

**Spectral properties of Galerkin and collocation Isogeometric matrices:  
a numerical comparison**

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In recent years, Isogeometric Analysis (IGA) have widely explored wave problems [1]. In our previous works, we studied the stability and convergence of IGA Collocation and Galerkin methods for the acoustic wave equation with absorbing boundaries and Newmark's time integration [2-3]. Since the mass matrices in both approaches are non-diagonal, solving the linear systems at each time step remains challenging, both in the explicit and implicit case. However, theoretical results on IGA matrix properties are still limited, with most estimates remaining conjectural. In this presentation, we provide a direct numerical comparison between the behavior of extreme eigenvalues and condition numbers of Collocation and Galerkin IGA matrices, varying polynomial degree  $p$ , mesh size  $h$ , regularity  $k$  [4,7]. We deduce some conjectures in addition to those available in the literature only for the Poisson problem with Dirichlet conditions in the case of IGA Galerkin. Our numerical results show that the condition numbers of the IGA Collocation and Galerkin mass and stiffness matrices exhibit similar trends with respect to  $h$ . When examining the behavior of the condition numbers as  $p$  increases, we observe that the condition numbers are generally more favorable for IGA Collocation than for IGA Galerkin. These results on spectral properties are significant not only for acoustic wave equations but also for a wide range of PDEs that trace back to the Laplacian operator. They also provide an important starting point for developing efficient preconditioning methods for iteration matrices, applicable to both explicit and implicit time-stepping schemes. In this regard, we propose a two-level Overlapping Schwarz (OS) preconditioner [5], accelerated with GMRES in the IGA collocation case and with the conjugate gradient method in the IGA Galerkin case. We conclude this presentation by showing some numerical results on the robustness of the OS preconditioner with respect to all discretization parameters [6].

## REFERENCES

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