berlin, Heidelberg, New York, Tokyo: Springer Verlag, 1985.
- 5. Microsoft C. Excel Handbook
- 6. Campbell MV. Excel macro library. Que Corporation - Indianapolis, Indiana.
- 7. Simerly DK. Excel per Macintosh, guida all'uso. Franco Muzzio Editore.

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CONTENTS

Why HIP International	1
<i>A. SURACE</i>	
What is "right", and what is "wrong" in hip arthroprothesis	3
<i>V. PIETROGRANDE</i>	
A new look at the forces acting on the hip joint	7
<i>R. BOMBELLI, N. KULLER, M. BOMBELLI</i>	
Exchange arthroplasty best for infected total hip replacement	17
<i>D. J. THOMAS, G. BANNISTER</i>	
Long term results of Charnley low-friction arthroplasty. A thirteen-year average follow-up study	21
<i>G. STRINGA, G.V. DI MURIA, M. MARCUCCI, R.P. PITTO</i>	
Cementless revision operations of loosened cemented hip arthroplasties: ten years experience	28
<i>R. WEINHART, TH. BUCHOLZ, P. RINDERLE, R. PARHOFER</i>	
The "elliptical" femoral prosthesis	39
<i>F. PIPINO, L. MOLFETTA</i>	
Computerized evaluation of the articular loads on the normal dynamic human hip: a study of biomechanics	45
<i>A. SURACE, A.M. PREVITERA, C. MICALE, G. MINEO</i>	